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OPTIMAL FISCAL POLICY WHEN PUBLIC CAPITAL IS PRODUCTIVE: A BUSINESS-CYCLE PERSPECTIVE

by Kevin J. Lansing

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Abstract

Recent empirical evidence suggests that the stock of public-sector capital may be an important input to private production. This paper examines the business-cycle implications of productive public capital in a two-sector, dynamic general-equilibrium model with endogenous fiscal policy. In the model, public capital is a direct input to the neoclassical production technology, and public consumption goods provide direct utility to households. The production of public and private goods takes place in separate sectors. At the optimum level of public capital, the rate of return on public investment is found to be less than that on private investment. In simulations, public investment and public consumption are procyclical, and the capital tax is more variable than the labor tax, features also observed in annual U.S. data. The introduction of stochastic shocks to households' preference for public consumption helps the model to match certain features of the data, namely, the high variability and low correlation of public expenditures relative to their private-sector counterparts over the business cycle.

1. Introduction

A growing body of research incorporates public capital into neoclassical growth models.[1] Furthermore, in recent years, business-cycle research has begun to address the effects of government fiscal policy on aggregate fluctuations. For example, Braun (1989) and McGrattan (1991) study the effects of distortionary taxation on business cycles, Aiyagari, Christiano, and Eichenbaum (1992) and Christiano and Eichenbaum (1992) examine the impact of time-varying public consumption expenditures, and Braun and McGrattan (1993) study wartime fluctuations in a model with government-owned (but privately operated) capital. Baxter and King (1993) discuss the temporary and long-run effects of changes in public consumption, public investment, and tax rates. A common feature of all these papers is that government policy is viewed as exogenous. In contrast, this paper undertakes the study of fiscal policy and business cycles in a model that endogenizes all variables of interest. I then subject the model to the same kind of quantitative comparisons with U.S. data that have been widely used in the real business-cycle literature.

The framework for the analysis is a two-sector, dynamic general-equilibrium model with an infinitely lived, representative household. The government solves a dynamic version of the Ramsey (1927) optimal tax problem in which the endogenous policy variables are public investment, public consumption, and tax rates on labor and capital income (for simplicity, a period-by-period balanced budget is assumed). A distinction is made between the production of private and public goods. Private-sector firms produce private goods, which households purchase using after-tax income. The government sector produces non-rival public goods. Public investment goods augment the stock of public capital, while public consumption goods provide direct utility to households. Public capital is a direct input to the constant-returns-to-scale production technologies in both sectors, as are sector-specific quantities of private capital and labor. The government does not assess fees for the use of public goods; rather, it levies distortionary taxes to finance their production.

In equilibrium, the rate of return on public investment (as measured by the marginal product of

[1]Recent models of endogenous growth with public capital include Barro (1990), Jones, Manuelli, and Rossi (1992), Barro and Sala-i-Martin (1992), and Glomm and Ravikumar (1993). Arrow and Kurz (1970) provide a comprehensive study of the existence and stability properties of equilibrium in a neoclassical growth model with public capital. Pestieau and Possen (1978) study optimal redistributive fiscal policy in a model with public capital and exogenous savings.

public capital) is found to be less than the rate of return on private investment. This result is due to the impact of distortionary capital taxation and the higher depreciation rate assumed for private versus public capital. Both effects reduce the steady-state level of private capital and hence raise its marginal product.

A primary finding is that optimal fiscal policy displays procyclical behavior for public investment and public consumption, in agreement with annual U.S. data. The presence of public capital in the constant-returns-to-scale production function causes private firms to realize positive profits after private factors are paid their marginal products. This, in turn, causes the optimal steady-state tax on capital to be positive. Essentially, the government uses the tax on private capital to collect a user fee for the productive services of public capital. Furthermore, in simulations, the optimal capital tax fluctuates over time to absorb budget shocks, while the optimal labor tax remains relatively stable. This characteristic can also be observed in estimates of average marginal tax rates for the U.S. economy.[2]

An important feature of the model is the introduction of stochastic shocks to household preferences for public consumption. These shocks are intended to represent unforeseen events that affect household demand for public goods, such as national defense, police protection, or disaster relief services. Specific examples might include the collapse of the Soviet Union (which reduced desired levels of U.S. defense spending) and the 1994 Los Angeles earthquake (which, so far, has resulted in more than $8 billion in federal disaster relief appropriations). The preference shocks cause adjustments along two margins: the mix of production between the private and government sectors, and the choice between public investment and public consumption. As a result, the shocks help the model to match certain features of U.S. data, namely, the high variability and low correlation of public expenditures relative to their private-sector counterparts over the business cycle. This result is similar to other examples in the business-cycle literature wherein a stochastic term in the household utility function can improve model performance, as demonstrated recently by Christiano and Eichenbaum (1992) and Bencivenga (1992).[3]

The remainder of this paper is organized as follows. Section 2 provides some facts about public

[2]Jones, Manuelli, and Rossi (1993) provide a formal treatment of the effect of profits on the optimal steady-state capital tax. The optimality of a state-contingent capital tax to absorb budget shocks is shown by Judd (1989) and Chari, Christiano, and Kehoe (1993).

[3]Flemming (1987) develops a model with shocks to the demand for public goods in a small open economy with exogenous wages and interest rates, and obtains procyclical government expenditures.

capital and public expenditures in the postwar U.S. economy. Sections 3 and 4 describe the model and the solution method. Section 5 describes the choice of parameter values. Section 6 presents quantitative results from steady-state analysis and dynamic simulations, and section 7 concludes.

2. The Data

The stock of public-sector capital in the United States is about one-third the size of the private capital stock. The largest single category of public capital comprises what is known as the economy's core infrastructure, namely, highways, streets, airports, transit systems, utilities, and the like. State and local governments own about two-thirds of the nation's public capital, while the federal government owns the stock of military capital, which represents about one-fifth of the total. A number of empirical studies (Ratner [1983], Aschauer [1989a], Munnell [1990, 1992], and Lynde and Richmond [1992]) suggest that public capital may be an important input to private production. The primary evidence for the public capital hypothesis can be seen in figure 1, which plots U.S. labor productivity versus the stock of public capital from 1947 to 1992.[4] The well-documented slowdown in the growth trend of U.S. productivity that began in the early 1970s coincides with a similar decrease in the growth trend of public capital. This observation is the basis for many claims regarding the productive effects of public capital. The association of these trends does not prove, however, the existence of a *causal* link running from public capital to output. Indeed, a number of studies cite evidence that disputes the public capital hypothesis. These include Tatom (1991), using aggregate U.S. data, Hulten and Schwab (1993), using regional data, and Holtz-Eakin (1992), using state-level data. Eberts (1990) provides evidence that causation between public capital and output runs in both directions, using metropolitan-level data.[5]

Theoretically, public capital could act as either a substitute or a complement to private capital in production. For example, public investment might "crowd out" private investment if firms rely increasingly on public capital for productive purposes rather than expanding their own capacity. On the

[4]In figure 1, output is real GNP in 1987 dollars from Citibase. Public capital is government-owned equipment, structures, and residential capital (with and without military capital) in 1987 dollars from *Fixed Reproducible Tangible Wealth in the United States*, U.S. Department of Commerce (1993). All series have been divided by the total labor hour series, LHOURS, in Citibase.

[5]See Munnell (1992) and Aschauer (1993) for a summary and critique of the various empirical studies on this topic.

other hand, public investment may raise the profitability of private capital, thus stimulating private investment. Aschauer (1989b) and Lynde and Richmond (1992) present empirical evidence (using postwar U.S. data) suggesting that the complementary effect dominates, although both effects appear to be operating. Furthermore, the empirical studies that find support for the public capital hypothesis are consistent with a Cobb-Douglas aggregate production function that exhibits constant returns to scale in labor, private capital, and public capital.

Figure 2 compares trends in U.S. private and public investment, while figure 3 compares the analogous consumption trends. During the postwar period, private investment averaged 21 percent of GNP, while public investment (including military) averaged 4.5 percent. The corresponding averages for private and public consumption are 56 percent and 17 percent, respectively.[6] The non-trivial size of the government sector relative to the private economy motivates a model in which government policy is not only included, but is endogenized in a full general-equilibrium framework.

Figures 4 and 5 depict the business-cycle movements of the annual investment and consumption series from 1947 to 1992, where cyclical components have been extracted by detrending each series with the Hodrick-Prescott filter.[7] Note that public investment and public consumption both tend to move procyclically. In figure 4a, the correlation coefficient between total public investment and real GNP is 0.59. If military investment is excluded, the correlation is 0.56. In the *private* sector, the correlation between investment and real GNP is 0.69. Even though both types of investment are procyclical, figure 4b shows that the contemporaneous correlation between private and public investment is quite low, ranging only from 0.04 to 0.25, with the higher value obtained when military investment is excluded. The standard deviation of total public investment is about 1.7 times larger than that of private investment, mostly due to large increases in military investment that occurred during the Korean and Vietnam Wars. Excluding military investment, the standard deviations of public and private investment

[6] In figure 2, the first series for private investment is fixed investment + consumer durables expenditures + residential investment, from *Fixed Reproducible Tangible Wealth in the United States*, U.S. Department of Commerce (1993). The second series is gross private investment (GPIQ), from Citibase. In figure 3, private consumption is expenditures on nondurables and services (GCNQ + GCDQ), from Citibase. Public consumption is government purchases of goods and services (GGEQ) from Citibase, minus public investment (this avoids double counting). All series are annualized and expressed in 1987 dollars.

[7] See Prescott (1986). The smoothing parameter for the filter was set at $\Lambda=100$, since all data are at annual frequency.

are about equal. For the consumption series (figure 5), the correlations with real GNP are 0.61 for public consumption and 0.71 for private consumption.[8] The contemporaneous correlation between the two consumption series is 0.16. The standard deviation of public consumption is about five times larger than the standard deviation of private consumption. The temporary spikes observed in the U.S. series for public investment and public consumption provide a justification for incorporating shocks to the demand for public goods in the model.

3. The Model

The model economy consists of many identical, infinitely lived households, identical private firms, and the government. Public and private goods are produced in separate sectors, each using a technology that exhibits constant returns to scale in the three productive inputs: labor, private capital, and public capital. The form of the technology implies that private firms earn an economic profit equal to the difference between the value of output and payments to private factor inputs. As owners of the firms, households receive net profits in the form of dividends, but consider them to be outside their control, similar to wages and interest rates. The government finances production of public goods by levying distortionary taxes on households and firms. It is assumed that profits are initially taxed at the firm level, then distributed as dividends and taxed again at the household level. This formulation is intended to capture the double taxation of corporate dividends in the U.S. economy. Furthermore, I assume that the government can distinguish between labor and capital income, but cannot distinguish between the different categories of capital income, such as profits, dividends, and interest. Consequently, there are only two types of distortionary taxes in the model: a labor tax and a capital tax.[9]

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[8]At quarterly frequency, the public consumption series (GGEQ from Citibase) displays much weaker procyclical behavior. From 1947:IQ to 1992:IVQ, the correlation with real GNP is 0.33. For the public investment series, data are available only at annual frequency. Because government budgets are enacted into law annually, the annual correlations would appear to be more relevant for a model with endogenous public expenditures.

[9]In a model with profits, Jones, Manuelli, and Rossi (1993) show that a restriction on the menu of tax instruments is needed for optimality of a positive tax on capital in the steady state. Guo and Lansing (1994) demonstrate that the structure of dividend taxation (double versus single) can affect the stability of the steady state in a one-sector version of this model.

3.1 The Households' Problem

Households maximize a discounted stream of within-period utility functions over consumption and leisure, subject to a sequence of budget constraints. The decision problem can be summarized as

$$\max_{\substack{c_t, h_{1t}, h_{2t} \\ k_{1t+1}, k_{2t+1}}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln c_t - A H_t + \exp(v_t) B \ln g_t \right] \qquad 0 < \beta < 1, \quad A, B \geq 0$$

$$\text{where} \quad H_t = h_{1t} + h_{2t} \qquad (1)$$

$$K_t = k_{1t} + k_{2t}$$

$$X_t = x_{1t} + x_{2t}$$

subject to

$$c_t + X_t \leq (1-\tau_{ht}) w_t H_t + (1-\tau_{kt})(r_t K_t + \hat{\pi}_t) + \tau_{kt} \delta K_t \qquad (2)$$

$$k_{1t+1} = (1-\delta) k_{1t} + x_{1t}, \qquad 0 < \delta < 1, \qquad k_{10}, k_{20} \text{ given} \qquad (3)$$

$$k_{2t+1} = (1-\delta) k_{2t} + x_{2t} \qquad (4)$$

$$v_{t+1} = \rho_v v_t + \xi_{t+1}, \qquad 0 < \rho_v < 1, \quad \xi_t \sim \text{iid}(0, \sigma_\xi^2), \quad v_0 \text{ given.} \qquad (5)$$

In the above equations, subscripts 1 and 2 refer to the private and government sectors, respectively. Households supply labor and capital to both sectors. The term c_t represents private consumption goods. Households are endowed with one unit of time each period and work a total of H_t hours during period t. Household preferences include a separable term representing the utility provided by public consumption goods g_t. Furthermore, these preferences are subjected to serially correlated exogenous shocks v_t that are revealed to agents at the beginning of period t. Examples of public consumption goods that might affect household utility are national defense, police protection, and government provision of food and housing during natural disasters. Public goods are assumed to be non-congestable and free of specific user charges. They are also nontradeable, i.e., no market exists wherein households may buy or sell public goods.

Households maximize the utility function in (1) over c_t, h_{1t}, and h_{2t}, but view g_t as outside their control. The logarithmic form of the within-period utility function has been chosen for tractability and for comparability with previous business-cycle literature. The separability in c_t and g_t implies that public consumption does not affect the marginal utility of private consumption, an assumption supported by parameter estimates in McGrattan, Rogerson, and Wright (1993). The symbol E_t is the expectation operator conditional on information available at time t, and β is the constant household discount factor.

The fact that utility is linear in hours worked reflects "indivisible labor," as described by Rogerson (1988) and Hansen (1985). This means that all variation in economywide hours worked is due to variations in the number of employed workers, as opposed to variations in hours per worker. In a decentralized economy, these authors show that the utility function in (1) can be supported by a lottery that randomly assigns workers to employment or unemployment each period, with the firm (or the government employer) providing full unemployment insurance. Wage contracts call for households to be paid based on their expected (rather than actual) number of hours worked. Real business-cycle models with indivisible labor are better able to match some key characteristics of aggregate labor market data. Specifically, U.S. data display a large volatility of hours worked relative to labor productivity and a weakly positive or even slightly negative correlation between hours and productivity.[10]

Equation (2) represents the period budget constraint of the household. The terms X_t and K_t represent total private investment and total private capital, respectively. Private capital is assumed to be "sector specific" in the sense that total capital cannot be reallocated freely between sectors at the start of each period, but instead must be reallocated via the investment process. This feature is reflected in the separate laws of motion for k_{1t} and k_{2t}, equations (3) and (4). The distribution of capital across sectors is relevant for household decisions in this model because, in equilibrium, the government's state-contingent policy rules are functions of both k_{1t} and k_{2t}. All private capital is assumed to depreciate at the constant rate δ. In equilibrium, the after-tax returns to labor and capital are equalized across sectors.

Households derive income by supplying labor and capital to both sectors at rental rates w_t and r_t, and pay taxes on labor and capital income at rates τ_{ht} and τ_{kt}, respectively. An additional source of income is the firms' net profits $\hat{\pi}_t$, which are distributed to households as dividends and are taxed at the same rate as interest income. The term $\tau_{kt}\,\delta K_t$ represents the depreciation allowance built into the

[10]See Hansen (1985), Christiano and Eichenbaum (1992), and Hansen and Wright (1992).

U.S. tax code. Households view tax rates, wages, interest rates, and dividends as determined outside their control. The household decision variables c_t, h_{1t}, h_{2t}, k_{1t+1}, and k_{2t+1} are all known at time t.

3.2 Household Optimality

The Lagrangian for the households' problem is defined as

$$\mathfrak{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - A H_t + \exp(v_t) B \ln g_t + \right.$$
$$\left. \lambda_t \left[(1-\tau_{ht}) w_t H_t + (1-\tau_{kt})(r_t - \delta) K_t + (1-\tau_{kt}) \hat{\pi}_t + K_t - K_{t+1} - c_t \right] \right\}. \tag{6}$$

The household first-order conditions with respect to the indicated variables and the associated transversality conditions are

$$\begin{aligned}
c_t: &\quad \lambda_t = \frac{1}{c_t} & \\
h_{it}: &\quad \lambda_t (1-\tau_{ht}) w_t = A & i = 1, 2 \\
k_{it+1}: &\quad \lambda_t = \beta E_t \lambda_{t+1} \left[(1-\tau_{kt+1})(r_{t+1} - \delta) + 1 \right] & i = 1, 2 \\
&\quad \lim_{t\to\infty} E_0 \beta^t \lambda_t k_{1t+1} = 0, \qquad \lim_{t\to\infty} E_0 \beta^t \lambda_t k_{2t+1} = 0. &
\end{aligned} \tag{7}$$

The government uses these equilibrium conditions to recover the appropriate tax rates, τ_{ht} and τ_{kt}, that support the household equilibrium allocations in a decentralized economy. Note that the first-order conditions for h_{it} and k_{it+1} yield the same equation for i=1, 2. This implies that if the before-tax prices w_t and r_t are the same across sectors (as assumed here), the government will announce τ_{ht} and τ_{kt} to also be the same across sectors. If w_t and r_t were allowed to differ across sectors, the government would announce sector-specific tax rates in order to maintain the equilibrium condition of equal *after-tax* returns across sectors.

3.3 Production Environment

Private-Sector Production

Private-sector output y_t is produced by identical firms that seek to maximize after-tax profits by using a technology that exhibits constant returns to scale in the three inputs, h_{1t}, k_{1t}, and k_{Gt}, where k_{Gt} is the stock of public capital. Private-sector output consists of private consumption goods c_t and private investment goods x_{1t} and x_{2t}. The firm's profits are taxed at rate τ_{kt}. The production technology is subjected to serially correlated exogenous shocks z_t that are revealed to agents at the beginning of period t. These shocks generate equilibrium business-cycle fluctuations in the model. Since the focus here is on the (detrended) business-cycle movements of variables, the model abstracts from exogenous technical progress. Private-sector production can be summarized as

$$\max_{k_{1t},\, h_{1t}} \; (1-\tau_{kt})(y_t - r_t k_{1t} - w_t h_{1t}) \tag{8}$$

subject to

$$y_t = \exp(z_t)\, k_{1t}^{\theta_1} h_{1t}^{\theta_2} k_{Gt}^{\theta_3} \qquad 0<\theta_i<1, \qquad \theta_1+\theta_2+\theta_3 = 1 \tag{9}$$

$$z_{t+1} = \rho_z z_t + \varepsilon_{t+1} \qquad 0<\rho_z<1, \qquad \varepsilon_t \sim \mathrm{iid}(0,\sigma_\varepsilon^2), \qquad z_0 \text{ given.} \tag{10}$$

The private firm's first-order conditions are[11]

$$r_t = \theta_1 \frac{y_t}{k_{1t}} \qquad\qquad w_t = \theta_2 \frac{y_t}{h_{1t}}. \tag{11}$$

The firm's after-tax profits, distributed to households in the form of dividends, are

$$\hat{\pi}_t = (1-\tau_{kt})(1-\theta_1-\theta_2)\, y_t. \tag{12}$$

[11]There is no need to distinguish between variables under the household's control and variables representing per capita quantities here, as is necessary when solving directly for a decentralized, competitive equilibrium. As noted by Lucas and Stokey (1983), the solution to the government's decision problem yields a set of policies that dictate household equilibrium allocations. These allocations determine the equilibrium prices r_t and w_t. Thus, prices are not outside the government's control as they are for households.

Government-Sector Production

Government-sector output y_{Gt} involves the employment of workers, the rental of private capital (for example, office buildings), and the use of public capital (such as infrastructure and government-owned facilities or equipment). The government-sector technology exhibits constant returns to scale in the inputs h_{2t}, k_{2t}, and k_{Gt}. Public capital is assumed to be "noncongestable" in the sense that the same capital stock is used as an input to both private- and government-sector production.[12] Government-sector output consists of public consumption goods g_t and public investment goods x_{Gt}. Since there is no formal market for public goods, there is no requirement for the government to maximize any notion of "profit." However, following Uzawa (1966), I assume that the government employs labor and private capital in such a way as to minimize external production costs. This assumption permits the definition of a shadow price of public goods relative to private goods P_{Gt}.[13] Technology shocks are assumed to be the same for both sectors, but are uncorrelated with preference shocks. Government-sector production is described as follows:

$$\min_{k_{2t},\, h_{2t}} \; r_t k_{2t} + w_t h_{2t} \tag{13}$$

subject to

$$y_{Gt} = \exp(z_t)\, k_{2t}^{\phi_1} h_{2t}^{\phi_2} k_{Gt}^{\phi_3} \qquad 0<\phi_i<1, \qquad \phi_1+\phi_2+\phi_3 = 1 \tag{14}$$

$$z_{t+1} = \rho_z z_t + \varepsilon_{t+1} \qquad 0<\rho_z<1, \qquad \varepsilon_t \sim \text{iid}(0,\sigma_\varepsilon^2), \qquad z_0 \text{ given.} \tag{15}$$

[12] Experiments with different versions of the model indicate that the results are not substantially changed if public capital is assumed to be fully congestable, i.e., if separate stocks of public capital k_{1Gt} and k_{2Gt} are employed in the two sectors.

[13] In a standard two-sector growth model, the sectors are typically identified with consumption and investment goods, and a formal market exists for both goods. Here, there is no formal market for goods produced in the government sector. This setup shares some features of a home production model in which the two sectors are identified with market and nonmarket (home-produced) goods. See Benhabib, Rogerson, and Wright (1991), Greenwood and Hercowitz (1991), and McGrattan, Rogerson, and Wright (1993) for home production models in a business-cycle framework.

The government's cost minimization condition yields an expression for the shadow price of public goods relative to private goods.[14] Also, since households earn income in both sectors, it is necessary to define what is meant by GNP. Here, GNP is defined as income and profits generated in the private sector (y_t), plus income earned in the government sector ($w_t h_{2t} + r_t k_{2t}$). The expressions are

$$P_{Gt} = \frac{r_t}{\frac{\partial y_{Gt}}{\partial k_{2t}}} = \frac{w_t}{\frac{\partial y_{Gt}}{\partial h_{2t}}} \tag{16}$$

$$GNP_t = y_t + w_t h_{2t} + r_t k_{2t}. \tag{17}$$

Combining (16) with the private firm's profit maximization conditions (11) yields the following constraint imposed on the government's choice of policy:

$$\frac{\theta_1 k_{2t}}{\phi_1 k_{1t}} = \frac{\theta_2 h_{2t}}{\phi_2 h_{1t}}. \tag{18}$$

3.4 The Government's Problem

The government chooses an optimal program of public expenditures and tax rates to maximize the discounted utility of the household. The vector $\mathcal{P}_t = \{x_{Gt}, g_t, \tau_{ht}, \tau_{kt}\}$ summarizes government policy implemented at time t. The problem is a dynamic version of the classic Ramsey case, involving a Stackelberg game between the government and households. To avoid time-consistency problems, I assume that the government can commit to a set of state-contingent policy rules announced at time zero. Also, to make the problem interesting, lump-sum taxes are ruled out. Otherwise, the government would elect to finance all future expenditures with an initial levy on private-sector assets. To simplify the formulation, I further assume that the government adheres to a period-by-period balanced budget

[14]The shadow price is the Lagrange multiplier on the production constraint in the cost minimization problem. Equivalently, the expression for P_{Gt} can be obtained from the following shadow profit maximization problem for a hypothetical competitive firm that produces public goods: max $\{P_{Gt} y_{Gt} - r_t k_{2t} - w_t h_{2t}\}$. The (unobservable) profits earned by such a firm are $(1 - \phi_1 - \phi_2) P_{Gt} y_{Gt}$.

constraint, i.e., government debt is ruled out.[15] With these assumptions, the government's problem is

$$\max_{k_{Gt-1},\, g_t,\, \tau_{ht},\, \tau_{kt}} E_0 \sum_{t=0}^{\infty} \beta^t \left[U(c_t, H_t, g_t) \right]$$

subject to

$$
\begin{aligned}
&(i) && \text{household first-order conditions and budget constraint,} \\
&(ii) && \text{private-firm profit maximization conditions,} \\
&(iii) && \text{government-sector cost minimization condition,} \\
&(iv) && w_t h_{2t} + r_t k_{2t} = \tau_{ht} H_t + (\tau_{kt} - \delta) K_t + (2\tau_{kt} - \tau_{kt}^2)(1 - \theta_1 - \theta_2) y_t , \\
&(v) && y_{Gt} = g_t + x_{Gt} , \\
&(vi) && k_{Gt+1} = (1 - \delta_G) k_{Gt} + x_{Gt} , \\
&(vii) && \lim_{t \to \infty} E_0 \beta^t \Lambda_{Gt} k_{Gt+1} = 0 .
\end{aligned}
\tag{19}
$$

Constraints (*i*) and (*ii*) summarize rational maximizing behavior on the part of private agents and constitute "implementability" constraints imposed on the government's choice of policy. Constraint (*iii*) is the cost minimization assumption for the government's use of private-sector inputs, i.e., equation (18). Constraint (*iv*) is the government budget constraint, where outlays on the left side consist of payments to labor and capital employed in the government sector. The last term on the right side of (*iv*) reflects the double taxation of firm dividends. Constraint (*v*) is the resource constraint governing the production of public goods. Constraint (*vi*) is the law of motion for public capital, where x_{Gt} is gross public investment and δ_G is the depreciation rate of public capital. Finally, (*vi*) is a transversality condition on the accumulation of public capital, where Λ_{Gt} is the marginal utility of public consumption g_t.

The summation of the household budget constraint (2) and the government budget constraint (19. *iv*) yields the following resource constraint for the private sector. Because the private-sector resource

[15] Adding government debt to the model introduces complications that I wish to avoid here. Specifically, equilibrium for a model with debt and capital imposes an ex ante arbitrage condition on the expected returns from government bonds and private capital. The steady-state level of debt is thus indeterminate (see Chamley [1985]). Furthermore, in a stochastic environment, the government can vary the ex post combination of the capital tax and the bond interest rate in many different ways to raise needed revenue, yet still satisfy ex ante arbitrage (see Zhu [1992] and Chari, Christiano, and Kehoe [1991]). One method of resolving these complications is to assume an exogenous ratio of steady-state debt to GNP and to restrict the government's ability to set the ex post bond interest rate independently (see Lansing [1993a]).

constraint and the government budget constraint are not independent equations, (20) will be used in place of (19.*iv*) in the recursive version of the problem.[16]

$$y_t = c_t + x_{1t} + x_{2t}. \tag{20}$$

4. Solution of the Model

The government's problem under commitment can be solved using the unique recursive algorithm developed by Kydland and Prescott (1980). A recursive structure is obtained by defining the household lagged shadow price λ_{t-1} to be a "pseudo-state variable." Including this price in the state vector provides a link to the past by which the policymaker at time t considers the fact that household decisions in earlier periods depend on current policy by means of expectations. This is the mechanism by which the commitment problem can be solved using dynamic programming.[17] Appendix A describes the procedure for formulating the recursive version of (19) and numerically solving the dynamic programming problem.

5. Calibration of the Model

To explore the quantitative predictions of the model, as many parameters as possible are assigned values in advance based on empirically observed features of postwar U.S. data. Parameter choices are also guided by the desire to obtain steady-state values for key model variables that are consistent with long-run averages in the U.S. economy. Table 1 summarizes the choice of parameter values and is followed by a brief description of how they were selected.

[16] An alternative formulation for the two resource constraints would be $y_t = c_t + x_{1t} + x_{2t} + x_{Gt}$ and $y_{Gt} = g_t$. This implies that public consumption goods g_t are produced in the government sector but that public investment goods x_{Gt} are purchased from the private sector using tax revenue. Experiments with this version of the model yielded less successful results (the correlation between public and private investment was too high). See Lansing (1993b) for further variations on the present model, including a one-sector model with adjustment costs for public investment and a two-sector model with sector-specific technology shocks.

[17] See Lansing (1993a) for a more detailed discussion of this method.

Table 1: Parameter Set

Agent	Parameters and Values			
Households	$\beta = 0.962$	$A = 2.60$ $B = 0.28$	$\rho_v = 0.85$	$\sigma_\xi = 0.04$
Firms	$\theta_1 = 0.30$ $\theta_2 = 0.62$ $\theta_3 = 0.08$	$\delta = 0.07$	$\rho_z = 0.85$	$\sigma_\varepsilon = 0.02$
Government	$\phi_1 = 0.30$ $\phi_2 = 0.62$ $\phi_3 = 0.08$	$\delta_G = 0.04$		

The time period in the model is taken to be one year. This is consistent with the frequency of available data on public investment and tax rates. The discount factor of β=0.962 implies an annual rate of time preference equal to 4 percent. The parameter A in the household utility function is chosen such that total hours worked is close to 0.3. This is in line with time-use studies, such as Juster and Stafford (1991), which indicate that households spend approximately one-third of their discretionary time in market work.[18] The value of B is chosen to yield a steady-state value of *g/GNP* near 0.18, the average ratio for the U.S. economy from 1947 to 1992. In computing this average, public consumption was estimated by subtracting public investment from an annualized series for government purchases of goods and services (GGEQ from Citibase). This was done to reduce double counting, since the series does not distinguish between government consumption and investment goods.

The exponents in the Cobb-Douglas production functions are chosen to be the same for both sectors. This is because estimates of private- versus government-sector output elasticities are not available. With these values, however, the shares of GNP in the model received by total private capital and total labor are close to levels estimated for the U.S. economy (see Christiano [1988]). Furthermore, these values yield a steady-state ratio of public investment to GNP near 0.045, consistent with the U.S. average (including military) from 1947 to 1992.[19] With constant returns to scale in all inputs, the value

[18] Total hours worked is $H=h_1+h_2$. The sector-specific values are h_1=0.23 and h_2=0.07. These values imply that employment in the private sector is about three times larger than in the government sector.

[19] The range of empirical estimates for θ_3 is quite large. Aschauer (1989a) and Munnell (1990) estimate values of θ_3=0.39 and 0.34, respectively, Tatom (1991) estimates values in the range of 0 to 0.13, and Holtz-Eakin (1992) estimates a value of θ_3=0 using state-level data. See Munnell (1992) and Aschauer (1993) for a summary and critique of the empirical findings.

of θ_3 $(=1-\theta_1-\theta_2)$ affects the steady-state level of firm profits. These profits in turn affect the steady-state level of τ_k, because the government uses the tax on *private* capital to collect an indirect user fee for the productive services of *public* capital. In the model, the steady-state ratio of profits to GNP is 0.06, and the resulting steady-state tax on capital is 0.27.

The private capital depreciation rate of δ=0.07 is based on the value estimated in Braun and McGrattan (1993) and is consistent with values commonly used in the real business-cycle literature. Together with the values of β and θ_1, this depreciation rate implies a steady-state ratio of private capital to GNP of 2.62 and a ratio of private investment to GNP of 0.18. The corresponding averages for the U.S. economy from 1947 to 1992 are 2.58 and 0.21. The public capital depreciation rate of δ_G=0.04 was estimated by regressing the linear law of motion on annual data for k_{Gt} and x_{Gt}.

The process governing technology shocks was estimated using annual data from 1947 to 1992. The series for z_t was constructed by computing the changes in output not accounted for by changes in the three productive inputs.[20] The estimated parameters, ρ_z=0.85 and σ_ε=0.02, represent values close to those estimated by other authors, such as Benhabib and Jovanovic (1991). In the simulations, the estimated value of σ_ε yields a standard deviation of output in the neighborhood of 2.5 percent, a value very close to the U.S. average of 2.46 percent over this period. The standard deviation of the preference shocks σ_ξ was chosen to match the relative variability of public versus private consumption in U.S. data. Over this period, the standard deviation of g_t is about fives times larger than the standard deviation of c_t. Due to the lack of any direct observations on the preference shock, the persistence parameter ρ_v is set equal to 0.85, the same value used for the technology shock. This choice reflects a belief that shocks that affect household demand for public consumption (such as wars or crime trends) tend to be highly correlated from one year to the next.[21]

[20]The production function residual was measured as $z_t = \ln GNP_t - 0.30\ln K_t - 0.62\ln H_t - 0.08\ln K_{Gt}$. The private capital stock K_t is defined as fixed private capital + stock of consumer durables + residential capital from *Fixed Reproducible Tangible Wealth in the United States*, U.S. Department of Commerce (1993). Real GNP and the labor input (H_t = LHOURS) are from Citibase. The public capital series includes military capital and is described in footnote 4.

[21]Flemming (1987) discusses the effect of changing the serial correlation properties of preference shocks.

6. Quantitative Properties of the Model

6.1 Steady-State Results

Table 2 shows the model's steady-state values versus the corresponding U.S. averages from 1947 to 1992. For the U.S. data, two values are shown for the average values of x_G/GNP and k_G/GNP. The first includes military items and the second excludes these items. Two values are also shown for the average marginal tax rates τ_h and τ_k. The first is computed using tax rate estimates in Barro and Sahasakul (1986) and Jorgenson and Sullivan (1981). The second uses tax rate estimates in McGrattan, Rogerson, and Wright (1993). For the most part, the choice of parameters yields steady-state values that are close to long-run averages in the U.S. economy. Under constant returns, the value of θ_3 ($= 1-\theta_1-\theta_2$) controls the level of firm profits. It is interesting that a relatively small value of θ_3 (which implies a ratio of profits to GNP of 0.06) is sufficient to induce an optimal capital tax of $\tau_k = 0.27$.

Table 2 does not report U.S. values for the rates of return on private versus public investment: r, r_{G1}, and r_{G2}. Empirical estimates of these rates span a wide range and are very sensitive to the level of data aggregation and the statistical techniques employed (see Aschauer [1993] and Holtz-Eakin [1993]). In the model, the rate of return on public investment (in both sectors) is less than the rate of return on private investment. This is due to the impact of distortionary taxation on firm profits and the higher depreciation rate of private capital in the calibration ($\delta > \delta_G$). Both effects tend to reduce the steady-state level of private capital and hence raise its marginal product.

Table 2: Model Steady State versus U.S. Averages

Variable	Model[a]	U.S. Economy[b]
x_G/GNP	0.043	0.045 / 0.030
X/GNP	0.184	0.216
g/GNP	0.178	0.171
c/GNP	0.613	0.557
K/GNP	2.63	2.56
k_G/GNP	1.08	0.787 / 0.605
τ_h	0.222	0.272 / 0.229
τ_k	0.268	0.300 / 0.568
Revenue/GNP from τ_h	0.136	0.159
Revenue/GNP from τ_k	0.068	0.067
$r=\partial y/\partial k_1 = P_G\,\partial y_G/\partial k_2$	0.124	-
$r_{G1}=\partial y/\partial k_G$	0.053	-
$r_{G2}=P_G\,\partial y_G/\partial k_G$	0.014	-

[a]These are steady-state values based on the parameters in table 1. For variables in the government sector, x_G, g, and k_G, the shadow price of public goods ($P_G = 0.903$ from equation [16]) was used to convert quantities into equivalent units of private goods.

[b]For the U.S. economy, investment, consumption, and capital averages are for 1947 to 1992. Data sources are described in footnotes 4, 6, and 20. The average labor tax rates (τ_h) are from Barro and Sahasakul (1986) for 1947-83 and from McGrattan, Rogerson, and Wright (1993) for 1947-87. The average capital tax rates (τ_k) are from Jorgenson and Sullivan (1981, table 11) for 1947-80 and from McGrattan, Rogerson, and Wright (1993) for 1947-87. Average tax revenue data are from various issues of *Revenue Statistics of OECD Member Countries, 1965-1990*, table 61. Labor tax revenue is defined to include federal and state individual income taxes and Social Security taxes. Capital tax revenue is defined to include federal and state corporate taxes, capital gains taxes, and property taxes.

6.2 Dynamic Simulation

6.2.1 Optimal Policy Rules

The (approximate) solution to the government's problem yields the following set of log-linear optimal policy rules, which are valid in the neighborhood of the deterministic steady state. The optimal decision rules for private investment and private consumption have also been included for comparison.

Table 3: Optimal Policy Rules

	Constant	z_t	v_t	$\ln(k_{1t})$	$\ln(k_{2t})$	$\ln(k_{Gt})$	$\ln(\lambda_{t-1})$
x_{Gt} =	0.233	0.116	-0.072	-0.133	0.066	-0.034	-0.118
x_{1t} =	0.859	0.276	-0.064	-0.624	0.061	0.164	-0.515
x_{2t} =	-0.870	0.211	0.104	0.645	-0.176	-0.172	0.480
g_t =	0.108	0.053	0.084	0.024	0.010	0.033	0.018
c_t =	0.637	0.060	0.001	-0.001	-0.013	0.004	-0.291
τ_{ht} =	-0.823	0.450	-0.054	0.392	0.193	0.099	1.133
τ_{kt} =	3.766	-1.451	0.119	-1.523	0.267	-0.256	-2.776

The optimal behavior of public versus private expenditures over the business cycle can be inferred by examining the coefficients on the technology shock z_t in the policy rules. For x_{Gt} and g_t, these coefficients are both positive, implying procyclical behavior. The coefficients on v_t indicate optimal responses to shifts in the demand for public consumption goods. As expected, g_t responds more strongly to preference shocks than does c_t, accounting for the higher variability of g_t relative to c_t in the model. The preference shocks affect the optimal trade-off between production in the two sectors as well as the optimal mix between public investment and public consumption. This feature is reflected by the coefficients on v_t in the investment rules. A positive value of v_t stimulates private investment in the government sector x_{2t} in order to increase production of g_t. As a result, x_{1t} and x_{Gt} both decline. In this way, preference shocks lower the contemporaneous correlation between public investment and *total* private investment X_t $(=x_{1t}+x_{2t})$ and improve the model's comparison with U.S. data.

The optimal response of tax rates to shocks is governed by the shock's impact on the government budget. In general, technology shocks alter the size of the tax base, affecting revenues, while preference shocks alter the level of required outlays. The government responds by adjusting the capital tax in an offsetting manner. This can be observed by examining the coefficients on z_t and v_t in the decision rule for τ_{kt}. The magnitude of these coefficients is the largest of any policy variable. A positive technology shock causes a large *decrease* in τ_{kt}. A positive z_t causes GNP and household income (the tax base) to rise, allowing the use of a lower tax on capital even though public expenditures

x_{Gt} and g_t increase. In contrast, a positive preference shock v_t calls for an increase in τ_{kt} to finance a higher level of outlays. Absorbing shocks in this way is efficient because capital is completely inelastic within a given period, unlike labor supply.[22] The shock-absorbing feature of τ_{kt} allows the government to maintain relatively stable tax rates on labor income, reminiscent of the tax-rate-smoothing hypothesis of Barro (1979). Finally, notice that tax rates (and household allocations) depend on the distribution of private capital across sectors, as reflected by the coefficients on $\ln(k_{1t})$ and $\ln(k_{2t})$ in the policy rules.

6.2.2 Policy Simulations

Figures 6 and 7 plot simulated public and private expenditures from the model and highlight the effect of preference shocks. Comparing these figures to those for the U.S. economy (figures 4b and 5b) reveals that preference shocks improve the model's performance. Without these shocks, x_{Gt} and X_t are too highly correlated (figure 6a) and g_t does not show enough variability relative to c_t (figure 7a). In figure 6a, the series for public and private investment diverge only slightly, due to the lower depreciation rate of public capital in the calibration.

Figures 8 and 9 plot simulated tax rates from the model versus estimates of average marginal tax rates for the U.S. economy. As noted earlier, the model predicts that the capital tax should be more variable than the labor tax. An eyeball comparison of the U.S. tax rate series seems to bear this out. Tables 4-7 provide a more quantitative comparison of the model with postwar U.S. data.[23]

[22] See Bohn (1988), Judd (1989), and Chari, Christiano, and Kehoe (1993) for related examples of optimal shock-absorbing policy.

[23] Figures 8 and 9 display the tax rate series *before* detrending. For quantitative comparisons (tables 6 and 7), detrending is necessary because the U.S. labor tax displays a distinct upward trend, while the U.S. capital tax displays a downward trend. These trends have no counterpart in the model. The trend in τ_{ht} is possibly linked to the phenomenon of "bracket creep," which existed before tax schedules were indexed for inflation in 1985. Regarding the trend in τ_{kt}, Auerbach and Poterba (1988) argue that the downward trend is due to increasingly generous investment tax credits and accelerated depreciation schedules.

Table 4a: Business-Cycle Statistics for the Model[a]

Variable	Std. Dev. (%)	Cross-Correlation of GNP with Variable at $t+i$				
		t-2	t-1	t	t+1	t+2
GNP	2.48	0.03	0.38	1.00	0.38	0.03
x_G	15.61	0.09	0.39	0.55	0.23	0.02
X	9.83	-0.14	0.19	0.96	0.44	0.13
g	3.28	0.12	0.25	0.43	0.09	-0.08
c	0.57	0.54	0.70	0.63	0.00	-0.30
H	1.32	-0.24	0.08	0.86	0.45	0.20
GNP/H	1.51	0.26	0.56	0.89	0.23	-0.12

[a]Statistics are mean values over 100 simulations, each 46 periods long. Before computing statistics, all series were logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 100. $GNP = y + rk_2 + wh_2$, $X = x_1 + x_2$, and $H = h_1 + h_2$.

Table 4b: Business-Cycle Statistics for U.S. Economy, 1947 to 1992[a]

Variable	Std. Dev. (%)	Cross-Correlation of GNP with Variable at $t+i$				
		t-2	t-1	t	t+1	t+2
GNP	2.46	0.05	0.56	1.00	0.56	0.05
x_G	9.99	0.33	0.61	0.59	0.42	0.20
X	5.96	-0.19	0.13	0.69	0.49	0.12
g	5.78	0.17	0.58	0.61	0.39	0.23
c	1.14	0.11	0.44	0.71	0.36	-0.20
H(hh)	1.76	-0.03	0.50	0.81	0.25	-0.16
H(est)	2.24	0.16	0.67	0.90	0.23	-0.24
GNP/H(hh)	1.45	0.12	0.34	0.71	0.64	0.26
GNP/H(est)	1.07	-0.25	-0.10	0.41	0.80	0.60

[a]Data sources for GNP, investment, and consumption are described in footnotes 4, 6, and 20. Public investment includes military items. Private investment is fixed investment + consumer durables expenditures + residential investment. All series have been annualized, logged, and detrended as in the model. *H*(hh) represents total labor hours from the household survey (LHOURS) in Citibase. *H*(est) represents total labor hours from the establishment survey (LPMHU) in Citibase.

Table 5: Comparison of Selected Statistics

Statistic	Model	U.S. Economy[a]
σ_{XG}/σ_X	1.59	1.68
corr (x_G, X)	0.49	0.04
σ_g/σ_c	5.76	5.05
corr (g, c)	0.37	0.16
$\sigma_H/\sigma_{Y/H}$	0.88	1.22 / 2.10
corr $(H, GNP/H)$	0.53	0.16 / -0.02

[a]For the labor market statistics, the two numbers refer to the household and establishment surveys, respectively. See the notes to table 4b.

The summary statistics in tables 4 and 5 reveal that the model performs reasonably well in capturing the correlation structure and *relative* standard deviations of the aggregate variables, but is less successful in matching the absolute standard deviations of the investment and consumption series. In agreement with the data, the model predicts that private expenditures are more strongly procyclical than public expenditures. Although the model standard deviations for x_G and X are much higher than in the data (table 4), the relative magnitudes (table 5) are about right. The model standard deviations could be reduced by introducing adjustment costs for both types of investment (see Lansing [1993b]). The standard deviations for the consumption series, g and c, display the opposite problem, i.e., the model series are *less* variable than the corresponding U.S. series. Once again, however, the relative variability in table 5 is close to the U.S. value. The two problems are no doubt linked; the excessive volatility of investment in the model allows for very smooth consumption.

In the labor market, the model predicts a lower standard deviation for total hours H than is observed in the U.S. data. Consequently, the model's variability of hours relative to labor productivity (table 5) is too low. This occurs despite the assumption of indivisible labor, which operates to increase the variability of H. Also, the correlation between hours and productivity in the model is higher than in the data. For these two statistics, the model does not do as well as other real business-cycle models that treat government policy as exogenous (see Christiano and Eichenbaum [1992], McGrattan [1991], and Braun [1989]). Although not shown in table 5, the contemporaneous correlation between hours worked in the two sectors, $\mathrm{corr}(h_1, h_2)$, averaged 0.37 in the simulations. This is consistent with conventional wisdom regarding the comovement of employment in different sectors over the business cycle.[24]

Tables 6 and 7 provide a closer examination of the behavior of policy variables. The model does well in matching the standard deviations and serial correlation of the tax rates (table 6), but is less successful regarding the cross-correlations with other policy variables (tables 7a and 7b). The model's prediction that the capital tax should display larger variability than the labor tax is generally confirmed by the data. The capital tax series estimated by Jorgenson and Sullivan (1981) has a much higher standard deviation than the series estimated by McGrattan, Rogerson, and Wright (1993). The values are 16.38 percent and 4.82 percent, respectively. The Jorgenson and Sullivan series is an estimate of the effective corporate tax rate, while the McGrattan, Rogerson, and Wright series also includes taxes paid by individuals on capital gains and dividends. The correlation between these two series (after detrending) is 0.45. The correlation between the two estimates of the U.S. labor tax is 0.84.

The correlation coefficients in the model match the signs in U.S. data for the majority of the cases in table 7. However, the model predicts signs opposite to those in the data for the correlation between x_G and g and the correlation between τ_h and τ_k. The predicted negative correlation between x_G and g (-0.36) reflects the impact of preference shocks on the optimal mix between public investment and public consumption. Without preference shocks, the correlation between x_G and g in the model is strongly positive (0.74) and close to the U.S. value (0.85). In this case, however, model performance suffers in other areas, such as the relative variability of g and c. The predicted negative correlation between τ_h and τ_k is also counter to U.S. data. The U.S data display a positive correlation, which suggests that there may be rigidities in the U.S. tax code (not accounted for in the model) that link the movement of tax rates over the business cycle.

[24] In a model version with different technology shocks in the two sectors, the correlation between h_1 and h_2 is negative (and hence counterfactual), as employment always flows into the most productive sector. This is also a characteristic of home production models that specify sector-specific technology shocks for the home and market sectors. See Benhabib, Rogerson, and Wright (1991).

Table 6: Comparison of Tax Rate Statistics (detrended)

		Model[a]	U.S Economy[b]	U.S. Economy[c]
τ_{ht}	Mean	0.222	0.272	0.229
	Std. Dev. (%)	3.82	5.65	3.65
	corr (-1)	0.41	0.43	0.55
	corr (-2)	0.05	-0.19	-0.09
	corr (-3)	-0.16	-0.67	-0.54
τ_{kt}	Mean	0.268	0.299	0.568
	Std. Dev. (%)	10.46	16.38	4.82
	corr (-1)	0.28	0.51	0.30
	corr (-2)	-0.02	-0.04	-0.20
	corr (-3)	-0.18	-0.24	-0.27

[a]Model statistics are means over 100 simulations, each 46 periods long. The tax rate series were detrended using the Hodrick-Prescott filter with a smoothing parameter of 100. The tax rate series were not logged before detrending.

[b]τ_{ht} is from Barro and Sahasakul (1986), 1947-83, and τ_{kt} is from Jorgenson and Sullivan (1981, table 11), 1947-80.

[c]τ_{ht} and τ_{kt} are from McGrattan, Rogerson, and Wright (1993), 1947-87.

Table 7a: Contemporaneous Correlation in Model[a]

	x_G	g	τ_h	τ_k	X	c	GNP
x_G	1.00	-0.36	0.82	-0.62	0.49	0.39	0.55
g		1.00	0.10	0.03	0.33	0.37	0.43
τ_h			1.00	-0.83	0.83	0.63	0.90
τ_k				1.00	-0.91	-0.37	-0.82

Table 7b: Contemporaneous Correlation in U.S. Economy[a]

	x_G	g	τ_h	τ_k	X	c	GNP
x_G	1.00	0.85	0.51 * 0.24 *	0.32 0.31	0.04 *	0.21 *	0.59 *
g		1.00	0.48 * 0.28 *	0.34 * 0.35 *	-0.02	0.16 *	0.61 *
τ_h			1.00	0.42 0.52	-0.18 -0.41	0.13 * -0.09	0.36 * 0.05 *
τ_k				1.00	-0.22 * 0.03	0.11 0.18	0.10 0.32

[a]An asterisk indicates that the correlation coefficient has the same sign as in the model. For the tax rate correlations, the top and bottom numbers in each cell are correlations using U.S. data described in footnotes b and c, respectively, of table 6. All variables have been detrended as in the model.

7. Concluding Remarks

The recent controversy surrounding the productive effects of public capital has focused on questions related to long-run growth. In contrast, this paper is concerned with the optimal behavior of public investment and other key elements of fiscal policy over the business cycle. While a number of researchers have incorporated fiscal policy into real business-cycle models, the government policymaker is generally treated as exogenous. This paper employs a two-sector real business-cycle model, augmented with preference shocks, to endogenize the choice of fiscal policy over time.

I subjected the model to comparisons with postwar U.S. data on public investment, public consumption, and tax rates and obtained reasonable success in capturing the observed behavior of the various time series. At the optimal level of public capital, the rate of return on public investment is less than the rate of return on private investment. The model predicts procyclical behavior for public investment and public consumption, in contrast to the Keynesian view of countercyclical fiscal policy. It also predicts a positive tax on capital that should be more variable than the labor tax. The model disagrees with the data, most notably, in predicting negative correlations between public investment and public consumption and between the capital tax and the labor tax.

A natural extension of this work would be to incorporate heterogeneity, since it is highly probable that key elements of fiscal policy, such as the capital tax, are driven by distributional considerations (see Lansing [1993a]). Further research is also needed to address the many empirical questions suggested by this work, including formal tests of model predictions and direct estimation of parameter values. In addition, much work remains to be done in modeling the institutional structure of the tax code and endogenizing the choice of tax instruments, topics that have been emphasized by Slemrod (1990).

APPENDIX A

A.1 Recursive Formulation of the Government's Problem

To formulate the recursive version of (19), we first substitute the household first-order conditions from (7) into the household budget constraint (2), the private-sector resource constraint (20), and the utility function $U(\cdot)$ to eliminate τ_{ht}, τ_{kt}, and c_t.[1] In addition, g_t can be eliminated using the government-sector resource constraint (19.v). The vector of state variables for the government's problem is $s_t = \{z_t, v_t, k_{1t}, k_{2t}, k_{Gt}, \lambda_{t-1}\}$. In the transformed problem, the government's decision variables are λ_t, h_{1t}, h_{2t}, k_{1t+1}, k_{2t+1}, and k_{Gt+1}. Using primes ($'$) to denote next-period quantities, the recursive version of the government's problem is shown in (A.1).

The Bellman equation in (A.1) summarizes the recursive nature of the problem. The first constraint is the household budget after substituting in the first-order conditions. The symbol u represents a composite error term that arises due to the presence of E_t in the first-order condition for k_{it+1}. The next constraint is the private-sector resource constraint. The government-sector resource constraint has been substituted into $U(\cdot)$. The remaining constraints define the cost minimization condition, the production technologies, the rental rate on private capital, and the laws of motion for the two exogenous shocks.

The dynamic programming problem applies for all $t > 0$. The problem at t=0 must be considered separately, as shown by Kydland and Prescott (1980), Lucas and Stokey (1983), and Chamley (1986). At t=0, the stock of private capital is fixed. Optimal policy thus implies a high initial tax on capital to take full advantage of this nondistortionary source of revenue. I assume that this form of lump-sum taxation is insufficient to finance the entire stream of future expenditures. The analysis here will focus on policy in stationary stochastic equilibrium, i.e., when t is very large. The linear-quadratic approximation method used to solve (A.1) is accurate only in the neighborhood of the deterministic steady state. Consequently, I do not solve the t=0 problem or compute the transition path to the stationary equilibrium.

---

[1] Due to the presence of the expectation operator in the first-order conditions for $k_{i\,t+1}$ (i=1, 2), the substitution is accomplished using the expression $E_{t-1} f_t(\cdot) = f_t(\cdot) - u_t$, where $f_t(\cdot)$ is a function of random variables and u_t is the forecast error. The assumption of rational expectations implies $E_{t-1} u_t = 0$.

$$V(s) = \max_{\substack{k_1', k_2', k_G', \\ h_1, h_2, \lambda}} E_{-1}\left\{U(\cdot) + \beta\left[V(s')|s\right]\right\}$$

$$\text{where} \qquad s = \{z, v, k_1, k_2, k_G, \lambda_{-1}\} \tag{A.1}$$

$$U(\cdot) = \ln(1/\lambda) - A(h_1 + h_2) + \exp(v) B \ln\left[y_G - k_G' + (1-\delta_G)k_G\right]$$

subject to

$$\frac{A}{\lambda}(h_1 + h_2) + \frac{\lambda_{-1}}{\beta\lambda}(k_1 + k_2) + (1-\theta_1-\theta_2)y\left[\frac{\frac{\lambda_{-1}}{\beta\lambda} - 1}{r-\delta}\right]^2 - (k_1' + k_2') - \lambda + u = 0$$

$$y - \frac{1}{\lambda} - k_1' - k_2' + (1-\delta)(k_1 + k_2) = 0$$

$$\frac{\theta_1 k_2}{\phi_1 k_1} - \frac{\theta_2 h_2}{\phi_2 h_1} = 0$$

$$y = \exp(z)\, k_1^{\theta_1} h_1^{\theta_2} k_G^{\theta_3}$$

$$y_G = \exp(z)\, k_2^{\phi_1} h_2^{\phi_2} k_G^{\phi_3}$$

$$r = \theta_1 \frac{y}{k}$$

$$z' = \rho_z z + \varepsilon'$$

$$v' = \rho_v v + \xi'.$$

Arrow and Kurz (1970) prove the existence of a unique, stationary equilibrium in a one-sector economy similar to the one developed here. Uzawa (1966) proves existence and uniqueness in a two-sector model with a production tax, but excludes public capital. Although I do not prove existence and uniqueness, analogy to these other models suggests that the model possesses a unique, stationary equilibrium. The computational algorithm (described below) always converged to the same value function regardless of the initial starting point. Equilibrium is defined as a value function $V(s)$ and an associated set of stationary decision rules that satisfy (A.1). The decision rules dictate a set of household allocations and prices at time t that can be implemented by means of the government's chosen policy. The government's explicit policy rules for tax rates and public expenditures are recovered by substituting

FIG 1: U.S. LABOR PRODUCTIVITY vs PUBLIC CAPITAL

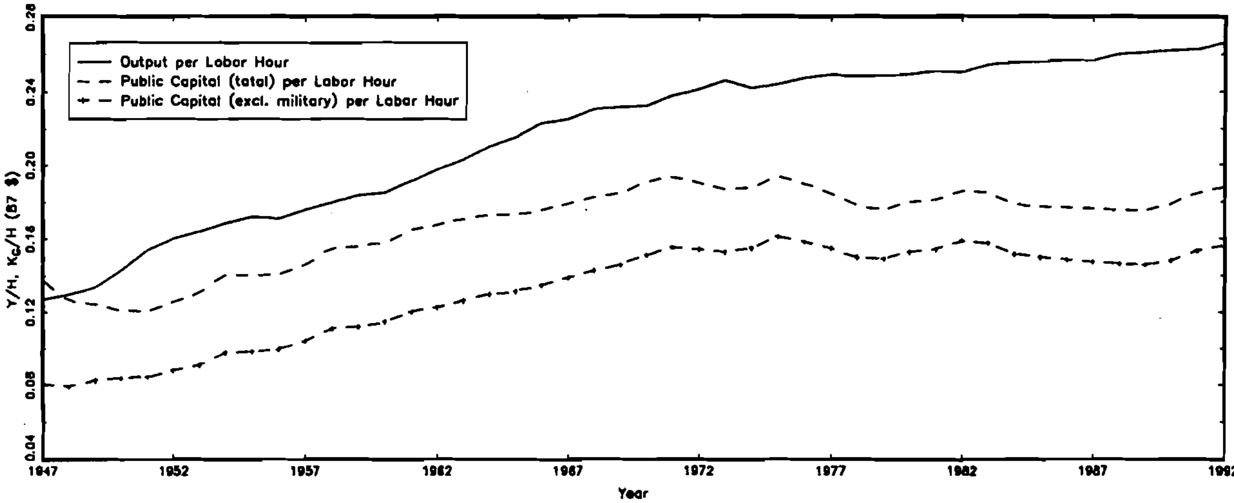


FIG 2: U.S. PRIVATE vs PUBLIC INVESTMENT

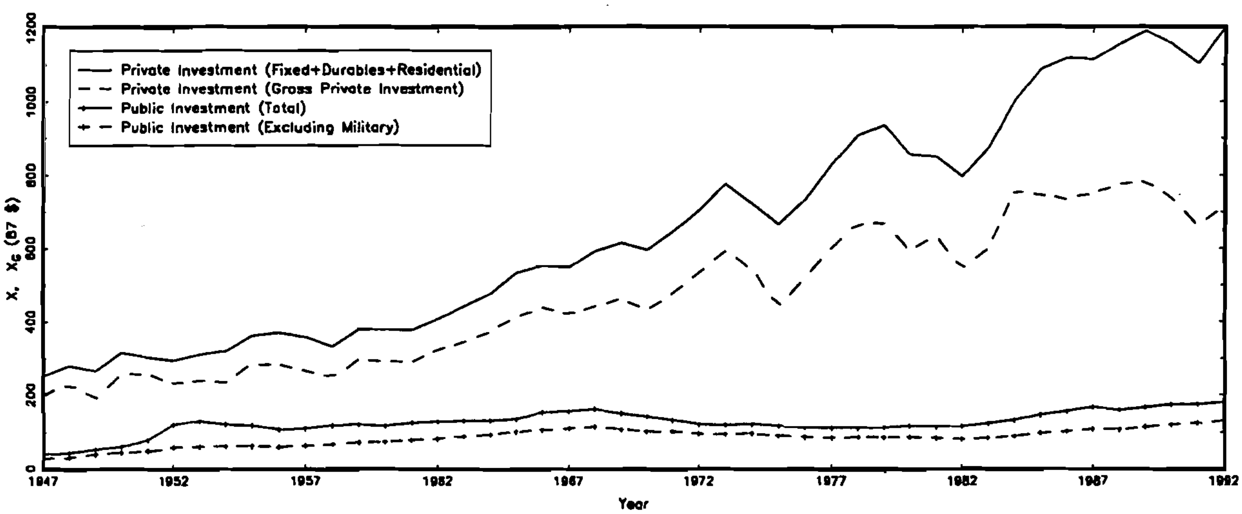
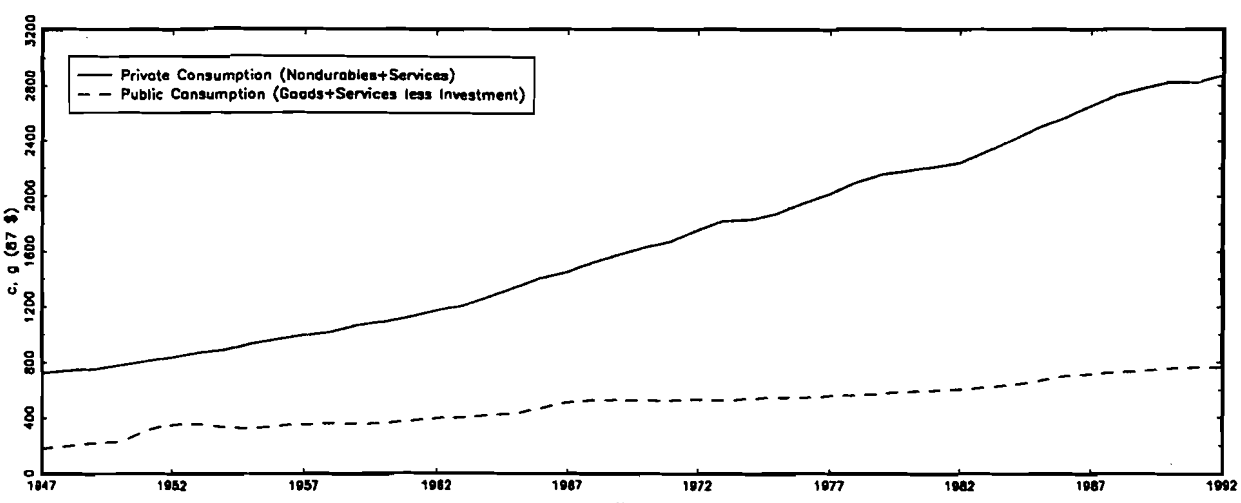


FIG 3: U.S. PRIVATE vs PUBLIC CONSUMPTION



Sources: U.S. Department of Commerce and Citibase.

FIG 4a: U.S. SERIES OF PUBLIC INVESTMENT AND GNP (Logged/Detrended)

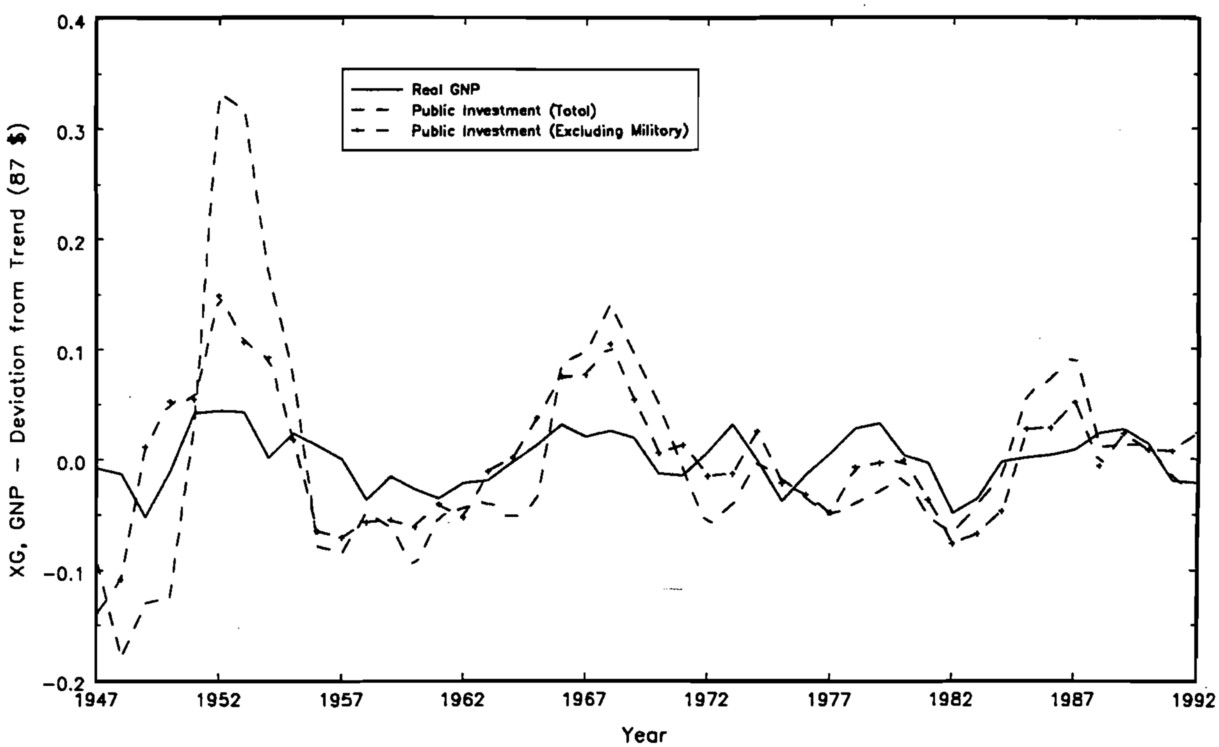
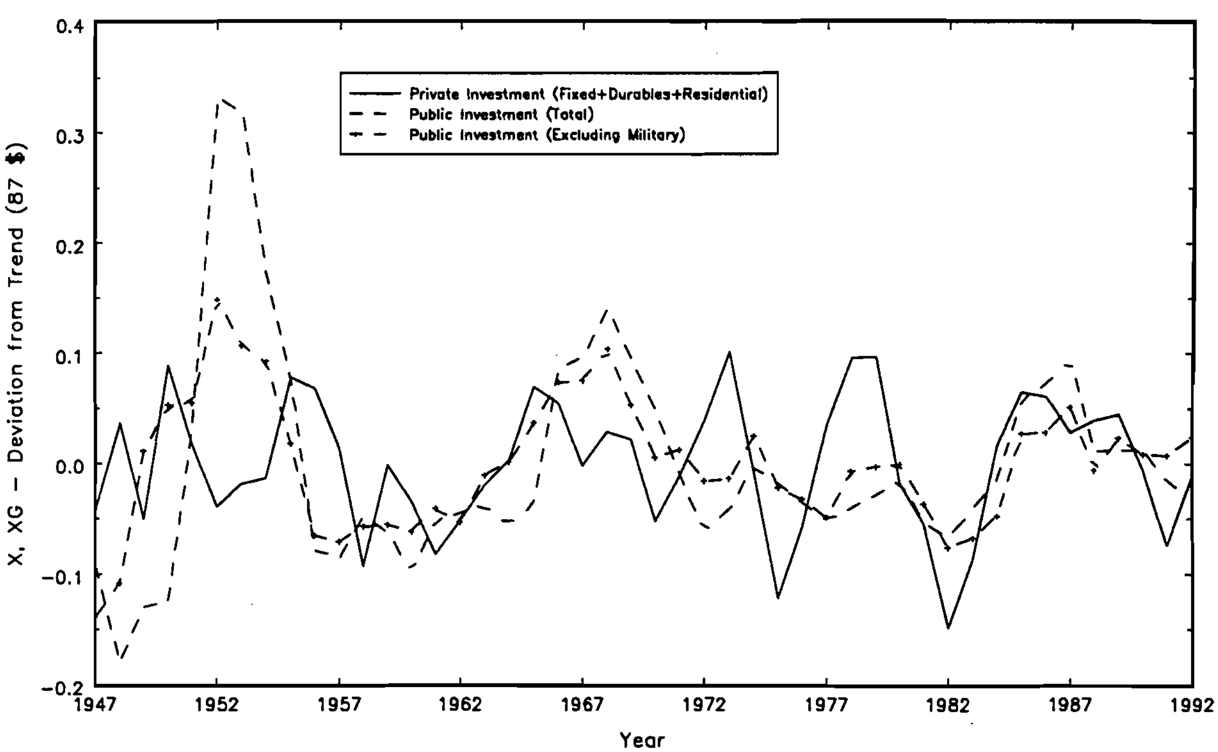


FIG 4b: U.S. SERIES OF PRIVATE & PUBLIC INVESTMENT (Logged/Detrended)



Sources: U.S. Department of Commerce and Citibase.

FIG 5a: U.S. SERIES OF PUBLIC CONSUMPTION AND GNP (Logged/Detrended)

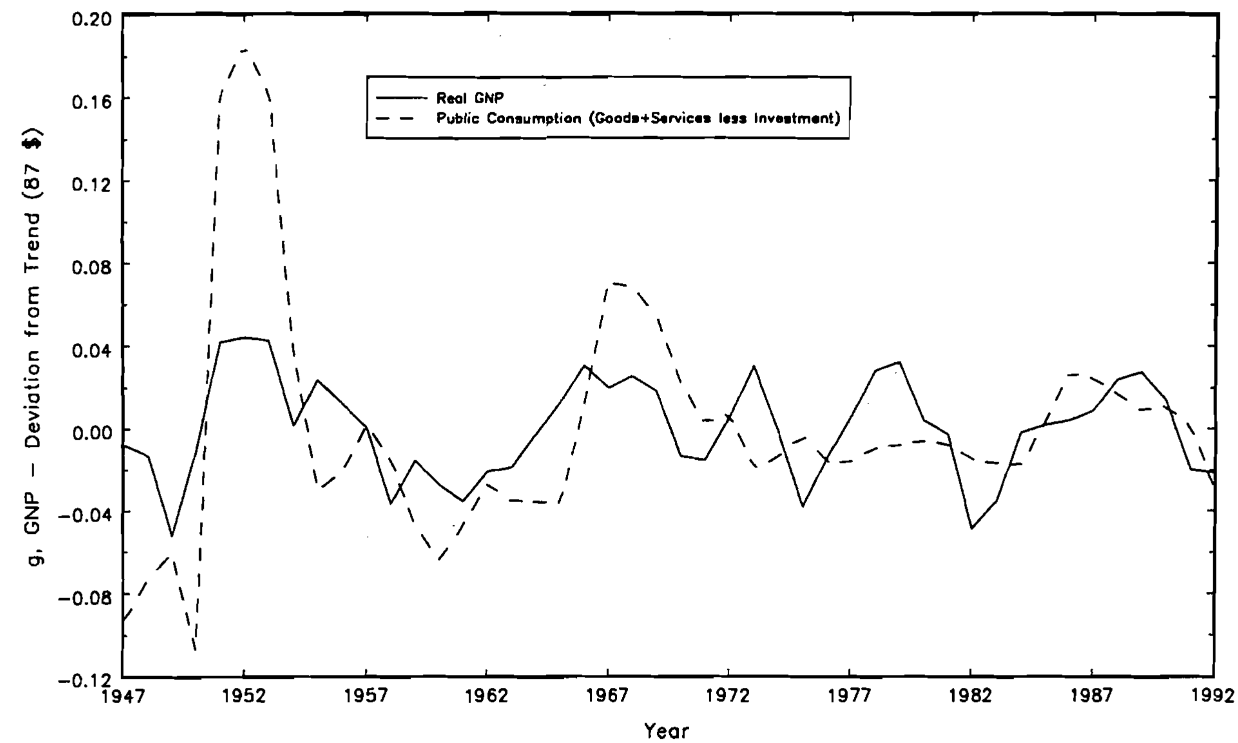
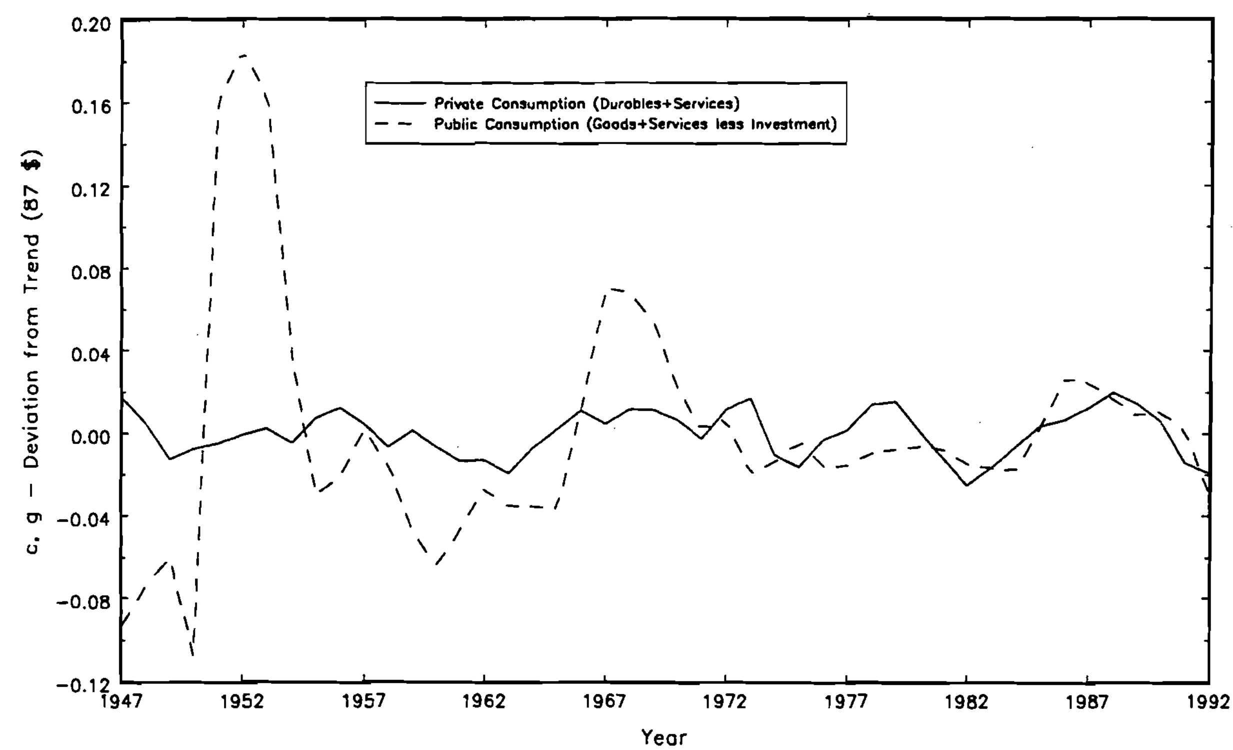


FIG 5b: U.S. SERIES OF PRIVATE & PUBLIC CONSUMPTION (Logged/Detrended)



Source: Citibase.

FIG 6a: SIMULATED PRIVATE & PUBLIC INVESTMENT – No Preference Shocks

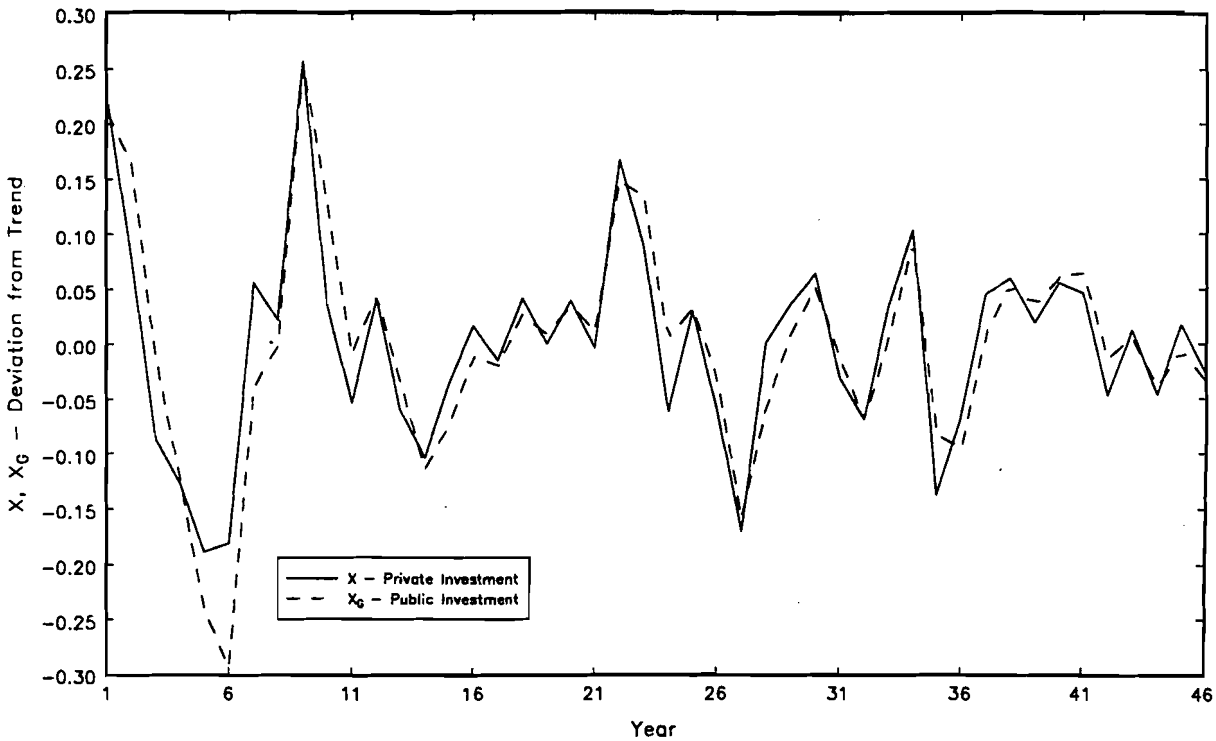
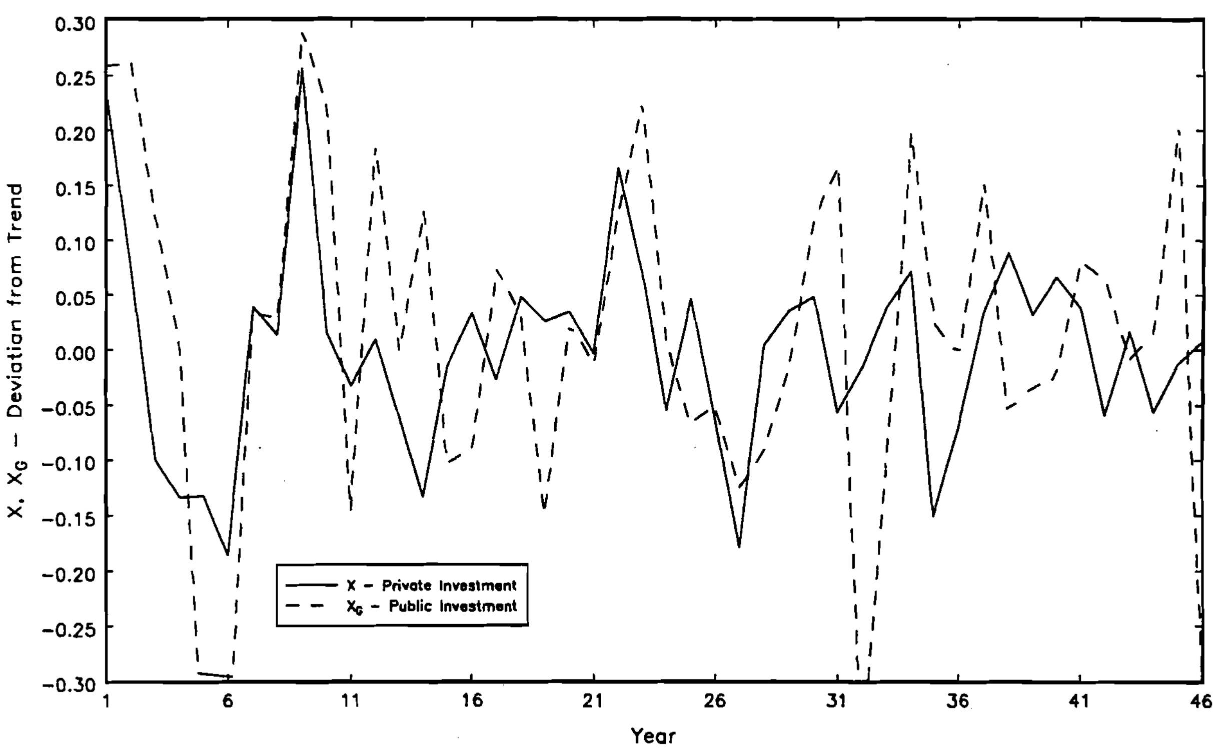


FIG 6b: SIMULATED PRIVATE & PUBLIC INVESTMENT – w/ Preference Shocks



Source: Author's calculations.

FIG 7a: SIMULATED PRIVATE & PUBLIC CONSUMPTION – No Preference Shocks

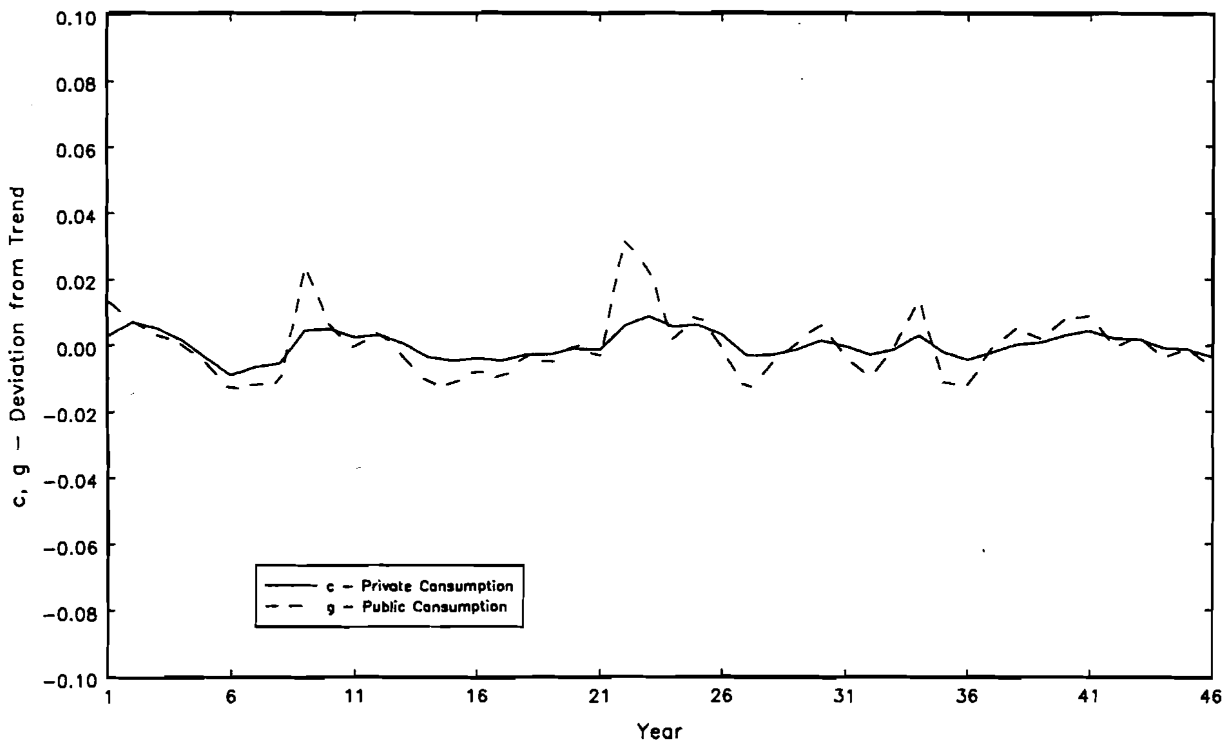
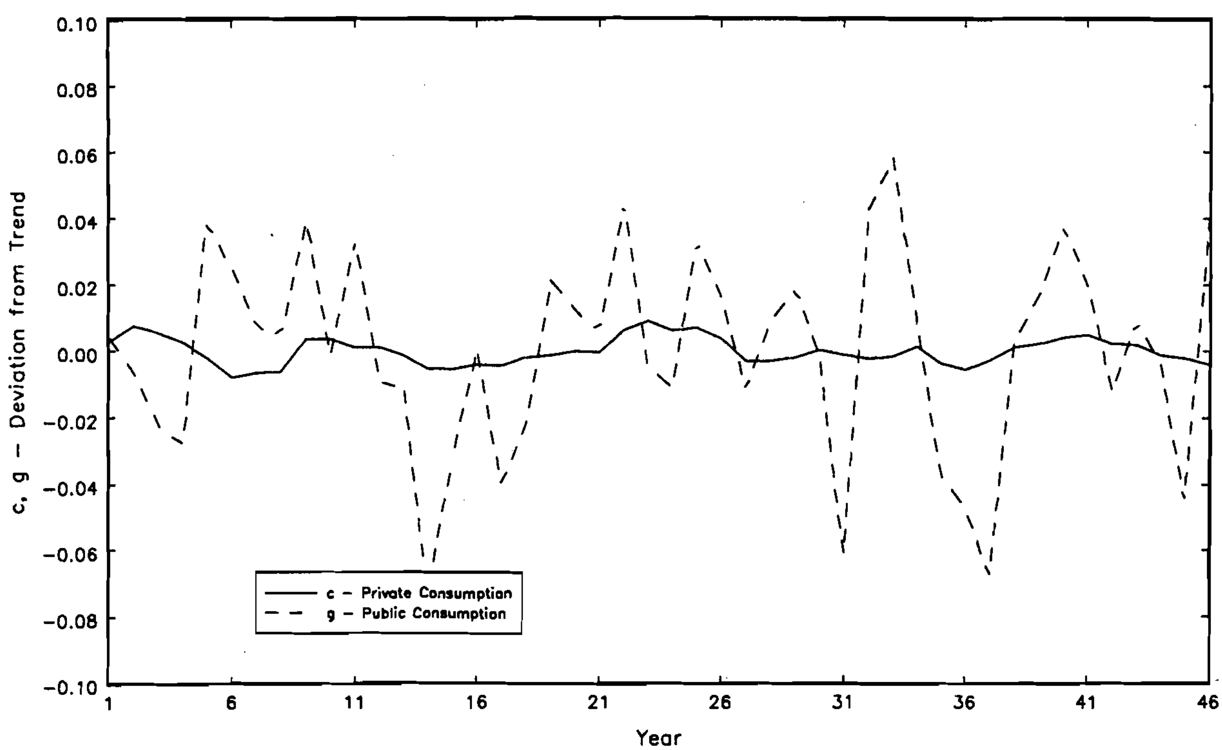


FIG 7b: SIMULATED PRIVATE & PUBLIC CONSUMPTION – w/ Preference Shocks



Source: Author's calculations.

FIG 8a: U.S. SERIES FOR LABOR TAX (Barro & Sahasakul–1986)

FIG 9a: U.S. SERIES FOR CAPITAL TAX (Jorgenson & Sullivan–1981)

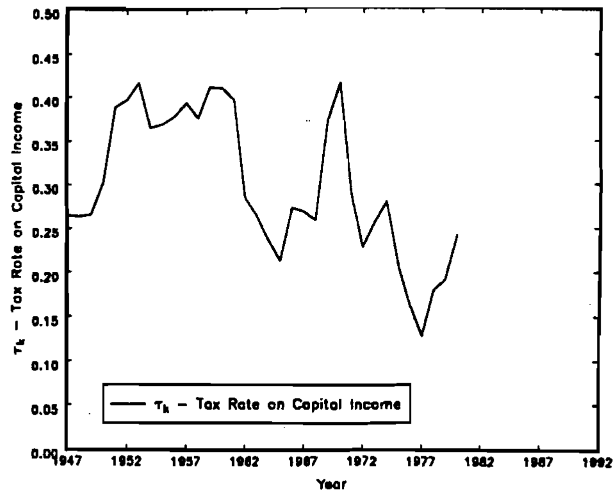


FIG 8b: U.S. SERIES FOR LABOR TAX (McGrattan et al. – 1993)

FIG 9b: U.S. SERIES FOR CAPITAL TAX (McGrattan et al. – 1993)

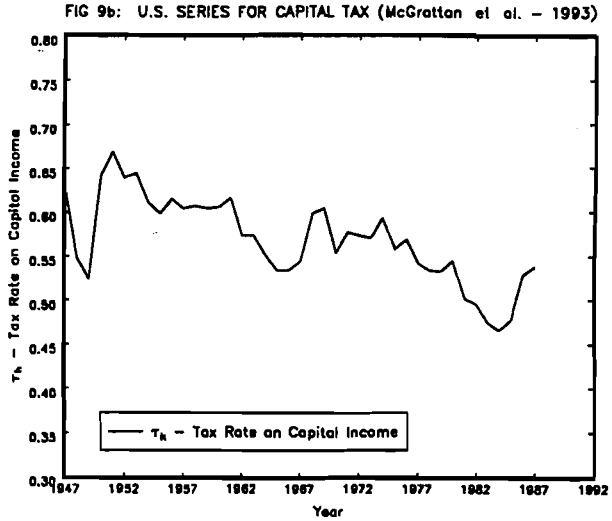


FIG 8c: SIMULATED SERIES FOR LABOR TAX – w/ Preference Shocks

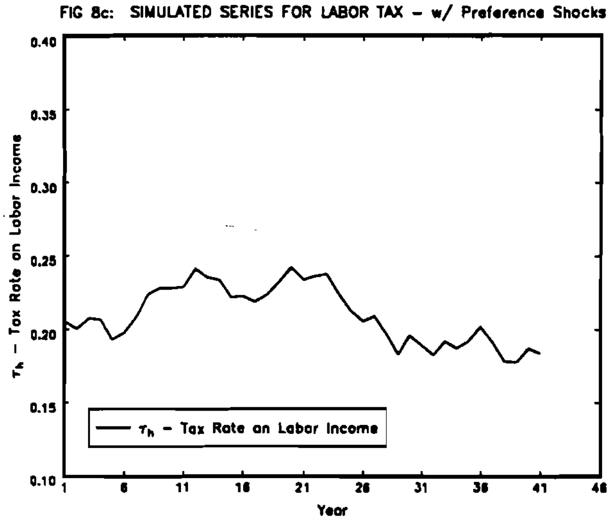
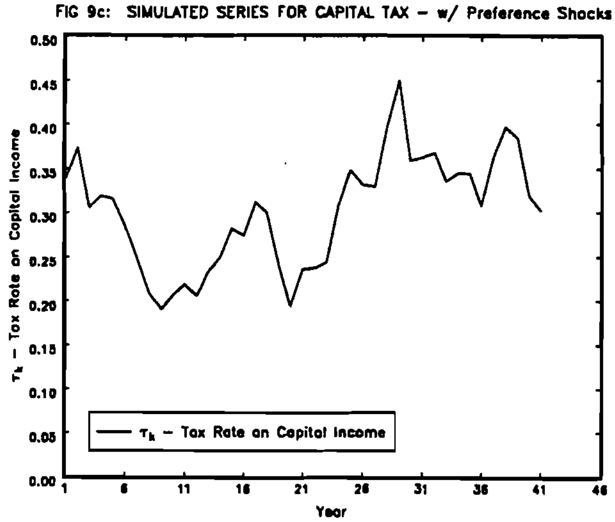


FIG 9c: SIMULATED SERIES FOR CAPITAL TAX – w/ Preference Shocks



Sources: As noted and author's calculations.