ZERO INFLATION: TRANSITION COSTS 
AND SHOE-LEATHER BENEFITS

by Charles T. Carlstrom and William T. Gavin

Charles T. Carlstrom is an economist and 
William T. Gavin is an assistant vice 
president and economist at the Federal Reserve 
Bank of Cleveland. The authors thank David 
Altig, John Carlson, Rik Hafer, and E.J. 
Stevens for useful comments.

Working papers of the Federal Reserve Bank of 
Cleveland are preliminary materials circulated 
to stimulate discussion and critical comment. 
The views stated herein are those of the 
authors and not necessarily those of the 
Federal Reserve Bank of Cleveland or of the 
Board of Governors of the Federal Reserve 
System.

October 1991
I. Introduction

The double-digit inflation rates of the late 1970s and early 1980s were extraordinarily high by American standards. The costs associated with this inflation were judged to be too high. Political recognition of these costs led the Federal Reserve, with the support of the Carter administration, to begin tightening monetary policy. The disinflationary policy was successful, and since 1982 the inflation rate has averaged about 4 percent. However, many have blamed this policy for bringing about the 1981-82 recession, the worst since the Great Depression.

Nevertheless, many would not hesitate to say that the United States is better off today with the lower inflation rate of around 4 percent. If the nation is indeed better off, it is natural to ask whether a move from 4 percent inflation to zero would be worth the costs of a possible recession. Howitt (1990) makes a persuasive case that zero inflation is probably the best inflation rate to inherit, but his analysis of the transition costs leaves him uncertain whether to recommend going all the way to zero. Aiyagari (1990) compares the costs of going to zero from a 5 percent inflation trend with the benefits of being there and concludes that the costs exceed the benefits. In this paper, we present some simple calculations to explain why we think that the benefits of achieving price stability would exceed the transition costs.

One benefit of zero inflation is the value of the extra real cash balances people would hold at a lower inflation rate. This benefit is sometimes referred to as the "shoe-leather" savings because, in a simple money demand model, holding larger amounts of cash saves shoe leather that would be worn down in making additional trips to the bank. Because inflation acts as a tax on cash, individuals spend time and resources (shoe leather) trying to economize on the cash they hold. Banks benefit if inflation is zero because reserves do not earn
interest and are therefore also taxed with inflation. Some economists consider
the shoe-leather costs so small relative to other costs of inflation that they
should not be considered by policymakers.¹ Other economists believe that the
welfare loss arising from a recession caused by tight monetary policy is so large
that it swamps all the benefits associated with zero inflation.²

In this paper, we show that the loss associated with a money-induced
recession is actually of the same order of magnitude as the gain attributable to
reduced shoe-leather costs if the inflation rate were reduced from 4 percent to
zero. This result is important because the transition costs associated with
ending inflation are thought to be large, capturing the main cost of a
disinflation policy, while the shoe-leather costs are generally thought to be
only a small, insignificant share of the total costs of inflation.

In order to make such comparisons useful, the costs and benefits of
reducing inflation must be in the same metric. Ideally, both should be measured
in welfare terms; that is, how much a person would have to be compensated in
order to be indifferent to a given policy change. Unfortunately, this is not
possible without a consistent model that explains both how disinflationary
policies cause recessions and why people are better off with zero inflation.
Instead, we measure the desirability of a zero inflation policy in terms of
resource costs and resource savings.

We begin the analysis in section II by estimating the costs of a
disinflationary policy. Then, in section III we show that the partial
equilibrium estimates of shoe-leather costs are equal to the value of resources
that would be saved if the price level were stable. This framework allows a
quantitative comparison of these offsetting effects. In section IV, we conclude
with a discussion of the policy implications.
II. The Costs of Disinflation

For the sake of the argument, we assume that the transition to zero inflation cannot be accomplished without causing a recession. In theory, the depth of the recession caused by disinflation can be reduced (or perhaps even eliminated) if the monetary authorities can make a credible commitment to achieving price stability. Actual losses will likely depend on the credibility of the Federal Reserve System, making the true cost/benefit calculation of a prospective disinflation quite difficult. In the absence of good information about the Fed's credibility, we assume that the transition costs of another 4 percent disinflation would be the same as the costs incurred in the early 1980s when the Federal Reserve engineered a 4 percent disinflation, reducing the inflation trend from approximately 8 percent to 4 percent.

The transition costs are measured as the accumulated value of consumption lost during the period from the beginning of the disinflation policy in 1979 until consumption returned to its trend level in 1985. Unlike Blinder (1990), we use consumption rather than output to measure the costs of disinflation. We assume that current consumption is a sufficient statistic for future output; that is, when the level of consumption returns to its long-run trend, we assume that the present discounted value of expected output has also returned to its long-run trend. This assumption is an implication of the permanent income hypothesis, which postulates that consumption is a constant fraction of permanent income (present discounted value of future output). Thus, when consumption returns to the same value that would have occurred without a recession, so has permanent income or expected future output.

To measure the present discounted value of consumption lost because of the
disinflation policy, one must know the path that consumption would have followed had the policy not been adopted. We assume that the trend in consumption would have grown at a constant rate of 2.7 percent; this is the trend growth in potential GNP for the 1980s as calculated by the research staff at the Board of Governors of the Federal Reserve System and as described in Braun (1990).

The expected trend in the inflation rate was around 8 percent per year in the summer of 1979, before the Fed adopted the disinflation policy. Since 1983, inflation has averaged just over 4 percent per year. The recession had ended by 1983, but consumption did not reach the level that would have occurred without the recession until 1985 (see figure 1). In the second half of the decade, the level of consumption spending was above our estimate of the sustainable long-run trend. We estimate the costs of disinflation by accumulating the discounted deviations of consumption below the trend level for each year until actual consumption returned to the trend:

\[
\lambda = \sum_{t=1980}^{1985} \beta^{t-1980}(c^p_t - c^*_t),
\]

where \( \lambda \) is the present value of the consumption lost, \( c^p_t \) is the trend (logarithmic) level of consumption in period \( t \), \( c^*_t \) is the actual logarithm of consumption in period \( t \), and \( \beta = 1/(1+r) \) is the discount rate. Assuming that the real interest rate was 4 percent and that the consumption trend was 2.7 percent, the present value of the consumption lost in the early 1980s was almost 18 percent of consumption, or 12 percent of the 1979 level of GNP.

Table 1 shows the sensitivity of our results to alternative assumptions about the real interest rate and the real consumption growth trend. The results are practically insensitive to alternative interest rate assumptions. The
welfare losses are shown with artificial precision, out to two decimal points, to illustrate a difference associated with a 1/2 percentage point change in the real rate. The assumption about the consumption growth trend, on the other hand, is quite important for the estimate of the transition costs. In the neighborhood of the 2.7 percent trend, each 0.1 percentage point increase in the assumed growth trend for consumption raises the estimate of the resource cost by about 1 to 1-1/4 percent of GNP.

Discussion of Assumptions

There are several possible objections to our procedure. First, some have argued that the transition costs are not linear as the inflation rate is reduced. They reason that going from 8 percent to 4 percent is credible, while going from 4 percent to zero is not. A rationale for this argument can be found in the reputation equilibrium model developed by Barro and Gordon (1983). However, there is no evidence that the disinflation in the early 1980s was anticipated. As mentioned above, at least one forecaster, Data Resources Inc. (DRI), guessed that inflation would average 8 percent in the 1980s. In 1978, Richard B. Hoey began surveying decision-makers for their 10-year inflation outlook. The average 10-year-ahead inflation expectation peaked in October 1980 at 8.82 percent.4

Second, some have suggested that high unemployment in Europe in the 1980s is evidence that reducing inflation may have long-run effects on output.5 However, this experience is recent and localized and may be due to industrial policies or labor laws. In a study aimed directly at this long-run issue, Boschen and Mills (1990) find that there is no effect of monetary factors on permanent movements in real GNP.

There are many reasons to think that this procedure will overstate the
costs of disinflation, because the recession of the 1980s was clearly associated with many different real shocks, including the structural adjustments in the auto and steel industries, the deregulation in transportation and financial industries, and the oil price shock of 1979. It is often difficult to disentangle real and monetary factors; however, we can use DRI's 1979 estimate of the impact of the oil price shock to adjust our calculations.

One way to adjust for this shock is to adjust the trend consumption growth downward. In 1979, DRI estimated that the oil price shock would cause consumption growth to decline by 2 percent in 1980. Consumption actually fell 0.2 percent that year; we assume the difference was due to monetary policy. After 1980, we assume that the consumption growth rate returns to the 2.7 percent trend and grows parallel to, but below, the trend shown in figure 1. This adjustment reduces the estimated resource cost of the disinflation to approximately 9.7 percent of GNP.

Our calculation of the transition costs of disinflation differs from others because we measure these costs in terms of lost consumption rather than in terms of additional point-years of unemployment or in terms of lost output. A recently published book by Blinder (1987) reports estimates of the transition costs that appear to be much larger than ours. Blinder measures output lost indirectly. He assumes a full-employment rate of unemployment of 5.8 percent (the actual rate in 1979) and calculates the cost of the disinflation as the amount by which actual unemployment exceeds 5.8 percent for the years 1980 through 1986. He calculates that the disinflation resulted in 12.5 point-years of unemployment. Using Blinder's estimate of Okun's Law, a rule of thumb relating unemployment and output, those 12.5 point-years of unemployment are equivalent to 30 percent of GNP.
This procedure probably overstates the cost of disinflation in two ways. First, the natural rate of unemployment is difficult to estimate. The large amount of sectoral reallocation that took place in the early 1980s makes it likely that the natural rate was higher than 5.8 percent.

The second way Blinder overstates the cost of the 1980-1982 recession is to add up lost output until unemployment (and, by Okun's Law, GNP) returns to its trend level. As we discussed earlier, however, the costs of disinflation should be measured as the accumulation of deviations of consumption from trend, and not the accumulation of deviations of GNP from trend. Measuring lost GNP involves a type of double counting. The measure of accumulated output lost includes forgone investment, which is a source of future consumption. By accumulating the consumption lost in each year until the level of consumption returns to its trend, we have implicitly included the investment that was lost in the recession.6

Blinder also assumes that the reduction of inflation in the early 1980s was 6 percent, not 4 percent as we have assumed. Thus, we may be overstating the costs of a 4 percent disinflation. Assuming that inflation was reduced by 6 percentage points in the early 1980s and that the costs of disinflation are linear, the estimated costs of reducing inflation are approximately two-thirds of those shown in table 1.

Our calculations put the costs of the 1980s disinflation between 8 percent and 15 percent of GNP. Of course these calculations are only rough estimates, but they indicate that, at least for this episode, the estimated costs of going to 4 percent inflation were substantial.
III. Shoe-Leather Costs

The shoe-leather benefit of eliminating inflation is the value society places on the extra money balances that would be held if inflation were expected to be zero. The demand for money (see figure 2) reflects the social value of an extra unit of cash balances. If the current nominal interest rate is 8 percent per year, society would value an additional dollar of real cash balances at eight cents per year. This value can be considered the increased utility of holding real balances, as in Sidrauski (1967), or the reduced shoe-leather costs, as in Baumol (1952) and Tobin (1956).

As is well known, the area under the demand curve for money, from the origin to a given quantity of money, is equal to the total value per year that society places on holding that amount of real cash balances. The shaded area (A+B in figure 2) is equal to the social value of the extra real balances that would be held if the inflation rate were reduced from \( \pi \) to zero. Area C in figure 2 shows the welfare loss that remains even at zero inflation.\(^7\)

This area is a partial-equilibrium measure of the welfare cost of zero inflation. The costs of disinflation are expressed in terms of resource costs, however, so that these benefits should also be expressed in similar terms. To see that the traditional measure of the welfare cost of inflation also equals the amount of resources that society wastes under positive inflation, consider a variant of the Tobin-Baumol model of money demand.

Suppose individuals get paid once per month and choose how many times they wish to go to the bank in that month. More trips allow the individual to hold a lower cash balance and a higher level of interest-earning assets, such as bonds. With \( n \) trips to the bank per period, the average amount of an
individual's real cash balances \((m)\) is given by \(y/2n\), where \(y\) is the amount of real earnings per month. The total cost of making \(n\) trips to the bank, \(C(n,y)\), depends on the income level, \(y\), as well as on the number of trips to the bank, \(n\). Individuals choose \(n\) in order to minimize the combined costs associated with holding currency and going to the bank:

\[
\text{Total Costs} = (r+\pi) \frac{y}{2n} + C(n,y), \quad \text{where} \quad \frac{\partial^2 C(n,y)}{\partial n^2} \geq 0. \tag{2}
\]

The first term represents the opportunity cost of holding currency, and the second term is the shoe-leather costs incurred making trips to the bank in order to minimize real cash holdings. The deadweight loss associated with an inflation rate of \(\pi\) is simply the total amount of resources that are spent trying to escape the inflation tax, \(C[n^*(i-r+\pi),y]\). The benefits of ending inflation equal the savings in brokerage fees or shoe-leather costs, \(C[n^*(i-r+\pi),y] - C[n^*(i-r),y]\). Appendix 1 shows that this difference is equal to area A+B in figure 2.

To derive a quantitative estimate of the benefits of going to zero inflation, we assume that the demand for real cash balances has the general functional form presented in Cagan (1956) and has a unitary income elasticity. The demand for money is given as

\[
\frac{M_t}{F_t} = m_t = ye^{\alpha-\beta i} \tag{3}
\]

or

\[
i = \frac{\alpha}{\beta} - \frac{1}{\beta} \ln(m_t) + \frac{1}{\beta} \ln(y_t). \tag{4}
\]

Figure 2 shows that integrating equation (3) from \(m_t\) (money demand at
i=r+π) to \( m'_{t} \) (money demand at i=r) gives us a framework for estimating the resources saved by adopting a zero inflation policy. The resource cost per year as a fraction of current GNP, \( b \), associated with an inflation rate of \( \pi \) (per year) is shown in appendix 2 to be approximately

\[
b = \frac{\beta \pi i + \beta \pi r}{(2-\beta \pi)V},
\]

where \( V \) is base velocity in a year, \( i \) is the nominal interest rate per year, and \( \beta \) is the semi-elasticity of money demand. We use the monetary base because it is the base for the inflation tax. Although there is a restriction against paying interest on the business demand deposits in M1, banks have long devised methods for paying implicit interest on these accounts. Given the structure of financial regulations in the United States, there is probably a welfare loss associated with inflation-induced distortions in the use of inside money. It is not appropriate, however, to measure the welfare loss as the area \( A+B \) under the demand curve for inside money as illustrated in figure 2.

The income velocity of the monetary base was approximately 19 in 1990. Assuming the real interest rate, \( r \), to be 3.5 percent, and the semi-elasticity of money demand to be 5, the reduction in the deadweight loss that would occur from eliminating a 4 percent inflation rate is approximately 0.064 percent of GNP each year.\(^6\)

This is seemingly a small amount, but one must remember these are only the one-year savings. By going to zero inflation and staying there, the savings would also include the present discounted value of all future savings. If the long-run income elasticity of money demand is approximately 1.0, the benefit will grow approximately one-for-one with the economy.\(^9\) The present discounted value
of the benefits of going to zero inflation would be $b/(r-g)$, where $g$ is the growth rate per year of output. Assuming trend growth in output is approximately 2.7 percent per year and the real interest rate is 3.5 percent per year (as assumed in the previous section), the total benefits of a zero inflation policy would be about 8.0 percent of current GNP.

Table 2 shows how the estimate of the welfare gain from going to zero inflation varies with changes in assumptions about the model’s parameters. The semi-elasticity of interest, $\beta$, varies between 3 and 7. This range encompasses the empirical estimates of the long-run interest elasticity of the monetary base reported in Hoffman and Rasche (1989). The size of the welfare gain depends importantly on the real interest rate and the real growth rate. This difference, $(r-g)$, is shown in the first column. We report results for three values of the real interest rate and three values of $\beta$. The estimate of the welfare gain ranges from 3.1 percent to 28.6 percent of GNP.

Discussion of Assumptions

To this point our analysis ignores the fact that eliminating inflation depletes tax revenue that must be replaced with some other distorting tax. Phelps (1973) notes that Friedman’s rule might not be optimal in a world with distortionary taxes. He argues that the government might find it advantageous to collect some revenue through inflation. Using our estimates of the resource costs of inflation (equation [6]), one can calculate the additional loss that occurs per additional dollar of revenue gained through inflation:

$$\frac{\partial \text{loss}}{\partial \text{Rev}} = \frac{\beta i}{1-\beta i}.$$  \hspace{1cm} (6)

Evaluated at zero inflation, the loss for each additional dollar gained by
increasing inflation is simply $\beta r/(1-\beta r)$. This implies that at zero inflation, every additional dollar of revenue that the government collects via inflation costs society an additional 25 cents in social loss.\textsuperscript{10} At 4 percent inflation, the last dollar redistributed costs society more than 66 cents! If the marginal welfare cost of raising revenue through some source is less than 25 cents, then the inflation tax would not be part of an optimal tax structure.

Our estimates of the resource costs of inflation are overstated because we do not include the amount of resources society must use to replace the revenue that accrues with 4 percent inflation. Any such estimates are beyond the scope of this paper. We proceed assuming they are negligible, but recognize that these costs should be estimated and included in a full cost/benefit analysis.

Aiyagari (1990) argues that much of the U.S. currency stock is held in the underground economy or by foreigners and, therefore, we should not include the full amount of the base in our resource cost estimate.\textsuperscript{11} Assuming, for example, that two-thirds of the monetary base is held by foreigners or the underground economy, effective velocity should be 57 instead of 19. This would reduce our welfare costs by a third.

However, this assumes that the interest elasticity of base demand is the same for everyone. The benefit of using currency is often the evasion of taxes or the avoidance of punishment for breaking the law. For the inflation rates that have historically prevailed in this country, the use of currency is likely to be very insensitive to the interest rate. Since the measured monetary base includes a mixture of these highly inelastic funds and the more interest-elastic funds held for legitimate purposes by U.S. citizens, the actual interest elasticity of base demand by aboveground holders of the monetary base must be much larger than the level estimated using the actual measured base. If that
part of the monetary base held by foreigners and by the underground economy is completely insensitive to the interest rate, then adjusting the size of the monetary base downward to exclude this portion causes a proportional upward adjustment in the estimated interest elasticity. Using equation (6) with a proportional increase in velocity and the interest elasticity leads to an increase in the estimate of the benefits of price stability. The present value of the shoe-leather savings rises from 8.0 percent to 10.3 percent of GNP for our base case when we assume the income velocity of money is equal to 57 and the semi-elasticity of money demand is 15.

IV. Conclusions

In summary, estimates of both the resource costs and benefits associated with disinflation can vary widely depending on the assumptions used in the analysis. The costs of an actual disinflation policy would range anywhere from something quite low if policy were credible and announced in advance to a high of around 15 percent of GNP. A comparison of the estimates in tables 1 and 2 shows that the transition costs of a disinflation policy from 4 percent to zero are in the same ballpark as the expected benefits of reducing shoe-leather costs. Our point is not to argue that these costs are identical, or even that one is greater than the other, but merely to show that they are probably of the same order of magnitude.

A measure of resources is not always a good measure of welfare. The resource costs of ending inflation are identical to the welfare costs of ending inflation if individuals care only about consumption and if the resulting losses are borne equally by all members of society. Clearly, the first assumption is false. Using a model that includes both consumption and leisure in utility would
imply that the welfare costs of ending inflation are smaller than the resource costs. The second assumption is also clearly not true. On average, those who become unemployed lose much more than those who keep their jobs. This would tend to make the resource costs of eliminating inflation less than the welfare costs. Thus, our measure of the costs of reducing inflation may either understate or overstate the actual welfare costs.

Our measure of the resource costs of ongoing inflation is derived from a partial equilibrium analysis. One would prefer, however, a full general equilibrium measure of these costs. Gillman (1990) and Benabou (1991), in a discussion of Cooley and Hansen (1991), both argue that partial equilibrium measures (the area under the money demand curve) will be less than those estimated in a general equilibrium model with a cash-in-advance constraint. A formal welfare analysis requires a general equilibrium model that can explain why ending inflation is costly and why people prefer zero inflation.

The purpose of this paper is to show that the transition costs of ending inflation, a major obstacle to monetary policy reform, are approximately equal to the shoe-leather benefits of having price stability. Summers (1991) convincingly argues that the shoe-leather costs of inflation pale in comparison to other costs. The most important measurable costs are those resulting from the interaction of inflation with our nominal tax system. Indexing does not seem to be a practical way to solve the problem. For example, despite the indexing provisions contained in the Economic Recovery Act of 1981, Altig and Carlstrom (1991b) estimate that bracket creep reduces steady-state output by 1.25 percent when the inflation trend is 4 percent. Bracket creep is the process by which inflation pushes individuals into higher tax brackets. If steady-state consumption also falls by 1.25 percent, then this resource cost of inflation is
more than 100 times larger than our estimate of the shoe-leather costs.

An even larger distortion occurs because we tax nominal interest income. Altig and Carlstrom (1991a) estimate that this practice reduces steady-state output by nearly 5 percent when the inflation trend is 4 percent. Again, these costs clearly swamp both the shoe-leather costs of maintaining a 4 percent inflation trend and the transition costs of ending inflation.
Footnotes

1. See Summers (1991) for a recent statement of this argument.

2. See, for example, Blinder (1987), chapter 2.

3. See Data Resources, Inc. (1979), which predicts 8 percent inflation for the next decade. The Consumer Price Index rose 7.6 percent in 1978 and 11.3 percent in 1979.

4. See Hoey (1989) for a list of the survey results going back to 1978.

5. See Blanchard and Summers (1986).

6. For the same reason that including investment expenditures represents a form of double counting, so will including durable goods in our measure of lost consumption. Since we do not have a good measure of the flow of services from durable goods, we decided to err on the side of making these costs appear larger.

7. See Bailey (1956) for an early exposition of the welfare costs associated with inflation. A loss remains at zero inflation because, as Friedman (1969) has argued, the optimal rate of inflation is achieved when prices fall at the real rate of interest so that the nominal interest rate is zero. This paper does not attempt to argue that zero, per se, is optimal.

8. This number is in line with those obtained by Fischer (1981). He estimates that a 10 percentage point decline in inflation would produce benefits of 0.3 percent of GNP. With our formula, we also obtain savings approximately equal to 0.3 percent per year. Fischer's estimates were obtained assuming that base demand is of the constant elasticity form, \( \ln(m) = a + \ln(y) - b \ln(i) \).


10. Marty (1976), using a constant elasticity form of base demand, estimated that the additional deadweight loss per dollar of revenue gained equals \( a \ln(i) / (1 - a) \). Using Fischer's (1981) estimate (see footnote 1) that the elasticity of money demand is approximately 0.25, at zero inflation society would also lose 25 cents per extra dollar of revenue gained. See also Barro and Fischer (1976) and McCallum (1989) for a discussion of this issue.

11. See Avery et al. (1987) for information about the distribution of currency among alternative users.
References


_____, "Bracket Creep in the Age of Indexing: Have We Solved the Problem?" Federal Reserve Bank of Cleveland, Working Paper 9108, June 1991b.


Table 1. The Resource Costs of Eliminating a 4 Percent Inflation
("transition" costs as a percent of GNP)

<table>
<thead>
<tr>
<th>Real Interest Rate</th>
<th>Trend Growth in Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r)</td>
<td>2.9%</td>
</tr>
<tr>
<td>.04</td>
<td>14.33%</td>
</tr>
<tr>
<td>.035</td>
<td>14.48%</td>
</tr>
<tr>
<td>.03</td>
<td>14.63%</td>
</tr>
</tbody>
</table>

Consumption just returns to the 2.9% trend in 1985 and then falls below the trend in 1986 and 1987. We assume the cost of the disinflation is over in 1985. Consumption returns to the 2.5% trend in 1984.

Source: Authors' calculations.
Table 2. The Welfare Gain from Eliminating a 4 Percent Inflation (*shoe-leather* gains as a percent of GNP)

<table>
<thead>
<tr>
<th>Real Interest Rate Minus Real Growth Rate (r - g)</th>
<th>Semi-Interest Elasticity of Money Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>.04 - .027</td>
<td>3.1%</td>
</tr>
<tr>
<td>.035 - .027</td>
<td>4.6%</td>
</tr>
<tr>
<td>.03 - .027</td>
<td>11.2%</td>
</tr>
</tbody>
</table>

Source: Authors' calculations.
FIGURE 1 PERSONAL CONSUMPTION EXPENDITURES (1982 dollars)

FIGURE 2  THE WELFARE COSTS OF INFLATION

Source: Authors.
Appendix 1

From equation (1), individuals choose the number of trips to the bank, n, such that

\[ \frac{\partial C(n,Y)}{\partial n} = \frac{iY}{2n^2}. \]  

(Al.1)

Solving the above equation for i and substituting m for Y/2n yields a general form of the Baumol money demand function. The benefit of going to zero inflation is the area under the money demand curve from \( m^*(i=r+\pi) \) to \( m^*(i=r) \):

\[ \frac{m^*(i=r)}{m^*(i=r+\pi)} \int \frac{n \ \partial C(n,Y)}{m} \ dm. \]  

(Al.2)

With a simple change of variables, this equals

\[ \frac{n^*(i=r)}{n^*(i=r+\pi)} \int \frac{-\partial C(n,Y)}{\partial n} \ dn = C[n^*(i=r+\pi)] - C[n^*(i=r)]. \]  

(Al.3)

which is simply the savings in shoe-leather costs associated with going from an inflation rate of \( \pi \) to zero.
From equation (3), the benefit of going to zero inflation is

\[
\frac{\partial C(n,Y)}{\partial n} = \frac{iY}{2n^2}.
\]  

(A2.1)

Integrating the above expression and using equation (3) yields the following welfare gain:

\[
(r+\frac{1}{B})m^*(i=r)-(r+\frac{1}{B}+\pi)m^*(i+r+\pi).
\]

(A2.2)

Rearranging terms, we get

\[
\frac{1+\beta r}{B}\Delta m - \pi m^*(i+r+\pi).
\]

(A2.3)

Since the semi-elasticity of money demand with respect to the interest rate is constant,

\[
\frac{\Delta m}{m^*(i=r+\pi)+\frac{\Delta m}{2}} \approx \beta \pi
\]

(A2.4)

or

\[
\Delta m = \frac{2\beta \pi}{2-\beta \pi} m^*(i=r+\pi).
\]

(A2.5)

Substituting from equation (A2.5) into expression (A2.3) and defining \( V = Y/m \) yields equation (5) in the text (\( Y \) enters because \( b \) is stated as a fraction of GNP).