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ON FLEXIBILITY, CAPITAL STRUCTURE, AND INVESTMENT DECISIONS FOR THE INSURED BANK

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Abstract

Most models of deposit insurance assume that the volatility of a bank's assets is exogenously provided. Although this framework allows the impact of volatility on bankruptcy costs and deposit insurance subsidies to be explored, it is static and does not incorporate the fact that equityholders can respond to market events by adjusting previous investment and leverage decisions. This paper presents a dynamic model of a bank that allows for such behavior. The flexibility of being able to respond dynamically to market information has value to equityholders. The impact and value of this flexibility option are explored under a regime in which flat-rate deposit insurance is provided.

I. Introduction

Almost all models of deposit insurance take the underlying source of risk, namely, the volatility of the bank's assets, to be exogenously provided.[1] Within this framework, the relative merits of the firm increasing its volatility and leverage can be easily explored. The disadvantage of this approach is that it is static and does not recognize the fact that equityholders can respond to market events by dynamically adjusting previous investment and leverage decisions. Such dynamic behavior can lead to changing levels of portfolio risk over time, with commensurate effects on the value of deposit insurance. This is the classic moral hazard problem.[2]

The objective of this paper is to establish a model that identifies how equityholders select a capital structure and investment policy under a flat-rate deposit insurance regime. The model we consider is dynamic and explicitly incorporates the *flexibility option* that allows shareholders to adapt their asset portfolio decisions to market events.[3] We investigate how this flexibility option affects portfolio decisions and risk-taking. Our findings show that with no opportunities to revise portfolio decisions, optimal bank financing and investment policies are bang-bang; that is, shareholders will either fully protect the charter value or fully exploit the insurance subsidy granted by the insurer. A special case of our one-period model reduces to the model developed by

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[1] The literature on deposit insurance using an option pricing framework was pioneered by Merton [1977]. For a review of the literature, see Flood [1990].

[2] The moral hazard problem has been well discussed by Kane [1985]. Fixed-rate deposit insurance gives bank owners strong incentives to increase risk. Kane illustrates that the incentive scheme can become so socially perverse that projects with a negative net present value may be optimally selected.

[3] The term "flexibility option" is derived from the asset option pricing literature and has been discussed by Brennan and Schwartz [1985], McDonald and Siegel [1985, 1986], Kester [1984], and Triantis and Hodder [1990].

Marcus [1984]. However, unlike his model, ours allows equityholders to select risks dynamically and therefore allows moral hazard to be incorporated. With a finite number of portfolio rebalance points remaining before an audit, bang-bang policies may no longer be optimal and interior solutions may exist. Finally, we investigate how the flexibility option granted to equityholders affects the value of deposit insurance. We show that ignoring the flexibility option leads to understating the value of deposit insurance. In particular, as the number of portfolio revisions allowed prior to an audit date increases, a bank's ability to exploit the insured-deposit base increases. This can only be to the detriment of the flat-rate deposit insuree.

This paper is organized as follows. Section II develops a one-period model of a banking firm in which the equityholders optimally select their capital structure and their investment policy over the time remaining before an audit. In this case, the firm invests either all or none of its wealth in risky assets. No interior solutions are preferable. Moreover, under certain assumptions, we show that the equityholders' interests are best served by supplying the minimum amount of capital. Section III extends the analysis to the two-period case and shows that interior solutions may be optimal. Section IV considers the case in which multiple portfolio-revision periods remain prior to the audit. Numerical illustrations are provided to highlight the fact that the value of deposit insurance increases with the number of portfolio-revision opportunities. Section V discusses policy implications and concludes the paper.

II. A One-Period Model of a Banking Firm

Consider an insured bank with one period remaining until an audit by the insuring agency. At the initial time, t=0, the deposit base is $1-\alpha$ and the capital supplied by the shareholders is α. Deposits are fully insured by the agency, which levies a fixed-rate premium per dollar deposited. Let P(t) be the value of this deposit insurance net of the premium. P(t) can be viewed as government-contributed capital. Since the deposits are insured, their value at the end of the period is $(1-\alpha)e^{r^*T}$, where r^* is the rate of return on the deposits. For simplicity, we assume that deposit inflows and outflows are equal over this period.

Depositors, unlike the bank, may be faced with high transaction costs and may be unable to hold the riskless asset directly. Moreover, bank deposits may have unique characteristics, such as convenience yields, that make them less-than-perfect substitutes for riskless assets. In either case, barriers to entry, such as the need for a government license or charter, allow banks to raise deposits at rates below the risk-free rate, r. This positive spread produces an intangible asset, or charter value, in the form of future monopoly rents. If the charter obtains its value solely from monopolistic rents attributable to the interest-rate spread, and if this spread remains constant or grows over time, then the charter value equals the deposit base, $D(0) = 1-\alpha$. In general, however, due to deregulation or increased competition from other financial intermediaries, monopolistic rents are likely to erode over time. Usually, the rents are taken to be some function of the deposit base at time t. For example, Marcus [1984] assumes that the charter value is a fraction of the deposit base. Let C(0) represent the present value of this charter. If the bank fails the audit, it loses its charter. Thus, at time 0, the bank holds a call option on the charter. Let G(0) be the value of this claim. In what follows, we assume that the liability grows at the risk-free rate; that is, $r^* = r$, with the capitalized value of the deposit spread reflected in the charter value.

We assume that the bank invests 1-q in riskless discount bonds and q in risky securities. Assuming no dividends, the risky portfolio follows a diffusion process of the form

$$dS/S = \mu dt + \sigma dz$$

where μ and σ are the instantaneous mean and volatility, respectively, and dz is the Wiener increment.

The bank's balance sheet at time 0 can be summarized as follows:

Assets		Liabilities and Net Worth	
Tangible Assets			
Riskless Asset	$1-q$	Deposits	$D(0)=1-\alpha$
Risky Asset	q	Shareholder-contributed Capital	α
Intangible Assets		Government-contributed Capital	$P(0)$
Government Subsidy	$P(0)$	Charter Value	$G(0)$
Charter Value	$G(0)$	Shareholder Equity	$E(0)$
Total = $1 + P(0) + G(0)$		Total = $1 - \alpha + E(0)$	

Clearly, $E(0) = \alpha + P(0) + G(0)$.

The initial value of the bank's tangible assets is $V(0) = 1$. Given q, the value of these assets follows the process

$$dV/V = (1 - q)rdt + qdS/S$$

Conditional on the capital structure decision, α, and the investment decision, q, the value of the tangible assets of the firm at time T is

$$V(T) = V(\alpha,q;T) = qe^{xT} + (1-q)e^{rT}$$

where x is a normal random variable with mean $\mu - \sigma^2/2$ and variance σ^2.

At the audit date, T, the deposit base is $D(T) = (1-\alpha)e^{rT}$. If the liquidation value of the marketable assets, $V(T)$, is less than the deposit base, then the bank is declared insolvent and the shareholders receive nothing. If, however, the bank is declared solvent, the equityholders receive a claim worth $V(T) - D(T) + G(T)$. Let $E(T)$ be the shareholders' equity at time T. Then, we have

$$E(T) = \begin{cases} V(T) - D(T) + G(T) & \text{if } V(T) > D(T) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Using standard option pricing methods, shareholder equity at time 0 is given by

$$E(0) \equiv E(\alpha,q;0) = \alpha + G(\alpha,q;0) + P(\alpha,q;0)$$

where

$$G(\alpha,q;0) = \begin{cases} C(0)N(d_2) & \text{if } q \geq \alpha \\ C(0) & \text{if } q < \alpha \end{cases} \tag{2}$$

$$P(\alpha,q;0) = \begin{cases} (q-\alpha)N(-d_2) - qN(-d_1) & \text{if } q \geq \alpha \\ 0 & \text{if } q < \alpha \end{cases} \tag{3}$$

$$d_1 = \frac{\ln[q/(q-\alpha)] + \sigma^2 T/2}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Shareholders will raise capital provided the marginal benefit of each incremental dollar raised is positive. Since we assume all financial assets are fairly priced, the tangible-asset portfolio has zero net present value, and the shareholders' objective is reduced to maximizing $Z(\alpha,q)$, where

$$\begin{aligned} Z(\alpha,q) &= E(\alpha,q;0) - \alpha \\ &= G(\alpha,q;0) + P(\alpha,q;0) \end{aligned} \tag{4}$$

Equation (4) clearly illustrates the trade-off faced by the shareholders. Specifically, in selecting the optimal capital and investment decisions, the shareholders trade off the value of the call option on the charter (which is maximized by reducing default risk) and the value of the put option (which is maximized by increasing default risk). Substituting for $G(\alpha,q;0)$ and $P(\alpha,q;0)$, we obtain

$$Z(\alpha,q) = \begin{cases} qN(d_1) - [q-\alpha-C(0)]N(d_2) - \alpha & \text{if } q \geq \alpha \\ C(0) & \text{if } q < \alpha \end{cases} \tag{5}$$

Let

$$Z(\alpha^*,q^*) = \underset{\substack{0\le\alpha\le1 \\ 0\le q\le1}}{\text{Max}} \{Z(\alpha,q)\} \tag{6}$$

Given that the insurer charges a flat-rate insurance premium independent of the portfolio composition, the equityholders' objective is to select the investment and capital parameters, q and α, such that $Z(\alpha,q)$ is maximized.

The Investment Decision

To investigate the optimal controls, first fix α and note that

$$\frac{\partial Z}{\partial q} = \begin{cases} [N(d_1) - N(d_2)] - \dfrac{\alpha C(0)}{(q-\alpha)^2} & \text{for } q \ge \alpha \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

If α were negative, then $\frac{\partial Z}{\partial q} > 0$ and hence $q^* = 1$. Insolvent banks are driven to extreme risk. This strategy is optimal because shareholders receive nothing unless the audit is passed. Indeed, for this case the firm may even select projects with a negative net present value to an all-equity firm, provided their volatilities are sufficiently large.

For $\alpha > 0$, the sign of $\frac{\partial Z}{\partial q}$ is indeterminate. By taking the second derivative of equation (7) for $q \ge \alpha$, we obtain

$$\frac{\partial^2 Z}{\partial q^2} = \frac{N(d_1)\alpha^2}{(q-\alpha)^2 q\sigma\sqrt{T}} + \frac{2C(0)\alpha}{(q-\alpha)^2} > 0 \tag{8}$$

Then, the function $Z(\alpha,q)$ is convex in q over the interval $[\alpha,1]$. Figure 1 illustrates possible functions for any given α.

Given that the function is flat in q over the interval $[0,\alpha]$, the optimal investment in risky assets, q^*, is either in that interval or at unity, depending on the value of α. Specifically,

$$Z(\alpha,q^*) = \text{Max } \{Z(\alpha,0),\ Z(\alpha,1)\}$$

$$= \begin{cases} N(d_1^*) - [1-\alpha-C(0)]N(d_2^*) - \alpha & \text{if } \alpha \le \alpha_B^* \\ G(0) & \text{if } \alpha > \alpha_B^* \end{cases} \tag{9}$$

where

$$d_1^* = [-\ln(1-\alpha) + \sigma^2 T/2] \;/\; (\sigma\sqrt{T})$$

$$d_2^* = d_1^* - \sigma\sqrt{T}$$

and α_B^* is that value of α chosen such that

$$\frac{N(-d_1^*)}{N(-d_2^*)} = 1 - \alpha - C(0) \tag{10}$$

We conclude that for any capital structure decision, the optimal investment decision is either $q = 0$ or $q = 1$.[4] Firms with capital lower than α_B^* will shift their portfolio out of the risk-free asset into the risky investment. Firms with capital greater than α_B^* will protect their charter value by increasing their risk-free holding and decreasing their investment in the risky portfolio.

As an example, assume the charter value is some fraction, f, of the deposit base. Then

$$C(0) = f(1-\alpha)$$

Figure 2 traces out the break-even point for given values of f and σ. Note that as σ increases, banks take on riskier positions. Therefore, for higher levels of asset risk, the range of capital structures and charter values over which the bank will risk its charter is larger. The graph highlights the fact that investment decisions depend critically on financing decisions in our model.

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[4]Actually, the optimal investment decision, q, is either anywhere in the interval $[0,\alpha]$ or 1. Since equityholders are indifferent between investments in the range $[0,\alpha]$, we restrict attention to 0. It is worth noting that if the risky investment is a positive net present value project, then the optimal investment, q^*, will be either at α or at unity, depending on which offers the greater value.

The Financing/Capital Decision

We now turn to the financing decision. From the above analysis, we have

$$Z(\alpha^*,q^*) = \text{Max}\ \{ \underset{0\leq\alpha\leq 1}{\text{Max}}\ \{Z(\alpha,0)\},\ \underset{0\leq\alpha\leq 1}{\text{Max}}\ \{Z(\alpha,1)\}\} \tag{11}$$

with

$$Z(\alpha,0) = C(0)$$
$$Z(\alpha,1) = N(d_1^*) - [1-\alpha-C(0)]N(d_2^*) - \alpha$$

Assume the charter value is some fraction f of the deposit base. Then

$$\begin{aligned} Z(\alpha,0) &= (1-\alpha)f \\ Z(\alpha,1) &= N(d_1^*) - (1-\alpha)(1-f)N(d_2^*) - \alpha \end{aligned} \tag{12}$$

For small charter value f, i.e., when $1-f \geq N(-\frac{\sigma}{2})/N(\frac{\sigma}{2})$, the $Z(\alpha,0)$ curve is uniformly higher than $Z(\alpha,1)$. The optimal capital structure should be $\alpha = 0$ with $q = 0$. On the other hand, when $1-f \leq N(-\frac{\sigma}{2})/N(\frac{\sigma}{2})$, the curves $E(\alpha,1)$ and $E(\alpha,0)$ have a unique intersection point for $0 \leq \alpha \leq 1$. Before the intersection, $Z(\alpha,1)$ is convex, decreasing, and above $Z(\alpha,0)$. Therefore, the optimal capital structure is again $\alpha = 0$ with $q = 1$, and the optimal financing decision is for equityholders to provide the minimal amount of capital; that is,

$$Z(\alpha^*,q^*) = \underset{\alpha,q}{\text{Max}}\ Z(\alpha,q) = \text{Max}\ \{Z(0,1),\ Z(0,0)\}$$

III. Extension to the Two-Period Case

We have seen that with no opportunities to revise portfolios, the optimal portfolio decision is always bang-bang. If a portfolio-revision opportunity exists prior to the audit date, then the optimal solution may not be bang-bang. This is illustrated below.

Let the current values of the bank's deposits and assets be 99 and 100, respectively, and let f equal 6 percent. For simplicity, assume that the risk-free rate and the deposit rate of return are both zero. Furthermore, assume that the risky-asset returns are either 20 percent or -20 percent in the next two periods. The probability of an up move in each period is 0.5. Finally, assume that the bank can revise its portfolio at the beginning of each period and that the audit is at the end of the second period.

TABLE 1: Comparison of Bang-Bang Strategies with an Interior Strategy

STRATEGY IN PERIOD 1	OPTIMAL STRATEGY IN PERIOD 2	EQUITY VALUE E_0
0	1 in upstate 1 in downstate	13.47
1	0 in upstate 0 in downstate	13.47
7/8	0 in upstate 1 in downstate	13.705

Table 1 shows the equity values associated with a few decisions in period 1, followed by optimal decisions in period 2. From our previous analysis, the optimal policy for period 2 is bang-bang. It is apparent that given an initial strategy $q_0 = 0$ (or $q_0 = 1$), the ability to switch decisions in the next period is valuable. Note that the values of the equity for the strategies $q_0 = 1$ and $q_0 = 0$, followed by optimal decisions in the next period, happen to be the same (13.47). However, the strategy $q_0^* = 7/8$, followed by optimal decisions in the next period, leads to a higher equity value of 13.705.

We now extend our model to two periods, where the time to an audit is t_2 and where portfolio-revision opportunities exist at times t_0 and

t_1, respectively. Let $V_j()$, $E_j()$, $D_j()$, and $C_j()$ be the portfolio value, shareholder equity, deposit level, and present value of the charter at times t_j, $j = 0,1,2$. Finally, let q_0 and q_1 be the fraction of funds invested in the risky portfolio at times t_0 and t_1.

When the risky portfolio follows a geometric Wiener process, then the value of the equity with one period to go, $E_1(V_1)$, is given by

$$E_1(V_1) = \begin{cases} V_1N(d^*_{11}) - (D_1 - G_1)N(d^*_{12}) & \text{for } V_1 \le V^*_1 \\ V_1 - D_1 + C_1 & \text{for } V_1 > V^*_1 \end{cases} \tag{13}$$

where

$$d^*_{11} = [\ell n(V_1/D_1) + \sigma^2(t_2-t_1)/2] \,/\, \sigma\sqrt{t_2-t_1}$$

$$d^*_{12} = d^*_{11} - \sigma\sqrt{t_2-t_1}$$

and V^*_1 satisfies the condition

$$V^*_1 = [D_1 - C_1]N(-d^*_{12})/N(-d^*_{11}).$$

The value of V_1, of course, depends on the initial decision q_0; that is,

$$V_1 = V_1(q_0) = q_0e^{x\tau} + (1 - q_0)e^{r\tau}$$

where $\tau = t_1-t_0$. Given an initial capital structure, α, and a portfolio decision, q_0, the initial equity value, $E_0(q_0|\alpha)$, is given by

$$\begin{aligned} E_0(q_0|\alpha) &= e^{-r\tau}\,\mathcal{E}_0[E_1(V_1)] \\ &= e^{-r\tau}\mathcal{E}_0[E_1(V_1)|V_1 \le V^*_1]P\{V_1 \le V^*_1\} + e^{-r\tau}[V_1 - D_1 + C_1]P\{V_1 \ge V^*_1\} \end{aligned}$$

where $\mathcal{E}_0$ is the expectation operator taken over the risk-neutralized process, $dS/S = rdt + \sigma dz$. The optimal q, q^*_0, is

$$E_0(q^*_0|\alpha) = \max_{0 \le q_0 \le 1} \{E_0(q_0)\}$$

Numerical methods are used to solve this optimization problem. Assuming capital structure decisions are made only at the initial period, the initial joint capital structure and investment problem is given by

$$Z(q^*,\alpha^*) = \underset{0\le\alpha\le 1}{\text{Max}} \{ \underset{0\le q\le 1}{\text{Max}} \{E_0(q_0|\alpha)\} \}$$

For the more general n-period problem, numerical procedures based on backward dynamic programming can be used to obtain the optimal value of the equity and the optimal control policy $q(\cdot)$.

IV. Numerical Results

In this section, we illustrate how the asset flexibility option affects the behavior of the banking firm under flat-rate deposit insurance. Consider a bank with deposits equal to $(1-\alpha)$ and a charter value equal to $f(1-\alpha)$. Assume the riskless rate, r, is 10 percent. Figure 3 depicts the net present value of the bank as a function of α for the cases where zero, two, and four revision opportunities are allowed before the audit date. The curved segment of the function corresponds to the range of α values where the bank optimally places the charter at risk. Conversely, the linear segment of the function corresponds to the range of α values where the bank's optimal portfolio decision is to set $q < \alpha$ to ensure that the charter value is captured.

Figure 3 illustrates how the number of portfolio-revision opportunities affects the net present value (NPV). Over the range of α where the NPV function curves, the charter is placed at risk. As n increases, two events occur. First, the range of α values over which the charter is placed at risk expands. Second, for any given α in this range, the NPV increases. The difference between the NPV curves with $n > 0$ and $n = 0$ represents the value of the flexibility option. The increase in the NPV of equity, due to the flexibility option, is obtained partly at the expense of the deposit insurer. Indeed, the fair value of deposit insurance increases with the number of portfolio revisions. As a result, empirical estimates that ignore the value of flexibility understate the true value of deposit insurance.

V. Conclusion

Optimal equityholder decisions involve trade-offs between risk-minimizing strategies, which reduce the likelihood of losing the charter, and risk-maximizing strategies, which exploit the insurance on the deposit base. Without the ability to respond dynamically to market information, optimal financing and investment policies are bang-bang; that is, the bank will select extreme positions.

Given any flat-rate insurance scheme, incentives will exist for firms to revise their portfolios dynamically in response to market information. These dynamic revisions are aimed at exploiting the insured-deposit base more fully, while mitigating the likelihood of bankruptcy. The additional value captured by equityholders responding dynamically to jointly maximize the charter value and deposit insurance subsidy, beyond the static value, is captured in the value of the asset flexibility option.

In the presence of the asset flexibility option, portfolio decisions may not be bang-bang and interior solutions may be optimal. The likelihood of an interior solution may increase as the number of portfolio-revision opportunities expands. Moreover, the value of the insured-deposit base, provided at a flat rate, increases with the number of portfolio-revision opportunities.

Our results suggest that the value of the deposit insurance may be significantly underestimated by static models because such models completely ignore the flexibility option. The findings also suggest that bank regulators should factor the flexibility option into any risk-adjusted capital guidelines, and also into closure policies.

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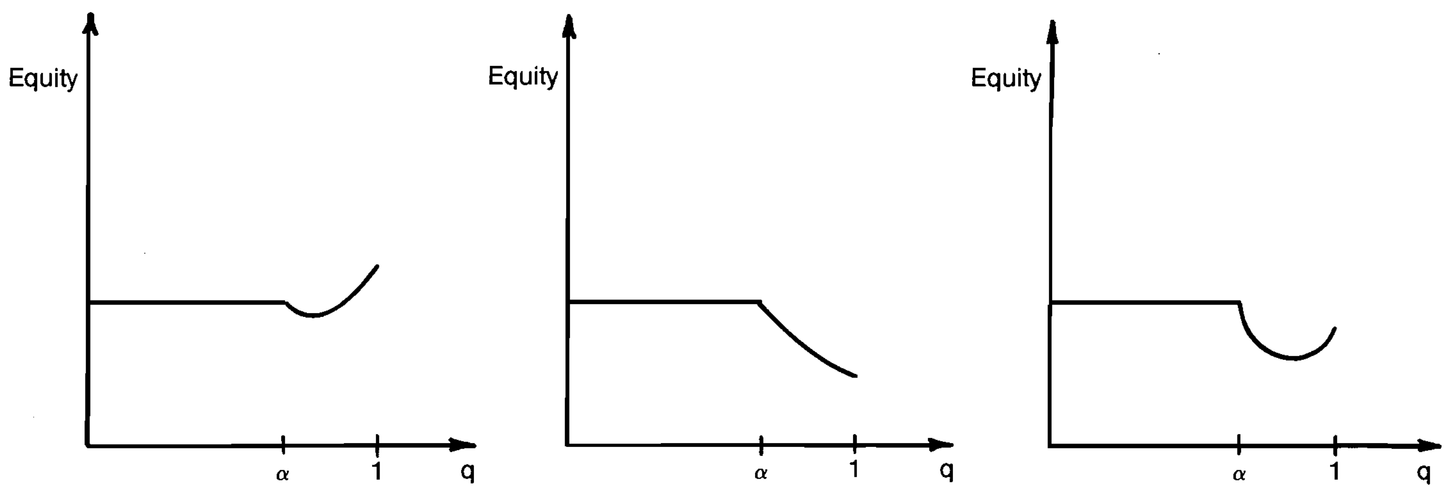

Figure 1. The value of equity as a function of the risky-asset portfolio weight, q. There are three possible equity functions. The first panel shows the case where the optimal q equals one. The second and third panels show the cases where the investor is indifferent between values of q in the interval $[0,\alpha]$.

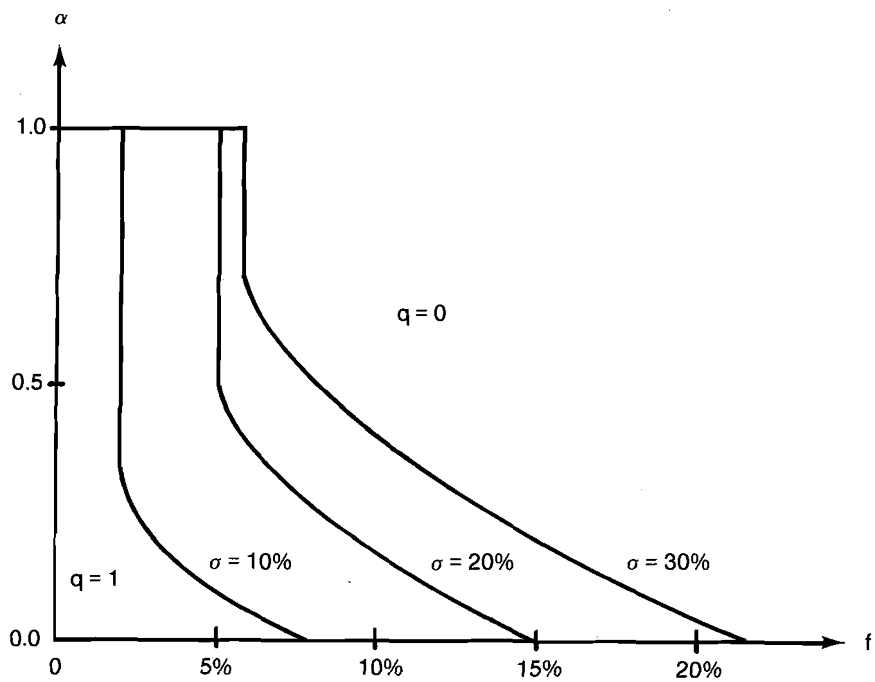

Figure 2. The break-even value of α as a function of the charter value, f, and asset volatility, σ. For a given σ, the values of α for which the bank is indifferent between setting $q = 0$ and $q = 1$ is a decreasing function of f. The range of (α, f) combinations over which it becomes optimal to risk the charter increases with σ.

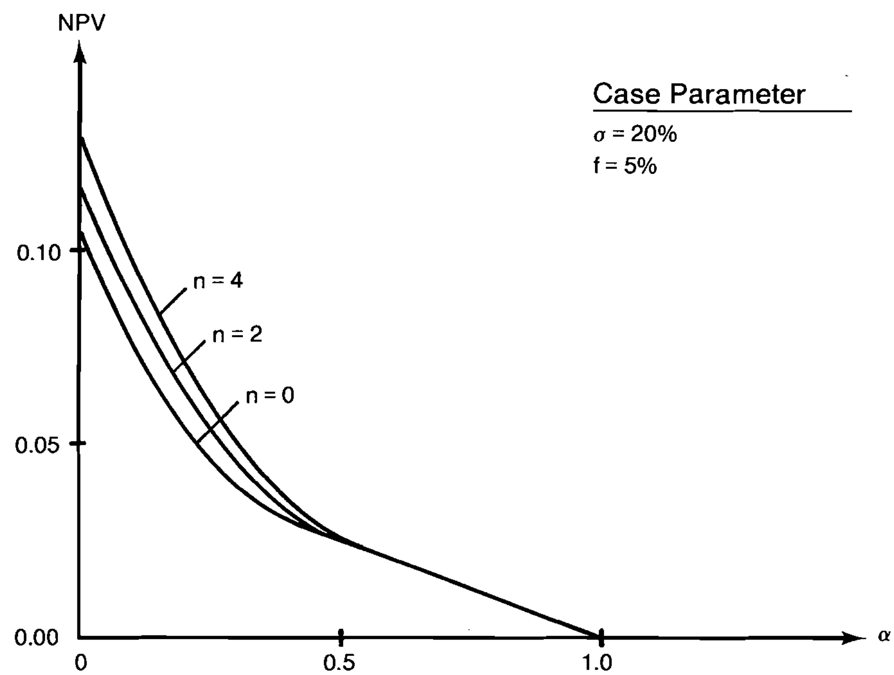

Figure 3. The impact of flexibility on the net present value (NPV) of equity. The NPV of equity is a decreasing function of initial shareholder-contributed capital, α. It is an increasing function of the number of revision opportunities for values of α where deposit insurance has value.