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ALTRUISM, BORROWING CONSTRAINTS, AND SOCIAL SECURITY

by David Altig and Steve J. Davis

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Abstract

We show how intergenerational altruism and borrowing constraints shape the interest rate, savings, and welfare response to funded and unfunded social security programs. Borrowing constraints pin down the optimal timing of altruistic intergenerational transfers and thereby alter the implications of intergenerational altruism for fiscal policy. Regardless of whether parent-to-child altruistic transfer motives operate, borrowing constraints imply effects of social security programs that deviate greatly from the effects in Ricardian and traditional life-cycle environments. If, however, child-to-parent altruistic gift motives operate in at least some families, social security programs are neutral in their impact on the interest rate, though not necessarily in their impact on consumption. This interest-rate neutrality result holds regardless of whether borrowing constraints bind, regardless of whether parent-to-child transfers operate, and regardless of whether exchange motives for intergenerational transfer are important.

1. Introduction

The implications of mandatory social security programs for the interest rate, for aggregate capital accumulation, and for economic welfare hinge critically on the nature and extent of intergenerational linkages and capital market imperfections. In this paper we develop the implications of social security when capital market imperfections take the form of an inability to borrow against future wage (or social security) income and altruism motivates intergenerational linkages. Within an overlapping generations framework populated by three-period-lived persons, we characterize the dynamic and steady-state response to funded and unfunded social security interventions. We consider the implications of binding borrowing constraints, parent-to-child altruistic transfer motives, and child-to-parent altruistic gift motives. A central theme of our analysis is that borrowing constraints and intergenerational linkages jointly determine the response to social security programs.

The interaction between borrowing constraints and intergenerational altruism is also a central theme in the analysis of government debt by Altig and Davis (1989a) and in the analysis of wealth accumulation and intergenerational transfer patterns by Laitner (1989). Aside from its focus on social security, this paper differs from our earlier work in three respects. First, we identify all equilibrium configurations of intertemporal and intergenerational linkages that can arise in our overlapping-generations framework. We find six equilibrium configurations, one of which corresponds to the standard life-cycle model with perfect capital markets, and one of which corresponds to Barro's dynastic model. Second, we analytically characterize the dynamic and steady-state effects of social security interventions on the capital stock when borrowing constraints bind. Our earlier work relied entirely on numerical simulations to characterize the capital stock response to nonneutral government debt policies.

Third, we prove that an operative child-to-parent gift motive (pre- and post-intervention) implies neutrality of the steady-state interest rate with respect to all lump-sum government interventions, including all social security interventions. This interest-rate neutrality result holds regardless of whether borrowing constraints bind and regardless of whether the young and middle-aged are connected by altruistic linkages. It also survives the introduction of non-altruistic agents into the economy, provided that the gift motive continues to operate for the altruists. It follows that, unlike neutrality results in the Barro-Becker-

Bernheim/Bagwell tradition, our interest-rate neutrality argument does not rely on direct or indirect altruistic linkages between persons who are taxed and/or subsidized in the government intervention. Our analysis of social security complements work on the interaction between imperfect annuity markets and intergenerational linkages by Kotlikoff and Spivak (1981), Sheshinski and Weiss (1981), Eckstein, Eichenbaum, and Peled (1985), Abel (1985,1986), and Kotlikoff, Shoven, and Spivak (1987). We show that borrowing constraints significantly alter the aggregate savings response to unfunded social security programs relative to the response in traditional life-cycle models like Feldstein (1974), Kotlikoff (1979), and Auerbach and Kotlikoff (1987) and relative to the response in models with intergenerational altruism and perfect capital markets like Barro (1974). We also show that Hubbard and Judd's (1987) argument for shifting the generational incidence of social security payroll taxes away from younger workers is greatly weakened by the introduction of a small degree of intergenerational altruism.

Underlying much of our analysis is a simple proposition regarding the interaction between borrowing constraints and intergenerational altruism: borrowing constraints pin down the optimal timing of altruistically motivated intergenerational transfers. Specifically, if children are borrowing-constrained when young and parents make positive transfers, parents make all transfers early in the life cycle. This timing proposition carries important implications for fiscal policy in economies with altruistic agents.

The determinate timing of intergenerational transfers implies that parents need not be connected to their children through operative linkages over the entire life cycle. Parents' marginal utility of consumption when old can exceed the discounted marginal utility of childrens' consumption when middle-aged – parents would choose to transfer resources from their children (and grandchildren) to themselves if a transfer mechanism was available. Unfunded social security provides such a transfer mechanism. Thus, unfunded social security interventions are nonneutral when borrowing constraints bind, despite altruistically motivated transfers from parents to children early in the life cycle. Of course, the borrowing constraints that drive the timing result also break the intertemporal (capital market) link between young and old persons. Hence, funded social security interventions that impinge on the budget constraints of the young are also nonneutral.

Our results are usefully juxtaposed against well-known results in the literature. As stressed by Feldstein (1974), an unfunded social security program depresses aggregate savings and the capital stock in a pure life-cycle environment characterized by perfect capital markets and an absence of intergenerational transfers. Barro (1974) shows that, when capital markets are perfect, the existence of altruistically motivated intergenerational transfers implies the complete neutrality of an unfunded social security program. (Other motives for intergenerational transfers carry profoundly different implications for the aggregate savings response to unfunded social security programs; see, for example, Cox (1987) and Bernheim, Schliefer, and Summers (1985).) We show that the introduction of binding borrowing constraints leads to quantitatively significant departures from the Ricardian benchmark, even when parents make altruistically motivated transfers to children. Indeed, the capital stock decline caused by an unfunded social security program is often larger in an environment with altruistic agents and borrowing constraints than in environments with (a) non-altruistic agents and perfect capital markets or (b) non-altruistic agents and borrowing constraints.

2. The Overlapping-Generations Framework

A. A Perfect Capital-Markets Economy and a No-Loan Economy

We describe an overlapping generations framework with three-period-lived persons and no government, postponing the discussion of fiscal policy variables to section 4. Within this framework we consider an economy with perfect capital markets and an economy with no consumption-loans market. Each person in these economies inelastically supplies homogeneous labor services according to a lifetime productivity profile, $(\alpha_1, \alpha_2, \alpha_3)$. Parents choose the timing and magnitude of altruistically-motivated transfers to children. (We defer consideration of child-to-parent gift motives to section 6.) Output is produced from capital and labor inputs according to a neoclassical production function.

We assume that an individual's productivity profile is hump-shaped, so that $\alpha_2 > \alpha_1$ and $\alpha_2 > \alpha_3$. We have shown elsewhere (Altig and Davis, [1989a]) that a life-cycle income profile that slopes up over the first two periods of life greatly reduces the degree of altruism necessary to generate transfers from parents to children. To make our discussion of borrowing restrictions nontrivial, we further assume that α_2 is sufficiently greater than α_1 so that the consumption-loans market influences the equilibrium capital stock and consumption profile. In other words, we focus on parameter configurations in which the equilibrium capital stock and consumption profile differ between the loan and no-loan economies.

In the consumption-loans economy with no government, a representative member of generation t chooses $(C_{1t}, C_{2t}, C_{3t}, x_{1t}, x_{2t}, b_{1,t+1}, b_{2,t+1}, b_{3,t+1})$ to maximize:

$$U_t = \sum_{i=1}^{3} \beta^{i-1} u(C_{it}) + \beta\gamma U^*_{t+1} \tag{1}$$

subject to:

$$C_{1t} + x_{1t} = \alpha_1 W_t + b_{1t}, \tag{2}$$

$$C_{2t} + (1+n)b_{1,t+1} + x_{2t} = (1+r_{t+1})x_{1t} + \alpha_2 W_{t+1} + b_{2t}, \tag{3}$$

$$C_{3t} + (1+n)(b_{2,t+1} + b_{3,t+1}) = (1+r_{t+2})x_{2t} + (1+r_{t+2})b_{3t} + \alpha_3 W_{t+2}, \tag{4}$$

$$C_{1t}, C_{2t}, C_{3t}, b_{1,t+1}, b_{2,t+1}, b_{3,t+1} \geq 0, \tag{5}$$

where:

C_{1t} = consumption by generation t when young,

C_{2t} = consumption by generation t when middle-aged,

C_{3t} = consumption by generation t when old,

x_{1t} = capital purchases (i.e., savings) by generation t when young,

x_{2t} = capital purchases by generation t when middle-aged,

$b_{i,t+1}$ = transfer made by a generation-t parent to each $(1+n)$ offspring in the children's i^{th} period of life (an inter vivos transfer for $i = 1, 2$, a bequest for $i = 3$),

β = intertemporal discount factor, $0 < \beta < 1$,

γ = interpersonal discount factor, $0 < \gamma \leq (1+n)/\beta$, which insures a positive steady-state interest rate when the transfer motive operates in the loans economy,

$u(\cdot)$ = period utility function, satisfying $u'(\cdot) > 0$, $u''(\cdot) < 0$, $\lim_{C\to 0} u'(C) = \infty$, and $\lim_{C\to\infty} u'(C) = 0$,

U^*_{t+1} = maximum utility attainable by a generation $t+1$ agent as a function of the transfer received,

$n =$ the population growth rate,

$W_t =$ the period-t wage in units of the good, and

$r_{t+1} =$ the one-period rate of return on physical capital (or consumption loans) held from t to $t + 1$.

The absence of nonnegativity constraints on savings by the young and middle-aged reflects the availability of a costless consumption-loans market.

In the no-loan economy a representative consumer of generation t maximizes (1) subject to (2) thru (5) and

$$x_{1t}, x_{2t} \geq 0. \tag{6}$$

This additional constraint reflects the absence of a viable enforcement mechanism to support the operation of a consumption-loans market.[1] We show below that, assuming the young choose to dissave in the consumption-loans economy, the constraint $x_{1t} \geq 0$ always binds in the corresponding no-loan economy.

Turning to the production side of the two economies, and normalizing so that generation 0 has one member, the aggregate period-t labor supply is

$$L_t = \left[\alpha_1 + \frac{\alpha_2}{1+n} + \frac{\alpha_3}{(1+n)^2}\right](1+n)^t \equiv \alpha(1+n)^t, \tag{7}$$

where α is per capita labor supply. Defining $k = K/L$ as the capital-labor ratio, we write the aggregate production function as

$$Y_t = \alpha(1+n)^t f(k_t), \tag{8}$$

where $f'(\cdot) > 0$, $f''(\cdot) < 0$, $\lim_{k\to 0} f'(k) = \infty$, and $\lim_{k\to\infty} f'(k) = 0$. The representative firm's competitive profit maximization conditions are

$$W_t = f(k_t) - k_t f'(k_t), \text{ and} \tag{9}$$

[1]The constraint (6) has more than one interpretation. First, borrowing constraints can arise from high costs of enforcing loan repayment, due partly to bankruptcy laws and other legal protections afforded to debtors. Second, the asymmetric tax treatment of interest income and interest payments on consumption loans can lead consumers to choose a corner outcome with respect to their borrowing and saving decision (see Altig and Davis [1989b]). Third, and somewhat further removed from our framework, sufficiently severe adverse selection effects can prevent the operation of a consumption-loans market. For empirical evidence on the incidence of binding borrowing constraints, see Zeldes (1989) and references therein.

$$r_t = f'(k_t). \tag{10}$$

The market-clearing conditions complete the specification of the two models. We obtain the goods market-clearing condition:

$$K_{t+1} - K_t + (1+n)^t \left[Clt + \frac{C_{2,t-1}}{} + \frac{C_{3,t-2}}{(1+n)^2} \right] = \alpha(1+n)^t f(k_t)$$

$$\Rightarrow \alpha(1+n)k_{t+1} - \alpha k_t + C_{1t} + \frac{C_{2,t-1}}{1+n} + \frac{C_{3,t-2}}{(1+n)^2} = \alpha f(kt), \tag{11}$$

and the capital market-clearing condition:

$$K_t = (1+n)^t \left[\frac{x_{1,t-1}}{1+n} + \frac{x_{2,t-2}}{(1+n)^2} + \frac{b_{3,t-2}}{(1+n)^2} \right]$$

$$\Rightarrow k_t = \frac{(1+n)x_{1,t-1} + x_{2,t-2} + b_{3,t-2}}{(1+n)^2 \alpha} \tag{12}$$

This completes the description of the loan and no-loan economies with no government. To introduce the government, one need only add the government budget constraint and make appropriate modifications to the consumer budget constraints and the goods market-clearing condition.

B. The Consumer's Optimization Problem

The consumer's intertemporal first-order conditions for own consumption are

$$u'(C_{1t}) \geq \beta(1 + r_{t+1})u'(C_{2t}), \tag{13}$$

$$u'(C_{2t}) \geq \beta(1 + r_{t+2})u'(C_{3t}). \tag{14}$$

Equations *(13)* and *(14)* hold with equality in the loan economy, and in the no-loan economy when equation (6) fails to bind. In these cases, equations (13) and *(14)* represent the familiar condition that the marginal rate of substitution between own current consumption and own future consumption equals the time-discounted gross rate of return to savings.

Using the envelope theorem, the first-order conditions governing intergenerational transfers are

$$u'(C_{it}) \geq \frac{7}{l+n} u'(C_{i-1,t+1}) \quad i = 2,3 \tag{15}$$

for inter vivos transfers and

$$u'(C_{3t}) \geq \frac{\gamma\beta}{1+n}(1 + r_{t+2})u'(C_{3,t+1}) \tag{16}$$

for bequests. Equations (15) and (16) state that when a transfer motive is operative, the discounted marginal rate of substitution of the parent's consumption for children's consumption equals the population -deflated interpersonal discount factor.

C. Equilibrium

An equilibrium in the consumption-loans economy is a sequence $\{C_{1t}, C_{2,t-1}, C_{3,t-2}, x_{1t}, x_{2,t-1}, b_{1t}, b_{2,t-1}, b_{3,t-2}, W_t, r_{t+1}, k_t, Y_t\}_{t=0}^{\infty}$ that satisfies equations (1)-(5) and (7)-(16) for all t, given the initial condition $(x_{1,-1}, x_{2,-2}, k_0)$. Similarly, an equilibrium in the no-loan economy is a sequence $\{C_{1t}, C_{2,t-1}, C_{3,t-2}, x_{2t}, b_{1t}, b_{2,t-1}, b_{3,t-2}, W_t, r_{t+1}, k_t, Y_t\}_{t=0}^{\infty}$ that satisfies equations (1)-(16).

We note one additional definitional matter here. In the perfect capital- markets economy with an operative transfer motive, the timing of intergenerational transfers is indeterminate–parents and children care only about the present value of intergenerational transfers. Because the timing of transfers is indeterminate, the volume of activity in the consumption-loans market is indeterminate. These indeterminacies have no bearing on the equilibrium capital stock or consumption profile, but they are a potential source of confusion in characterizing the influence of the consumption-loans market on the equilibrium outcome. We use the term "active consumption-loans market" to refer to an economy with an active consumption- loans market in *every* equilibrium, including the equilibrium in which parents make all transfers during their second period of life.

With this definition in mind, we now state a preliminary proposition. Assuming uniqueness of the steady-state equilibrium in the loan economy, we have

Proposition *1:* If the consumption-loans market is active in the loan economy, borrowing constraints bind in the corresponding no-loan economy.

Proof: Follows directly from equation (13) and from the uniqueness assumption.

The result in Proposition 1 is independent of whether the transfer motive operates. Thus, intergenerational transfers can never be large enough to overcome 'borrowing restrictions when dissaving is optimal in the steady state of the loan economy.[2]

[2]It is possible for transfer motives to be strong enough in the loans economy to eliminate the young's desire to dissave. In this case the consumption-loans market is redundant and

One further preliminary proposition will prove useful in the analysis below.

Proposition 2: Let $\bar{r}$ and $\tilde{r}$ denote steady-state interest rates in the loan and no-loan economies, respectively.

(a) If the transfer motive operates in the loan economy, then

$$\bar{r} = \frac{(1+n)}{\gamma\beta} - 1 \equiv r^*. \tag{17}$$

(b) If borrowing constraints bind in the no-loan economy, then $\tilde{r} < r^*$.

Proof: Part (a) follows immediately by combining the equality versions of equations (13) and (15). Part (b) follows by combining the strict inequality version of (13) with (15).

Part (a) of this proposition contains the standard result for the dynastic model, showing that the steady-state capital stock satisfies the modified golden rule. Part (b) states that binding borrowing constraints drive the steady-state capital stock above the level implied by the modified golden rule, regardless of whether the transfer motive operates.

3. Borrowing Restrictions and the Timing of Transfers

A. The Optimal Life-Cycle Timing of Altruistic Transfers

We turn now to a discussion of the optimal life-cycle timing of intergenerational transfers in the no-loan economy. While the budget expressions in equations (2)-(4) allow for any combination of inter **vivos** transfers and bequests, we show that transfers early in the life cycle dominate transfers later in the life cycle. We begin by proving

Proposition 3: If the consumption-loans economy has an active consumption-loans market, then bequests and inter vivos transfers from the old to the middle-aged equal zero in the corresponding no-loan economy.

Proof: Suppose bequests or transfers from the old to middle-aged are positive. Then equations (14)-(16) imply that

$$r = \frac{(1+n)}{\beta\gamma} - 1 = r^*. \tag{P3}$$

But, by Propositions 1 and **2(a),** (P3) violates the hypothesis of an active loan market.

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the loans and no-loans equilibria are identical.

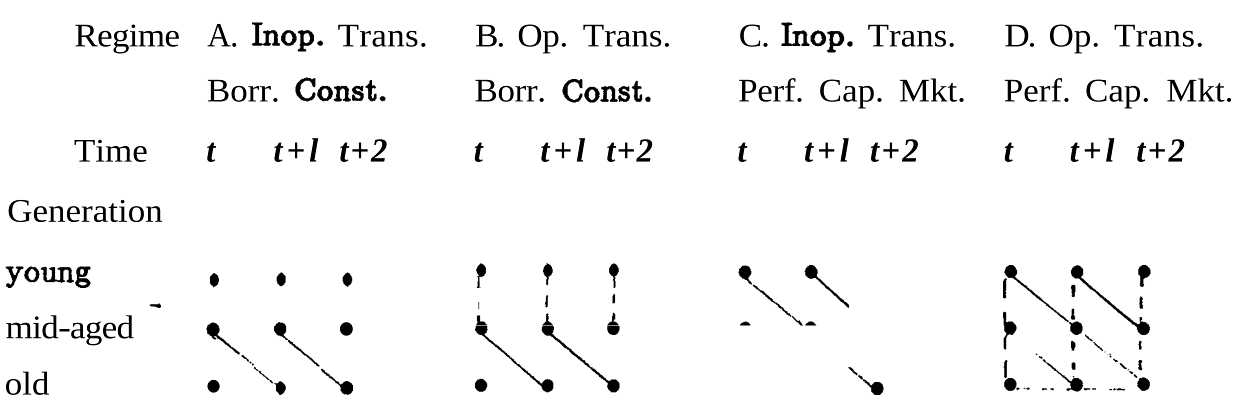
In light of Proposition 1, we can interpret Proposition 3 to say: if borrowing constraints bind on the young, then parents make no bequests upon death or transfers when old. Can binding borrowing constraints on the young co-exist with positive transfers by middle-aged parents? Applying Proposition 2, the answer is yes. When the transfer motive operates, the steady-state marginal rate of substitution between consumption by the young and consumption by the middle-aged equals $\frac{(1+n)}{\gamma}$. But by Proposition 2, $\frac{(1+n)}{\gamma}$ exceeds the young's desired marginal rate of substitution in the na-loan economy. Hence, positive transfers from middle-aged parent to young child can co-exist with binding borrowing constraints on the young. Indeed, we show in Altig and Davis (1989a) that binding borrowing constraints weaken the conditions under which parents make transfers to children. We summarize this discussion in

Proposition 4: If borrowing restrictions bind in a steady-state equilibrium, then any intergenerational transfers occur from middle-aged parents to young children.

B. Patterns of Intertemporal and Intergenerational Linkages

Using Proposition 4, we now describe the patterns of intertemporal (capital market) and intergenerational linkages that can emerge as steady-state equilibria in the no-loan economy. The following simple diagrams illustrate these patterns and show the relationship of our environment with binding borrowing constraints to traditional life-cycle and Ricardian environments.[3]

Patterns of Intertemporal and Intergenerational Linkages



Dashed lines in the diagram depict altruistically motivated intergenerational linkages, and

[3] We thank Doug Bernheim for suggesting this expositional device.

solid lines depict intertemporal linkages operating through the capital market. More precisely, a line connecting two dots indicates that the relevant first- order condition holds with equality.

Regime C in the diagram corresponds to a pure life-cycle economy with perfect capital markets. Regime D depicts the Ricardian environment, characterized by perfect capital markets and an operative transfer motive. Regimes C and D represent the range of equilibrium linkage patterns in the loan economy. When the borrowing constraint is non-binding, these two regimes can also arise in the no-loan economy. Two other linkage patterns arise in the no-loan economy when borrowing constraints bind. The linkage pattern in Regime A arises when borrowing constraints bind and the transfer motive is inoperative. The linkage pattern in Regime B, which reflects the result in Proposition 4, arises when borrowing constraints bind and transfers are positive.

Proposition 1 informs us that Regime B always emerges (in the new steady state) when borrowing constraints are imposed on a Ricardian environment with an active loan market. Altig and Davis (1989a) show that either Regime A or B can arise when borrowing constraints are introduced into Regime C.

4. The Capital-Accumulation Effects of Social Security

In this section we analytically characterize the capital accumulation effects of social security interventions when borrowing constraints bind. As in Diamond (1965), the key ingredients of the analysis are an aggregate savings function and a stability condition that characterizes the dynamic behavior of the economy along the transition path to a steady-state equilibrium.

A. Social Security Interventions

Let $T_{i,t}$ denote lump-sum taxes (subsidies, if negative) levied on members of generation t during the ith period of life. Let d_t denote the time-t issue of one-period government debt per middle-aged person. The government budget constraint is

$$\frac{(1+r_t)}{1+n}d_{t-1} = (1+n)T_{1t} + T_{2,t-1} + \frac{T_{3,t-2}}{1+n} + d_t.$$

We define a funded social security intervention as a forced savings program that pays a market rate of return. That is, a funded social security program obeys

$$-T_{3,t} = (1 + r_{t+1})(1 + r_{t+2})T_{1,t} + (1 + r_{t+2})T_{2,t}. \tag{18}$$

Note that the government runs a budget surplus under a funded social security program.

We define an unfunded or pay-as-you-go social security intervention as a forced inter-generational transfer program that satisfies

$$-T_{3,t} = (1 + n)^2 T_{1,t+2} + (1 + n)T_{2,t+1}. \tag{19}$$

Note that, in a steady state, unfunded social security programs offer the individual a rate of return equal to the population growth rate.

B. The Private-Sector Savings Function

We first derive the savings function of the middle-aged in an economy with binding borrowing constraints and no transfers. Defining $x_{2t} + d_{t+1} \equiv s_{t+1}$, use the budget constraint equations (3) and (4) to write equation (14) as

$$u'[\alpha_2 W_{t+1} - T_{2t} - s_{t+1}] = \beta(1 + r_{t+2})u'[\alpha_3 W_{t+2} + (1 + r_{t+2})s_{t+1} - T_{3t}].$$

This equation implies the existence of a savings function for the middle-aged,

$$s_{t+1} = s[\alpha_2 W_{t+1} - T_{2t}, \alpha_3 W_{t+2} - T_{3t}, r_{t+2}], \tag{20}$$

with partial derivatives satisfying

$$0 < s_1 < 1, \qquad -1 < s_2 < 0, \qquad s_1 - (1+r_{t+2})s_2 = 1, \quad \text{and} \qquad s_3 \gtrless 0. \tag{21}$$

Thus, in the no-transfer economy, savings by the middle-aged is an increasing function of after-tax labor income during middle-age and a decreasing function of after-tax labor income during old age. Savings by the middle-aged increases (decreases) in the interest rate if the-substitution (income) effect dominates.

In the transfer economy the savings function has similar properties, but its derivation is more complicated. From the transfer-motive first-order condition (15) and the intertemporal first-order condition (14), we have

$$C_{1,t+1} = \psi[\alpha_3 W_{t+2} - T_{3t} + (1 + r_{t+2})s_{t+1}, r_{t+2}],$$

where $\psi(\cdot)$ is the inverse marginal utility function, and $\psi_1 > 0$. Using this expression for $C_{1,t+1}$ and the household budget constraints, we write equation (14) as

$$u'[\alpha_2 W_{t+1} - T_{2t} - s_{t+1} - (1+n)\psi(\cdot) + \alpha_1(1+n)W_{t+1} - (1+n)T_{1,t+1}] =$$
$$\beta(1+r_{t+2})u'[\alpha_3 W_{t+2} + (1+r_{t+2})s_{t+1} - T_{3t}].$$

This equation implies a savings function for the middle-aged,

$$s_{t+1} = s\{[\alpha_1(1+n) + \alpha_2]W_{t+1} - (1+n)T_{1,t+1} - T_{2t}, \alpha_3 W_{t+2} - T_{3t}, r_{t+2}\}, \tag{22}$$

with partial derivatives satisfying equation (21).

The form of these savings functions is easily understood in terms of the analysis in section 3. Recall the pattern of intertemporal and intergenerational linkages in the no-transfer economy with binding borrowing constraints–at the margin, the middle-aged are connected only to their own old age. Thus, as equation (20) indicates, social security directly affects the savings behavior of the middle-aged only insofar as it alters their current taxes or their anticipated old-age benefits. In the transfer economy with binding borrowing constraints, the middle-aged are also connected at the margin to their young children. Thus, in line with equation (22), changes in social security taxes levied on their children when young also directly affect the savings behavior of the middle-aged.

C. Stability Analysis

We now combine the private-sector savings function, the government budget constraint, and the capital market-clearing condition to characterize the dynamic behavior of the aggregate capital stock. The evolution of the aggregate capital stock between t + 1 and t + 2 obeys

$$\alpha(1+n)^2 k_{t+2} = s_{t+1}(\cdot,\cdot,\cdot) - d_{t+1} \equiv S_{t+1}, \tag{23}$$

where S_{t+1} denotes the aggregate savings function at t + 1. $s_{t+1}(\cdot,\cdot,\cdot)$ is given by equation (20) in the no-transfer economy and equation (22) in the transfer economy.

Equation (23) implies a relationship between k_{t+2} and k_{t+1} that, following Diamond, we refer to as the savings locus. Differentiate equation (23) to obtain the slope of the savings locus,

$$\frac{dk_{t+2}}{dk_{t+1}} = \begin{cases} \dfrac{-\alpha_2 s_1 k_{t+1} f''(k_{t+1})}{\alpha(1+n)^2 - \alpha_3 s_2 k_{t+2} f''(k_{t+2}) - s_3 f''(k_{t+2})}, & \text{in the no-transfer economy;} \\ \dfrac{-[\alpha_1(1+n)+\alpha_2] s_1 k_{t+1} f''(k_{t+1})}{\alpha(1+n)^2 - \alpha_3 s_2 k_{t+2} f''(k_{t+2}) - s_3 f''(k_{t+2})}, & \text{in the transfer economy.} \end{cases} \tag{24}$$

The numerator-is unambiguously positive, but the denominator can be positive or negative. If $\alpha_3 = 0$, so that the old supply no labor services, the middle term in the denominator vanishes, and the expression for the slope of the savings locus has exactly the same form as in Diamond.

What does equation (24) imply about the transition path to the steady-state equilibrium? Restricting attention to stable steady states, there are two cases to consider. If $0 < dk_{t+2}/dk_{t+1} < 1$ (in the neighborhood of the steady-state equilibrium), then the capital stock converges monotonically to its steady-state value. Alternatively, if $0 > dk_{t+2}/dk_{t+1} > -1$, then the capital stock oscillates around the steady-state value along the transition path. Savings loci corresponding to the monotonic and oscillatory transition paths are illustrated by curves A and B, respectively, in Figure 1.

Equation (24) not only characterizes dynamic behavior along the transition path, but it determines the steady-state capital stock response to nonneutral social security interventions. This is an example of Samuelson's (1947) correspondence principle. As we show in Appendix 1, when the denominator in equation (24) is positive, the partial equilibrium response of aggregate savings to social security interventions carries over, in qualitative terms, to the general equilibrium effect. In contrast, when the denominator in equation (24) is negative, the partial equilibrium effect of social security on aggregate savings is reversed in general equilibrium. Hence, we refer to steady-state equilibria that satisfy $0 < dk_{t+2}/dk_{t+1} < 1$ as stable and regular.

D. Linkage Patterns and the Effects of Social Security

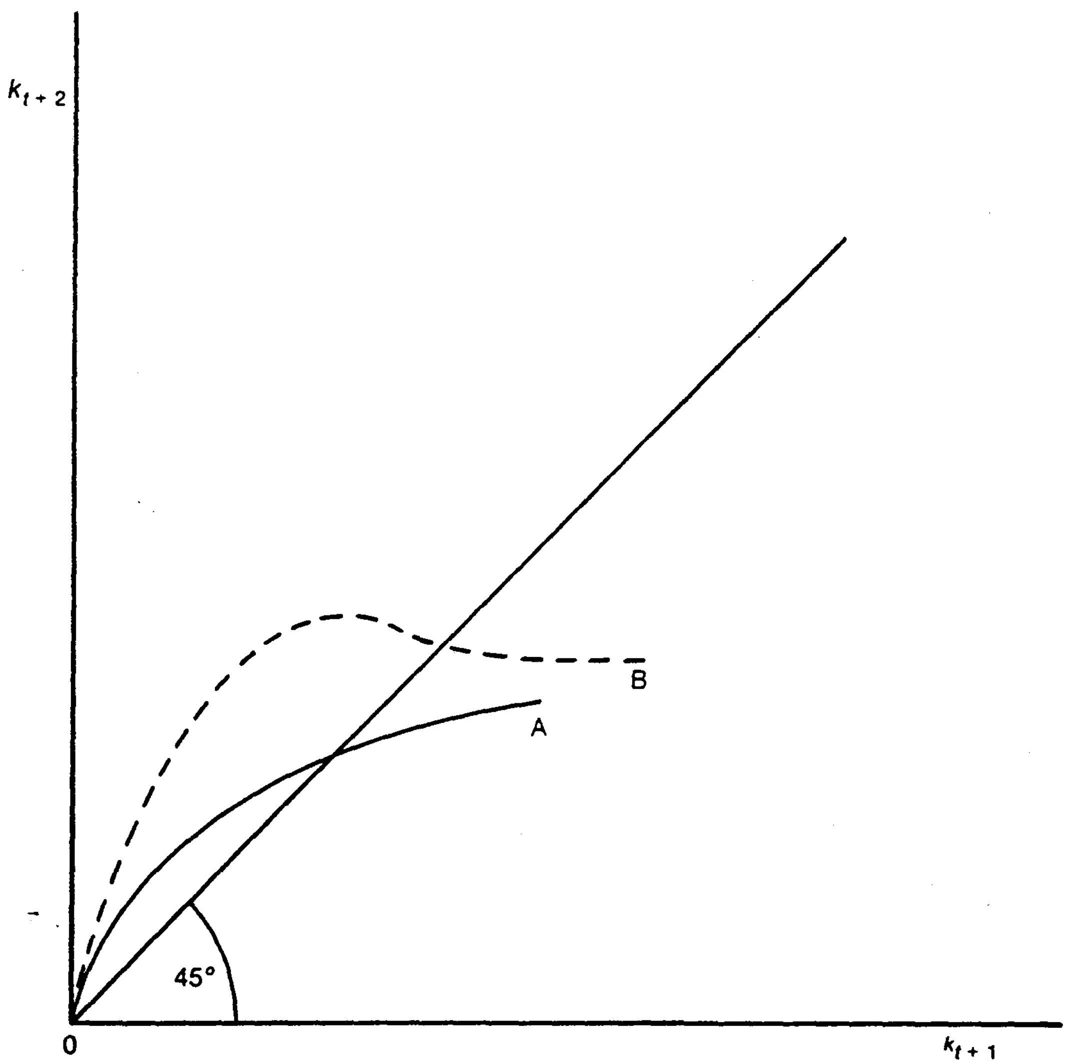
We are now prepared to characterize the effects of social security interventions on capital accumulation when the borrowing constraint binds. We first describe the steady-state effects.

Proposition 5: Consider the overlapping-generations framework with binding borrowing constraints on the young. Assume that the steady-state equilibrium is stable, regular, and unique (pre- and post-intervention).

(a) A funded social security system financed by taxes on the middle-aged has no effect on capital accumulation.

(b) A funded social security system financed by taxes on the young increases the steady-

Figure 1
The Savings Locus and Steady-State Equilibrium



state (per capita) capital stock.

(c) An unfunded social security system decreases the steady-state capital stock.

(d) If the transfer motive operates, the generational incidence of the taxes used to finance old-age benefits under an unfunded system is irrelevant to the determination of the. capital stock. If the transfer motive is inoperative, a shift in taxes from the middle-aged to the young increases the capital stock.

Proof: See Appendix 1.

If we drop the uniqueness assumption in Proposition 5, then the results apply in some neighborhood of the initial steady-state equilibrium. If we drop the regularity assumption, then the qualitative responses to nonneutral interventions are reversed.

The intuition behind Proposition 5 can be understood as follows: The neutrality result in part (a) reflects the intertemporal link between the middle-aged and the old in Regimes A and B. Since the middle-aged are already trading-off own current consumption for own future consumption at the rate $(1+r)$, they fully offset the funded social security intervention. In this respect, the borrowing-constraint economies mirror the behavior of the standard life-cycle economy depicted in the diagram by Regime C.

Likewise, the irrelevance result in part (d) of the proposition for the transfer economy reflects the intergenerational link between the middle-aged and young as illustrated in the diagram for Regime B. When the transfer motive operates, the young and middle-aged are trading-off consumption at the rate $(1+n)$, which is identical to the trade-off implied by shifts in the generational incidence of taxes under an unfunded social security system. This logic holds regardless of whether the young are borrowing-constrained.

Turning to the nonneutral interventions, consider a funded social security program financed by a one dollar tax on each young person. There are distinct impact and secondary effects here, both of which lead to an increase in the capital stock. First, aggregate saving rises because the government forces each of the $(1+n)$ young persons to save one dollar. This impact effect is mitigated, but not reversed, when the transfer motive operates, because middle-aged parents adjust transfers to partially compensate the young for their disposable income loss. Hence, when the transfer motive operates, the partial equilibrium impact effect on aggregate savings is $(1+n)(1-s_1)$. Second, after the funded program has been in operation for more than one period, each middle-aged person experiences a $(1+r)$

dollar increase in own wealth over the last two periods of life. This effect leads to a further increase in aggregate savings in the amount of $(1+r)$ times the marginal propensity to save out of middle-aged income. Thus, in the no-transfer economy, the partial equilibrium effect is to increase aggregate savings by $(1+n)+s_1(1+r)$. In the transfer economy, the partial equilibrium effect is to increase aggregate savings by only $(1+n)(1-s_1)+s_1(1+r)$. The regularity condition, $0 < dk_{t+2}/dk_{t+1} < 1$, insures that these partial equilibrium effects carry over to the general equilibrium. In terms of Figure 1, the aggregate savings locus A shifts up and to the left.

Now, consider the effects of an unfunded social security program. An unfunded social security program weakens the life-cycle motive for saving by shifting the timing of income receipt to a later period of life. The increase in after-tax income during old age leads to a partial equilibrium reduction in aggregate savings. This is the only effect when taxes fall entirely on the borrowing-constrained young and the transfer motive is inoperative. If taxes fall on the young and the transfer motive operates, then altruistic transfers from the middle-aged to the young rise. Hence, the net-of-transfer income of the middle-aged falls, and there is a further depressive effect on aggregate savings. If the tax falls on the middle-aged, then the decline in the after-tax income of the middle-aged is an additional effect contributing to the reduction in savings. Under all of these scenarios, an unfunded social security program depresses savings.

Note the sharply contrasting implications of altruistic intergenerational linkages in the loan and no-loan economies. With perfect capital markets, altruistic transfers are the mechanism that neutralizes the aggregate savings effects of an unfunded social security program. With binding borrowing constraints on the young, altruistic transfers exacerbate the decline in aggregate savings relative to the no-transfer case. This additional depressive effect on aggregate savings reflects the efforts by altruistic parents to offset the reduction in after-tax income of their borrowing-constrained children. Thus, the fiscal policy implications of altruistic intergenerational linkages hinge critically on the issue of whether borrowing constraints bind.

We can use Proposition 5 to draw a sharp distinction between our no-loan economy with operative transfers and that of Laitner (1989). In our no-loan economy, the non-neutrality of unfunded social security programs entirely reflects the effects of government-mandated transfers between persons who are members of the same family line. Furthermore, in regime B, nonneutrality holds despite altruistic linkages that connect each person to his parent and children at some stage of the life cycle. In Laitner's model, government-mandated transfers between persons who are members of the same family line are neutral. Neutrality of these transfers holds in Laitner's model, because each person weights his parent's and child's utility as heavily as his own. It follows that the nonneutrality of unfunded social security in Laitner's model entirely reflects the effects of government-mandated transfers between persons who are members of different family lines. Presumably, a sufficiently rich model would capture both the intra-family and inter-family effects of unfunded social security.

Drawing on our stability analysis, we can also characterize the dynamic capital accumulation response to nonneutral social security interventions.

Proposition 6: Consider a one-time, permanent social security intervention in the no-loan economy with binding borrowing constraints. Assume that the initial and new steady-state equilibria are stable, regular, and unique. If the intervention is nonneutral, then.(per capita) capital accumulation is monotonic along the transition path from the initial to the new steady-state equilibrium.

Proof: (Sketch) The proof is implicit in the preceding discussion. The analysis in sections 4.b and 4.c shows that, under the hypotheses of the proposition, the transition path to a steady-state equilibrium is monotonic. It remains only to check that any secondary effects of a social security intervention shift the savings locus in the same direction as the impact effect. For interventions involving changes in an unfunded program, there are no secondary effects. For interventions involving changes in a funded program, the wealth effect on the savings behavior of the middle-aged reinforces the impact effect.

5. The Magnitude of the Response to Social Security Interventions

A. *Description of the Numerical Simulation Ezperiments*

In this section, we parametrize the economy and numerically simulate its dynamic response to lump-sum interventions under Regimes A-C. (The dynamic response is trivial in Regime D.) The simulations help gauge the magnitudes of the nonneutralities identified above. They also illustrate how the interaction between borrowing constraints and intergenerational altruism shape the aggregate savings and welfare response to social security.

Within the economic environments of Regimes A-C, we consider funded and unfunded social security interventions under polar assumptions about the generational incidence of social security taxes. Upon introduction of an unfunded intervention, the government subsidizes the old and levies taxes on the middle-aged or young in a way that satisfies equation (19). Upon introduction of a funded system at time t, the government levies taxes on the middle-aged or young; benefit payments commence in period $t+1$ if the middle-aged pay into the system, or in period $t+2$ if only the young pay into the system. The path of the government's budget surplus under a funded intervention is determined by substituting equation (18) into the government budget constraint.

For simplicity, we assume that the economy is initially at a steady-state equilibrium, and that the interventions represent unanticipated, permanent changes to the structure of the social security program.[4] Our numerical simulation technique, described in Appendix 2, can easily accommodate relaxations of these assumptions.

We parameterize the economies as follows: All of our simulations assume that capital's share is equal to .25 in a Cobb- Douglas production function; a lifetime productivity profile $(\alpha_1, \alpha_2, \alpha_3) = (1.5, 6.0, 2.5)$; no government taxes or subsidies at the initial steady state; an intervention that introduces an old-age benefit payment equal to 6% of the old's wage income in the initial steady state; a population growth rate, n, equal to $(1+.01)^{25} - 1$; and an intertemporal discount factor, β, equal to $.99^{25}$. Here, we interpret a period in the model as corresponding to twenty-five years. The period utility function is iso-elastic:

$$u(C_{it}) = \begin{cases} C_{it}^{1-\frac{1}{\sigma}} & \sigma > 0, \quad \sigma \neq 1 \\ \ln C_{it}, & \sigma = 1, \end{cases}$$

where a equals the intertemporal substitution elasticity in consumption. Our baseline parameter specification assumes $a = 2/5$, which accords well with most estimates in the empirical literature; see Auerbach and Kotlikoff (1987, pp. 50-51). In the borrowing

[4]The linkage patterns in Regimes A and B imply that the evolution of the capital stock solves an initial value problem. At time t no agent is connected, at the margin, through intergenerational or capital-market linkages to consumption levels in period $t+2$ and beyond. This observation implies, among other things, an identical dynamic response to anticipated and unanticipated funded social security interventions under Regimes A and B.

constraint economy with operative transfers, we set 7, the interpersonal discount factor, equal to .1.

Appendix 3 reports the results of repeating our simulation exercises for values of the intertemporal substitution elasticity that range from 1/3 to 1, and values of the interpersonal discount factor that range from .10 to .52. At least within these ranges, the basic messages of our simulation exercises are not sensitive to the parameter specifications.

B. The Dynamic Response of the Capital Stock

Our reported simulation results highlight the aggregate capital stock response to social security interventions. We measure the crowding-out ratio t periods after the intervention as

$$R_t = \frac{\alpha(1+n)^2(k_0 - k_t)}{.06\alpha_3 W_0}, \qquad t = 0 \ldots T,$$

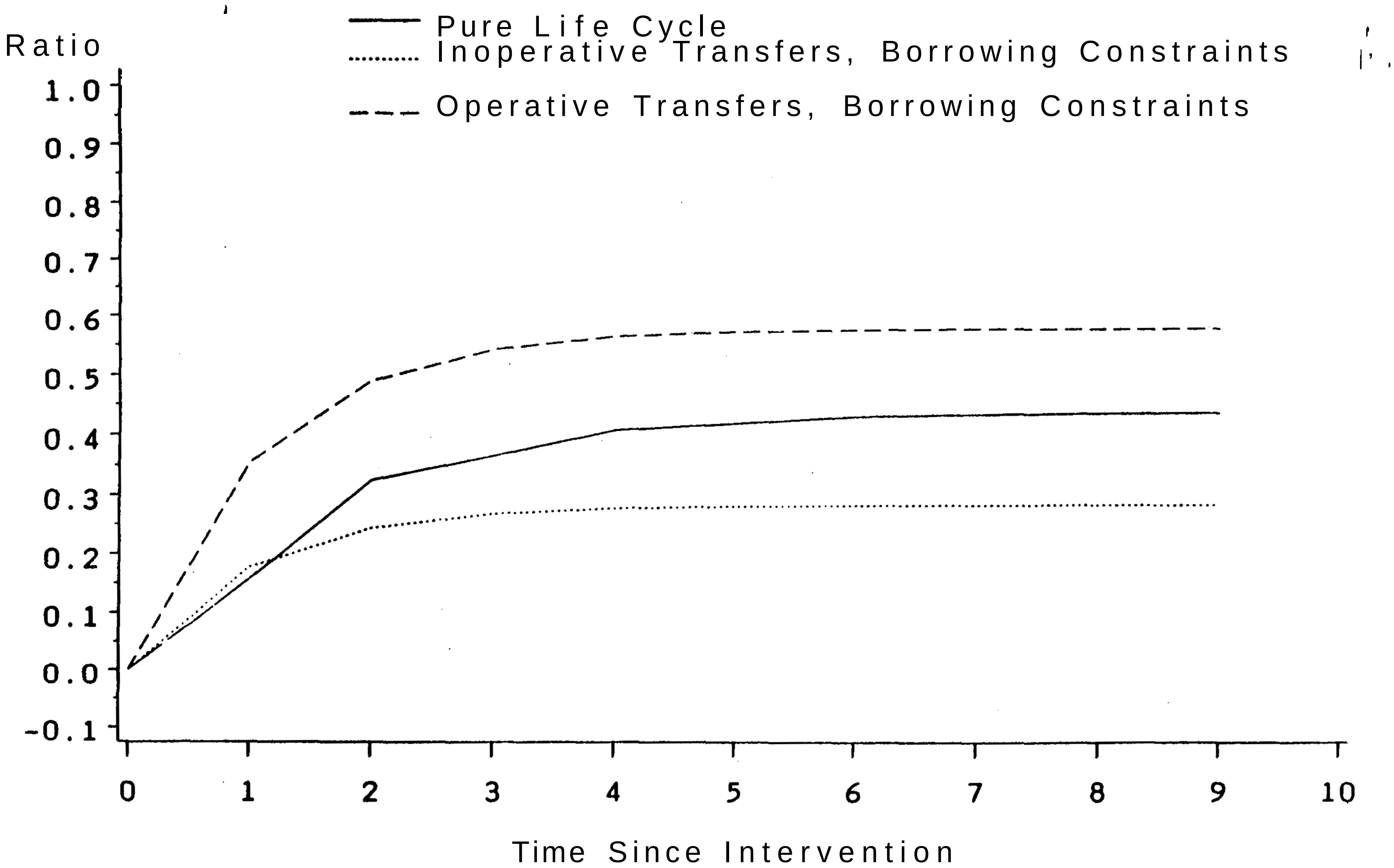
where $\alpha(1+n)^2 k_0$ equals the (per old person) capital stock in the pre-intervention steady state, and $.06\alpha_3 W_0$ is the size of the social security benefit (per old person). A positive value for R_t indicates that the capital stock is smaller at t as a result of the intervention.

Figures 2-4 illustrate the dynamic response of the capital stock to social security interventions under Regimes A-C. We measure the capital stock response in terms of the crowding-out ratio defined above. Figures 2 and 3 illustrate the dynamic response to an unfunded social security intervention, assuming, respectively, that the young and middle-aged pay all taxes. Figure 4 illustrates the dynamic response to a funded social security intervention, assuming the young pay all taxes. As an example of how to interpret the figures, consider the pure life-cycle case in Figure 2. According to Figure 2, the capital stock declines in the long run by an amount equal to 43% of the increase in the benefit payment to the old. One period after the intervention, the decline equals 16% of the increase in the benefit payment to the old.

Four interesting results emerge from Figures 2-4. First, the crowding-out response to unfunded social security interventions is small to large in magnitude, ranging (in the long run) from 5% to 64% of the benefit payment. The lower end of this range corresponds to the life-cycle regime in which the middle-aged pay the taxes, and the upper end corresponds to the no-transfer/borrowing-constraint regime with taxes on the middle-aged.

Second, large crowding-out ratios are fully consistent with altruistic intergenerational

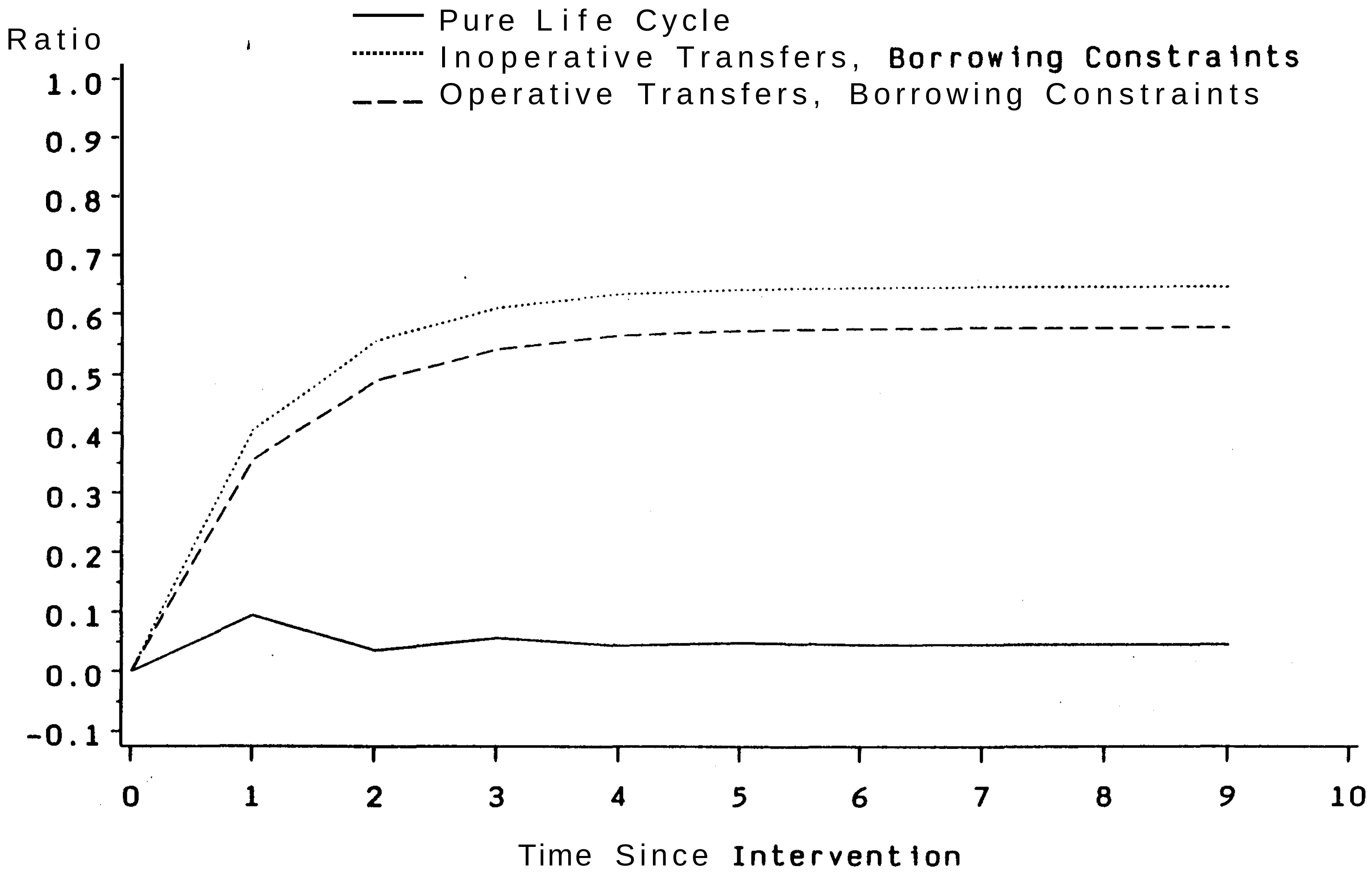
Figure 2
DYNAMIC RESPONSE TO AN UNFUNDED SOCIAL SECURITY INTERVENTION--YOUNG PAY TAXES



Crowding-out ratios for the baseline specification described in the text.

Figure 3

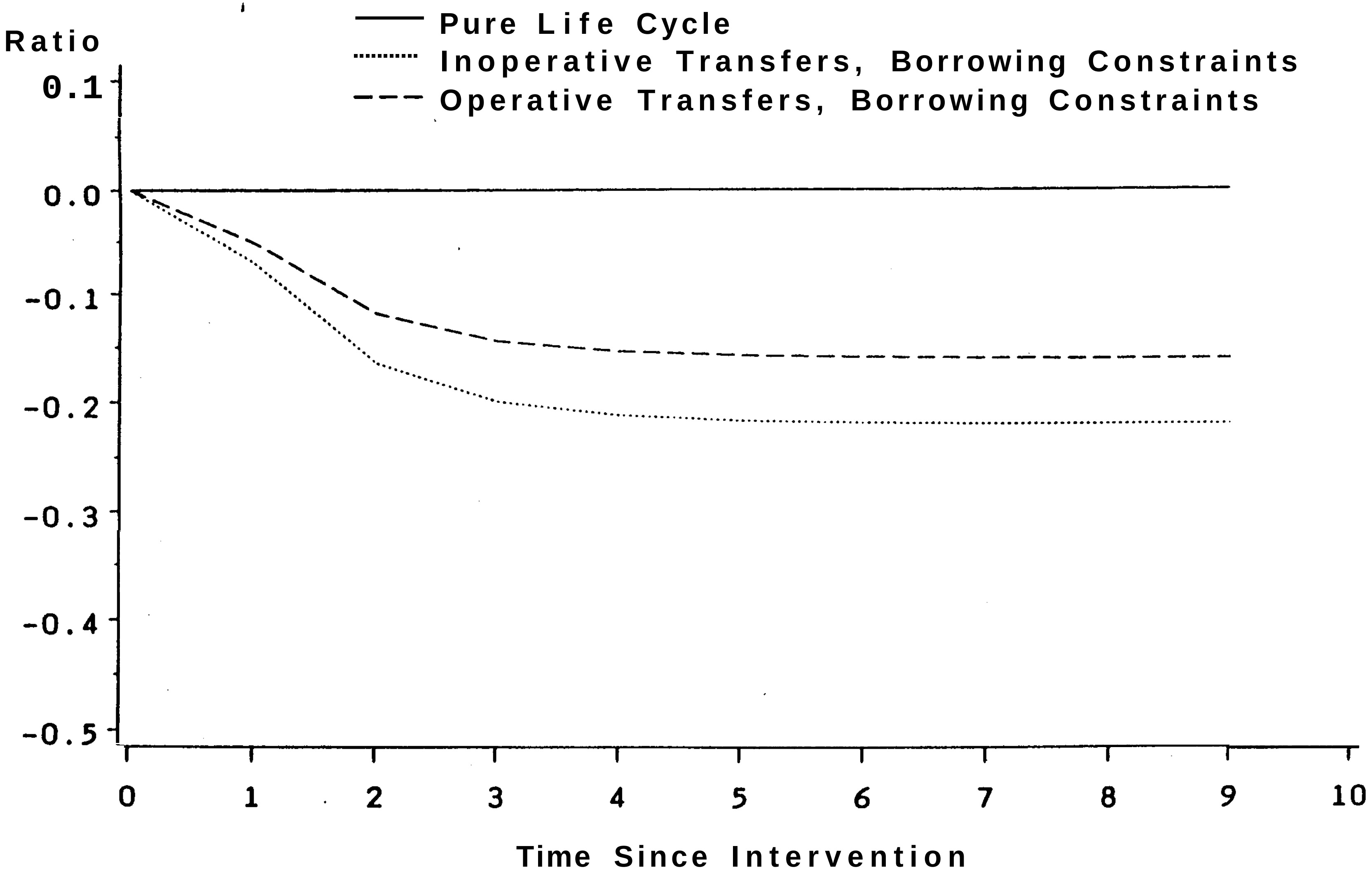
DYNAMIC RESPONSE TO AN UNFUNDED SOCIAL SECURITY INTERVENTION--MIDDLE-AGED PAY TAXES



Crowding-out ratios for the baseline specification described in the text.

Figure 4

DYNAMIC RESPONSE TO A FUNDED SOCIAL SECURITY INTERVENTION—YOUNG PAY TAXES



Crowding-out ratios for the baseline specification described in the text.

linkages. In the regime with operative transfers and borrowing constraints, the unfunded social security interventions cause a long-run capital stock decline equal to 58% of the old-age benefit payment. Thus, borrowing constraints imply a quantitatively significant departure from Ricardian equivalence.

Third, the magnitude of the crowding-out response in the regime with operative transfers and borrowing constraints is closer to the response in the life-cycle regime than the response in the Ricardian regime. Figure 2 indicates that, when the young pay the taxes, the crowding-out ratio is moderately larger in the transfer regime than in the life-cycle regime. For sufficiently high values of the intertemporal substitution elasticity, this ranking is reversed. As Figure 3 indicates, when the middle-aged pay the taxes, the crowding-out ratio is much larger in the regime with operative transfers and borrowing constraints than in the life-cycle regime.

Fourth, viewed from the perspective of either borrowing-constraint regime, life-cycle models provide highly misleading implications about the capital stock response to shifts in the generational incidence of social security taxes. Under an unfunded intervention, a shift from taxes on the young to taxes on the middle-aged reduces the crowding-out ratio from .43 to .05 in the life-cycle regime. In the operative transfer regime, the shift has no effect (Proposition 6[b]). In the regime with borrowing constraints but no transfers, the shift *increases* the crowding-out ratio from .28 to .64. Under a funded intervention, social security is neutral in all regimes when the middle-aged pay the taxes. But a shift in taxes to the young causes a modest increase in the capital stock when borrowing constraints bind. The shift has no effect in the life-cycle scenario.

In summarizing our results on the capital stock response to social security interventions, we stress two points. First, borrowing constraints imply large deviations from Ricardian equivalence. Second, neither Ricardian models nor traditional life-cycle models provide good approximations to the aggregate savings effects of social security interventions in economies with binding borrowing constraints.

C. The Welfare Consequences of Shifting the Generational Incidence of Taxes

Hubbard and Judd (1987) develop an argument for shifting the generational incidence of the social security payroll tax from younger to older workers. In a setting with borrowing constraints and no altruistically motivated intergenerational linkages, the argument is compelling. An intertemporal shift in the burden of payroll taxes from younger to older workers mitigates the adverse consequences of borrowing constraints on lifetime welfare. We now investigate whether this argument retains its force in our setup with altruistically motivated transfers.

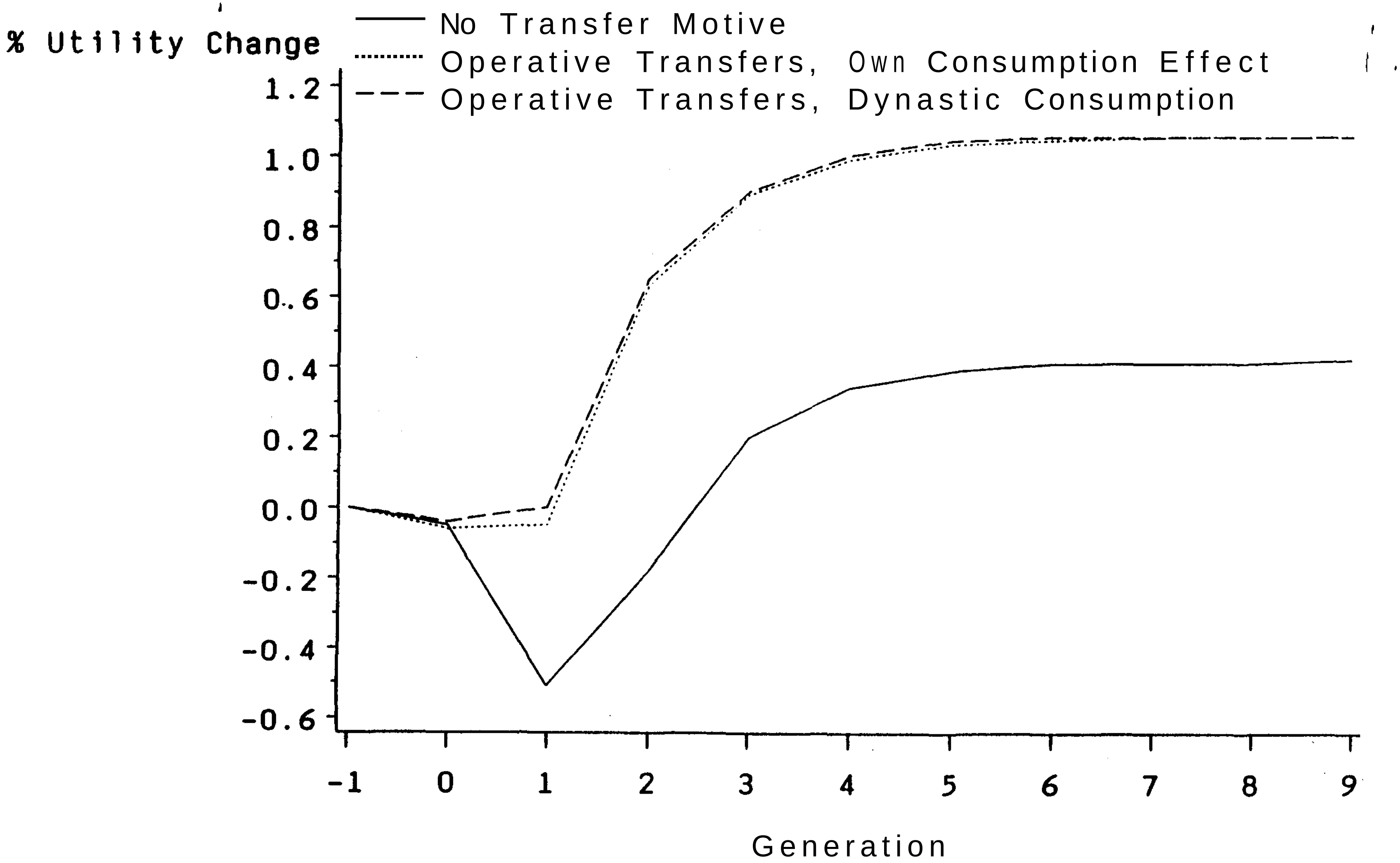
Consider first the welfare implications of shifting the generational burden of social security taxes under an unfunded system. We know from Proposition 5(d) that the financing regime in an unfunded social security system is fully neutral, if altruistic transfers are positive. Thus, under an unfunded system, shifts in the generational burden of social security taxes yield no welfare gains, despite borrowing constraints on the young.

While the United States operates a largely unfunded social security system, Hubbard and Judd develop their analysis in the context of a funded system. Under a funded system, the financing regime does affect welfare in the no-loan economy. To assess the magnitude of these welfare effects and their sensitivity to altruistic linkages, we use numerical simulations.

Figure 5 plots the percentage change in utility associated with shifting the burden of taxation from the middle-aged to the young in a funded system. The utility changes are relative to the outcomes that would have occurred had there been no change in the financing regime. Figure 5 compares the generational welfare response to the financing switch for the no-transfer-motive and operative-transfer-motive $(7 = .1)$ cases. For the operative-transfer-motive case, the figure illustrates the direct utility effect of the switch on own lifetime consumption and the full utility effect that takes into account the changes in descendants' consumption.

The details of our numerical simulation are as follows: At the initial steady state, the social security program is financed entirely by taxes on the middle-aged. As a result of the unanticipated intervention, which occurs when generation -1 is old, the social security program becomes entirely financed by taxes on the young. The size of the social security program-and all parameter settings are identical to the baseline specification used in our previous simulations. Note first that steady-state lifetime welfare rises as a result of shifting taxes from the unconstrained middle-aged to the constrained young. In the long-run, the utility gains associated with a larger capital stock more than compensate for the utility losses due to less complete consumption smoothing. The long run gains are greater when

Figure 5
THE WELFARE EFFECTS OF SHIFTING SOCIAL SECURITY TAXES FROM THE UNCONSTRAINED MIDDLE-AGED TO THE CONSTRAINED YOUNG IN A FUNDED SYSTEM



The intervention occurs under a funded system when generation -1 is old.

transfer motives operate.

Consider next the welfare impact on members of generation 1, who are young when the financing switch takes place. In the no-transfer case, the young bear the full brunt of a reduced ability to smooth consumption, but members of generation 1 benefit little by eventual increases in the capital stock. Hence, members of generation 1 suffer a relatively large utility loss. Finally, consider the most striking aspect of Figure 5. The welfare losses suffered by persons who are young when the financing switch occurs are greatly mitigated by an operative altruistic transfer motive. Members of generation 0, who are middle-aged when the financing switch occurs, increase their transfers to young children and thereby offset most of the potential utility losses from taxes on the young. Thus, from a welfare perspective, altruistically motivated transfers within the family serve as a good substitute for consumption smoothing in the market.[5] This result is reminiscent of Kotlikoff and Spivak's (1981) finding that insurance arrangements within the family can achieve most of the welfare gains associated with perfect annuity markets.

Thus, in the context of a funded system, the force of Hubbard and Judd's argument for switching the generational burden of payroll taxes is greatly reduced by an operative altruistic transfer motive. In the context of an unfunded system, altruistic transfers completely vitiate the argument for shifting the generational burden of taxes, as we noted above. It is worth stresssing that our critique of the Hubbard and Judd argument relies on a small degree of parental altruism: in Figure 5, parents weight children's utility only 10% as heavily as their own.

One caveat should be borne in mind when interpreting our critique of Hubbard and Judd's argument. The ability of altruistic linkages within the family to substitute for an absent consumption-loans market, or to offset social security taxes on borrowing-constrained young persons, hinges critically on the optimal timing proposition. Aspects of the economic environment that mitigate against the timing proposition might also restore some of the force to Hubbard and Judd's argument. For example, imperfect annuity markets provide parents with incentives to defer transfers, as they await the resolution of uncertainty about their own longevity and the longevity of living ancestors. To the extent that

[5] The pattern of generational welfare effects in Figure 5 holds in the other numerical simulations we have conducted with alternative values of a and 7.

parents delay transfers, transfers become less useful in smoothing consumption over the life cycle.

6. Two-Sided Altruism and the Effects of Social Security

A. A Model with Two-Sided Altruism

It is reasonable to ask whether a child-to-parent altruistic gift motive restores intergenerational linkages later in the life cycle and thereby neutralizes social security interventions. An operative gift motive clearly implies the neutrality of social security when capital markets are perfect–at issue is whether gift motives imply social security neutrality in the face of borrowing constraints.

To examine the implications of a gift motive, we extend the preference specification (1) as follows:

$$U_t = \sum_{i=1}^{3} \beta^{i-1} u(C_{it}) + \beta\gamma U^*_{t+1} + \frac{\rho}{\beta} U^*_{t-1}. \tag{1'}$$

We follow Abel (1987) in equation (1') and assume that the gift decision is made taking the gifts of siblings as given.[6] We also note that $\rho\gamma \leq 1 + n$ is a necessary condition for the existence of a steady-state equilibrium.[7]

We modify the budget constraints for a member of generation t to include gifts from children to parents, denoted by $g_{i,t-1}$, $i = 1, 2$. Since we assume borrowing constraints bind, Proposition 4 allows us to omit b_{2t} and b_{3t}.

$$C_{1t} + g_{2,t-1} = \alpha_1 W_t + b_{1t}, \tag{2'}$$

$$C_{2t} + (1+n) b_{1,t+1} + x_{2t} + g_{3,t-1} = \alpha_2 W_{t+1} + (1+n) g_{2t}, \tag{3'}$$

$$C_{3t} = \alpha_3 W_{t+2} + (1 + r_{t+2}) x_{2t} + (1+n) g_{3t}. \tag{4'}$$

[6] In an environment with perfect capital markets and operative intergenerational linkages–that is, an environment with dynastic families–living persons' treatment of deceased ancestors' utility calculations bears on both the existence and form of a solution; see Kimball (1988). When borrowing constraints bind, the dynastic character of the representative person's problem is destroyed, so that the treatment of deceased ancestors' utility has no bearing on the solution.

[7] If this condition fails to hold, the transfer motive and gift motive first-order conditions contradict each other; see Abel (1987) for elaboration on this point.

The new first-order conditions implied by the introduction of a gift motive are

$$u'(C_{it}) \geq \rho u'(C_{i+1,t-1}), \qquad i = 1,2. \tag{25}$$

B. Gifts and the Pattern of Linkages

We now identify the additional linkage patterns that can arise from the introduction of a gift motive. The following useful proposition follows directly from the intertemporal first-order conditions and equation (25).

Proposition 7:

(a) *If* the gift motive operates, $r = \frac{\rho}{\beta} - 1$ in a steady-state equilibrium.

(b) If borrowing constraints bind, the gift motive does not operate when children are young.

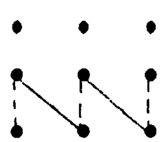
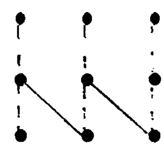
Proposition 7(b) rules out gifts early in the life cycle when borrowing constraints bind. There remains the question of whether gifts late in the life cycle can co-exist with borrowing constraints. The next two examples answer this question in the affirmative.

Ezample 1-Binding Borrowing Constraints, Operative Gift Motive, Inoperative Transfer Motive: Consider a parametric version of the gift-motive economy with log utility and capital's share equal to .1 in a Cobb-Douglas production function. Set 7 = n = 0, p = 1.0, $\beta = .5$, and $(\alpha_1, \alpha_2, \alpha_3) = (.15, .6, .25)$. Supposing that the gift motive operates, Proposition 7(a) implies that $r = 1$. It then follows that k = .0774, $w = .6968$, and $(1+r)k = .1548$. Consider the consumption profile (.1045, .3348, .3348), the savings profile (0, .0774), the gift profile (0, .0058), and the transfer/bequest profile (0,0,0). The reader can verify that these profiles represent an equilibrium in which borrowing constraints bind on the young, the gift motive operates only for the middle-aged, and the transfer motive is inoperative.

Example 2-Binding Borrowing Constraints, Operative Gift Motive, Operative Transfer Motive: Modify the previous example by setting $\gamma = .35$. Consider the consumption profile (.1153, .3294, .3294), the savings profile (0, .0774), the gift profile (0, .0005), and the transfer/bequest profile (.0108, 0, 0). The reader can verify that these profiles represent an equilibrium in which borrowing constraints bind on the young, the gift motive operates only for the middle-aged, and the transfer motive operates only for the middle-aged.

In terms of the diagrams introduced in section 3.b, the introduction of a gift motive implies two new linkage patterns:

Additional Linkage Patterns with an Operative Gift Motive

	Regime	E. Inop. Trans. Borr. Const. Op. Gifts	F. Op. Trans. Borr. Const. Op. Gift
	Time	t $t+1$ $t+2$	t $t+l$ $t+2$
Generation **young** mid-aged old			

C. Implications of Gift Motives for the Effects of Social Security

The following proposition states conditions under which social security interventions are fully neutral, despite the existence of binding borrowing constraints.

Proposition 8: Assume that borrowing constraints bind, and that the gift motive operates.

(a) Any (small) social security intervention that fails to impinge on the budget constraint of the young is neutral in its impact on capital accumulation, the consumption profile, and welfare.

(b) If the transfer motive operates, any (small) social security intervention is neutral in its steady-state impact on capital accumulation, the consumption profile and welfare.

Proof: Part (a) follows immediately from the linkage diagrams for Regimes E and F. To prove part (b), note from Proposition **9(b)** that steady-state aggregate consumption (per capita) is unaffected. It then follows from the transfer- and gift-motive first-order conditions that the steady-state **consumption** profile is unaffected.

Proposition 8 is entirely in the spirit of the neutrality results that appear in Proposition 5 and standard Ricardian neutrality results in environments with operative gift or transfer motives and perfect capital markets. The parallel nature of these results suggests a symmetry between the effects of operative gift motives and the effects of operative transfer motives. Despite these aspects of symmetry, an operative gift motive carries much stronger implications for fiscal policy than an operative transfer motive, when borrowing constraints bind. As a corollary to Proposition 7(a), an operative gift motive pins down the steady-state interest rate in the face of ***any*** (small) lump-sum fiscal policy intervention. With inelastic labor supply, the level of the capital stock is also invariant to (small) lump-sum fiscal interventions. Thus,

Proposition 9: Assume that the gift motive operates.

(a) Then all (small) social security interventions are neutral in their impact on the steady-state interest rate.

(b) If labor supply is inelastic, all (small) social security interventions are neutral in their impact on the steady-state capital stock.

An operative gift motive does not imply full neutrality when borrowing constraints bind and parent-to-child altruistic transfer motives are inoperative. In this case, social security interventions that impinge on the budget constraint of the young affect the shape of the lifetime consumption profile. (If we allow for elastic labor supply, they also affect aggregate consumption and the capital stock.) As an example, consider an unfunded social security intervention financed by taxes on the young. The reader can easily verify that this intervention affects the shape of the lifetime consumption profile in Example 1 but not in Example 2. (The key is to observe that unfunded social security interventions are isomorphic to a-compensated changes in the shape of the lifetime productivity profile.)

Possible effects on consumption notwithstanding, Proposition 9 is a remarkably robust neutrality result. It applies regardless of whether parent-to-child transfer motives operate early in the life cycle. It applies regardless of whether young persons are borrowing-constrained. Provided that the gift motive remains operative for the altruists, Proposition 9 survives the introduction of non-altruistic agents into the economy. By the same token, Proposition 9 survives the introduction of exchange motives (as in Cox [1987]) for intergenerational transfers. (However, see the discussion in footnote 9 below.)

To place this surprising neutrality result in perspective, several comments are in order. First, Proposition 9 differs in an essential way from the neutrality results that appear in Barro (1974), Becker (1974), Bernheim and Bagwell (1988), Altig and Davis (1989a), and the many related papers in the literature. The neutrality results in the Barro-Becker-

Bernheim/Bagwell tradition rest upon an extensive interconnected network of budget constraints. Hence, these neutrality results break down, partially or completely, if operative altruistic linkages are insufficiently pervasive to maintain the fully interconnected network of budget constraints. In contrast, our interest-rate neutrality result follows immediately from the intertemporal and gift-motive f.o.c.'s of the middle-aged.[8] Thus, Proposition 9 directly exploits the properties of altruistic preferences, unlike neutrality results in the Barro-Becker-Bernheim/Bagwell tradition, which exploit the implications of altruistic preferences for connections among budget constraints.[9]

Second, in light of our strong neutrality result, it is natural to inquire whether gift motives operate under "reasonable" conditions. In the analytical framework of this paper, it turns out that equilibria with positive gifts can arise only if the gift motive is quite strong:

Proposition 10: $\rho > \beta$ is a necessary condition for an operative gift motive.

Proof: The Inada conditions require a positive interest rate. Hence, using Proposition 7(a), an operative gift motive can occur only when $r = (\rho/\beta) - 1 > 0$.

Proposition 10 states that children must care about their parent's current utility more than their own future utility for gift motives to operate. This necessary condition is

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[8] To the best of our knowledge, Summers (1982) and Altig and Davis (1989b) are the only other writers to exploit the first-order conditions in this way to obtain steady-state neutrality results. Neither of these papers derive a neutrality result in the presence of borrowing constraints.

[9] Proposition 9 fails if we sufficiently relax the separability assumptions embodied in equation (1'). Consider the the general form for preferences

$$U_t = u(C_{1t}, C_{2t}, C_{3t}, U^*_{t-1}),$$

where we ignore parental altruism for simplicity. By combining the steady-state versions of equations (14) and (25), assuming an operative gift motive, we obtain

$$1 + r = u_4(C_1, C_2, C_3, U^*).$$

Now, in the context of regime E, consider a social security intervention that impinges on the budget constraint of the young. If $u_{41} \neq 0$, then interest-rate neutrality fails to hold. But, note that either intertemporal *or* interpersonal separability implies $u_{41} = 0$. Even if u_{41} is nonzero, interest-rate determination in our framework is radically different than in Ricardian and life-cycle models. We thank Jim Davies for directing our attention to the separability assumption that underlies Proposition 9.

a strong one, and it might prompt one to dismiss Proposition 9 as a theoretical curiosity. This dismissal would be inappropriate. In Altig and Davis (1989b), we show that analogs to Proposition 9 hold in environments with quite modest degrees of altruism and small imperfections in the consumption-loans market. Thus, Proposition 9 is one example of a class of interest-rate neutrality theorems that hold in environments with altruistic linkages and capital market imperfections.

7. Concluding Remarks

The interaction between capital market imperfections and intergenerational altruism carries important implications for the life-cycle timing of intergenerational transfers and for the response of the interest rate, capital stock, and lifetime consumption profiles to social security interventions. Our analysis provides a thorough characterization of these implications when capital market imperfections take the form of borrowing constraints on the young and altruistic preferences do not engender strategic behavior. However, several important questions remain open.

First, given the frequently strong results in this paper, it is natural to ask whether they survive in environments with milder imperfections in the consumption-loans market. In Altig and Davis (1989b), we consider environments with intergenerational altruism and small imperfections in the capital market. The imperfections take the form of a wedge between borrowing and lending rates that stems from the asymmetric tax treatment of interest income and interest payments on consumption loans. The timing proposition survives completely intact in this environment, and the interest-rate neutrality proposition emerges in an even more powerful form. Surprisingly, however, a dichotomy arises between the short-run and long-run capital accumulation responses to social security when altruistic linkages are present. In the short run, an unfunded social security program- crowds out capital just as in the no-loan economy of this paper, but eventually the economy returns to the initial-equilibrium capital intensity.

Second, we abstracted from individual uncertainty about lifetime earnings and longevity. Coupled with less-than-perfect insurance and annuity markets, these factors imply incentives for altruistic parents to defer transfers to children, even borrowing-constrained children, as they await the resolution of uncertainty. Thus, uncertainty about earnings and longevity mitigates against the optimal timing proposition. Furthermore, to the extent that social security influences the magnitude of precautionary savings in an uncertain environment, the argument underlying our interest-rate neutrality proposition may be undercut. While we have yet to formally address these issues, straightforward modifications of our analytical framework provide a useful vehicle for doing so. Issues associated with annuity market imperfections, for example, are easily introduced into our framework by assuming that persons face uncertainty about whether they live for two or three periods. In future research, we hope to determine how the interaction among borrowing constraints, imperfect annuity markets, and altruistically motivated intergenerational linkages shapes the aggregate savings and welfare response to social security programs.

Finally, much recent research focuses on strategic aspects of altruistically motivated interpersonal transfers. See Bernheim and Stark (1988), Bruce and Waldman (1988), Lindbeck and Weibull (1988), and Kotlikoff, Razin and Rosenthal (1988). The Samaritan's dilemma modelled by the first three sets of authors cannot arise in our framework with binding borrowing constraints and parental altruism only. Since parents want borrowing-constrained children to consume the entire transfer, over-consumption by the young is not an issue. There is scope for the Samaritan's dilemma in the gift-motive economy we consider, because parents might over-consume during middle-age to elicit larger gifts from children during old age. By ignoring this possibility in section 6, we implicitly assumed the existence of a technology or device that enables children to credibly precommit when young to a certain level of gifts when middle-aged. We believe, however, that this assumption is inessential to the derivation of steady-state interest rate neutrality in the gift motive economy. Only the exact form–and not the essential nature-of the intertemporal and interpersonal first-order conditions underlying interest-rate neutrality seems to depend on whether parents engage in this type of strategic behavior.

In contrast, strategic behavior in a framework of cooperative bargaining between altruistic parents and children is likely to undercut the interest-rate neutrality proposition. This conjecture is based on the observation, stressed by Kotlikoff, Razin and Rosenthal, that government redistributions alter the strategic postures (that is, threat points) of parents and children in a cooperative bargaining framework, and that strategic postures in turn influence the magnitude of net transfers. Whether the optimal timing proposition

carries over directly to a cooperative bargaining framework is not clear, but the factors underlying the timing result in our noncooperative environment would seem to be present in a cooperative environment as well.

Appendix 1—Proof To Proposition 5

Part (a): In this intervention, $T_{3t} = -(1+r_{t+2})T_{2t}$. Using (20)-(22), the time $t+1$ partial equilibrium response of savings by the middle-aged is $-s_1T_{2t} + (1+r_{t+2})s_2T_{2t} = T_{2t}[(1+r_{t+2})s_2 - s_1] = -T_{2t}$. But from (23) and the government budget constraint, government savings rises by T_{2t}. Hence, the net effect on aggregate savings is nil.

Part (b): Consider a shift in the financing of a funded social security system from taxes on the middle-aged to taxes on the young. Since we want to deduce the steady-state effect of this intervention, assume that it has been in operation for more than one period as of $t+1$. Using the government budget constraint, and the steady-state condition $T_{1t} = T_{1,t+1}$, the accumulation of capital between $t+1$ and $t+2$ obeys

$$s[\alpha_2 W_{t+1} + (1+r_{t+1})T_{1,t+1}, \alpha_3 W_{t+2}, r_{t+2}] + T_{1,t+1} = \alpha(1+n)k_{t+2}$$

in the no-transfer economy,

$$s[(\alpha_1(1+n)+\alpha_2)W_{t+1} + (r_{t+1}-n)T_{1,t+1}, \alpha_3 W_{t+2}, r_{t+2}] + T_{1,t+1} = \alpha(1+n)k_{t+2}$$

in the transfer economy.

Now, calculate the partial equilibrium effect of the increase in $T_{1,t+1}$ on aggregate savings at $t+1$:

$$\frac{\partial S_{t+1}}{T_{1,t+1}} = \begin{cases} (1+r_{t+1})s_1 + (1+n) > 0, & \text{in the no-transfer economy;} \\ s_1 r_{t+1} + 1 + (1-s_1)n > 0, & \text{in the transfer economy.} \end{cases}$$

We can use this result to determine how the savings locus shifts. Differentiate the savings locus, holding k_{t+1} constant, to obtain

$$\frac{dk_{t+2}}{dT_{1,t+1}} = \frac{\partial S_{t+1}/\partial T_{1,t+1}}{\alpha(1+n)^2 - \alpha_3 s_2 k_{t+2} f''(k_{t+2}) - s_3 f''(k_{t+2})}.$$

By the regularity assumption, this expression exceeds zero. Hence, the intervention shifts the savings locus A upwards in Figure 1, and the steady-state capital stock rises. Combining this result with the neutrality result in part (a) proves part (b).

Part c: Consider an unfunded intervention financed by taxes on the middle-aged. That is, $T_{1t} = 0$ and $T_{3t} = -(1+n)T_{2,t+1}$. Using the steady-state condition $T_{2,t+1} = T_{2t}$

and the aggregate savings function, we obtain the partial equilibrium effect on savings in both economies:

$$\frac{\partial S_{t+1}}{\partial T_{2,t+1}} = -s_1 + (1+n)s_2 < 0,$$

using (21). Differentiating the aggregate savings locus for a fixed k_{t+1}, yields

$$\mathrm{d}k_{t+2}/\mathrm{d}T_{2,t+1} > 0,$$

using the regularity assumption.

When the unfunded intervention is financed by taxes on the young, the partial equilibrium response of aggregate savings is given by

$$\frac{\partial S_{t+1}}{\partial T_{1,t+2}} = \begin{cases} (1+n)^2 s_2 < 0, & \text{in the no- transfer economy;} \\ -(1+n)s_1 + (1+n)^2 s_2 < 0, & \text{in the transfer economy.} \end{cases}$$

Differentiating the aggregate savings locus as before, and using the regularity assumption, yields

$\mathrm{d}k_{t+2}/\mathrm{d}T_{1,t+2} < 0$. This proves part (c).

Part d: Compare the partial equilibrium savings responses for the two different methods of financing an unfunded system. In the no-loan economy, $(1+n)\partial S_{t+1}/\partial T_{2,t+1} < \partial S_{t+1}/\partial T_{1,t+2}$, so that a shift to taxes on the young, for a fixed old-age benefit, increases the capital stock. In the loan economy, $(1+n)\partial S_{t+1}/\partial T_{2,t+1} = \partial S_{t+1}/\partial T_{1,t+2}$, so that the generational incidence of the tax is irrelevant.

Appendix 2–Numerical Simulation Technique

Our numerical simulation technique is the same as the one used in Auerbach and Kotlikoff (1987). The procedure involves the following steps. (1) At the pre-intervention steady-state equilibrium ($t = 0$), calculate the aggregate capital stock, government debt, consumption loans, and the asset holdings for representative members of each cohort. This step essentially involves solving a system of equations that can be reduced to one nonlinear equation in one unknown, k. (2) Calculate the post-intervention steady-state equilibrium, and assume that the economy converges to the post-intervention steady-state after $T < \infty$ periods. (3) Conjecture a time path, $\{k_t^0\}_{t=0}^T$, for the capital stock, constraining the path to pass through the steady-state values calculated in step one. (4) Given the factor prices implied by the conjectured path for k (and given agents' initial pattern of asset holdings), solve the consumers' problems to obtain time paths for transfers, consumption, and saving. (5) Aggregate the solutions to the consumers' problems to obtain the implied time path for k, $\{\hat{k}_t^0\}_{t=0}^T$. (6) Construct a new path $\{k_t^1\}_{t=0}^T$ where $k_t^1 = \delta k_t^0 + (1-\delta)\hat{k}_t^0$ for $t = 0, \ldots T$, and $0 < \delta < 1$. (7) Using the new path for k, repeat steps (3)-(6) to obtain $\{k_t^n\}_{t=0}^T$, $n = 2, \ldots$ Continue until, for all $t \in [0,T]$, $|k_t^n - \hat{k}_t^n| < a$.

Appendix 3—Further Simulation Results

This appendix describes how our numerical simulation results are affected by varying 7 and σ. We first discuss the rationale for varying the degree of altruism with the intertemporal substitution elasticity.

The smaller the intertemporal substitution elasticity, the greater the desire to "flatten" the life-cycle consumption profile. In a pure life-cycle scenario, consumption smoothing is implemented entirely through the capital market, that is, savings decisions. With an altruistic preference specification, another consumption-smoothing device potentially operates: intergenerational transfers. Consumption-smoothing within the family can supplement or displace consumption-smoothing through the capital market. The two consumption-smoothing devices are (imperfect) substitutes, so that a greater desire to smooth consumption increases the scope for an operative transfer motive. By the same token, a greater degree of altruism reduces the scope for an active consumption-loans market. See Altig and Davis (1989a) for further discussion on this point.

Table A1 illustrates the interaction between the consumption-loans market (or borrowing constraints) and the operativeness of the transfer motive. The interaction effects are quite dramatic. Transfer motives are inoperative for values of 7 as large as .45 when utility is logarithmic. For the same value of 7, transfer motives are strong enough to overcome borrowing restrictions for values of a as large as 2/3. When a equals 1/2, the transfer motive operates for values of 7 as small as .15. Since we are interested in scenarios that fall under Regime B, we vary a and 7 simultaneously to ensure that these equilibria obtain.

Under alternative assumptions about these parameters, Tables A2-A4 show the long-run crowding-out ratios associated with the social security interventions considered in Figures 1-3. The most pronounced effect of varying the intertemporal substitution elasticity occurs in the context of unfunded interventions in the life-cycle regime: smaller intertemporal substitution elasticities significantly reduce the crowding-out ratio. This result occurs because government-imposed intergenerational redistributions have little impact on the lifetime consumption profile when individuals have a strong desire to smooth consumption intertemporally and have access to perfect capital markets. The importance of access to capital markets is easily seen by noting that the crowding-out ratio is not very sensitive to the intertemporal substitution elasticity when borrowing constraints bind.

Table A1: Interaction Between Altruism and Intertemporal Substitution

7	.15	.20	.25	.30	.35	.40	.45	.50
σ								
1	I	I	I	I	I	I	I	O
2/3	I	I	I	O	O	O	N	N
1/2	O	O	O	O	N	N	N	N
2/5	O	O	O	N	N	N	N	N
1/3	O	O	N	N	N	N	N	N

Notes: I indicates that borrowing constraints bind and transfers are zero.
O indicates that borrowing constraints bind and transfers are positive.
N indicates that borrowing restrictions do not bind and transfers are positive.

Table A2: Crowding-out Ratios: Tax on Young
Unfunded Social Security Intervention

	σ	1	2/3	1/2	2/5	1/3
	7	.52	.3	.2	.1	.1
Life-cycle		.8015	.7072	.5785	.4347	.3059
Inoperative		.2195	.2489	.2679	.2815	.2916
Operative		.5490	.5645	.5609	.5761	.5367

Notes: All crowding-out ratios refer to the outcome in the post-intervention steady state. See the text for the definition of the crowding-out ratio. The displayed values of 7 apply to the operative transfer regime; $\gamma = 0$ for the other two regimes.

Table A3: Crowding-out Ratios: Tax on Middle-Aged
Unfunded Social Security Intervention

	σ	1	2/3	1/2	2/5	1/3
	7	.52	.3	.2	.1	.1
Life-cycle		.2341	.1558	.0907	.0459	.0209
Inoperative		.6155	.6285	.6379	.6448	.6498
Operative		.5490	.5645	.5609	.5761	.5367

Note: See notes to Table A2.

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	Table A4: Crowding-out Ratio: Tax on Young Funded Social Security Intervention					
	σ 7	1 .52	2/3 .3	1/2 .2	2/5 .1	1/3 .1
Life-cycle		0	0	0	0	0
Inoperative		-.2104	-.1749	-.1502	-.1326	-.1186
Operative		-.1461	-.1256	-.1049	-.0963	-.0728

Notes: See notes to Table A2. Negative numbers indicate increases in the steady-state capital stock.

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