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USING SMVAM AS A LINEAR APPROXIMATION TO A NONLINEAR FUNCTION: A NOTE

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The statistical market value accounting model (SMVAM) has been proposed by Unal and Kane [6] as a way to model the components of the market value of a firm. Their model is as follows,

$$MV = U_e + kBV + e. \tag{1}$$

In equation (1), MV is the market value of the firm's shares or market value of equity and BV is the book value of equity. The constant term in the equation, U_e, represents the nonbooked part of equity, which is the value of the firm as an ongoing entity (see Thomson [5]). The slope coefficient, k, represents the adjustment factor the market applies to book equity. To put it another way, one minus k represents the discount (premium) the market places on one dollar of book equity. Finally, e is the random error term in the regression.[1]

More simply put, SMVAM is the relationship between the price of a firm's stock and its two components: the book value per share of the stock and the per-share value of the off-balance-sheet options and activities of the firm. SMVAM can be rewritten this way by dividing both sides of equation (1) by the number of shares outstanding.

SMVAM is a linear approximation to the call option that represents equity (see Black and Scholes [1]). That is, the market value of the firm's stock is the value of the option stockholders have to buy the firm back from the debtholders for the face value of the liabilities. The relationship between

market and book values is shown in figure 1. The broken line represents the value of the option on the expiration date when the option is in the money. The x-axis to the left of point A is the value of the option at expiration when it is out of the money. Finally, the solid curved line represents the value of the option before expiration.

The two lines that comprise the value of the call at expiration form the asymptotes for the unexpired call. As BV increases (decreases) to the right (left) of point A the curve approaches the broken line (x-axis) asymptotically and the slope of the function approaches b (zero), where b (zero) is the slope of the right (left) asymptote. At point A, the market value of the firm is comprised entirely of the off-balance-sheet options and activities of the firm. The value of these activities is represented by the point C on the y-axis. Demirguc-Kunt [2] shows that this curve can be approximated using a rectangular hyperbola.

$$MV = 0.5b(BV-A) + \sqrt{0.25b^2(BV-A)^2 + C^2} + u \qquad (2)$$

She proposes the rectangular hyperbola as an alternative to SMVAM. Unlike SMVAM, the rectangular hyperbola takes into account the nonlinearity of options and the nonnegativity constraint on option values.

I. The Relationship Between SMVAM and the Rectangular Hyperbola

Comparing SMVAM and the rectangular hyperbola as approximations to the equity call option in figure 1 allows us to investigate the properties of the SMVAM estimates. Specifically, it allows us to determine the nature and severity of the bias that arises as a result of using a linear approximation to a nonlinear function. As we shall see, SMVAM works fairly well when $BV > D$. Note that although we use the rectangular hyperbola to represent the nonlinear function, the overall results do generalize to other nonlinear approximations to the call option in figure 1.

To show the relationship between SMVAM and the rectangular hyperbola, we rewrite equation (2) as follows,

$$MV = C + b(BV - A) + \phi + u. \tag{3}$$

ϕ is the factor that takes into account the nonlinearity in the relationship between market and book values of equity, where

$$\phi = \sqrt{0.25b^2(BV-A)^2 + C^2} - (C + 0.5b(BV - A)). \tag{4}$$

Rearranging (3) gives us

$$MV = C - bA + bBV + \phi + u, \tag{5}$$

which is simply (1) with $U_e = C - bA$, $k = b$, and $e = \phi + u$.

There are two sources of bias to the coefficients of the **SWAM** regression. The first source is the nonlinear term, ϕ. Its impact on $\hat{k}$ and $\hat{U}_e$ will be discussed in section II. The second source of bias in the **SMVAM** regression occurs when A is not zero. This bias only has consequences for the intercept term, U_e. A positive (negative) value for A causes U_e to understate (overstate) the true C. Furthermore, when A is positive (negative) the intercept term will be negatively (positively) correlated with k.

II. Potential Biases in SMVAM Coefficients

From (5), it is clear that **SMVAM** suffers from a misspecification problem since estimating a linear approximation of a nonlinear function is equivalent to omitting relevant "explanatory variables" from the regression equation. That is, the error term in (1) includes the random error u and the omitted variable ϕ. If the omitted variable, ϕ, is correlated with the included explanatory variable, BV, the estimators of U_e and k will be biased and inconsistent. If ϕ is not correlated with BV, the estimator of U_e will still be biased and inconsistent but the estimator of k will be unbiased and consistent. However, the estimator of the variance of $\hat{k}$ will be biased upward, so that the usual tests of significance and confidence intervals for k are biased towards accepting the null hypothesis (see **Kmenta** [3]). Unfortunately, in this case ϕ and BV are correlated since ϕ is a function of BV.

This is not to say that **SMVAM** is not a valid approximation of the nonlinear function over certain ranges. As we will show, for BV > D (where D is some positive constant greater than A) **SMVAM** is a reasonable approximation to the nonlinear function. To see this, let $x = 0.5b(BV-A)/C$ and $y = \phi/C$. Dividing both sides of (4) by C and substituting in x and y yields

$$y = \sqrt{x^2 + 1} - (x + 1). \tag{6}$$

The size of the nonlinear term, y, is given as the difference between two functions. The first function is the first term on the right-hand side of (6). This function is a rectangular hyperbola defined for positive y. It is plotted as a function of x and y in figure 2. The second term on the right-hand side of (6) is a straight line that crosses the x-axis at -1.0 and the y-axis at 1.0 in figure 2. The difference between the two functions is the size of the nonlinear term, y.

As seen in figure 2, for large positive x (large BV relative to A) the size of the nonlinear term is bounded and approaches a constant value. That is, as x increases ϕ and BV become orthogonal. Therefore, if **SMVAM** is estimated over a sample where BV is always greater than some threshold value D, then $\hat{k}$ will be both unbiased and consistent. However, ϕ will cause U_s to be both biased and inconsistent.

The direction of the bias of $\hat{k}$ depends on the sign of the coefficient of ϕ, and the direction of the correlation between ϕ and BV. The bias is

positive when the coefficient of ϕ and the correlation between ϕ and BV have the same sign, otherwise it is negative. Since BV and ϕ are negatively correlated, the direction of the bias in $\hat{k}$ is determined by the coefficient of ϕ. The coefficient of 4 is the partial correlation between MV and 4. At large negative BV's, ϕ prevents MV from becoming negative and at large positive BV's, 4 again prevents MV from declining below the asymptote. Thus MV and ϕ are positively correlated and the coefficient of ϕ is positive resulting in a negative bias in $\hat{k}$.

Intuitively, the direction of the bias of $\hat{k}$ should be negative because SMVAM does not take into account the nonnegativity constraint on MV. The consequences of estimating SMVAM over the data is analogous to estimating ordinary least squares (OLS) over a sample truncated at zero. Maddala [4] shows that OLS in this case results in low-biased estimates of the slope coefficient. That is, failure to account for the truncation in the sample (in this case, the nonnegativity of MV), results in an estimated regression line that is flatter than the true line (see figure 3).[2]

Since the omission of ϕ results in a low-biased $\hat{k}$ and $\hat{U}_e$ is simply the projection of the SMVAM regression line on the y-axis, the omission of ϕ causes $\hat{U}_e$ to be a high-biased estimate of C (when A equals zero). As seen in figure 4, the bias in $\hat{U}_e$ is not only caused by the omission of ϕ but also nonzero A. Positive (negative) A biases $\hat{U}_e$ in the opposite (same)

direction as the omitted variable 4. Therefore, when A is positive (negative) the direction of bias of $\hat{U}_s$ is indeterminate (positive).

III. Conclusions

The statistical market-value accounting model is a reasonable approximation of the relationship between market and book equity for firms whose balance sheet has a positive liquidation value. For example, this should be true for Unal and Kane [6] where their data consists of portfolios of large commercial banks. Therefore, it is only the intercept term in their study that is biased and inconsistent while their slope coefficient is unbiased and consistent. However, when the sample includes firms whose balance sheet has low and even negative liquidation value, such as thrifts (see Thomson [5]), the linear approximation is no longer adequate since both the slope and intercept term of SMVAM are biased and inconsistent.

Footnotes

1) Unal and Kane apply SMVAM to banks where the interpretation of U_e is the net value of off-balance-sheet assets and liabilities (including federal deposit guarantees) and k is the adjustment factor the market applies to on-balance-sheet assets and liabilities.

2) Note the results of estimating SMVAM are similar to that of truncated regression because MV has a lognormal distribution.

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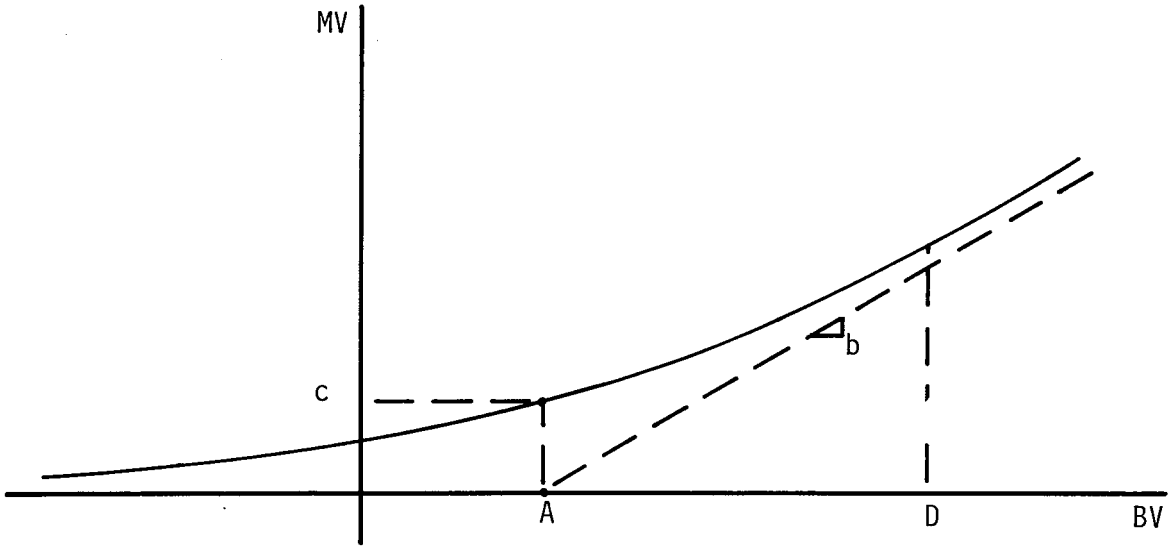
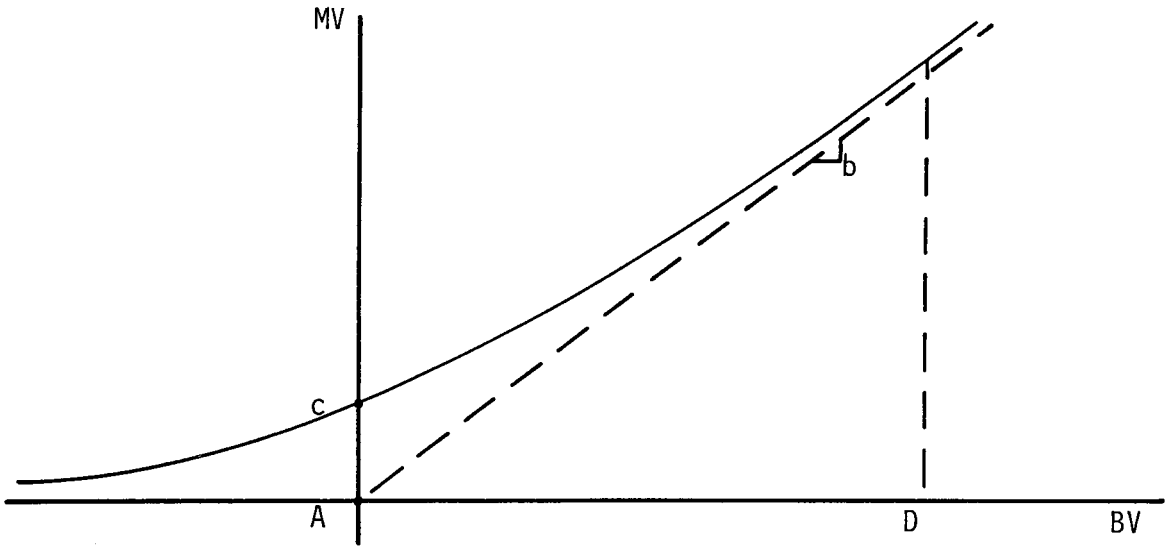
Figure 1: Equity Call Option

A



B



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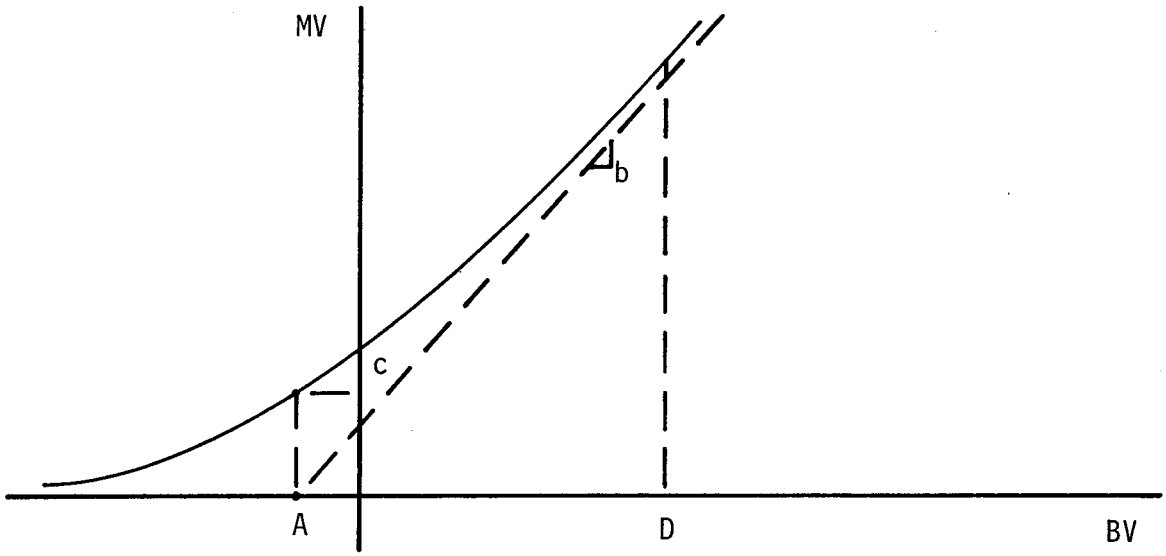

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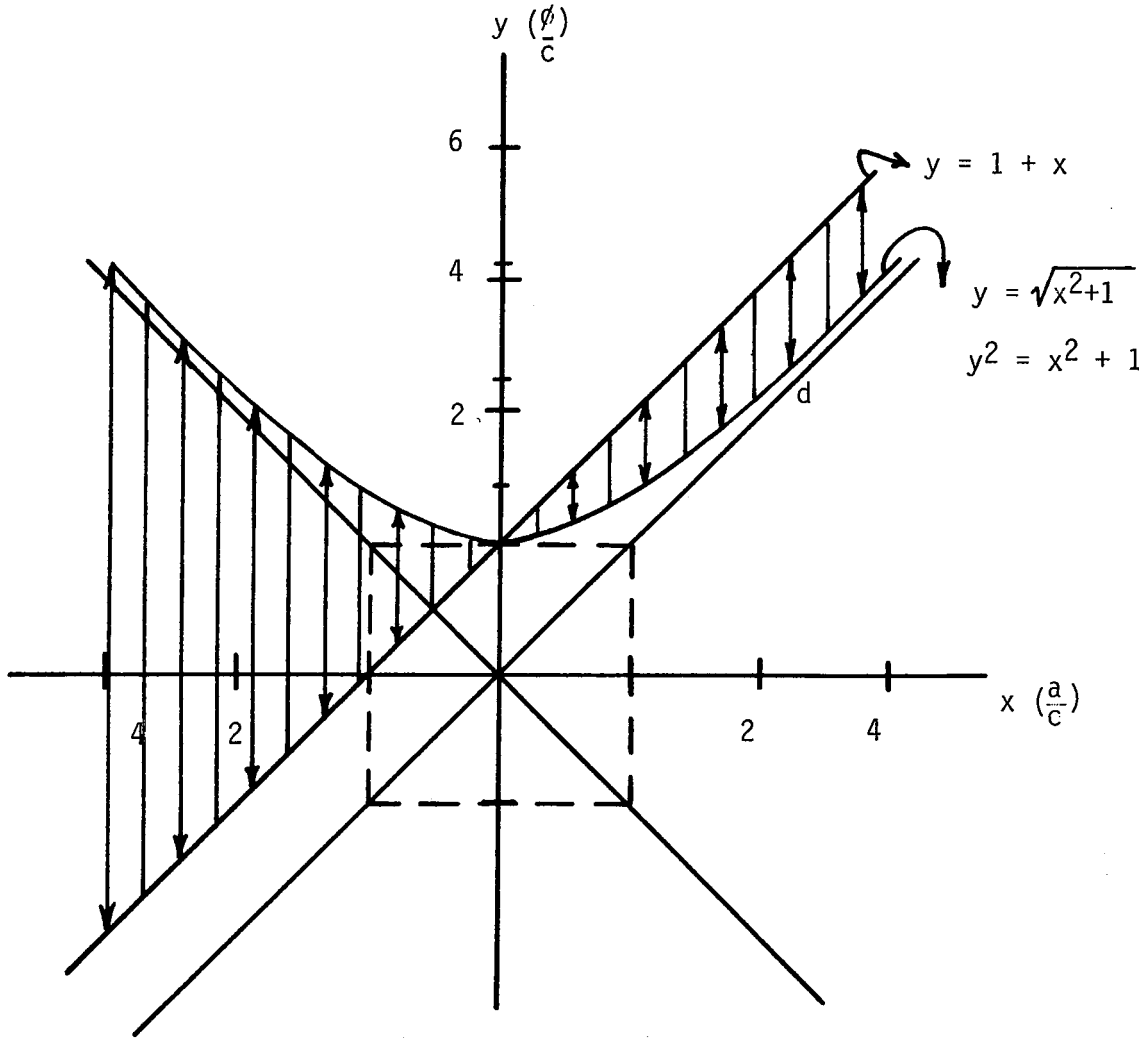
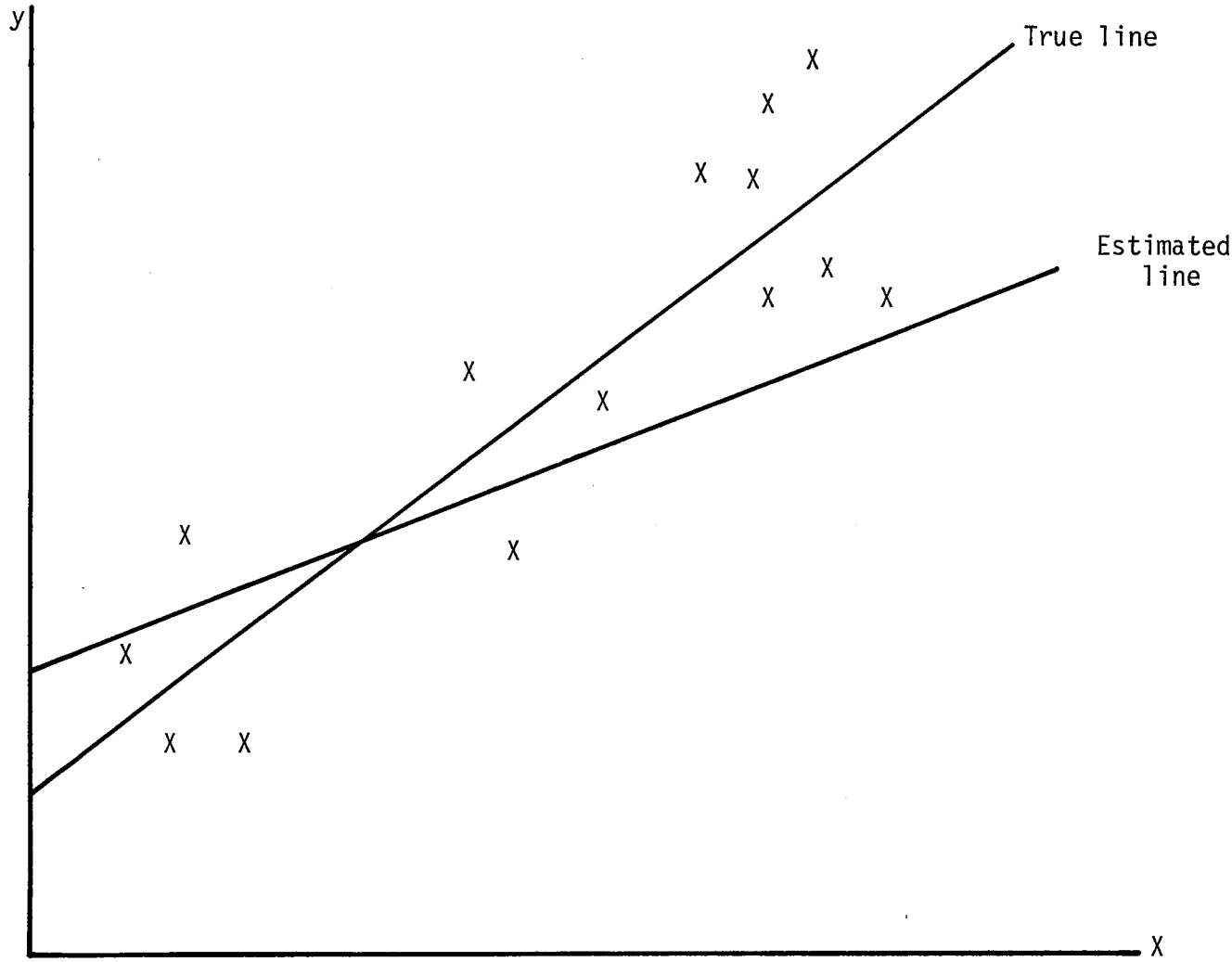
Figure 2: The size of the Omitted Nonlinear Term



Source: Authors

Figure 3: Truncated regression model



Source: Authors

Figure 4: Bias in U_e From nonzero A