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MONETARY POLICY UNDER RATIONAL EXPECTATIONS WITH MULTIPERIOD WAGE STICKINESS AND AN ECONOMY-WIDE CREDIT MARKET

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ABSTRACT

The effects of money supply responses to both the current interest rate and the lagged state of the economy are analyzed in a dynamic IS-LM model with multiperiod wage stickiness. Allowing private agents to use the current interest rate in forming expectations creates some potentially counter-intuitive effects of shocks, but leaves policy implications largely unaffected. A conflict between firm-level and aggregate-level output stabilization is found to be inherent in the multiperiod wage contracting scheme. Policy rules that stabilize output or serve other objectives are explicitly derived. Because of supply shocks, procyclical money growth may be required to stabilize firm output, prices, real wages, or output relative to its full-information level.

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I. Introduction

This article reconsiders the role of monetary policy in a Keynesian model that incorporates rational expectations and displays the natural rate property. Important features are multiperiod wage stickiness and the availability, to both private agents and the policymaker, of current information from observations of the nominal interest rate. The model is essentially a log-linearized version of that of Fischer (1977), augmented by an (implicit) economy-wide credit market. As in Fischer's influential paper, feedback from the lagged state of the economy is consequential for output behavior because of the conjunction of multiperiod wage stickiness and autocorrelation of shocks. At the same time, because of incomplete current information, policy responses to the current interest rate are also of consequence for output, as in the famous analysis of Poole (1970).

The analysis of this article is of interest from three different standpoints. First, it addresses the analytical problem of current information sets under rational expectations. The analysis finds that private use of current information from the interest rate to update expectations does not necessarily have the dramatic implications for policy that Dotsey and King (1983, 1986) obtain in completely flexible-price models. In particular, while policy responses to the interest rate are irrelevant for output in flexible-price models, such responses are potentially important influences on the behavior of output in sticky-wage models.

Second, this article extends the classic analysis of optimal policy under uncertainty, inaugurated by Poole, to an expanded IS-LM model in which prices are not fixed, the natural rate proposition holds, and supply shocks occur. Surprisingly, policy rules to stabilize output, or to serve other objectives, in models with plausible specifications of commodity supply behavior apparently have not heretofore been derived.

A third reason this article is of interest is that it treats the problem of inconsistency between three potential policy objectives: (1) stabilization of total output, (2) stabilization of typical individual firm output, and (3) stabilization of output around its perfect foresight or full-information level. The inconsistency between the first and second objectives has not been noted before, and is inherent in the staggered wage contracting setup. The conflict between the first and third objectives has been noted before, but its implications for policy rules in sticky-wage models have not been developed as fully as in this article. The appropriate response of policy to a commodity supply shock will depend on which objective is embraced. If the third objective is chosen, money should be increased in response to supply shocks, and the same may also be true under the second objective. Numerical examples show that the behavior of money appropriate to either of these objectives can easily involve a positive correlation between money and output.

In studying the effects of private agents' use of current information from the interest rate, this article joins Barro (1980), King (1983), Canzoneri, Henderson, and Rogoff (1983), and Dotsey and King (1983, 1986). These prior studies make simplifying assumptions to render the analysis tractable, including, most importantly, limits on policy alternatives, allowance for only a few economic disturbances, or both. Even these simplifications have not always permitted explicit solutions to the model. The present article offers closed-form solutions without imposing any arbitrary restrictions on monetary policy, enabling derivation and qualitative characterization of policies that satisfy various objectives. Also, the analysis here includes three distinct aggregate disturbances, for commodity supply, commodity demand, and money demand.

The expectational effects of current observation of the interest rate create the potential for counterintuitive responses of the economy to some disturbances. For example, a positive shock to the money supply may reduce output and prices. However, in contrast to the other studies, this article finds that current information does not appear to change the behavior of output in the model in a way that is likely to be quantitatively important, nor does it seem to have very important or surprising implications for monetary policy. The conclusions and intuitions of Poole and Fischer concerning output-stabilizing policy need not be importantly affected by the presence of current information.

II. Review of Recent Literature

This section presents a stylized perspective of the literature on policy effectiveness, rational expectations, current information, and sticky-wage models as they relate to the issues of this article.

A major problem in rational expectations macroeconomic models is that, if private agents have partial information about the current state of the economy, then unambiguous qualitative conclusions about the effects of monetary policy are often difficult to deduce. Indeed, unless the structure of the model is severely restricted, it is often difficult to derive analytical solutions to the model. From the standpoint of monetary policy analysis, the problem of contemporaneous information appears especially critical for whether or how money supply responses to the nominal interest rate affect the behavior of output and other variables of concern.

Traditionally, as in Poole (1970), the monetary authority has been assumed to use the current interest rate as well as delayed information on the state of the economy in setting the money supply. An optimal policy generally takes the form

$$m_t=qR_t+\underline{\mu}S_{t-1}, \tag{1}$$

where m_t is the money stock, R_t is the current nominal interest rate, and S_{t-1} is the lagged state of the economy. q and $\underline{\mu}$ are policy response coefficients relating the money supply to current and lagged information. In Poole's analysis, the relevance of q--the response of policy to the interest rate--arises from the policymaker's incomplete information concerning the state of the economy, while the relevance of $\underline{\mu}$--the response of policy to the lagged state of the economy--arises from autocorrelation in economic shocks, together with implicit, perfectly elastic supply behavior (fixed prices). Poole's particular contribution was to explicitly relate structural parameter and stochastic assumptions to the characteristics of a stabilization policy.

Sargent and Wallace (1975), in a model that introduced an explicit supply sector, brought the relevance of $\underline{\mu}$ into question, but preserved the relevance of q.[1] They imposed rational expectations on private agents, who used S_{t-1}, the lagged state, but not R_t, the current interest rate, as information in forming expectations of future prices. Private agents knew R_t but did <u>not</u> use it to update inflation expectations. The relevance of q in the model of Sargent and Wallace depended on this violation of Muthian rational expectations: agents wasted available information contained in the interest rate.

Subsequently, Barro (1976) demonstrated that policy effectiveness in then-extant macromodels with the natural rate property generally depended on superior information of the policymaker relative to private agents. It seemed that, unless the policymaker knew more than private agents, both q and μ would be irrelevant for the behavior of output. Because private agents do actually have equal access to information on current interest rates and to virtually the same information on the lagged state of the economy, the relevance of monetary policy appeared dubious.

Subsequently, King (1983) and Dotsey and King (1983, 1986) found that, via a mechanism involving private, heterogeneous information sets that included local prices as well as the global interest rate, policy was an effective determinant of output behavior after all.[2] But only money-supply responses to the past state of the economy, μ, were effective; q was irrelevant for real variables.[3] Dotsey and King argued that rational expectations dramatically reverses the conclusions of Poole on the relevance of q, and also reverses the conclusions of Sargent and Wallace on the irrelevance of μ. However, μ is relevant via a very different mechanism than envisioned by Poole. Also, the heterogeneous information mechanism creates analytical intractabilities and ambiguities that prevent derivation of explicit solutions for the model or qualitative knowledge of how various values of μ would influence economic outcomes, making it difficult to design appropriate state-dependent money rules. Certainly, Dotsey and King's results provided no basis for conventional countercyclical policies, and indeed were used to suggest that rational expectations destroys the case for such policies.

However, these provocative arguments of Sargent and Wallace, Barro, King, and Dotsey concerning policy were illustrated in flexible wage and price models, in which output deviations from optimal levels were due strictly to incomplete information. Fischer (1977) showed that the relevance of μ for output behavior follows from the assumption that wages are sticky for longer than the global information lag, or for at least two periods. However, lacking a credit market, the interest rate did not enter Fischer's model, so that the relevance of q could not be assessed.

Canzoneri, Henderson, and Rogoff (1983) added further ambiguity to the issue of whether q and μ were relevant. Given that wages are sticky for one period and that agents making demand decisions ("investors") use all the information contained in the current interest rate, Canzoneri, et al. derive two sets of values for (q,μ) that result in the minimum output variance: one of these has $(q=0,\ \mu\neq 0)$, while the other has $(q\neq 0,\ \mu=0)$.[4]

In the next section, a model is presented with two-period wage contracts that makes both q and μ relevant, and in much the same way as in the analysis of Poole, despite the addition of rational expectations, information sets that include the current interest rate, a supply sector that displays the natural rate property, and supply shocks. In effect, the analysis invokes Fischer's multiperiod wage stickiness to rescue Poole's policy from vulnerability to the ineffectiveness propositions arising from recent work in expectations formation. Although Fischer himself has already done so with regard to μ, the more recent arguments concerning contemporaneous information require that the relevance of q be reexamined.

III. A Prototype Keynesian Model

The particular model was chosen to implement three major characteristics as straightforwardly as possible. First, the model generates a substantial role for countercyclical monetary policy via a Keynesian multiperiod sticky-wage mechanism. Second, it incorporates an equilibrium wage dynamic and does not rely on money illusion, persistent expectational biases, or ignored information among private agents. As a result, it displays the natural rate property: mean values of real variables are invariant with respect to monetary factors, including monetary policy.[5] Third, the model possesses an implicit economy-wide credit market that clears simultaneously with the commodity market, as in the IS-LM model, introducing the interest rate as a variable that is jointly endogenous with other real and nominal variables. This facilitates analysis of monetary policies that condition the money supply on the interest rate, as in Poole's (1970) famous analysis. These three features are implemented by combining the dynamic IS-LM model with the supply sector of Fischer (1977) and by imposing rational expectations.

The output supply sector is a straightforward log-linear adaptation of Fischer's. Firms decide upon output after observing current prices and taking into account predetermined wages, set either one or two periods ago. To fix ideas, and without loss of generality, the economy can be seen as composed of two groups of firms. The first group consists of firms signing contracts at the end of period t-1, and the second group consists of firms that signed contracts at the end of period t-2. Any particular firm will be in group one in the first year of its contract and in group two in the second year. For example, the contract signed by a firm at the end of period t-2 provided for wages in t-1, when it was a group one firm, and for wages in t, when it is a group two firm. Likewise, the contract signed at the end of period t-1 provides for group one wages in period t and for group two wages in period t+1. Wages are always set to match the expected price level, thus attempting to stabilize real wages in the face of uncertain inflation. This scheme calls for wages to change for every firm in each period, but only the most recently contracting firms, group one, have wages that reflect knowledge of the state of the economy in period t-1. Furthermore, no wages are adjusted in immediate response to contemporaneous shocks, but only after a new contract is signed.

Equations (2) state these assumptions formally.

$$w_{1,t-1}=E_{t-2}p_{t-1} \text{ and } w_{2t}=E_{t-2}p_t;$$

$$w_{1t}=E_{t-1}p_t \text{ and } w_{2,t+1}=E_{t-1}p_{t+1}; \tag{2}$$

where w_{it} is the log of the nominal wage at group i firms and p_t is the log of the price level.

Firms are assumed to have a constant, positive elasticity of supply with respect to real wages. Formally, firm supply functions are:

$$y_{it} = \alpha+\beta(p_t-w_{it})+u_t, \quad i=1,2, \ \beta>0, \tag{3}$$

where y_i is output of group i, and u_t is an economy-wide supply shock. The preservation of linearity of the model is achieved by approximating the log of the sum of group outputs as:

$$y_t=k_0+(1/2)(y_{1t}+y_{2t}). \tag{4}$$

Finally, a supply function or Phillips Curve that relates output to price-level surprises is found from combining (2), (3), and (4):

$$y_t = (k_0+\alpha)+(1/2)\beta\Sigma_{i=1}^{2}(p_t-E_{t-i}p_t)+u_t. \tag{5}$$

The key feature of the supply function (4) is that output responds equally to deviations of the price level from its expectation formed both one and two periods ago, when contracts currently in effect were signed.

The commodity demand (or IS) equation, (6), reflects the assumption that the commodity market and the credit market clear simultaneously in the sense that agents know the economy-wide interest rate and the current price in their local market when making demand decisions. Demand depends on the (ex ante) real rate of interest.

$$y_t = b_0-b_1[R_t-(E^{+}_{t-1}p_{t+1}-p_t)]+x_t \tag{6}$$

where

$$E^{+}_{t-1}p_{t+1} = E[p_{t+1}|\Omega_t], \tag{7}$$

Ω_t = observable state of economy at time t

$= \{R_t;S_{t-1}\}$,

and S = state vector (given a specific identity in the next section).

The nominal interest rate, R_t, is measured as the natural logarithm of unity plus the coupon rate of return. The future price expectation, $E^+_{t-1}p_{t+1}$, is conditioned on the observed state of the economy, Ω_t, an information set that includes the current economy-wide interest rate, R_t, and the lagged state vector, S_{t-1}.[6] $E^+_{t-1}p_{t+1}$ can differ from $E_{t-1}p_{t+1}$ because agents are permitted to know the nominal interest rate to update their inflation expectations. x_t is a stochastic demand shock.

Money demand is conventional.

$$m_t-p_t = a_0 - a_1R_t+a_2y_t+v_t, \tag{8}$$

where v_t is a random disturbance.

Monetary policy is characterized by a decision rule for the quantity of money as a function of the observed state of the economy, or a money supply rule. The simplest adequate form of this rule is

$$m_t=qR_t+\mu_0+\mu_1u_{t-1}+\mu_2v_{t-1}+\mu_3x_{t-1}. \tag{9}$$

The shocks u, v, and x are first-order autoregressive processes with innovations ε, η, and λ, respectively. All shocks are mutually uncorrelated.

$$u_t=\rho_1u_{t-1}+\varepsilon_t, \quad 0<\rho_1<1, \quad \varepsilon_t \sim N(0,\sigma^2_\varepsilon)$$

$$v_t=\rho_2v_{t-1}+\eta_t, \quad 0<\rho_2<1, \quad \eta_t \sim N(0,\sigma^2_\eta)$$

$$x_t=\rho_3x_{t-1}+\lambda_t, \quad 0<\rho_3<1, \quad \lambda_t \sim N(0,\sigma^2_\lambda)$$

$$E(\varepsilon_t\eta_t)=E(\eta_t\lambda_t)=E(\varepsilon_t\lambda_t)=0$$

$$E(\varepsilon_t\varepsilon_{t-i})=E(\eta_t\eta_{t-i})=E(\lambda_t\lambda_{t-i})=0, \quad i>0. \tag{10}$$

IV. Analysis of the Model

Although the model is extremely simple, the conjunction of contemporaneous information and rational expectations makes the solution rather complex. Many uncouth details of the solution method are banished to an appendix, but some fundamental points are discussed in the text of this section.

A. The Endogenous Variables and the State Vector

The endogenous variables of essential concern are total output, group 1 and group 2 output, the price level, the money stock, and the interest rate (y_t, y_{1t}, y_{2t}, p_t, m_t, and R_t). Using the method of undetermined coefficients, each endogenous variable will be assigned its own trial equation in terms of the state vector. From these trial equations are derived implied linear reduced-form equations for the endogenous expectations variables, $E_{t-1}p_t$, $E_{t-2}p_t$, and $E^{*}_{t-1}p_{t+1}$.

The state vector is determined as follows. First, obvious candidates for inclusion are all predetermined and exogenous variables explicitly appearing in the structural equations, including the autoregressive processes (9). These are the lagged level of structural disturbances, u_{t-1}, v_{t-1}, and x_{t-1}, and the current innovations, ϵ_t, η_t, and λ_t. A unit "variable" is also included among the state vector to allow for intercepts in the reduced-form equations. (The constants are of no interest for cyclical analysis and will not appear beyond this section.) The appearance of the predetermined prior expectations, $E_{t-2}u_{t-1}$, $E_{t-2}v_{t-1}$, and $E_{t-2}x_{t-1}$, is required even though they do not appear explicitly in the structural equations. This is because, if the state vector of the trial solution does not include these terms, then the required identities cannot hold.[7] Expectations formed two periods ago generally matter for the endogenous variables because of the existence of contracts made two periods ago, based on then-available information.

This appearance of predetermined prior expectations of the exogenous shock processes raises the question of why the policy equation of the previous section was able to omit these artificial state variables. The reason is that, under any of the policy criteria examined in this article, money supply coefficients on these state variables are redundant and, hence, their values are indeterminate. They have been arbitrarily set to zero in this study in order to determine the policy vector uniquely and to simplify analysis. If it were desired to study policies directed toward a mixed or compound objective--such as both output and price stability, or policies to stabilize both interest rates and prices--then the policy rule should generally include nonzero coefficients on $E_{t-2}u_{t-1}$, $E_{t-2}v_{t-1}$, and $E_{t-2}x_{t-1}$.

Thus, the conclusion is that the state vector

$S_t=\{u_{t-1},\ v_{t-1},\ x_{t-1};\ \varepsilon_t,\ \eta_t,\ \lambda_t;\ E_{t-2}u_{t-1},\ E_{t-2}v_{t-1},\ E_{t-2}x_{t-1}\}$

is adequate to describe the position of the system, ruling out bootstraps.

Table 1 provides a glossary of endogenous and state variables that will appear in the trial solution.

B. The Trial Solution

Then the trial solution includes the following equations. (11) through (16) are implications of the assumed linearity and of the state vector S_t. (17) and (18) are direct implications of (13).

$$y_{1t}= \Pi_{10}+\Pi_{11}u_{t-1}+\Pi_{12}v_{t-1}+\Pi_{13}x_{t-1}+\Pi_{14}\varepsilon_t+\Pi_{15}\eta_t+\Pi_{16}\lambda_t +\Pi_{17}E_{t-2}u_{t-1}+\Pi_{18}E_{t-2}v_{t-1}+\Pi_{19}E_{t-2}x_{t-1} \quad (11)$$

$$y_{2t}= \Pi_{20}+\Pi_{21}u_{t-1}+\Pi_{22}v_{t-1}+\Pi_{23}x_{t-1}+\Pi_{24}\varepsilon_t+\Pi_{25}\eta_t+\Pi_{26}\lambda_t +\Pi_{27}E_{t-2}u_{t-1}+\Pi_{28}E_{t-2}v_{t-1}+\Pi_{29}E_{t-2}x_{t-1} \quad (12)$$

$$p_t= \Pi_{30}+\Pi_{31}u_{t-1}+\Pi_{32}v_{t-1}+\Pi_{33}x_{t-1}+\Pi_{34}\varepsilon_t+\Pi_{35}\eta_t+\Pi_{36}\lambda_t +\Pi_{37}E_{t-2}u_{t-1}+\Pi_{38}E_{t-2}v_{t-1}+\Pi_{39}E_{t-2}x_{t-1} \quad (13)$$

$$m_t= \Pi_{40}+\Pi_{41}u_{t-1}+\Pi_{42}v_{t-1}+\Pi_{43}x_{t-1}+\Pi_{44}\varepsilon_t+\Pi_{45}\eta_t+\Pi_{46}\lambda_t +\Pi_{47}E_{t-2}u_{t-1}+\Pi_{48}E_{t-2}v_{t-1}+\Pi_{49}E_{t-2}x_{t-1} \quad (14)$$

$$R_t= \Pi_{50}+\Pi_{51}u_{t-1}+\Pi_{52}v_{t-1}+\Pi_{53}x_{t-1}+\Pi_{54}\varepsilon_t+\Pi_{55}\eta_t+\Pi_{56}\lambda_t +\Pi_{57}E_{t-2}u_{t-1}+\Pi_{58}E_{t-2}v_{t-1}+\Pi_{59}E_{t-2}x_{t-1} \quad (15)$$

$$y_t= \Pi_0+\Pi_1u_{t-1}+\Pi_2v_{t-1}+\Pi_3x_{t-1}+\Pi_4\varepsilon_t+\Pi_5\eta_t+\Pi_6\lambda_t+ + \Pi_7E_{t-2}u_{t-1}+\Pi_8E_{t-2}v_{t-1}+\Pi_9E_{t-2}x_{t-1} \quad (16)$$

TABLE 1

GLOSSARY OF VARIABLES IN THE TRIAL SOLUTION

ENDOGENOUS VARIABLES

y_t. Log of total output

y_{1t}. Log of output of group one

y_{2t}. Log of output of group two

p_t. Log of price level

m_t. Log of money stock

R_t. Log of [1 + nominal interest rate]

Endogenous Expectations

$E_{t-1}p_t = E[p_t | S_{t-i}]$, $i=1,2$. (Prior expectations of p_t)

$E^+_{t-1}p_{t+1} = E[p_{t+1} | \Omega_t] = E[p_{t+1} | R_t, S_{t-1}]$. (Updated expectation of p_{t+1})

STATE VARIABLES

Predetermined

u_{t-1}. Observed level of aggregate supply

v_{t-1}. Observed level of money demand

x_{t-1}. Observed level of aggregate demand

$E_{t-2}u_{t-1}$.

$E_{t-2}v_{t-1}$.

$E_{t-2}x_{t-1}$.

Exogenous

ε_t. Innovation to aggregate supply

η_t. Innovation to money demand

λ_t. Innovation to aggregate demand

$$E_{t-1}p_t=\Pi_{30}+\Pi_{31}u_{t-1}+\Pi_{32}v_{t-1}+\Pi_{33}x_{t-1}+\Pi_{37}E_{t-2}u_{t-1}+\Pi_{38}E_{t-2}v_{t-1}+\Pi_{39}E_{t-2}x_{t-1} \qquad (17)$$

$$E_{t-2}p_t=\Pi_{30}+(\Pi_{31}+\Pi_{37})E_{t-2}u_{t-1}+(\Pi_{32}+\Pi_{38})E_{t-2}v_{t-1}+(\Pi_{33}+\Pi_{39})E_{t-2}x_{t-1} \qquad (18)$$

The derivation of the trial solution for $E^+_{t-1}p_t$ deserves special comment. This expectation is composed of two orthogonal components:

$$E^+_{t-1}p_{t+1}=E_{t-1}p_{t+1}+[E^+_{t-1}p_{t+1}-E_{t-1}p_{t+1}]. \qquad (19)$$

The first component of (19) is the prior expectation of next period's price level, which is conditioned on last period's full realization of the state of the economy, S_{t-1}. Using (13), and the assumptions about the error processes, (10), this is seen to be

$$E_{t-1}p_{t+1}=\Pi_{30}+\rho_1(\Pi_{31}+\Pi_{37})u_{t-1}+\rho_2(\Pi_{32}+\Pi_{38})v_{t-1}+\rho_3(\Pi_{33}+\Pi_{39})x_{t-1}. \qquad (20)$$

The second component of (19) is the revision to the future-price expectation that occurs after agents observe the current nominal interest rate, R_t. This second term, then, constitutes an updating of the future price expectation based on the information content of the nominal interest rate. From the trial solution specification (13), it is clear that the influence of current (period-t) shocks on p_{t+1} must be

$$(p_{t+1}-E_{t-1}p_{t+1}) = (\Pi_{31}\varepsilon_t+\Pi_{32}\eta_t+\Pi_{33}\lambda_t). \qquad (21)$$

The expectation of this component of the future price level formed at time t-1 is clearly zero. But at time t, agents can condition this component on the innovation in the nominal interest rate,

$$(R_t-E_{t-1}R_t)=(\Pi_{54}\varepsilon_t+\Pi_{55}\eta_t+\Pi_{56}\lambda_t). \qquad (22)$$

The future price expectation updating coefficient, $\theta=d[E^+_{t-1}p_{t+1}-E_{t-1}p_{t+1}]/d[R_t-E_{t-1}R_t]$, is essentially the slope of the least-squares regression line relating $(p_{t+1}-E_{t-1}p_{t+1})$ to $(R_t-E_{t-1}R_t)$.

$$\theta = \frac{E\{[p_{t+1}-E_{t-1}p_{t+1}][R_t-E_{t-1}R_t]\}}{E[R_t-E_{t-1}R_t]^2}$$

$$= \frac{(\Pi_{31}\Pi_{54}\sigma^2_\varepsilon+\Pi_{32}\Pi_{55}\sigma^2_\eta+\Pi_{33}\Pi_{56}\sigma^2_\lambda)}{(\Pi^2_{54}\sigma^2_\varepsilon+\Pi^2_5\text{ sO:+\textasciitilde: }{}_6\sigma^2_\lambda)} \qquad (23)$$

Then the updated future price expectation is

$$E^{+}_{t-1}p_{t+1}=\Pi_{30}+\rho_1(\Pi_{31}+\Pi_{37})u_{t-1}+\rho_2(\Pi_{32}+\Pi_{38})v_{t-1}+\rho_3(\Pi_{33}+\Pi_{39})x_{t-1}$$
$$+\theta(\Pi_{54}\varepsilon_t+\Pi_{55}\eta_t+\Pi_{56}\lambda_t). \quad (24)$$

The effect of the assumption that agents use the information content of the interest rate is to add the θ-term, with θ defined by (23), to the right-hand side of (24). The effect of the assumption can be seen in the solution for the model, in the presence of θ.

C. General Properties

The appendix presents the identities among Π-coefficients implied by the structure, the solutions for these coefficients, and the final form solution of the model. These solutions, (A.18) through (A.21), reflect some important general properties of the model, due to its two-period Fischer contracts. The effect of money demand or commodity demand shocks two periods or more earlier is only on the nominal variables such as the price level, p_t, nominal wages, w_t, the interest rate, R_t, and the money stock, m_t, and not on real variables such as output, y_t, the real rate of interest, $[R_t-(E^{+}_{t-1}P_{t+1}-P_t)]$, and real wages, $w_{it}-p_t$, $i=1,2$. Group one output, y_{1t}, is determined by the entire history of commodity supply innovations, $\{\varepsilon_\tau\}^{t}_{\tau=-\infty}$, and on period-t money demand and commodity demand innovations, η_t and λ_t. But y_{1t} is unaffected by anticipated levels of money demand and commodity demand shocks, and hence is independent of lagged values of η and λ. Similarly, group two output, y_{2t}, is determined by the entire history of commodity supply innovations and on period-t and (t-1) money demand and commodity demand innovations: η_t, η_{t-1}, λ_t, and λ_{t-1}. The effects of each innovation on nominal variables die out, beyond the second lag, geometrically with a rate of decay strictly dependent on the appropriate autocorrelation coefficient.

Policy parameters $\{q,\mu_1,\mu_2,\mu_3\}$ are all generally consequential for the behavior of output. Monetary policy can influence the output effects of current (period-t) shocks and can eliminate output effects of shocks occurring in the previous period (t-1). It is infeasible for monetary policy to affect the component of the business cycle due to supply shocks after their price level implications become reflected in all wage contracts (two periods). Hence, supply shocks occurring in $t-j$, $j>1$, have effects on real variables, including output, that policy cannot modify. Money supply shocks or control errors need not be formally introduced into the model; they have effects identical in magnitude but opposite in sign to money demand shocks. Constant terms can also be excluded from further consideration, since they have no implications for cyclical behavior.

V. Constant Money Supply

Examining the hypothetical constant-money case helps make the model's properties more transparent. It is useful to review the familiar properties of conventional IS-LM-type models under a constant money policy. Usually, commodity supply shocks have positive effects on output and negative effects on prices. Commodity demand shocks have positive effects on output, prices, and the interest rate. Money demand shocks have positive effects on the interest rate, but negative effects on output and prices. The model can be regarded as displaying these usual properties of IS-LM models in a very broad and general way. However, some of the signs of coefficients of the final form are formally ambiguous, and some anomalous cases are plausible.

In the model at hand, under the constant money policy, $\{q,\mu_1,\mu_2,\mu_2\}=0$, the solution for prices, output, and the interest rate reduces to the equations below.

$$y_t=\{1-\beta[a_1+a_2b_1(1-\theta)]J\}\varepsilon_t+\rho_1[1-\beta(1+a_1)(a_1+a_2b_1)G_1]\varepsilon_{t-1}+\Sigma_{j=2}^{\infty}\rho_1^j\varepsilon_{t-j}$$

$$-b_1\beta(1-\theta)J\eta_t-b_1\beta\rho_2(1+a_1)G_2\eta_{t-1}+\beta a_1J\lambda_t+\beta a_1(1+a_1)\rho_3G_3\lambda_{t-1} \quad (25)$$

$$p_t = -[a_1+a_2b_1(1-\Theta)]J\varepsilon_t - 2(1+a_1)\rho_1(a_1+a_2b_1)G_1\varepsilon_{t-1}$$
$$-\Sigma_{j=2}^{\infty}\rho_1^j(a_1+a_2b_1)[2(1+a_1)+(\beta/b_1)(a_1+a_2b_1)]G_1\varepsilon_{t-j} - b_1(1-\Theta)J\eta_t$$
$$-2b_1\rho_2(1+a_1)G_2\eta_{t-1} - \Sigma_{j=2}^{\infty}\rho_2^j[\beta(a_1+a_2b_1)+2b_1(1+a_1)]G_2\eta_{t-j}$$
$$+a_1J\lambda_t + 2a_1(1+a_1)\rho_3G_3\lambda_{t-1} + \Sigma_{j=2}^{\infty}\rho_3^j[2(1+a_1)+(\beta/b_1)(a_1+a_2b_1)]a_1G_3\lambda_{t-j} \tag{26}$$

$$R_t = -(1-a_2b_1)J\varepsilon_t + (1/a_1)\rho_1[a_2-(2+a_2\beta)(1+a_1)(a_1+a_2b_1)G_1]\varepsilon_{t-1}$$
$$+\Sigma_{j=2}^{\infty}\rho_1^j\{(a_2/a_1)-(a_1+a_2b_1)[(\beta/b_1)(1-a_2b_1)+(1/a_1)(2+a_2\beta)(1+a_1)]G_1\}\varepsilon_{t-j}$$
$$+(b_1+\beta)J\eta_t + \rho_2\{(\beta+2b_1)[1+a_1(1-\rho_2)]-\rho_2b_1(2+a_2\beta)\}G_2\eta_{t-1}$$
$$+\Sigma_{j=2}^{\infty}\rho_2^j\{(\beta+2b_1)[1+a_1(1-\rho_2)]-\rho_2b_1(2+a_2\beta)-\beta(1-a_2b_1)\}G_2\eta_{t-j}$$
$$+(1+a_2\beta)J\lambda_t + \rho_3(2+a_2\beta)(1+a_1)G_3\lambda_{t-1} + \Sigma_{j=2}^{\infty}\rho_3^j[2(1+a_1)+(\beta/b_1)(a_1+a_2b_1)]G_3\lambda_{t-j} \tag{27}$$

where

$$J = [a_1(b_1+\beta)+b_1(1-\Theta)(1+a_2\beta)]^{-1} \tag{28}$$

$$G_i = \{[1+a_1(1-\rho_i)][\beta(a_1+a_2b_1)+2b_1(1+a_1)]\}^{-1} \tag{29}$$

and

$$\Theta = \frac{[b_1(1+a_1)+\beta(a_1+a_2b_1)]Z}{\Sigma+b_1(1+a_2\beta)Z} \tag{30}$$

where

$$Z = 2(1+a_1)\left\{\begin{array}{l}(1-a_2b_1)(a_1+a_2b_1)G_1\rho_1\sigma_\varepsilon^2 \\ -b_1(b_1+\beta)G_2\rho_2\sigma_\eta^2 \\ +a_1(1+a_2\beta)G_3\rho_3\sigma_\lambda^2\end{array}\right\} \tag{31}$$

and

$$\Sigma = (1-a_2b_1)^2\sigma_\varepsilon^2 + (b_1+\beta)^2\sigma_\eta^2 + (1+a_2\beta)^2\sigma_\lambda^2 > 0. \tag{32}$$

Some preliminary analysis of 8 and Z will facilitate understanding the behavior of this system. The future price expectation updating coefficient, 8, will take the sign of the term in the large brackets in the definition of Z, (31). Z can be positive or negative, and generally depends on all of the parameters of the system, including error variances. For example, Z, hence Θ, is more likely to be positive if the variance and autocorrelation in money demand shocks, σ_η^2 and ρ_2, are large relative to the variance and autocorrelation of commodity demand shocks, σ_λ^2 and ρ_3. θ is more likely to be positive if the elasticity of commodity demand with respect to the real interest rate, b,, is large, and if the product of this elasticity and the income elasticity of money demand, a_2b_1, exceeds unity. Finally, the effect on 8 of the elasticity of money demand with respect to the nominal interest rate, a,, depends on the relative sizes of shocks and on other parameters. In most reasonable cases, however, increases in a_1 are associated with increases in θ.

In extreme cases, 8 may not only be positive, but greater than unity.

$$\theta>1 \text{ iff } a_1(b_1+\beta)Z>\Sigma. \tag{33}$$

Because the condition (33) involves Z multiplicatively, the likelihood of this case may be enhanced by the same structural conditions just mentioned as determining the sign of 8 and Z. The potential importance of high values of the elasticity of money demand, a,, is seen in the fact that for some a_1 sufficiently high, θ will exceed unity. But an upper bound can be placed on 8. The definitions of J and 8 can be manipulated to show that, given $\Sigma>0$ (since Σ is a weighted sum of the disturbance variances, it is necessarily positive), J>0 and

$$8 < 1+a_1(b_1+\beta)/b_1(1+a_2\beta) \leqq 1+a_1/a_2b_1 \tag{34}$$

(using $a_2 \leqq 1$).

These inequalities help determine signs of total derivatives, or coefficients in the final form. If $\theta>1$, then money demand or supply shocks will have anti-intuitive effects on the price level and output. For example, an increase in money demand under this constant money policy will actually increase the price level and output in the contemporaneous period. In the following period, and beyond, the effect on prices and output is negative.

The effect of agents' use of the interest rate to update future price expectations can be illustrated with a hypothetical temporary increase in the money supply of one unit at time **t**. (This shock has the same effects as a one-unit decrease in money demand, $\eta_t=-1$.) The interest rate, R_t, unambiguously falls by $(b_1+\beta)J$. The expected future price level, $E^+_{t-1}p_{t+1}$, is changed by the same amount times 8, which can be positive or negative. The current price level, p_t, changes by $b_1(1-\Theta)J$, which can be negative **if** $\Theta>1$. The expected rate of inflation, $[E^+_{t-1}p_{t+1}-p_t]$, changes by $-(b_1+\Theta\beta)J$, which is negative unless $\Theta<-(b_1/\beta)$. The real rate of interest, $r_t=[R_t-(E^+_{t-1}p_{t+1}-p_t)]$, changes by $-\beta(1-\Theta)J$, which is negative unless $\Theta>1$.

If the real interest rate declines in response to a temporary money increase, then aggregate demand increases, which is associated with higher current prices and output. The counterintuitive case, in which the real interest rate increases with a money supply shock, arises **if** the necessary decline in the nominal interest rate is overshadowed by an even larger decline in inflation expectations based on observation of that interest-rate decline. In the borderline case of $\Theta=1$, observed rises in R_t generate equal rises in $E^+_{t-1}p_{t+1}$, so that inflation expectations rise by an amount equal to the rise in the nominal interest rate. Therefore, the real rate, output, and the current price level are unchanged. **If** $\Theta>1$, then observed rises in R_t are associated with larger rises in $E^+_{t-1}p_{t+1}$, so that expected inflation rises by more than the rise in the nominal interest rate. Then, real rates fall with observed positive innovations in the nominal rate, increasing aggregate demand and raising output and the current price level.

It is interesting to consider that, under a simple elastic money-supply policy, such as $m_t=qR_t$, Θ will eventually exceed unity as q is increased without limit. (Such a policy might be motivated by a naive effort to stabilize the interest rate, although this method of doing so would be unnecessarily costly in terms of other potential objectives.) Then money demand and supply shocks would have counterintuitive effects on prices and output.

The importance of the cases of counterintuitive contemporaneous effects of shocks is inherently limited by the structure of the model. If the sum of the elasticities of money supply and money demand, a_1+q, is high enough, these cases arise. But, then, money supply and demand shocks have little contemporaneous effect on output, as can be seen by noting that the coefficient J in the reduced forms approaches zero as a_1+q rises without limit. So, if a_1+q is high enough to create anti-intuitive responses of output and the price level to money shocks, then these responses will tend to be small. Likewise, these cases can arise due to a preponderance of commodity demand shocks relative to money demand and money supply shocks. But then, by hypothesis, the influences that create the anti-intuitive effects are relatively small.

Beyond the contemporaneous period, the effect of an innovation on any endogenous variable depends, not on 8, nor on relative sizes of error variances, but strictly on structural elasticities and on autocorrelation coefficients of shocks. The most surprising possibility is that output can fall in the wake of a positive supply innovation. Formally,

$dy_t/d\varepsilon_{t-1}<0$ iff

$$[1+a_1(1-\rho_1)][\beta(a_1+a_2b_1)+2b_1(1+a_1)] < \beta(1+a_1)\rho_1(a_1+a_2b_1).^8 \qquad (35)$$

The behavior of the interest rate in response to shocks displays several interesting characteristics. The effect of all innovations on the nominal interest rate is generally nonzero at all lags, although it dies out at fixed rates, beyond the second lag, that depend strictly on the ρ_i. The effect of supply innovations on the interest rate is ambiguous. This effect of supply innovations in the contemporaneous period is negative if the product of the income elasticity of money demand, a_1, and the real interest rate elasticity of commodity demand, b_1, is less than unity, but positive if $a_2b_1>1$. At one lag, supply innovations may raise or lower the interest rate. If ρ_1 is near unity, ß is large, or a_2 is small, then this effect is more likely to be negative.

Although money demand innovations unambiguously raise the interest rate contemporaneously, they may actually decrease the interest rate in the subsequent period. Such a case is fostered by high autocorrelation, ρ_2, and high values of a, and b_1.

The influence of private agents' use of the current interest rate to update inflation expectations is seen in the presence of 8 in the solution equations. If agents did not use the interest rate, the economy would behave as if θ were zero in the solution equations. Obviously, the influence of agents' use of the interest rate is small if θ is reasonably close to zero, as will be illustrated with a numerical example. The differences in the behavior of the system under the alternative assumptions about private information pertain strictly to the contemporaneous effects of innovations. The effect of current supply innovations on output, $dy_t/d\varepsilon_t$, is smaller if private agents use the information content of the interest rate, if and only if the sign of $[(1-a_2b_1)\theta]$ is negative. The effect of current money demand innovations on output, $dy_t/d\eta_t$, is always negative if agents do not use R,, but can be positive if agents use R_t and $\theta>1$. If $\theta<0$, then the absolute magnitude of the output effect of money demand shocks is magnified. The effect of current demand innovations on output, $dy_t/d\lambda_t$, is always positive. It is increased for $\theta>0$ if agents use R_t in expectation formation.

VI. Output Stabilization Policy

The economy-wide output stabilization policy involves feedback from the current interest rate and past economic shocks in a manner that is broadly consistent with the well-known analyses of Poole and Fischer.

In deriving this policy, it will be easier to begin by noting that the effects on output of innovations of more than one period ago cannot be modified by policy. t-2 or earlier supply innovations will have full impact, while distant demand and money demand innovations have no effect. Then $\Pi_7=\Pi_8=\Pi_9=0$ in equation (16). Stabilization policy operates by minimizing the effects of t and t-1 innovations. It is clear from (16) that t-1 shocks are prevented from influencing y_t if $\Pi_1=\Pi_2=\Pi_3=0$, or

$$\Pi_1=[\rho_1+(1/2)\beta\Pi_{31}]=0, \quad \Pi_2=(1/2)\beta\Pi_{32}=0, \quad \text{and } \Pi_3=(1/2)\beta\Pi_{33}=0, \tag{36}$$

using the relevant equations among (A.1), (A.2), and (A.6) in the appendix. (36) requires the following relations between the μ_i and q, in view of the expressions for Π_{31}, Π_{32}, and Π_{33} among equations (A.10).

$$\mu_1=\frac{-\rho_1\{[1+(a_1+q)(1-\rho_1)][\beta(a_1+a_2b_1+q)+2b_1(1+a_1+q)]-\beta(1+a_1+q)(a_1+a_2+q)\}}{\beta(1+a_1+q)b_1}$$

$$\mu_2=\rho_2>0, \qquad \mu_3=-(a_1+q)\rho_3/b_1<0 \tag{37}$$

The contribution of period-t shocks to output variance cannot be eliminated, but is minimized, given the Gaussian stochastic assumption, if the expectation of the output innovation is zero. Given availability of current observations on the interest rate, this implies that output and interest rate innovations will be uncorrelated, or

$$COV[(y_t-E_{t-1}y_t),(R_t-E_{t-1}R_t)] = \Pi_4\Pi_{54}\sigma_\varepsilon^2+\Pi_5\Pi_{55}\sigma_\eta^2+\Pi_6\Pi_{56}\sigma_\lambda^2$$

$$=(\beta\Pi_{34}+1)\Pi_{54}\sigma_\varepsilon^2+\beta\Pi_{35}\Pi_{55}\sigma_\eta^2+\beta\Pi_{36}\Pi_{56}\sigma_\lambda^2 = 0. \tag{38}$$

The substitution of Π_{3i} terms for Π_i terms in (38) uses the relevant equations among (A.1), (A.2), and (A.6). (38) can, with the model, be used to determine the value of q in the following way. There are three equations in (A.7) for Π_4, Π_5, and Π_6. There are also three equations in (A.12) for Π_{54}, Π_{55}, and Π_{56}. Together with the definition of 8, (23), these constitute seven equations in eight unknowns:

$$\{\Pi_{34},\ \Pi_{35},\ \Pi_{36},\ \Pi_{54},\ \Pi_{55},\ \Pi_{56},\ q,\ \theta\}.$$

The last equality of (38) provides an eighth and determining equation. The solutions for q and 8 are as follows.

$$q=(A_1A_2-A_3A_4)/(A_3A_5-A_6A_2)$$

where

$$A_1=2\rho_1(1-a_2b_1)[a_1(b_1+\beta)+b_1(1+a_2\beta)]\sigma_\varepsilon^2$$

$$A_2=b_1[(1-a_2b_1)\sigma_\varepsilon^2+\beta(b_1+\beta)\sigma_\eta^2]$$

$$A_3=[\beta(1-a_2b_1)^2+2\rho_1(1-a_2b_1)b_1(1+a_2\beta)]\sigma_\varepsilon^2+(b_1+\beta)^2\sigma_\eta^2+(1+a_2\beta)^2\sigma_\lambda^2$$

$$A_4=b_1(1-a_2b_1)(1+a_1)\sigma_\varepsilon^2+b_1\beta(b_1+\beta)\sigma_\eta^2-\beta a_1(1+a_2\beta)\sigma_\lambda^2$$

$$A_5=b_1(1-a_2b_1)\sigma_\varepsilon^2-\beta(1+a_2\beta)\sigma_\lambda^2$$

$$A_6=2\rho_1(1-a_2b_1)(b_1+\beta)\sigma_\varepsilon^2 \qquad (39)$$

$$\theta = \frac{2\rho_1(1-a_2b_1)[(a_1+q)(b_1+\beta)+b_1(1+a_2\beta)]\sigma_\varepsilon^2}{(1-a_2b_1)[(1-a_2b_1)+2\rho_1b_1(1+a_2\beta)]\sigma_\varepsilon^2+(b_1+\beta)^2\sigma_\eta^2+(1+a_2\beta)^2\sigma_\lambda^2} \qquad (40)$$

θ will take the sign of $(1-a_2b_1)$.

In the simplifying case of $(1-a_2b_1)=0$ (or, with identical consequences, $\sigma_\varepsilon^2=0$), 8 equals zero and

$$q = \frac{b_1(b_1+\beta)\sigma_\eta^2}{(1+a_2\beta)\sigma_\lambda^2} - a_1. \qquad (41)$$

If, further, $b_1=a_2=1$, then

$$q=[\sigma_\eta^2-a_1\sigma_\lambda^2]/\sigma_\lambda^2=\sigma_\eta^2/\sigma_\lambda^2-a_1. \qquad (42)$$

This last result is reminiscent of Poole's analysis of a fixed-price IS-LM model under comparable assumptions about the elasticity of demand for goods with respect to the interest rate. The output-stabilizing policy provides for a money supply with interest rate elasticity that has a lower bound of $-a_1$ and no upper bound. It is directly related to the ratio of money demand variance to commodity demand variance. q decreases one-for-one with increases in a_1.

The output-stabilizing response of the money supply to commodity supply shocks, μ_1, is of indeterminate sign. However, even if μ_1 is positive, money will respond negatively to observed output shocks when the indirect effects via interest rate changes are considered.'

The final-form equations for total output, the price level, and the interest rate are shown below. J, q, and θ are as given in (A.14), (39), and (40), respectively.

$$y_t=\{1-\beta[a_1+q+a_2b_1(1-\theta)]J\}\varepsilon_t+\Sigma_{i=2}^{\infty}\rho_1^i\varepsilon_{t-1}-\beta b_1(1-\theta)J\eta_t+\beta(a_1+q)J\lambda_t \qquad (43)$$

$$p_t=-[a_1+q+a_2b_1(1-\theta)]J\varepsilon_t-2(\rho_1/\beta)\varepsilon_{t-1}$$
$$-[(2/\beta)+(a_1+a_2b_1+q)b_1^{-1}(1+a_1+q)^{-1}]\Sigma_{i=2}^{\infty}\rho_1^i\varepsilon_{t-i}+b_1(1-\theta)J\eta_t+(a_1+q)J\lambda_t \qquad (44)$$

$$R_t=-(1-a_2b_1)J\varepsilon_t+(a_1+q)^{-1}[a_2\rho_1-\mu_1-(2+a_2\beta)(\rho_1/\beta)]\varepsilon_{t-1}$$
$$+\{(a_1+q)^{-1}[a_2\rho_1-\mu_1-(2+a_2\beta)(\rho_1/\beta)]-(\rho_1/b_1)(1-a_2b_1)(1+a_1+q)^{-1}\}\Sigma_{i=2}^{\infty}\rho_1^{(i-1)}\varepsilon_{t-i}$$
$$+(b_1+\beta)J\eta_t+(1+a_2\beta)J\lambda_t+(1/b_1)\Sigma_{i=1}^{\infty}\rho_3^i\lambda_{t-i} \qquad (45)$$

VII. Firm Output Stabilization Policy

Although output determination under the wage-contracting scheme is necessarily Pareto-suboptimal, it is not clear that a policy to minimize economy-wide output fluctuations will improve economic welfare. Several other criteria would seem more adequate in measuring the welfare losses from wage stickiness. The first, dealt with in this section, is the variance of firm, as opposed to economy-wide, output. If workers and firms dislike variations in their own output and employment more than they dislike variations in aggregate output and employment, then economic welfare will be higher under a policy aimed at stability of output at the firm level.

The primary difference between policies designed to stabilize firm output and those designed to stabilize aggregate output is in the policy response to observed supply, μ_1. In the aggregate output policy, μ_1 is chosen so that the effect of last period's supply innovation on economy-wide output is zero, or $dy_t/d\varepsilon_{t-1}=0$. Given that the effect on output of group one is $dy_{1t}/d\varepsilon_{t-1}=\rho_1>0$, this necessarily requires $dy_{2t}/d\varepsilon_{t-1}=-\rho_1<0$, since (in view of (5)) dy_t is a simple average of dy_{1t} and dy_{2t}. Thus, the aggregate output policy uses its power to decrease group two output, in response to positive supply shocks in t-1, by a magnitude equal to the necessary increase in group one output. With regard to supply shocks, then, such a policy succeeds in stabilizing aggregate output partly by destabilizing firm-level output. Last period's supply shock, ε_{t-1}, which unavoidably caused y_{1t} to vary, has now caused y_{2t} to vary as well, via an excessively deflationary policy response. A less countercyclical response to supply shocks, or a numerically higher μ_1, is appropriate for firm-level output stabilization. The appropriate response will have the effect of allowing enough deflation to offset the effects of the supply shock on group two firms' output decisions.

This policy is derived in the following way. An appropriate measure of the variance of output for a typical firm over a contract interval, during which a firm will have one period as a group one firm, and another period as a group two firm, is $E[y_{1t}-Ey_{1t}]^2+E[y_{2t}-Ey_{2t}]^2$. Using the relevant equations among (25) and (26), and rearranging,

$$\begin{aligned} E[y_{1t}-Ey_{1t}]^2+E[y_{2t}-Ey_{2t}]^2 &= \rho_1^2(1+\rho_1^2)/(1-\rho_1^2)]\sigma_\varepsilon^2 \\ &+[(\beta\Pi_{31}+\rho_1)^2\sigma_\varepsilon^2+(\beta\Pi_{32})^2\sigma_\eta^2+(\beta\Pi_{33})^2\sigma_\lambda^2] \\ &+2[(\beta\Pi_{34}+1)^2\sigma_\varepsilon^2+(\beta\Pi_{35})^2\sigma_\eta^2+(\beta\Pi_{36})^2\sigma_\lambda^2]. \end{aligned} \tag{46}$$

This expression is the sum of three components. First, $\rho_1^2(1+\rho_1^2)/(1-\rho_1^2)\sigma_\varepsilon^2$

is a constant, or deadweight term, and can be ignored in policy choice. A second component involves the terms with Π_{31}, Π_{32}, and Π_{33}, and is reduced to zero if

$$\beta\Pi_{31}+\rho_1=0,\ \beta\Pi_{32}=0,\ \beta\Pi_{33}=0. \tag{47}$$

(Comparing (47) with (36), it is seen that Π_{31} differs between firm and economy-wide output policies, but Π_{32} and Π_{33} are identically zero.) These, in conjunction with the relevant equations among (A.10), imply

$$\mu_1=\rho_1[(a_2/2)-(1/\beta)-(a_1+q)\{[(1-\rho_1)/\beta]-(1/2)-[\rho_1(a_1+a_2b_1+q)/2b_1(1+a_1+q)]\}]$$

$$\mu_2=\rho_2>0,\quad \mu_3=-\rho_3(a_1+q)/b_1<0. \tag{48}$$

It remains to determine q. The third component of firm variance, $[2(\beta\Pi_{34}+1)^2\sigma_\varepsilon^2+2(\beta\Pi_{35})^2+2(\beta\Pi_{36})^2]$, is equal to $2E[y_t-E_{t-1}y_t]^2$. Given the Gaussian error structure, this component is minimized if the policy vector is such that condition (38) of the previous section is fulfilled. Then, (38), (47), (23), and the relevant six equations from (A.10) and (A.12) can be solved for q and Θ. This condition, in conjunction with the above expressions for Π_{31}, Π_{32}, and Π_{33}, and the model, implies

$$q=(C_1C_2-C_3C_4)/(C_3C_5-C_6C_2)$$

and $\Theta=(C_1C_5-C_6C_4)/(C_3C_5-C_6C_2)$

where

$$C_1=\rho_1(1-a_2b_1)[a_1(b_1+\beta)+b_1(1+a_2\beta)]\sigma_\varepsilon^2$$

$$C_2=-b_1(1-a_2b_1)\sigma_\varepsilon^2-b_1\beta(b_1+\beta)\sigma_\eta^2$$

$$C_3=[\beta(1-a_2b_1)+\rho_1b_1(1+a_2\beta)](1-a_2b_1)\sigma_\varepsilon^2+(b_1+\beta)^2\sigma_\eta^2+(1+a_2\beta)^2\sigma_\lambda^2$$

$$C_4=-(1-a_2b_1)b_1(1+a_1)\sigma_\varepsilon^2-b_1\beta(b_1+\beta)\sigma_\eta^2+a_1\beta(1+a_2\beta)\sigma_\lambda^2$$

$$C_5=-b_1(1-a_2b_1)\sigma_\varepsilon^2+\beta(1+a_2\beta)\sigma_\lambda^2$$

$$C_6=\rho_1(1-a_2b_1)(b_1+\beta)\sigma_\varepsilon^2. \tag{49}$$

In the simplifying case of $(1-a_2b_1)\sigma_\varepsilon^2=0$, then q is given by (41) and Θ is zero, precisely as for the economy-wide output stabilizing policy.

In comparison with the economy-wide output stabilizing policy, the firm output stabilization policy is characterized by a (numerically) higher value of μ_1. The difference between these policies with regard to q is ambiguous. μ_2 is identical. μ_3 differs only to reflect any differences in q, leaving the total responses of money to a commodity demand shock, $dm_t/d\lambda_{t-j}$, $i>0$, the same.

VIII. Price Stabilization Policy

Another interesting form of output stabilization is minimizing the variance of deviations in output from its full-information level. This policy criterion has been advocated by, among others, Barro (1976) and McCallum and Whitaker (1979). The full-information level of output is that which would obtain if price expectations were realized exactly. If, barring unexpected inflation, output is chosen by the firms in an optimal manner, then an appropriate measure for the performance of an economy is the degree to which output "tracks," or tends to match, that optimal level. Supply-shock effects on output may be appropriate responses to changing opportunities, especially to the extent that such shocks represent productivity shifts.

The variance of the deviation of output from the full-information level is minimized if the variances of price expectation errors over one- and two-period horizons are minimized. If the variances of price expectation errors over <u>all</u> horizons are minimized, then the policy is unique and is identical to the price stabilization policy. This same policy also necessarily minimizes the variance of both real and nominal wages. This section derives the unique price stabilization policy.

The price level has minimum feasible variance if the policy rule is such that, given the observable state of the economy, the expectation of the price level is its constant mean. Formally, select the q and $\{\mu_i\}_{i=1}^{3}$ for which

$$E^{+}_{t-1}p_t=E[p_t|\Omega_t]=0. \qquad (50)$$

This conditional expectation is composed of two necessarily orthogonal components:

$$E^{+}_{t-1}p_t=E_{t-1}p_t+[E^{+}_{t-1}p_t-E_{t-1}p_t]. \qquad (51)$$

The first of these terms can be reduced to a value of zero only if the effect of t-1 disturbances on p_t is removed. It is clear from inspection of (13) that this requires

$$\Pi_{31}=\Pi_{32}=\Pi_{33}=0. \qquad (52)$$

(52) and the relevant equations among (A.10) imply

$$\mu_1=(\rho_1/b_1)(a_1+a_2b_1+q)>0, \quad \mu_2=\rho_2>0, \text{ and } \mu_3=-\rho_3(a_1+q)/b_1<0. \qquad (53)$$

Given (53), it is readily seen from the last three equations of (A.10) that $\Pi_{37}=\Pi_{38}=\Pi_{39}=0$. Therefore, the effects of all past shocks on p_t are eliminated. The first term on the right-hand side of (51), $E_{t-1}p_t$, is made equal to zero and prices are rendered nonautocorrelated.

Condition (52) and the definition of 0, (23), imply

$$\theta=0. \qquad (54)$$

The second term in the conditional expectation (51) can be reduced to zero,

$$[E^{+}_{t-1}p_t-E_{t-1}p_t]=0, \qquad (55)$$

by appropriate choice of q. (55) implies, given the stochastic assumptions, (10), that

$$COV[(p_t-E_{t-1}p_t),(R_t-E_{t-1}R_t)]=\Pi_{34}\Pi_{54}\sigma^2_\varepsilon+\Pi_{35}\Pi_{55}\sigma^2_\eta+\Pi_{36}\Pi_{56}\sigma^2_\lambda=0. \qquad (56)$$

Then, (54), (56), and the six relevant equations from (A.10) and (A.12) constitute a system of eight equations in the eight unknowns, $\{\Pi_{34}, \Pi_{35}, \Pi_{36}, \Pi_{54}, \Pi_{55}, \Pi_{56}, q, \theta\}$.

The solution for q is

$$q=\frac{-(a_1+a_2b_1)(1-a_2b_1)\sigma^2_\varepsilon+b_1(b_1+\beta)\sigma^2_\eta-a_1(1+a_2\beta)\sigma^2_\lambda}{(1-a_2b_1)\sigma^2_\varepsilon+(1+a_2\beta)\sigma^2_\lambda}, \qquad (57)$$

which completes the characterization of the price stabilization policy.

The sign of q is ambiguous and will depend strictly and directly on the numerator of the right-hand side of (57). It is easy to imagine structures with either positive or negative values for q arising from a price stabilizing policy. The particularly simple case of $a_2=b_1=1$ has

$$q=(1+\beta)(\sigma_\eta^2-a_1\sigma_\lambda^2)/2\sigma_\lambda^2>0 \text{ iff } \sigma_\eta^2/\sigma_\lambda^2>a_1. \qquad (58)$$

Interestingly, this sign condition for q under the price stabilization policy is identical to that of the output stabilization policy with $a_2=b_1=1$. However, the scale or magnitude of q is smaller or larger in the price policy relative to the output policy, as $(1+\beta)/2$ is more or less than unity, respectively.

The minimum price-level variance attainable is

$$\text{Min}\quad E[p_t-Ep_t]^2=[(a_1+q+a_2b_1)^2\sigma_\varepsilon^2+b_1^2\sigma_\eta^2+(a_1+q)^2\sigma_\lambda^2]J^2. \qquad (59)$$

The final-form equations for total output, the price level, the interest rate, and the money stock are shown below. J, q, and θ are as given by (A.14), (54), and (57), respectively.

$$y_t = [1-\beta(a_1+a_2+q)J]\varepsilon_t+\Sigma_{i=1}^{\infty}\rho_1^i\varepsilon_{t-i}-b_1\beta J\eta_t+\beta(a_1+q)J\lambda_t \qquad (60)$$

$$p_t = -(a_1+a_2+q)J\varepsilon_t-b_1J\eta_t+(a_1+q)J\lambda_t \qquad (61)$$

$$R_t = -(1-a_2b_1)J\varepsilon_t-\Sigma_{i=1}^{\infty}(\rho_1^i/b_1)\varepsilon_{t-i}+(b_1+\beta)J\eta_t$$
$$+(1+a_2\beta)J\lambda_t+\Sigma_{i=1}^{\infty}(\rho_3^i/b_1)\lambda_{t-i} \qquad (62)$$

$$m_t = -q(1-a_2b_1)J\varepsilon_t+(a_2+a_1/b_1)\Sigma_{i=1}^{\infty}\rho_1^i\varepsilon_{t-i}+q(b_1+\beta)J\eta_t$$
$$+\Sigma_{i=1}^{\infty}\rho_2^i\eta_{t-i}+q(1+a_2\beta)J\lambda_t-(a_1/b_1)\Sigma_{i=1}^{\infty}\rho_3^i\lambda_{t-i} \qquad (63)$$

The final form for the money stock calls for positive responses of money to commodity supply and money demand shocks, and negative responses to commodity demand shocks. All of the signs of the total derivatives in the final form are of determinate signs, except $dR_t/d\varepsilon_t$, whose sign depends on whether a_2b_1 exceeds unity. The interest rate always falls in response to observed supply shocks ($dR_t/d\varepsilon_{t-i}$, $i>0$) and always rises in response to demand shocks, even contemporaneously ($dR_t/d\lambda_{t-i}>0$, for nonnegative i).

IX. A Numerical Example

An example structure illustrates the workings of the model economy under various policies. It is specified by the following set of values for the structural parameters.

$$\{ a_1=2,\ a_2=2/3,\ b_1=1,\ \beta=1;\ \rho_i=.8,\ i=1,2,3;\ \sigma_\varepsilon^2=1,\ \sigma_\eta^2=5,\ \sigma_\lambda^2=2 \} \tag{64}$$

The value of a, implies, for example, that an increase in the nominal interest rate from 5 percent to 6 percent would, for given levels of income and prices, lower real money demand by approximately 1.9 percent. A value of a, somewhat less than one is suggested by a priori theorizing on the transactions demand for money. The commodity supply and demand elasticities of unity were chosen merely because, of all (equally arbitrary) values, they are the most straightforward choices. (Econometric evidence currently available does not provide direct knowledge of supply and demand elasticities.) The relative sizes of the disturbances attempted to give considerable scope to demand-side influences on output, and to allow for a relatively unstable money demand function.

Table 2 presents the policy parameters and some measures of the system's resulting cyclical behavior under the policies satisfying the four policy criteria. The signs of coefficients in the final forms are displayed in table 3. The example is free of counterintuitive anomalies that might occur with other numerical structures or with arbitrary policies.

Interesting features of the example are that stabilization of firm-level output calls for a positive response of money to lagged supply shocks and a positive correlation between money and output. Under the policy to stabilize prices, which also minimizes output deviations from the full-information level, money will respond even more strongly to supply shocks, resulting

TABLE 2

OUTCOMES UNDER ALTERNATIVE POLICIES

IN THE EXAMPLE STRUCTURE

Policy Parameters	Policies Designed to Stabilize: Money	Economy-Wide Output	Firm Output	Price Level
q	0	+.92	+1.16	+.67
μ_1	0	-.82	+1.16	+2.67
μ_2	0	+.80	+.80	+.80
μ_3	0	-2.33	-2.53	-2.13
Correlation Between Money and Output	-	-.33	+.14	+.59
θ	-.23	+.15	+.08	0
Variance of:				
Economy-Wide Output	2.43	1.80	1.96	2.44
Firm Output[1]	3.22	2.44	2.12	2.40
Prices	15.47	12.86	3.69	0.62
Real Wages[2]	2.22	1.84	0.88	0.62
Money	0	26.91	24.42	36.00
Interest Rate	3.26	4.33	4.81	5.86

1. $E[y_{1t}^2]+E[y_{2t}^2]$

2. $E[p_t-w_{1t}]^2+E[p_t-w_{2t}]^2=\Sigma_{i=1}^2 E[p_t-E_{t-i}p_t]^2$

TABLE 3: RESPONSES OF ENDOGENOUS VARIABLES TO DISTURBANCE INNOVATIONS UNDER ALTERNATIVE POLICIES

POLICY CRITERION	DISTURBANCE INNOVATION and Period of Occurrence								
	COMMODITY SUPPLY			MONEY DEMAND			COMMODITY DEMAND		
Endogenous Variable	t	t-1	t-2	t	t-1	t-2	t	t-1	t-2
CONSTANT MONEY									
Output	.53	.27	.64	-.20	-.20	0	.33	.40	0
Prices	-.47	-1.05	-1.22	-.20	-.40	-.46	.33	.79	.91
Interest Rate	-.06	-.44	-.40	.33	.14	.09	.28	.53	.46
ECONOMY-WIDE OUTPUT STABILIZATION									
Output	.52	0	.64	-.12	0	0	.40	0	0
Prices	-.48	-1.60	-1.87	-.12	0	0	.40	0	0
Interest Rate	-.05	-.27	-.27	.28	0	0	.23	.80	.64
Money Stock	-.04	-1.07	-.90	.25	.80	.64	.21	-1.60	-1.28
FIRM OUTPUT STABILIZATION									
Output	.52	.40	.64	-.12	0	0	.40	0	0
Prices	-.48	-.80	-.93	-.12	0	0	.40	0	0
Interest Rate	-.04	-.53	-.45	.25	0	0	.21	.80	.64
Money Stock	-.05	.54	.40	.30	.80	.64	.25	-1.60	-1.28
PRICE LEVEL STABILIZATION									
Output	.52	.80	.64	-.14	0	0	.38	0	0
Prices	-.48	0	0	-.14	0	0	.38	0	0
Interest Rate	-.05	-.80	-.64	.29	0	0	.24	.80	.64
Money Stock	-.03	2.13	1.71	.19	.80	.64	.16	-1.60	-1.28

in a correlation of +.59 between money and output. This policy actually increases gross, economy-wide output variance relative to a constant-money policy, reflecting the importance of supply shocks for output behavior, despite the relative smallness of supply variance compared to demand and money demand variance. However, the price-stabilization policy actually reduces firm output variance relative to the policy that stabilizes economy-wide output, illustrating the conflict between stability of aggregate output versus firm output.

The effect of private agents' use of the information content of the interest rate on output behavior is relatively slight in this example. Under a constant money policy, the effect of current supply innovations, $dy_t/d\varepsilon_t$, is virtually unaffected, taking the value .529 if agents do not use R, versus .534 if they do. The effect of money demand innovations, $dy_t/d\eta_t$, is -.176 if agents do not use R_t and -.203 if they do. The effect of demand innovations, $dy_t/d\lambda_t$, is lowered from .353 to .331 if agents use the interest rate. The behavior of prices and interest rates, like that of output, is only slightly modified by the alternative assumptions about use of current information.[10]

Output objectives, either local or global, require somewhat different values for policy parameters depending on whether agents use the information in the interest rate, but the behavior of the controlled system is largely unaffected. For example, the economy-wide output stabilization policy is $(q, \mu_1, \mu_2, \mu_3) = (1.44, -.66, .8, -2.76)$ if agents do not use the information in the interest rate, compared with (.92, -.82, .8, -2.34) if they do. Similarly, the firm-level output stabilization policy is $(q, \mu_1, \mu_2, \mu_3) = (1.44, 1.31, .8, -2.76)$ if private agents do not use the information in the interest rate, compared with (1.16, 1.16, .8, -2.53) if they do. Price, output, and ex ante real interest rate behavior is entirely unaffected by private use of the information content of the interest rate, given that the policy rule is chosen appropriately. Nominal interest rate behavior is modified by private use of the information content of the interest rate, but the modification is slight if 8 is reasonably close to zero. The response of the nominal interest rate to any contemporaneous innovation is $(1-\theta)$ times greater if agents do not use the interest rate than if they do, where θ is the updating coefficient when agents do use the interest rate to update expectations. In the example structure, the relevant values of 8 are .15 and .08 in the aggregate-level output and firm-level output stabilization policies, so that the contemporaneous responses of the interest rate to shocks are reduced by 15 and 8 percent, respectively, if private agents ignore the information in the interest rate. As is always the case under any policy objective, innovations in period t have exactly the same effect on all endogenous variables in subsequent periods $t+j$, $j>0$, regardless of whether agents use the interest rate.

If the price stabilization policy is chosen, then the future price expectation is its unconditional mean and therefore cannot change with current shocks, even if the latter are known. Therefore, variations in the interest rate, while they continue to be informative about current shocks, are uninformative about future prices, so $\theta=0$. Consequently, if the policy authorities choose to stabilize the price level, then allowing private agents to observe the current interest rate has no effect on expectations, and hence no effect on the behavior of any endogenous variables or on the appropriate policy rule parameters.

X. Summary and Conclusion

While use of contemporaneous information is an important and problematic theoretical issue, the analysis here suggests that its policy implications may not be quantitatively important. Indeed, if the objectives are limited to price stabilization, real wage stabilization, and stabilization of output around its full-information level, then policy authorities need not concern themselves with the issue (assuming they know the economic parameters and variances), because the appropriate policy rule is invariant to whether or not private agents use contemporaneous information.

The analysis of this article reaffirms the conventional effectiveness of money supply responses to the current interest rate, q, and responses to the lagged state of the economy, $\underline{\mu}$, in a model with two-period wage stickiness, even if private agents use the information content of the nominal interest rate to form rational expectations. Thus, neither superior information of the policymaker relative to private agents nor irrational expectations are necessary to make both q and $\underline{\mu}$ relevant, unless perfectly flexible wages and prices are assumed.

This article also analyzes a conflict between stabilization of total output, aggregate output, and output relative to its perfect-foresight level--a conflict that arises in the presence of supply shocks. Stabilization of output around its full-information level, rather than its unconditional mean, may be more appropriate and is likely to imply a positive correlation between money and output. The policy rule that stabilizes output around its full-information level has the additional advantage of being consistent with minimization of price and real wage variances. Firm-output stabilization may also require positive money-supply responses to supply shocks. Hence, while multiperiod wage stickiness provides a conventional mechanism for the effectiveness of policy (both q and μ), it does not necessarily, or even probably, provide a persuasive rationale for countercyclical monetary policy, if the latter is interpreted as a negative correlation between money and output.

Appendix

This appendix contains details of the derivation of the solution to the model. Some intermediate results are referred to in the text in proofs of propositions and derivations of policies satisfying various criteria. The set of ten equations {(11),(12),(13),(14),(15),(16),(17),(18),(23),(24)} constitutes the trial solution. Then certain identities among the parameters of the trial solution are implied by the structural equations (3), (6), (8), and (9), and the accounting identity (4). The first two sets of identities, numbered (A.1) and (A.2), are implied by the output supply equations (3). (A.3) is implied by the money demand equation, (8); (A.4) is implied by the aggregate demand equation, (6); (A.5) is implied by the money supply function, (9); and (A.6) is implied by the accounting identity, (4).

$$\begin{aligned}
&\Pi_{10}=\alpha &&\Pi_{20}=\alpha\\
&\Pi_{11}=\rho_1 &&\Pi_{21}=\beta\Pi_{31}+\rho_1\\
&\Pi_{12}=0 &&\Pi_{22}=\beta\Pi_{32}\\
&\Pi_{13}=0 &&\Pi_{23}=\beta\Pi_{33}\\
&\Pi_{14}=\beta\Pi_{34}+1 &&\Pi_{24}=\beta\Pi_{34}+1\\
&\Pi_{15}=\beta\Pi_{35} &&\Pi_{25}=\beta\Pi_{35}\\
&\Pi_{16}=\beta\Pi_{36} &&\Pi_{26}=\beta\Pi_{36}\\
&\Pi_{17}=0 &&\Pi_{27}=-\beta\Pi_{31}\\
&\Pi_{18}=0 &&\Pi_{28}=-\beta\Pi_{32}\\
&\Pi_{19}=0 \qquad \text{(A.1)} &&\Pi_{29}=-\beta\Pi_{33} \qquad \text{(A.2)}
\end{aligned}$$

$$\Pi_{40}=\Pi_{30}+a_0-a_1\Pi_{50}+a_2\Pi_0$$
$$\Pi_{41}=\Pi_{31}-a_1\Pi_{51}+a_2\Pi_1$$
$$\Pi_{42}=\Pi_{32}-a_1\Pi_{52}+a_2\Pi_2+\rho_2$$
$$\Pi_{43}=\Pi_{33}-a_1\Pi_{53}+a_2\Pi_3$$
$$\Pi_{44}=\Pi_{34}-a_1\Pi_{54}+a_2\Pi_4$$
$$\Pi_{45}=\Pi_{35}-a_1\Pi_{55}+a_2\Pi_5+1$$
$$\Pi_{46}=\Pi_{36}-a_1\Pi_{56}+a_2\Pi_6$$
$$\Pi_{47}=\Pi_{37}-a_1\Pi_{57}+a_2\Pi_7$$
$$\Pi_{48}=\Pi_{38}-a_1\Pi_{58}+a_2\Pi_8$$
$$\Pi_{49}=\Pi_{39}-a_1\Pi_{59}+a_2\Pi_9 \qquad (A.3)$$

$$\Pi_0=b_0-b_1\Pi_{50}$$
$$\Pi_1=-b_1\Pi_{51}+b_1(\Pi_{31}+\Pi_{38})\rho_1-b_1\Pi_{31}$$
$$\Pi_2=-b_1\Pi_{52}+b_1(\Pi_{32}+\Pi_{39})\rho_2-b_1\Pi_{32}$$
$$\Pi_3=-b_1\Pi_{53}+b_1(\Pi_{33}+\Pi_{3,10})\rho_3-b_1\Pi_{33}+\rho_3$$
$$\Pi_4=-b_1\Pi_{54}+b_1\theta\Pi_{54}-b_1\Pi_{34}$$
$$\Pi_5=-b_1\Pi_{55}+b_1\theta\Pi_{55}-b_1\Pi_{35}$$
$$\Pi_6=-b_1\Pi_{56}+b_1\theta\Pi_{56}-b_1\Pi_{36}+1$$
$$\Pi_7=-b_1\Pi_{57}-b_1\Pi_{37}$$
$$\Pi_8=-b_1\Pi_{58}-b_1\Pi_{38}$$
$$\Pi_9=-b_1\Pi_{59}-b_1\Pi_{39} \qquad (A.4)$$

$$\Pi_{40}=q\Pi_{50}+\mu_0$$
$$\Pi_{41}=q\Pi_{51}+\mu_1$$
$$\Pi_{42}=q\Pi_{52}+\mu_2$$
$$\Pi_{43}=q\Pi_{53}+\mu_3$$
$$\Pi_{44}=q\Pi_{54}$$
$$\Pi_{45}=q\Pi_{55}$$
$$\Pi_{46}=q\Pi_{56}$$
$$\Pi_{47}=q\Pi_{57}$$
$$\Pi_{48}=q\Pi_{58}$$
$$\Pi_{49}=q\Pi_{59} \qquad (A.5)$$

$$\Pi_0=(1/2)(\Pi_{10}+\Pi_{20})+k_0$$
$$\Pi_1=(1/2)(\Pi_{11}+\Pi_{21})$$
$$\Pi_2=(1/2)(\Pi_{12}+\Pi_{22})$$
$$\Pi_3=(1/2)(\Pi_{13}+\Pi_{23})$$
$$\Pi_4=(1/2)(\Pi_{14}+\Pi_{24})$$
$$\Pi_5=(1/2)(\Pi_{15}+\Pi_{25})$$
$$\Pi_6=(1/2)(\Pi_{16}+\Pi_{26})$$
$$\Pi_7=(1/2)(\Pi_{17}+\Pi_{27})$$
$$\Pi_8=(1/2)(\Pi_{18}+\Pi_{28})$$
$$\Pi_9=(1/2)(\Pi_{19}+\Pi_{29}) \qquad (A.6)$$

These identities can be solved for the undetermined coefficients of the trial solution, the Πs.

Total output coefficients:

$\Pi_0=\alpha+k_0$

$\Pi_1=\rho_1-\beta(1+a_1+q)[\rho_1(a_1+a_2b_1+q)-b_1\mu_1]G_1$

$\Pi_2=-\beta(\rho_2-\mu_2)(1+a_1+q)G_2$

$\Pi_3=\beta(1+a_1+q)[\rho_3(a_1+q)+b_1\mu_3]G_3$

$\Pi_4=1-\beta[a_1+q+a_2b_1(1-\theta)]J$

$\Pi_5=-\beta b_1(1-\theta)J$

$\Pi_6=\beta(a_1+q)J$

$\Pi_7=-(\beta^2/2b_1)[(a_1+a_2b_1+q)^2\rho_1-b_1\mu_1]G_1$

$\Pi_8=-(\beta^2/2)(\rho_2-\mu_2)(a_1+a_2b_1+q)G_2$

$\Pi_9=(\beta^2/2b_1)(a_1+a_2b_1+q)[\rho_3(a_1+q)+b_1\mu_3]G_3$ (A.7)

Group one output:

$\Pi_{10}=\alpha$

$\Pi_{11}=\rho_1$

$\Pi_{12}=0$

$\Pi_{13}=0$

$\Pi_{14}=1-\beta[a_1+q+a_2b_1(1-\theta)]J$

$\Pi_{15}=-\beta b_1(1-\theta)J$

$\Pi_{16}=\beta(a_1+q)J$

$\Pi_{17}=0$

$\Pi_{18}=0$

$\Pi_{19}=0$

Group two output:

$$\Pi_{20}=\alpha$$

$$\Pi_{21}=\rho_1-2\beta(1+a_1+q)[\rho_1(a_1+a_2b_1+q)-b_1\mu_1]G_1$$

$$\Pi_{22}=-2\beta\rho_2b_1(1+a_1+q)G_2$$

$$\Pi_{23}=2\beta(1+a_1+q)[\rho_3(a_1+q)+b_1\mu_3]G_3$$

$$\Pi_{24}=1-\beta[a_1+q+a_2b_1(1-\theta)]J$$

$$\Pi_{25}=-\beta b_1(1-\theta)J$$

$$\Pi_{26}=\beta(a_1+q)J$$

$$\Pi_{27}=(\beta^2/b_1)[\rho_1(a_1+a_2b_1+q)^2-b_1\mu_1]G_1$$

$$\Pi_{28}=\rho_2\beta^2(a_1+a_2b_1+q)G_2$$

$$\Pi_{29}=-(\beta^2/b_1)(a_1+a_2b_1+q)[\rho_3(a_1+q)+b_1\mu_3]G_3 \qquad (A.9)$$

Price level:

$$\Pi_{30}=\mu_0+[(a_1+q)/b_1](b_0-\alpha-k_0)-[a_0+a_2(\alpha+k_0)]$$

$$\Pi_{31}=-2(1+a_1+q)[\rho_1(a_1+a_2b_1+q)-b_1\mu_1]G_1$$

$$\Pi_{32}=-2b_1(\rho_2-\mu_2)(1+a_1+q)G_2$$

$$\Pi_{33}=2(1+a_1+q)[\rho_3(a_1+q)+b_1\mu_3]G_3$$

$$\Pi_{34}=-[a_1+q+a_2b_1(1-\theta)]J$$

$$\Pi_{35}=-b_1(1-\theta)J$$

$$\Pi_{36}=(a_1+q)J$$

$$\Pi_{37}=-(\beta/b_1)(a_1+a_2b_1+q)[\rho_1(a_1+a_2b_1+q)-b_1\mu_1]G_1$$

$$\Pi_{38}=-(\rho_2-\mu_2)\beta(a_1+a_2b_1+q)G_2$$

$$\Pi_{39}=(\beta/b_1)(a_1+a_2b_1+q)[\rho_3(a_1+q)+b_1\mu_3]G_3 \qquad (A.10)$$

Money stock:

$$\Pi_{40}=\mu_0+(q/b_1)(b_0-\alpha-k_0)$$

$$\Pi_{41}=\mu_1+q(a_1+q)^{-1}\{(a_2\rho_1-\mu_1)-(2+a_2\beta)(1+a_1+q)[\rho_1(a_1+a_2b_1+q)-b_1\mu_1]G_1\}$$

$$\Pi_{42}=\mu_2+q(\rho_2-\mu_2)\{(\beta+2b_1)[1+(a_1+q)(1-\rho_2)]-\rho_2b_1(2+a_2\beta)\}G_2$$

$$\Pi_{43}=\mu_3(1-q)+q(a_1+q)^{-1}(2+a_2\beta)(1+a_1+q)[\rho_3(a_1+q)+b_1\mu_3]G_3$$

$$\Pi_{44}=-q(1-a_2b_1)J$$

$$\Pi_{45}=q(b_1+\beta)J$$

$$\Pi_{46}=q(1+a_2\beta)J$$

$$\Pi_{47}=q(\beta/b_1)[\rho_1(a_1+a_2b_1+q)-b_1\mu_1](1-a_2b_1)G_1$$

$$\Pi_{48}=-q(\rho_2-\mu_2)\beta(1-a_2b_1)G_2$$

$$\Pi_{49}=(q/b_1)[\rho_3(a_1+q)+\mu_3](1-a_2b_1)\beta G_3 \qquad \text{(A. 11)}$$

Interest rate:

$$\Pi_{50}=(b_0-\alpha-k_0)/b_1$$

$$\Pi_{51}=(a_1+q)^{-1}\{(a_2\rho_1-\mu_1)-(2+a_2\beta)(1+a_1+q)[\rho_1(a_1+a_2b_1+q)-b_1\mu_1]G_1\}$$

$$\Pi_{52}=(\rho_2-\mu_2)\{(\beta+2b_1)[1+(a_1+q)(1-\rho_2)]-\rho_2b_1(2+a_2\beta)\}G_2$$

$$\Pi_{53}=-\mu_3/(a_1+q)^{-1}+(2+a_2\beta)(a_1+q)^{-1}\{(1+a_1+q)[\rho_3(a_1+q)+b_1\mu_3]\}G_3$$

$$\Pi_{54}=-(1-a_2b_1)J$$

$$\Pi_{55}=(b_1+\beta)J$$

$$\Pi_{56}=(1+a_2\beta)J$$

$$\Pi_{57}=-(\beta/b_1)[\rho_1(a_1+a_2b_1+q)-b_1\mu_1](1-a_2b_1)G_1$$

$$\Pi_{58}=-(\rho_2-\mu_2)\beta(1-a_2b_1)G_2$$

$$\Pi_{59}=[(\rho_3/b_1)(a_1+q)+\mu_3](1-a_2b_1)\beta G_3 \qquad \text{(A.12)}$$

where

$$G_i=\{[1+(a_1+q)(1-\rho_i)][\beta(a_1+a_2b_1+q)+2b_1(1+a_1+q)]\}^{-1}, \quad i=1,2,3, \qquad \text{(A. 13)}$$

$$J=[(a_1+q)(b_1+\beta)+b_1(1-\theta)(1+a_2\beta)]^- \tag{A.14}$$

$$\theta=\frac{[b_1(1+a_1+q)+\beta(a_1+a_2b_1+q)]Z}{\Sigma - b_1(1+a_2\beta)Z} \tag{A.15}$$

$$Z=2(1+a_1+q)\left\{\begin{array}{l}(1-a_2b_1)[\rho_1(a_1+a_2b_1+q)-b_1\mu_1]G_1\sigma_\varepsilon^2\\ -(\rho_2-\mu_2)b_1(b_1+\beta)G_2\sigma_\eta^2\\ +[\rho_3(a_1+q)+b_1\mu_3](1+a_2\beta)G_3\sigma_\lambda^2\end{array}\right\} \tag{A.16}$$

$$\Sigma = (1-a_2b_1)^2\sigma_\varepsilon^2+(b_1+\beta)^2\sigma_\eta^2+(1+a_2\beta)^2\sigma_\lambda^2 \tag{A.17}$$

The Final Form

The final form solution, which relates endogenous variables strictly to disturbance innovations, is given below for output variables.

$$\begin{aligned} y_t=&\alpha+k_0+\{1-\beta[a_1+q+a_2b_1(1-\theta)]J\}\varepsilon_t+\{\rho_1-\beta(1+a_1+q)[\rho_1(a_1+a_2b_1+q)-b_1\mu_1]G_1\}\varepsilon_{t-1}\\ &+\Sigma_{j=2}^{\infty}\rho_1^j\varepsilon_{t-j}-\beta b_1(1-\theta)J\eta_t-(\rho_2-\mu_2)\beta b_1(1+a_1+q)G_2\eta_{t-1}+\beta(a_1+q)J\lambda_t\\ &+\beta(1+a_1+q)[\rho_3(a_1+q)+b_1\mu_3]G_3\lambda_{t-1}\end{aligned} \tag{A.18}$$

$$y_{1t}=\alpha+\{1-\beta[a_1+q+a_2b_1(1-\theta)]J\}\varepsilon_t+\Sigma_{j=1}^{\infty}\rho_1^j\varepsilon_{t-j}-\beta b_1(1-\theta)J\eta_t+\beta(a_1+q)J\lambda_t \tag{A.19}$$

$$y_{2t}=y_{1t}+2\beta(1+a_1+q)\left\{\begin{array}{l}-[\rho_1(a_1+a_2b_1+q)-b_1\mu_1]G_1\varepsilon_{t-1}\\ -\rho_2b_1G_2\eta_{t-1}\\ +[\rho_3(a_1+q)+b_1\mu_3]G_3\lambda_{t-1}\end{array}\right\} \tag{A.20}$$

The final forms for prices, money, and the interest rate are:

$$\begin{aligned} X_t(j)=&\Pi_{j0}+\Pi_{j4}\varepsilon_t+\Pi_{j1}\varepsilon_{t-1}+(\Pi_{j1}+\Pi_{j7})\Sigma_{i=2}^{\infty}\rho_1^{(i-1)}\varepsilon_{t-i}+\Pi_{j5}\eta_t+\Pi_{j2}\eta_{t-1}\\ &+(\Pi_{j2}+\Pi_{j8})\Sigma_{i=2}^{\infty}\rho_2^{(i-1)}\eta_{t-i}+\Pi_{j6}\lambda_t+\Pi_{j3}\lambda_{t-1}+(\Pi_{j3}+\Pi_{j9})\Sigma_{i=2}^{\infty}\rho_3^{(i-1)}\lambda_{t-i}\end{aligned}$$

where $X_t(3)=p_t$, $X_t(4)=m_t$, and $X_t(5)=R_t$. (A.21)

Footnotes

1. Sargent and Wallace did not derive the output stabilizing value of q or analyze its dependence on the structure. Instead, they contrasted strict money-supply rules (e.g., money supply rules with q=0) with interest rate rules (e.g., $R_t=f(S_{t-1})$). One important result was that prices and the money stock were indeterminate under the latter rule. As argued in Hoehn (1987), an interest rate rule is not a feasible policy option in models such as that of Sargent and Wallace. In later sections of this article, a complete analysis of the output-stabilizing and other objective-seeking values of q and their dependence on the structure is provided.

2. Other important papers contributing to analysis of contemporaneous, heterogeneous information in models with a credit market include Barro (1980), Weiss (1980), and King (1982). The latter provides an excellent discussion of the general implications of heterogeneous current information for the policy effectiveness proposition in flexible-price models, as well as some fully worked-out numerical examples.

3. Dotsey and King (1986) also argued, in effect, that the behavior of output and other variables depends on whether q is finite or infinite, and identified the latter case with an interest rate rule (e.g., R_t is a strict function of S_{t-1}, as in $R_t=f(S_{t-1})$). But Hoehn (1987) argues that interest rate rules are infeasible in models such as theirs and that policies with infinite q cannot be made operational in any model. In any case, variations in q are irrelevant in the flexible-price model of Dotsey and King, so long as q is finite. The discussion in the text of this article rules out infinite q.

4. This nonuniqueness of the output-stabilizing policy could have been eliminated if, as in the present analysis, the policy rule's arguments exclude elements that augment the minimal state vector, S, beyond those necessary to achieve policy objectives. Then, nonzero $\underline{\mu}$ would have been ruled out and the conclusion would have been that q is relevant for output while $\underline{\mu}$ is irrelevant for output.

5. McCallum (1987) argues convincingly that Keynesian sticky-wage models that incorporate equilibrium wage dynamics represent a significant advance over earlier models that did not.

6. The formulation of the real ex ante interest rate employed here is similar to that of Canzoneri, Henderson, and Rogoff (1983). Indeed, the updating superscript, "+", is taken from their notation. However, they use $E^{+}_{t-1}p_t$ for the current price term, whereas I have assumed that demanders know the current price they pay when making purchasing decisions. The economy-wide price level, p_t, is the appropriate current price term for aggregate demand if agents know the price they pay when making purchasing decisions, even if individuals' purchasing prices differ. Although agents are allowed to know their purchasing price, it is assumed for simplicity that they do not use that information to update the future price expectation. This assumption is a reasonable approximation if individual prices are highly variable so that they contain little global information. The assumption could be relaxed only in the context of a disaggregate model that accounts for differences in information available to agents in different markets. Such a model requires a set of assumptions about local markets, intermarket trading, factor mobility, market clearing, and so forth. A special problem is that

these assumptions would have to be consistent with the aggregate supply behavior involving wage stickiness. Such an analysis, while desirable, is beyond the scope of this article, and does not seem particularly necessary for an analysis of the use of global information from the interest rate, which is the most important source of current information available to monetary authorities.

King (1982, 1983) and Dotsey and King (1983) constructed disaggregate market-clearing models along lines suggested by Lucas and Barro. Their analysis focused on the effects of private agents' use of idiosyncratic information from local prices in generating a kind of policy effectiveness from feedback to the lagged state of the economy, μ. Dotsey and King (1986) also analyze a model in which some agents know the full state, other agents know only the lagged state, and a single commodity market clears. Again, they find μ relevant for output via a heterogeneous information mechanism, although little can be said about the influence of μ other than that it exists.

A heterogeneous information mechanism is not incorporated into the present analysis for technical reasons, but it might coexist with the mechanisms of policy effectiveness analyzed in this article, in a fully developed disaggregate version of the macroeconomic model employed.

7. In particular, the trial solution for the endogenous expectation of the price level at time t formed at time t-2, $E_{t-2}p_t$, necessarily involves terms in $E_{t-2}u_{t-1}$, $E_{t-2}v_{t-1}$, and $E_{t-2}x_{t-1}$, even if these are not included in the state vector. This fact is apparent from equation (18) of the trial solution, in which $\Pi_{31}E_{t-2}u_{t-1}$, $\Pi_{32}E_{t-2}v_{t-1}$, and $\Pi_{33}E_{t-2}x_{t-1}$ will appear even if Π_{37}, Π_{38}, and Π_{39} are set to zero, as would be the case if $E_{t-2}u_{t-1}$, $E_{t-2}v_{t-1}$, and $E_{t-2}x_{t-1}$ were excluded from S_t.

8. For example, if $\beta=2$, $a_1=5$, $a_2=2/3$, and $b_1=1/2$, then $dy_t/d\varepsilon_{t-1}$ approaches -2.84 as ρ_1 approaches unity.

9. For an example in which μ_1 is positive, consider $a_2=b_1=1$, $a_1=5$, $\beta=1/2$, $\{\rho_i=1/2;\ i=1,2,3\}$, $\sigma_\eta^2=10$, and $\sigma_\lambda^2=1$. Then optimal policy is given by $\{q,\ \mu_1,\ \mu_2,\ \mu_3\} = \{5,\ 3,\ 112,\ -5\}$, which involves $\mu_1>0$. In this example, $dm_t/d\varepsilon_{t-1}= \mu_1+(dR_t/d\varepsilon_{t-1})q=-3.06$, showing that money contracts in response to the observed (t-1) supply shock.

10. Ironically, this economy would, under a constant money policy, produce output typically closer to the natural rate if private agents did not use the interest rate to update inflation expectations. The variance of deviations of output from its natural rate is raised by .016 as agents use the interest rate.

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