ESTIMATING TOTAL FACTOR PRODUCTIVITY 
IN A GENERALIZED COST SYSTEM

by Philip Israilevich and K. J. Kowalewski

Philip Israilevich and K. J. Kowalewski are economists at the Federal Reserve Bank of Cleveland.

Working papers of the Federal Reserve Bank of Cleveland are preliminary materials circulated to stimulate discussion and critical comment. The views stated herein are those of the authors and not necessarily those of the Federal Reserve Bank of Cleveland or of the Board of Governors of the Federal Reserve System.

March 1987
I. Introduction

Measuring the impact of regulation on electric utilities is of considerable interest to economists and regulators. Courville (1974), Spann (1974), Peterson (1975), Cowing (1978), and Nelson and Wohar (1983), for example, find evidence of an overcapitalization bias using variations of the Averch and Johnson (1962) model. These models are criticized by Atkinson and Halvorsen (1984) because the impact of additional regulatory constraints is ignored. Atkinson and Halvorsen (A-H) estimate a generalized cost model with a cross-section of electric utilities and find a regulatory bias on the utilization of noncapital factor inputs. Moreover, Fare and Logan (1983) show that a rate-of-return constraint alone invalidates the use of Shephard's Lemma for noncapital inputs.

Joskow (1974) argues that regulation may also bias the rate of technical change implemented by utilities. The only empirical paper to have considered such an impact is Nelson and Wohar. They find a regulatory impact on the total factor productivity (TFP) experienced by electric utilities during the late 1970s. However, in addition to considering only a rate-of-return constraint, their approach can be criticized because it considers the regulatory impact on TFP to be independent of the returns-to-scale and
technical-change components of TFP. Because TFP is the sum of returns-to-scale and technical-change terms in a cost function framework, the Nelson and Wohar model implies a deus ex machina regulatory impact on TFP. Obviously, the authors cannot test for a regulatory impact on technical change.

In this paper we develop a model that can be used with time-series data to test for a regulatory impact on all the components of TFP. The foundation of our model is the A-H generalized cost function, modified with time variables to capture the dynamic aspect of TFP. To this, we add an equation for TFP. Although this equation is not necessary to test for the impact of regulation on TFP and its components, its use will presumably increase the efficiency of parameter estimates because it is additional behavioral information and because TFP is measured as a rate of change. Two alternative TFP equations are derived; one is considerably easier to estimate than the other.

In the next section of this paper, an alternative derivation of the A-H model is given. After that, the two TFP equations are formulated and discussed. Finally, the translog version of our generalized cost model is given.

1. The TFP equation in the Nelson and Wohar paper is:

\[ \dot{W} = \dot{\nu}_T + \nu_0 \dot{Q} + \lambda [\sum_i \dot{M}_i P_i - (sX_n/C)s], \]

where

- \( \dot{W} \) = rate of change of TFP
- \( \dot{\nu}_T \) = technical change
- \( \nu_0 \) = returns to scale
- \( \dot{Q} \) = output
- \( \dot{M}_i \) = i-th input share
- \( P_i \) = input price
- \( s \) = rate of return
- \( X \) = capital input
- \( C \) = cost.
II. The Atkinson and Halvorsen Model

Atkinson and Halvorsen show that general regulatory constraints alter the nature of the cost-minimizing, first-order conditions. Instead of equating the marginal costs of each factor input to its market price, a regulated firm finds it optimal to equate the marginal costs of each factor input to its shadow price. These shadow prices are market prices adjusted for the impact of regulation, and their specification depends on the exact nature of the regulatory constraints. Atkinson and Halvorsen approximate these shadow prices, $P^*_i$, with simple proportional relationships to market prices, $P^*_i = k_i P_i$, for each input $i$. The generalized, or shadow, cost function is simply the actual cost function, but with shadow prices instead of market prices. Because shadow costs are not observed, the shadow-cost function must be rewritten in terms of observable variables.

Accounting identities for the actual ($C^a$) and shadow ($C^s$) total costs are respectively:

\begin{equation}
C^a = \sum P_i X_i \quad \text{and} \quad C^s = \sum k_i P_i X_i,
\end{equation}

where $X_i$ is an input factor and $P_i$ is its market price. The shadow-cost-share equations are:

\begin{equation}
M^s_i = \frac{k_i P_i X_i}{C^s} \quad \text{for each } i.
\end{equation}

Instead of the traditional system of share equations, the following system can be derived:

\begin{equation}
P_i X_i = \frac{C^s M^s_i}{k_i} \quad \text{for each } i.
\end{equation}
The sum of these equations is:

\[(3) \quad C^g = C^g \Sigma (M^g_i / k_i),\]

and taking the logarithm of both sides yields:

\[(3') \quad \ln C^a = \ln C^g + \ln \Sigma (M^g_i / k_i).\]

Equation \((3')\) is estimable after substituting the shadow translog cost function for \(\ln C^g\) and the derived shadow cost shares \(MQ = \partial \ln C^g / \partial \ln (k_i P_i).\)

Observable cost-share equations can be derived by dividing both sides of equation \((2)\) by \(C^a:\)

\[(4) \quad M^g_i = \frac{C^g M^g_i}{C^a k_i},\]

where \(M^g_i\) is the actual cost share for input \(i\), and by substituting equation \((3)\) into equation \((4)\) to obtain:

\[(5) \quad M^g_i = k_i^{-1} M^g_i / \Sigma k_i^{-1} M^g_i.\]

The A-H shadow-cost system is \((3')\) and \((5)\). Because the sum of the actual cost shares is one, one of the actual cost-share equations in equation \((5)\) can be dropped.

**III. The Total Factor Productivity Equations**

The cost function employed by Atkinson and Halvorsen uses only shadow input factor prices and output as arguments. In order to estimate TFP using the A-H model, time must be added as an additional explanatory variable. The derivative of shadow cost with respect to time is then:

\[
\frac{dC^g}{dT} = \frac{\partial C^g}{\partial P^*_i} \frac{dP^*_i}{dT} + \frac{\partial C^g}{\partial Q} \frac{dQ}{dT} + \frac{\partial C^g}{\partial T} \frac{dT}{dT}.
\]

The first term on the right-hand side can be simplified by dividing both sides of the equation by \(C^g\), by using Shephard's Lemma for the shadow cost.
function \((i.e., \frac{\partial C^s}{\partial P^s}) = X_i\), and by noting that \(k_i\) is constant.

Then multiply and divide the second term by \(Q\) to obtain:

\[
C^s = \sum_i \dot{P}_i + v^s \dot{Q} + v^s,
\]

where the superscript \(\dot{}\) indicates the rate of change, for example \(\dot{C}^s = \frac{d\ln C^s}{dT}\). \(v^s\) is the shadow elasticity of shadow cost with respect to output. \(v^s\) is usually called technical change. It is the rate of change of shadow cost, holding constant all cost function arguments except time.

The rate of change in shadow TFP \(W^s\) can be defined similarly to that of actual TFP using a Divisia index of factor-input-shadow shares:

\[
W^s = \sum_i \dot{X}_i.
\]

Using the accounting identity (1)

\[
(6') \quad C^s = \sum_i \dot{X}_i + \sum_i \dot{P}_i,
\]

\(W^s\) can be expressed as:

\[
(7) \quad W^s = \dot{C}^s - \sum_i \dot{X}_i = \dot{v}^s \dot{Q} + v^s.
\]

Equation (7) is analogous to the equation derived by Nelson and Wohar for the rate of change in actual TFP \(W^a\):

\[
(7') \quad W^a = \dot{C}^a - \sum_i \dot{X}_i = \dot{v}^a \dot{Q} + v^a.
\]

The important difference between equations (7) and (7') is that equation (7) was derived by applying Shephard's Lemma to shadow cost, while (7') uses Shephard's Lemma for actual cost.

The difference between \(W^a\), defined in (7'), and \(W^s\) can be derived analytically. Substituting the identity \(\dot{C}^a = \dot{C}^s + \frac{\dot{C}^s}{C^s} \dot{C}^s\) into (7')
\( W^a \). The advantage of using (12') is that it is considerably easier to estimate than (9). The disadvantage of using (12') is that, in case of \( k_i = 1 \), it does not use an actual value of \( W^a \) as in equation (7'). Instead, \( W^a \) derived from (12'), with \( k_i = 1 \), is \( W^a = \hat{C}^a - \Sigma \hat{M}_i^a P_i \), where the \( \hat{M}_i^a \) are estimated actual cost shares.

IV. The Translog Specification of the Generalized Cost System

The translog form of the shadow-cost function is:

\[
\ln C^a = \alpha_o + \Sigma B_i \ln k_i P_i + B_0 \ln Q + B_T T
+ .5 \Sigma \Sigma Y_{i,j} \ln k_i P_i \ln k_j P_j + \Sigma Y_{i,0} \ln k_i P_i \ln Q
+ \Sigma Y_{i,T} (\ln k_i P_i) T + .5 Y_{00} (\ln Q)^2 + Y_{0T} (\ln Q) T + .5 Y_{TT} T^2.
\]

The shadow-cost function is restricted to be linearly homogeneous with respect to shadow prices using the coefficient restrictions:

\[
(13') \quad \Sigma B_i = 1, \quad \Sigma Y_{i,0} = 0, \quad \Sigma Y_{i,T} = 0,
\]

\[ \Sigma Y_{i,j} = 0, \text{ and } Y_{i,j} = Y_{j,i}. \]

The logarithmic partial derivative of equation (13) with respect to \( \ln k_i P_i \), using the modified Shephard's Lemma, yields the translog cost-share equations:

\[
(14) \quad M_i^a = (\partial \ln C^a)/\partial \ln (k_i P_i)
= B_i + \Sigma Y_{i,j} \ln k_j P_j + Y_{i,0} \ln Q + Y_{i,T} T.
\]

Substituting equations (13) and (14) into (3') yields the translog version of the cost function:
The translog cost-share equations are obtained by substituting equation (14) into (5): 

\[ (15) \quad \ln C^g = \alpha_o + \sum B_i \ln k_i P_i + B_o \ln Q + B_T T \]

\[ + \sum \sum \gamma_{ij} \ln k_i P_i \ln k_j P_j + \sum \gamma_{i0} \ln k_i P_i \ln Q \]

\[ + \sum \gamma_{iT} (\ln k_i P_i) T + \gamma_{oT} (\ln Q) T \]

\[ + \gamma_{oT} T^2 \]

\[ + \ln \left[ \Sigma k_{i}^{-1} (\beta_i + \Sigma \gamma_{ij} \ln k_j P_j + \gamma_{i0} \ln Q + \gamma_{iT} T) \right]. \]

The returns to scale \((v_o^g)\), technical change \((v_i^g)\), and shadow TFP equations \((9)\) and \((12')\) are:

\[ (16) \quad M^g = \left[ k_{i}^{-1} (\beta_i + \Sigma \gamma_{ij} \ln k_j P_j + \gamma_{i0} \ln Q + \gamma_{iT} T) \right] / \]

\[ \left[ \Sigma k_{i}^{-1} (\beta_i + \Sigma \gamma_{ij} \ln k_j P_j + \gamma_{i0} \ln Q + \gamma_{iT} T) \right]. \]

The returns to scale \((v_o^g)\), technical change \((v_i^g)\), and 
\[ d\Sigma(M_{15}/k_i)/dT \] expression for the translog shadow TFP equations \((9)\) and \((12')\) are:

\[ (17) \quad v_o^g = (\partial \ln C^g)/\partial \ln Q \]

\[ = \beta_o + \sum \gamma_{i0} \ln k_i P_i + \gamma_{oT} T + \gamma_{o0} \ln Q, \]

\[ (17') \quad v_i^g = (\partial \ln C^g)/\partial T \]

\[ = \beta_T + \sum \gamma_{iT} \ln k_i P_i + \gamma_{oT} \ln Q + \gamma_{iT} T, \]

and

\[ (17'') \quad d\Sigma(M_{15}/k_i)/dT = \Sigma [k_i^{-1} (\partial^2 \ln C^g)/\partial \ln (k_i P_i) \partial T] \]

\[ + \Sigma [k_i^{-1} (\partial^2 \ln C^g)/\partial \ln (k_i P_i) \partial Q] \cdot \]

\[ + \Sigma [k_i^{-1} (\partial^2 \ln C^g)/\partial^2 \ln (k_i P_i)] \cdot P_i, \]

\[ = \frac{\Sigma k_i^{-1} \gamma_{iT} + \Sigma k_i^{-1} \gamma_{i0} Q + \Sigma \Sigma k_i^{-1} \gamma_{ij} P_i}{\Sigma k_i^{-1} M_i^g} \]

\[ = \frac{\Sigma k_i^{-1} \gamma_{iT} + \Sigma k_i^{-1} \gamma_{i0} Q}{\Sigma k_i^{-1} M_i^g}. \]

Also, for the following discussion, note that:
(18) $v_0^\circ = (\partial \ln C^a) / \partial \ln Q = v_0^\circ + (\Sigma k_i^1 \gamma_{iO}) / \Sigma k_i^1 M_i^2$, and
(19) $v_1^\circ = (\partial \ln C^a) / \partial T = v_1^\circ + (\Sigma k_i^1 \gamma_{iT}) / \Sigma k_i^1 M_i^1$.

Thus, equation (9) can be rewritten as:
(20) $C^a = v_0^\circ Q + v_1^\circ + \Sigma M_i^2 p_i$.

Similarly, the translog form of the second shadow TFP equation is obtained by substituting (17), (17'), and (14) into (12'), which is obviously a much shorter expression than equation (20) in translog form.

V. Estimating the Regulatory Bias

Atkinson and Halvorsen showed that equations (15) and (16) are homogeneous of degree zero (h.d.z.) with respect to the $k_i$. Therefore, one of $k$, can be chosen arbitrarily, and one is a natural and convenient normalization value. An estimate of the effect of regulation on total cost and other components is obtained by comparing the fitted values of the desired variable generated by the estimated model, with all of the $k$, equal to their estimated values (estimated regulatory impact included), with the fitted values of the same variable but with all of the $k$, set to one (no regulation).

It is important to note that this procedure works only for variables whose equations are h.d.z. with respect to the $k_i$; otherwise, the magnitude of the regulatory bias depends on the value of the $k_i$ normalization.

The $\ln C^a$ equation (13) is not h.d.z. in the $k_i$. All of the terms associated with the $\gamma_i$ coefficients are h.d.z. with respect to $k$, from the coefficient restrictions (13'), but the terms related to the $\beta_i$.
coefficients are not; if the $k_i$ are multiplied by some constant $t$, then
\[ \Sigma B \cdot \text{Int} = \text{Int} \] using (13').

The shadow share equations (14) are h.d.z. in the $k_i$ because they have no terms involving both the $\beta_i$ and $k_i$. Hence, the equations (16) are h.d.z. with respect to the $k_i$ because the $t$ factors for the $k_i^{-1}$ will cancel out in the numerator and denominator. Actual cost (equation (15)) is h.d.z. in the $k_i$ because the Int term for the nonhomogeneous component of $\ln C^s$ will cancel out with the $-\text{Int}$ term of the nonhomogeneous component of $\ln M_{i/k_i}$. Both $v_0^s$ and $v_i^s$ are h.d.z. in the $k_i$ because they have no terms involving both the $\beta_i$ and $k_i$; $v_0^s$ and $v_i^s$ (equations (18) and (19)) are h.d.z. in the $k_i$ because the terms added to $v_0^s$ and $v_i^s$ are h.d.z. in the $k_i$.

Both TFP equations, (20) and (12'), are h.d.z. in the $k_i$. The first two components of (20) are h.d.z. in $k_i$; the third term also is h.d.z. in the $k_i$ because $M^s$ is h.d.z. in the $k_i$. All of the terms in equation (10) have already been shown to be h.d.z. in the $k_i$. Thus, the effect of regulation on TFP is computed by adding the individual regulatory effects on returns to scale, technical change, and shadow shares. This is an improvement over the Nelson and Wohar approach, which assumed that returns-to-scale and technical-change components are independent of any regulatory effect.
References


