

Working Paper 8612

DESIGNING MONETARY POLICY UNDER RATIONAL EXPECTATIONS:
ANALYSIS AND PRACTICAL IMPLICATIONS

By James G. Hoehn

James G. Hoehn is an economist at the Federal Reserve Bank of Cleveland. The author thanks **William T. Gavin**, Mark S. Sniderman, and Alan C. Stockman for comments on drafts.

Working papers of the Federal Reserve Bank of Cleveland are preliminary materials circulated to stimulate discussion and critical comment. The views stated herein are those of the author and not necessarily those of the Federal Reserve Bank of Cleveland or of the Board of Governors of the Federal Reserve System.

December 1986

DESIGNING MONETARY POLICY UNDER RATIONAL EXPECTATIONS:
ANALYSIS AND PRACTICAL IMPLICATIONS

This paper describes and attempts to generalize the practical implications for the design of monetary policy of some of the most popular macroeconomic models incorporating rational expectations. Perhaps the most important of these implications is a shift in the focus of policy from output or interest-rate stabilization toward price-level stabilization.

The rational expectations assumption rules out systematic expectational errors. The assumption proved necessary to ensure consistency of models with the natural rate property--that the average level of output is invariant with respect to monetary policy and other monetary phenomena. Introduction of rational expectations overturns the case for conventional countercyclical policies.

New macroeconomic models, which combine rational expectations with either incomplete information or nominal contracts, offer a seemingly unintelligible variety of results and implications for policy. The purpose of this paper is to describe the ways in which rational expectations fundamentally change monetary policy analysis and to attempt to generalize the implications of such analysis. To do so effectively, some careful development of mathematical concepts is indispensable. For example, in rational expectations models, expectations are forward-looking rather than backward-looking. Therefore, policy must be specified as a contingent rule of behavior; that is, an equation relating instruments to observed outcomes.

In practice, the analytical problems of finding the optimal policy rule are virtually insurmountable except in simplified cases. The dimensions of

the models or the range of policy rules considered must be severely reduced in some arbitrary way to obtain results. To place various results in perspective, it is essential that the significance of particular simplifying assumptions be understood. Otherwise, interpretations are insecure, ambiguous, and seemingly contradictory. In principle, such problems exist in any class of models, but the dynamics of rational expectations models pose new and ill-understood analytical problems. A willingness to grapple with some tricky analytical difficulties is essential to the practical application of rational expectations models to policy analysis.

The analytical problems may in part account for the continued popularity of pre-rational-expectations IS-LM models. These are the source of persistently popular notions concerning policy and form the basis of much empirical work, including the large-scale macroeconomic models. In these models, a reasonably well-defined policy can improve welfare by stabilizing aggregate demand. Under this conventional countercyclical policy, the money stock depends upon the last observations of the (currently unknown) state of the economy. Also, under plausible assumptions about parameters and relative disturbance variances, the money stock should respond positively to the (currently known) interest rate. These IS-LM models had serious problems, symptomatic of which was inconsistency with the natural rate property, that could be corrected only by introducing rational expectations.

Sargent and Wallace (1975) introduced rational expectations into an otherwise conventional IS-LM model. The main result was that, once policy effects operating through systematic expectational errors were ruled out, money supply responses to the state of the economy were of no consequence for output behavior. Nevertheless, positive money stock responses to the current interest rate could still be helpful, as in the pre-rational-expectations

IS-LM model. But an "interest rate rule," or money supply policy that makes the money stock infinitely elastic with respect to the interest rate, would leave the price level and the money stock indeterminate.

Subsequent rational expectations models considered different information assumptions and different structural characteristics, such as long-term contracts or intertemporal substitution of leisure. Policies to improve welfare in these models depend too much on particulars to provide unambiguous descriptions of optimal policy rules. Indeed, derivation of the optimal policy becomes an analytically intractable task, without somewhat arbitrary restrictions on either the structure of the economy or the range of policy choices considered.

However, rational expectations models (except certain cases that do not possess the natural rate property) have two characteristics with practical policy implications: (1) an optimal policy is equivalent to one that minimizes the price level uncertainty of suppliers over various horizons that are determined by information lags and/or contract lengths, and (2) "interest rate rules" make the price level and money stock indeterminate.

The first section to follow considers the way policy objectives and choices are specified in rational expectations models and notes some limitations and unresolved analytical problems, which are illustrated in later sections. Next, the basis of prevailing concepts of policy is shown to be rooted in the pre-rational-expectations IS-LM models, and the shortcomings of those models are discussed. In the next section, it is shown that rational expectations destroys the case for the conventional countercyclical policies, but does not lead to very specific conclusions about the optimal policy rule without particular assumptions about information available to private and public agents. Non-market-clearing models, such as those in which sellers of

goods or labor agree to accommodate demand at a predetermined nominal price, are found to have unconventional, but few general, policy **implications**. Finally, similar ambiguities are found in intertemporal substitution models. A final section interprets the practical policy **implications** of rational expectations models as arguing for price stability and against policies that seek to stabilize output or interest rates.

The Policy Problem

What Should Monetary Policy's **Objective(s)** Be?

At the most general level, the policy objective can be taken as the enhancement of the welfare of the representative agent (**consumer/factor supplier**). Implicit in different macroeconomic models that give a role for monetary policy are different constraints that impede agents' attainment of the **first-best** economic outcomes. Then policy can improve welfare by reducing the effectiveness of these constraints.

Implicit in all the major competing models is a common set of micro-economic assumptions, which can be briefly described as follows. The welfare of the representative individual in the economy is specified by his utility function, which he maximizes subject to various constraints. His utility depends positively upon the amounts of consumption and leisure he enjoys in each period, with future amounts discounted according to how soon they will occur. In addition, consumption and leisure provide declining marginal **utility**, so that individuals display risk aversion--they tend to prefer, for example, more stable patterns of consumption and **leisure over** unstable ones, for given present discounted values of consumption and leisure streams. Individuals have access to a production function either directly, or

indirectly through a labor market in which profit-maximizing firms are buyers. The production function has labor and at least one other (capital or fixed) factor input, and displays decreasing marginal productivity and constant returns to scale. In the absence of the other constraints to be discussed, individuals can trade freely in various markets, subject to budget constraints.

Changes in productivity and technology alone are entirely capable of generating cycles of the kind actually observed in developed, market economies, even if agents optimize, and even without the additional constraints assumed in monetary models of the business cycle. Recent work on real business cycle models suggests important limits on the scope of fluctuations attributable to monetary phenomena in general and monetary policy in particular. However, these models are not the subject of study here because their implications for the conduct of monetary policy in a cyclical framework are relatively speculative and do not fit well within the discussion.

Within the class of models that do give an important role to monetary policy in generating cycles, the critical differences between the alternative models do not involve the assumptions about utility and production functions. Instead, the main differences lie in certain additional constraints faced by agents. Usually, it is assumed that business cycles reflect some failure of the market economy to reach a Pareto-optimum. To explain this failure, models have placed agents under constraints of one of two types: (1) incomplete information, of which money illusion can be considered an extreme special case, or (2) failure of markets to clear, of which nonexistence of markets can be thought of as an extreme special case. These constraints determine the mechanism by which policy exerts its influence, in ways to be explicated below.

Given the utility, production function, and market failure assumptions, each model **implies** a set of decision rules describing how agents respond to information available to them. These decision rules imply market demand and supply functions, usually expressed as **linear** approximations, that can be used to relate the behavior of the aggregate of individuals to the state of the economy. **It** is these representations of the models--linear supply and demand functions--that have proven most analytically tractable.' This representation is termed the "structural" form of the model.

A serious problem inherent in these representations is that they are rarely, **if** ever, invariant with respect to the class of policy interventions considered. This point was made forcefully by Lucas (1976), who demonstrated that the orthodox IS-LM models were not invariant to changes in policy in the presence of rational expectations. As a principle, this point is **uncontrover-**sial, although its practical implications are troubling: either policy **analy-**sis must be regarded as impossible, or the sensitivity of the representation to the range of policies considered must be assessed.² This sensitivity analysis cannot be performed without making explicit the microeconomic foundations of the model. Recognition of this principle gave added vigor to the ongoing search for microeconomic foundations.

What Is Monetary Policy?

The very existence of monetary policy requires some set of regulations and/or legal tender restrictions affecting the financial and payments systems. The nature of these regulations and restrictions is a critical part of the model in which policy choices are made. The regulations and restrictions are, however, not well understood and are taken as fixed in the most common form of policy analysis. Sufficient assumptions for the existence of

monetary policy are that the monetary policymaker **(a)** controls base money through a role as sole provider and **(b)** simultaneously controls the rate of return of base money relative to other assets through reserve requirements and other controls over the payments system. The first assumption gives policy a nominal quantity **it** can control; the second one gives rise to a demand function for that quantity, which gives manipulation of that quantity conceivable leverage over the macroeconomy.

Given this legal and institutional framework, some program of changes in the stock of money (or base money) constitutes an instrument for minimizing the effect of the market failures on the representative private agents' welfare.³ This optimization takes place with reference to **all** conceivable contingent behaviors for money. These behaviors can be specified most generally in terms of parameters of a rule linking the quantity of money to the set of information available to the policymaker.

To formalize, consider policy rules that are a linear function of the information set. Then the rule may be written as:

$$(1) \quad m_t = HI_t,$$

where m_t is the log of the money stock as of time period t , H is a vector of coefficients characterizing the policy responses, and I_t is the information set available to the policymaker at time t . I_t might include the "variable" 1 (one) and any powers of t (time index). By convention, I_t excludes m_t itself. Although the latter is observable, **it** is already included on the left-hand side--merely a kind of normalization. However, I_t may include m_{t-1} or other lagged money terms, and generally will.

Equation (1) states the obvious truth that policymakers can only respond to the information they have at a point in time. Choice of policy is represented by choice of values of H . Furthermore, choice of the elements of H

that are coefficients of unity and powers of time will be irrelevant for nearly all purposes, at least in rational expectations models, or any others displaying the neutrality of money.

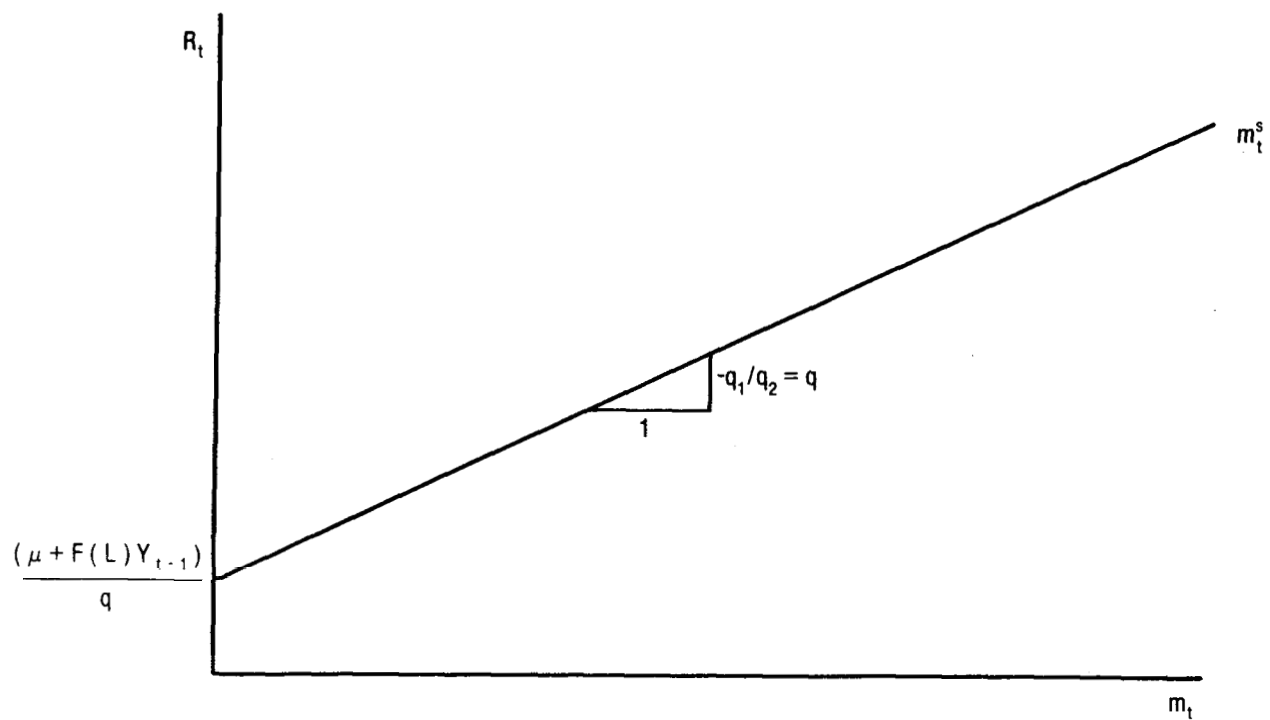
As will be seen, it is analytically useful in the following discussion to distinguish carefully between policy responses to contemporaneous information versus responses to delayed information. This distinction can be effected by segmenting the information set, \mathbf{I}_t , into current (period- t) realizations and previous (period $t-1, t-2, \dots$) realizations. In particular, the current interest rate ought to be considered contemporaneously observable, while output and prices are known only with a lag. Then the policy rule may be written as:

$$(2) \quad m_t + qR_t = \mu + F(L)Y_{t-1},$$

where R is the nominal interest rate, Y is a vector of state variables, q is a scalar, and $F(L)$ is an n -dimensional vector polynomial in the lag operator, L (defined such that $L^k Y_{t-1} = Y_{t-1-k}$). Trends or polynomials in t have been excluded in (2) because they only clutter the results with uninteresting terms. μ is treated as an exogenous constant; its value does not bear on the issues addressed. The policy choice is then represented by the joint choice of q and $F(L)$. In each period, the policymaker observes Y_{t-1} and, in light of Y_{t-1}, Y_{t-2}, \dots , chooses a linear sum of money and the interest rate that will serve as the criterion for money provision in period t .

Rules of form (2) are commonly encountered in the literature on monetary policy, and are often termed money supply functions.⁴ This function is the basis of the money supply curve in figure 1. The positive slope there reflects a negative q , indicating that the money stock is increased contemporaneously with rises in the nominal interest rate, for given realizations of Y_{t-1}, Y_t, \dots . The intercept, $(\mu + F(L)Y_{t-1})/q$, varies with the

Figure 1
Money Supply Function



observed (lagged) state of the economy. For example, the choices of F-elements might tend to reduce the intercept in response to output declines and increase it after output increases. These two features, especially the cyclical intercept, characterize what can be termed the conventional stabilization policy.

What Range of Policy Choices Is Relevant for Analysis?

This representation of the range of policy choices must be simplified carefully in the context of each model in order to proceed with analysis. Unfortunately, no general procedure exists for determining an adequate, yet sufficiently parsimonious specification of an optimal policy rule under rational expectations. The appropriate specification will depend on both the structure and the assumptions about information sets available to the policy-maker and private agents. This problem seems to limit analysis to cases in which an adequate policy specification can be confidently determined. In practice, this has meant certain restrictions on the structure and information sets that limit the generality of results. Some considerations in the choice of the appropriate specification are the subject of this section.

First, the specification of the policy rule might include as arguments only variables in the set I_t that are also in the set of minimal state variables, denoted M_t . M_t contains all variables, treating lags as distinct variables, appearing explicitly in (nonpolicy) equations of the model in its structural form (see McCallum [1983]). In models in which period $t-1$ variables appear in the structural equations, but variables dated earlier do not, the minimal-state-variable criterion will serve to truncate $F(L)$ to a vector of scalars. Then equation (2) can be rewritten as:

$$(3) \quad m_t = \mu - qR_t + FY_{t-1}^s,$$

where $\{R_t, Y_{t-1}^s\} = \{M_t \cap I_t\}$. Except where otherwise noted, this representation shall be treated as the appropriate representation of the policy rule.

The minimal-state-variable approach has practical advantages and may serve as an appropriate starting point for identifying relevant variables for the policy rule. Certainly, variables in M_t are prime candidates for inclusion in the policy rule. And limiting those included to the minimal-state-variable set rules out an indefinitely large number of trivial variables, which analysis would ultimately find to be irrelevant anyway (their optimal coefficients in the policy rule would be zero). The limitation to minimal state variables is also thought to rule out inclusion of intrinsically irrelevant variables, termed bootstrap variables, that analysis would find relevant only **if** they were included in the setup of the problem, either in the policy rule or in private rational expectations formation.

Unfortunately, the minimal-state-variable approach will not necessarily result in an adequate policy rule. Ambiguities about which variables are relevant generally arise unless particular assumptions are made concerning private agents' information sets, denoted S_t . M_t may not contain all the variables providing relevant conditioning information that agents use to form rational expectations. Hence, some variables not in $\{M_t \cap I_t\}$ may be relevant state variables after all and should appear in the representation of the policy rule. For example, prices in the last period may not appear in the "structural" form of the model (supply and demand equations); yet, **if** private agents forming expectations have an information set containing only lagged prices, then those prices will generally influence supply and demand decisions via expectations, and should appear in the optimal policy rule. But **if**, on the other hand, private agents were endowed with a different information set,

including the current price level, they might find lagged prices uninformative. Then lagged prices might not be a relevant state variable and need not appear in the policy rule. The importance of this example is that, regardless of the specification of S_t , lagged prices are not a minimal state variable, because the latter depends only on the structural equations, not on S_t . Yet, a change in S_t affects the relevant set of variables that generally appear in the optimal policy rule. Hence, inclusion of all minimal state variables does not assure a sufficient representation for an optimal rule.

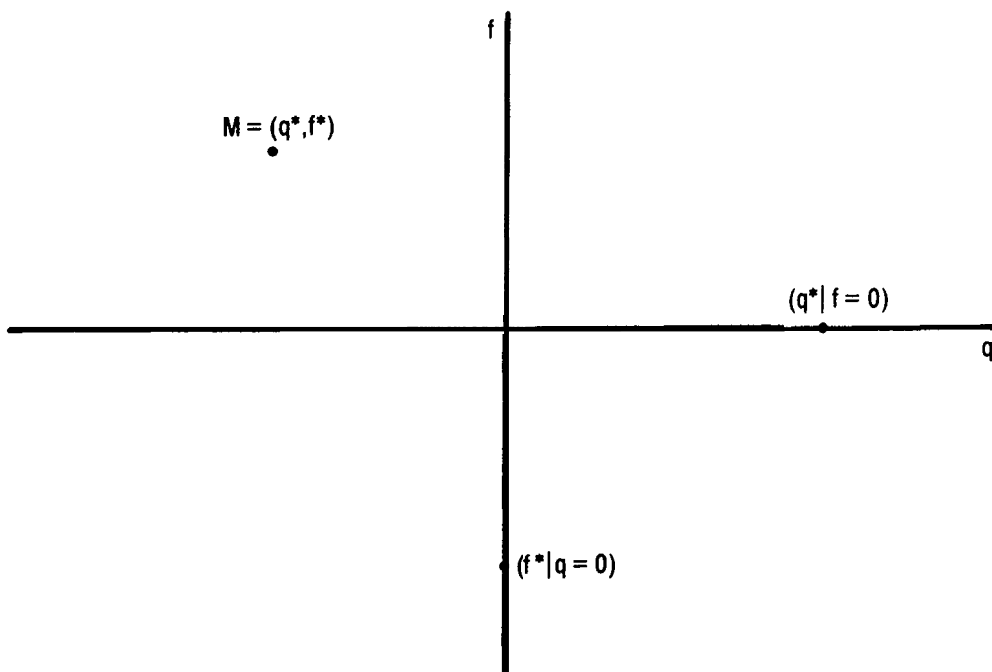
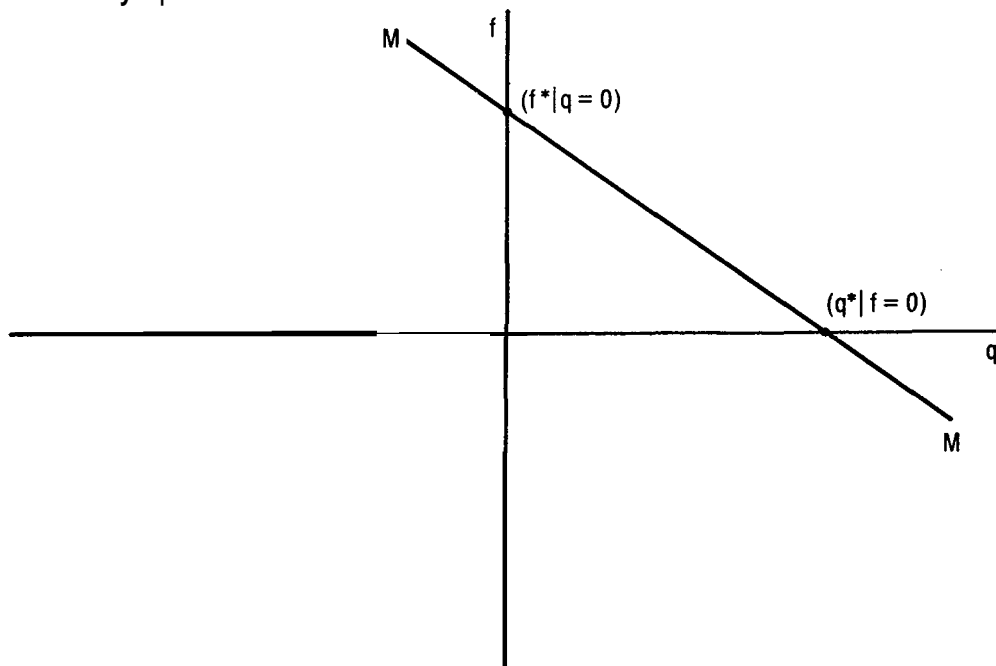
All told, parsimony in the setup of the optimization problem, and particularly the variables included in the policy rule, is both essential and fraught with dangers. Chief among these dangers is that relevant variables (including various lags) may inadvertently be left out of the policy rule. Yet, unless the analyst is sure that (at least) all the relevant variables are included, the form of the rule postulated may exclude the optimal policy or policies altogether. Then, the optimal policy or policies are ruled out in the setup of the analysis. Or, an unduly restricted policy space may include only some of the members of the class of optimal policies, but not all. Then, feasible policies with very different characteristics may be just as desirable as the best within the restricted policy space, yet the analyst might incorrectly argue against them on the basis of his limited results.

Hypothetical illustrations of the problems attending undue restriction of the policy space are conveyed by figure 2 for two different economic models. For simplicity, the appropriate specification of the policy rule is assumed to be

$$(4) \quad m_t = -qR_t + fy_{t-1},$$

where y is output. The unrestricted policy space is $q \times f$, or R^2 .

Figure 2
The Policy Space



If the exclusion restriction $f=0$ or $q=0$ is arbitrarily imposed, the restricted optimum is $(q^*|f=0)$ or $(f^*|q=0)$, respectively. Without these undue restrictions, the optimal policy set is the line MM in the model corresponding to the upper panel, and the point M in that of the lower panel. In the first case, partial analysis leads to an optimum; but the analyst might, on the basis of his results, argue fallaciously against other optimal policies that had either q or f negative. In the second case, partial analysis does not arrive at a global optimum, or even a correct evaluation of the signs of optimal policy parameters. Problems of both kinds can easily occur. Formal examples of the first kind will be given in what follows. The second type of problem was informally illustrated above; it is likely to arise without particular restrictions on information and the structure. These problems are a major, if inadequately acknowledged, pitfall of analysis of optimal policy under rational expectations. Often, analysis has avoided this problem only by somewhat arbitrary restrictions on information sets I_t and/or S_t .

Another, valid, restriction on the relevant policy space is that which rules out indeterminacy of important variables. The values of q and F of the policy rule specified by the general form (3) cannot be specified arbitrarily, for (3) must suffice to complete the economic model in the sense of rendering all the endogenous variables determinate. The force of this restriction obviously depends on the other aspects of the model. An important example to be given relates to indeterminacy of money and prices under a "policy" of pegging the interest rate.

Throughout the formal analysis that follows, it will be further assumed that q and F are not functions of time.' This assumption implies that the policymaker is able to make a commitment to a time-consistent rule of behavior. Under rational expectations, the ability to so commit is necessary to

the attainment of the optimal outcome; otherwise, policy will be unable to enlist the support of private expectations. This assumption is far from innocuous and its realism is doubtful. A new and growing literature attempts to deal with the design of second-best policies in models in which the policy-maker is constrained by an inability to precommit. (See, for example, Barro [1986]). Nevertheless, even if the first-best policy is infeasible, the macroeconomic issues involved in its design will still be relevant.

The analysis to follow will restrict attention to steady-state properties of alternative stochastic models, because only these properties are determined by rational expectations models. For a given model and a given objective function, there is a mapping from each element in the policy space to the value of the objective function. The optimal policy is characterized by the element (or, if nonunique, set of elements) in the policy space associated with the optimization of the objective function. This policy will serve to minimize the effectiveness of, or utility loss pursuant to, the constraints on information or market-clearing that prevent the economy from attaining a Pareto-optimal allocation of resources. In the following analysis, the variations in the constraints on private utility maximization that differentiate prominent macroeconomic models are shown to imply variations in the optimal policy rule.

Pre-Rational-Expectations IS-LM Models

For a number of related reasons, it is useful to begin analysis with pre-rational-expectations IS-LM models. First, they have pedagogical value in that their analytical simplicity sets the stage for easier understanding of more complex models. Second, these IS-LM models generate most conventional

views on optimal policy. They serve as essential representations of most macroeconometric models. An assessment of the shortcomings of these pre-rational IS-LM models helps motivate the assumption of rational expectations. Finally, understanding how optimal policy is designed in pre-rational models will allow insight-provoking contrasts with the rational expectations models.

The Fixed-Price Model

In the influential treatment of monetary policy of Poole (1970), the model was of the simple textbook IS-LM form, with fixed, or at least exogenous, prices. The aggregate commodity demand function, or IS curve, was

$$(5) \quad y_t^d = d_0 + d_1 R_t + d_2 y_{t-1} + u_{1t}, \quad d_1 < 0 < d_2 < 1,$$

where y was output, and the money demand or LM curve was

$$(6) \quad m_t = a_0 + a_1 R_t + a_2 y_t + e_t, \quad a_1 < 0 < a_2.$$

u_t and e_t were disturbances. Output was determined strictly by demand:

$$(7) \quad y_t = y_t^d.$$

Generally, if, as in this case, there are no expectations in a model, the policy rule need include, at most, the minimal set of state variables that are also in the policymakers' information set. The minimal set includes y_t , y_{t-1} , R_t , and m_t , but y_t is not contemporaneously observable to the policymaker. Therefore, an optimal policy will take the form:

$$(8) \quad m_t = \mu_0 - qR_t + f_1 y_{t-1},$$

where q and f_1 are scalars. Since output is determined strictly by demand, reflecting the fixed-price assumption, and since the utility function embodies risk aversion, the appropriate criterion is minimization of deviations of output around its optimal level, where the latter depends on implicit and fixed productivity and tastes. An appropriate value of μ is needed to make

average output equal to the optimal output level, because in this model, the average level of output depends on the average level of money. (Obviously, this is a particularly crude violation of the natural rate property.) Then, using a quadratic local approximation for the utility function, the appropriate objective is the minimization of output variance around this optimal output average. The optimization problem is thus separable into a level and a variance problem, and the latter will occupy the following discussion.

Then the optimization can be represented as that of minimizing the variance of output with respect to (q, f_1) , subject to equations (5), (6), and (7).

The reduced form solution for output is

$$(9) \quad y_t = d_1(\mu - a_0)J_1 + (a_1d_2 + q + d_1f_1)J_1y_{t-1} + (a_1 + q)J_1u_t - d_1J_1e_t,$$

$$\text{where } J_1 = (a_1 + a_2d_1 + q)^{-1}$$

with a steady-state variance of

$$(10) \quad \sigma_y^2 = [(a_1 + q)^2 \sigma_1^2] + d_1^2 \sigma_e^2 \quad \mathbf{J}_2,$$

$$\text{where } \sigma_y^2 = E(y_t - Ey_t)^2,$$

$$\sigma_1^2 = E(u_{1t} - Eu_{1t})^2,$$

$$\sigma_e^2 = E(e_t - Ee_t)^2, \text{ and}$$

$$\mathbf{J}_2 = (a_1 + a_2d_1)^2 - (a_1d_2 + df_1)^2 + 2q(a_1 + a_2d_1 - a_1d_2 - d_1f_1)^2.$$

The policy space $q \times f_1$ is R^2 excluding $q = -a_1 - a_2d_1$. Assuming the disturbances are uncorrelated, the first-order conditions imply the optimal q and f_1 are given by

$$(11) \quad q = (d_1/a_2)(\sigma_e^2/\sigma_1^2) - a_1$$

and

$$(12) \quad f_1 = -a_1d_2/d_1 + q/d_1.$$

These expressions show that the static IS-LM model supports conventional

views on the appropriate design of monetary policy. The optimal value of q is negative if, as commonly supposed, the variance of money demand disturbances is "large" (in a loose sense that depends on d_1/a_2 and a_1) relative to commodity demand disturbances. Then money supply should be positively related to the current interest rate (given y_{t-1}). The value of f_1 will be negative, implying that countercyclical variations in money help stabilize output. It is noteworthy, for comparisons with later models, that the optimal q and f_1 are unique. Also, the choice of q (contemporaneous responses) is separable from the choice of f_1 (lagged responses), in the sense that the optimal choice of q can be found without considering the optimal value of f_1 . The optimal choice of q is also unaffected by the magnitude of d_2 , the coefficient linking commodity demand to its past. In other words, the dynamics of this model are such that they do not become a consideration in the choice of the slope of the money supply function depicted in figure 1, but only in the choice of its state-dependent intercept.

One of the most glaring shortcomings of the static IS-LM model is that it leaves prices undetermined, or exogenous. One simple, and conventional, means of making prices endogenous is to introduce the "law of supply and demand,"

$$(13) \quad p_t = p_{t-1} + v(y_t - y^c), \quad v > 0,$$

under which inflation varies directly with "demand pressure," equal to real demand, y_t , minus "full employment" or "capacity" output, y^c . This alteration invites placement of p_t in the money demand function,

$$(14) \quad m_t - p_t = a_0 + a_1 R_t + a_2 y_t + e_t, \quad a_1 < 0 < a_2.$$

However, this method of making the price level endogenous has severe problems. If neither buyers nor sellers can be forced to transact, then the demand-determination of output implies that prices must be too high to clear

the market at full employment. To make rudimentary sense of this model, some additional explanation of supply behavior is required. Suppliers must either face money illusion or be under some type of non-price rationing constraint, or prices and the interest rate would fall immediately to the level that would clear the commodity market and output would be constrained by supply, rather than by demand. Even with rationing or money illusion, prices will fall persistently over time, as long as aggregate demand is the constraint on output. The optimal policy would seem to be an increase in the money stock adequate to force interest rates down low enough and drive output up to its supply constraint, at which time deflation would halt. It is interesting to note that there is no trade-off between maintaining full employment and stabilizing the price level in this model. Falling prices invariably reflect less-than-capacity output levels. It is also noteworthy that aggregate demand is misspecified in this model, and ought to have the real rate of interest rather than the nominal rate as its argument, unless expectations of inflation (actually, deflation) are fixed. This problem can be resolved only by introducing price-level expectations.

The Adaptive Expectations Model

An explicit commodity supply function, together with some mechanism for reconciling demand and supply--either by market-clearing price and interest-rate adjustments, or by some rationale for price stickiness other than rationing of demand among suppliers--seemed necessary elements for macroeconomic models with desirable microeconomic (and empirical) implications. The first major attempt, attributable to Phelps (1967), Friedman (1968), and Lucas and Rapping (1969), relied on workers' confusions between real and nominal wages which were exploited by employers. Wages were slow to adjust to actual

inflation caused by monetary shocks because workers were less than fully informed about the price level. Thus, they were tricked into working harder, at lower actual real wages, whenever the price level rose relative to previous expectations. This notion is incorporated in the following dynamic version of the IS-LM model, in which expectations adapt slowly and mechanically.

In the IS-LM model with autoregressive expectations, p_t appears in the money demand function, as in (14). The aggregate demand function becomes

$$(15) \quad y_t^d = d_0 + d_1 r_t + d_2 y_{t-1} + u_{1t}, \quad d_1 < 0 < d_2 < 1,$$

where

$$(16) \quad r_t = R_t - E_t(p_{t+1} - p_t)$$

and the aggregate supply function becomes

$$(17) \quad y_t^s = s_0 + \lambda y_{t-1} + s(p_t - E_t p_t) + u_{2t}, \quad s > 0, \quad 0 \leq \lambda < 1.$$

The lagged output term, y_{t-1} , in (17) can represent capacity effects of previous output levels, or costs of adjustment in employment levels, as in Sargent (1979). To (15) and (17) is added a market-clearing equation

$$(18) \quad y_t^s = y_t^d$$

and expectations equations of the adaptive type, such as

$$(19) \quad E_t p_t = \phi_0 + \phi p_{t-1}$$

$$\text{and } E_t p_{t+1} = \phi_0(1-\phi) + \phi^2 p_{t-1}, \quad 0 < \phi < 1.$$

According to (19), agents predict prices according to a first-order autoregression. It is noteworthy, for later comparisons, that price expectations are backward-looking: they are uniquely determined at time t by the initial condition p_{t-1} .

In general, an adequate yet parsimonious representation of an optimal policy rule is easy to determine in models with ad hoc expectations formations. An adequate policy rule need include only those variables in the

policymaker's information set that are either minimal state variables or variables upon which private expectations are conditioned. p_{t-1} is the only expectations-conditioning variable, according to (18) and (19). So the rule should include m_t , R_t , y_{t-1} , and p_{t-1} :

$$(20) \quad m_t = \mu - qR_t + f_1 y_{t-1} + f_2 p_{t-1}.$$

Many economists initially treated output stabilization as the appropriate criterion for policy in this model. That criterion is inappropriate, as will be argued later, when the microeconomics of the supply function are considered. Nevertheless, to understand the implications for theory and policy of rational expectations--as distinct from the advances in microeconomic foundations of supply behavior that occurred more or less concomitantly--it is useful to consider how policies might be designed to control output in the adaptive expectations model.

The reduced-form equation for output is

$$(21) \quad y_t = \Pi_0 + \Pi_1 y_{t-1} + \Pi_2 p_{t-1} + \Pi_3 u_{1t} + \Pi_4 u_{2t} + \Pi_5 e_t$$

for $\Pi_0 = dsJ\mu + (dk + d_0 - \phi\phi_0)$,

$$\text{where } k = [s(-a_0 + 1 + \phi_0 - a_2 d_0 + a_2 \phi\phi_0) - d_0 + \phi\phi_0]J,$$

$$\Pi_1 = d_1(\lambda + sf_1 - d_2(1 + a_2 s))J + d_2,$$

$$\Pi_2 = [s(f_2 - \phi) + d_1\phi(\phi - 1)(1 - a_2 s)]J + d_1\phi(1 - \phi),$$

$$\Pi_3 = s(a_1 + q)J,$$

$$\Pi_4 = dJ,$$

and $\Pi_5 = -sdJ,$

where

$$J = [d_1 + s(q + a_1 + a_2 d_1)]^{-1}.$$

In this model, the relevant policy space includes all combinations of (q, f_1, f_2) except those for which $q = -d_1/s - a_1 - a_2 d_1$, because the latter would

violate the completeness restriction. The policy that minimizes output variance is specified by:

$$(22) \quad q = (d_1\sigma_2^2 + s^2\sigma_e^2) / (1 + a_2s)\sigma_1^2 - a_1,$$

$$(23) \quad f_1 = -s^{-1}[\lambda - d_2(1 + a_2s)] - d_2s^{-1}d_1^{-1}J^{-1},$$

and

$$(24) \quad f_2 = \phi[1 + (\phi - 1)(d_1/s)(1 - a_2s + J^{-1})].$$

Somewhat surprisingly, the signs of the optimal values of f_1 and f_2 are ambiguous, without extensive empirical information. However, a counter-cyclical policy, by which is meant nonzero $F = [f_1 \ f_2]$, can obviously be effective in this model. Indeed, the business cycle--characterized by persistent high and low values of output relative to trend--can be eliminated, given complete knowledge of the structural parameters. The minimization of output variations (corresponding to these choices of q , f_1 , and f_2) is consistent with this elimination of the business cycle. So, the properly designed policy will **both** make output innovations (stochastic fluctuations not attributable to tendencies of previous such fluctuations to persist) as small as possible **and** eliminate any tendency toward persistence. That these two properties are found simultaneously in the optimal rule for this--and other pre-rational-expectations models--may seem trivial. However, this coincidence is **not** a general feature of rational expectations models.

As in the static IS-LM model, the optimal choice of q can be determined without reference to the choice of f_1 and f_2 . Also like the static IS-LM model, the output-variance-minimizing value of q is independent of the dynamics of the model (d_2 , ϕ , f_1 , and f_2 do not appear in equation (22)), while F depends on both the dynamic elements (d_2 and ϕ) as well as q .

It is noteworthy that conventional macroeconomic models are essentially an admixture of the static and dynamic IS-LM models described in this

section. The aggregate supply behavior is much like that of the dynamic IS-LM model with autoregressive expectations (see McCallum [1980] and the references cited there). Instead of suppliers' prices responding to adapting price expectations, though, price behavior is described as responding to labor and product market conditions. This difference in description is essentially inconsequential for the qualitative analysis of optimal policy. However, the aggregate demand specification of conventional models is that of the static model, with the nominal rather than the real interest rate as an argument. This latter difference is consequential.

The IS-LM model with autoregressive expectations and an expectational Phillips curve seemed, initially, to satisfy objections to the earlier simple IS-LM model. Prices are no longer exogenous, but respond to the same set of shocks as does output. If e_t and $u_{1,t}$ have variances that are "large" relative to that of $u_{2,t}$ (in a loose sense that depends upon the structural parameters d_1 , s , a_1 , a_2 , q , and ϕ), then output and prices will be positively correlated. Yet, an occasionally "large" supply shock $u_{2,t}$ could result in coincidence of high inflation and low output. These features gave the model greater empirical credibility than the earlier IS-LM models in which supply behavior was not made explicit.

Yet, despite these improvements, the IS-LM model with autoregressive expectations retained one fatally implausible microeconomic implication: it was inconsistent with the natural rate hypothesis. In the representation shown above, in which price-level expectations are stationary, any regular increase in the money stock would bring about a permanent increase in output. In the more popular "accelerationist" representation, price-level expectations are stationary in growth rates. In these, a regular increase in the growth rate of money would bring about a permanent increase in output. If auto-

regressive expectations are specified as stationary in the d th difference, then specification of a policy rule of $(d+1)$ th-order stationarity will be sufficient to make the model inconsistent with the natural rate hypothesis. This inconsistency arises because policy can render expectations biased. To eliminate this inconsistency, economists have found it necessary to adopt the assumption of rational expectations.

The IS-LM Model with Rational Expectations

The fundamental policy insight of rational expectations is that, to the extent policy effects depend on expectational errors, they cannot be systematic. This proposition follows, in large part, from the expectational Phillips curve, or supply function (17), under which output is entirely inelastic with respect to expected inflation. Therefore, a supposedly countercyclical policy of, for example, increasing money growth when a recession is observed, will not be effective in stabilizing output because sellers will fully anticipate the implied variations in prices. This proposition was frequently described as "policy ineffectiveness," seeming to suggest that choice of any one policy rule is as good as another (at least within the class of rules serving to complete the model). However, careful analysis below will show that the relevance or irrelevance of Q and F depends on particular assumptions about information availability or endowments. The resulting ambiguities are largely a result of problems related to the aggregate demand function.

Rational expectations, in their strong Muthian form, are those generated by using the information available and in full knowledge of the model, including the money supply rule. Formally, rational expectations of current and

future prices are

$$(25) \quad E_t p_{t+1} = E[p_{t+1} | S_t], \quad i=1,2,$$

where the symbol E denotes the (true) mathematical expectation derived within the model and S_t is the information set conditioning expectations at time t . The rational expectations assumption prevents any systematic errors in price expectations. In particular, it replaces the autoregressive expectations mechanism of the previous section with a mechanism that is explicitly dependent on the structure, including the policy rule. This ensures that the effects of contemplated changes in the policy rule do not rely on exploitation of systematic expectational errors.

Sargent and Wallace (1975) demonstrated that replacement of autoregressive with rational expectations in the IS-LM model described above implied two radical implications about the optimal policy rule: (1) that the values of f_1 and f_2 were irrelevant for output variance, and (2) that prices and the money stock were indeterminate under a pure "interest rate rule."

To show the first proposition, consider that reduced-form solutions for p_t (expressions for the latter are functions, necessarily linear, of the state variables, or predetermined and exogenous variables entering model equations) must take the trial solution form:

$$(26) \quad y_t = \pi_{10} + \pi_{11} y_{t-1} + \pi_{12} p_{t-1} + \pi_{13} u_{1t} + \pi_{14} u_{2t} + \pi_{15} e_t$$

$$\text{and } p_t = \pi_{20} + \pi_{21} y_{t-1} + \pi_{22} p_{t-1} + \pi_{23} u_{1t} + \pi_{24} u_{2t} + \pi_{25} e_t$$

for some π_{ij} s, where the latter are functions of parameters of the model.⁶ The complete reduced-form solutions for all endogenous variables can be obtained by assigning them trial solution forms, substituting them into model equations, and solving the implied identities for the π_{ij} s. Rational expectations are imposed by applying (25) to (26). In this application,

it was assumed by Sargent and Wallace that the information set contained only lagged realizations of variables,

$$(27) S_t = \{p_{t-1}, y_{t-1}, \dots\}.$$

Agents were ignorant of u_{1t} , u_{2t} , and e_t , which for simplicity are assumed here to be nonautocorrelated and independent.

$$(28) E_t u_{1t} = E_t u_{2t} = E_t e_t = E u_{1t} u_{2t} = E u_{2t} e_t = E u_{1t} e_t = 0.$$

Then, (25) through (28) imply

$$(29) E_t p_t = \pi_{20} + \pi_{21} y_{t-1} + \pi_{22} p_{t-1}$$

$$\text{and } E_t p_{t+1} = (\pi_{10} + \pi_{11} \pi_{20} + \pi_{12} \pi_{10}) + \pi_{11} (\pi_{21} + \pi_{12}) y_{t-1} + (\pi_{11} \pi_{22} + \pi_{12}^2) p_{t-1}.$$

It is by the derivation of the expectations expressions in (29), and their use in the trial solution, that rationality of expectations is imposed on the model.

The supply function (17) and the trial solutions (26) and (29) imply

$$(30) y_t = s_0 + \lambda y_{t-1} + s \pi_{13} u_{1t} + (1 + s \pi_{14}) u_{2t} + s \pi_{15} e_t.$$

y_{t-1} appears in this expression with the fixed coefficient λ , and p_{t-1} does not appear at all, hence, f_1 and f_2 do not influence $\partial y_t / \partial y_{t-1}$ or $\partial y_t / \partial p_{t-1}$. The partial derivatives

$$(31) \quad \partial y_t / \partial u_{1t} = s \pi_{13} \quad = \Pi_3 = s(a_1 + q)J,$$

$$\partial y_t / \partial u_{2t} = 1 + s \pi_{14} \quad = \Pi_4 = dJ,$$

$$\text{and } \partial y_t / \partial e_t = s \pi_{15} \quad = \Pi_5 = -sdJ,$$

turn out to be identical to those for the version with autoregressive expectations, shown in equation (21).

Two implications for (q, f_1, f_2) of these results are immediate.

First, the optimal f_1 and f_2 are nonunique; indeed, those parameters are irrelevant for output, as claimed by the first proposition of Sargent and Wallace. Second, the output-variance-minimizing value of q is the unique value given by (22), and has the properties attributed to it there, including

its independence of dynamic elements. In fact, these two implications for (q, f_1, f_2) are entirely unaffected by additional dynamic elements, such as the introduction of additional lagged terms or autocorrelations of error terms in the model's equations.

In this model, even nonmonetary influences on aggregate demand that can be forecast by agents in advance of their occurrence are impotent, being neutralized via changes in prices. For example, fiscal policy that operates through the mechanism of changing aggregate demand (as part of $u_{1,t}$) is irrelevant for output if announced in advance (regardless of the monetary policy rule adopted). In IS-LM type models, only unexpected monetary or fiscal policies matter for output, aside from the automatic stabilizers inherent in graduated-rate tax systems, in the case of fiscal policies, and the choice of q , in the case of monetary policies.'

"Interest Rate Rules"

The second major result in Sargent and Wallace's model is that an "interest rate rule" does not serve to complete the model in that it leaves prices and money indeterminate. This result will require a modified analysis to derive, because such "rules" are not representable in the policy space (q, f_1, f_2) in R^3 . Consider the "rules" of form:

$$(32) \quad R_t = g_0 + g_1 y_{t-1} + g_2 p_{t-1}.$$

(The indeterminacy result would also occur in any generalization of (32) in which m_t is lacking). (32) does not specify corresponding elements in the (q, f_1, f_2) space, because there is no unique transformation from (g_0, g_1, g_2) into (q, f_1, f_2) . Therefore, the solutions for endogenous variables under the money supply rule (20) cannot be used to determine outcomes under (32).'

Before turning directly to the problem of indeterminacy of nominal variables, it is worth noting that a kind of policy irrelevance continues to hold true. In fact, output under (32) would be

$$(33) \quad y_t = s_0 + \lambda y_{t-1} + s u_{1t}$$

(which happens to coincide with the limit of expression (30) as q approaches infinity), an expression devoid of the g_i 's.

To show that p_t and m_t are indeterminate in this case, it simplifies matters to notice first that the money demand equation, (14), does not, in view of (32), help determine p_t . It only determines m_t if p_t is determined by the other equations of the model. So (14) need not be part of the analysis of determinateness of prices if the specification of "policy" is (32).

Now the determinacy of prices could be established by demonstrating that the trial solution of (26) for p_t is unique; that is, that the π_{2i} 's are finite and unique. The restrictions on the π_{2i} 's implied by the model can be inferred in the following way. Equating y_t^d , (15), and y_t^s , (17), and using (32),

$$(34) \quad s p_t = (d_0 + d_1 g_0 - s_0) + d_1 y_{t-1} + (d_2 - \lambda) y_{t-1} + (u_{1t} - u_{2t}) - d E_t p_{t+1} + (d+s) E_t p_t.$$

Then, substitution of (26), (29), and (33) into (34) implies the following identities:

$$(35) \quad s \pi_{20} = (d_0 + d_1 g_0 - s_0) - d_1 (\pi_{20} (1 + \pi_{22}) + s_0 \pi_{21}) + (d_1 + s) \pi_{20}$$

$$s \pi_{21} = (d_1 g_1 + d_2 - \lambda) - d_1 \pi_{21} (\lambda + \pi_{22}) + (d_1 + s) \pi_{11}$$

$$s \pi_{22} = -d_1 \pi_{22}^2 + (d_1 + s) \pi_{22} + d_1 g_2$$

$$s \pi_{23} = 1$$

$$s \pi_{24} = -1$$

$$s \pi_{25} = 0$$

These identities obviously provide unique values for π_{23} , π_{24} , and π_{25} .⁹

However, the third identity, for π_{22} , is quadratic, so π_{22} can take on two distinct values. This nonuniqueness is contagious because the solution values for π_{20} and π_{21} depend on π_{22} . Hence, it is already clear that the model with (32) does not place sufficient restrictions on the π_{2i} s to provide determinacy. (However, the solution for output remains uniquely determinate.)

To simplify further study of the identities in (35), consider the restriction of g_2 , which appears in the third identity, to zero. This restriction is, in view of (33), irrelevant for output. Under the minimal-state-variable approach, this restriction would be imposed by eliminating p_{t-1} from the trial solution, so that $\pi_{22}=0$, by assumption. Then the "solution" for prices is

$$(36) \quad p_t = (\infty) + (d_0 + d_1 g_0 - s_0) s_0^{-1} d_1^{-1} y_{t-1} + s^{-1} u_{1t} - s^{-1} u_{2t},$$

where the (∞) symbol indicates that the intercept π_{20} is undefined.

This indefinite intercept indicates p_t is indeterminate.

On the other hand, if the minimal-state-variable set is augmented by inclusion of p_{t-1} , π_{22} can take on the values of zero or unity. In the former case, (36) is again the solution. If $\pi_{22}=1$, then the solution is

$$(37) \quad p_t = [(d_0 - s_0) + d_1(g_0 - s_0 g_1) - s(d_2 - \lambda)](2 + 2d_1)^{-1} \\ + [g_1 + (d_2 - \lambda)d_1^{-1}]y_{t-1} + p_{t-1} + s^{-1}u_{1t} - s^{-1}u_{2t}.$$

This solution, having $\partial p_t / \partial p_{t-1} = 1$, implies a time series for p_t that is stationary only in first differences. Nevertheless, this solution is determinate.

McCallum (1986) shows that a reduced form for R_t specified by (32) can result from two different money supply rules, one which is stationary in m_t , as in (20), and one which is stationary only in first differences, as if m_t , the left-hand-side variable in (20), were prefixed by the difference operator

(1-L). However, it could also be shown that there are an indefinitely large number of money supply rules, in higher order differences of money, that would make R_t behave as specified by (32), as the bootstrap variables p_{t-1} , p_{t-2} , p_{t-3}, \dots are allowed to enter the solution. Consequently, as noted at the start of this section, "interest rate rules" do not adequately specify the policy rule and hence fail to complete the model.

In summary, either there is no determinate solution for prices, as when bootstraps are ruled out, or there are an indefinitely large number of multiple solutions, including one in which the intercept is indeterminate. It is hard, indeed, to regard these two possibilities as meaningfully distinct. Any behavior of prices can be consistent with the model under "policy rule" (32) and rational expectations.

Analytical Problems Under Alternative Information Constraints

Money supply rules, except in pathological cases, leave all variables determinate. But, for analytical reasons, characterization of an optimal rule or set of rules is difficult or impossible except in a very restrictive class of models, of which the Sargent and Wallace model is an example. The assumptions about information constraints on private agents made policy implications easy to derive in that model. Alternative information structures in which private agents observe current realizations of variables, such as R_t , in forming the expectations of the current and future price level, can lead to surprising implications for the optimal policy rule.

This point is neatly illustrated by an insightful analysis by Canzoneri, Henderson, and Rogoff (1983). First, they simplify the structural form of the Sargent and Wallace model by dropping terms in y_{t-1} from both commodity supply and demand functions, and eliminating supply shocks ($u_{2t}=0$ for all t).

Then they consider alternative assumptions about private-agent use of the nominal interest rate. If the expectation of the current price level, present in the supply function, is conditioned on

$$(38) \quad S_t^{(+)} = \{ R_t, \text{ all lagged realizations of state variables} \},$$

then the policy rule is irrelevant for output. In this case, even the optimal q is indeterminate! In essence, suppliers, whose behavior could be systematically influenced by policy only because of contemporaneous policy responses to the interest rate, are able to estimate such responses by looking at the interest rate, and the market will therefore adjust prices to neutralize them after all.

An alternative information specification assumes that expectations of the rate of inflation, present in the demand function, are made with knowledge of the current interest rate:

$$(39) \quad E_t(p_{t+1} - p_t) = E[(p_{t+1} - p_t) | S_t^{(+)}].$$

In this case, tractability requires further ad hoc and unmotivated restrictions on the policy space. In the examples explored by Canzoneri, Henderson, and Rogoff, either q or F is restricted to zero or a zero vector, respectively. If $F=0$ is imposed, as in the policy rule

$$(40) \quad m_t = \mu - qR_t,$$

then the expression for the optimal q is the same as for the Poole model, given in equation (11). The other example they explore considers policy rules of the form

$$(41) \quad m_t = \mu + f_3 m_{t-1} + f_4 R_{t-1}.$$

In the latter case, Canzoneri, Henderson, and Rogoff show that, if f_3 is restricted to unity, the optimal f_4 is

$$(42) \quad f_4 = 1 + [a_1(1+a_2s)/a_1s][\sigma_1^2/\sigma_e^2],$$

whose sign cannot be determined without further knowledge about parameters and

disturbance variances.

Interestingly, the behavior of output is the same in these cases: either with (40) and (11), or with (41), $f_3=1$, and (42). The resulting output variance is probably the lowest attainable, but other policy rules without these values of q and F are also likely to result in the same variance. This case is reminiscent of the upper panel of figure 2.

This example shows that the optimal policy rule may not be unique, and that this characteristic may not always be easy to discover, unless the analyst is willing to try out many different representations, some of which may not be obvious or intuitive. The restriction $f_3=1$ or $f_3=0$ has commonly been necessary to impose even in very simple models, with few behavioral parameters and disturbances.¹⁰ And if the supply shock, u_{2t} , is reintroduced and autocorrelation in various shocks or lagged output terms are allowed in structural equations, the optimal values of q and F become hopelessly ambiguous. Even their relevance or irrelevance and optimal signs will depend on too many particulars.

Although ambiguities concerning policy effectiveness arise from generalizations of the Sargent and Wallace model, it was important in dramatically demonstrating that rational expectations, whose imposition was required to make the conventional IS-LM models consistent with the natural rate property, had quite radical implications for those models, rather than representing a technical advance those models might or might not usefully incorporate. In particular, previous demonstrations that conventional policies could stabilize output were found dependent on implausible exploitation of biased expectations.

Non-Market-Clearing Models

Market clearing is the equation of commodity demand and supply via adjustments in prices and interest rates. This assumption has been at the heart of most microeconomic theory, and might be considered the natural mechanism by which supply and demand are reconciled. But price changes are only one conceivable means of reconciling quantities demanded and supplied. Many economists see the economy as laboring under constraints that can be usefully described as constraints on adjustments of prices. As this constraint is imposed on the model, **potential** (notional) demanders and suppliers are frustrated, unable to make transactions they both desire at mutually agreeable prices, because transactions at these prices are ruled out. This frustration can be quite persistent, **if** price adjustments are slow, causing persistent output fluctuations.

This constraint cannot be taken literally. **It** is intended only as a useful representation of some hard-to-specify problems. **It** is not necessarily that agents are somehow forced to transact only at sticky prices or to sign contracts because of constraints other than those arising from technology or tastes. **It** is, instead, that aspects of technology or tastes are not adequately captured by neoclassical production and utility functions. Then, sticky prices or contracts may help, or reflect attempts by, agents to optimize. For **example**, nominal contracting may reflect "technological" difficulties in developing credible Pareto-optimal agreements arising from some kind of information, monitoring, enforcement, or coordination problems. Further work into the microfoundations of "sticky prices" may succeed in replacing price adjustment or contracting "constraints" with a more satisfactory

representation of the underlying problems. Analysis at this deeper level may show that the true constraints are poorly represented by present sticky-price macroeconomic models. Hence, any practical policy conclusions of present models should be regarded as speculative. Also, the irony of treating such phenomena as voluntary contracts as a "constraint" preventing, rather than aiding, optimization should be noted.

Proceeding on the more superficial or descriptive level, recent work on non-market-clearing models has incorporated rational expectations and imposed a tendency of prices to adjust to market-clearing levels, albeit slowly. These two features have made them consistent with the natural rate property. Consequently, that policy responses to the state of the economy can sometimes be effective in these models is interesting, and they seem to provide a potentially persuasive negation of the first proposition of Sargent and Wallace.

Of course, the relevance of this negation depends on whether markets actually clear. Rational expectations models with incomplete information have altered economists' views concerning the plausibility of the market-clearing assumption. Failure of markets to clear had often been considered apparent from the slow movements **and/or** discontinuity of individual prices. However, incomplete information among buyers and sellers can lead to slowly moving prices under market-clearing, or to changes at discrete intervals when new information is received. Skepticism regarding market-clearing had also arisen from the persistence of above- or below-average measured unemployment rates. Then again, **it** is not obvious that such phenomena reflect failure of wages to adjust to clear "the" labor market. Labor **immobility** and interindustry labor demand shifts can combine to create such fluctuations, even **if** wages are not sticky. And, again, some cyclical behavior of unemployment could be observed

if information problems existed, as in the Sargent and Wallace model. In short, **it** is difficult or impossible to determine empirically whether observed price behavior reflects non-market-clearing. The issue may be purely meta-physical.

Yet, consider the view that the markets do not clear. The implications for policy cannot be discovered without specifying the nonprice elements of the mechanism by which the reconciliation between demand and supply is effected. In a perfectly competitive, many-good economy, **if** the vector of prices for all commodities is somehow not equal to the vector (or not an element of the set of vectors, **if nonunique**) that achieves market-clearing, outputs and leisure will diverge from a Pareto-optimum. For the economy to obtain such a result would certainly be remarkable. Hypothetical methods of doing so, such as the Walrasian auctioneer, are literally implausible and often ridiculed. Also, **it** is reasonable to suppose that, **if** exchange is voluntary, such discrepancies of prices from the market-clearing prices **will** result in a shortfall of commodity output from its Pareto-optimum (full employment?). This is because either some marginal sellers or some marginal buyers in each good market will balk at the terms of exchange, and transactions are two-sided. Monetary policy can then affect the workings of the economy depending on how the real quantity of money appears in the production and utility functions, the nature of price adjustments, and how monetary policy is specified. But adequate models at this level of generality have not been constructed and **will** probably prove elusive.

At a less general **level**, non-market-clearing has been **coupled** with the assumption that sellers satisfy the demand at current prices. The similarity of this type of model with the earlier IS-LM models lies in the **demand-determination** of output. The newer types of non-market-clearing models are

different, however, in that they allow prices to be determined in a manner that, in conjunction with rational expectations, allows model consistency with the natural rate property. In particular, output will fluctuate around a "natural" or average rate that is uninfluenced by the monetary policy rule and that is frequently supposed to be the optimum output level. In this kind of model, the nature of price determination is crucial.

It is analytically useful to distinguish between two types of price adjustment mechanisms: those in which all prices change in each period, and those in which some prices do not change in each period. Stabilization policy appears to have little or no scope for effectiveness in the first case, but it can generally be effective in the latter. One instructive example of the first type is the case for which prices are completely predetermined one period in advance. To enforce consistency with the natural rate property, it will be sufficient to assume that sellers set period- t prices at the level rationally expected to clear the market, conditioned on realizations of variables in period $t-1$:

$$(43) \quad p_t = E_t p_t^c,$$

where price expectations are conditioned on S_t and p_t^c is the solution to

$$(44) \quad y_t^s(p_t^c, E_t M_t^{(-)}) = y_t^d(p_t^c, E_t M_t^{(-)}, E_t P_{t+1}).$$

In the latter equation, $M^{(-)}$ denotes the minimal state vector truncated by omission of p_t . By substitution of (15), (16), and (17) into (44), and using (43) and its obvious implication $p_t = E_t p_t$, it can be shown that prices are determined according to

$$(45) \quad p_t = E_t p_{t+1} - E_t R_t + (s_0 - d_0) d_1^{-1} + (\lambda - d_2) d_1^{-1} y_{t-1}.$$

Then, solving the system of equations (7), (14), (15), (16), (20), and (45), and imposing rational expectations, it can be shown that output has the representation:

$$(46) \quad y_t = s_0 + \lambda y_{t-1} + (q + a_1) J_1 u_{1t} - d_1 J_1 e_t \\ (J_1 = (a_1 + a_2 d_1 + q)^{-1}).$$

It is immediately apparent that f_1 , and f_2 are irrelevant for output determination, implying that the first proposition of Sargent and Wallace holds in this model. Interestingly, the conditional mean $E_t y_t$ is determined uniquely by supply behavior, while the deviation $(y_t - E_t y_t)$ is determined uniquely by demand behavior, and in precisely the same fashion as in the static IS-LM model (compare equations (46) and (9)). The optimal value of q is given in equation (11).

As for the second proposition of Sargent and Wallace, indeterminacy of prices and money under an "interest rate rule" of the form (32), it might seem that predetermination of prices would imply determinateness. It turns out, however, that such an intuition is incorrect. The solution for the price level is indeterminate, as in the market-clearing case, with one solution having an undefined intercept, and an infinite number of alternative solutions if bootstrap components are not ruled out. The ironic lack of determinateness despite predetermination arises from the forward-looking nature of expectations in this (or any) rational expectations model. Prices expected to clear the market are dependent one-for-one on expected prices of the period after that, and so on into the indefinite future. An interest rate rule does not anchor any of these expectations. This indeterminateness also occurs in other non-market-clearing models. Indeed, it seems a necessary feature of all models in which rational expectations play a nontrivial role.¹¹

Another type of price-adjustment equation in which all prices change each period is the familiar partial-adjustment mechanism,

$$(47) \quad (p_t - p_{t-1}) = \rho(p_t^c - p_{t-1}) + \eta_t, \quad 0 < \rho < 1,$$

where η is a nonautocorrelated random variable. McCallum (1978) considers

this case at length, and finds that, given $q=0$, f_i and f , are irrelevant for output.¹² Unfortunately, the optimal policy rule or, more likely, set of policy rules, cannot be adequately characterized, because the ad hoc restriction $q=0$ is needed to render analysis tractable.

In the second set of non-market-clearing models, some prices do not adjust each period. Fixed-price models were an example treated above. Models of staggered, multiperiod contracts are particularly interesting because they can simultaneously possess the natural rate property and imply policy effectiveness, even when the authorities have no superior information. In these models, sellers, usually of labor services, agree to accept wages or prices predetermined for more than one period in advance.

To illustrate, suppose that the economy comprises two equally numbered groups of perfectly competitive firms. In each period, output consists of the sum of the outputs of group 1, Y_{1t} , and of group 2, Y_{2t} :

$$(48) \quad Y_t = Y_{1t} + Y_{2t}.$$

Without loss of generality, let group 1 consist of the firms that signed contracts at the end of period $t-1$, to remain in effect during period t and $t+1$, while group 2 consists of those whose contracts were signed at the end of period $t-2$, and expire at the end of period t . Wages are set for group i based on available information at the end of period $t-i$. Further assume the production function:

$$(49) \quad Y_{it} = Z_t N_i^\gamma, \quad i=1,2, \quad \gamma > 0.$$

Z is a global productivity variable whose log is $Z_t = k + \varepsilon_t$, for some constant k and disturbance ε_t . The latter is nonautocorrelated and homoscedastic. N is the (unlogged) employment level. In the spirit of the contracting models, let the wages contracted for period t by group i equal the expected marginal (physical) product times the price level:

$$(50) \quad W_{i,t} = E_{t-1}(Z_t \gamma N_i^{\gamma-1} P_t), \quad i=1,2,$$

where P is the unlogged price level. Firms make output decisions based on contemporaneous knowledge of p , W , and Z . Then each firm's profits are maximized by equating the predetermined wage with the actual marginal (physical) product times the price level:

$$(51) \quad W_{i,t} = Z_t \gamma N_i^{\gamma-1} P_t, \quad i=1,2.$$

These two equations, the production function (49), and the linear approximation:

$$(52) \quad y_t = (y_{1,t} + y_{2,t})/2$$

imply

$$(53) \quad y_t = s_0 + s(p_t - E_t p_t) + s(p_t - E_{t-1} p_t) + u_{2,t},$$

where $s = \gamma/2(1-\gamma) > 0$,

$$\text{and } u_{2,t} = (1-\gamma)^{-1} \varepsilon_t.$$

Equation (53) suggests, ironically, that multiperiod contract models can be represented as a particular kind of "generalization" of the Sargent and Wallace market-clearing model.

The design of the optimal policy rule is an intractable problem in **this** model, unless some ad hoc restriction, such as $q=0$, is imposed. Nevertheless, some characteristics of the optimal policy can be deduced. In this model, as in the Sargent and Wallace model, the choice of q to reduce output deviations is dependent strictly on the variances of disturbances and the structural parameters linking them to current output. The more interesting property, however, is that f_1 and f_2 are relevant for output determination, unlike in the Sargent and Wallace model. This is because **firm/worker** combinations under the older contracts are not making full use of the information set S_t , but, in a particular sense, are acting as if they knew only S_{t-1} .¹³ The policymaker can use the information that is "ignored," in effect, by the

firm/worker combinations with older contracts, thereby stabilizing output. To explain simply, consider that policy can influence output, according to (53), via price level surprises over one- and two-period horizons. Then the relevance for output of f_1 and f_2 and like terms in the policy rule can be indirectly assessed by asking whether output is influenced by expected prices. In fact, any tendency toward persistence of output fluctuations could be eliminated by a policy rule that made expected prices vary in the following way:

$$(54) \quad E_t p_t = E_{t-1} p_t - [\lambda(1-\gamma)/\gamma] y_{t-1}.$$

As stated above, the money supply rule or set of rules that effects this predetermined variation in prices is difficult to derive.

Despite the existence of contracts, many economists suggest they are largely facades. Perhaps they are merely means of exchanging information between firms and workers, but do not create undesired fluctuations in output. Certainly, incentives exist to eliminate, or at least minimize, these undesired fluctuations. Nonprice mechanisms for raising and lowering real wages may, at little cost in efficiency, substitute for changes in explicit wages. Contract provisions regarding overtime pay, hiring practices, and other aspects affecting labor costs may tend to face the firm with a marginal labor cost schedule nearly matching the rising disutility of work. To the extent these match, labor contracts are consistent with optimal output determination. If the match is imperfect, then the optimal policy will depend on the details of the mismatch, how the mismatch is affected by the policy rule, and variances and parameters. While optimal q , f_1 , and f_2 will generally take nonzero values, even their signs will be hard to assess.

Output Versus Price Stabilization as the Objective

As explained in the previous section, the multiperiod contracting model restores a trade-off between output and price stability somewhat reminiscent of pre-rational-expectations models. Yet, implicit in both the contracting models and the Sargent and Wallace model is a source of doubt concerning the relevance of this trade-off. This doubt arises from reconsideration of output variance as an adequate representation of the objective. As mentioned earlier, real business-cycle models can describe fluctuations in output as optimizing responses to changing production opportunities facing individual agents. This possibility is either ruled out or obscured in models in which output is demand-determined (determined by exogenous spending propensities and monetary and fiscal policy).

However, in the models in which supply behavior plays a nontrivial role, it becomes important to ask what role productivity changes play. For example, in the supply equation (53) of the contracting model, the terms $s(p_t - E_t p_t)$ and $s(p_t - E_{t-1} p_t)$ represent the deviations in output resulting from the inability of workers and firms to develop Pareto-optimal contracts; ones in which output is determined by the appropriate marginal conditions. Likewise, under incomplete information among suppliers that implied the aggregate supply function (17), the term $s(p_t - E_t p_t)$ represents the deviation of output from its optimum. In either case, the effectiveness of the additional constraint on the economy that prevents full optimization is related to the component of prices that could not be anticipated in advance. If multiperiod contracts are considered important, then their length will help determine the horizon over which price-level uncertainty should be minimized.

The somewhat ironic conclusion is that multiperiod nominal labor contracts appear to argue even more strongly for price stability over longer horizons as a policy criterion, rather than suggesting a policy trade-off between output and price stability. In general, it is intuitive that the degree of efficiency of (incompletely indexed) nominal contracts of length n , whether for labor, capital, or other factor services, will depend on price-level predictability over horizons from one to n periods. At least, this statement is obviously true if contracts couple predetermination (or incomplete indexation) of nominal factor payments with Pareto-suboptimal demand-determination of factor quantities, as in the labor contracts supposed empirically relevant for unionized firms.

This conclusion does not immediately provide the optimal policy rule, of course, because, at least in the models considered, price-level stabilization cannot be perfect. Also, there may be trade-offs between price stabilization over various horizons. For example, attempts to return prices quickly to a long-established target might clash with the desire to avoid problems under existing contracts, which may already reflect the existing deviation of prices. The weight to be put on price uncertainty over various horizons will depend on the constraints that incomplete information and contracting models describe the economy as being under. If both kinds of constraints are relevant, both kinds of models can contribute to our understanding of how monetary policy should be designed.

One "small" change in the models could make the optimal policy virtually unambiguous. If the policymaker is allowed to observe the price level p_t contemporaneously, it can, to any arbitrarily exact degree of accuracy, set it on any pre-announced course that eliminated forecast errors over the relevant horizons. Such a rule is:

$$(55) \quad m_t = \mu - \theta p_t,$$

where θ is a positive magnitude large enough to prevent significant price movements, but not so large as to imply a contradiction: some deviations in prices must be observed in order to practice this policy.

Even modest measurement and information-delay problems concerning prices would seem to call for something less than complete reliance on current prices. Then, price stability would be best achieved by some more complicated rule, about which there is inadequate knowledge, and about which economists with different models could disagree. Nevertheless, consensus among competing models on the price-stability criterion is useful in the absence of full knowledge or agreement.

Intertemporal Substitution Models

The models discussed in previous sections are variants of IS-LM models. Characteristic of these is an asymmetry between agents' behavior as suppliers and demanders in the commodity market. This might be rationalized, as implicitly in contracting models, by inefficiencies on the supply side arising from principal-agent problems. But firms maximizing the welfare of the representative owner-worker would make output respond to (ex ante) real rates of return to labor (the variable input). Models in which both supply and demand respond to real rates of return are termed intertemporal substitution models and have an appealing basis in microeconomic theory. Such behavior would make supply behavior symmetric with respect to demand behavior, as in:

$$(56) \quad y_t^s = s'_0 + s'_1 [R_t - E_t(p_{t+1} - p_t)] + \lambda y_{t-1} + u_{2t}.$$

Then the classical dichotomy would hold--that is, output would not be influenced by monetary factors--as shown in the solution for output:

$$(57) \quad y_t = \left(\frac{s'_1 d_0 - s'_0 d_1}{s'_1 - d_1} \right) + \left(\frac{s'_1 d_2 - \lambda d_1}{s'_1 - d_1} \right) y_{t-1} + \frac{s'_1 u_{1t} - d_1 u_{2t}}{s'_1 - d_1}.$$

A particularly interesting aspect of this solution is that **it** is invariant to the specification of expectations formation, so long as they are formed in the same manner by agents when making supply and demand decisions.

On the other hand, differences in expectation formation affecting supply versus demand decisions would generate a real-nominal interaction. However, some rationale for such a seemingly bizarre double-consciousness among agents would be needed to provide plausibility. Alternatively, some agents may possess information others do not have. Various forms of heterogeneity in the information sets available to agents, S_t , have been used to generate disparate implications for the optimal policy rule; none appear to have generality. In most analyses, the policy space is arbitrarily restricted in ways unmotivated, except by analytical tractability. For example, policy rules are often confined to those providing trend-stationarity to nominal variables, or ruling out cycles in them. Also, just as in the Sargent and Wallace model, seemingly small changes in the information assumptions can create intractabilities or reverse the signs of optimal policy parameters.

Very recently, some deeper analysis has been undertaken of the micro-economic underpinnings of potential real-nominal interactions in intertemporal substitution models. Models involving a real balance effect can destroy the classical dichotomy in intertemporal substitution models. A real-nominal interaction arises **if** the transaction services of money are considered, and **if** demand and supply are differentially sensitive to the real balance and real rate arguments. But transactions services have played little role in the business cycle models and are thought to be empirically unimportant as determinants of cyclical variations in output.

If only the ordinary wealth (nontransactions service) effects of real money balances on individuals' wealth are considered, the positive relation between real variables and money surprises disappears. False (but rational) individual perceptions of higher real wealth, due to a surprise increase in nominal balances, would have two effects on employment and output that tend to be offsetting. First, a direct wealth effect increases leisure and reduces work. But, second, this effect would create an excess demand for credit at the initial (ex ante) real interest rate. Therefore, a higher interest rate is necessary to clear the commodity market. The effect of this higher real rate on individual decisions is to increase employment via an intertemporal substitution effect. (Whether this rise will mainly take the form of changes in nominal rates or of expected inflation will depend, among other things, upon the policy rule.) Barro and King (1984) show that if there are no storable goods and utility is time-separable, then the wealth and substitution effects must identically cancel, so that the classical dichotomy is confirmed. Relaxing the assumptions to allow for capital goods seems to suggest real rates will respond by less in the face of initial real money balance changes, implying that output would actually fall from money increases not perceived by incompletely informed agents. Hence, conventional stabilization policies are not indicated.

Practical Policy Implications

The role of monetary policy in rational expectations models arises from constraints on private optimization in the form of incomplete information or limits on price adjustments. Properties of policy rules that minimize the welfare losses associated with these constraints are very sensitive to

particular aspects of the constraints, about which adequate information is unavailable. Also, there are unresolved analytical problems. Thus, the optimal policy rule is unknown, and probably unknowable.

A characteristic of a policy that would be ideal, albeit infeasible, is that it would prevent any price expectation errors over relevant horizons. Information lags facing private agents and the length of contracts would largely determine the relevant horizons. Long-term contracts, for example, argue for policies that achieve longer-term price predictability. But this hypothetical ideal is not entirely feasible, because the policymaker faces information constraints regarding current prices.

The uncertainties and analytical problems in designing an optimal policy, however, seem to roughly correspond to the problem of designing a means of minimizing price-level uncertainty. That minimization of price-level uncertainty will be a property of an optimal policy does not necessarily help determine an optimal policy, if prices are not contemporaneously observable. However, the price stability criterion may help rank concrete policy alternatives and evaluate actual policy performance. And, to the extent that the price level can be observed by the monetary authorities without significant information delays, price stabilization may be considered not only a good policy but a reasonably specific and practical one. Certainly, it is more specific and practical than vague notions that policy should "lean against the wind" of undesired output fluctuations. Adequate knowledge does not exist to differentiate desirable from undesirable fluctuations, or to know how to offset them. Hence, such a "policy" is too obscure to be a practical, discussable alternative.

The major practical alternatives seem to be constant money growth rules and predetermined price-level targets. The former have often been chosen over

the latter because of the supposedly superior relative controllability of money. This consideration cannot be represented formally in models of the sort exhibited in this paper and can have little practical force in the current environment, in which money stock measurement, or even conceptualization, is fraught with difficulties.

The problems of a policy of close price control are made less formidable by rational expectations. To the extent that such a policy is practiced consistently, private agents will tend to make decisions that neither reflect expectations of, nor serve to encourage, fluctuations in prices. And such a policy can be practiced without concern for any supposed output-inflation trade-off.

Existing macroeconomic models with rational expectations provide little support for conventional monetary policies. Output-stabilization policies are not only extremely difficult to design, but, to the extent that they are "effective," tend to interfere with the economy's efficient responses to changing productive opportunities. Finally, policies naively directed toward interest-rate stabilization or stated in terms of "interest rate rules" are either infeasible or invite unknown, and probably undesired, consequences.

FOOTNOTES

1. The linearity restriction, here and elsewhere in this analysis, implies a general restriction on utility and production functions and on disturbance distributions. For example, quadratic utility functions, linear production functions, and Gaussian disturbances (in logarithms of variables) may suffice. Unless this general restriction is upheld, linearity must be regarded as a local approximation.
2. If the representation is not so sensitive, then it is considered, according to a newer terminology, a structural model with respect to that range of interventions.
3. Because, in existing macroeconomic models, the distinction between base money and money is irrelevant, the term money will be used here without loss of generality. See Hoehn (1984) for a discussion of some ways in which the distinction between base money (actually, bank reserves) and money becomes important in the optimization problem under a number of interesting or historically relevant regulatory and institutional frameworks.
4. Hoehn (1984) deals at length with the derivation of money supply functions in empirically relevant and relatively detailed institutional settings.
5. Actually, this assumption was implicit in the reduction of equation (1) to equation (2). The requirement that policy involve an absolute commitment to a time-consistent rule implies a restriction on the policy space considered. The unrestricted policy space is $(Q \times F) \times T$, where T is the infinite-dimensional time vector. The point (Q, F) could be specified for each period of time. Indeed, the optimal policy would involve a specification of (Q, F) as a function of time, where that function depended on the initial conditions. But, in subsequent periods, the initial conditions would change in a way that can partly be predicted on the basis of the current state. Then it will be optimal to make a new specification of (Q, F) as a function of time. But agents, under rational expectations, will expect this replanning and take it into account, rendering unattainable the macroeconomic outcome envisioned when the (Q, F) as a function of time was originally specified. Then, the usual methods of attempting to determine the optimal policy--optimal control theory--are inapplicable. Here, attention is restricted to policies that are time-consistent and must be chosen prior to observation of the initial conditions. The astute reader will note the irony that the lack of a constraint on policymakers to precommit reduces the policy space over which analysis need search for an optimum. This provides another example of the way in which rational expectations poses new analytical issues.
Incidentally, the optimal values of q and F could vary over time without implying time inconsistency, if the model is modified in certain ways. For example, if the structural parameters were known, nonstochastic functions of time, then a precommitment to fixed paths for q_t and F_t would generally be appropriate. If parameters were subject to stochastic variation, and agents lacked perfect foresight regarding such variation, then the optimal policy would involve precommitment to a rule for changing q_t and F_t in response to available information. Finally, if structural parameters are unknown, but

agents, including the policymaker, are capable of adaptive learning behavior, then the optimal policy again involves a precommitment to a rule for changing q_t and F_t . (Some potential problems of model nonconvergence are here ignored.) Hence, the identification of time consistency of a policy with time constancy of q and F is specific to the class of models examined in this paper, which impose structural parameter constancy and parameter certainty for convenience. In general, time constancy of q and F is sufficient, but not necessary, for time consistency of policy.

6. Bootstrap components in p_{t-2}, p_{t-3}, \dots are omitted in (26). This omission is inconsequential for study of the behavior of y or the irrelevance of F . Such bootstraps would matter for price behavior, as discussed later in this section.

7. Incidentally, preannounced changes in fiscal policy are effective in non-IS-LM models, such as the intertemporal substitution and real business cycle models with rational expectations and incomplete information. Indeed, in these models, changes in taxes or spending programs may have even larger effects on private agents if the latter have time to plan their responses to the changes in incentives implied by fiscal policy changes.

8. Sometimes, solutions under (32) are taken to be the limiting cases of expressions for the solutions under (20), as q approaches infinity. The mathematical concept of a limit is appropriate to use when the value of a function is undefined at some point in the range and values arbitrarily close to that point are of interest. Unfortunately, its use cannot be formally justified as representing the value of the function at that very point in the range, if the value is undefined there. In the case at hand, if q were actually infinite, rather than arbitrarily large in magnitude, then a money supply function such as (20) would not exist. Hence, any solutions derived using (20) would be irrelevant, and the limits of such expressions as q approaches infinity cannot be regarded as outcomes under an "interest rate rule." As noted, other analysts have not taken this view, and indeed do analyze an "interest rate rule" as a limiting case of a money supply rule, as q approaches infinity. This difference of view leads to some rather subtle differences of interpretation. In particular, McCallum interprets his results (1981) as that interest rate rules are feasible, but only as limiting cases of money supply rules. Further, these money supply rules are not unique (McCallum [1986]), hence the associated interest rate rule is not an adequate specification of policy. My interpretation is that "interest rate rules" are not actually policies, but merely represent outcomes for the interest rate that occur under (nonunique) money supply rules, where only the latter are admissible policies. These differences in interpretation are probably without practical significance for two reasons. First, either interpretation recommends policy rules that include m_t as an argument, that is, money supply rules. Second, an optimal policy would generally have a finite, non-zero value of q even if determinacy were not at issue. In other words, policies that completely predetermine money or interest rates--often described as policies using money or interest rate "instruments"--generally are suboptimal as long as the policymaker can observe both contemporaneously, even aside from problems of determinacy.

9. To avoid inconsistency, in the form of two different values for π_{21} , g_0 and g_1 must obey: $g_0 = s_0 d_0 (d_1 g_1 + d_2 - \lambda) + d_1 (1 - \lambda) (d_0 - s_0) d_1^{-1} (\lambda - 1)^{-1}$. This restriction essentially ensures equality between the average real rate of interest and the natural real rate of interest, determined in the commodity market.
10. Goodfriend (1985, 1986) considers a policy space in which f_3 is not constrained in this way.
11. "Rational expectations models" in which prices are exogenous are conceivable. Expectations would then be effectively exogenous as well, hence anchored. Other examples of rational expectations models without the indeterminacy problem may or may not exist, but are unlikely to have the natural rate property or other acceptable microeconomic implications.
12. Nominal indeterminacy under an "interest rate rule" could also be shown to be a feature of this model.
13. The modifier "in a particular sense" is included because firm/worker combinations which determined output in a Pareto-efficient manner, subject to the constraint that they could not observe S_t , would not behave in quite the same manner as in the nominal contracting models, which implicitly assume Pareto-inefficiency.

REFERENCES

- Barro, Robert J. "A Capital Market in an Equilibrium Business Cycle Model," Econometrica, vol. 48, no. 6 (September 1980), pp. 1393-1417.
- _____. Macroeconomics. New York: John Wiley and Sons, 1984.
- _____. "Recent Developments in the Theory of Rules Versus Discretion," University of Rochester Working Paper No. 12, April 1985.
- _____, and Robert G. King. "Time-Separable Preferences and Intertemporal Substitution Models of Business Cycles," Quarterly Journal of Economics, vol. 99 (November 1984), pp. 817-39.
- Canzoneri, Matthew B., Dale W. Henderson, and Kenneth S. Rogoff. "The Information Content of the Interest Rate and Optimal Monetary Policy," Quarterly Journal of Economics, vol. 98, no. 4 (November 1983), pp. 545-66.
- Fischer, Stanley. "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," Journal of Political Economy, vol. 85, no. 1 (February 1977), pp. 191-205.
- Friedman, Milton. "The Role of Monetary Policy," American Economic Review, vol. 58, no. 1 (March 1968), pp. 1-17.
- Goodfriend, Marvin. "Rational Expectations, Interest Rate Smoothing, and the 'Optimality' of a Non-Trend-Stationary Money Supply Rule," unpublished working paper, Federal Reserve Bank of Richmond, 1983.
- _____. "Interest Rate Smoothing and Price-Level Trend-Stationarity," Federal Reserve Bank of Richmond Working Paper 86-4, 1986.
- Hoehn, James G. "Monetary Policy Regimes: A Synthesis of the Monetary Control and Rational Expectations Literatures," Federal Reserve Bank of Cleveland Working Paper 8407, December 1984.
- Lucas, Robert E. Jr. "Some International Evidence on Output-Inflation Trade-offs," American Economic Review, vol. 63, no. 3 (June 1973), pp. 326-34.
- _____. "Econometric Policy Evaluation: A Critique," Journal of Monetary Economics, Supplementary Series, vol. 1 (1976), pp. 19-46.
- _____, and Leonard A. Rapping. "Real Wages, Employment, and Inflation," Journal of Political Economy, vol. 77, no. 5 (September-October 1969), pp. 721-54.
- McCallum, Bennett T. "Price Level Adjustments and the Rational Expectations Approach to Macroeconomic Stabilization Policy," Journal of Money, Credit, and Banking, vol. 10, no. 4 (November 1978), pp. 418-36.

- _____. "Rational Expectations and Macroeconomic Stabilization Policy: An Overview," Journal of Money, Credit, and Banking, vol. 12, no. 4 (November 1980 part 2, pp. 716-46.
- _____. "Price Level Determinacy with an Interest Rate Policy Rule and Rational Expectations," Journal of Monetary Economics, vol. 8., no. 3 (November 1981, pp. 319-29.
- _____. "On Non-Uniqueness in Rational Expectations Models: An Attempt at Perspective," Journal of Monetary Economics, vol. 11, no. 2 (March 1983, pp. 139-68.
- _____. "Some Issues Concerning Interest Rate Pegging, Price Level Determinacy, and the Real Bills Doctrine," Journal of Monetary Economics, vol. 17, no. 1 (January 1986, pp. 135-60.
- Phelps, Edmund S. "Phillips Curves, Expectations of Inflation, and Optimal Unemployment over Time," Economica, vol. 34 (August 1967, pp. 254-81.
- Poole, William. "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model," Quarterly Journal of Economics, vol. 84, no. 2 (May 1970, pp. 197-216.
- Sargent, Thomas J., and Neil Wallace. "'Rational' Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule," Journal of Political Economy, vol. 83, no. 2 (April 1975, pp. 241-54.