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VELOCITY: A MULTIVARIATE TIME-SERIES APPROACH

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Abstract

The Federal Reserve announces targets for the monetary aggregates that are implicitly conditioned on an assumption about future velocity for each of the monetary aggregates. In this paper we present explicit models of velocity for constructing rigorous tests to determine whether the behavior of velocity has changed from what was expected when the targets were chosen. We use time-series methods to develop alternative forecasts of velocity. Multivariate time-series models of velocity that include information about past interest rates produce significantly better out-of-sample forecasts than do univariate methods. Using this multivariate time-series framework, we analyze the Federal Reserve's decisions to change, miss, and switch targets from 1980:IQ to 1984:IIQ. For this period, we find that when the Federal Reserve deviated from its announced target, velocity deviated significantly from its predicted value.

I. Introduction

In the last two years, inflation forecasts have consistently been too high, particularly forecasts based on the quantity theory of money in which inflation is estimated to be an explicit function of growth in M-1 (the narrow definition of the money stock). Throughout 1982 and early in 1983, M-1 grew at double-digit rates, while inflation decelerated to less than 4 percent. This unexpected shift in the relation between inflation and M-1 has complicated the Federal Reserve's monetary targeting approach to ending inflation.

The Federal Reserve began announcing annual targets for monetary aggregates in 1975. These targets are not the ultimate goals of monetary policy, but merely intermediate targets conditioned on economic forecasts and long-term goals, such as price stability and economic growth. The announcements of monetary targets are used by the public as indicators of policy intentions. However, the intentions of policy are more accurately defined in terms of the ultimate objectives. Each member of the Federal Reserve Open Market Committee (FOMC), the deliberating body of the Federal Reserve responsible for monetary policy, has a unique model that relates the intermediate targets to the final goals. The individuals on the FOMC make decisions about the monetary targets based on forecasts (assumptions) about the relationship between the monetary targets and other economic variables. As even the most casual observer knows, economic forecasts are subject to large errors and frequent revision. Understanding this is basic to understanding the role of the monetary targets and why Congress allows the Federal Reserve so much discretion in choosing and changing the targets.

Implicit in the choice of a monetary target is an assumption about the expected behavior of velocity--that is, the ratio of nominal GNP to the monetary aggregate. Uncertainty about future velocity behavior is one reason that monetary targets are presented as ranges. In the past few years, the Federal Reserve has stated more explicitly how desirable monetary growth depends upon the unexpected growth of velocity. To quote from a recent Monetary Policy Report to Congress, "Growth around the midpoint of the (M-1) range would appear appropriate on the assumption of relatively normal velocity growth; if velocity growth remains weak compared with historical experience, M-1 growth might appropriately be higher in the range" (Board of Governors of the Federal Reserve System 1984, p. 72).

While monetarists such as Karl Brunner (1983) have argued that the Federal Reserve should ignore temporary deviations of velocity in implementing monetary policy, no one would deny that the targets should be changed when there is a fundamental change in the behavior of velocity growth.

In this paper, the expected behavior of velocity is defined as the forecast from a time-series model. We use a recent development in time-series modeling by Tiao and Box (1981) to construct multivariate models of velocity. Univariate Box-Jenkins (1976) models are used as the standard against which we compare these multivariate models. The time-series models are reduced-form models that may be consistent with many different structural models of the economy. Our goal in this paper is limited: to develop models of velocity for constructing rigorous tests to determine whether velocity behavior has changed. A by-product of this exercise is a better forecasting model for velocity.

Although we use reduced-form time-series models, we must rely on economic theory to decide which variables to include, how to measure them, and generally how they are expected to be related in a structural model. These decisions are necessary for setting up a multivariate time-series model because the way one transforms the variables (whether one takes differences, logarithms, etc.) affects the processes that generate the error terms. Also, the choice of the sample period may depend on knowledge about the economic structure. While one generally uses all available information, knowledge about special circumstances or structural changes may suggest using less than the full period for which data are available.

In this empirical study of velocity, we select a sample that starts in 1959. This year marked the beginning of the Federal Reserve's historical data set on the most recent versions of M-1 and M-2. We assume that there was a stable stochastic process generating velocity from 1959 through 1979. The estimation period ends in 1979:IVQ, because in that quarter the Federal Reserve announced its determination to restrain monetary growth and adopted a new operating procedure to lend credibility to the announcement. This change in procedures was the first of several events that may have induced a structural change in the economy and in the stochastic process generating velocity. Other events that may have induced a structural change in the economy were the imposition and subsequent relaxation of credit controls in 1980; deregulation of interest-rate restrictions in deposit markets in 1981, 1982, and 1983; and another change in operating procedures in late 1982.

We use univariate Box-Jenkins (1976) models and the Tiao-Box (1981) multivariate procedure to measure the behavior of velocity growth. We construct explicit models of velocity as well as trivariate models of money, nominal GNP, and interest rates from which a velocity forecast can be derived. We includc money and nominal GNP separately, because both money and nominal GNP are endogenous variables in periods as short as one quarter. By including these two variables separately, we hope to sort out their dynamic behavior, which may become obscured if we look at the ratio of the two.

We use the quantity theory of money as the analytical framework for selecting and scaling variables in this study. We set aside the problem of sorting out nominal versus real effects of monetary growth and look only at nominal GMP. Growth rates of nominal GNP and the money stock are approximated by changes in the logarithm. Previous research suggests that past interest rates contain important information about future money growth (see Bagshaw and Gavin 1983). Studies in money demand also suggest that the interest rate should be an important determinant of the ratio of income to money.

In section II, we present univariate and multivariate models of velocity growth. We include models for M-1 and M-2 velocity growth because the Federal Reserve has alternately used one or the other of these aggregates as its primary target. The Federal Reserve makes use of both aggregates in the policy process. Section III includes a comparison of the out-of-sample forecasting properties of the different models. In section IV, we use the estimated time-series models to monitor whether and when the actual behavior of velocity deviated from what was expected during the period from 1980:IQ to 1984:IIQ. Section V contains a summary and concluding comments.

II. Models of Velocity Growth

We begin by estimating univariate autoregressive integrated moving average (ARIMA) models of velocity growth for M-1 and M-2 (see table 1). For the 1959:IIQ to 1979:IVQ period, M-1 velocity growth can be represented by a constant growth trend (3.1 percent annually) plus a white noise process.

Brunner (1983) has used this result to support the case for a constant money-growth rule.

M-2 velocity growth is identified as a first-order moving average process. There is a 3 percent information gain over the naive model.' The naive model is just the average growth rate for the sample period. (We saw above that the univariate model for M-1 velocity was the naive model.)

Bivariate models of velocity are estimated using procedures developed in Tiao and Box (1981). These procedures are used to estimate the parameters of a multivariate simultaneous equation model. This method is interactive, similar in principle to that of single-equation Box-Jenkins modeling. The steps are: (1) tentatively identify a model by examining autocorrelations and cross-correlations of the series, (2) estimate the parameters of this model, and (3) apply diagnostic checks to the residuals. If the residuals do not pass the diagnostic checks, then the tentative model is modified, and steps 2 and 3 are repeated. This process continues until a satisfactory model is obtained. This is basically a forecasting procedure; contemporaneous correlation among the variables is not explained or taken into account, but relegated to the error matrix. The time-series procedure effectively filters out autocorrelation and dynamic cross-correlation among the errors. For a more detailed description of how to identify and estimate the vector autoregressive moving average (ARMA) model, see Tiao and Box (1981).

There is a controversy about the amount of differencing that should be used in multivariate time-series analysis. In univariate procedures, the variable is differenced if the series is not stationary. In multivariate procedures, Tiao and Box (1981) suggest not differencing to avoid specification error. Clowever, this does not rule out differencing if economic theory suggests a relationship in the differenced data. In this paper, we difference the monetary variables and GMP, but not the interest rate, to conform with a priori economic theory. From one period of equilibrium to the next, we expect money growth to be proportional to income growth and approximately proportional to the logarithm of 1 plus the nominal yield on short-term riskless assets. Therefore, the raw data are taken to be first differences in the natural logarithm of velocity and the logarithm of 1 plus the quarterly bond-equivalent yield on Treasury bills with three months to maturity.

The bivariate M-1 velocity growth model includes a lagged error from the interest-rate equation (see table 2). Like the univariate model, this model includes a constant equal to the average growth of velocity during the sample period. The information gain from the inclusion of the interest-rate variable is 3.4 percent.

The multivariate M-2 velocity growth model also includes the lagged error from the interest-rate equation. M-2 velocity growth is more sensitive to deviations of the interest rate from trend than is M-1 velocity growth. The information gain in the M-2 velocity growth equation is 7.2 percent, somewhat larger than for M-1 velocity. These multivariate velocity growth models represent an improvement over the univariate models, although they may not detect a systematic dynamic relationship between money and nominal GMP that would help explain the velocity trend. We try to do this by using trivariate models that include money and GNP separately.

The trivariate models are shown in table 3. M-1 growth is estimated to depend on past M-1 growth and the lagged error from the interest-rate equation. The interest rate is estimated to be a function of the lagged interest rate and the error in the previous period's interest-rate forecast. According to this equation, a set of information that excludes past values of

M-1 and nominal GNP appears sufficient to predict future interest rates. The coefficient on the lagged interest rate is not significantly different from 1. GNP is estimated to be a function of past M-1 growth and the lagged error from the M-1 equation. The Treasury bill rate influences GNP through its effect on M-1.

A forecast for velocity can be derived from these trivariate models. For M-1 we get the following equation:

$$\nabla \ln \ln VM1_t = .630 \quad \nabla \ln M1_{t-1} - .898\, a_{1,t-1} + 1.202\, a_{2,t-1} + (a_{3,t} - a_{1,t}).$$

The difference between this model and the bivariate M-1 velocity model is the implication for the behavior of velocity. In the bivariate M-1 model of table 2, the trend in M-1 velocity growth is a constant growth rate--3.1 percent annually. In the derived-velocity model, velocity is determined by M-1 growth. In the steady state, higher M-1 growth implies faster velocity growth. This implication is consistent with a standard economic model that includes non-interest-bearing money. When money growth exceeds real economic growth, inflation and higher interest rates raise the opportunity cost of holding money, and people devise ways to manage money balances more closely. This model is also consistent with the hypothesis stated in Meltzer (1983) that a policy-induced supply shock to money growth is associated with a temporary decline in velocity. The reason is simply that a shock to money growth affects GNP growth with a lag.

The M-2 velocity equation derived from the M-2 model is shown below:

$$\nabla \ln VM2_t = .021 \quad \nabla \ln M2_{t-1} - .329\, a_{1,t-6} + 2.436\, a_{2,t-1} + .119\, a_{2,t-6} + (a_{3,t} - a_{1,t}).$$

The coefficient on lagged M-2 growth is very small.

III. Forecast Performance

Post-sample forecasts from the models shown in tables 1, 2, and 3, are used to examine the advantages of these models in predicting velocity from 1980:IQ to 1984:IIQ. The statistics in table 4 compare velocity forecasts of different models. Clearly, the bivariate velocity model produces the best forecasts for M-1 velocity. The root mean square error (RMSE) is reduced from 1.73 percent in the univariate model to 1.17 percent in the bivariate model. The RMSE of the velocity forecasts derived from the trivariate M-1 model is 1.55 percent, better than the univariate velocity forecast but substantially worse than forecasts from the bivariate velocity model.[2]

All of the M-1 velocity growth forecasts are badly biased. The bias occurs in the forecasts for 1982 and 1983. The bivariate model includes a large effect from the lagged error in the interest-rate equation that causes the model to track movement in velocity witkout bias through 1981:IVQ. The RMSE from this bivariate model is 0.88 percent for the first eight quarters of our post-sample period. This is only one-half the RMSE from the univariate model (1.62) and about equal to the in-sample error for the bivariate model.

The accuracy of the M-1 velocity growth forecast in 1980 and 1981 is surprising, because interest rates were more volatile in the post-1979 period than during any comparable period in the sample. Similar results are obtained using the trivariate M-1 model. Furthermore, the contemporaneous correlation between the M-1 and interest-rate forecast errors from the trivariate model is strong and positive (0.41), while the in-sample correlation is weak and negative (-0.14). The change in monetary policy operating procedures is most likely responsible for the high positive correlation between the forecast errors (see Hoehn 1983).

The negative correlation between contemporaneous values of M-1 and interest rates during the period before 1979 has been interpreted as a money demand relationship and was most likely caused by the Federal Reserve's shifting the money supply curve to smooth interest rates. As a result, the scatter of points in the interest-rate M-1 space tended to trace a relatively stable demand curve. In October 1979, the Federal Reserve adopted a nonborrowed-reserve operating procedure in which the nonborrowed-reserve path was constructed on a stable money-supply path. When money demand took M-1 above (below) path, interest rates were forced up (down) by the nonborrowed-reserve operating procedure. Under this regime, the scatter of points in the interest-rate M-1 space tended to trace out a relatively stable supply curve. While the change in monetary control procedures was associated with a different contemporaneous correlation between M-1 and the interest rate, the change does not seem to have affected the relationship between the interest-rate error lagged one quarter and M-1 velocity growth.

In table 4, we show that the forecasts from the bivariate velocity model are better than the forecasts from the univariate models. This result implies that the preferred specification of a velocity model should include information about interest rates. In a recent paper, Ashley, Granger, and Schmalensee (1980) describe in detail a test statistic that we use to determine whether the bivariate model is significantly better than the univariate model. Because time-series procedures require mining the data to identify the model, in-sample statistics are inappropriate for specification testing. The proposed specification test is based on out-of-sample forecasting performance in which the criterion for performance is the mean square error (MSE) of the forecast. The test statistics are calculated by regressing the difference between the out-of-sample forecast errors on a constant and the sum of out-of-sample forecast errors. In particular, we construct a test of the bivariate model, as follows:

Let:

$$d_t = u_t - b_t,$$

and:

$$s_t = u_t + b_t,$$

where u_t is the forecast error from the univariate model, and b_t is the forecast error from the bivariate model. Estimate the following regression:

$$d_t = c_0 + c_1 (s_t - \bar{s}) + e_t,$$

where e_t is treated as if it were independent of s_t and $\bar{s}$ is the mean of the s_t's. The difference between MSEs is equal to the sum of two components: the difference between the mean of the errors squared and the difference between the variances. This regression provides a test of whether the difference between MSEs is significant. The ordinary least squares (OLS) estimate, $\hat{c}_0$, is an estimate of the difference between the mean of the error terms from each model. The OLS estimate, $\hat{c}_1$, is proportional to the difference between the variances of the error terms from each model. The mean of errors is negative for both univariate and bivariate models of M-1 and M-2 (see table 4). Therefore, we can reject the bivariate model if $\hat{c}_0$ is positive and significant, or if $\hat{c}_1$ is negative and significant. If $\hat{c}_0 < 0$, and $c_1 > 0$, we can use an F-test of the joint hypothesis that both $\hat{c}_0$ and c_1 are not significantly different than zero.

Ashley, Granger, and Schmalensee (1980) note that this F-test is four-tailed; it does not take into account the signs of the estimated coefficients. When the signs are taken into account, the appropriate significance level is one-half that obtained from the tables. The regression results using one-step-ahead errors from 1980:IQ to 1984:IIQ are shown in table 5. In both cases, taking interest rates into account improves the forecasts: for M-1 the improvement is highly significant at a 0.2 percent critical level; for M-2, the improvement is not statistically significant.

IV. Monitoring the Velocity Assumption

The monetary targets announced each year by the Federal Reserve are implicitly conditioned on an assumption about the expected behavior of velocity. Given a goal for inflation and an assumption about the trend in real output growth, whether money grows on average along the midpoint of the target range should depend on whether new information indicates that the assumption about velocity is accurate. To make that judgment, we must have a model of velocity and a notion about the probability distribution describing deviations of velocity from its expected value.

Since we cannot know <u>the</u> model for the FOMC's implicit forecast of velocity, we assume that the predicted value from our time-series model is the same as the FOMC expectation. Under this assumption, tests about model adequacy provide a method of monitoring the velocity assumptions that were made when the targets were chosen. To see whether this is a reasonable assumption, we compare the four-step-ahead forecast for velocity growth with the <u>ex ante</u> M-1 velocity assumption implied by the FOMC forecasts of nominal GNP and the midpoints of the M-1 target ranges (information presented to Congress by the Federal Reserve Chairman in February of each year, 1980 through 1984). A summary of the forecasts and the implied velocity assumptions are listed in table 6 with the four-step-ahead M-1 velocity forecast (using the bivariate models from table 2 in the text).

The four-step-ahead forecast of M-1 velocity growth falls within the range predicted by the FOMC in three of the five years shown. In 1981:IVQ, the actual interest rate was 1 percent (at quarterly rates) below the forecast. This led to a much lower velocity forecast in early 1982. The actual velocity growth in 1982 was -5.7 percent, well below both the FOMC and the time-series forecast. In spite of some obvious differences between the FOMC's implied assumption of M-1 velocity and our time-series forecasts, we proceed as if our tine-series model forecasts of velocity were the same as the FOMC's assumption.

We use the bivariate velocity models of M-1 and M-2 to evaluate the behavior of velocity over the period 1980:IQ to 1983:IVQ. This evaluation is based on the one-step-ahead forecasts from the model estimated for the period 1959:IIQ to 1979:IVQ. Under the null hypothesis that the estimated model is an adequate representation for the post-sample period, the one-step-ahead forecasts are distributed randomly with zero mean and covariance matrix, $\hat{\Sigma}$. The sum of errors is approximately distributed as:

$$\frac{1}{\hat{\sigma}_i \sqrt{T}} \sum_{t=1}^{T} a_{it} \sim N(0,1).$$

The sum of the squared errors is approximately distributed as:

$$\frac{1}{\hat{\sigma}_1^2} \sum_{t=1}^{T} a_{it}^2 \sim \chi^2_{(T)}.$$

Tables 7 and 8 include statistics for testing the hypothesis that the forecast errors of velocity growth from the bivariate models have zero mean and variance equal to the estimated variance of the sample errors. The tests are calculated for forecast errors accumulated over four quarters, beginning with the forecast error in the first quarter of each year. This test can be constructed from any point in time to examine the stability of velocity growth.

In table 7, we compare the univariate and bivariate forecast errors for M-1 velocity. If we had used the univariate model, we would have rejected the hypothesis that velocity was stable in 1981. The error was positive; the Federal Reserve elected to aim at the low end of the target ranges (see chart 1). If we had used the bivariate model, we would not have rejected the hypothesis that M-1 velocity was stable. A decision to restrain M-1 growth at the end of 1980 was implemented by choosing a lower path for reserves and, consequently, inducing an unexpected rise in the interest rates. This unpredicted jump in interest rates explains the subsequent rise in velocity in the bivariate model.

Taking interest rates into account does not completely explain the large decline in velocity in 1982.[3] Preliminary information about velocity in the 1982:IQ was available in March, but was not finalized until June 1982.[4] By that time, however, the evidence was convincing, and at its July meeting, the FOMC voted to allow M-1 growth to exceed the upper limit of the target range. The M-1 velocity model continued to produce large negative forecast errors throughout the first quarter of 1983. Since then the errors have been small and offsetting. Clearly, the bivariate model failed to explain M-1 velocity growth in 1982. Whether the breakdown was permanent or temporary is a subject of continuing research.

The end-of-year cumulative M-1 errors shown in chart 1 are important because they are incorporated permanently into the base for the next year's target range. The Federal Reserve has been criticized for this practice, but shifts in the base for the target since 1979 can be justified because they offset an unexpected drift in velocity.

The forecast errors for M-2 velocity are shown in table 8. Using the univariate model led us to reject the hypothesis that the velocity trend was stable in 1981 and 1982. Using the bivariate M-2 velocity growth model, we could not reject the hypothesis that the velocity trend was stable until 1983:[10] The stability of M-2 velocity through 1982 led the FOMC to switch its primary emphasis from M-1 to M-2 in October 1982. This change in emphasis occurred just before the only significant forecast error for M-2 velocity growth, which was associated with the introduction of money market deposit accounts (MMDAs). However, in anticipation of this error, the FOMC chose the 1983 February-to-March average as the base for the M-2 target range.

V. Conclusion

In this paper, we have shown that multivariate time-series procedures produce significantly better forecasts of M-1 velocity than univariate procedures do. The best model of M-1 velocity growth is a bivariate model that includes M-1 velocity growth and the Treasury bill rate. This model, estimated from a period during which the Federal Reserve used an interest-rate operating target, did an exceptionally good job of forecasting velocity in 1980 and 1981 and continued to produce forecasts that varied with actual values in 1982 and 1983. The forecasts were badly biased in these last two years, although not as badly biased as the forecasts from the univariate model or the derived velocity model.

The best model of M-2 velocity is derived from the trivariate model that includes M-2, nominal GNP, and the Treasury bill rate. The estimated effect of the lagged interest-rate error on M-2 velocity growth is approximately one-third larger than the impact on M-1 velocity. Taking interest rates into account does improve the out-of-sample forecast for M-2 velocity, but the improvement is not statistically significant. The bivaridte model is similar to the velocity model derived from the trivariate model and leads to similar out-of-sample forecasts. The n-step-ahead forecast for changes in M-2 velocity is zero for n greater than 1 in the bivariate model, and very close to zero for the trivariate model.

The unusual behavior of velocity in 1982 and 1983 has been attributed to deregulation and the rapid decline of inflation. Constructing and implementing monetary targets during this period required several major changes in the monetary targets. In the absence of a complete structural model of the economy, we will never be able to predict all the shifts in velocity, but we have presented evidence that relatively simple models of velocity that incorporate information about interest rates yield significantly better forecasts than do univariate models. in the last four years, these models would have warned of a shift in velocity. Furthermore, for the period since 1980, they show that deviations of the money stock from announced targets have offset unexpected shifts in velocity.

Footnotes

1. The information gain of model B over model A is calculated as:

$$I(B,A) = [1 - SEE(B)/SEE(A)] \times 100,$$

where SEE is the standard error of the equation. This method of comparing models was suggested by James Hoehn. See Hoehn, Gruben, and Fomby (1984).

2. Our univariate forecast errors are comparable in size to the univariate forecast errors presented in Hein and Veugelers (1983).

3. There are several explanations for the decline in velocity. One is that there was a shift in money demand associated with the introduction of interest-bearing checkable deposits (see Simpson 1984). Judd (1983) argues that the shift in money demand was caused by a sudden lowering of inflation expectations. See the proceedings from a conference held at the Federal Reserve Bank of San Francisco (1983), for other papers attempting to explain the unusual behavior of velocity in 1982 and early 1983.

4. These data have been revised. However; the money supply and GNP data that were available at the time resulted in an even more dramatic breakdown in all the M-1 velocity models.

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Table 1 Univariate Velocity Models for 1959:IIQ to 1979:IVQ

M-1 velocity

$$\nabla \ln VM1_t = .0077 + a_t$$

SEE = .0087

I(U,N) = 0

M-2 velocity

$$\nabla \ln VM2_t = .270\, a_{t-1} + a_t$$

SEE = .0097

I(U,N) = 3.0

NOTE: SEE is the standard error of the equation. I(U,N) is the information gain of the univariate model over the naive model. The M-1 velocity model is the naive model; that is, velocity growth forecast is equal to the mean growth rate of the sample.

Table 2 Bivariate Velocity Models for 1959:IIQ to 1979:IVQ

M-1 velocity model

$$\nabla \ln VM1_t = .0076 + 1.656\, a_{2,t-1} + a_{1,t}$$

$$\ln(1+RTB3)_t = 1.021 \ln(1+RTB3)_{t-1} + .541\, a_{2,t-1} + a_{2t}$$

$$\text{Error correlation matrix} = \begin{bmatrix} (.0084) & \\ .03 & (.0014) \end{bmatrix}$$

M-2 velocity model

$$\nabla \ln VM2_t = .246\, a_{1,t-1} + 2.326\, a_{2,t-1} + a_{,t}$$

$$\ln(1+RTB)3_t = 1.016 \ln(1+RTB3)_{t-1} + .508\, a_{2,t-1} + a_{2,t}$$

$$\text{Error correlation matrix} = \begin{bmatrix} (.0090) & \\ .09 & (.0014) \end{bmatrix}$$

NOTE: The standard deviations of the error term are shown in parentheses on the diagonal of the error correlation matrix.

Table 3 Trivariate Models for Nominal GNP, the Treasury Bill Rate, and the Money Stock for 1959:IIQ to 1979:IVQ

M-1 model

$$\nabla \ln M1_t = .923\ \nabla \ln M1_{t-1} + a_{1,t} - 1.202\ a_{2,t-1}$$

$$\ln(1+RTB3)_t = 1.031\ \ln(1+RTB3)_{t-1} + a_{2,t} + .498\ a_{2,t-1}$$

$$\nabla \ln GNP_t = 1.553\ \nabla \ln M1_{t-1} - .898\ a_{1,t-1} + a_{3,t}$$

$$\text{Error correlation matrix} = \begin{bmatrix} (.0055) & & \\ -.14 & (.0014) & \\ .45 & -.21 & (.0092) \end{bmatrix}$$

M-2 model

$$\nabla \ln M2_t = .973\ \nabla \ln M2_{t-1} + a_{1,t} + .329\ a_{1,t-6} - 2.436\ a_{2,t-1} - .119\ a_{2,t-6}$$

$$\ln(1+RTB3)_3 = 1.027\ \ln(1+RTB3)_{t-1} + a_{2,t} + .385\ a_{2,t-1} + .049\ a_{2,t-5} + .019\ a_{2,t-6} - .135\ a_{1,t-5} - .052\ a_{1,t-6}$$

$$\nabla \ln GNP_t = .952\ \nabla \ln M2_{t-1} + a_{3,t}$$

$$\text{Error correlation matrix} = \begin{bmatrix} (.0044) & & \\ -.29 & (.0014) & \\ .25 & .02 & (.0093) \end{bmatrix}$$

NOTE: The standard deviation of the error term is listed on the diagonal of the error correlation matrix.

Table 4 Comparison of Forecast Errors for 1980:IQ to 1984:IIQ

		Velocity forecast errors	
Model		M-1	M-2
Univariate	mean	-.0054	-.0017
Velocity	RMSE	.0173	.0155
Bivariate	mean	-.0046	-.0011
Velocity	RMSE	.0117	.0123
Trivariate	mean	-.0082	-.0002
Velocity	RMSE	.0155	.0120

Table 5 Ashley, Granger, and Schmalensee Specification Tests

Dependent variable	Estimation results		
	$\hat{c}_0$	$\hat{c}_1$	F(2,14)
M-1 velocity growth	-.075	.217	7.94[a]
	(-.51)	(3.95)	
M-2 velocity growth	-.055	.141	0.75
	(-.19)	(1.21)	

NOTE: The t-statistics are shown in parentheses.

a. The F-statistic rejects the hypothesis that $\hat{c}_0$ and $\hat{c}_1$ are not significantly different from zero at the 0.002 critical level.

Table 6 M-1 Velocity: Implied Assumptions and Time-Series Forecasts [a]

Year	GNP forecast central tendency	M-1 midpoint	Implied velocity assumption	4-Step-ahead velocity forecast
1980	7.5-11	5.25	2.25-5.75	3.9
1981	9-12	4.75[b]	4.25-7.25	4.6
1982	8-10.5	4	4-6.5	1.6
1983	8-9	6	2-3	2.9
1984	9-10	6	3-4	2.7

a. All figures in percent growth rates.

b. The M-1 midpoint was adjusted for expected growth in negotiable order of withdrawal (NOW) accounts by the staff of the Board of Governors of the Federal Reserve System.

SOURCE: "Monetary 'Policy Report to Congress," Federal Reserve Bulletin, various issues.

Table 7 Tests for Changes in the Trend of M-1 Velocity Growth

	Velocity forecast error cumulated over the year		N(0,1) test for change in mean growth rate		$\chi^2_{(T)}$ test for a change in the variance of the error	
Year: tq	Univariate	Bivariate	Univariate	Bivariate	Univariate	Bivariate
1980:IQ	.21	- .57	.25	-.68	.06	.46
II	.78	- .12	.63	-.10	.48	.75
III	-1.58	- .69	-1.04	-.47	7.78	1.20
IV	-1.32	-1.09	- .76	-.65	7.87	1.43
1981:IQ	3.07	1.59	3.53[a]	1.89	12.44[a]	3.56
II	1.79	.92	1.46	.77	14.60[a]	4.20
III	3.33	2.03	2.21[a]	1.40	17.71[a]	5.93
IV	2.03	1.04	1.16	.62	19.91[a]	7.32
1982:IQ	-3.37	-1.97	-3.87[a]	-2.34[a]	14.93[a]	5.46[a]
II	-3.52	-3.20	-2.86[a]	-2.69[a]	14.96[a]	7.60[a]
III	-5.19	-4.01	-3.44[a]	-2.77[a]	18.64[a]	8.52[a]
IV	-8.78	-6.60	-5.05[a]	-3.93[a]	35.56[a]	17.98[a]
1983:IQ	-1.87	-1.71	-2.14[a]	-2.03[a]	4.58[a]	4.10[a]
II	-2.61	-2.54	-2.12[a]	-2.14[a]	5.30	5.09
III	-3.30	-3.23	-2.19[a]	-2.23[a]	5.94	5.76
IV	-2.76	-2.93	-1.59	-1.74	6.33	5.89
1984:IQ	.94	1.31	1.07	1.55	1.16	2.40
II	1.18	1.28	.96	1.07	1.24	2.40

NOTE: The errors are in percent at quarterly rates cumulated from the first to the fourth quarter.

a. Using the 5 percent critical region, we can reject the null hypothesis that the process generating velocity has not changed.

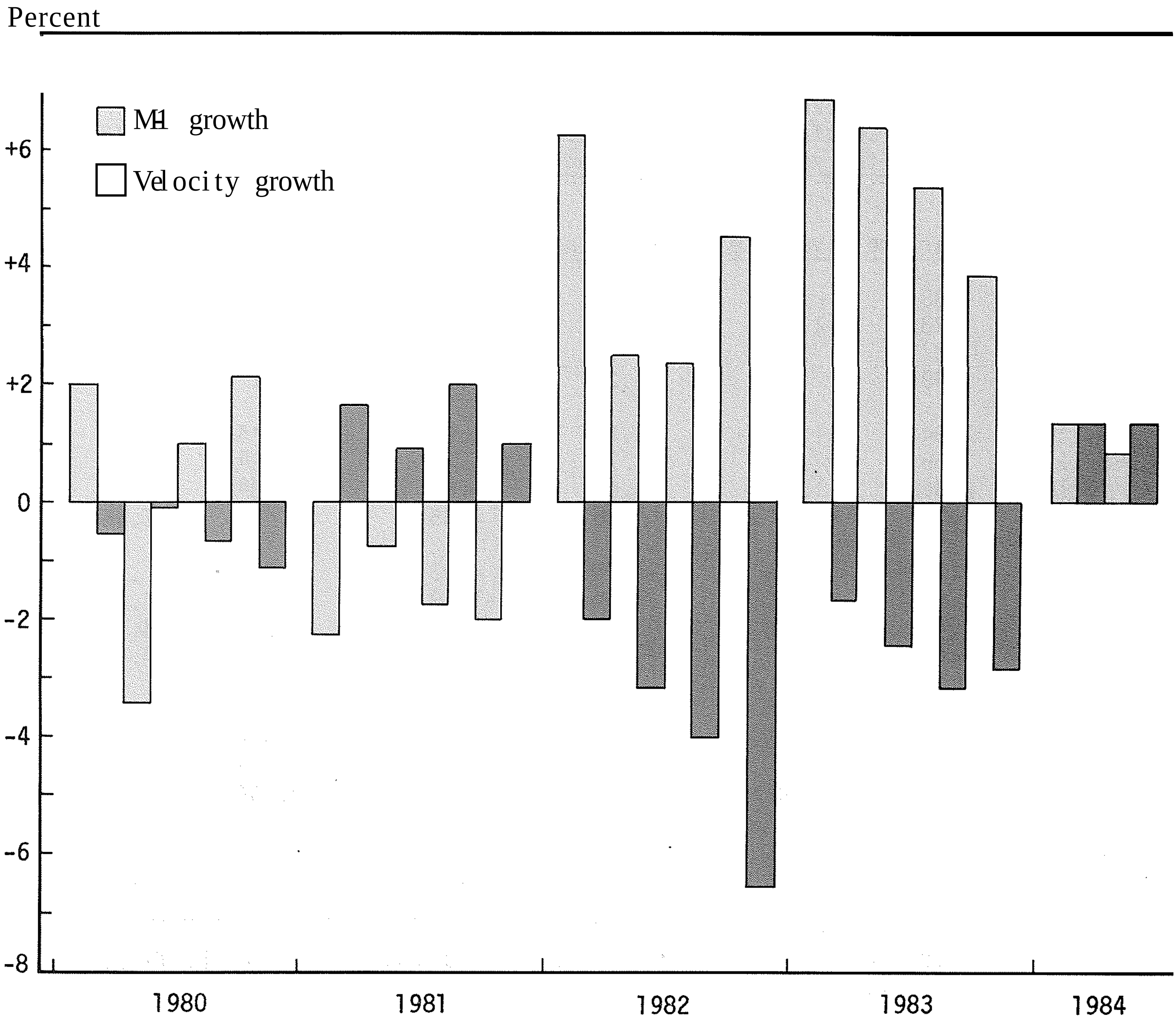
Table 8 Tests for Changes in the Trend of M-2 Velocity Growth

Year: tq	Velocity forecast error cumulated over the year[a]		N(0,1) test for change in mean growth rate		$\chi^2_{(T)}$ test for a change in the variance of the error	
	Univariate	Bivariate	Univariate	Bivariate	Univariate	Bivariate
1980:IQ	-.88	-.21	-.98	-.21	.95	.05
II	-.23	-1.26	-.18	.92	2.47	1.22
III	-.97	.46	-.62	.27	3.13	4.34
IV	.49	.42	.27	.21	5.71	4.34
1981:IQ	2.84	1.06	3.16[a]	1.09	9.84[a]	1.20
II	.96	.47	.75	.34	14.18[a]	1.57
III	2.56	1.12	1.64	.67	17.33[a]	2.03
IV	.15	-.68	.08	-.35	24.42[a]	5.46
1982:IQ	-1.78	-.03	-1.98[a]	-.03	3.87[a]	.01
II	-1.96	-2.18	-1.54	-1.59	3.91	4.91
III	-3.61	-2.25	-2.31[a]	-1.34	7.23	4.91
IV	-4.82	-2.44	-2.68[a]	-1.26	9.01	4.95
1983:IQ	-2.63	-2.70	-2.92[a]	-2.78[a]	8.41[a]	7.76[a]
II	-1.63	-1.90	-1.28	-1.39	9.62[a]	8.43[a]
III	-1.20	-1.43	-.77	-.85	9.84[a]	8.67[a]
IV	-.90	-1.48	-.50	-.76	9.95[a]	8.68
1984:IQ	1.68	2.25	1.87	2.32[a]	3.44	5.39[a]
II	2.10	2.19	1.65	1.60	3.66	5.39

NOTE: The errors are in percent at quarterly rates cumulated from the first to the fourth quarter.

a. Using a 5 percent critical region, we can reject the null hypothesis that the process generating velocity had not changed.

Chart 1 Deviations of M-1 and Velocity from Expected values[a]



NOTE: Quarterly deviations are cumulated over the calendar year.

a. Expected values of M-1 growth are based on the midpoint of the M-1 target range. Expected values of velocity are one-step-ahead forecast errors from the bivariate model.