April 15, 2002

Federal Reserve Bank of Cleveland

A Beautiful Theory

by Ed Nosal and Peter Rupert

Long before the book, the movie, the Golden Globe, and the Academy Award, John Forbes Nash, Jr., the central character in *A Beautiful Mind*, was a legend of epic proportions in economics. For his contributions to economics, Nash was awarded the 1994 Nobel Prize. This *Commentary* explains why Nash's work had such a profound influence on the way economists both think about and do economics. His influence has been felt in virtually every field of study in economics.

Background

Nash was awarded a Nobel Prize for his contributions to game theory. Game theory is used to analyze problems that feature "strategic interaction" between individuals or "players." Strategic interaction refers to a situation in which the actions of one party affect—directly or indirectly—the well-being of other parties. For example, an airline company that lowers the price of its tickets will affect not only its own profitability, but also the profitability of its competitors, since a lower price will draw consumers away from the competition. This example points to the *competitive* nature of strategic interaction. But many strategic encounters also involve a *cooperative* element. To continue with the example, airline carriers may compete aggressively for certain routes but may form alliances—cooperate—for other routes. Game theory turns out be a useful tool for understanding economic behavior precisely because most social encounters entail an element of strategic interaction. Our example points out that a complete theory of games should contain elements of both competition and cooperation.

Nash, of course, was not the first person to recognize the importance of strategic interaction between social actors: The first inhabitants of our planet no doubt realized that! But, in a series of articles between 1950 and 1953, Nash laid the groundwork that helped economists and other social scientists think fruitfully about and model strategic aspects of social engagement. In fact, some of the most important and pathbreaking work in economics undertaken in the latter half of the twentieth century simply could not have taken place without the benefit of Nash's insights.

Pre-1950 Game Theory

To put Nash's contributions in perspective, one must appreciate the state of game theory before he entered the scene. In 1928 the brilliant mathematician John von Neumann published the first important paper on game theory. In that paper, von Neumann recognizes that social interaction typically involves elements of both competition and cooperation. However, he also realizes that any model that embeds these elements will not always be solvable. Given this limitation, a major success of von Neumann's 1928 paper was that it completely solved a game that featured competition and excluded the possibility of cooperation. This game of pure strategic competition is known as a "two-person zero-sum" game because there are only two players, and whatever one player gains, the other player loses.

When we say that von Neumann was able to "solve" a game, what we mean is that he was able to characterize each player's "winning strategy" or, in other words, how each player intended to play the game. Von Neumann was able to prove that each player's winning strategy provided the highest guaranteed payoff. By guaranteed payoff we mean the lowest payoff the player could ever receive, no matter how his opponent plays. It turns out—and this is the linchpin to von Neumann's proof—this winning strategy also minimizes the highest payoff that his *opponent* can possibly receive. Hence, the strategy that maximizes the guaranteed payoff is a winning one because if a player chooses to play something else, he will be made worse off. Because von Neumann was able to show that two-person zero-sum games are always solvable, they became an important vehicle for modeling strategic interactions for many years.

It wasn't *A Beautiful Mind*—the book or the movie—that made John Forbes Nash, Jr., famous. It was his work in game theory, a theory that models strategic interactions between people as games. Before Nash, the only games theorists could get a handle on were artificial ones with no real-world applications. Nash's insights enabled economists to expand the use of game theory to interesting practical problems.

Without doubt, von Neumann's solution to two-person zero-sum games was a major building block in the theory of games. However, one must recognize that in most strategic social encounters, it is generally *not* the case that whatever one person gains, the other person loses. A basic problem with the two-person zero-sum game is that it is incapable of describing interesting or realistic social encounters. In most social encounters, players can typically take actions that are mutually beneficial, mutually destructive, or beneficial to one player and harmful to the other.

ISSN 0428-1276

To study such behavior, that is, to capture the competitive and cooperative elements that are observed when people interact, non-zero-sum games are required. In these kinds of games, however, von Neumann's "linchpin" is destroyed: The strategy that maximizes the guaranteed payoff to one player no longer minimizes the highest payoff that his opponent can receive. This implies that if a player chooses to play something other than the strategy that maximizes the guaranteed payoff, he can actually make himself *better off*. So having players choose strategies that maximize their guaranteed payoffs is not a solution to these games. What strategies should players use?

■ Nash 1950–53

Between 1950 and 1953, Nash published four remarkable papers that effectively redirected research efforts in game theory in a fruitful direction. Two papers focused on competitive or "noncooperative" non-zero-sum games; another paper focused on cooperative two-person games; and the final paper examined the relationship between cooperative and noncooperative games and suggested how elements of competition and cooperation could be incorporated into a general game.

Noncooperative Game Theory and the Nash Equilibrium (1950,1951)

Strategies that maximize guaranteed payoffs do not solve two-person non-zero-sum games. What, then, is the solution to a non-zero-sum game? This is where Nash comes in. His insight was to have each player imagine how his opponent intended to play and to have players use this information in helping them formulate their own strategies. Nash envisioned that a player would take his "best response" to other players' intended strategies. A solution or *Nash equilibrium* to a game is when all players' intended strategies *are* their best responses. Nash used the term "noncooperative" to describe his approach: Although in non-zero-sum games there may be gains from cooperation, Nash assumed it is "impossible for players to communicate or collaborate in any way."

Nash was able to show that for *any* non-zero-sum game that has a finite number of players and actions, an equilibrium to the game always exists. This is a very powerful result. It implies that researchers could now write down a game that captured realistic aspects of social interaction—because the game is non-zero sum—and could always be assured that the game had a solution. It is rather comforting to know that the Nash equilibrium for a two-person zero-sum game corresponds to von Neumann's solution. In this way, Nash's result extended and generalized von Neumann's result to non-zero-sum games with potentially many players.

There are a huge number of social phenomena that can best be thought of as noncooperative games. For example, firms compete for consumers in a variety of ways: They can compete through the prices they charge, through the quality of their services, and so on. Firms may have an incentive to cooperate with one another: But it is illegal for firms to form alliances when these kinds of formal agreements ultimately hurt consumers. So, in the context of trying to understand how firms interact with one another, one can view them as playing a noncooperative game with each other. This is not to say that cooperation is unimportant.

Tariff wars provide a good example. In competing for consumers, countries have been known to use a tariff to raise the price of imports, thereby making the home good relatively cheaper. In response, the other country imposes tariffs. In the end, both countries are generally worse off. Organizations such as the World Trade Organization can be interpreted as a coordinating or cooperative mechanism that helps prevent such destructive competitive behavior. Similarly, the United Nations can be viewed as a coordinating mechanism that helps prevent other kinds of destructive behavior between countries. Although in some contexts it is appropriate, or at least not misleading, to ignore cooperation, in other contexts of social engagement it is not.

The Bargaining Problem and the Nash Bargaining Solution (1950)

Many strategic situations require explicit cooperation. Consider, for example, the case of a union and a firm bargaining over a contract. It may be that the union and firm are opposed on many issues, but cooperation is required for both parties to agree to sign the contract. How exactly would the union and firm agree to split the benefits associated with their relationship? Nash provided the first solution to this kind of bargaining problem.

Nash looked at the problem of two players having to agree to split a "pie." Both people want as much of the pie as they can get, but if the players fail to agree on a split, the pie disappears. Nash's approach to solving this problem was revolutionary. Anything short of revolutionary was doomed to fail: For years, many great economists and game theorists tried to solve the two-person bargaining problem without success. Instead of attempting to determine the solution to an explicit bargaining game, Nash suggested a number of reasonable properties that any solution to a bargaining problem should satisfy. For example, two of his properties are: Players should never leave any "pie on the table," and identical players should split the pie evenly. Nash was able to show that for the set of properties that he viewed as being reasonable, there would be a unique split of the pie and the actual split could be described by a simple mathematical equation, which is now known as the "Nash bargaining solution." Nash's solution to the bargaining problem was an extremely important development because it provided an answer to a longstanding, unresolved question, and his approach to solving the problem was quite novel.

But, in a way, the novelty of the approach also carried with it a liability. Nash's approach to solving a (two-person) cooperative game is fundamentally different from the approach he used to solve a (non-zero-sum) noncooperative game. The problem, it appears, is that one kind of "theory" is needed to understand (or solve) noncooperative strategic interactions and a completely different kind of theory is needed to solve cooperative strategic interactions. But, in practice, social engagements are generally not exclusively cooperative or exclusively noncooperative—but rather a mixture of both. How, then, should one think about problems that have both elements?

The Nash Program (1953)

When people cooperate in a strategic situation, they first discuss or negotiate what they would like to receive, emphasizing the actions they may take if their opponent does not agree, and then, typically, they sign an agreement. Consider, once again, the situation of a firm and union bargaining over a new contract. The firm and union may bargain over wages, working conditions, and so on.

The firm may threaten to lock out workers if the union does not agree to its proposal; or workers can threaten to strike, providing the firm with no labor until the contract dispute is resolved.

Nash demonstrated that one could "reduce" this cooperative situation to a noncooperative game. Specifically, players first announce "threats"—actions they would take in the event that no settlement is reached. Then, players announce their settlement offers: If the settlement offers are not feasible or are incompatible, then players will undertake their threat actions. This is a noncooperative game because players do not sign contracts or make prior agreements. In the case of splitting a pie, settlement offers would not be compatible if the players' desired shares added up to more the whole pie. Nash showed that there is a solution, that is, a "Nash equilibrium," to this noncooperative game, where players announce certain threats, and settlement offers are compatible. So, players will end up "splitting a pie" in a noncooperative manner. What is remarkable about the solution to this noncooperative game is that it corresponds to the (Nash bargaining) solution of a purely cooperative game; that is, it yields the same split of the pie. This is an important result because, although the cooperative and noncooperative approaches to game theory are quite different, Nash showed they could lead to the same solution.

How should one think about the resolution of strategic encounters that have elements of *both* competition and cooperation? Nash's 1953 paper suggests that a cooperative situation could be reduced to a noncooperative one. Therefore, a suitably specified noncooperative game is able to characterize elements of competition and cooperation and the solution to this game is given by the Nash equilibrium.

Conclusions

In the 1940s, a group of influential economists and mathematicians were convinced that the most fruitful way to view economic and social interactions was through the lens of game theory. However, by the end of the 1940s enthusiasm had stalled because a main engine of game theory, the two-person zero-sum game, was incapable of describing relevant problems. In short, Nash showed us how to think about and solve the more interesting non-zero-sum games. He was the first to make a distinction between noncooperative games and cooperative games. Both of these frameworks are capable of addressing interesting economics problems, and Nash showed that solutions existed for each type. Although noncooperative games are fundamentally different from cooperative games, Nash demonstrated that the different approaches could lead to the same solution. In the end, Nash provided us with a framework—the noncooperative game—and a solution—the Nash equilibrium—that are capable of analyzing both the cooperative and competitive aspects of social interaction. There is virtually no area in economics or the social sciences that has not benefited from Nash's "beautiful theory."

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