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# Credit Misallocation and Macro Dynamics with Oligopolistic Financial Intermediaries

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## Abstract

Bank market power shapes firm investment and financing dynamics and hence affects the transmission of macroeconomic shocks. Motivated by a secular increase in the concentration of the US banking industry, I study bank market power through the lens of a dynamic general equilibrium model with oligopolistic banks and heterogeneous firms. The lack of competition allows banks to price discriminate and charge firm-specific markups in excess of default premia. In turn, the cross-sectional dispersion of markups amplifies the impact of macroeconomic shocks. During a crisis, banks exploit their market power to extract higher markups, inducing a larger decline in real activity. When a “big” (i.e., non-atomistic) bank fails, the remaining banks use their increased market power to control the supply of credit, worsening and prolonging the recession. The results suggest that bank market power could be an important concern when formulating appropriate bail-out policies.

*JEL Codes:* D43, E44, G12, G21, L11.

*Keywords:* Dynamic Financial Oligopoly, Endogenous Financial Markups, Heterogeneous Firms, Firm Dynamics, Micro-Founded Financial Frictions, Price Discrimination.

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# 1 Introduction

The banking industry has become increasingly concentrated over the past two decades, with the asset market share of the five largest US banks rising from 26% in 1996 to 50% in 2018. Moreover, the Lerner index increased from 0.2 in 1996 to 0.33 in 2014, pointing to a sizable increase in markups.<sup>1</sup> A large and influential literature has studied the interactions between financial markets, firm, and aggregate dynamics but has typically assumed perfectly competitive financial intermediaries; thus, it does not speak to this trend of increasing banking sector consolidation and markups.<sup>2</sup>

In this paper, I study the role of imperfect competition in the financial intermediation sector for firm investment and financing dynamics, as well as for the transmission of macroeconomic shocks. I develop a novel dynamic general equilibrium model that incorporates an oligopolistic financial sector with heterogeneous firms. Imperfect competition enables financial intermediaries to charge firm-specific markups that depend on the idiosyncratic characteristics of the firms to which they lend. In particular, banks exert a higher degree of market power on firms that are more financially constrained and have a high marginal productivity of capital. These firms have worse outside options (e.g., a high cost of non-bank finance) and one additional unit of investment in physical capital can contribute significantly to their future production; hence, they exhibit a higher and less elastic demand for credit. The resulting dispersion of markups (i) induces credit – and thus capital – misallocation, reducing aggregate productivity and (ii) plays a significant role in shaping the transmission of macroeconomic shocks.

During a crisis, banks exploit their market power to extract higher markups, inducing a larger decline in real activity. Notably, since my model features non-atomistic banks, I can study market structure changes in the intermediation sector (e.g., the failure of a large intermediary). When a single “big” bank fails, surviving banks utilize their increased level of market power to control the supply of credit contributing to amplifying and prolonging the recession. The results suggest that banks’ market power should be an important source of concern for policymakers deciding whether to bail out a large intermediary.

Succinctly, the model works as follows. Firms make optimal capital structure decisions

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<sup>1</sup>See Appendix A.4. See [Corbae and D’Erasmus \(2021\)](#) for related and more detailed evidence.

<sup>2</sup>See, for instance, [Bernanke and Gertler \(1989\)](#), [Covas and den Haan \(2011\)](#), [Jermann and Quadrini \(2012\)](#), [Khan and Thomas \(2013\)](#), and, more recently, [Elenev, Landvoigt, and Van Nieuwerburgh \(2021\)](#).

by balancing equity and debt financing, generating an endogenous dynamic demand for loans. Banks are large (i.e., non-zero mass) players and firms are a continuum of followers in a Stackelberg fashion (i.e., each financial intermediary takes firms’ dynamic demand for loans as given and competes to supply funding to each individual firm). Intermediaries make strategic decisions by internalizing the effect of their actions on present and future banks, firms’ decisions, and on the aggregate economy. In such an environment, the bank’s optimal equilibrium choice of loan supply is determined by solving so-called generalized Euler equations.<sup>3</sup> Each generalized Euler equation is an otherwise standard Euler equation, except that it contains a firm-specific elasticity that measures the sensitivity of the future interest rate with respect to the current supply of loans. The model generates firm-specific credit spreads that accrue to banks, which include default premia and markups.

My analysis proceeds in two main steps. First, I develop a stylized two-period model that I use to derive analytical insights on the role of oligopolistic intermediaries for firm and aggregate dynamics. Second, I build a quantitative infinite-horizon version of the model that I use to gauge the quantitative importance of these connections. I calibrate the model to match several financial and macroeconomic variables using Federal Deposit Insurance Corporation (FDIC) data on Commercial and Industrial (C&I) Loans. The calibrated model yields an annualized aggregate financial markup of approximately 24bps, in excess of default premia. This translates into an average TFP loss of approximately 0.2%, suggesting that imperfect competition in the financial sector can have a sizable effect on long-run macroeconomic outcomes. I then use the calibrated model to investigate the role that banks’ market power plays in the transmission of three aggregate shocks: (i) a credit quality deterioration (in the model, a sudden increase in the aggregate firms’ default probability), (ii) a “big” bank failure (in the model, a sudden exit of a large bank), and (iii) a cut to the bank funding rate. I conduct the experiments by comparing the dynamic response of the oligopolistic economy against one with perfectly competitive financial intermediaries. I find that in each of these cases, the endogenous cross-sectional dispersion of markups plays a key role in shaping — and in particular, amplifying — the economy’s response to the exogenous shock.

An increase in the aggregate firms’ default probability induces a higher proportion of

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<sup>3</sup>Note that the optimal fiscal policy literature uses generalized Euler equations and Markov perfect equilibria in macroeconomics (Klein and Ríos-Rull, 2003; Krusell, Martin, and Ríos-Rull, 2004; Klein, Krusell, and Ríos-Rull, 2008; and Clymo and Lanteri, 2020).

more financially constrained firms with a high marginal productivity of capital. A more concentrated banking sector can control the supply of credit more tightly by extracting higher markups from these firms, leading to higher credit spreads. This mechanism allows banks to compensate for the larger losses due to defaults, but it leads to a larger decline in real activity, amplifying the recession. Quantitatively, when the aggregate firms' default rate matches that observed in the Great Recession, bank market power induces a larger peak output decline of about 30% relative to the case of perfect competition.

When the economy also experiences the failure of a large intermediary, there are two key effects: (i) in the short run, the change in market structure lowers the supply of credit to firms, slowing down the economy; and (ii) in the long run, the resulting increase in banks' market power further amplifies and prolongs the recession. When a "big" bank fails, the surviving banks extend more credit to firms in order to capture the market share of the defaulted bank. However, the speed of this adjustment is dampened by the decreased level of competition. As a result of both credit constraints and market power, the aggregate volume of credit drops sharply in the short run. In the long run the economy stabilizes at a lower level of total credit, which results in less investment, output, and productivity. The result suggests that banks' market power may be an important source of concern for policymakers deciding whether to bail out a "big" bank.

Finally, following a cut to the bank funding rate, the oligopolistic intermediation sector acts as a potent transmission channel. Financial intermediaries exploit markups to dampen the aggregate loans' interest rate response. Quantitatively, these effects dampen the peak of the expansion by about one-third relative to the competitive benchmark.

**Related Literature** This paper is mainly related to two strands of literature: (i) firm dynamics in the face of financing frictions and (ii) macroeconomics with financial intermediaries. Indeed, one of the paper's main contributions is to link the first literature, which typically assumes a perfectly competitive credit market, with the second, which typically abstracts from firm dynamics and the heterogeneous effects of imperfectly competitive financial intermediaries across firms with varying characteristics.

**Credit Markets and Firm Dynamics.** An important literature has studied the impact of credit market frictions (e.g., borrowing constraints) on firm and aggregate dynamics, but

typically assumes that firms face a perfectly competitive credit market. My paper contributes to this line of work by jointly studying firms’ financing and investment decisions in a credit market characterized by imperfectly competitive, non-atomistic banks that compete strategically and focusing on how banks’ market power shapes the cross-sectional behavior of firms. Classical papers in this literature are [Kocherlakota \(2000\)](#); [Gomes \(2001\)](#); [Cooley and Quadrini \(2001\)](#); [Cordoba and Ripoll \(2004\)](#); [Hennessy and Whited \(2005\)](#); [Hennessy and Whited \(2007\)](#); [Covas and den Haan \(2011\)](#); and [Jermann and Quadrini \(2012\)](#). [Khan and Thomas \(2013\)](#) and [Khan, Senga, and Thomas \(2016\)](#) study models of heterogeneous firms in a dynamic stochastic general equilibrium environment in which firms can source their financing from a perfectly competitive intermediation sector.

Relatedly, the dynamic financial oligopoly I develop generates endogenous firm-level financial frictions that lead to time-varying second moments, such as the dispersion of loan rates (directly linked to the dispersion of marginal products of capital) and aggregate productivity. In agreement with other work and empirical findings (e.g., [Lanteri 2018](#) and [David, Schmid, and Zeke 2022](#)), the model generates an increasing dispersion of loan rates during recessions and hence, an increasing dispersion of marginal products of capital that shapes the dynamic behavior of aggregate productivity. Thus, the model uncovers a new channel for credit (hence, capital) misallocation linked to banks’ market power.<sup>4</sup>

[Burga and Céspedes \(2022\)](#) empirically estimate the effect of changes in bank market power by exploiting a merger episode using a sample of small Peruvian firms and find that, in agreement with the predictions of the model, the change in bank market structure results in (i) a reduction of capital concentrated among small firms with a high marginal return and (ii) an increase in capital misallocation.<sup>5</sup> To conclude, another related literature in firm dynamics studies financing constraints and irreversibility (e.g., [Caggese 2007](#)).

**Macroeconomics with Financial Intermediaries.** Several papers analyze the role of financial intermediaries in macroeconomics, either with a focus on banks’ imperfect competition (e.g., [Corbae and D’Erasmo 2021](#)) or focusing on the interaction between credit con-

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<sup>4</sup>Another literature studies constrained optimal dynamic contracts in partial equilibrium (e.g., [Albuquerque and Hopenhayn 2004](#); [Clementi and Hopenhayn 2006](#); [Brusco, Lopomo, Roperio, and Villa 2021](#)).

<sup>5</sup>The empirical literature about relationship lending – [Rajan and Zingales \(1998\)](#); [Black and Strahan \(2002\)](#); [Cetorelli and Gambera \(2001\)](#); [Cetorelli \(2004\)](#); [Cetorelli and Strahan \(2006\)](#); and [Cetorelli and Peretto \(2012\)](#) (theoretically)– is discussed in detail in Subsection 4.3.

straints and the financial intermediation sector (e.g., [Elenev, Landvoigt, and Van Nieuwerburgh 2021](#)).

There has been increasing interest in macroeconomics in analyzing the role of market power.<sup>6</sup> [Corbae and D’Erasmus \(2021\)](#) are among the first to investigate the effects of imperfect competition in loan markets by building a rich quantitative model of banking industry dynamics to study the effects of financial regulations. My paper complements their seminal work by embedding an imperfectly competitive banking sector in a heterogeneous firm environment, in which each firm makes optimal capital structure decisions and each bank extracts endogenous firm-specific markups. Hence, the focus of my work is on the impact of bank market power on macroeconomic outcomes with endogenously evolving heterogeneity in borrower types.

[Elenev, Landvoigt, and Van Nieuwerburgh \(2021\)](#) investigate financial intermediaries’ capital requirements in a model with both financially constrained firms and intermediaries. Similarly to my paper, their model focuses on the sudden and persistent fall in macroeconomic outcomes and credit supply during the financial crisis. My work complements their analysis by investigating the role of intermediary market power which, through time-varying endogenous firm-specific markups, leads to the amplification of macroeconomic shocks. As a direction for future research, it would be interesting to combine intermediaries’ market power with collateral constraints in order to investigate the economic mechanism through which they interact and quantitatively disentangle the two joint effects.

Other papers studying the interaction between credit constraints and financial intermediation include [Kiyotaki and Moore \(1997\)](#) and [Gertler and Karadi \(2011\)](#).<sup>7</sup> [He and Krishnamurthy \(2013\)](#) introduce a stochastic model that explains how intermediary capital affects risk premia variation. [Rampini and Viswanathan \(2018\)](#) propose a dynamic model whereby financial intermediaries provide a superior collateralization service to households. In contrast to these papers, my focus is on the transmission of macroeconomic shocks when intermediaries have market power.

Last, recent papers have highlighted a key role of intermediary market power in shaping

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<sup>6</sup>For example, see [Gutiérrez and Philippon \(2016\)](#); [Farhi and Gourio \(2018\)](#); [De Loecker, Eeckhout, and Unger \(2020\)](#); [Corhay, Kung, and Schmid \(2020\)](#) and [Berger, Herkenhoff, and Mongey \(2022\)](#).

<sup>7</sup>Other relevant papers are [He and Krishnamurthy \(2013\)](#); [Brunnermeier and Sannikov \(2014\)](#); and [Rampini and Viswanathan \(2018\)](#).

the transmission of monetary policy shocks.<sup>8</sup> For example, Wang, Whited, Wu, and Xiao (2022) analyze the impact of banks’ deposit market power on the loan maturity structure and assess the relevance (for the transmission of monetary policy) of banks’ market power in both the deposit and loan markets. The salient distinctive feature of my approach is that it allows for endogenous firm-specific markups, creating an equilibrium cross-sectional dispersion of markups, that I show is a key transmission channel of macroeconomic shocks.

**Outline.** The rest of the paper is organized as follows. Section 2 presents a stylized version of the model with analytical insights. Section 3 describes the quantitative model and discusses various aspects of its solution in detail. Section 4 explains the calibration and results of the *oligopolistic stationary equilibrium*. Section 5 illustrates the results of the three aforementioned macroeconomic shocks, such as the failure of one “big” bank and its interaction with the production sector’s default rate. Section 6 concludes.

## 2 Stylized Model

In this section, I analyze a two-period model designed to provide preliminary intuition for the quantitative model presented in Section 3. An oligopolistic banking sector interacts with a continuum of heterogeneous firms in the presence of idiosyncratic total factor productivity (TFP) and default shocks. I provide analytical results on the effects of an increase of the number of banks  $B$  on several financial and macroeconomics variables of interest (including aggregate loans, interest rates, expected returns on equity, physical investment, aggregate leverage, dispersion of capital, dispersion of loan interest rates, dispersion of expected returns, and aggregate TFP). In this stylized version of the model, there are two dates denoted  $t = 0, 1$ .

**Preferences.** There are  $B$  identical banks, each owned by a continuum of identical savers; hence there are  $B$  representative savers. Each saver’s preferences are represented by the

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<sup>8</sup>Malamudy and Schrimpf (2017); Drechsler, Savov, and Schnabl (2017); Li, Loutskina, and Strahan (2019); and Wang, Whited, Wu, and Xiao (2022).



following linear utility function:

$$C_{b,0} + \beta \cdot C_{b,1},$$

where  $C_{b,t}$  is the saver's consumption at time  $t$  and  $\beta \in (0, 1)$  is the discount factor.<sup>9</sup>

There is a continuum of firms, each owned by a continuum of identical entrepreneurs; hence there is one representative entrepreneur. The entrepreneur is risk-neutral and has preferences represented by the utility function:

$$C_{E,0} + \beta \cdot C_{E,1},$$

where  $C_{E,t}$  is the entrepreneur's aggregate consumption at time  $t$ .

**Ownership Structure.** Each representative saver owns a bank. In equilibrium, the saver is indifferent between financing the bank's loans with debt or equity. The representative entrepreneur owns the entire mass of firms  $j \in [0, 1]$ . Each firm  $j$  is characterized by its state vector

$$x(j) \equiv \{\{l_b(j)\}_b^B, r_l(j), k(j), z(j), \mathcal{I}(j)\},$$

where  $l_b(j)$  denotes the firm's loan by bank  $b$ ,  $r_l(j)$  is the interest rate (charged by all banks),  $k(j)$  is the firm's capital stock,  $z(j)$  is the firm's productivity, and  $\mathcal{I}(j)$  is an indicator function that takes value 1 if the firm has not defaulted. Let  $\phi(x)$  denote the density function of firms in the economy.

Each saver cannot own any firm's equity and thus needs to save through banks.

**Markets.** There are five markets in the economy: banks' debt, banks' equity, firms' loans, firms' equity, and the market for the representative good.

*Banks' equity and debt markets.* Each saver  $b$  invests in the production sector by supplying

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<sup>9</sup>The risk-neutrality assumption in the stylized model is made to simplify the analysis and isolate effects that do not depend on the savers' risk-aversion. In the quantitative model of Section 3 the savers are risk-averse.

equity or debt to banks and faces the budget constraints:

$$\begin{aligned} C_{b,0} + p_b \cdot S_{b,1} + D_{b,1} &= (p_b + \pi_{b,0}) \cdot S_{b,0} \\ C_{b,1} &= \pi_{b,1} \cdot S_{b,1} + R_D \cdot D_{b,1}, \end{aligned}$$

where  $p_b$ ,  $S_{b,0}$ ,  $S_{b,1}$ ,  $D_{b,1}$ ,  $\pi_{b,0}$ , and  $\pi_{b,1}$  are, respectively, the bank's share price, the share holdings at  $t = 0, 1$ , the debt holdings at  $t = 1$ , and the bank's profit at  $t = 0, 1$ . Bank  $b$  demands equity and debt from saver  $b$ , in order to finance loans to firms.

*Firms' equity market.* The entrepreneur invests in the production sector by supplying equity to the firms and faces budget constraints:

$$\begin{aligned} C_{E,0} + \int [\mathcal{I} \cdot p_0 \cdot S_1 + (1 - \mathcal{I}) \cdot p_0 \cdot S_1] d\Phi &= \int \mathcal{I} \cdot (p_0 + \tilde{d}_0) \cdot S_0 d\Phi \\ C_{E,1} &= \int \mathcal{I} \cdot \tilde{d}_1 \cdot S_1 d\Phi, \end{aligned}$$

where  $p_0$ ,  $S_0$ ,  $S_1$ ,  $\tilde{d}_0$ , and  $\tilde{d}_1$  are, respectively, the share price, share holdings at  $t = 0, 1$ , and the dividend of each firm (net of equity issuance cost) at  $t = 0, 1$ . Firms demand equity from, or distribute dividend to, the entrepreneur. If a firm decides to issue equity, it incurs a quadratic equity issuance cost at  $t = 0$  (with  $\lambda_0$  being a positive constant):

$$\lambda(d_0) = \begin{cases} \lambda_0 \frac{d_0^2}{2} & \text{if } d_0 \leq 0 \\ 0 & \text{if } d_0 > 0 \end{cases},$$

where  $d_0$  is a firm dividend at  $t = 0$ , defined below. The convexity of  $\lambda(\cdot)$  captures the idea of increasing marginal underwriting cost, or the increasing threat posed by a moral-hazard problem when a greater amount of equity is demanded.

*Firms' loan market.* A finite (and exogenous) number  $B$  of (identical) banks supply loans to a continuum of firms. Each bank  $b = 1 \dots B$  can issue non state-contingent loans  $l_{b,1}$  to each firm. Loans are due for repayment in the next period, unless the firm defaults. A firm  $j$  takes the interest rate  $r_1(j)$  as given and chooses how much to invest and how much to borrow from each bank. Banks take each firm's demand schedule as given and compete à la Cournot, i.e, simultaneously and independently choose their loan portfolios.

*Goods market.* The representative entrepreneur and the  $B$  representative savers demand goods supplied by all firms.

**Technology.** In each period  $t = 0, 1$ , the output  $y_t(j)$  produced by each firm  $j \in [0, 1]$  is given by the production function  $y_t(j) = z_t(j) \cdot k_t(j)^\alpha$ , where  $0 < \alpha < 1$ .

**Shocks.** At time 0, firms are heterogeneous with respect to their capital stock  $k_0$  and their idiosyncratic total factor productivity (TFP)  $z_0$ . At time 1, there are two types of idiosyncratic shocks: the firm can default, with exogenous probability  $1 - \rho$  and, if it survives,  $z_1$  realizes according to  $z_1 = \rho_z z_0 + \xi_1$  where  $\xi_1 \sim \mathcal{N}(0, \sigma_z^2)$  and  $\rho_z > 0$ .

**Timing.** All decisions are taken at  $t = 0$ . Given the initial distribution of firms with pdf  $\phi(x_0)$  (and cdf  $\Phi(x_0)$ ), the timing is as follows: (1) each firm produces output  $y_0 = z_0 k_0^\alpha$ ; (2) each bank finances its supply of loans,  $\int l_b(x_0) d\Phi(x_0)$ , by issuing equity and/or debt; (3) each firm takes the interest rate  $R_1(x_0)$  as given and chooses how much to invest and the amount of loan to demand from each bank; (4) banks take each firm's demand schedule as given and compete with each others to supply the loans. The outcome is a contract establishing: loan amount  $l_b(x_0)$ , interest rate  $R_1(x_0)$ , and the new level of capital  $k_1(x_0)$ ; and (5) firms distribute dividends  $d_0 = z_0 k_0^\alpha + (1 - \delta)k_0 - k_1 + \sum_b^B l_b$  to the entrepreneur. At  $t = 1$ , the  $1 - \rho$  mass of defaulting firms exits the market. For the surviving firms,  $z_1$  is realized and: (1) firms produce output  $y_1 = z_1 k_1^\alpha$ ; (2) firms repay their outstanding debt plus interest  $R_1(x_0) \cdot \sum_b^B l_b(x_0)$ ; (3) each bank distributes its profit  $\int \rho R_1(x_0) l_{1,b}(x_0) d\Phi(x_0)$  to the saver; and (4) firms distribute dividend  $d_1 = z_1 k_1^\alpha + (1 - \delta)k_1 - R_1 \sum_b^B l_{1,b}$  to the entrepreneur.

## 2.1 Agents' Optimization Problems

The *representative saver*  $b$  maximizes its intertemporal utility subject to their budget constraint, yielding an Euler equation that pins down the price of banks' equity:  $\forall b : \beta \pi_{b,1} = p_{b,0}$ . The *representative entrepreneur* maximizes its intertemporal utility subject to their

budget constraint, yielding an Euler equation that pins down the price of each firm's equity:

$$\rho\beta\mathbb{E}_0\left[\frac{d_1}{p_0}\right] = 1 - \lambda_d(d_0), \quad (1)$$

where  $p_0$  is the price of the share of a firm at time 0. *Firms* maximize the net present value of dividends  $d_0 + \beta \cdot \mathbb{E}_0[\mathcal{I} \cdot d_1]$ , where dividends in each period are given by:

$$d_0 = z_0 k_0^\alpha + (1 - \delta)k_0 - k_1 + \sum_b^B l_{1,b}, \quad d_1 = z_1 k_1^\alpha + (1 - \delta)k_1 - R_1 \sum_b^B l_{1,b}.$$

The firm's first-order condition with respect to capital requires that the future interest rate equals the expected marginal productivity of capital net of depreciation:

$$R_1 = \mathbb{E}_0[1 + \alpha z_1 k_1^{\alpha-1} - \delta]. \quad (2)$$

The firm's optimality condition with respect to the loan requires that the discounted future expected interest rate be one net of the equity issuance cost:

$$\rho\beta R_1 = 1 - \lambda_d(d_0). \quad (3)$$

*Banks'* strategies map firm characteristics ( $x_0$ ) onto the current quantity and future interest rate of loans. Given the probability density function  $\phi(x_0)$  (and cumulative distribution function  $\Phi(x_0)$ ), each bank  $b$  chooses  $l_{1,b}(x_0)$  to best respond to other banks' strategies  $l_{1,-b}(x_0)$ , such that

$$\max_{l_{1,b}(x_0)} \pi = - \int l_{1,b}(x_0) d\Phi(x_0) + \beta \int \rho R_1(x_0) l_{1,b}(x_0) d\Phi(x_0),$$

subject to equations (1), (2), and (3) for all firms in the distribution.

Each bank's best response is characterized by the following generalized Euler equation (GEE)

$$\forall x_0 : \frac{\partial \pi}{\partial l_{1,b}(x_0)} = -1 + \rho\beta \frac{\partial R_1(x_0)}{\partial l_{1,b}(x_0)} l_{1,b}(x_0) + \rho\beta R_1(x_0) = 0, \quad (4)$$

where  $\frac{\partial R_1(x_0)}{\partial l_{1,b}(x_0)}$  can be determined by the implicit function theorem on equations (2) and (3). Equation (4) is a *generalized* Euler equation because it contains the derivative of the firm's

policy functions. Each bank best responds by internalizing the effect of loans on the firms' capital choice  $\frac{\partial k_1}{\partial l_{1,b}}$  as well.

**Inverse Elasticity.** For ease of notation, I drop the dependency of all optimal choices from  $x_0$ . The inverse elasticity implicitly contained in the GEE  $\frac{\partial R_1}{\partial l_{1,b}} \frac{l_{1,b}}{R_1}$  requires the determination of the term  $\frac{\partial R_1}{\partial l_{1,b}}$ . From equation (2)

$$\frac{\partial R_1}{\partial l_{1,b}} = \mathbb{E}_0 \left[ \alpha(\alpha - 1) z_1 k_1^{\alpha-2} \frac{\partial k_1}{\partial l_{1,b}} \right]$$

it is clear that the inverse elasticity depends on the expectation of the second derivative of the production function. Note that this also captures the effects of banks' decisions on the investment choice of each firm (through the term  $\frac{\partial k_1}{\partial l_{1,b}}$ ). The previous equation can be rewritten as

$$\frac{\partial R_1}{\partial l_{1,b}} \frac{l_{1,b}}{R_1} = \mathbb{E}_0 \left[ (\alpha - 1) k_1^{-1} \cdot \underbrace{\alpha z_1 k_1^{\alpha-1}}_{\text{MPK}} \cdot \frac{\partial k_1}{\partial l_{1,b}} \frac{l_{1,b}}{R_1} \right]. \quad (5)$$

Note that the term  $\alpha - 1$  is always negative. This yields an inverse elasticity  $\frac{\partial R_1}{\partial l_{1,b}} \cdot \frac{l_{1,b}}{R_1}$  that is always negative. Banks exert higher market power when  $\frac{\partial R_1}{\partial l_{1,b}} \cdot \frac{l_{1,b}}{R_1}$  is smaller. The formula suggests that banks incorporate two components in their decision making when they extend loans to firms.

First, the higher the marginal productivity of capital (MPK) of a firm, the higher the markup that banks can extract (since for that firm the marginal value of one unit of investment is higher than for an established firm with high capital and low MPK).

Second, banks think strategically by internalizing the effects their actions have on firms' investment decisions. This second effect is captured by the cross-elasticity  $\frac{\partial k_1}{\partial l_{1,b}} \cdot \frac{l_{1,b}}{R_1}$ .

Expressions for the two cross-derivatives can be found jointly by taking the total derivatives of equations (2) and (3):

$$\frac{\partial R_1}{\partial l_{1,b}} = \frac{1 - \rho\beta R_1}{\rho\beta l_{1,b}}, \quad \frac{\partial k_1}{\partial l_{1,b}} = \frac{1 - \rho\beta R_1}{\rho\beta l_{1,b}} \cdot \frac{1}{\alpha(\alpha - 1)\mathbb{E}_0[z_1]k_1^{\alpha-2}}.$$

In equilibrium, for the mass of financially constrained firms ( $d_0 < 0$ ), the degree of imperfect competition (number of banks  $B$ ) matters. For each firm, the equilibrium is a

vector  $(k_1^*, R_1^*, l_{1,b}^*, p_0^*)$  such that equations (1), (2), (3), and (4) hold simultaneously. For the mass of firms that, in equilibrium, is not financially constrained, the degree of imperfect competition does not matter. For these firms, the solution is given by  $(k_1^*, R_1^*, p_0^*)$  such that equations (1), (2), and (3) hold simultaneously. Note that for these firms, the Modigliani-Miller theorem holds; hence,  $l_{1,b}^*$  is undetermined.

## 2.2 Characterization of the Equilibrium

I now describe intuitively the main mechanism that drives the analytical results presented in this section. First, Equation (5) suggests that the higher the marginal productivity of capital (MPK) of a firm, the higher the marginal value of one unit of loan for that firm that translates in a lower inverse elasticity  $\frac{\partial R_1}{\partial l_{1,b}} \cdot \frac{l_{1,b}}{R_1}$ . Second, as explained above, the degree of imperfect competition (number of banks  $B$ ) matters only for the mass of financially constrained firms. Hence, banks endogenously exert a higher degree of market power on firms that are financially constrained and with a high marginal productivity of capital. Intuitively, these firms have worse outside options (e.g., a high cost of non-bank finance) and one additional unit of investment in physical capital can contribute significantly to their future production; hence, they exhibit a less elastic demand for credit. An imperfectly competitive financial sector internalizes that the same financial resources are more valuable for this type of firms and for their future growth paths; therefore, can charge higher markups.

This mechanism leads financially constrained firms to grow slower, the higher the degree of imperfect competition. As a result, the dispersion of marginal productivity of capital is higher when there are fewer banks  $B$  in the economy. At the same time, a lack of competition in the financial intermediation reduces aggregate productivity, since firms grow toward their efficient level of capital on slower trajectories. This intuitive mechanism is at the base of Proposition I. See Appendix C.2 for the proof.

## Proposition I

Assume that the distribution  $\phi(x_0)$  is such that there is a non-zero measure of financially constrained firms:<sup>a</sup>

$$\mathcal{P} = \int \mathbf{1}[d_0(x_0, k_1^*, l_{1,b}^*) \geq 0] d\Phi(x_0) < 1.$$

A higher number of banks (i.e., a higher  $B$ ) has the following effects:

1. aggregate loans per bank  $\int l_b^* d\Phi$  decreases;
2. average loan interest rate  $\int R_l^* d\Phi$  decreases;
3. aggregate physical investment  $\int k_1^* - (1 - \delta)k_0 d\Phi$  increases;
4. aggregate share of expected returns  $\int \mathbb{E}[d_1^*] / p^* d\Phi$  decreases;
5. aggregate loans  $\int \sum_b^B l_b^* d\Phi$  increases;
6. aggregate leverage  $\int \sum_b^B l_b^* / k_1^* d\Phi$  increases;
7. aggregate TFP  $\int k_1^{*\alpha} d\Phi / (\int k_1^* d\Phi)^\alpha$  increases;
8. variance of capital  $\int k_1^{*2} d\Phi - (\int k_1^* d\Phi)^2$  decreases;
9. variance of loan interest rates  $\int R_l^{*2} d\Phi - (\int R_l^* d\Phi)^2$  decreases;
10. variance of expected returns  $\int (\mathbb{E}[d_1^*] / p^*)^2 d\Phi - (\int \mathbb{E}[d_1^*] / p^* d\Phi)^2$  decreases.

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<sup>a</sup>For subpoints 7, 8, 9 and 10, I assume that the mass of financially constrained firms  $1 - \mathcal{P}$  are all ex-ante identical.

## 3 Quantitative Model

In the two-period model, banks' choices are static. In the infinite-horizon model, each bank faces a dynamic problem that: (i) depends on the same bank's future strategies and other banks' current and future strategies, and (ii) is subject to all firms' dynamic demand for loans; also both the current and future distributions of firms matter. The equilibrium concept used

in this section is a Markov perfect equilibrium (e.g., [Maskin and Tirole, 2001](#)). Specifically, I characterize the equilibrium using generalized Euler equations in a similar fashion to the optimal fiscal policy literature (see, for instance, [Klein and Ríos-Rull, 2003](#); [Krusell, Martin, and Ríos-Rull, 2004](#); [Klein, Krusell, and Ríos-Rull, 2008](#); and [Clymo and Lanteri, 2020](#)).

In this section, I build a dynamic framework to study firms’ financing-investment decisions when banks are big, strategically interact with each other, and face idiosyncratic firms’ default risk.<sup>10</sup> Households derive utility from a non-durable consumption good, own the shares of the banks, and supply deposits. Banks issue debt and use both their internal resources and debt to purchase firms’ loans. Firms make investment decisions, taking into account the fact that debt provides a tax shield and issuing new equity is increasingly costly. The key feature of the framework is the simultaneous presence of strategic interactions among financial institutions, general equilibrium, macroeconomic shocks, and heterogeneous firms. Note that each firm stipulates an idiosyncratic contract with the banks: in equilibrium, banks have different degrees of market power on each single firm in function of its idiosyncratic characteristics.

I will now describe the model and proceed to define the *stationary oligopolistic equilibrium*, in which all aggregates quantities and prices are constant over time. I overcome the computational challenge by proposing algorithms to solve for the oligopolistic stationary equilibrium and the related transitional dynamics in the presence of strategic interactions, general equilibrium, and heterogeneous firms. The algorithms are discussed in detail in [Appendix B](#).

### 3.1 Environment

Time is discrete  $t = 0, 1, \dots$  and the horizon is infinite.

**Preferences.** There is an exogenous number  $B$  of identical banks. Each bank is owned by a continuum of identical and infinitely lived savers, equivalent to  $B$  representative savers. Each saver  $b$  ranks stream of consumption  $C_{b,t}$  according to the following lifetime utility

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<sup>10</sup>Specifically, I interpret the financial intermediation sector as a succession of decision makers – one at each date  $t$  – without commitment to future realized quantity of loans supplied.



function:

$$\sum_{t=0}^{\infty} \beta_b^t \cdot u(C_{b,t}), \quad (6)$$

where  $\beta_b \in (0, 1)$  is the saver's discount factor, and  $u_c > 0$ ,  $u_{cc} < 0$ .

There is a continuum of firms, each owned by a continuum of identical infinitely lived entrepreneurs, equivalent to one representative entrepreneur. The entrepreneur ranks stream of consumption  $C_{E,t}$  according to the following lifetime utility function:

$$\sum_{t=0}^{\infty} \beta_E^t \cdot u(C_{E,t}), \quad (7)$$

where  $\beta_E \in (0, 1)$  is the entrepreneur's discount factor. Note that I treat the two discount factors identically (i.e.,  $\beta_b = \beta_E \equiv \beta$ ) in the paper with the exception of Subsection 5.3.

**Ownership Structure.** Each representative saver owns a bank. In equilibrium, the saver is indifferent between financing the bank's loans with debt or equity. The representative entrepreneur owns the entire mass of firms  $j \in [0, 1]$ . Each firm  $j$  is characterized by its state vector

$$x(j) \equiv \{\{l_b(j)\}_b^B, r_l(j), k(j), \mathcal{I}(j)\},$$

where  $l_b(j)$  denotes the firm's loan by bank  $b$ ,  $r_l(j)$  is the interest rate (charged by all banks),  $k(j)$  is the firm's capital stock, and  $\mathcal{I}(j)$  is an indicator function that takes value 1 if the firm has not defaulted. Let  $\phi(x)$  denote the density function of firms in the economy.

Each saver cannot own any firm's equity and thus needs to save through banks.

**Markets.** There are six markets in the economy: banks' debt, banks' equity, firms' loans, firms' equity, interbank market, and the market for the representative good.

*Banks' equity and debt markets.* Each saver  $b$  invests in the production sector by supplying equity or debt to banks and faces the budget constraint:

$$C_{b,t} + p_{b,t} \cdot S_{b,t+1} + D_{b,t+1} = (p_{b,t} + \pi_{b,t}) \cdot S_{b,t} + R_{D,t} \cdot D_{b,t}$$

where  $p_{b,t}$ ,  $S_{b,t}$ ,  $S_{b,t+1}$ ,  $D_{b,t}$ ,  $D_{b,t+1}$ ,  $R_{D,t}$  and  $\pi_{b,t}$  are, respectively, the bank's share price, the share holdings at  $t$  and  $t + 1$ , the bank's debt holdings at  $t$  and  $t + 1$ , the interest rate on the bank's debt, and the bank's profit at  $t$ . Bank  $b$  demands equity and debt from saver  $b$ , in order to finance loans to firms.

*Firms' equity market.* The entrepreneur invests in the production sector by supplying equity to the firms, and faces budget constraints:

$$C_{E,t} + \int [\mathcal{I} \cdot p_t \cdot S_{t+1} + (1 - \mathcal{I}) \cdot p_t \cdot S_{t+1}] d\Phi = \int \mathcal{I} \cdot (p_t + \tilde{d}_t) \cdot S_t d\Phi$$

where  $p_t$ ,  $S_t$ ,  $S_{t+1}$ , and  $\tilde{d}_t$  are, respectively, the share price, share holdings at  $t$ , and the dividend of each firm (net of equity issuance cost) at  $t$ . Firms demand equity from, or distribute dividends to, the entrepreneur. If a firm decides to issue equity, it incurs a quadratic equity issuance cost  $\lambda(d_t)$  (see Section 2), where  $d_t$  is a firm dividend at time  $t$ , defined below.

*Firms' loan market.* A finite (and exogenous) number  $B$  of (identical) banks supply loans to a continuum of firms. Each bank  $b = 1 \dots B$  can issue non-state-contingent loans  $l_{b,t+1}$  to each firm. Loans are due for repayment in the next period, unless the firm defaults. A firm  $j$  takes the interest rate  $r_{l,t+1}(j)$  as given and chooses how much to invest and how much to borrow from each bank. Banks take each firm's demand schedule as given and compete à la Cournot, i.e, simultaneously and independently choose their loan portfolios. The process determines the total amount of loans banks supply to each firm which, together with the firm's demand schedule, pins down the firm-specific interest rate  $r_{l,t+1}(j)$ . At time  $t$ , each bank and firm commit to such an interest rate.

*Interbank market.* A bank  $b$  can lend  $M_{b,t}$  to other banks that will be repaid in the following period at rate  $r_{M,t+1}$ . Since all banks are identical, in equilibrium  $\forall b : M_{b,t} = 0$ .

*Goods market.* The representative entrepreneur and the  $B$  representative savers demand goods supplied by all firms.

**Technology.** In each period  $t$ , the output  $y_t(j)$  produced by each firm  $j \in [0, 1]$  is given by the production function  $y_t(j) = z_t(j) \cdot k_t(j)^\alpha$ , where  $0 < \alpha < 1$ .

**Shocks.** At time  $t$  the firm can default with exogenous probability  $1 - \rho$ . A new mass of firms re-enters the economy with exogenous characteristics  $x_0$  so that the total mass is

constant over time. I relax this assumption in Appendix A.1.

**Government.** The government imposes proportional taxes  $\tau$  on all firms' production. Firms can deduct loan interest and depreciated capital from their taxes. Government runs a balance budget constraint. That is, the government uses the aggregate revenue from taxes  $T_t = \tau \int \left( z_t k_t^\alpha - \sum_{b=1}^B r_{l,t} l_{b,t} - \delta k_t \right) d\Phi$  to finance an exogenous government expenditure that exactly balances  $T_t$  at each point in time.

**Timing.** The aggregate state space of the economy at time  $t$  is

$$X_t \equiv \{ \{D_{b,t}\}_b^B, r_{D,t}, \{M_{b,t}\}_{b=0}^B, r_{M,t}, B, \rho, \phi(x_t) \}.$$

Given  $X_t$ , the timing is as follows: (1) a mass  $1 - \rho$  of firms defaults, (2) each surviving firm produces output  $y_t = z_t k_t^\alpha$ ; (3) each bank finances its supply of loans,  $\int l_{b,t+1}(x_t) d\Phi(x_t)$ , by issuing equity and/or debt; (4) each firm takes the interest rate  $r_{l,t+1}(x_t)$  as given and chooses how much to invest and the amount of loan to demand from each bank; (5) banks take each firm's demand schedule as given and compete with each others to supply the loans. The outcome is a contract establishing: loan amount  $l_{b,t}(x_t)$ , interest rate  $r_{l,t+1}(x_t)$ , and new level of capital  $k_{t+1}(x_t)$ ; (6) firms distribute dividends  $d = (1 - \tau) [z k^\alpha - \sum_b r_l l_b] + \tau \delta k - \tilde{i}$  to the entrepreneur; (7) bank  $b$  distributes profit to saver  $b$ . To simplify notation, in the following subsections I avoid explicitly sub-scripting each variable with time  $t$  and  $t + 1$ , but it is understood that  $(x, X)$  refers to  $(x_t, X_t)$ , and  $(x', X')$  refers to  $(x_{t+1}, X_{t+1})$ .

### 3.2 Household: Saver

I now describe the saver's problem in recursive form. Let  $V_S(X)$  be the value function of the saver with debt holdings  $\mathbf{D} = [D_1 \dots D_B]$  and equity holdings  $\mathbf{S} = [S_1 \dots S_B]$  in each bank  $b$ . This function satisfies the following functional equation:

$$V_S(X) = \max_{\mathbf{S}', \mathbf{D}'} u(C_b) + \beta \cdot V_S(X') \quad (8)$$

subject to the budget constraint:

$$C_b + p_b \cdot S'_b + D'_b = (p_b + \pi_b) \cdot S_b + R_D \cdot D_b. \quad (9)$$

The left-hand side of budget equation (9) reports the saver's expenditures: household aggregate consumption, banks  $b$ 's equity and debt purchases. The right-hand side reports the saver's resources: bank  $b$ 's equity holdings and debt.

The saver takes the future banks' debt market rate  $r'_D$  as given, together with future banks' profits, and purchases banks' debt and equity according to:

$$\forall b : \quad 1 = M'_S \cdot \frac{p'_b + \pi'_b}{p_b} \quad (10)$$

$$\forall b : \quad 1 = M'_S \cdot R'_D, \quad (11)$$

where  $M'_S \equiv \beta \frac{u_c(C'_b)}{u_c(C_b)}$ , and  $\pi_b$  is the profit of bank  $b$  distributed as a dividend to the saver.

### 3.3 Firms

I now characterize firm  $j$ 's problem in recursive form. For convenience, I omit the index notation  $j$ . Let  $V_F(x, X)$  be the value function of the firm  $j$  with loan holdings  $\mathbf{l} = [l_1 \dots l_B]$  from each bank  $b$  and capital  $k$ . This function satisfies the following functional equation:

$$V_F(x, X) = \max_{\{l'_b\}_{b=1}^B, k'} d - \lambda(d) + \mathbb{E}[\mathcal{I}' \cdot M'_E \cdot V_F(x', X') \mid (x, X)],$$

subject to

$$\begin{aligned} k' &= k(1 - \delta) + i \\ i &= \tilde{i} + \sum_b (l'_b - l_b) \\ d &= (1 - \tau) \left[ zk^\alpha - \sum_b r_l l_b \right] + \tau \delta k - \tilde{i}, \end{aligned}$$

where  $M'_E$  is the discount factor of the entrepreneur, as described in the following subsection. Each firm takes the future loans' market rate  $r'_l$  as given and finances itself through internal financing (production and equity issuance) and external financing (loans from banks). The

first-order condition with respect to  $k'$  is

$$1 - \lambda_d(d) = \mathbb{E} \left[ \mathcal{I}' \cdot M'_E \cdot \left( 1 + (1 - \tau) \left( z' \alpha k'^{\alpha-1} - \delta \right) \right) \cdot (1 - \lambda_d(d')) \mid (x, X) \right]. \quad (12)$$

The first-order condition with respect to  $l'_b$  is

$$1 - \lambda_d(d) = \mathbb{E} [ \mathcal{I}' \cdot M'_E \cdot (1 + (1 - \tau)r'_l) \cdot (1 - \lambda_d(d')) \mid (x, X) ]. \quad (13)$$

### 3.4 Household: Entrepreneur

I now describe the entrepreneur's problem in recursive form. Let  $V_E(X)$  be the value function of the representative entrepreneur with shares holding  $S(\mathbf{S})$ . This function satisfies the following functional equation:

$$V_E(X) = \max_{S(\cdot)'} u(C_E) + \beta \cdot V_E(X') \quad (14)$$

subject to the budget constraint:

$$C_E + \int \mathcal{I} \cdot p \cdot S' + (1 - \mathcal{I}) \cdot p(x_0) \cdot S'(x_0) d\Phi = \int \mathcal{I} \cdot (p + \tilde{d}) \cdot S d\Phi, \quad (15)$$

where  $\tilde{d}$  is the firm-specific dividend  $d$  net of the equity issuance cost  $\lambda(d)$ . Hence, each firm's share value is priced according to:

$$1 = \mathbb{E} \left[ \mathcal{I}' \cdot M'_E \cdot \frac{p' + \tilde{d}'}{p} \mid (x, X) \right], \quad (16)$$

where  $M'_E \equiv \beta \frac{u'(C'_E)}{u(C_E)}$ .

### 3.5 Banks

A bank  $b$  chooses the new level of debt to demand from the saver and the new level of loans to offer to each firm. Formally, the strategy space is defined as:

$$\mathcal{S}'_b(x, X) \equiv \{D'_b(X), l'_b(x, X)\}.$$

The new amount of debt issued ( $\Delta D'_b = D'_b - D_b$ ) and internal financing  $F$  is chosen to provide enough coverage for the change in interbank lending and aggregate loans:

$$F + \Delta D'_b = \Delta M'_b + \rho \int \Delta l'_b(x, X) d\Phi. \quad (17)$$

I now describe the bank's problem in recursive form. Let  $V_b(X)$  be the value function of a bank  $b$ . This function satisfies the following functional equation:

$$V_b(X) = \max_{\{D'_b, r'_D\}, M'_b, \{l'_b(x, X), r'_l(x, X)\}} \pi_b + M'_S(X, X') \cdot V_b(X') \quad (18)$$

subject to: (i) equation (17), (ii) the household's interest rate-quantity schedule jointly defined by equation (10) and (11), (iii) each firm's interest rate-quantity schedule jointly defined by equations (12), (13). Bank  $b$ 's profit  $\pi_b$  is given by:

$$\pi_b = \rho \int r_l \cdot l_b d\Phi + r_M M_b - r_D D_b - F. \quad (19)$$

Future market rates  $r'_D(X)$  and  $r'_l(x, X)$  adjust consistently with the interest rate-quantity schedules. Each bank  $b$  issues bank debt according to a generalized Euler equation:

$$1 = M'_S(X, X') \cdot R'_D(X, X') \cdot (1 + \eta'_D(X, X')), \quad (20)$$

where  $\eta'_D$  is the inverse elasticity  $\frac{\partial R'_D}{\partial D'_b} \cdot \frac{D'_b}{R'_D}$  between debt and its rate. In principle, equation (20) is a best response function that captures the trade-off that a bank faces issuing new debt. Every new unit of debt increases today financing capacity but needs to be repaid tomorrow at the contracted interest rate. Moreover, since  $\eta'_D$  is non-negative, when a bank issues new debt it is also increasing the market rate of deposits, incurring an additional future marginal cost. In equilibrium,  $\eta'_D$  is zero, as implied by equations (20) and (11). Without aggregate risk, savers and banks are completely indifferent to the financing structure of the banks.

A similar generalized Euler equation arises from the loans' first-order condition:

$$1 = \mathbb{E} [\mathcal{I}' \cdot M'_S(X, X') \cdot R'_l(x, X, x', X') \cdot (1 + \eta'_l(x, X, x', X')) \mid (x, X)], \quad (21)$$

where  $\eta'_l(x, X, x', X') \equiv \frac{\partial R'_l}{\partial l'_b} \cdot \frac{l'_b(x, X)}{R'_l(x, X, x', X')} < 0$  is the firm-specific inverse elasticity between

loans and their rates. Note that equation (21) is a functional equation that depends on the idiosyncratic characteristics of each firm. See Appendix C.1 for details on how to calculate  $\eta'_i(x, X, x', X')$ . Equation (21) is a best response function that captures the trade-off that a bank faces issuing a new unit of loan to a specific firm. Every new unit of loan decreases the current bank's dividend but produces a marginal income tomorrow at the contracted interest rate. Moreover, since  $\eta'_i$  is non-positive, when banks issue new loans they are also decreasing the future market rate of loans, incurring a marginal loss in the future. Note that banks best respond internalizing the effects that their actions have on aggregate quantities and all firms' choices (e.g., if a bank changes the quantity of loan offered to a firm, that firm might decide to re-optimize and adopt a different capital structure as a function of the credit market conditions. Banks internalize all these effects in their decisions). Equation (21), together with an Euler equation that regulates the banks' behavior on the interbank market

$$1 = M'_S(X, X') \cdot R'_M(X, X'), \quad (22)$$

captures the decision making behavior of each bank. The outcome of the game played by the banks at time  $t$  is a contract that pins down the firm-specific intermediation margin  $R'_i(x, X, x', X') - R'_D(X, X')$ . In principle, this margin can be decomposed into: (i) firm-specific loan's intermediation margin ( $R'_i(x, X, x', X') - R'_M(X, X')$ ) and (ii) debt's intermediation margin ( $R'_M(X, X') - R'_D(X, X')$ ). Note that the debt intermediation margin is zero, since  $\eta'_D = 0$ , hence  $R'_D(X, X') = R'_M(X, X')$ . The following paragraph explains the decomposition of the loan's intermediation margin in greater detail.

**Loan's Intermediation Margin** The spread between the firm-specific loan's rate and the interbank market rate can be obtained by combining equations (21) and (22):

$$R'_M(X, X') = \mathbb{E}[T' \cdot R'_i(x, X, x', X') \cdot (1 + \eta'_i(x, X, x', X')) \mid (x, X)]. \quad (23)$$

The generalized Euler equation (23) can be manipulated

$$\begin{aligned}
R'_l(x, X, x', X') - R'_M(X, X') &= - \underbrace{\frac{\eta'_l(x, X, x', X')}{1 + \eta'_l(x, X, x', X')}}_{\text{(1) Rents}} \cdot \underbrace{\frac{1}{M'_S(X, X')}}_{\text{MC}} \\
&+ \underbrace{\frac{1 - \rho}{\rho} \cdot \frac{1}{1 + \eta'_l(x, X, x', X')}}_{\text{(2) Risk Premia}} \cdot \frac{1}{M'_S(X, X')},
\end{aligned}$$

to reveal a non-linearly separable decomposition of the loan's intermediation margin in (1) markup over marginal cost and (2) risk premia.<sup>11</sup>

Since  $\eta'_l$  is non-positive, and the reciprocal of  $M'_S$  is the interbank market rate, this formula is an inter-temporal markup rule (markup over marginal cost, the marginal cost being  $R'_M(X)$  or  $R'_D(X)$ ). The more banks are introduced in the economy, the more the financial sector becomes competitive and the inverse elasticity  $\eta'_l$  tends to decrease in module; hence, the expected loan's rate tends to the bank funding rate.

### 3.6 Oligopolistic Equilibrium

The government aggregate income from taxes is:

$$T = \tau \int \left( zk^\alpha - \sum_{b=1}^B r_l(x, X) \cdot l_b(x, X) - \delta k \right) d\Phi.$$

The aggregate resource constraint of the economy is:

$$\sum_{b=1}^B C_b + C_E + \int i(x, X) + \lambda(x, X) d\Phi + T = \int zk^\alpha d\Phi. \quad (24)$$

The total production of the economy on the right-hand side of equation (24) can: (i) be consumed by the savers, (ii) be consumed by the entrepreneur, (iii) be used for aggregate investment in physical capital (in case some dividends are negative, some resources are spent to pay the equity issuance cost  $\lambda$ ), (iv) or be paid in taxes. The inverse elasticity  $\eta'_D$  is zero

<sup>11</sup>It is non-linearly separable in the sense that risk premia contains  $\eta'_l(x, X, x', X')$  which, in turn, contains the default probability.



and  $\eta'_i(x, X, x', X')$  of equation (21), is calculated as described in Appendix C.1.

A formal definition of the notion of *Recursive Stationary Oligopolistic Equilibrium* is presented in Definition 3.1, and its extension to the dynamic case is discussed in Section 5.

**Definition 3.1.** A **Recursive Stationary Oligopolistic Equilibrium** is a Markov perfect equilibrium where i) the banks' debt holdings  $\{D_b\}_{b=1}^B$  and the relative market rate  $R_D$ ; ii) the banks' share holdings  $\{S_b\}_{b=1}^B$  and the relative market prices  $\{p_b\}_{b=1}^B$ ; iii) the interbank debt holdings  $\{M_b\}_{b=1}^B$  and the relative market rate  $R_M$ ; iv) the saver's consumption  $C_b$  and the entrepreneur's consumption  $C_E$ ; v) the distribution  $\phi(x)$ ; vi) the policy functions:  $k'(x)$ ,  $l'(x)$  and  $R'_i(x)$ ; are such that i) the saver's problem is solved—i.e, equations (10) and (11); ii) the entrepreneur's problem is solved—i.e, equation (16) holds; iii) each firm's problem is solved—i.e, equations (12) and (13); iv) each bank is best responding to all other banks—i.e, equations (20), (21) and (22) hold; v) and all markets clear: 1) the good market clears—i.e., equation (24) holds; 2) each bank's equity market clears—i.e.,  $\forall b : S_b = 1$ ; 3) each firm's equity market clears—i.e.,  $S(x) = 1$ ; 4) the interbank market clears—i.e.,  $\forall b : M_b = 0$ .

## 4 Calibration

In this section, I calibrate the *Stationary Oligopolistic Equilibrium* of the model to match the credit spreads of the Commercial & Industrial (C&I) Loans.<sup>12</sup> C&I credit spreads are calculated as the difference between the weighted-average C&I effective loan rate and the 3-Month T-bill rates.<sup>13</sup> I use the C&I Loans charge-off rates as a proxy for default risk, which can be retrieved from the *loan performance* archive of the FDIC Quarterly Banking Profile.<sup>14</sup> I choose this asset class for two reasons: (i) it is an important component of US banks' balance sheets (as of February 2022, the amount outstanding is USD 2.48 trillion) and (ii) it is composed of short-term loans, making maturity premium a minor concern.

### 4.1 Data

As shown in Table IV in Appendix A.4, data reveals a positive correlation (significant at the 1% level) between C&I credit spreads and banks' asset market concentration, even after

<sup>12</sup>C&I Loans: <https://fred.stlouisfed.org/series/BUSLOANS>.

<sup>13</sup>Weighted-average C&I effective loan rate: <https://fred.stlouisfed.org/series/EEANQ>.

<sup>14</sup>FDIC QBP: <https://www.fdic.gov/analysis/quarterly-banking-profile/index.html>.

controlling for default risk (identified with the charge-off rate on C&I Loans), quantity of loans outstanding, and average maturity. Moreover, the correlation between C&I credit spreads and the quantity of C&I Loans outstanding is also significant at the 1% level and negative, consistent with the sign of the model’s inverse elasticity  $\eta_L$  between loan and loan’s rate contained in equation (23). In Table IV, banks’ asset market concentration is calculated as the market share of the five largest US banks, which can be obtained by aggregating the assets of each single bank from the FDIC aggregate balance sheets.

Figure 1 reports the quarterly net charge-off to loans, used as a proxy for default rate.<sup>15</sup> Figure 1 also reports the aggregate time series for C&I credit spreads.<sup>16</sup> As a proxy for the interbank market rate ( $r_M$ ), I use the 3-Month T-bill rates. Note that there is not a quantitatively significant difference when the Fed funds rates are used instead of the 3-Month T-bill rates. Hence, credit spreads reported in Figure 1 are calculated as interest rates on C&I Loans ( $r_L$ ) minus 3-Month T-bill rates ( $r_M$ ). Consistently with the model, interest expenses and holdings are converted in real terms.

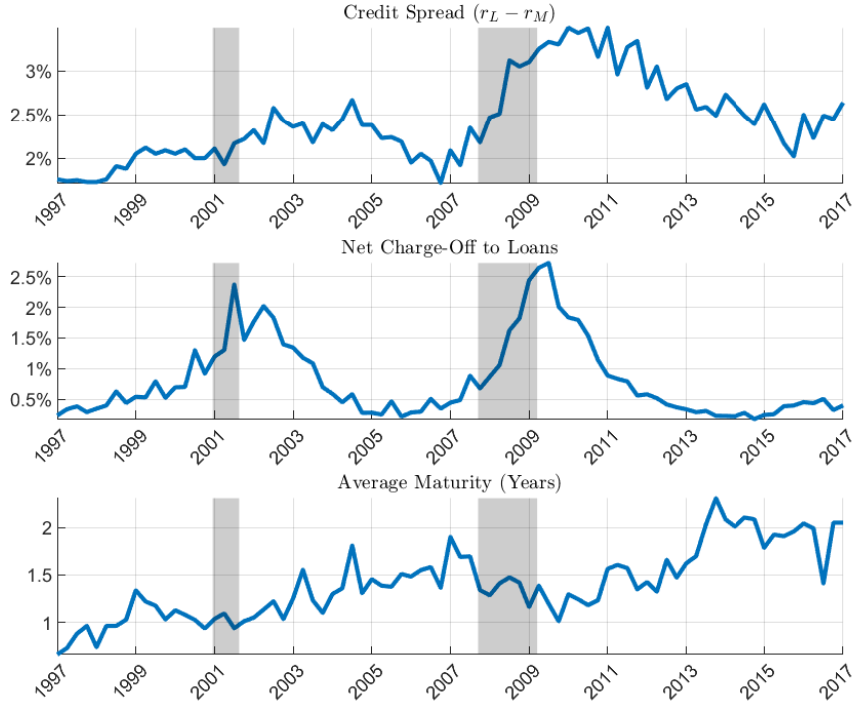
The counterparts in the model of the aggregate credit spreads reported in Figure 1 are equilibrium firm-specific (i) risk premia and (ii) markup, according to the generalized Euler equation (23). As previously shown, these two components are not linearly separable. In order to separate the aggregate markup from risk premia, I solve the *Recursive Stationary Oligopolistic Equilibrium* with an increasing number of banks.

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<sup>15</sup>Charge-off rates are broadly consistent with default rates reported by Moody’s and Standard & Poor’s, but have the advantages to refer specifically to C&I Loans.

<sup>16</sup>Alternatively to using the weighted-average C&I effective loan rate, it is possible to compute credit spreads from the interest rates obtained by combining the FDIC aggregate balance sheets and income statements. Hence, obtain interest rates on loans as the ratio between interest income and loans. This procedure yields a closely similar time series.

Figure 1. C&I LOANS DATA



*Notes:* The figure shows the evolution over time of the (annualized) credit spreads (top panel), the (annualized) net charge-off to loans (central panel) and the average maturity (bottom panel) of the C&I Loans. The average annualized credit spread between 1997 and 2017 is  $\sim 2.2\%$ . The average annualized net charge-off to loan between 1997 and 2017 is  $\sim 0.84\%$ . The average maturity between 1997 and 2017 is  $\sim 1.4$  years.

I choose the number of banks in the economy to match an annualized credit spread of 2.16%, obtained as the average of the time-series illustrated in Figure 1. Hence, the aggregate markup can be identified as the difference between (i) credit spreads in the calibrated economy and (ii) credit spreads under perfect competition. This procedure is illustrated in detail in the following subsection and yields an annualized aggregate markup of 0.24%.

## 4.2 Calibration of the Oligopolistic Equilibrium

I now describe the choices of parameters values for preferences for the production sector and the number of banks. Table II in Appendix A.2 summarizes all parameter values.

**Household Preferences.** A period in the model coincides with a quarter, consistent with the frequency of interest expenses in the data. I set  $\beta = 0.9936$  to match a quarterly

stationary bank’s debt rate (or interbank debt rate) of 0.64%, which is consistent with the average of the 3-month T-bill rates calculated in the time window considered in Figure 1, and reflects the prolonged period of low interest rates. I use a constant relative risk aversion utility function

$$u(C) = \begin{cases} \frac{C^{1-\gamma}}{1-\gamma} & \gamma \geq 0 \wedge \gamma \neq 0 \\ \log(C) & \gamma = 1 \end{cases},$$

where the parameter  $\gamma$  is the degree of relative risk aversion. I use a unitary  $\gamma$  in the baseline calibration. I benchmark the baseline calibration against the same equilibrium calculated with  $\gamma = 5$  in Table III in Appendix A.2.

**Firms.** I set a depreciation rate  $\delta$  equal to 2.5%, consistent with an annualized depreciation rate of 10%. The effective capital share is set to 0.41, obtained as the average of the time series from 1990 to 2018 reported by the Bureau of Labor Statistics, which reflects the secular decline in the labor share. Firms’ income tax is set to 24%, obtained as the average from 1990 to 2018 of the ratio between taxes on corporate income and corporate profit (both time series obtained from FRED). I use the C&I Loans charge-off rates to identify the risk of default, set to 0.21% (quarterly) consistent with the average of the time series reported in Figure 1. The equity issuance cost  $\lambda_0$  is set to 1.5 to match an annualized frequency of equity issuance of 0.04. The frequency of equity issuance is computed from a sample of non-financial, unregulated firms from Compustat. Note that, more generally,  $\lambda_0$  can be seen as a non-bank financing cost. For robustness (and to capture the idea that firms could have other outside options and not just equity issuance), I benchmark the baseline calibration against the same equilibrium calculated with  $\lambda_0 = 0.75$  in Table III. I also fix a maximum age  $\bar{N}$  in the life cycle of the firm. In particular, I consider an average lifespan of a company of 19.5 years, as calculated using the Standard and Poor’s 500 Index.<sup>17</sup> Note that 19.5 years corresponds to 78 quarters; hence,  $\bar{N}$  is such that:

$$\sum_{\text{age}=0}^{\bar{N}} \text{age} \cdot \rho^{\text{age}} = 78 \cdot \sum_{\text{age}=0}^{\bar{N}} \rho^{\text{age}}.$$

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<sup>17</sup>This is consistent with an average of 19.6 years between 1997 and 2017 computed from the Standard and Poor’s 500 Index available here: <https://www.statista.com/statistics/1259275/average-company-lifespan/>.

This yields a maximum age of  $\bar{N} = 167$  quarters. In other words, the oldest firms in the model have an age of about 42 years. In each period, I also assume that a new mass of firms replaces the mass of firms that defaulted  $1 - \rho$ , starting from age zero with zero capital and zero debt. I relax this assumption in Appendix A.1.

**Banks.** The number of banks is calibrated to match the average aggregate credit spread (calculated as the average of the data reported in Figure 1). For the sake of transparency, Figure 12 in Appendix A.3 illustrates the process as a function of the number of banks. A model with two banks predicts a credit spread of 2.2%, close to the value in the data of 2.16% (calculated as the average of the time-series reported in Figure 1). The figure also shows that, as the number of banks increases, the model converges to the perfectly competitive case with a credit spread of 1.96%. Hence, the model predicts an annualized aggregate markup of 0.24%. In Table I, I report the corresponding aggregate stationary equilibrium moments. As explained, the aggregate credit spread and the frequency of equity issuance are targeted by controlling the number of banks and the equity flotation cost, respectively. All remaining moments are untargeted and reported for validation. The model performs fairly well on untargeted moments such as capital-to-GDP ratio and investment-to-capital ratio.<sup>18</sup> Notably, the financial frictions that arise from the strategic interactions among intermediaries render the debt adjustment cost to capital ratio significantly smaller (0.2%) than in the perfectly competitive case (0.9%) and toward the value observed in the data (0.1%). The more banks are introduced in the economy, the cheaper the interest rate on debt. For this reason, a more competitive oligopoly induces firms to substitute internal financing with external resources. As a result, leverage increases with the number of banks in the economy and tends to stabilize in a perfectly competitive financial market. Note that, even under perfect competition, firms still make well-defined optimal capital structure decisions because of the presence of the tax shield. Notably, the calibrated economy yields a leverage of 30%, close to the average in the data, which is 34% between 1997 and 2018. Leverage in the data is computed as the non-financial corporate business debt as a percentage of the market value of corporate equities, retrievable from FRED.

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<sup>18</sup>Capital and investment in the data are computed from nonresidential current-cost net stock of fixed assets, retrievable from the U.S. Bureau of Economic Analysis.

Table I. STATIONARY EQUILIBRIUM AND QUARTERLY MOMENTS

Description	Moment	Model			Data
		<i>Monopoly</i>	<i>Duopoly</i>	<i>P.C.</i>	
Credit Spread	$\int r_l(x, X)d\Phi - r_M$	0.70%	0.55%	0.49%	0.54%
Freq. of Equity Issuance	$\int (d(x, X) < 0)d\Phi$	3%	1%	0%	1%
Capital to GDP Ratio	$K/Y$	10.9	11.1	11.3	11.8
Investment to K Ratio	$I/K$	3.24%	3.23%	3.21%	3.5%
Debt Adjust. to K Ratio	$B \int \Delta l'_b(x, X) d\Phi / K$	0.1%	0.2%	0.9%	0.1%
Market Leverage	$B \int l_b(x, X) / V_F(x, X) d\Phi$	17%	30%	132%	34%

*Notes:* This table reports the targeted and untargeted aggregated quarterly moments in function of the number of banks. The aggregate credit spread and the frequency of equity issuance are targeted by controlling the number of banks and the equity flotation cost, respectively. Other moments are untargeted. Column *monopoly* refers to an economy with 1 bank. Column *duopoly* refers to an economy with 2 banks. Column *P.C.* refers to an economy with 10 banks, as a proxy for perfect competition.

For illustration purposes and robustness, in Table III, I report the moments in the calibrated stationary equilibrium benchmarked against two cases: (i) an equity flotation cost reduced from  $\lambda_0 = 1.5$  (calibrated) to  $\lambda_0 = 0.75$  (left column) and (ii) a risk-aversion increased from  $\gamma = 1$  (calibrated) to  $\gamma = 5$  (right column). Note that the risk-aversion  $\gamma$  plays a role in the stationary equilibrium since it enters all firms elasticities  $\eta'_i(x, X, x', X')$ .

First, the calibrated credit spreads are stable. Moreover, when the credit spreads are subtracted from the credit spreads calculated under perfect competition in the three corresponding cases, I get markups of about 24bps, 24bps, and 25bps, respectively. A higher  $\gamma$  increases the consumption smoothing desire of the entrepreneur, with an effect on markups similar to the equity issuance cost  $\lambda_0$ . The difference being that  $\gamma$  acts at the level of the aggregate dividend and the equity issuance cost acts at a firm-level. A higher  $\gamma$  increases banks' markups by 1bp. Hence, markups are stable as well.

Second, a lower  $\lambda_0$  intuitively increases the aggregate frequency of equity issuance as the firms' outside option of issuing equity becomes more appealing. Although in aggregate terms the stationary credit spread remains stable, as an additional check, I also run the macroeconomic shocks presented in Section 5 with  $\lambda_0 = 0.75$ . The results are qualitatively and quantitatively similar to those presented in Section 5 with the calibrated model.

### 4.3 Credit Misallocation in the Stationary Equilibrium

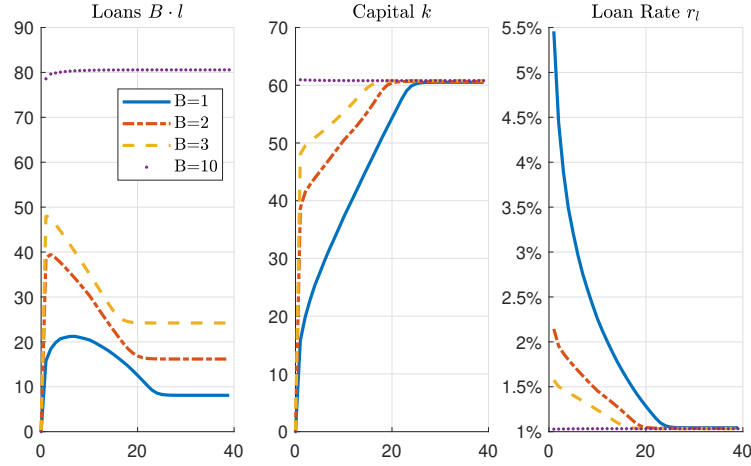
I now describe the key properties of the stationary equilibrium of the calibrated model, with a greater focus on the role of strategic interactions. Figure 2 reports the life cycle of a firm in the stationary equilibrium. Firms can reach their capital objectives by (i) investing internal resources, (ii) issuing equity or (iii) demanding external financing resources on the loan market. A more concentrated banking sector reduces the credit availability in the economy. Firms with a high marginal productivity of capital and fewer outside options, likely smaller or highly leveraged, exhibit higher credit demand. Therefore, this type of firm is more exposed to the negative effects of the lack of competition in the banking sector. This is the same intuition captured by equation (5) in the stylized model of Section 2.

Along the life cycle of firms, since markups are endogenous in the cross-section of firms, banks endogenously exert a higher degree of market power on firms with a high marginal productivity of capital and lower internal resources; hence, fewer outside options. These firms need banks' credits and would otherwise incur an equity issuance cost to finance their growth, in case their current production alone would not be sufficient to sustain the desired level of physical investment. An imperfectly competitive financial sector internalizes that the same financial resources are more valuable for firms with a higher marginal productivity of capital and fewer outside options (e.g., a higher equity issuance cost) and, therefore, can charge higher markups. This creates a mechanism of endogenous financial friction, as captured by the central panel of Figure 2. Through this mechanism, the lack of competition in the financial sector not only induces credit misallocation that forces firms to grow slower, but also induces lower aggregate productivity. This outcome is illustrated in Figure 3. Notably, [Burga and Céspedes \(2022\)](#) use a dataset of small firms from Peru to estimate the effect of banks' market power. In particular, they exploit a merger episode and find that the change in banks' market structure results in “a contraction of capital among small firms with high marginal returns, which increases capital misallocation,” in agreement with the predictions of (i) the central panel of Figure 2 and (ii) the dispersion of marginal products of capital,  $\sigma(r_L)$ , reported on the left-axis of Figure 3.<sup>19</sup>

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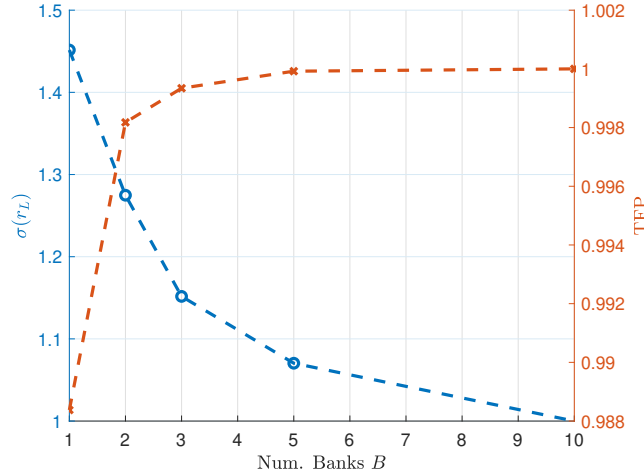
<sup>19</sup>Relatedly, also [Cavalcanti, Kaboski, Martins, and Santos \(2021\)](#) study the effects of intermediation costs and market power on the dispersion of credit spreads using Brazilian data.

Figure 2. STATIONARY EQUILIBRIUM AND FIRMS' LIFE CYCLE



Notes: This figure reports the equilibrium policies for loan quantity (left panel) and loan interest rate (right panel), along the life cycle of a firm in the stationary equilibrium. Younger firms need financing resources to reach their capital objectives, and since markups are endogenous in the cross-section, banks naturally extract higher markups from this type of firms, characterized by a higher marginal productivity of capital and low internal resources. This creates a mechanism of *endogenous financial friction*, as captured by the central panel. X-axes report the firms' age.

Figure 3. STATIONARY EQUILIBRIUM AND CREDIT MISALLOCATION



Notes: This figure reports the standard deviation of interest rates and TFP, both calculated as a ratio to those obtained in an economy with 10 banks. The figure illustrates the mechanism: lack of competition in the financial sector induces misallocation of credits, which is linked to misallocation of capital and a reduced productivity.

Figure 13 in Appendix A.3 reports the inverse elasticities  $\eta'_i(x, X, x', X')$  contained in the generalized Euler equation (21), along the life cycle of a firm in the stationary equilibrium.



These elasticities, which are endogenous and depend on current and future firm-level characteristics, are directly linked to the equilibrium trajectories of markups. A lower inverse elasticity translates into a higher financial markup. Furthermore, Figure 13 in Appendix A.3 shows that the higher the concentration of the banking sector, the lower the inverse elasticities and the longer it takes for a firm to reach its efficient level of capital. Under perfect competition, the inverse elasticity would be constant in the cross-section of firms (long-lived firms still experience a non-zero elasticity because of the tax shield).<sup>20</sup> A more concentrated banking sector can extract higher rents out of financially constrained firms with a high marginal productivity of capital. This mechanism endogenously creates slower growth trajectories as a function of the banks' market structure, as shown in the central panel of Figure 2. The intensity of this mechanism is time-varying, acting as an amplification channel in the transition paths of the macroeconomic shocks reported in Section 5.

To conclude this subsection, it is important to highlight that an extensive empirical literature has investigated the impact of bank competition on the cross-section of firms. Early seminal contributions include [Rajan and Zingales \(1998\)](#); [Black and Strahan \(2002\)](#); [Cetorelli and Gambera \(2001\)](#); [Cetorelli \(2004\)](#); and [Cetorelli and Strahan \(2006\)](#). Although evidence is mixed, the general consensus is that, consistent with my model, banks' market power reduces the total amount of credit available in the economy, but importantly this effect is not constant across firms. This same literature has suggested that two countervailing forces might be in play and would explain the mixed results.

On the one hand, younger firms exhibit higher credit demand and, therefore, are more exposed to the negative effects of lack of competition in the banking sector than established firms. This is the classic industrial organization perspective (see, [Freixas and Rochet 1997](#)) and is fully captured by my model.

On the other hand, a more concentrated banking sector has an incentive to sustain one of its established clients and refrain from extending credit to young firms. The less competitive the conditions in the credit market, the lower the incentive for lenders to finance newcomers as documented by [Petersen and Rajan \(1995\)](#) and [Cetorelli and Strahan \(2006\)](#). Although my model does not focus on capturing complex features of relationship banking, the desire

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<sup>20</sup>For the sake of comparison, other papers (e.g., [Jamilov and Monacelli 2021](#) and [Wang, Whited, Wu, and Xiao 2022](#)) capture banks' market power via constant elasticity of substitution. Differently, my approach allows for endogenous firm-specific markups, which is key to the transmission of macroeconomic shocks.

for inter-temporal smoothing of banks’ profits and households’ consumption – embedded in the dynamic contract of equations (12), (13), and (21) – captures the fact that creditors, when contracting markups, take into account the expected stream of future dividends of the firms, as well as their own future profits. This important economic force is consistent with Petersen and Rajan (1995) and Cetorelli and Strahan (2006). In my calibrated model, as is clear from Figure 2, the first force dominates.

## 5 Macroeconomic Dynamics

This section analyzes the role that banks’ market power plays in the transmission of macroeconomic shocks. The section includes three macroeconomic shocks: (i) a credit quality deterioration (in the model, a sudden increase to the aggregate firms’ default probability), (ii) a sudden change to the market structure of the banking sector (in the model, a sudden decrease of the number of banks in the economy), and (iii) a cut to the bank funding rate. I compute the transitional dynamics of the model initialized at the stationary equilibrium as defined in Section 4. Then, I hit the economy with unexpected aggregate shocks and, depending on the type of shock, the economy converges to the old or a new stationary equilibrium in the long run. Several papers assume agents did not foresee the aggregate shocks of the Great Recession (e.g., Guerrieri and Lorenzoni, 2017). Along the transitional dynamics, after the shock, I assume all agents can perfectly foresee the paths of all aggregate variables. In order to compute the equilibrium dynamics, I find sequences of: (i) aggregate savers’ consumption  $\{C_{b,t}\}_{t=0}^T$ , (ii) aggregate entrepreneurs’ consumption  $\{C_{E,t}\}_{t=0}^T$ , and (iii) firms’ distributions  $\{\phi_t(x_t)\}_{t=0}^T$ ; such that both households maximize utilities, all markets clear in each period and the firms’ distributions evolve according to: (i) the firms’ policy functions, (ii) the banks’ generalized Euler equations (21) and (iii) the idiosyncratic default shocks. See Appendix B for additional computational details. The main computational challenge is to solve for an equilibrium path simultaneously characterized by general equilibrium, banks’ strategic interactions, and heterogeneous firms: (i) firms’ decisions are affected by other firms’ decisions through the aggregate variables, (ii) banks’ decisions are affected by each single firm’s decisions (banks issue idiosyncratic firm-level optimal contract), aggregate variables (banks internalize the effects that their actions have on aggregate dividends), and other banks’ decisions (each bank is best responding to other banks).

## 5.1 Credit Quality Deterioration

This section investigates the effects of banks' market power when the aggregate firms' default probability is hit by an aggregate shock that pushes it to suddenly increase as shown in Figure 14 in Appendix A.3.<sup>21</sup>

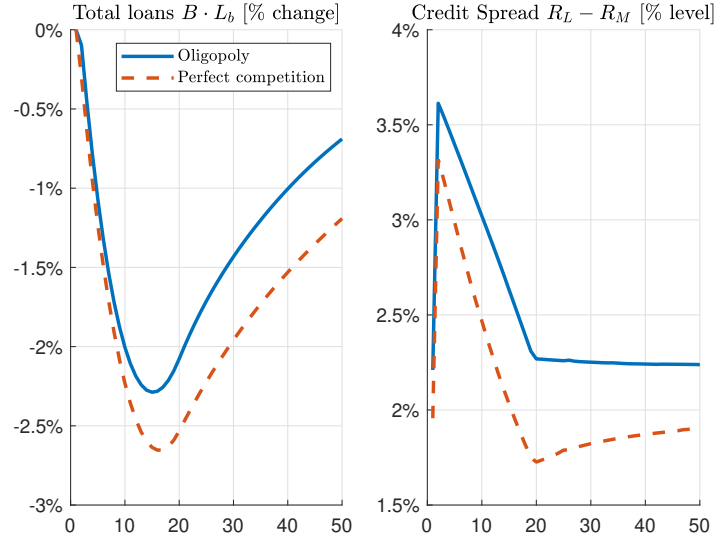
Figures 4 and 5 report the dynamic responses calculated (i) with the calibrated oligopolistic banking sector of Section 4 (solid line) and (ii) with the corresponding perfectly competitive banking sector (dashed line). As discussed in Section 4.3, a more concentrated banking sector can extract higher rents out of the financially constrained firms with a high marginal productivity of capital. In the stationary equilibrium, this mechanism endogenously creates slower growth trajectories as a function of the banks' market structure. In the dynamics, this mechanism of endogenous financial frictions interacts with the higher default probability, generating a higher credit demand that drives up markups as shown in Figure 15 in Appendix A.3. Therefore, during the shock, credit spreads rise more under oligopolistic competition than under perfect competition, as shown in the right panel of Figure 4.

Moreover, as shown by the left panel of Figure 4, total loans decline less under oligopolistic competition. Banks exploit their market power not only to extract higher markups, but also to boost their profits. Through this mechanism, banks effectively exploit their market power to face the increased losses due to the higher default rate, generating countercyclical financial markups and bank profits. In particular, during the Great Recession, the annualized aggregate C&I credit spread (see data in Figure 1) spiked up by a magnitude comparable to the one reported in the right panel of Figure 4. In summary, when credit quality deteriorates, a concentrated banking sector exploits its market power to extract higher markups as shown in Figure 15. This mechanism induces a larger and more persistent decline in real activity in terms of aggregate investment and output, as shown in Figure 5.

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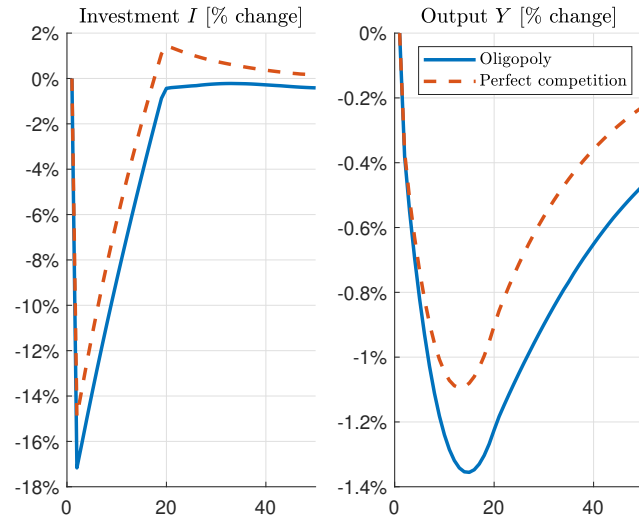
<sup>21</sup>The shock is calibrated on a magnitude similar to that of the Great Recession.

Figure 4. CREDIT QUALITY SHOCK AND FINANCING



*Notes:* This figure reports the transitional dynamics of aggregate loans  $B \int l_{b,t}(x_t, X_t) d\Phi_t$  (in % change from the initial stationary equilibrium aggregate value) and the annualized aggregate credit spread  $\int r_{l,t}(x_t, X_t) d\Phi_t - R_{M,t}(X_t)$  (in % level) following an unexpected credit quality shock (as reported in Figure 14 in Appendix A.3). X-axes report time  $t$ .

Figure 5. CREDIT QUALITY SHOCK AND REAL ACTIVITY



*Notes:* This figure reports the transitional dynamics (in % change from the initial stationary equilibrium aggregate values) of aggregate physical investment  $\int i_t(x_t, X_t) d\Phi_t$  and aggregate output  $\int y_t(x_t, X_t) d\Phi_t$  following an unexpected credit quality shock (as reported in Figure 14 in Appendix A.3). X-axes report time  $t$ .

Finally, note that the decline in real activity captured by Figure 5 further constrains

firms by restraining households’ capacity to support firms with equity issuance. The lack of outside options creates a vicious cycle that further reduces the interest rate-quantity loan elasticities  $\eta'_l(x, X, x', X')$  and boosts banks’ markups. Throughout the paper, I assume that the defaulting mass of firms is replaced, at each date  $t$ , by an equal mass of new entrant firms. In Appendix A.1, I relax this assumption. As shown by Figure 11 in Appendix A.3, when the firms’ default rate increases but not all firms immediately re-enter the market (in the spirit of the Great Recession), then imperfect competition in the financial intermediation sector leads to a lower amplification effect at the peak but is much more persistent. See Appendix A.1, for more details.

## 5.2 Changes in Bank Market Structure

The salient ingredient of my framework is the presence of non-atomistic financial intermediaries. Thanks to this feature, I can study market structure changes, such as banking industry consolidation and a bank failure. I view the number of banks in the economy as exogenous, in the sense that this industry has historically been heavily regulated and my primary purpose is to study its effect on the real economy. Among other examples, the reader can think of the acquisition of Wachovia by Wells Fargo in 2008 (following a government-forced sale to avoid Wachovia’s failure).

Through the lens of the model, a “big” bank is a financial firm with non-zero mass. Figures 6 and 7 report the dynamic response to a shock that combines: the credit quality deterioration shock of Section 5.1 with a market structure shock (i.e., the market structure changes from 2 banks to 1 bank). At the beginning of date  $t$ , when the failure occurs, firms that do not default repay their entire amount of loans outstanding to each bank. All loans are repaid before the market structure change, since agents trade one-period securities. Hence, there is a market structure change.<sup>22</sup>

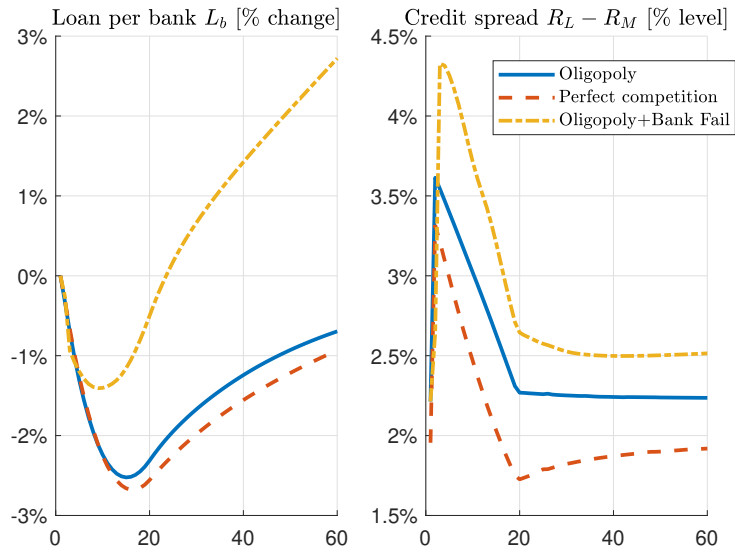
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<sup>22</sup>Initially, I assume there is one continuum of households that trades shares of both banks. Note that, in the model, the bank failure is not equivalent to a merger, because banks are not consolidated and the bank that stops its activity does not pay dividends. Since Modigliani-Miller holds on the bank side, savers are indifferent between equity and deposits. I assume a Debt-to-Equity Ratio of 1, roughly in agreement with US data. When the bank fails, savers recover deposits but not equity. In order to focus the paper, I omitted the results but I also simulated a M&A between two “big” banks. In the long run, it yields identical results to the bank failure. In the short run, it significantly softens the impact with a peak GDP drop of approximately 3%, instead of almost 7%, as shown in Figure 7 for the bank failure.

After the change, all agents make decisions according to the new structure of the economy. Succinctly, when the number of banks changes the model captures two ideas: (i) in the short term, the effects of market power of the surviving bank contributes to lowering the supply of credit to firms, further slowing down the economy and (ii) in the long run, the heightened banks’ market power further contributes to amplifying and prolonging the recession and, ultimately, leads to a lower level of available credit, less investment, output, and productivity. My approach offers a different angle – related to market power – complimentary to typical arguments (e.g., avoid a bank run) to explain why governments may want to bail out a “big” bank.

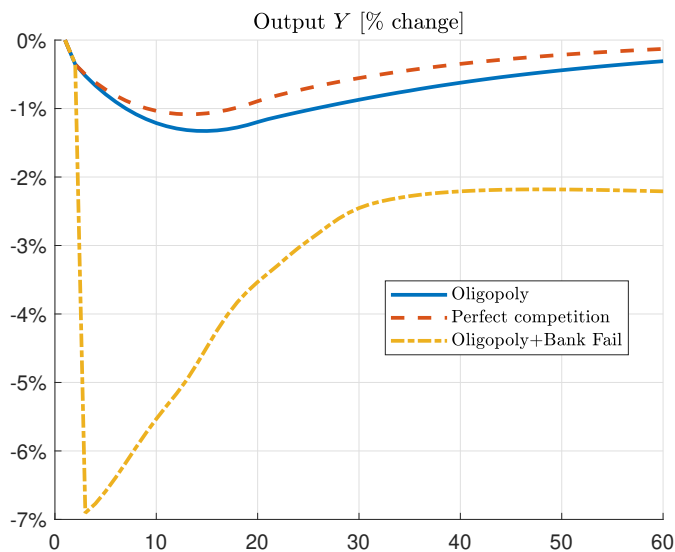
As shown in the left panel of Figure 6, when a “big” bank fails, the surviving bank starts to slowly extend more credit to firms in order to capture the market share of the defaulted bank. However, the speed of this adjustment is dampened by the decreased level of competition among surviving banks. The surviving bank’s market power interacts with credit constraints, yielding a sharp drop in the aggregate volume of credit in the short run.

Figure 6. MARKET STRUCTURE AND FINANCING



*Notes:* This figure reports the transitional dynamics of aggregate loan per bank  $\int l_{b,t}(x_t, X_t) d\Phi_t$  (in % change from the initial stationary equilibrium aggregate value) and the annualized aggregate credit spread  $\int r_{l,t}(x_t, X_t) d\Phi_t - R_{M,t}(X_t)$  (in % level) following an unexpected credit quality shock (as reported in Figure 14 in Appendix A.3) and, simultaneously, a market structure change. X-axes report time  $t$ .

Figure 7. MARKET STRUCTURE AND REAL ACTIVITY



*Notes:* This figure reports the transitional dynamics (in % change from the initial stationary equilibrium aggregate values) and aggregate output  $\int y_t(x_t, X_t) d\Phi_t$ , following an unexpected credit quality shock (as reported in Figure 14 in Appendix A.3) and, simultaneously, a market structure change. X-axis reports time  $t$ .

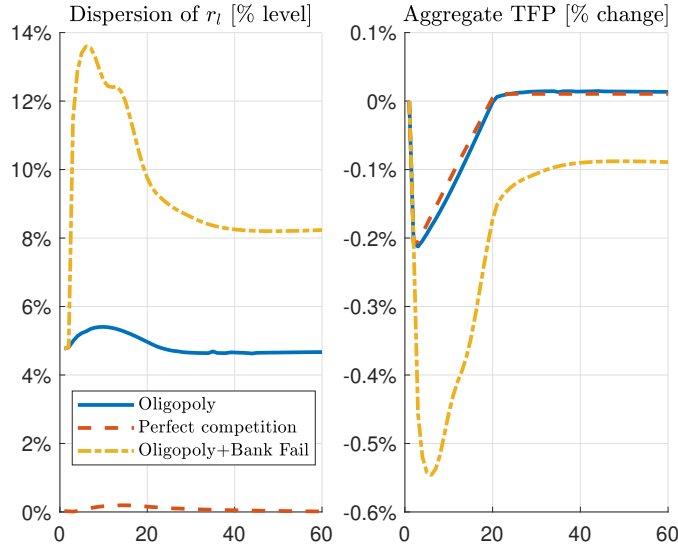
Moreover, because of the general equilibrium effects and the reduced level of competition, in the long run, the economy stabilizes at a lower level of volume of credit, which results in less investment, output, productivity, and more credit and capital misallocation (as shown by the next subsection). In this sense, my analysis suggests that banks’ market power may be an important source of concern for policymakers deciding whether to bail out a “big” bank.

### 5.2.1 Dispersion of Loan Rates and Aggregate TFP

The dynamic financial oligopoly generates firm-level endogenous financial frictions that create time-varying second moments, such as the dispersion of loan rates (directly linked to the dispersion of marginal products of capital) and TFP. The left panel of Figure 8 reports the dynamic of the standard deviation of loan rates, expressed in percentage levels. In agreement with empirical evidence (e.g., Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry 2018 and David, Schmid, and Zeke 2022), the model produces a dynamic with an increasing dispersion of loan rates during recessions; hence, an increasing dispersion of marginal productivity of capital which, in turn, shapes the dynamic behavior of aggregate TFP. The

right panel of Figure 8 reports the dynamic of aggregate TFP (calculated as the residual of an aggregate production  $Y_t = \text{TFP}_t \cdot K_t^\alpha$ ). Figure 8 suggests that, after the failure of a large player, banks' market power contributes significantly to the misallocation of credits (hence, dispersion of marginal products of capital) and induces a persistent decline in aggregate TFP.

Figure 8. MARKET STRUCTURE SHOCK AND CREDIT MISALLOCATION



*Notes:* This figure reports the transitional dynamics of the dispersion of loans' interest rates (in % levels), calculated as the square root of  $\int r_{l,t}^2(x_t, X_t) d\Phi_t - (\int r_{l,t}(x_t, X_t) d\Phi_t)^2$ , and aggregate TFP (in % change from the initial stationary equilibrium aggregate value), calculated as  $\int k_t^\alpha(x_t, X_t) d\Phi_t / (\int k_t(x_t, X_t) d\Phi_t)^\alpha$ , following an unexpected credit quality shock (as reported in Figure 14 in Appendix A.3) and, simultaneously, a market structure shock. X-axes report time  $t$ .

### 5.3 Bank Funding Rate Cut

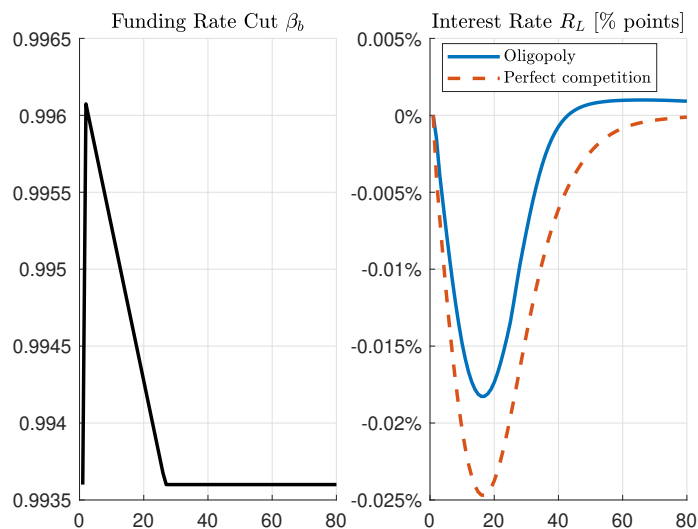
This subsection analyzes the effects of banks' market power when the banks' funding rate suddenly decreases, as shown in the left panel of Figure 9.

Initially, the economy is in stationary equilibrium with  $R_M = M_S'^{-1}(X, X') = \beta_b^{-1}$ , where  $\beta_b$  denotes the discount factor of the saver in equation (6).<sup>23</sup> Following a cut to the bank funding rate, the oligopolistic financial intermediation sector, which has the power to affect aggregate prices, acts as a potent transmission channel. Oligopolistic financial intermediaries exploit markups to dampen the aggregate loans' interest rate response.

<sup>23</sup>Note that  $\beta_E$  in equation (7) is held fixed during the funding rate shock.

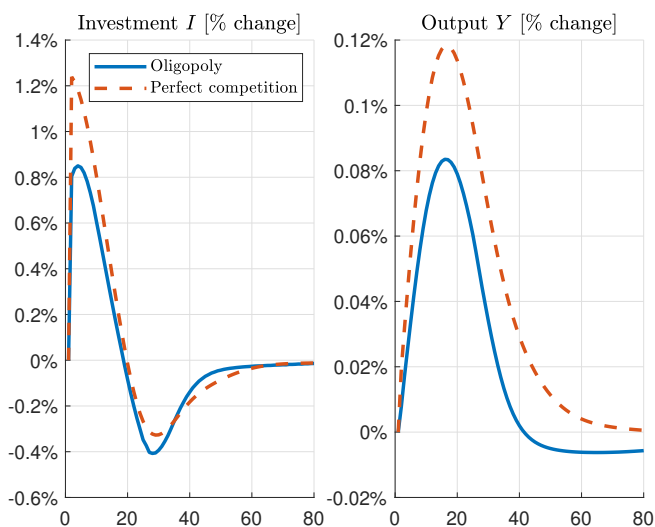


Figure 9. FUNDING RATE CUT AND LOAN RATE



*Notes:* The left panel of this figure reports the shock: following a sudden unexpected 25 basis points increase in the  $\beta_b$  discount rate of the banker (time  $t = 2$ ), the economy mean-reverts to its original level. After the unexpected shock, all agents can perfectly forecast the mean-reversion path. The right panel reports the transitional dynamics of the annualized aggregate interest rate on banks' loans  $\int r_{l,t}(x_t, X_t) d\Phi_t$  (in % points with respect to the initial stationary equilibrium aggregate value), correspondent to the shock reported in the left panel. X-axes report time  $t$ .

Figure 10. FUNDING RATE CUT AND REAL ACTIVITY



*Notes:* This figure reports the transitional dynamics (in % change from the initial stationary equilibrium aggregate values) of aggregate physical investment  $\int i_t(x_t, X_t) d\Phi_t$  (left panel) and aggregate output  $\int y_t(x_t, X_t) d\Phi_t$ , correspondent to the shock reported in the left panel of Figure 8. X-axes report time  $t$ .

Quantitatively, this effect is shown in the right panel of Figure 9. As a consequence, the associated boom in output is dampened by roughly 30% at the peak. These effects are shown in the left and the right panel of Figure 10 for aggregate investment and aggregate output, respectively.

## 6 Conclusion

Motivated by several pieces of evidence, in this paper I study banks' market power through the lens of a new dynamic general equilibrium model that combines oligopolistic banks and heterogeneous firms. The distinctive feature of my model is that financial intermediaries charge firm-specific markups, practicing price discrimination. The lack of competition in the financial sector creates an endogenous cross-sectional dispersion of markups, yielding a mechanism of endogenous financial friction which induces misallocation of credits and reduces productivity. I find that this mechanism plays a significant role in the transmission of macroeconomic shocks. Notably, since my model features non-atomistic banks, I can study banks' market structure changes, such as a "big" bank failure. When a "big" (i.e., non-atomistic) bank fails, surviving banks' market power lowers the total supply of credit, contributing to amplifying and prolonging the recession. In this sense, my analysis offers a different angle, complimentary to typical arguments (e.g., avoid a bank run), to explain why policymakers may want to bail out a "big" bank. A natural direction for future research is to combine intermediaries' market power with collateral constraints, in order to investigate the economic mechanism through which they interact and disentangle the two joint effects. Another interesting direction for future research would consider integrating endogenous entry of firms in this framework. The empirical evidence (e.g, [Cetorelli and Strahan 2006](#)) highlights that in markets characterized by higher bank concentration, potential entrant firms face greater difficulty in gaining access to credit, since banks have an incentive to favor incumbent firms. Taking a further step to incorporate forces of relationship lending in the model would allow one to study if and how these forces affect the propagation of macroeconomic shocks.

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# ONLINE APPENDIX

This online appendix is organized as follows. First, Appendix [A](#) contains additional materials such as additional robustness checks. Second, Appendix [B](#) is the computational appendix that contains details about the algorithms to solve for (i) the stationary equilibrium and (ii) the transitional dynamics. Third, Appendix [C](#) contains mathematical details.

## A Additional Material

This section is divided in four parts: (i) the first part contains the extension where I check the effects of the entry rate to the credit quality deterioration shock described in Subsection [5.1](#), (ii) the second part contains additional tables, (iii) the third part contains additional figures, and (iv) the fourth part contains additional empirics.

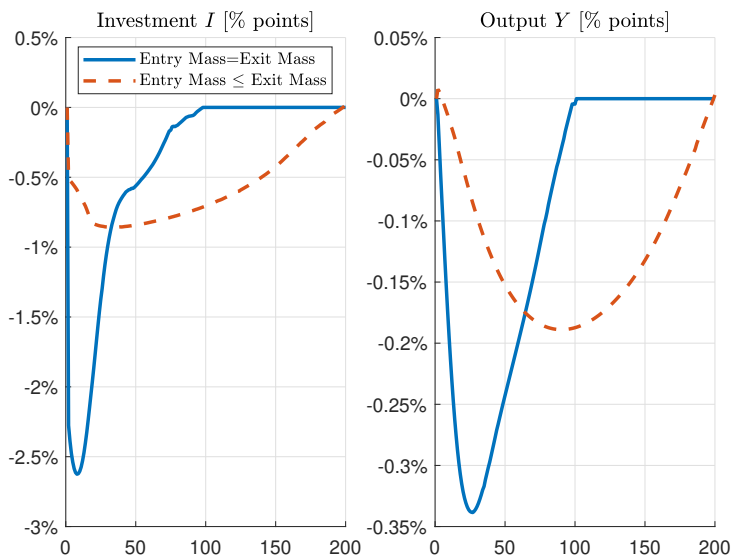
### A.1 The Effects of the Entry Rate

Throughout the paper, I assume that the defaulting mass of firms is replaced, at each date  $t$ , by an equal mass of new entrants firms. In this subsection, I relax this assumption. As before, I let the mass of exiting firms evolves according to the evolution of the default rate in the credit deterioration shock of Figure [14](#). Differently from before, I keep the entering mass of firms constant to its initial stationary equilibrium value. As a consequence, along the shock, the mass of firms in the production sector drops in the short-run and returns to 1 in the long-run. In the spirit of the parallelism with the Great Recession, this produces (in the model) a decline in the mass of firms of about -3.2% at the lower peak, aligned with the percentage decline in the number of firms between 2007 and 2009.<sup>24</sup> In Figure [11](#), I refer to this type of shock with the label  $Entry\ Mass \leq Exit\ Mass$ . In contrast, the label  $Entry\ Mass = Exit\ Mass$  refers to the shock already analyzed in the previous Section [5.1](#), whose effects on real activity are reported in Figure [5](#).

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<sup>24</sup>See, for instance, <https://www.federalreserve.gov/econres/feds/firm-entry-and-employment-dynamics-in-the-great-recession.htm>.

Figure 11. AMPLIFICATION OF CREDIT QUALITY SHOCK AND FIRMS' ENTRY



*Notes:* This figure reports the *amplification effects* during the transitional dynamics (in % points) of aggregate physical investment  $\int i_t(x_t, X_t) d\Phi_t$  and aggregate output  $\int y_t(x_t, X_t) d\Phi_t$ , following an unexpected credit quality shock (as reported in Figure 14) with and without the assumption  $Entry\ Mass = Exit\ Mass$ . The *amplification effects* are computed as follow. First, compute the transitional dynamics (in % change from the initial stationary equilibrium aggregate values) with and without perfect competition, as in Figure 5. Second, calculate the *amplification effects* as % points difference between the responses under oligopolistic competition and under perfect competition. X-axes report time  $t$ .

Figure 11 suggests that, when the firms' default rate increases but not all firms immediately re-enter the market, then imperfect competition in the financial intermediation sector leads to a lower amplification effect at the peak but is much more persistent. The intuition is linked to the economic mechanism discussed in the Section 5.1.

On the one side, a more concentrated banking sector extracts higher rents out of the financially constrained firms with a high marginal productivity of capital, since these firms have worse outside options (e.g., a higher equity issuance cost) and one additional unit of investment in physical capital can contribute significantly to their production. When not all defaulting firms immediately re-enter the market, there is a decline of smaller size firms which tend to be more financially constrained and with a high marginal productivity of capital. Hence, banks partly postpone the extraction of markups to the future, when there will be a recovery.

On the other side, the additional decline in real activity due to the decline in the mass



of producing firms restrains the household’s capacity to support firms with equity issuance as the aggregate dividend falls. This economic force tends to increase banks’ market power in the short run.

## A.2 Additional Tables

Table II. PARAMETER VALUES

		Parameter	Value	Target/Source
<b>Household</b>	Time Discount	$\beta$	0.9936	Match Deposit Rate (Source: FDIC)
	Risk Aversion	$\gamma$	1	
<b>Firms</b>	Depreciation Rate	$\delta$	0.025	Bureau of Economic Analysis
	Effective Capital Share	$\alpha$	0.41	Bureau of Labor Statistics
	Corporate Tax Rate	$\tau$	0.241	Tax Corp. Income/ Corp. Profit (Source: FRED)
	Default Rate	$1 - \rho$	0.21%	Quarterly Net Charge-off to Loan (Source: FDIC)
	Equity Flotation Cost	$\lambda_0$	1.5	Internally calibrated (see Table I)
<b>Banks</b>	Number of Banks	$B$	2	Internally calibrated (see Table I)

*Notes:* The table reports the parameter values.

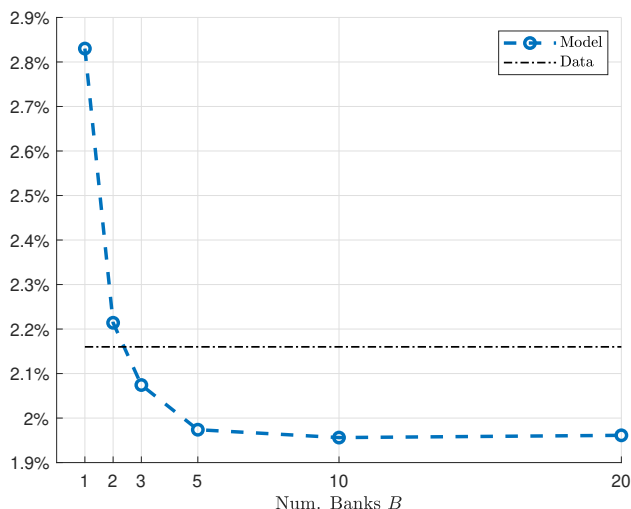
Table III. STATIONARY EQUILIBRIUM AND MOMENTS ROBUSTNESS

Description	Moment	Model		
		$\lambda_0 = 0.75$	<i>Calibrated</i>	$\gamma = 5$
Credit Spread	$\int r_l(x, X)d\Phi - r_M$	0.55%	0.55%	0.56%
Freq. of Equity Issuance	$\int (d(x, X) < 0)d\Phi$	3.5%	1%	2%
Capital to GDP Ratio	$K/Y$	11.13	11.13	11.12
Investment to K Ratio	$I/K$	3.23%	3.23%	3.23%
Debt Adjust. to K Ratio	$B \int \Delta'_b(x, X) d\Phi / K$	0.2%	0.2%	0.2%
Market Leverage	$B \int l_b(x, X) / V_F(x, X) d\Phi$	30.40%	30.41%	30.23%

*Notes:* This table reports the same moments of Table I. The *calibrated* column is the same column of Table I under *Duopoly*, with baseline parameters  $\lambda_0 = 1.5$  and  $\gamma = 1$ . The right column reports the moments when  $\gamma = 5$ , everything else equal to the the *calibrated* column. The left column reports the moments when  $\lambda_0 = 0.75$ , everything else equal to the the *calibrated* column.

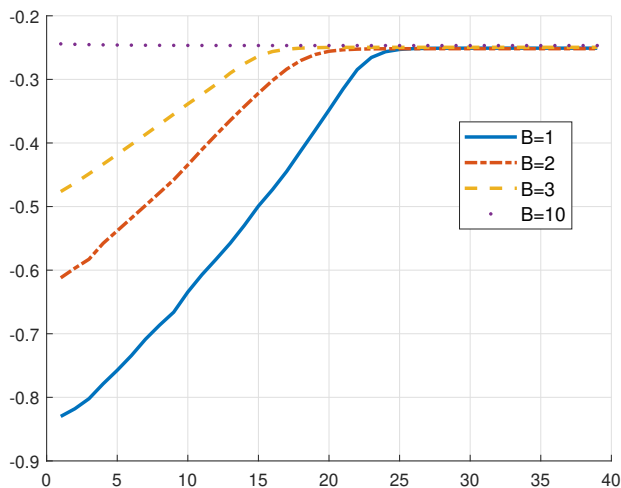
### A.3 Additional Figures

Figure 12. STATIONARY EQUILIBRIUM AND CREDIT SPREADS



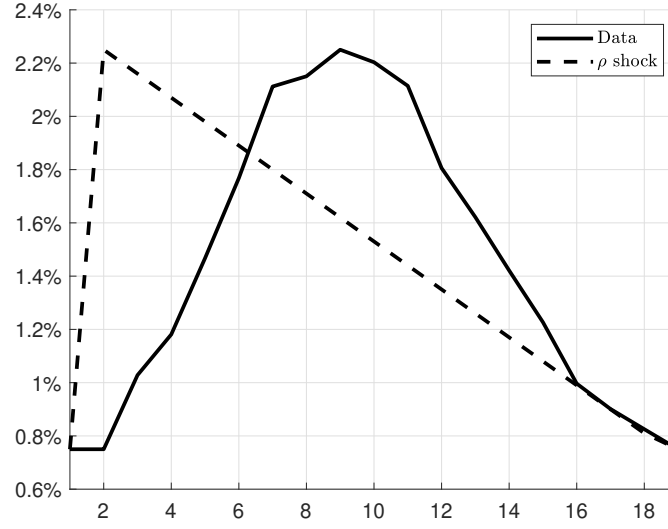
*Notes:* This figure reports the average annualized credit spreads predicted by the model in various stationary equilibria, calculated with an increasing number of banks. The figure shows that the model with two banks predicts an annualized credit spread of about 2.2%, close to the value of 2.16% observed in the data (calculated as average of the time series reported in Figure 1). Moreover, the more banks are introduced in the oligopoly, the more the economy converges to the perfectly competitive case with credit spread of about 1.96%. Hence, the model predicts an annualized aggregate markup of about 0.24%.

Figure 13. STATIONARY EQUILIBRIUM AND CROSS-SECTIONAL MARKUPS



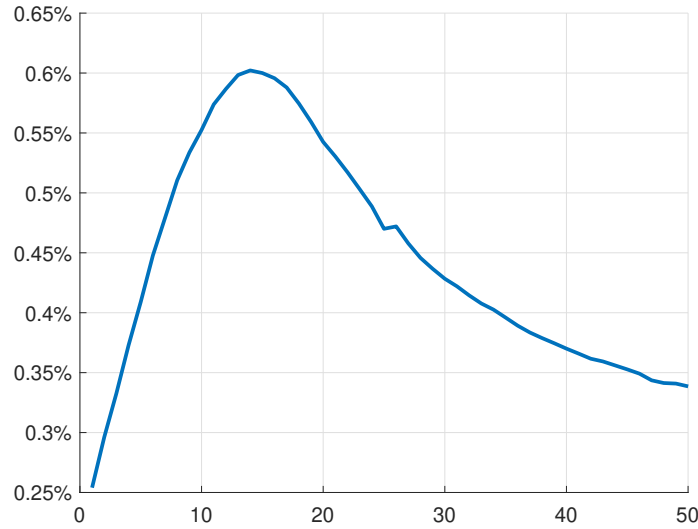
*Notes:* This figure reports the inverse elasticities  $\eta'_L(x, X, x', X')$  contained in the generalized Euler equations (21) along the life cycle of a firm in the stationary equilibrium. These are the elasticities of the future loans' interest rate with respect to loans' quantity. The X-axis reports the firms' age.

Figure 14. ANNUALIZED CHARGE-OFF RATE  $1 - \rho$  (2008Q1-2012Q2)



*Notes:* This figure reports the annualized net charge-off rates during the Great Recession. The X-axis reports time  $t$ , expressed in quarters. The shock is represented by the dashed line: following a sudden unexpected increase in the aggregate firms' default probability (time  $t = 2$ ) the economy mean-reverts to its original level. After the unexpected shock, all agents can perfectly forecast the mean-reversion path.

Figure 15. CREDIT QUALITY SHOCK AND AGGREGATE MARKUP

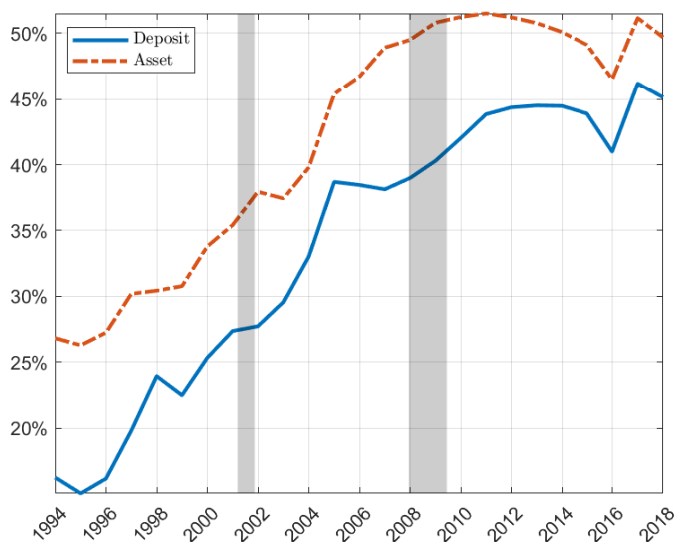


*Notes:* This figure reports the transitional dynamics of the annualized aggregate markup (in % level) following an unexpected credit quality shock (as reported in Figure 14). The evolution over time of the aggregate markup is calculated as the difference between (i) the annualized aggregate credit spread  $\int r_{l,t}(x_t, X_t) d\Phi_t - R_{M,t}(X_t)$  under the calibrated oligopoly and the (ii) the annualized aggregate credit spread  $\int r_{l,t}^{PC}(x_t, X_t) d\Phi_t^{PC} - R_{M,t}^{PC}(X_t)$  under perfect competition. Note that the dynamics of both credit spreads are reported in the right panel of Figure 4. The X-axis reports time  $t$ .

## A.4 Additional Empirics

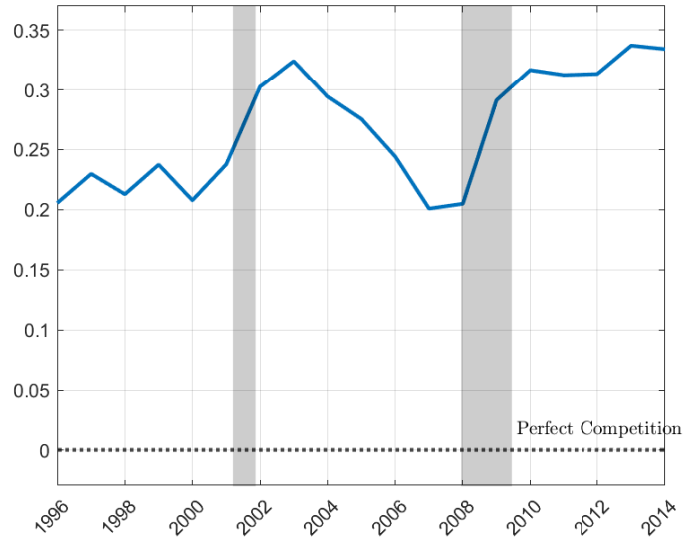
This section contains additional empirical evidences of imperfect competition in the banking sector. Figure 16 shows the time evolution of the 5-Bank Asset Concentration for the United States, which has been increasing both in terms of deposits and assets since 1995. This data are obtained using the Federal Deposit Insurance Corporation (FDIC) summary of deposits survey of branch from 1994 to 2018, from bank's level data. Banks deposits and assets are aggregated at the level of the holding bank and divided by the deposits and assets of the entire industry. As an example, in 2018, the biggest five banks in terms of assets were: (1) JP Morgan Chase & Co (14.45%), (2) Bank of America Corp. (11.73%), (3) Wells Fargo & Company (11.17%), (4) Citigroup Inc. (9.32%) and (5) U.S. Bancorp (3.02%).

Figure 16. MARKET SHARE OF THE TOP 5 US BANKS



*Notes:* The figure shows the evolution over time of the deposit and asset market share of the top 5 US banks. The source of the data is the FDIC summary of deposits survey of branch.

Figure 17. LERNER INDEX (US BANKS)



*Notes:* This figure reports the Lerner index, which is the difference between output prices and marginal costs divided by prices: 0 indicates perfect competition and 1 indicates monopoly. Output prices are calculated as total bank revenue over assets. Marginal costs are calculated as estimated translog cost function with respect to output. The source of the data is the World Bank Global Financial Development Database.

Another evidence of imperfect competition in the banking sector is provided by the Lerner Index. As shown by Figure 17, the Lerner index has been increasing. Moreover, it is significantly different from zero, where zero is the perfect competition benchmark.

Another relevant statistics in this context is the Rosse-Panzar H index, also reported in the World Bank Global Financial Development Database. The value has fluctuated around 0.45 between 2010 and 2015. It is a measure of the elasticity of bank revenues relative to input prices: 1 indicates perfect competition, 0 (or less) indicates monopoly. Overall, these evidences suggest that there is a significant degree of imperfect competition in the banking sector.

To conclude the analysis, I now turn the attention to the data used to calibrate the model. Table IV reports the effects that the market share of the top 5 US banks and the net charge-off rates have on C&I credit spreads. As a reminder, credit spreads are calculated as the difference between the weighted-average effective loan rate for all C&I Loans ( $R_L$ ) and 3-Month T-bill rates ( $R_M$ ). Two other measures are added to the analysis: (i) outstanding quantity of C&I Loans (\$tn) and (ii) the weighted-average maturity for all C&I Loans. Each

period  $t$  is a quarter between 1997Q2 and 2017Q2. The results of the following regression

$$R_{L,t} - R_{M,t} = \beta_0 + \beta_1 \times C_{5,t} + \beta_2 \times (1 - \rho_t) + \beta_3 \times L_t + \beta_4 \times M_t,$$

are reports in Table IV.

Table IV. CREDIT SPREAD AND BANKS MARKET CONCENTRATION

	<i>Dependent variable:</i>		
	Commercial & Industrial Loan Rates Spreads over intended federal funds rate		
	(1)	(2)	(3)
Market share of top 5 banks (%)	0.040*** (0.004)	0.053*** (0.006)	0.056*** (0.007)
Net Charge-Off Rate (%)	0.337*** (0.051)	0.295*** (0.051)	0.272*** (0.059)
C&I Loans (\$tn)		-0.391*** (0.139)	-0.345** (0.152)
Maturity			-0.121 (0.157)
Constant	0.434** (0.183)	0.384** (0.177)	0.406** (0.179)
Observations	81	81	81
R <sup>2</sup>	0.644	0.677	0.680
Adjusted R <sup>2</sup>	0.635	0.664	0.663
Residual Std. Error	0.291 (df = 78)	0.279 (df = 77)	0.280 (df = 76)
F Statistic	70.500*** (df = 2; 78)	53.802*** (df = 3; 77)	40.292*** (df = 4; 76)

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

*Notes:* There is a strong significant positive correlation between credit spread, banks market concentration, and net charge-off rate. The correlation with the quantity of C&I Loans outstanding is significant and negative, consistently with the model (the elasticity between loan and loan rate is negative).

## B Computational Online Appendix

In this section, I describe the algorithm to solve both the stationary equilibrium and the dynamics with the MIT shocks. I highlight the novel methodology used to solve for both General Equilibrium and strategic interactions. Since banks optimize over the optimal choices of the firms, solving this problem using value function iteration would require to nesting two value function iterations inside each other and iterating on the nested value function system given guesses for the aggregate dynamics. Moreover, accounting for strategic interactions with value function iteration would require to solving this system of two nested VFIs given other banks' strategies and finding the fixed point of the resulting policies. This brute force

approach is clearly not viable. To avoid this, I use projection methods jointly on the generalized Euler equations (21) and the loan firms optimality conditions (12) and (13). Hence, I leverage the fact that the elasticities can be calculated applying the implicit function theorem as described in Appendix C.1. Note that the aggregate quantities are not only contained in the discount factors, but also in the elasticities of the generalized Euler equations (see Appendix C.1). In order to account for strategic interactions and solve the generalized Euler equations (which can also be interpreted as best response functions), I impose ex-post symmetric strategies between banks after calculating the elasticity as described in Section C.1; hence, I proceed to calculate the root of the resulting equation with time iteration and projection methods, as any other Euler equation.<sup>25</sup> Note also that the projection step with time iteration can be efficiently computed parallelizing the calculation of the policy functions, fixing state variables. In particular, on a grid of  $\mathcal{K} \times \mathcal{L}$ , such that  $\mathcal{K} = [0, k_1, \dots, k_{\bar{K}}]$  and  $\mathcal{L} = [0, l_{b,1}, \dots, l_{b,\bar{L}}]$ , this corresponds to have  $\bar{K} \times \bar{L}$  parallel subproblems at each step of the time iteration. The code is written in C/C++, each subproblem can be efficiently parallelized using the [OpenMP](#) API specification for parallel programming. Moreover, each subproblem is solved using the Levenberg-Marquardt algorithm contained in [ALGLIB](#).

## B.1 Oligopolistic Stationary Equilibrium

Here are the main steps to solve for the *oligopolistic stationary equilibrium* (see Definition 3.1). Create grids  $\mathcal{K} = [0, k_1, \dots, \bar{k}]$  and  $\mathcal{L} = [0, l_{b,1}, \dots, l_{b,\bar{L}}]$ . Initialize the policy functions for investment and loan to the solution of the corresponding steady-state model without firms heterogeneity; i.e.,  $\forall (k, l_b) \in \mathcal{K} \times \mathcal{L}$ ,  $l'_b(k, l_b) = l_b^*$  and  $k'(k, l_b) = k^*$ . Create an iterator  $j$  and set  $j = 0$ ; hence, proceed as follow.

1. Guess an aggregate dividend  $\tilde{D}^j = \int \tilde{d} d\Phi$  (e.g., use the steady-state dividend calculated without firm heterogeneity).<sup>26</sup>
2. Create an iterator  $w$  and set  $w = 0$ .
  - (a) Fork the program in  $\bar{k} \times \bar{L}$  parallel threads. Each thread solves the subproblem associated with a fixed pair  $(k, l_b) \in \mathcal{K} \times \mathcal{L}$ .<sup>27</sup>

<sup>25</sup>This is equivalent to finding the fixed-point of the banks' strategies.

<sup>26</sup>Note that the aggregate dividend  $\tilde{D}^j$  is contained in the elasticity  $\eta_l$  of equation (21).

<sup>27</sup>This can be achieved either by using POSIX Threads directly or, at a higher level, using [OpenMP](#) API.

- (b) Each thread uses Levenberg-Marquardt to jointly solve for  $k'^{w+1}$ ,  $l'_b{}^{w+1}$ ,  $R'_l{}^{w+1}$  such that the firms' first-order conditions (12) and (13) hold, and such that the generalized Euler equation (21) hold, given future policy functions  $k''^w(k'^{w+1}, l'_b{}^{w+1})$ ,  $l''^w(k'^{w+1}, l'_b{}^{w+1})$ , and  $R''_l{}^w(k'^{w+1}, l'_b{}^{w+1})$ .<sup>28</sup> The elasticity  $\eta'_l$  of equation (21) is calculated according to equations (25), (26), (27), (28) and the condition of symmetry among bank's strategies  $l'_1{}^{w+1} = \dots = l'_b{}^{w+1} = \dots = l'_B{}^{w+1}$ , which is imposed ex-post.
- (c) Wait for all parallel threads to complete their tasks and update the policy functions accordingly  $\forall (k, l_b) \in \mathcal{K} \times \mathcal{L}$ ,  $k'(k, l_b) = k'^{w+1}$ ,  $l'_b(k, l_b) = l'_b{}^{w+1}$ ,  $R'_l(k, l_b) = R'_l{}^{w+1}$ .<sup>29</sup>
- (d) If the policy functions converged (i.e.,  $\max(\sup |k'^{w+1} - k'^w|, \sup |l'_b{}^{w+1} - l'_b{}^w|) < \epsilon$ ) proceed to step 3. Otherwise, set  $w = w + 1$  and restart from step 2.

3. The probability density function over age is given by

$$\phi(\text{age}) = \frac{\rho^{\text{age}}}{\sum_{\text{age}=0}^{\bar{N}} \rho^{\text{age}}}.$$

Start from age = 0 and simulate the policy functions up to age  $\bar{N}$ . This yields a mapping between  $\phi(\text{age})$  and  $\phi(x)$ .

4. Compute the implied aggregate dividend  $\tilde{D}^{j+1} = \int \tilde{d} d\Phi(x)$ .
5. If the aggregate dividend converged (i.e.,  $|\tilde{D}^{j+1} - \tilde{D}^j| < \epsilon$ ), the program terminates. Otherwise, set  $j = j + 1$  and restart from step 2. Use a quasi-Newton method to correct the guess of the aggregate dividend  $\tilde{D}^j$ , given the implied aggregate dividend  $\tilde{D}^{j+1}$ .<sup>30</sup>

## B.2 Transitional Dynamics

The economy is initially in its stationary equilibrium when all agents discover a sudden change in a model parameter at  $t = 0$ . In order to compute the equilibrium dynamics, I need to find sequences of: (i) aggregate saver's consumption  $\{C_{b,t}\}_{t=0}^T$ , (ii) entrepreneur's

<sup>28</sup>I use  $\bar{k} = 10$ ,  $\bar{L} = 10$  and perform piece-wise bilinear interpolation.

<sup>29</sup>The update requires dampening. For example, I update the policy function for capital according to  $k'^{w+1} = (1 - \omega) \cdot k'^w + \omega \cdot k'^{w+1}$ , with  $\omega = 0.01$ .

<sup>30</sup>This update also requires dampening, similarly to the update of the policy functions.



aggregate consumption  $\{C_{E,t}\}_{t=0}^T$ , and (iii) firms distributions  $\{\phi_t(x)\}_{t=0}^T$ ; such that both households maximize utilities, all markets clear in each period and the firms distributions evolve according to: (i) the firms' policy functions, (ii) the banks generalized Euler equations and (iii) the idiosyncratic default shocks. First, compute the two stationary equilibria associated with the configuration of parameters before and after the shock, as described previously.<sup>31</sup> Second, create an iterator  $j$  and set  $j = 0$ ; hence, proceed as follow.

1. Guess sequences of: (i) aggregate saver's consumption  $\{C_{b,t}^j\}_{t=0}^T$ , (ii) entrepreneur's aggregate consumption  $\{C_{E,t}^j\}_{t=0}^T$ .<sup>32</sup>
2. Create an iterator  $t$  and set  $t = T - 1$ . Hence, use projection with backward time iteration from  $t = T - 1$  to  $t = 0$ . The policy functions at  $t = T$ , are the ones associated with the ending stationary equilibrium, previously calculated. At each time  $t$  proceed similarly to before.
  - (a) Fork the program in  $\bar{k} \times \bar{L}$  parallel threads. Each thread solves the subproblem associated with a fixed pair  $(k, l_b) \in \mathcal{K} \times \mathcal{L}$ .
  - (b) Each thread uses Levenberg-Marquardt to jointly solve for  $k^{t+1}, l_b^{t+1}, R_l^{t+1}$  such that the firms' first-order conditions (12) and (13) hold, and such that the generalized Euler equation (21) hold, given future policy functions  $k^{t+2}(k^{t+1}, l_b^{t+1}), l_b^{t+2}(k^{t+1}, l_b^{t+1})$ , and  $R_l^{t+2}(k^{t+1}, l_b^{t+1})$ , always performing piece-wise bilinear interpolation when needed. The elasticity  $\eta_l^t$  of equation (21) is calculated according to equations (25), (26), (27), (28) and the condition of symmetry among bank's strategies  $l_1^{t+1} = \dots = l_b^{t+1} = \dots = l_B^{t+1}$ , which is imposed ex-post.
  - (c) Wait for all parallel threads to complete their tasks and update the policy functions accordingly  $\forall (k, l_b) \in \mathcal{K} \times \mathcal{L}, k^{t+1}(k, l_b) = k^{t+1}, l_b^{t+1}(k, l_b) = l_b^{t+1}, R_l^{t+1}(k, l_b) = R_l^{t+1}$ .
3. Now, start from  $t = 0$  and iterate forward up to  $t = T$ . The probability density

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<sup>31</sup>If there are not permanent change to the parameters, the two stationary equilibria coincides.

<sup>32</sup> $T$  should be long enough, so that after the shock the economy converges to its long-run stationary equilibrium. In this paper, I use  $T=100$  quarters or  $T=200$  quarters, depending on the shock considered.

function over age is now time-varying

$$\phi_t(\text{age}) = \frac{\rho_t^{\text{age}}}{\sum_{\text{age}=0}^{\bar{N}} \rho_t^{\text{age}}}.$$

At each time  $t$ , start from age = 0 and simulate the time  $t$  policy functions up to age  $\bar{N}$ . This yields a mapping between  $\phi_t(\text{age})$  and  $\phi_t(x)$ .

4. For each time  $t$ , compute the implied aggregate dividend  $C_{E,t}^{j+1} = \int \tilde{d}_t d\Phi_t(x)$ , which is consumed by the entrepreneur. Compute the implied aggregate bank's profit  $C_{b,t}^{j+1} = \int \pi_b d\Phi_t(x)$ , which is consumed by the saver.
5. If the sequences for aggregate consumptions converged; i.e.,

$$\max(\sup\{|C_{E,t}^{j+1} - C_{E,t}^j|\}_{t=0}^T, \sup\{|C_{b,t}^{j+1} - C_{b,t}^j|\}_{t=0}^T) < \epsilon,$$

the program terminates. Otherwise, set  $j = j + 1$  and restart from step 2, after heaving updated the sequences with a heavy dampening parameter.

## C Mathematical Online Appendix

This section contains: (i) the calculation of the elasticity  $\eta_l$  in the generalized Euler equation (21), and (ii) the proofs of the statements in the proposition of Section 2.

### C.1 Firm-Specific Inverse Elasticity $\eta_l$

This subsection contains the calculation of the elasticity in the generalized Euler equation (21). Combine the firms' first-order conditions (12) and (13) to define functions

$$f(x, X, x', X') \equiv \pi_k(k') - \delta - R'_l + 1 = 0,$$

$$g(x, X, x', X') \equiv \rho \cdot M'_E \cdot (1 - \lambda'_d) ((1 - \tau)(R'_l - 1) + 1) - 1 + \lambda_d = 0.$$

Compute the total derivatives of these two functions with respect to  $k'$  and  $l'_b$ . Hence, solve the resulting linear system to get the following expression:<sup>33</sup>

$$\frac{\partial R'_l}{\partial l'_b} = \frac{f_{k'} \cdot g_{l'_b} - f_{l'_b} \cdot g_{k'}}{f_{R'_L} \cdot g_{k'} - f_{k'} \cdot g_{R'_L}}.$$

Note that  $f_{l'_b} = 0$ ,  $f_{R'_l} = -1$ , and  $f_{k'} = \pi_{kk}(k')$ . Therefore, the elasticity  $\eta'_l$  is given by

$$\eta'_l = \frac{\partial R'_l}{\partial l'_b} \frac{l'_b}{R'_l} = -\frac{f_{k'} \cdot g_{l'_b}}{g_{k'} + f_{k'} \cdot g_{R'_l}} \frac{l'_b}{R'_l}, \quad (25)$$

where the partial derivatives of the  $g$  function are

$$g_{k'} = \rho \frac{\partial M'_E}{\partial k'} (1 - \lambda_d(d')) ((1 - \tau)r'_l + 1) - \rho M'_E \frac{\partial \lambda_d(d')}{\partial k'} ((1 - \tau)r'_l + 1) + \frac{\partial \lambda_d(d)}{\partial k'}, \quad (26)$$

$$g_{l'_b} = \rho \frac{\partial M'_E}{\partial l'_b} (1 - \lambda_d(d')) ((1 - \tau)r'_l + 1) - \rho M'_E \frac{\partial \lambda_d(d')}{\partial l'_b} ((1 - \tau)r'_l + 1) + \frac{\partial \lambda_d(d)}{\partial l'_b}, \quad (27)$$

$$g_{R'_l} = \rho \frac{\partial M'_E}{\partial R'_l} (1 - \lambda_d(d')) ((1 - \tau)r'_l + 1) - \rho M'_E \frac{\partial \lambda_d(d')}{\partial R'_l} ((1 - \tau)r'_l + 1) + \rho M'_E (1 - \tau) + \frac{\partial \lambda_d(d)}{\partial R'_l}. \quad (28)$$

Note that equations (26), (27), and (28) contain all other banks strategies  $\mathbf{l}_{-b} = [l_1, \dots, l_B] \setminus \{l_b\}$  in the discount factor  $M'_E$  and in the marginal equity issuance costs  $\lambda_d(d)$ ,  $\lambda_d(d')$ .

## C.2 Proofs

This section contains the proofs of the statements 1-10 in the main proposition of Section 2.

*Proof. Statements 1,2 and 3.* Note that the number of banks matters only for the financially constrained firms, so that each integral can be rewritten as follow

$$\frac{\partial}{\partial B} \int x^* d\Phi = \int \frac{\partial x^*}{\partial B} \cdot \mathbf{1}_{[d_1^* < 0]} d\Phi + \int \frac{\partial x^*}{\partial B} \cdot \mathbf{1}_{[d_1^* \geq 0]} d\Phi = \int \frac{\partial x^*}{\partial B} \cdot \mathbf{1}_{[d_1^* < 0]} d\Phi,$$

where  $x^*$  is a place holder for  $l_{1,b}^*$ ,  $R_1^*$ ,  $p_0^*$  and  $k_1^*$ . Hence, a sufficient condition to establish the sign of  $\frac{\partial}{\partial B} \int x^* d\Phi$  is to determine the sign of  $\frac{\partial x^*}{\partial B} \cdot \mathbf{1}_{[d_1^* < 0]}$ . Total differentiation of the optimality conditions (1), (2) and (3) of the financially constrained firms yields the following

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<sup>33</sup>This is equivalent to apply the implicit function theorem.

linear system

$$\begin{bmatrix} \kappa_1 & \kappa_2 & \frac{\partial R_1^*}{\partial l_{1,b}} \cdot \kappa_2 \\ -\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} & 1 & 0 \\ -1 & \rho\beta & B \end{bmatrix} \begin{bmatrix} \frac{\partial k_1^*}{\partial B} \\ \frac{\partial R_1^*}{\partial B} \\ \frac{\partial l_{1,b}^*}{\partial B} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -l_{1,b}^* \end{bmatrix}.$$

Note first that equation (2), for firms with  $d_0 < 0$ , implies  $R_1^* = \frac{1-\lambda_0 d_0}{\rho\beta} > \frac{1}{\rho\beta}$  for  $\lambda_0 > 0$ . Hence, the GEE implies  $\frac{\partial R_1^*}{\partial l_{1,b}} < 0$  for financially constrained firms. Hence, by concavity of the production function and since  $0 < \alpha < 1$ ,  $\kappa_1 = \frac{\partial R_1^*}{\partial l_{1,b}} \frac{\lambda_0(\alpha-2)}{\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-1}} < 0$ . It also follows that  $\kappa_2 = \frac{1}{l_{1,b}^*} \left( \frac{\lambda_0}{\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2}} - \rho\beta \right) < 0$ . The determinant of the matrix is therefore:

$$\mathcal{D} = \frac{\partial R_1^*}{\partial l_{1,b}} \kappa_2 + \kappa_1 B + \kappa_2 \alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} B - \frac{\partial R_1^*}{\partial l_{1,b}} \kappa_2 \alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} \rho\beta.$$

Direct inversion yields:

$$\begin{bmatrix} \frac{\partial k_1^*}{\partial B} \\ \frac{\partial R_1^*}{\partial B} \\ \frac{\partial l_{1,b}^*}{\partial B} \end{bmatrix} = \begin{bmatrix} \frac{\partial R_1^*}{\partial l_{1,b}} \kappa_2 l_{1,b}^* \\ \frac{\partial R_1^*}{\partial l_{1,b}} \kappa_2 \alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} l_{1,b}^* \\ -l_{1,b}^* (\kappa_1 + \kappa_2 \alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2}) \end{bmatrix} \cdot \mathcal{D}^{-1}.$$

Note that if  $\kappa_1 + \kappa_2 \alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} > 0$ , then we can conclude that:  $\frac{\partial k_1^*}{\partial B} > 0$ ,  $\frac{\partial R_1^*}{\partial B} < 0$  and  $\frac{\partial l_{1,b}^*}{\partial B} < 0$ . This is equivalent to showing:

$$\frac{1 - \rho\beta R_1^*}{\rho\beta l_{1,b}^*} \frac{\lambda_0(\alpha-2)}{\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-1}} + \frac{\lambda_0}{l_{1,b}^*} - \frac{1}{l_{1,b}^*} \rho\beta \alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} > 0.$$

Note that equation (1) of the optimalities of the constrained firm can be rewritten as

$$\lambda_0 \frac{1 - \rho\beta R_1^*}{\rho\beta l_{1,b}^*} \frac{1}{\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-1} k_1^{*-1}} = \rho\beta \frac{1 - \rho\beta R_1^*}{\rho\beta l_{1,b}^*} + \lambda_0.$$

Using this equivalence *the want to show* can be rewritten as

$$\begin{aligned} & \frac{1 - \rho\beta R_1^*}{\rho\beta l_{1,b}^*} \frac{\lambda_0(\alpha-2)}{\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-1}} + \frac{\lambda_0}{l_{1,b}^*} - \frac{1}{l_{1,b}^*} \rho\beta \alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} \\ &= \rho\beta \frac{1 - \rho\beta R_1^*}{\rho\beta l_{1,b}^*} (\alpha-2)k_1^{*-1} + \lambda_0(\alpha-2)k_1^{*-1} + \frac{\lambda_0}{l_{1,b}^*} - \frac{1}{l_{1,b}^*} \rho\beta \alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} > 0. \end{aligned}$$

Multiply everything by  $l_{1,b}^* > 0$ , to get:

$$(1 - \rho\beta R_1^*)(\alpha - 2)k_1^{*\alpha-1} + \lambda_0(\alpha - 2)\frac{l_1^*}{k_1^*} + \lambda_0 - \rho\beta\alpha(\alpha - 1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} > 0.$$

Hence, use equation (3) of the optimalities of the constrained firms to back out an expression for  $l_{1,b}^*$  in function of  $R_1^*$  and  $k_1^*$ , and rewrite

$$(1 - \rho\beta R_1^*)(\alpha - 2)k_1^{*\alpha-1} + (\alpha - 2)\frac{1 - \rho\beta R_1^* - \lambda_0(z_0k_0^\alpha + (1 - \delta)k_0 - k_1^*)}{Bk_1^*} + \lambda_0 - \rho\beta\alpha(\alpha - 1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} > 0.$$

The left-hand side can be rearranged as

$$\begin{aligned} & (\alpha - 2)\frac{(1 - \rho\beta R_1^*)(B + 1) - \lambda_0(z_0k_0^\alpha + (1 - \delta)k_0 - k_1^*)}{Bk_1^*} + \lambda_0 - \rho\beta\alpha(\alpha - 1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} \\ & = (\alpha - 2)[(1 - \rho\beta R_1^*)(B + 1) - \lambda_0(z_0k_0^\alpha + (1 - \delta)k_0 - k_1^*)] + \lambda_0Bk_1^* - \rho\beta\alpha(\alpha - 1)\mathbb{E}_0[z_1]Bk_1^{*\alpha-1}. \end{aligned}$$

Divide by  $(\alpha - 2) < 0$  (changing sign because it is always negative), the previous *want to show* is equivalent to show

$$\underbrace{(1 - \rho\beta R_1^*)(B + 1)}_{<0 \text{ when } d_0 < 0} - \underbrace{\lambda_0(z_0k_0^\alpha + (1 - \delta)k_0)}_{>0} + \underbrace{\lambda_0k_1^* \frac{\alpha - 2 + B}{\alpha - 2}}_{<0 \text{ if } B > 1} - \underbrace{\rho\beta\alpha \frac{\alpha - 1}{\alpha - 2} \mathbb{E}_0[z_1]Bk_1^{*\alpha-1}}_{>0} < 0.$$

Consider two cases. If  $B > 1$  (oligopoly), this last inequality is always satisfied. For  $B = 1$  (monopoly), the inequality collapses to

$$\underbrace{(1 - \rho\beta R_1^*)2}_{<0 \text{ when } d_0 < 0} - \underbrace{\lambda_0(z_0k_0^\alpha + (1 - \delta)k_0)}_{>0} + \underbrace{(\lambda_0k_1^* - \rho\beta R_1^*)}_{<0} \underbrace{\frac{\alpha - 1}{\alpha - 2}}_{>0} < 0.$$

Finally, rearrange the Euler  $\rho\beta R_1^* = 1 - \lambda_0d_0$  to get

$$\lambda_0k_1^* - \rho\beta R_1^* = \lambda_0(z_0k_0^\alpha + (1 - \delta)k_0 + l_{1,b}^*) - 1,$$

which yields the result

$$\underbrace{(1 - \rho\beta R_1^*)}_<0 \text{ when } d_0 < 0 \cdot 2 + \left( \frac{\alpha - 1}{\alpha - 2} - 1 \right) \underbrace{\lambda_0(z_0 k_0^\alpha + (1 - \delta)k_0)}_{>0} + \underbrace{(\lambda_0 l_{1,b}^* - 1)}_{<0} \underbrace{\frac{\alpha - 1}{\alpha - 2}}_{>0} < 0.$$

□

*Proof. Statements 4, 5 and 6.* Following a similar logic as the one of equation (5) the proof focuses in studying the signs of the optimal choices of the financially constrained firms. Hence, all equations that follow refer to those firms such that  $d_0(k_0, z_0, k_1^*, l_{1,b}^*) < 0$ .

For the expected return of the shares, equating the Eulers for loans and the price of shares provides the following non-arbitrage condition  $\mathbb{E}_0 \left[ \frac{d_1^*}{p_0^*} \right] = R_1^*$ . Hence, for financially constrained firms  $\frac{\partial}{\partial B} \mathbb{E}_0 \left[ \frac{d_1^*}{p_0^*} \right] = \frac{\partial R_1^*}{\partial B} < 0$ , which is always negative by previous result.

Before studying the effect of the number of banks on leverage, first note that the effect on total debt is ambiguous:

$$\frac{\partial}{\partial B} B \cdot l_{1,b}^* = B \frac{\partial l_{1,b}^*}{\partial B} + l_{1,b}^*.$$

As shown previously, as the number of banks increases  $l_{1,b}$  decreases. Plugging the formula for  $\frac{\partial l_{1,b}^*}{\partial B}$  found previously can resolve this ambiguity:

$$\frac{\partial}{\partial B} B \cdot l_{1,b}^* = l_{1,b}^* \left( 1 - B \frac{\kappa_1 + \kappa_2 \alpha (\alpha - 1) \mathbb{E}_0[z_1] k_1^{*\alpha-2}}{\mathcal{D}} \right) = l_{1,b}^* \left( 1 - \frac{1}{1 + \frac{\frac{\partial R_1^*}{\partial l_{1,b}^*} \kappa_2 (1 - \alpha (\alpha - 1) \mathbb{E}_0[z_1] k_1^{*\alpha-2} \rho \beta)}{B \kappa_1 + B \kappa_2 \alpha (\alpha - 1) \mathbb{E}_0[z_1] k_1^{*\alpha-2}}} \right),$$

since  $l_{1,b}^* > 0$  and  $\frac{\frac{\partial R_1^*}{\partial l_{1,b}^*} \kappa_2 (1 - \alpha (\alpha - 1) \mathbb{E}_0[z_1] k_1^{*\alpha-2} \rho \beta)}{B \kappa_1 + B \kappa_2 \alpha (\alpha - 1) \mathbb{E}_0[z_1] k_1^{*\alpha-2}} > 0 \implies \frac{\partial}{\partial B} B \cdot l_{1,b}^* > 0$ .

In order to prove that the leverage increases with the number of banks, it remains to show that the following inequality is always satisfied for the financially constrained firms:

$$\begin{aligned} \frac{\partial}{\partial B} \frac{B \cdot l_{1,b}^*}{k_1^*} &= \left( B \frac{\partial l_{1,b}^*}{\partial B} + l_{1,b}^* \right) \frac{1}{k_1^*} - \frac{B \cdot l_{1,b}^*}{k_1^{*2}} \cdot \frac{\partial k_1^*}{\partial B} \\ &= \frac{l_{1,b}^*}{k_1^*} \left( 1 - \frac{B \kappa_1 + B \kappa_2 \alpha (\alpha - 1) \mathbb{E}_0[z_1] k_1^{*\alpha-2} - B \frac{l_{1,b}^*}{k_1^*} \kappa_2 \frac{\partial R_1^*}{\partial l_{1,b}^*}}{\mathcal{D}} \right) > 0. \end{aligned}$$

Since  $l_{1,b}^*/k_1^* > 0$ ,  $\mathcal{D} > 0$  and  $\kappa_2 \frac{\partial R_1^*}{\partial l_{1,b}} > 0$ , this is equivalent to show:

$$-B \frac{l_{1,b}^*}{k_1^*} \kappa_2 \frac{\partial R_1^*}{\partial l_{1,b}} < \frac{\partial R_1^*}{\partial l_{1,b}} \kappa_2 - \frac{\partial R_1^*}{\partial l_{1,b}} \kappa_2 \alpha (\alpha - 1) \mathbb{E}_0[z_1] k_1^{*\alpha-2} \rho \beta \iff -B \frac{l_{1,b}^*}{k_1^*} < 1 - \alpha (\alpha - 1) \mathbb{E}_0[z_1] k_1^{*\alpha-2} \rho \beta,$$

which is always true since  $-B \frac{l_{1,b}^*}{k_1^*}$  is always negative and  $1 - \alpha (\alpha - 1) \mathbb{E}_0[z_1] k_1^{*\alpha-2} \rho \beta$  is always positive. □

*Proof. Statements 7, 8, 9 and 10.* For statements 7, 8, 9 and 10, I assume that the mass of financially constrained firms  $1 - \mathcal{P}$  are all ex-ante identical. For TFP, the *want to show* is

$$\frac{\partial}{\partial B} \frac{\mathbb{E}[k_1^{*\alpha}]}{(\mathbb{E}[k_1^*])^\alpha} = \frac{\alpha \mathbb{E}\left[k_1^{*\alpha-1} \frac{\partial k_1^*}{\partial B}\right]}{(\mathbb{E}[k_1^*])^\alpha} - \alpha \frac{\mathbb{E}[k_1^{*\alpha}] \mathbb{E}\left[\frac{\partial k_1^*}{\partial B}\right]}{(\mathbb{E}[k_1^*])^{\alpha+1}} > 0.$$

This is equivalent to show that

$$\mathbb{E}\left[k_1^{*\alpha-1} \frac{\partial k_1^*}{\partial B}\right] \mathbb{E}[k_1^*] - \mathbb{E}[k_1^{*\alpha}] \mathbb{E}\left[\frac{\partial k_1^*}{\partial B}\right] > 0.$$

Which is again equivalent to

$$\begin{aligned} k_1^{*\alpha-1} \frac{\partial k_1^*}{\partial B} (1 - \mathcal{P}) (k_1^* (1 - \mathcal{P}) + \bar{k} \mathcal{P}) - (k_1^{*\alpha} (1 - \mathcal{P}) + \bar{k}^\alpha \mathcal{P}) \frac{\partial k_1^*}{\partial B} (1 - \mathcal{P}) &> 0 \\ \iff k_1^{*\alpha-1} (k_1^* (1 - \mathcal{P}) + \bar{k} \mathcal{P}) - (k_1^{*\alpha} (1 - \mathcal{P}) + \bar{k}^\alpha \mathcal{P}) &> 0 \\ \iff k_1^{*\alpha} (1 - \mathcal{P}) + k_1^{*\alpha-1} \bar{k} \mathcal{P} - k_1^{*\alpha} (1 - \mathcal{P}) - \bar{k}^\alpha \mathcal{P} &> 0 \\ \iff k_1^{*\alpha-1} \bar{k} \mathcal{P} - \bar{k}^\alpha \mathcal{P} &> 0 \\ \iff k_1^{*\alpha-1} > \bar{k}^{\alpha-1}. \end{aligned}$$

Since  $k_1^* < \bar{k}$ , the last inequality is always verified.

For the dispersion of capital, the *want to show* is

$$\frac{\partial}{\partial B} \mathbb{E}[(k_1^* - \mathbb{E}[k_1^*])^2] = \mathbb{E}\left[\frac{\partial}{\partial B} (k_1^* - \mathbb{E}[k_1^*])^2 | d_0 < 0\right] (1 - \mathcal{P}) + \mathbb{E}\left[\frac{\partial}{\partial B} (\bar{k}_1 - \mathbb{E}[k_1^*])^2 | d_0 \geq 0\right] \mathcal{P} < 0,$$

where  $\mathcal{P}$  is the mass of the firms not financially constrained and  $\bar{k}$  is the optimal choice of

capital of the non financially constrained firms. Hence:

$$\begin{aligned} \frac{\partial}{\partial B} \mathbb{E} [(k_1^* - \mathbb{E}[k_1^*])^2 | d_0 < 0] &= 2(k_1^* - k_1^*(1 - \mathcal{P}) - \bar{k}\mathcal{P}) \left( \frac{\partial k_1^*}{\partial B} - \frac{\partial k_1^*}{\partial B}(1 - \mathcal{P}) - \underbrace{\frac{\partial \bar{k}}{\partial B}}_{=0} \mathcal{P} \right) \\ &= 2\mathcal{P}(k_1^* - \bar{k}) \frac{\partial k_1^*}{\partial B} \mathcal{P} < 0. \end{aligned}$$

Note that the last inequality follows from the fact that  $k_1^* < \bar{k}$ , otherwise the mass of firms  $1 - \mathcal{P}$  would not be financially constrained.  $\frac{\partial k_1^*}{\partial B} > 0$  is positive from the previous proof. Note that the second term is always negative

$$\mathbb{E} \left[ \frac{\partial}{\partial B} (\bar{k}_1 - \mathbb{E}[k_1^*])^2 | d_0 \geq 0 \right] = 2\mathbb{E} \left[ \underbrace{(\bar{k}_1 - \mathbb{E}[k_1^*])}_{>0} \left( \underbrace{\frac{\partial \bar{k}_1}{\partial B}}_{=0} - \underbrace{\frac{\partial \mathbb{E}[k_1^*]}{\partial B}}_{>0} \right) | d_0 \geq 0 \right] < 0.$$

Furthermore, note that  $R_1^* = \mathbb{E}_0[1 + \alpha z_1 k_1^{*\alpha-1} - \delta]$  and

$$\sigma(R_1^*) = \sigma^2 (1 + \alpha \mathbb{E}_0[z_1] k_1^{*\alpha-1} - \delta) = \alpha^2 \mathbb{E}_0^2[z_1] \sigma(k_1^{*\alpha-1}).$$

Hence:

$$\begin{aligned} \frac{\partial \sigma^2(R_1^*)}{\partial B} &= \alpha^2 \mathbb{E}_0^2[z_1] \frac{\partial \sigma^2(k_1^{*\alpha-1})}{\partial B} \\ &= \alpha^2 \mathbb{E}_0^2[z_1] \left( 2\mathcal{P} \underbrace{(k_1^{*\alpha-1} - \bar{k}^{\alpha-1})}_{>0} \underbrace{(\alpha - 1)}_{<0} k_1^{*\alpha-2} \frac{\partial k_1^*}{\partial B} \mathcal{P} + 2\mathbb{E} \left[ \underbrace{(\bar{k}_1^{\alpha-1} - \mathbb{E}[k_1^{*\alpha-1}])}_{<0} \left( \underbrace{\frac{\partial \bar{k}_1^{\alpha-1}}{\partial B}}_{=0} - \underbrace{\frac{\partial \mathbb{E}[k_1^{*\alpha-1}]}{\partial B}}_{<0} \right) | d_0 \geq 0 \right] \right) < 0. \end{aligned}$$

Equating the two Euler equations for the price of the shares of the firms and the price of the bonds yields:  $\frac{\partial}{\partial B} \mathbb{E} \left[ \frac{d_1^*}{p_0^*} \right] = \frac{\partial R_1^*}{\partial B} < 0$ .

□