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Optimal Monetary Policy in an Open Emerging Market Economy

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Abstract

The majority of households across emerging market economies are excluded from the financial markets and cannot smooth consumption. I analyze the implications of this for optimal monetary policy and the corresponding choice of domestic versus external nominal anchor in a small open economy framework with nominal rigidities, aggregate uncertainty, and financial exclusion. I find that, if set optimally, monetary policy smooths the consumption of financially-excluded agents by stabilizing their income. Even though CPI inflation targeting approximates optimal monetary policy when financial inclusion is high, targeting the exchange rate is appropriate if financial inclusion is limited. Nominal exchange rate stability, upon shocks that create trade-offs for monetary policy, directly stabilizes the import component of financially-excluded agents' consumption baskets, which smooths their consumption and reduces macroeconomic volatility. This study provides a counterpoint to Milton Friedman's long-standing argument for a float.

Keywords Asymmetric Risk-Sharing, Fixed Exchange Rates, Financial Exclusion, Optimal Monetary Policy, Emerging Market Economies

JEL classification F21, F31, E24, E52, F43

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"In developing countries where financial markets are underdeveloped, the poor do not have access to credit and [are thus restricted to consuming disposable income]... The challenge is how to stabilize output more effectively and reduce the burden on the poor... Central Banks need to resort to instruments other than the interest rate."

- The 2014 Global Economic Symposium (GES, 2014)

1 Introduction

The Global Financial Crisis of 2007-09 led to macroeconomic volatility in many emerging market economies (EMEs) and, in particular, adversely affected poor agents within these countries (World Bank, 2009). One significant factor that contributed to the vulnerability of the poor was their inability to insure against the crisis-related contraction of incomes by drawing on savings held in the financial markets. The macroeconomic consequences of this are not trivial, as on average, financially-excluded households constitute 60% of households in EMEs, compared to the corresponding average of under 5% in advanced economies (World Bank, 2015; see Figure 1).[1] This was highlighted by the crisis, which renewed interest in the following interlinked macroeconomic stabilization questions. Can monetary policy be effective while paying attention to financially-excluded agents? Relatedly, should the supporting nominal anchor be domestic or external?

A major channel in the monetary transmission mechanism, through which nominal interest rate changes affect the real economy, is the intertemporal optimization by private agents who react to interest rate surprises by changing their asset portfolios. This channel is considerably restricted in economies where the majority of households do not hold assets, and the appropriate monetary policy in this case has been subject to much recent debate in policy circles (summarized in Prasad, 2013 and IMF, 2015a). To complement the policy discussions, however, the corresponding theoretical analysis has been fairly limited. I seek to fill in this gap by providing benchmark results on the implications of limited financial inclusion for optimal monetary policy and the corresponding choice of a nominal anchor in an open emerging market economy.

The economy in this study is small and open, and specifically of measure zero, as EMEs are typically price takers for tradable goods (Frankel, 2010). Openness further permits a quantitative comparison of domestic versus external nominal anchors in approximating optimal monetary policy. Financially excluded agents fully consume their income each period, as they do not have the economic means to save and borrow from financial institutions. The Euler Equation does not hold for these agents, implying that a fraction of the economy cannot insure against shocks through international risk-sharing arrangements. I characterize the optimal monetary policy for the model in the presence of distortions caused by monopolistic competition and staggered price setting, and rank simple monetary rules in terms of lowest welfare losses away from the optimum. The implications of different

[1] Regional abbreviations used in Figure 1 are as follows. MENA: Middle East and North Africa. SSA: Sub-Saharan Africa. ASIA: East, South, and Central Asia. LAC: Latin America and the Caribbean. EUR: Europe. CIS: Commonwealth of Independent States.

shocks are analyzed, with the focus primarily on adverse supply (or cost-push) shocks. This is of particular interest since over the past few years, cost-push shocks, due to domestic food price volatility (driven by weather conditions) and fuel input price volatility (driven by world market conditions), have been predominant in EMEs (Frankel, 2010).

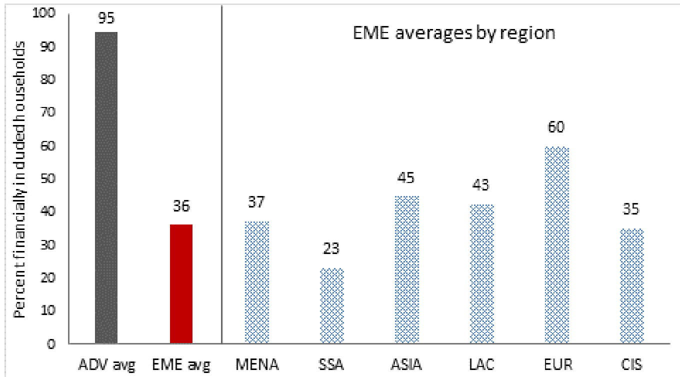

Figure 1: Financial Inclusion Across the World
Data: Percent Households with Account at Any Financial Institution
Sources: World Bank 2014 financial inclusion database (World Bank, 2015), and IMF 2015 country classification scheme (IMF, 2015b)

Two main findings emerge from the analysis in this paper. First, optimal monetary policy smooths the consumption of financially-excluded agents. The intuition for this result stems from the fact that hand-to-mouth agents are not able to privately smooth consumption or insure themselves through risk-sharing arrangements with foreign agents in the model. Their consumption is thus far more volatile than that of financially-included agents. To mitigate the resulting macroeconomic volatility upon shocks, optimal policy smooths the consumption of financially-excluded agents by stabilizing their income. This can be interpreted as provision of insurance by monetary policy.

Welfare losses are minimized when the weight placed on minimizing output gap volatility, relative to domestic inflation volatility, increases with financial exclusion and openness.[2] Minimizing output gap volatility stabilizes labor demand and hence real wages, which in turn stabilizes hand-to-mouth consumption upon shocks. The relative weight on stabilizing the output gap increases with the degree of financial exclusion as there is greater need to smooth the consumption of progressively more disposable-income driven agents. The relative weight also increases as the economy becomes more open. To see this, note that greater openness implies that domestic agents consume more imports relative to home goods. This increases the consequences of exchange rate volatility, which adversely affects hand-to-mouth agents as unlike asset holders, they cannot smooth their consumption against the increasingly volatile price of imported goods. Hence, optimal policy makes up for their inability to privately insure themselves by stabilizing their wage income through the output gap.

[2]Another way to interpret this is that as financial exclusion increases - the relative weight on stabilizing domestic inflation decreases, since inflation erodes the value of financial assets, and there are fewer asset holders now to consider. This is discussed also in Bilbiie (2008).

The second result is that the insurance properties of the optimal monetary plan are internalized through the external anchor of a fixed nominal exchange rate. I find that exchange rate stability is desirable in an economy with a large fraction of financially-excluded agents. Upon cost-push shocks, which simultaneously decrease output and increase inflation, thus creating a trade-off for monetary policy, targeting the exchange rate approximates the efficient dynamics through two channels. It leads to more muted real appreciation, which prevents the consumption of financially-included agents from falling as much. This leads to a relative output expansion, which closes the output gap, thus stabilizing real wages and hand-to-mouth consumption. Further, lower exchange rate volatility directly stabilizes the import component of the financially-excluded consumption basket and hence smooths their consumption. These results provide a counterpoint to Milton Friedman's long-standing argument for flexible exchange rates, which provides the underlying intuition for the optimality of a float in the more recent complete financial inclusion reference model of Gali and Monacelli (2005).

Friedman advocated a float on the premise that it would allow for greater relative price adjustment between domestic and foreign goods, given that nominal rigidities constrain real adjustment in practice. A peg would only exacerbate the lack of relative price adjustment and thus lead to greater macroeconomic volatility. However, this does not account for high financial exclusion. My results extend Friedman's classic arguments to an economic setting where the majority of agents cannot smooth consumption. Even though a float continues to allow for greater relative price adjustment with nominal rigidities, this actually leads to greater loss in the presence of high financial exclusion, as it instigates a corresponding rise in hand-to-mouth consumption volatility. For EMEs, however, Friedman advocated a peg due to the gain in credibility by anchoring domestic monetary policy to a stable, advanced country's regime (Hanke, 2008). I corroborate these political economy reasons by providing a new theoretical rationale for the desirability of a peg in EMEs.

Related Literature This paper contributes to three strands of research: monetary policy in emerging market economies, the dynamics of general equilibrium models with hand-to-mouth agents, and optimal monetary policy in open economies. The former literature is burgeoning, and in a recent study, Prasad and Zhang (2015) build a multi-sector model with traded and non-traded sectors. They find that exchange rate targeting benefits households in the tradable sector as it fixes the domestic price of traded goods. This paper differs in two main ways. It shows that the distribution of household asset income in EMEs has significant implications for exchange rate stabilization. Further, it employs rigorous perturbation techniques in micro-founding the loss function, with corresponding sharp, closed-form solutions. Other related papers, while not having a specific monetary policy focus, have investigated business cycles in EMEs. I discuss a few of these below.

Aguiar and Gopinath (2007) estimate a frictionless RBC model to find that trend growth shocks to output can account for a significant proportion of economic fluctuations in EMEs. These findings are challenged and argued to be specific to a short data time series by Garcia-Cicco et al. (2010), who pursue a similar analysis with an extended dataset. Frankel (2010) provides an extensive review of short-run policy in EMEs, and suggests that cost-push shocks, due to food and fuel input price fluctuations, are a significant source of concern in driving business cycles. This paper complements the existing literature on business cycles in EMEs by focusing on dynamics with cost-push shocks, and also providing a comparison with productivity shocks. It diverges by incorporating market imperfections such as nominal rigidities and financial exclusion, and providing new theoretical results on optimal policy in the presence of financial exclusion in the familiar New Keynesian framework.

The literature on hand-to-mouth agents in New Keynesian models started with Galí et al. (2007), who showed that simple rules can lead to indeterminacy with a high fraction of hand-to-mouth agents, λ, in a closed economy with capital accumulation. Bilbiie (2008) further analyzes determinacy for a version of that closed economy model without capital accumulation and derives the optimal monetary policy. He finds that stabilizing inflation becomes less important as λ increases, and like Galí et al. (2007), that equilibrium indeterminacy with simple rules arises if λ is above a particular threshold. Eser (2009) and Ascari et al. (2011) find that hand-to-mouth agents in a currency union, and with sticky wages, affect determinacy and also make output stabilization more important. This paper extends the current literature in four different dimensions.

First, I analyze the implications of a fluctuating nominal exchange rate for optimal monetary policy with financial exclusion, and show the importance of paying attention to external volatility. This is because exchange rate movements affect the heterogeneous agents in a highly asymmetric manner. While $1-\lambda$ fraction of agents can share risk internationally, so that changes to real income from exchange rate fluctuations are offset through the proceeds from state-contingent claims, exchange rate volatility leads to an immediate rise in consumption volatility (in the presence of nominal rigidities) by the other λ fraction. The implied net foreign asset position in the domestic economy is non-zero, and thus the equilibrium consequences are non-trivial.

Second, I consider the appropriate choice of an implementable nominal anchor that can approximate the optimal monetary policy. This quantitative analysis yields some new, unconventional results including those that provide a counterpoint to Friedman's argument for a float. Third, I develop a technically rigorous derivation of the loss function, which makes use of analytical perturbation techniques, and does not require an ad-hoc subsidy to eliminate the linear terms - an "unappealing assumption" (abstract, Benigno and Woodford, 2005) used for convenience by much of the optimal policy literature. Fourth, due to a non-zero net foreign asset position as λ fraction of agents do not hold assets, the resulting open economy equilibrium and optimal policy are not isomorphic to the closed economy case, complementing other instances such as Monacelli (2005) and Farhi and Werning (2012).

A detailed review of the third strand of related research, on optimal monetary policy in open economies, is found in Corsetti et al. (2010). This literature generally approaches optimal policy through a linear quadratic framework, following the perturbation techniques discussed in Benigno and Woodford (2012). The terms and weights in these papers' quadratic micro-founded loss functions play key roles in driving the results. This paper diverges in two ways. Technically, I do not use a convenient ad-hoc subsidy to eliminate the linear term in the loss function, and instead take second-order approximations of the structural equations to replace out for it with the correct quadratic terms.

I also model an austere form of financial market incompleteness, as relevant in EMEs, where a large proportion of households are not able to engage in financial transactions. Only a fraction of the population is allowed to insure against aggregate uncertainty through international risk-sharing arrangements. This fundamental asymmetry in the ability to pool risk with foreign agents is new to the open economy optimal policy literature, and has striking implications for the exchange rate. It is worth noting that the form of market incompleteness that has typically been considered, ie. international trade in a single risk-free bond as opposed to state-contingent securities, is also applicable to EMEs. This would imply that financially-included agents engage in imperfect risk-sharing. I leave open for future research an analysis of optimal policy with financial exclusion when even financially-included agents cannot fully insure against uncertainty.

The rest of the paper proceeds as follows. Section 2 develops an open economy model with financial exclusion. Optimal monetary policy for the model is characterized in Section 3. Section 4 compares optimal policy with simple monetary rules, and considers the choice of an implementable nominal anchor in the presence of financial exclusion. Section 5 concludes, and discusses possible extensions. The appendix contains additional figures.

2 A Small Open Economy with Financial Exclusion

I develop an open economy general equilibrium framework with nominal rigidities, aggregate uncertainty, and two types of households. The model incorporates financially-excluded agents in addition to the standard financially-included agents considered in the open economy DSGE literature, and builds upon the small open economy framework of Gali and Monacelli (2005). Section 2 is cast in non-linear terms for expositional ease, and the resulting equilibrium will be linearized in Sections 3 and 4 to solve for optimal monetary policy using perturbation techniques.

2.1 Households

There exists a continuum of households indexed by $l \in [0,1]$. Fraction $1-\lambda$ of households participate in the financial markets, which are assumed to be complete both within and across countries. Through a complete set of state-contingent securities available internationally, they are able to share risk with foreign agents. However, a fraction λ of domestic agents do not engage in financial transactions as they do not have the economic means to do so, similar to the "hand-to-mouth" consumers in Mankiw (2000). Their sole source of income is wage income. Thus, although financially-excluded agents can optimize labor supply, they are unable to share risk internationally or smooth consumption. Throughout the paper, financially-included households are denoted with the symbol, $\hat{}$, while financially-excluded households are denoted with the symbol, $\check{}$.

The domestic economy, with its heterogeneous consumers, is of measure zero compared to the world. This implies that domestic monetary policy decisions do not have any impact on the rest of the world, which consists of a continuum of economies, $j \in [0,1]$, populated by identical, financially-included households. While different economies are subject to asymmetric and imperfectly correlated shocks, initial net foreign asset positions are symmetrically zero across countries, and preferences and market structures are identical. In each period $t \geq 0$, a stochastic event, $s_t \in S$, is realized. I denote by $s^t = (s_0, ..., s_{t-1}, s_t)$ the history of events until period t. The unconditional probability, as of period 0, of observing any particular history s^t is $\mu\left(s^t\right)$. The probability of history s^{t+1}, conditional on s^t, is given by $\mu\left(s^{t+1}|s^t\right)$. The initial realization, s_0, is taken as given so that $\mu\left(s_0\right) = 1$ for a particular $s_0 \in S$.

2.1.1 Financially-Excluded Agents

Financially-excluded agents, $l \in [0, \lambda]$, fully consume their income each period and cannot smooth consumption through the financial markets. These agents gain utility from consumption, $\check{C}\left(s^t\right)$, and disutility from hours worked, $\check{N}\left(s^t\right)$. The representative household chooses its period t allocation after the realization of the event, s_t, to maximize its utility, $\check{U}(s^t) = \sum_{s^t} \mu(s^t) U\left\{\check{C}\left(s^t\right), \check{N}\left(s^t\right)\right\}$, through the following static optimization problem

$$\underset{\left\{\check{C}(s^t), \check{N}(s^t)\right\}}{Max} \check{U}(s^t) = \sum_{s^t} \mu(s^t) \left\{ \frac{\check{C}\left(s^t\right)^{1-\sigma}}{1-\sigma} - \frac{\check{N}\left(s^t\right)^{1+\phi}}{1+\phi} \right\} \tag{1}$$

$$s.t.\ P\left(s^t\right) \check{C}\left(s^t\right) \leq W\left(s^t\right) \check{N}\left(s^t\right) \tag{2}$$

where σ is the inverse intertemporal elasticity of substitution (IES), ϕ is the inverse Frisch elasticity of labor supply, $P\left(s^t\right)$ is the consumer price index (CPI), $W\left(s^t\right)$ denotes nominal wages, $\check{N}\left(s^t\right)$ represents hours of labor supplied, and $\check{C}\left(s^t\right)$ is consumption of domestically produced and imported goods from a constant elasticity of substitution (CES) consumption basket. All these variables are denominated in units of domestic currency.

Preferences are locally non-satiated, so that the budget constraint binds. This implies that the consumption of excluded households derives directly from (2). Further, as these agents do not hold bonds and thus cannot optimize intertemporally through an Euler Equation, real interest rate changes are not internalized by λ fraction of the economy. Relatedly, it is worth noting that empirical evidence on the inverse IES, σ, is unavailable for these agents. I will address this by assuming a standard calibration for σ in the optimal monetary policy analysis.

Financially-excluded households trade off leisure for consumption goods, through their optimal labor supply condition, (3), where $w\left(s^t\right)$ denotes the real wage rate, $w\left(s^t\right) = \frac{W\left(s^t\right)}{P\left(s^t\right)}$. While nominal wages, $W\left(s^t\right)$, are the same for both consumers and producers, *real* wages faced by households, $w\left(s^t\right)$, differ from the real wages faced by domestic firms, $\frac{W\left(s^t\right)}{P_H\left(s^t\right)}$, where $P_H\left(s^t\right)$ is the domestic (monopolistically set) price index. This is because households take into account the CPI, $P\left(s^t\right)$, which consists of both domestic and imported good prices. The labor supply condition, derived by combining the first-order conditions on consumption and labor, is

$$\omega\left(s^t\right) = \check{N}\left(s^t\right)^{\phi} \check{C}\left(s^t\right)^{\sigma} \tag{3}$$

2.1.2 Financially-Included Agents

Financially-included agents, $l \in [\lambda, 1]$, gain utility from consumption, $\hat{C}\left(s^t\right)$ and disutility from hours worked, $\hat{N}\left(s^t\right)$. These agents own the domestic firms, and have access to a complete portfolio of state-contingent bonds, which are traded sequentially in spot markets with foreign households in each period, $t \geq 0$, before s_{t+1} occurs. The equilibrium that arises from this assumption of sequential trading is equivalent to that which would arise from a time 0 trading of a complete set of history-contingent bonds. Notably, although financial trades are executed before uncertainty realizes, the period t allocation is chosen after event s_t occurs.

I let $B\left(s^t, s_{t+1}\right)$ denote the representative financially-included agent's holdings of a complete vector of state-contingent claims at time t, given history s^t, on time $t+1$ domestic *currency*. One unit of this vector pays off one unit of domestic currency if the particular state s_{t+1} occurs and 0 otherwise. $B\left(s^t, s_{t+1}\right)$ is denominated in units of domestic currency and each of its units is priced at $Z\left(s^{t+1}|s^t\right)$, where $s^{t+1} = \left(s^t, s_{t+1}\right)$. Taking $Z\left(s^{t+1}|s^t\right)$ as given, the representative household maximizes its utility, $\hat{U}\left(s^t\right) = \sum_{s^t} \mu(s^t) U\left\{\hat{C}\left(s^t\right), \hat{N}\left(s^t\right)\right\}$, subject to a sequence of budget constraints for $t \geq 0$, by solving the following dynamic optimization problem

$$\underset{\left\{\hat{C}(s^t), \hat{N}(s^t), B(s^t, s_{t+1})\right\}}{Max} \sum_{t=0}^{\infty} \beta^t \hat{U}(s^t) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) \left\{ \frac{\hat{C}\left(s^t\right)^{1-\sigma}}{1-\sigma} - \frac{\hat{N}\left(s^t\right)^{1+\phi}}{1+\phi} \right\}$$

$$s.t. \quad P\left(s^t\right) \hat{C}\left(s^t\right) + \sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right) B\left(s^t, s_{t+1}\right) \leq B\left(s^t\right) + W\left(s^t\right) \hat{N}\left(s^t\right) + \Omega\left(s^t\right) - T\left(s^t\right) \tag{4}$$

where $\beta^t \in [0, 1]$ is the subjective discount factor, $\hat{N}\left(s^t\right)$ represents hours of labor supplied, $\hat{C}\left(s^t\right)$ is consumption of domestically produced and imported goods from a CES consumption basket, $\Omega\left(s^t\right)$ denotes profits received from ownership of domestic firms, and $T\left(s^t\right)$ is a nominal lump-sum tax. All these variables are denominated in units of domestic currency, and the initial condition, $B(s_0)$, is taken as given. The budget constraint binds, and as standard in the optimal policy literature, the lump-sum tax is used to finance a constant wage subsidy, τ^e, to offset the steady state monopolistic distortion

$$T\left(s^t\right) = \tau^e W\left(s^t\right) N\left(s^t\right) \tag{5}$$

where $N\left(s^t\right)$ is labor demanded by the monopolistic producers. Each period, financially-included agents trade off leisure for consumption goods through their optimal labor supply condition, (6), derived by combining the first-order conditions on consumption and labor

$$\omega\left(s^t\right) = \hat{N}\left(s^t\right)^{\phi} \hat{C}\left(s^t\right)^{\sigma} \tag{6}$$

The first-order conditions on state-contingent bonds and consumption can be combined to yield the Euler Equation

$$Z\left(s^{t+1}|s^t\right) = \beta\mu\left(s^{t+1}|s^t\right)\left[\frac{\hat{C}\left(s^{t+1}\right)}{\hat{C}\left(s^t\right)}\right]^{-\sigma}\frac{1}{\Pi\left(s^{t+1}\right)} \tag{7}$$

where $\Pi\left(s^{t+1}\right) = \frac{P\left(s^{t+1}\right)}{P\left(s^t\right)}$ is CPI inflation. A similar equilibrium condition with respect to domestic currency state-contingent bonds, adjusted for the presence of the nominal bilateral exchange rate, $\varepsilon^j\left(s^t\right)$, to ensure price equalization across the world in contingent claims, holds for the representative household in country j

$$Z\left(s^{t+1}|s^t\right) = \beta\mu\left(s^{t+1}|s^t\right)\left[\frac{C^j\left(s^{t+1}\right)}{C^j\left(s^t\right)}\right]^{-\sigma}\frac{\varepsilon^j\left(s^{t+1}\right)}{\varepsilon^j\left(s^t\right)}\frac{1}{\Pi^j\left(s^{t+1}\right)} \tag{8}$$

where $C^j\left(s^t\right)$, $\Pi^j\left(s^t\right)$, and $\varepsilon^j\left(s^t\right)$ denote the consumption basket, CPI inflation, and bilateral nominal exchange rate of country j.[3] Combining the Euler Equation of the domestic household, (7), with that of each foreign household $j \in [0,1]$, (8), and integrating across foreign households, $C^*\left(s^t\right) = \int_0^1 C_j\left(s^t\right) dj$, yields the Backus-Smith (Backus and Smith, 1993) international risk-sharing condition from the perspective of the domestic small open economy

$$\hat{C}(s^t) = vC^*Q(s^t)^{\frac{1}{\sigma}} \tag{9}$$

which assumes that agents across the world make appropriate ex-ante international insurance transfers through complete financial markets, to ensure that risk is pooled internationally, ie. $v = 1$, and that world consumption is exogenous and constant. The precise value of the insurance transfers to the domestic economy is given in equation (32) in section 2.6. International risk-sharing, (9), means that the marginal utility of consumption, weighted by the real effective exchange rate, $Q(s^t) = \varepsilon(s^t)\frac{P^*(s^t)}{P(s^t)}$, is equalized across countries, where $P^*(s^t)$ is the world price index and $\varepsilon(s^t)$ is the nominal effective exchange rate ie. an index of the prices of foreign currencies $j \in [0,1]$ in terms of domestic currency.

Efficient risk-sharing thus implies that demand, by financially-included agents worldwide, is directed at countries where it is cheaper to consume. Risk-sharing is possible because of the timing of financial trades. Financial markets open before monetary policy decisions are made, implying that financially-included agents are able to smooth consumption in the face of uncertainty implied by choice of monetary regime. Further, this pooling of risk with foreign agents insures financially-included agents in an open economy, compared to the closed economy case (without capital) where it is not possible to insure against aggregate uncertainty.

[3]The representative household in country j faces the following budget constraint in each period $t \geq 0$: $P^j\left(s^t\right)C^j\left(s^t\right) + \varepsilon^j\left(s^t\right)\sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right)B^j\left(s^t, s_{t+1}\right) \leq \varepsilon^j\left(s^t\right)B^j\left(s^t\right) + W^j\left(s^t\right)N^j\left(s^t\right) + \Omega^j\left(s^t\right) - T^j\left(s^t\right)$, where $B^j\left(s^t\right)$ represents the foreign household's holdings of the state-contingent bond denominated in units of domestic currency and $\varepsilon^j\left(s^t\right)$ is the bilateral nominal exchange rate with respect to the domestic economy ie. the price of domestic currency in units of foreign currency. $\varepsilon^j\left(s^t\right)$ serves to convert the domestic currency payoffs into foreign currency.

I assume that financial asset market arbitrage opportunities do not exist, as these would lead to indeterminacy in the international portfolio allocation problem. This implies that the equilibrium prices - in domestic currency - of risk-free one-period uncontingent nominal bonds at home and in foreign country j, are related to their gross returns as follows

$$\sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right) = \frac{1}{1+i\left(s^t\right)} \qquad \sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right) \varepsilon_j\left(s^{t+1}\right) = \frac{\varepsilon\left(s^t\right)}{1+i_j\left(s^t\right)} \tag{10}$$

where the domestic currency price of a domestic bond, $\sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right)$, is inversely related to the gross domestic nominal interest rate, $1+i\left(s^t\right)$, and the domestic currency price of a foreign bond, $\sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right)\varepsilon_j(s^{t+1})$, is inversely related to the gross foreign nominal interest rate, $1+i_j\left(s^t\right)$, adjusted by the bilateral nominal exchange rate. Domestic monetary policy has direct leverage over $i\left(s^t\right)$. Combing the domestic and foreign bond pricing equations, and aggregating across countries $j \in [0,1]$, yields the uncovered interest rate parity (UIP) condition

$$\sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right)\left\{\left(1+i\left(s^t\right)\right) - \left(1+i^*\left(s^t\right)\right)\frac{\varepsilon\left(s^{t+1}\right)}{\varepsilon\left(s^t\right)}\right\} = 0 \tag{11}$$

where $i^*\left(s^t\right) = \int_0^1 i_j\left(s^t\right) dj$ is the world nominal interest rate, taken as given by the domestic small open economy. UIP formalizes the lack of arbitrage opportunities in the international asset markets, by implying that the nominal exchange rate is expected to adjust upon shocks to equalize the domestic currency returns on domestic and foreign contingent bonds.

2.1.3 Aggregation

A perfectly competitive firm uses CES technology to produce an aggregate consumption good, $C\left(s^t\right)$, from domestic and foreign aggregates, respectively $C_H(s^t)$ and $C_F(s^t)$. The aggregate consumption good is purchased by financially-excluded and financially-included agents according to $C\left(s^t\right) = \lambda\check{C}\left(s^t\right) + (1-\lambda)\hat{C}\left(s^t\right)$, and is produced as follows

$$C\left(s^t\right) = \left[(1-\alpha)^{\frac{1}{\varepsilon_I}} C_H\left(s^t\right)^{\frac{\varepsilon_I-1}{\varepsilon_I}} + \alpha^{\frac{1}{\varepsilon_I}} C_F\left(s^t\right)^{\frac{\varepsilon_I-1}{\varepsilon_I}}\right]^{\frac{\varepsilon_I}{\varepsilon_I-1}} \tag{12}$$

where $\alpha \in [0,1]$ is the share of imports in the CES basket and represents the degree of openness to international trade in goods (conversely, $1-\alpha$ is the degree of home bias), and ε_I is the elasticity of substitution between domestic and foreign goods. The associated price index is

$$P\left(s^t\right) = \left[(1-\alpha)P_H^{\varepsilon_I}\left(s^t\right) + \alpha P_F^{1-\varepsilon_I}\left(s^t\right)\right]^{\frac{1}{1-\varepsilon_I}} \tag{13}$$

where $P\left(s^t\right)$ is the consumer price index (CPI), $P_H\left(s^t\right)$ is the price index for domestic goods, and $P_F\left(s^t\right)$ is the import price index. Minimizing expenditure on the CES consumption basket (12) gives rise to the following downward-sloping demand functions for domestic and imported aggregates

$$C_H\left(s^t\right) = (1-\alpha)\left[\frac{P_H\left(s^t\right)}{P\left(s^t\right)}\right]^{-\varepsilon_I} C\left(s^t\right) \qquad C_F\left(s^t\right) = \alpha\left[\frac{P_F\left(s^t\right)}{P\left(s^t\right)}\right]^{-\varepsilon_I} C\left(s^t\right) \tag{14}$$

where $C_H\left(s^t\right) = \lambda\check{C}_H\left(s^t\right) + (1-\lambda)\hat{C}_H\left(s^t\right)$ and $C_F\left(s^t\right) = \lambda\check{C}_F\left(s^t\right) + (1-\lambda)\hat{C}_F\left(s^t\right)$. The imported good, $C_F\left(s^t\right)$ comprises of goods from each foreign country, j, and ε_F denotes the elasticity of substitution between imported goods

$$C_F\left(s^t\right) = \left[\int_0^1 C_j\left(s^t\right)^{\frac{\varepsilon_F-1}{\varepsilon_F}} dj\right]^{\frac{\varepsilon_F}{\varepsilon_F-1}} \tag{15}$$

The subcomponents of the domestic and imported good indices measure domestic consumption of individual varieties of goods, i. These varieties are produced at home as well as in the continuum of foreign small open economies, $j \in [0,1]$, so that $C_H\left(i,s^t\right)$ is the consumption of domestic variety i and $C_j\left(i,s^t\right)$ is the consumption of variety i imported from country j

$$C_H\left(s^t\right) = \left[\int_0^1 C_H\left(i,s^t\right)^{\frac{\varepsilon_p-1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p-1}} \qquad C_j\left(s^t\right) = \left[\int_0^1 C_j\left(i,s^t\right)^{\frac{\varepsilon_p-1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p-1}} \tag{16}$$

where, due to the assumption of identical market structures across countries, ε_p is the elasticity of substitution between individual varieties, i, produced in any country j, including home. The price indices associated with the demand for domestic and country j's varieties are

$$P_H\left(s^t\right) = \left[\int_0^1 P_H\left(i,s^t\right)^{1-\varepsilon_p} di\right]^{\frac{1}{1-\varepsilon_p}} \qquad P_j\left(s^t\right) = \left[\int_0^1 P_j\left(i,s^t\right)^{1-\varepsilon_p} di\right]^{\frac{1}{1-\varepsilon_p}} \tag{17}$$

2.2 Relative Prices and Exchange Rates

The model equilibrium will be defined in terms of the effective terms of trade, $X\left(s^t\right) = \frac{P_F\left(s^t\right)}{P_H\left(s^t\right)}$, which is an index of the bilateral terms of trade between the domestic economy and all foreign economies $j \in [0,1]$. That is, $X\left(s^t\right) = \left(\int_0^\infty X_{jH}\left(s^t\right)^{1-\varepsilon_F} dj\right)^{\frac{1}{1-\varepsilon_F}}$, where $X_{jH}\left(s^t\right) = P_j\left(s^t\right)/P_H\left(s^t\right)$ denotes the price of country j's goods in terms of domestic goods.

To put prices in terms of $X\left(s^t\right)$, I first normalize all prices by the CPI, (13), to define them in relative terms. These are the relative domestic price index, $p_H\left(s^t\right) = P_H\left(s^t\right)/P\left(s^t\right)$, and the relative import price index, $p_F\left(s^t\right) = P_F\left(s^t\right)/P\left(s^t\right)$. As functions of the effective terms of trade, $X\left(s^t\right)$, these are written as

$$p_H\left(s^t\right) = \left[1-\alpha+\alpha X\left(s^t\right)^{1-\varepsilon_I}\right]^{-\frac{1}{1-\varepsilon_I}} \qquad p_F\left(s^t\right) = X\left(s^t\right)\left[1-\alpha+\alpha X\left(s^t\right)^{1-\varepsilon_I}\right]^{-\frac{1}{1-\varepsilon_I}} \tag{18}$$

The effective terms of trade, $X\left(s^t\right)$, can thus be written as a ratio of relative prices

$$X\left(s^t\right) = \frac{p_F\left(s^t\right)}{p_H\left(s^t\right)} \tag{19}$$

The nominal effective exchange rate, $\varepsilon\left(s^t\right) = \left(\int_0^1 \varepsilon_j\left(s^t\right)^{1-\varepsilon_F} dj\right)^{\frac{1}{1-\varepsilon_F}}$, is an index of the nominal bilateral exchange rates between foreign countries, $j \in [0,1]$, and the domestic economy. The nominal bilateral exchange rate between home and any country j, $\varepsilon_j\left(s^t\right)$, is the price of foreign currency, j, in units of domestic currency. $P^*\left(s^t\right) = \int_0^1 P^j\left(s^t\right) dj$ is the world price index, where $P^j\left(s^t\right)$ is the CPI in country j. The real effective CPI exchange rate

$$Q\left(s^t\right) = \frac{\varepsilon\left(s^t\right) P^*\left(s^t\right)}{P\left(s^t\right)}$$

is defined as the domestic currency price of a foreign basket of consumption, $\varepsilon\left(s^t\right) P^*\left(s^t\right)$, relative to the domestic currency price of a domestic basket of consumption, $P\left(s^t\right)$. $Q\left(s^t\right) = \left(\int_0^1 Q_j\left(s^t\right)^{1-\varepsilon_F} dj\right)^{\frac{1}{1-\varepsilon_F}}$, where $Q_j\left(s^t\right)$ is the real bilateral exchange rate between home and country j. $Q\left(s^t\right)$ can be expressed in terms of $X\left(s^t\right)$. Assuming that purchasing power parity holds in the imported goods market, so that $P_F\left(s^t\right) = \varepsilon\left(s^t\right) P_F^*\left(s^t\right)$, and that from the perspective of the domestic economy, which is small, the world as a whole behaves like a closed economy, so that $P^*\left(s^t\right) = P_F^*\left(s^t\right)$, I derive

$$Q\left(s^t\right) = \left[(1-\alpha)X\left(s^t\right)^{\varepsilon_I - 1} + \alpha\right]^{\frac{1}{\varepsilon_I - 1}} \tag{20}$$

Note that the real exchange rate is a function of the effective terms of trade in this framework, so that these two relative prices co-move upon shocks.

2.3 Firms

Goods are supplied to domestic and foreign consumers. Firms, $i \in [0,1]$, are monopolistic and set prices in a staggered fashion. In any given period and independent of time elapsed since last reset, a fraction $(1-\theta)$ of (randomly selected) firms can re-optimize prices. Fraction θ of firms cannot re-optimize and instead adjust labor demand to meet changes in output demand upon shocks. Firms that do reset prices upon shocks take into account that the probability of keeping this period's price k periods ahead is given by θ^k.

With production function $Y(i,s^t) = A\left(s^t\right) N\left(i,s^t\right)$, each reoptimizing firm i sets its optimal reset price as a markup over current and expected marginal costs, where $MC\left(i,s^t\right) = W\left(s^t\right)/A\left(s^t\right)$, giving rise to domestic inflation. Noting that a firm that reoptimizes in period t will choose the price $P_H^*\left(i,s^t\right)$ that maximizes current and future expected discounted profits until period $t+k$ while this price remains effective, so that $P_H^*\left(i,s^{t+k}\right) = P_H^*\left(i,s^t\right)$ for $k = 0,...,\infty$, the optimal reset price at time t

solves the following problem

$$\underset{P_H^*(i,s^t)}{Max} \sum_{k=0}^{\infty} \sum_{s^{t+k}} Z\left(s^{t+k}|s^t\right)\theta^k \left\{(1+\tau^e)P_H^*\left(i,s^t\right)Y\left(i,s^{t+k}\right) - W\left(s^{t+k}\right)N\left(i,s^{t+k}\right)\right\}$$

$$s.t.\;\; Y\left(i,s^{t+k}\right) = \left(\frac{P_H^*\left(i,s^t\right)}{P_H\left(s^t\right)}\right)^{-\varepsilon_p}\left(C_H\left(i,s^{t+k}\right) + \int_0^1 C_H^j\left(i,s^{t+k}\right)dj\right) \tag{21}$$

where $Z\left(s^{t+k}|s^t\right)$ is the stochastic discount factor (as households own the firms) in period $t+k$ given history s^t (recall that $s^{t+k} = \left(s^t, s_{t+k}\right)$), τ^e is a steady state wage subsidy, $Y\left(i,s^{t+k}\right)$ and $W\left(s^{t+k}\right)N\left(i,s^{t+k}\right)$ are respectively the output and total cost in period $t+k$ for a firm that last reset its price in period t, and $C_H\left(i,s^{t+k}\right)$ and $\int_0^1 C_H^j\left(i,s^{t+k}\right)dj$ represent demand for good i in period $t+k$ respectively by domestic consumers and foreign consumers in countries j. In a symmetric equilibrium, the same price is chosen by all firms that can re-optimize so that $P_H^*\left(i,s^t\right) = P_H^*\left(s^t\right)\forall i$. The first-order condition is

$$\sum_{k=0}^{\infty}\sum_{s^{t+k}} Z\left(s^{t+k}|s^t\right)\theta^k Y\left(i,s^{t+k}\right)\left[P_H^*\left(s^t\right) - \frac{1}{1+\tau^e}\frac{\varepsilon_P}{\varepsilon_P - 1}\frac{W\left(s^{t+k}\right)}{A\left(s^{t+k}\right)}\right] = 0 \tag{22}$$

Using (21), the labor-market clearing condition $N_t = \int_0^1 N\left(i,s^t\right)di$, the price index associated with the demand for monopolistic goods $P_H\left(s^t\right) = \left[\int_0^1 P_H\left(i,s^t\right)^{1-\varepsilon_p}di\right]^{\frac{1}{1-\varepsilon_p}}$, and the definition of price dispersion, $\Delta\left(s^t\right) \equiv \int_0^1\left(\frac{P_H\left(i,s^t\right)}{P_H\left(s^t\right)}\right)^{-\varepsilon_p}di$, which follows law of motion

$$\Delta\left(s^t\right)^{\frac{\varepsilon_P-1}{\varepsilon_P}} = \theta\pi_H\left(s^t\right)^{\varepsilon_P-1}\Delta\left(s^{t-1}\right) + (1-\theta)\frac{1-\theta\Pi_H\left(s^t\right)^{\varepsilon_P-1}}{1-\theta} \tag{23}$$

it is possible to derive (24), which resembles an aggregate production function, but takes into account the technological restrictions and distributional inefficiencies associated with price dispersion

$$Y\left(s^t\right)\Delta\left(s^t\right) = A\left(s^t\right)N\left(s^t\right) \tag{24}$$

where productivity shocks, $A\left(s^t\right)$, are determined relative to their steady state value A and follow the following stationary autoregressive process

$$Ln\left(1+A\left(s^t\right)\right) - Ln(1+A) = \rho_A Ln\left(1+A\left(s^{t-1}\right)\right) - Ln(1+A) + s_t^A$$

with $\rho_A \in (0,1)$ and $s_t^A \sim N(0,\sigma_A^2)$. Using the price index and definition of price dispersion, the optimal reset price relates to the domestic inflation rate, $\Pi_H\left(s^t\right) = \frac{P_H\left(s^t\right)}{P_H\left(s^{t-1}\right)}$, as follows

$$\frac{P_H^*\left(s^t\right)}{P_H\left(s^t\right)} = \left(\frac{1-\theta\Pi_H\left(s^t\right)^{\varepsilon_P-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon_P}}$$

From the first-order condition, (22), the optimal reset price is related to aggregate variables as

$$\frac{P_H^*\left(s^t\right)}{P_H\left(s^t\right)}=\frac{F\left(s^t\right)}{K\left(s^t\right)} \tag{25}$$

where $F\left(s^t\right)$ is the present discounted value of total costs in real terms

$$F\left(s^t\right)=\frac{1}{1+\tau^e}\frac{\varepsilon_P}{\varepsilon_P-1}\frac{Y\left(s^t\right)}{C\left(s^t\right)^\sigma}\frac{MC\left(s^t\right)}{X\left(s^t\right)^\alpha}+\beta\theta\mu\left(s^{t+1}|s^t\right)\Pi_H\left(s^{t+1}\right)^{\varepsilon_P}F\left(s^{t+1}\right) \tag{26}$$

and $K\left(s^t\right)$ is the present discounted value of total revenues in real terms

$$K\left(s^t\right)=\frac{Y\left(s^t\right)X\left(s^t\right)^{-\alpha}}{C\left(s^t\right)^\sigma}+\beta\theta\mu\left(s^{t+1}|s^t\right)\Pi\left(s^{t+1}\right)^{\varepsilon_P-1}K\left(s^{t+1}\right) \tag{27}$$

Equations (25) to (27) summarize the recursive representation of the non-linear Phillips Curve, as in Benigno and Woodford (2005).[4] The monopolistic sector faces exogenous cost-push shocks which directly increase domestic inflation without excess aggregate demand pressures. These disturbances represent unanticipated food price or fuel input price shocks that arise in markets outside the authority's control. As standard in the literature, and discussed further in Sutherland (2005), $V\left(s^t\right)$ is the net monopolistic markup, $\frac{\varepsilon_P}{\varepsilon_P-1}\frac{1}{1+\tau^e}$. Markup shocks, s_t^V, are assumed to arise from random changes in the production subsidy or the degree of monopoly power. Cost-push/ markup shocks are determined relative to their steady state value V and follow the following stationary autoregressive process

$$Ln\left(1+V\left(s^t\right)\right)-Ln(1+V)=\rho_V Ln\left(1+V\left(s^{t-1}\right)\right)-Ln(1+V)+s_t^V$$

where $\rho_V\in(0,1)$ and $s_t^V\sim N(0,\sigma_V^2)$.

2.4 Central Bank

Monetary policy is set according to either simple rules or a model-specific optimized rule. The strict simple rules (in linearized terms) are domestic inflation targeting (DIT), $\pi_H\left(s^t\right)=0$, CPI inflation targeting (DIT), $\pi\left(s^t\right)=0$, and a fixed exchange rate, $\Delta e\left(s^t\right)=0$, and the flexible ones are Taylor-type rules where the nominal interest rate responds to a measure of inflation and the output gap.

[4] The corresponding linearized version of the Phillips curve takes the familiar form: $\pi_{H,t}=\beta\pi_{H,t+1}+\xi mc_t+v_t$, where $\xi=\frac{(1-\beta\theta)(1-\theta)}{\theta}$ and v_t is a cost-push shock that is appended on, as standard in the literature.

2.5 Market-Clearing and Accounting

The demand for each monopolistic good $i \in [0,1]$ is[5]

$$Y_t\left(i,s^t\right) = C_H\left(i,s^t\right) + \int_0^1 C_H^j\left(i,s^t\right)dj$$

where $C_H\left(i,s^t\right)$ is consumption of home good i by domestic consumers and $C_H^j\left(i,s^t\right)$ denotes consumption of home good i by country j. Replacing in the expressions for the consumption of individual varieties, i, international risk-sharing, and the definitions of the bilateral and effective terms of trade, I write the above as

$$Y\left(s^t\right) = \left(\frac{P_H\left(s^t\right)}{P\left(s^t\right)}\right)^{-\varepsilon_I}\left[(1-\alpha)C\left(s^t\right) + \alpha\hat{C}\left(s^t\right)\int_0^1 Q_j\left(s^t\right)^{\varepsilon_I-\frac{1}{\sigma}}\left(X_j\left(s^t\right)X^j\left(s^t\right)\right)^{\varepsilon_F-\varepsilon_I}dj\right] \tag{28}$$

where $X_j\left(s^t\right)$ and $X_j\left(s^t\right)$ denote bilateral variables for the domestic economy, and $X^j\left(s^t\right)$ denotes the effective terms of trade for country j. For $\sigma = \varepsilon_I = \varepsilon_F = 1$, as in the welfare analysis, $Y\left(s^t\right) = X\left(s^t\right)^{\alpha}\left[(1-\alpha)C\left(s^t\right) + \alpha\hat{C}\left(s^t\right)\right]$. The labor market is Walrasian, with the real wage, $w\left(s^t\right)$, moving instantly to clear demand and supply imbalances

$$\lambda\check{N}\left(s^t\right) + (1-\lambda)\hat{N}\left(s^t\right) = N\left(s^t\right) = \int_0^1 N\left(i,s^t\right)di \tag{29}$$

where $\int_0^1 N_t\left(i,s^t\right))di$ is the demand for labor by each firm, i. Aggregate consumption is a weighted average of consumption by financially-included and financially-excluded agents

$$C\left(s^t\right) = \lambda\check{C}\left(s^t\right) + (1-\lambda)\hat{C}\left(s^t\right) \tag{30}$$

The real trade balance, in terms of the CPI, $P\left(s^t\right)$, is defined as the imbalance between domestic production and consumption, and given by

$$NX\left(s^t\right) = p_H\left(s^t\right)Y\left(s^t\right) - \left(\lambda\check{C}\left(s^t\right) + (1-\lambda)\hat{C}\left(s^t\right)\right) \tag{31}$$

The trade balance is non-zero, unlike Gali and Monacelli (2005), which is nested when $\lambda = 0$ in this framework. This point is independent of calibration, but is seen more easily for $\sigma = \varepsilon_I = \varepsilon_F = 1$, as in the welfare analysis, so that $NX\left(s^t\right)$ boils down to $NX\left(s^t\right) = \alpha\lambda\left(\hat{C}\left(s^t\right) - \check{C}\left(s^t\right)\right)$ upon replacing out for (28) in (31). The fact that $NX\left(s^t\right)$ is away from zero is driven by $\lambda > 0$, ie. a positive degree of financial exclusion, and has implications for the domestic net foreign asset position.

Net foreign assets, $NFA\left(s^t\right)$ are non-zero upon shocks (they are zero in the initial steady state), and quantify the net present value of insurance transfers to the domestic financially-included house-

[5] $C_H\left(i,s^t\right) = (1-\alpha)\left(\frac{P_H(i,s^t)}{P_H(s^t)}\right)^{-\varepsilon_p}\left(\frac{P_H(s^t)}{P(s^t)}\right)^{-\varepsilon_H}C\left(s^t\right)$ and $C_H^j\left(i,s^t\right) = \alpha\left(\frac{P_H(i,s^t)}{P_H(s^t)}\right)^{-\varepsilon_p}\left(\frac{P_H(s^t)}{\varepsilon_j(s^t)P_F^j(s^t)}\right)^{-\varepsilon_F}\left(\frac{P_F^j(s^t)}{P^j(s^t)}\right)^{-\varepsilon_H}C\left(s^t\right)$

holds when uncertainty realizes. In each period, the stock of $NFA\left(s^t\right)$, normalized by the CPI, $P\left(s^t\right)$, derived by iterating the financially included household's budget constraint, is given by

$$\begin{aligned} NFA\left(s^t\right) &= -\sum_{k=0}^{\infty}\sum_{s^{t+k}} Z\left(s^{t+k}|s^t\right)\Big[NX\left(s^{t+k}\right) \\ &\quad + \left(C\left(s^{t+k}\right)-\hat{C}\left(s^{t+k}\right)\right) - w\left(s^{t+k}\right)\left(N\left(s^{t+k}\right)-\hat{N}\left(s^{t+k}\right)\right)\Big] \end{aligned} \tag{32}$$

Equation (32) is a useful summary of the difference between closed economies with financially-excluded agents, for example, Galí et al. (2007) and Bilbiie (2008), and an open economy. It arises because of (i) international risk-sharing and the associated insurance transfers, possible only in an open economy and (ii) the fact that the trade balance is not zero with $\lambda > 0$. If $\lambda = 0$, then $NFA\left(s^t\right) = 0$ in each period, as in Gali and Monacelli (2005), and the open economy model is isomorphic to the closed case.

In the current framework, however, as only a fraction of domestic agents can share risk with foreign agents, open economy elements do not affect the two types of agents in a symmetric fashion. In particular, financially-included agents can share risk and smooth consumption upon exchange rate fluctuations, whereas financially-excluded agents cannot. This fundamental asymmetry, connected with international risk-sharing, is why $NFA\left(s^t\right) \neq 0$ and optimal policy in an open economy model with financially-excluded agents is not isomorphic to the closed economy case.

2.6 Equilibrium

For a particular specification of monetary policy (which determines the nominal interest rate, $i\left(s^t\right)$), an equilibrium for the model is a state-contingent sequence of prices

$$\left\{X\left(s^t\right), Z\left(s^{t+1}|s^t\right), \Pi\left(s^t\right), \Pi_H\left(s^t\right), MC\left(s^t\right), \Delta\left(s^t\right), \varepsilon\left(s^t\right)\right\}_{t=0}^{\infty}$$

and quantities

$$\left\{\check{C}\left(s^t\right), \hat{C}\left(s^t\right), \check{N}\left(s^t\right), \check{N}\left(s^t\right), C_H\left(s^t\right), C_F\left(s^t\right), C\left(s^t\right), N\left(s^t\right), Y\left(s^t\right), F\left(s^t\right), K\left(s^t\right)\right\}_{t=0}^{\infty}$$

such that

- Goods, (28) and labor, (29) markets clear
- International-risk sharing, (9), and no asset market arbitrage, (11), conditions hold
- Financially-excluded agents, $l \in [0, \lambda]$, optimize: (2) and (3)
- Financially-included agents, $l \in [\lambda, 1]$, optimize: (6) and (7)
- Aggregate consumer optimization of domestic and foreign goods yields: (12) and (14)
- Firms, $j \in [0, 1]$, optimize: (24), (25), (26), and (27)

taking as given initial conditions, $B\left(s^0\right)$, $\Delta\left(s^0\right)$, $X\left(s^0\right)$, and exogenous processes for shocks and foreign quantities $\left\{A\left(s^t\right), V\left(s^t\right), i^*\left(s^t\right), P^*\left(s^t\right)\right\}_{t=0}^{\infty}$. The effective terms of trade, $X\left(s^t\right)$, given by (19), is the only relative price that matters for the characterization of equilibrium.

3 Optimal Monetary Policy

I proceed to characterize the optimal monetary policy for this model. I provide a generalized analysis of the optimum, before specifically analyzing cost-push shocks and ranking simple rules. Some salient results emerge. These include the insurance role played by optimal monetary policy in the presence of nominal rigidities and financial exclusion, and the desirability of stabilizing the nominal exchange rate in these circumstances. I focus on the Cole-Obstfeld parameterization (Cole and Obstfeld, 1991), $\sigma = \varepsilon_I = \varepsilon_F = 1$, to keep the analysis tractable. Two flexible price allocations are characterized, to serve as references, before turning to the constrained efficient case where monetary policy plays a role.

3.1 Flexible Prices

I begin by describing the efficient allocation of economic resources in the absence of market imperfections. This is the allocation away from which optimal monetary policy, in the presence of real and nominal rigidities, seeks to minimize deviations. The efficient allocation corresponds to the solution of a Planner's problem, who maximizes a weighted sum of household utilities and faces the flexible price and perfectly competitive versions of the constraints in the optimal monetary policy problem. The efficient allocation can be decentralized through a wage subsidy, funded through lump-sum taxes on the financially-included household, (5).

I will show that the efficient allocation does not coincide with the flexible price version of the model, sometimes called the natural allocation, implying that the flexible price business cycle and steady state of the model are inefficient. I choose an appropriate wage subsidy, τ^e, to implement the efficient allocation at the steady state. To derive the efficient allocation, the Planner maximizes a weighted sum of household utilities, $U\left(s^t\right) = \lambda \check{U}\left(s^t\right) + (1-\lambda)\hat{U}\left(s^t\right)$, with weights given by λ, and where $\chi_1\left(s^t\right) - \chi_8\left(s^t\right)$ are Lagrange multipliers attached to the constraints.

$$\underset{\Sigma(s^t)}{Max}\, U\left(s^t\right) = \lambda\left(Ln\check{C}^e\left(s^t\right) - \frac{\check{N}^e\left(s^t\right)^{1+\phi}}{1+\phi}\right) + (1-\lambda)\left(Ln\hat{C}^e\left(s^t\right) - \frac{\hat{N}^e\left(s^t\right)^{1+\phi}}{1+\phi}\right)$$

$$
\begin{array}{llcl}
s.t. & \left[\chi_1\left(s^t\right)\right] \quad C^e\left(s^t\right) & = & \lambda \check{C}^e\left(s^t\right)+(1-\lambda)\hat{C}^e\left(s^t\right) \\
& \left[\chi_2\left(s^t\right)\right] \quad N^e\left(s^t\right) & = & \lambda \check{N}^e\left(s^t\right)+(1-\lambda)\hat{N}^e\left(s^t\right) \\
& \left[\chi_3\left(s^t\right)\right] \quad Y^e\left(s^t\right) & = & A\left(s^t\right) N^e\left(s^t\right) \\
& \left[\chi_4\left(s^t\right)\right] \quad w^e\left(s^t\right) & = & \check{C}^e\left(s^t\right) \check{N}^e\left(s^t\right)^\phi \\
& \left[\chi_5\left(s^t\right)\right] \quad w^e\left(s^t\right) & = & \hat{C}^e\left(s^t\right) \hat{N}^e\left(s^t\right)^\phi \\
& \left[\chi_6\left(s^t\right)\right] \quad \check{C}^e\left(s^t\right) & = & w^e\left(s^t\right) \check{N}^e\left(s^t\right) \\
& \left[\chi_7\left(s^t\right)\right] \quad \hat{C}^e\left(s^t\right) & = & X^e\left(s^t\right)^{1-\alpha} \\
& \left[\chi_8\left(s^t\right)\right] \quad Y^e\left(s^t\right) & = & \alpha X^e\left(s^t\right)^\alpha \hat{C}^e\left(s^t\right)+(1-\alpha) X^e\left(s^t\right)^\alpha C^e\left(s^t\right)
\end{array}
$$

where $\Sigma\left(s^t\right)=\left\{\hat{C}^e\left(s^t\right), \check{C}^e\left(s^t\right), C^e\left(s^t\right), \hat{N}^e\left(s^t\right), \check{N}^e\left(s^t\right), N^e\left(s^t\right), \omega^e\left(s^t\right), Y^e\left(s^t\right), X^e\left(s^t\right)\right\}$. The optimum derives as a non-linear system of seventeen endogenous variables, which solve an equilibrium with nine first-order conditions and eight constraints. Further to some manipulations, it is possible to summarize the equilibrium in a more compact system of three equations with three unknowns: $X^e\left(s^t\right)$, $\hat{N}^e\left(s^t\right)$, and the shadow value of the resource constraint, $\chi_8\left(s^t\right)$.

$$
\begin{aligned}
X^e\left(s^t\right)-A\left(s^t\right) \hat{N}^{e-\phi}\left(s^t\right) & = 0 \\
\frac{1}{X^e\left(s^t\right)}-\chi_8\left(s^t\right)\left(\lambda \hat{N}^e\left(s^t\right)^\phi+(1-\lambda)+\frac{\alpha}{1-\alpha}\right) & = 0 \\
\frac{\lambda \phi \hat{N}^e\left(s^t\right)^{-1}-(1-\lambda) \hat{N}^e\left(s^t\right)^\phi-\chi_8\left(s^t\right) A\left(s^t\right)(1-\lambda)}{X^e\left(s^t\right)(1-\alpha) \phi \lambda}-\chi_8\left(s^t\right) & = 0
\end{aligned}
$$

Proposition 1 gives the closed-form expressions for the efficient labor allocation, $N^e\left(s^t\right)$, the financially-included agent's consumption, $\hat{C}^e\left(s^t\right)$, and the financially-excluded agent's consumption, $\check{C}^e\left(s^t\right)$.

Proposition 1. **(**Efficient Allocation) *In the first-best allocation for the economy when prices are flexible and firms operate in perfect competition, employment is given by*

$$
\begin{aligned}
& \lambda \phi\left(\frac{N^e\left(s^t\right)}{1-\lambda}-\frac{\lambda}{1-\lambda}\right)^{-1}-(1-\lambda)\left(\frac{N^e\left(s^t\right)}{1-\lambda}-\frac{\lambda}{1-\lambda}\right)^\phi \\
= & \frac{(1-\alpha) \phi \lambda-(1-\lambda)\left(\frac{N^e\left(s^t\right)}{1-\lambda}-\frac{\lambda}{1-\lambda}\right)^\phi}{\lambda\left(\frac{N^e\left(s^t\right)}{1-\lambda}-\frac{\lambda}{1-\lambda}\right)^\phi+(1-\lambda)+\frac{\alpha}{1-\alpha}}
\end{aligned} \tag{33}
$$

and financially-included and financially-excluded consumption are functions of aggregate employment

$$\begin{aligned}\check{C}^e\left(s^t\right) &= A\left(s^t\right)\left(\frac{N^e\left(s^t\right)}{1-\lambda}-\frac{\lambda}{1-\lambda}\right)^{\phi\alpha} \\ \hat{C}^e\left(s^t\right) &= A\left(s^t\right)\left(\frac{N^e\left(s^t\right)}{1-\lambda}-\frac{\lambda}{1-\lambda}\right)^{-\phi(1-\alpha)}\end{aligned}$$

Closed-form solutions for the other endogenous variables, as well as the Lagrange multipliers, are backed out from the remaining constraints and optimality conditions.

The solution to the Planner's problem with heterogeneous agents, when $\lambda = 0$, nests Gali and Monacelli (2005) where $N^e\left(s^t\right) = (1-\alpha)^{\frac{1}{1+\phi}}$. Furthermore, note that the consumption of financially-included and financially-excluded agents differ as only $1-\lambda$ fraction of the economy shares risk internationally. This is in contrast to the closed-economy case of Bilbiie (2008), where consumption of both agents are limited to be identical in both flexible price allocations. The competitive allocation for the flexible price economy derives from the definition of equilibrium provided in the previous section, and is given in closed-form in Proposition 2.

Proposition 2. (Natural Allocation) *In the second-best allocation with flexible prices and monopolistic competition, employment is given by*

$$N^n\left(s^t\right)^{\phi}\left(N^n\left(s^t\right)-(1-\alpha)\lambda\frac{\varepsilon_P-1}{\varepsilon_P}\right) = \frac{\varepsilon_P-1}{\varepsilon_P}(1-\lambda)^{\phi}\left((1-\alpha)(1-\lambda)+\alpha\right) \tag{34}$$

Other variables are backed out in closed-form from the decentralized equilibrium conditions. The subsidy τ^e that restores steady state efficiency is of size $\tau^e = 1-\frac{\varepsilon_P-1}{\varepsilon_P}/(1-\alpha)$.

Monetary policy does not play a stabilization role in either of the flexible price allocations since prices can instantly jump. When prices are sticky, however, the relative price distortions that arise due to sluggish real adjustment, justify policy intervention. Upon trade-off creating shocks, welfare with sticky prices is strictly lower than with flexible prices, and for the purposes of comparison, it will be useful to express the allocation in presence of nominal rigidities as log-deviations, $\tilde{x}_t$, from the flexible price reference allocations.

3.2 Sticky Prices

It is convenient to work with a linearized version of the model hereafter, as computing analytical solutions to the non-linear problem with nominal rigidities becomes intractable. This approach is also followed by much of the analytical literature on optimal monetary policy, for eg. Farhi and Werning (2012). I characterize the constrained efficient allocation away from a symmetric deterministic steady state with zero inflation. This is done by taking a linear approximation of the constraints and a quadratic approximation of the welfare loss function. The optimal targeting rule that results is internally consistent and a locally linear approximation of the non-linear optimal policy. These perturbation techniques are described further in Benigno and Woodford (2012).

3.2.1 Constraints

The constraints in the Central Bank optimization problem are the first-order linear approximations, in log-deviation or "gap" terms, of the equilibrium conditions defined in section 2.6. A linearized variable, $f\left(s^t\right)$, is related to its true non-linear value and non-linear steady state, $F\left(s^t\right)$ and F, approximately as $f\left(s^t\right) \approx \frac{F\left(s^t\right)-F}{F}$.[6] A variable in log-deviation terms, $\tilde{f}\left(s^t\right)$ is defined as the sticky price linearized variable, $f\left(s^t\right)$, in deviation from the efficient linearized variable, $f\left(s^t\right)^e$, so that $\tilde{f}\left(s^t\right) = f\left(s^t\right) - f\left(s^t\right)^e$. Note that each linearized variable is measured in deviations from its *efficient* version, as the first-best case is the relevant welfare benchmark. Henceforth, I refer to $\tilde{f}\left(s^t\right)$ as an "efficient gap". I also use $\tilde{f}\left(s^t\right) \equiv \tilde{f}_t$, and the expectations operator, $E_t\left\{.\right\}$, to save on notation.

It is possible to summarize the constraints in demand (dynamic IS Equation) and supply blocks (New Keynesian Phillips Curve, or NKPC). Notably, in the presence of financially-excluded agents, it is still possible to derive a condition that resembles the canonical IS Equation for the small open economy. This is done by substituting the resource constraint, (28), into the aggregate Euler Equation. The latter combines the consumption of financially-excluded agents, (2), and the intertemporal optimality condition of financially-included agents, (7), so thus summarizes the joint consumption evolution of the heterogeneous households in the model

$$\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\tilde{y}_t - \alpha\tilde{x}_t = \frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}E_t\tilde{y}_{t+1} - \alpha E_t\tilde{x}_{t+1} - \left(i_t - E_t\pi_{t+1}\right)$$

The Phillips Curve is derived by combining the evolution of domestic inflation resulting from firm optimization, $\pi_{Ht} = \beta\pi_{Ht+1} + \xi\tilde{mc}_t$, where $\xi = \frac{(1-\beta\theta)(1-\theta)}{\theta}$, and $\tilde{mc}_t = \frac{\phi}{1-\lambda}\tilde{y}_t + \tilde{x}_t$, which is a version of the marginal cost condition derived by replacing the resource constraint, (28), risk-sharing condition (9), financially-excluded consumption, (2), and the optimal labor supply conditions, (3) and (6), into real marginal costs given by $\tilde{mc}_t = \tilde{\omega}_t + \alpha\tilde{x}_t$.[7] In the NKPC, I also make use of the relation between the terms of trade and output, $\tilde{x}_t = \Upsilon\tilde{y}_t$, which is derived by combining the resource constraint with risk-sharing. Proposition 3 formalizes the constraints in the optimal monetary policy problem.

[6] At the symmetric and efficient steady state with zero inflation, shocks $\{A, V, C^*, i^*\}$ are normalized to 1. Quantities and relative prices are endogenous. As financial assets and profits are zero in the steady state, the budget constraints of both households coincide. Then due to the same functional form for preferences, $C = \hat{C} = \check{C}$ and $N = \hat{N} = \check{N}$. It is possible to derive that $X = 1$ by combining the resource constraint, $Y = CX^{1-\alpha}$, risk-sharing condition, $C = X^{1-\alpha}$, production function, $Y = N$, marginal cost condition, $1 = wX^{\alpha}$, and labor supply condition, $w = CN^{\phi}$. The remaining variables are directly backed out.

[7] Real marginal costs, in terms of output, in an open economy differ from a closed economy due to the existence of a wedge between output and consumption, and thus domestic prices and the CPI. So, $mc_t = \omega_t - \hat{p}_{H,t} - a_t \rightarrow mc_t = \omega_t + \alpha x_t - a_t$ using the relation between relative domestic prices and the terms of trade, $p_{H,t} = -\alpha x_t$. And in gap terms: $\tilde{mc}_t = \tilde{\omega}_t + \alpha\tilde{x}_t$.

Proposition 3. (Canonical Equilibrium) *For a particular specification of monetary policy, the equilibrium with inflexible prices is*

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \frac{1}{\Upsilon}\left(i_t - E_t\pi_{Ht+1} - r_t^e\right) \tag{35}$$

$$r_t^e = -\Upsilon\left(1-\rho_a\right)a_t \tag{36}$$

$$\pi_{H,t} = \beta\pi_{H,t+1} + \frac{\xi}{\Lambda}\tilde{y}_t + v_t \tag{37}$$

where $\Upsilon = \frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}$, $\xi = \frac{(1-\theta)(1-\beta\theta)}{\theta}$, $\Lambda = \frac{1-\lambda}{1-\lambda-\lambda\phi(1-\alpha)+\phi}$, v_t *is an exogenous cost-push shock in the NKPC, and* r_t^e *is the equilibrium efficient rate of interest.*

These constraints nest Gali and Monacelli (2005) when $\lambda = 0$. Note, however, that α enters the canonical equilibrium when $\lambda > 0$. In the presence of financial exclusion, openness, α (or conversely home bias, $1-\alpha$), plays an explicit role in linking inflation and output. As financially-excluded agents cannot smooth consumption, they are adversely affected by the increased consequences of exchange rate volatility as the economy opens up.

Notably, Divine Coincidence (Blanchard and Galí, 2007) is present, implying that the central planner can simultaneously stabilize domestic inflation and the *efficient* output gap. In the absence of cost-push shocks, setting $\pi_{H,t} = 0$ closes the *natural* output gap, $y_t - y_t^n$, when $\tilde{f}_t = f_t - f_t^n$ in Proposition 3. But natural and efficient output are scalar functions of each other, so that strict domestic inflation targeting (DIT), $\pi_{H,t} = 0$, is able to achieve the first-best allocation of $\pi_{H,t} = y_t - y_t^e = 0$ in the absence of cost-push shocks. This has implications for the optimal monetary policy response to productivity shocks, as discussed further in Section 4.2.

Equilibrium Determinacy The equilibrium is determinate for all degrees of financial exclusion, λ, regardless of the values of the other parameters, as long as the optimal targeting rule is implemented, which corroborates the analogous closed economy optimal policy result in Bilbiie (2008). This point will be discussed further after giving the optimal targeting rule, (42), in Proposition 5. With standard active simple rules, the equilibrium is indeterminate for extreme values of λ, and determinacy requires that financial exclusion is below a certain threshold, λ^*, as in other DSGE models with hand-to-mouth agents, for instance Galí et al. (2007) and Bilbiie (2008). This threshold depends on other structural parameters, ie. $\lambda < \lambda^* = \frac{1}{1+\phi(1-\alpha)}$, as shown in the appendix. Indeed, it is possible to have λ^* as high as 0.8 with a standard range of calibrations for ϕ and α, so that the threshold value does not restrict my analysis or affect any of the results or conclusions. It is worth noting that the percentage of financially excluded agents, λ, in the vast majority of emerging market economies, is much less than 80%. Intuition for why the equilibrium is determinate when $\lambda < \lambda^*$ is as follows.

When $\lambda < \lambda^*$, the elasticity of output with respect to the nominal interest rate, $-\frac{1}{\Upsilon}$, from the IS Equation, (35), is negative so that ceteris paribus, contractionary monetary policy would lead to a real contraction as one would expect. Of course, this point ignores the simultaneous effect on output of movements in expected output, inflation, and the exchange rate, so that in general equilibrium, output and the nominal interest rate may may not end up being negatively correlated. However, when $\lambda \geq \lambda^*$, the interest elasticity of output is positive, ie. $-\frac{1}{Y} > 0$, a point that was originally discussed in Bilbiie (2008), and continues to hold in an open economy. Implied by this, when $\lambda \geq \lambda^*$, simple active rules lead to indeterminacy (discussed in the appendix). Although optimal monetary policy leads to equilibrium determinacy with all λ, I prefer to keep $\lambda < \lambda^*$ in this paper (so that λ is always less than around 80%), making it possible to compare optimal policy with active simple rules in a unique, rational expectations equilibrium.

3.2.2 Micro-founded Objective

The objective of optimal monetary policy is to maximize the expected utility of households. Motivated by this, Benigno and Woodford (2005) show that it is possible to derive a quadratic approximation to household welfare that takes the form of a discounted quadratic loss function with terms including those in inflation and the output gap, in the case where exogenous disturbances are sufficiently small in amplitude. The loss function is minimized to yield a local approximation to optimal policy. The advantage of using this method to "micro-found" the monetary policy objective is that is affords a precise characterization of which terms appear in the loss function, with relative weights that depend on the specific monetary transmission mechanism in the model considered.

In this framework, the Central Bank is required to maximize the expected utilities of *two* types of households. It is still possible, however, to derive a purely quadratic approximation of a weighted average of household utilities, by replacing for linear terms with second-order approximations of the structural equations. This means that I can cast the Central Bank's optimization problem in the familiar linear-quadratic form: here, the quadratic welfare loss function is minimized with respect to the linear aggregate demand and supply constraints. The solution to this problem is a flexible inflation targeting rule, (42), that maximizes a weighted sum of household utilities.

To derive the quadratic Central Bank loss function, I begin by taking a second-order approximation of the weighted sum of household utility functions: $U\left(s^t\right) = \lambda \check{U}\left(s^t\right) + (1-\lambda)\hat{U}\left(s^t\right)$. As an initial step, this yields some non-zero linear terms, where $t.i.p.$ stand for "terms independent of policy" (constants and functions of disturbances), and $o(3)$ contains terms of third-order and higher (to place a bound on the amplitude of the perturbations)

$$\mathscr{L}_t = -E_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{1}{2} \frac{\varepsilon}{\xi} \pi^2_{H,t+s} + \frac{U_N}{U_C} \frac{N}{C} \tilde{y}_{t+s} + \tilde{c}_{t+s} \right\} + t.i.p. + o(3)$$

To be able to correctly evaluate optimal policy upto second-order, the linear terms in $\tilde{y}_{t+s}$ and $\tilde{c}_{t+s}$ need to be eliminated. I follow the method of Benigno and Woodford (2005) to eliminate $\tilde{y}_{t+s}$ and $\tilde{c}_{t+s}$ through the analytical approach of replacing for these terms through second-order approximations of the following non-linear equilibrium conditions: the consumption of financially-excluded agents, (2), labor supply by both agents, (3) and (6), risk-sharing, (9), evolution of price dispersion, (23), the aggregate "production function", (24), equations from the recursive representation of the NKPC, (25), (26), and (27), the resource constraint, (28), and finally aggregate labor and consumption, (29) and (30). The resulting loss function is an expression with *purely quadratic* terms, described in Proposition 4.

Proposition 4. (Micro-founded Quadratic Loss Function) *Central Bank preferences in a small open economy with financially-excluded agents, with $\tilde{x}_t = x_t - x_t^e$ and $\tilde{y}_t = y_t - y_t^e$, are represented as*

$$\mathscr{L}_t = -\frac{1}{2} E_t \sum_{s=0}^{\infty} \beta^s \left\{ \Psi_\pi \pi_{H,t+s}^2 + \Psi_y \tilde{y}_{t+s}^2 + \Psi_s \tilde{x}_{t+s}^2 \right\} \tag{38}$$

with weights on domestic inflation, output gap, and terms of trade gap written as

$$\begin{aligned}
\Psi_\pi &= \frac{\varepsilon_P}{\xi} \left(\Xi \Lambda (1+\nu) - (1 + (1-\alpha)\Phi)\, \nu\lambda \right) - \frac{\varepsilon_P}{\xi} \frac{U_N}{U_C} \frac{N}{C} \\
\Psi_y &= \lambda(1-\alpha)^2 \nu^2 - (1-\alpha) - \lambda(1-\lambda)\nu^2 - \Xi\Lambda\lambda(1-\alpha)\nu^2 \\
&\quad + \Xi \left(\Lambda^{-1} + \Lambda \right) - \frac{1+\phi}{1-\lambda} \frac{U_N}{U_C} \frac{N}{C} \\
\Psi_x &= (1-\alpha)(1-2\alpha) + 2(1-\alpha)\omega\Upsilon - \Xi\Lambda(1-2\alpha) - 2\Xi\Lambda\omega\Upsilon
\end{aligned}$$

and composite parameters written as

$$\begin{aligned}
\Upsilon &= \frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda} \\
\Lambda &= \frac{1-\lambda}{1-\lambda-\lambda\phi(1-\alpha)+\phi} \\
\omega &= \frac{\lambda\phi(1-\alpha)^2 + \alpha(1-\lambda)}{1-\lambda} \\
\nu &= \frac{\phi}{1-\lambda} \\
\Xi &= (1-\alpha)\Upsilon + \lambda\nu + \frac{U_N}{U_C} \frac{N}{C} \\
\Phi &= \Xi\Lambda - (1-\alpha)
\end{aligned}$$

where the efficient variables x_t^e and y_t^e are functions of structural parameters and shocks, as in Proposition 1.

The loss function in an open economy model with financial exclusion features a quadratic term in the terms of trade beyond what is usually found in models where all households are financially-included. When $\lambda = 0$, the criterion collapses to that in Gali and Monacelli (2005) where $\tilde{x}_t^2 = \tilde{y}_t^2$: $\Psi_\pi|_{\lambda=0} = \Psi_\pi^{GM} = \frac{\varepsilon_P}{\xi} \frac{U_N}{U_C} \frac{N}{C}$ and $\Psi_y|_{\lambda=0} + \Psi_x|_{\lambda=0} = \Psi_y^{GM} = (1+\phi) \frac{U_N}{U_C} \frac{N}{C}$. I give the loss function below with $\lambda = 0$, ie. the nested Gali and Monacelli (2005) case, as this serves as a useful point of comparison for some of the results. (39) requires that $\frac{U_N}{U_C} \frac{N}{C} = -(1-\alpha)$, ie. the steady state is rendered efficient.

$$\mathscr{L}_t|_{\lambda=0} = \mathscr{L}_t^{GM} = -\frac{1-\alpha}{2} E_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{\varepsilon}{\xi} \pi_{H,t+s}^2 + (1+\phi) \tilde{y}_{t+s}^2 \right\} \tag{39}$$

The key driver for the differences in Central Bank preferences in the presence of financial exclusion is the asymmetry in the ability of domestic households to share risk. Recall that financially-included households can pool risk with foreign agents, (9), by receiving (or making) appropriate international insurance transfers, (32). This smooths their consumption upon fluctuations in their income. In sharp contrast, financially-excluded agents must fully consume their income, (2). This implies that, unlike the $\lambda = 0$ case, (39), that the stabilization of the terms of trade gap, $\tilde{x}_t^2$, is required in (38) to minimize welfare loss.

Result 1. *Terms of trade fluctuations have first-order distortionary effects in the presence of financial exclusion.*

In an economy where all agents can smooth consumption, changes to purchasing power from exchange rate fluctuations are offset through the proceeds from state-contingent claims. For instance, import price volatility, caused by exchange rate movements, does not affect financially-included agents by much as they adjust their financial asset holdings in a manner that consumption is smoothed. However, changes to the purchasing power of financially-excluded agents when import prices fluctuate cause an equivalent rise in consumption volatility, and hence greater aggregate welfare loss. This gives the Central Bank an incentive to stabilize the terms of trade.

Policy Trade-offs Result 1 is one dimension of the loss-minimizing objective, but what do Central Bank preferences look like overall? It is useful to provide more intuition on the micro-founded objective, (38), as this drives most of the results in the optimal policy analysis. However, the weights in (38) are fairly complicated functions of structural parameters, so that their implications are not transparent. This can be addressed by re-writing the loss function in terms of only domestic inflation, $\pi_{H,t}$, and the output gap, $\tilde{y}_t$, as (40), by using the proportionality between the terms of trade gap and the output gap, $\tilde{x}_t = \Upsilon \tilde{y}_t$. The optimal policy trade-offs that arise are then captured by the relative weight, (41).[8]

$$\mathscr{L}_t = -\frac{1}{2} E_t \sum_{s=0}^{\infty} \beta^s \left\{ \pi_{H,t+s}^2 + \Phi^{\lambda} \tilde{y}_{t+s}^2 \right\} \tag{40}$$

where the relative weight on $\tilde{y}_t$, Φ^{λ}, is given by

$$\Phi^{\lambda} = \frac{\Xi \left(\Lambda^{-1} + \Lambda \right) - (1-2\alpha)\, \Phi \Upsilon^2 - \left((1-\lambda) + (1-\alpha)\Phi \right) \lambda \nu^2 - 2\Phi \omega \Upsilon - (1-\alpha) - \frac{1+\phi}{1-\lambda} \frac{U_N}{U_C} \frac{N}{C}}{\frac{\varepsilon}{\xi} \left(\Xi \Lambda (1+\nu) - (1 + (1-\alpha)\Phi)\, \nu \lambda \right) - \frac{\varepsilon_P}{\xi} \frac{U_N}{U_C} \frac{N}{C}} \tag{41}$$

[8] A numerical check yields that the relative weight, Φ^{λ}, is strictly positive when the structural parameters in the loss function, $\alpha, \lambda, \phi, \varepsilon, \beta, \theta$, jointly satisfy the following system of inequalities: $0 < \alpha < 0.95$, $0 < \lambda < 0.9$, $0 < \theta < 1$, $0 < \beta < 1$, $\varepsilon_P > 0$, $\phi > 0$. The positive weight implies that the quadratic loss function is convex so that the second-order sufficient conditions are satisfied. Note that convexity ensues for a wide range of parameter values in this model due to the logarithm and isoelastic functional forms in utility, as also in Benigno and Woodford (2005).

Result 2. *The relative weight on output gap stabilization, Φ^{λ}, is increasing in α and λ.*

It is instructive to compare Φ^{λ} with the Gali and Monacelli (2005) model, where the relative weight, $\Phi^{GM} = \frac{1+\phi}{\frac{\varepsilon}{\xi}}$, is invariant with respect to openness, α. In contrast, Φ^{λ} in this model increases with α. To see this, note that as the economy becomes more open, domestic agents consume more imports relative to home goods. This increases the consequences of exchange rate volatility, which in an economy with financial exclusion, results in an equivalent rise in hand-to-mouth consumption volatility. To mitigate the ensuing macroeconomic volatility and hence greater loss, it is optimal to stabilize the output gap by more as α increases. Doing so stabilizes labor demand and hence real wages, which in turn stabilizes the consumption of financially-excluded agents. $\frac{\partial \Phi^{\lambda}}{\partial \alpha} \neq 0$ is thus completely dependent on $\lambda > 0$, ie. a fraction of agents not being able to insure against uncertainty.

Similarly, Φ^{λ} increases with λ to minimize the greater loss as progressively more agents cannot smooth consumption. This analysis is depicted in Figure 2 which plots Φ^{λ} as a function of λ and α for $\lambda, \alpha \in [0, 0.6]$ (Φ^{λ} increases disproportionately for higher calibrations). The steady state is rendered efficient ie. $\frac{U_N}{U_C}\frac{N}{C} = -N^{1+\phi} = -(1-\alpha)$, to better permit comparison with the baseline case of Gali and Monacelli (2005) (the analogous relative weight there does not change with either α or λ).

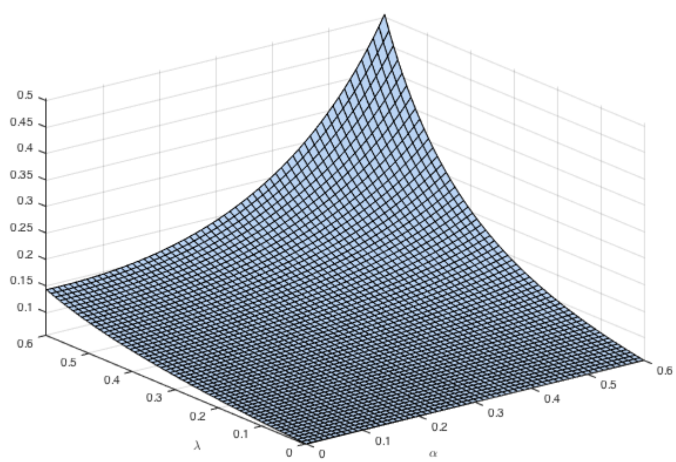

Figure 2: Optimal Relative Weight on Output Gap

Result 3. *The closed economy case is not isomorphic for $\lambda > 0$.*

This point is of independent interest to the question at hand, but deserves a mention. Optimal policy in the open economy model with financially-excluded agents does not necessarily converge to the closed economy case when $\alpha \to 0$. Usually considered desirable from a modeling perspective, other instances can be found in Monacelli (2005) and Farhi and Werning (2012). There is an irreversible open economy asymmetry in this model compared to the closed economy version of Bilbiie (2008), as only a fraction of domestic consumers share risk with foreign agents. Exchange rate movements affect the heterogeneous agents in completely different ways, implying that consumption patterns fundamentally diverge over the business cycle, and overriding the type of closed economy isomorphism that characterizes Gali and Monacelli (2005).

This can be seen as follows. In the closed economy limit ie. as $\alpha \to 0$, a fixed nominal exchange rate, $e_t - e_{t-1} = 0$ amounts to a fixed nominal interest rate (equal to the exogenous foreign interest rate, i_t^*) through the UIP condition, $i_t = i_t^* + E_t e_{t+1} - e_t$. However, it is known that a fixed nominal interest rate implies equilibrium indeterminacy in a closed economy (Galí, 2009). I correspondingly find that a fixed nominal interest rate, $i_t = 0$, results in indeterminacy in my model when $\alpha = 0$, and indeed for all α. When $\alpha > 0$, however, an exchange rate peg, $e_t - e_{t-1} = 0$, ensures a unique equilibrium and moreover approximates the optimal policy, as will be discussed in Section 4. Technically, the equilibrium correspondence with a fixed exchange rate is not lower hemi-continuous in the parameter, α. This argument is similar to that in Farhi and Werning (2012), and corroborates their conclusion on being cautious in using closed economy models to approximate the open.

Targeting Rule I now derive the flexible inflation targeting rule for the model under dynamically consistent timeless commitment, where the optimal plan set in later periods is the same as the one that would have been set initially (Woodford, 2011). This requires minimizing the quadratic loss function (38) with respect to the linear aggregate supply relation, (37). The IS Equation does not bind since the nominal interest rate is unconstrained. I attach the Lagrange multiplier, $\chi_{\pi,t}$, on the NKPC and take first-order conditions with respect to domestic inflation, $\pi_{H,t}$, and the output gap, $\tilde{y}_t$

$$\begin{aligned} 2\Psi_\pi \pi_{H,t} + \chi_{\pi,t} - \chi_{\pi,t-1} &= 0 \\ 2\Psi_y \tilde{y}_t - \xi \Lambda^{-1} \chi_{\pi,t} &= 0 \end{aligned}$$

These can be combined to yield the optimal plan.

Proposition 5. (Optimal Policy) *The optimal plan under timeless commitment is implemented as follows*

$$\pi_{H,t} = -\frac{\Lambda}{\xi} \Phi^\lambda \left(\tilde{y}_t - \tilde{y}_{t-1}\right) \tag{42}$$

where y_t^e is efficient output as defined in Proposition 1.

The optimal plan, (42), implies the classic "leaning against the wind" analogy of contracting the output gap to bring down domestic inflation, whenever the latter is inefficiently high. The targeting rule holds only if prices are slow to adjust, $\xi \neq \infty$, as it would be redundant in a flexible price environment where inflation creates no real distortions.

Determinacy It is useful to note that determinacy with the optimal plan, (42), ensues for all $\lambda \in [0, 1)$, which corroborates the closed economy analog in Bilbiie (2008). This can be seen by combining (42) with the NKPC (37) to get the following second-order difference equation

$$E_t \tilde{y_{t+1}} = \left[1 + \frac{1}{\beta}\left(1 + \frac{\xi^2}{\Lambda^2}\frac{1}{\Phi^\lambda}\right)\right]\tilde{y}_t - \frac{1}{\beta}\tilde{y}_{t-1} + \frac{1}{\beta}\frac{\xi}{\Lambda}\frac{1}{\Phi^\lambda}v_t$$

whose roots are on the opposite sides of the unit circle, for all $\lambda \in [0,1)$, as required for a unique, rational expectations equilibrium. Thus, following through with the optimal plan prevents the indeterminacy issues associated with very high λ for standard, active simple rules.

Closed-form solutions for relative domestic prices, $\hat{p}_{H,t}$, and the output gap, $\tilde{y}_t$, are derived by combining the targeting rule (42) and the Phillips Curve (37). The corresponding choice of nominal interest rate, i_t, is derived by replacing (43) and (44) in the IS Equation (35). Defining $\delta = \frac{1-\sqrt{1-4\alpha^2\beta}}{2\alpha\beta} \in (0,1)$, $\gamma_1 = \left(\frac{\Upsilon\xi}{\Lambda\Phi^\lambda} - 1\right)(1-\delta)$, $\gamma_2 = \left(\frac{\Upsilon\xi}{\Lambda\Phi^\lambda} - 1\right)\delta$, and $\alpha = \left(1 + \beta + \frac{\xi^2}{\Lambda^2\Phi^\lambda}\right)^{-1}$, it follows that

$$\hat{p}_{H,t} = \delta\hat{p}_{H,t-1} + \delta E_t \sum_{k=0}^{\infty}(\beta\delta)^k v_{t+k} \tag{43}$$

$$\tilde{y}_t = -\frac{\xi}{\Lambda\Phi^\lambda}\hat{p}_{H,t} \tag{44}$$

$$i_t = r_t^e + \gamma_1\hat{p}_{H,t} - \gamma_2 E_t \sum_{k=0}^{\infty}(\beta\delta)^k v_{t+k} \tag{45}$$

Closed-form solutions for the remaining endogenous variables can be backed out as follows: the employment gap from the production function (46), the financially-included agent consumption and labor supply gaps from the risk-sharing condition (47) and intratemporal optimality condition (48), the financially-excluded agent consumption and labor supply gaps from the budget constraint (49) and intratemporal optimality condition (50), the real wage gap from the firm's marginal cost condition (51), the terms of trade gap from the resource constraint (52), and finally CPI inflation from its linkage with domestic inflation and the terms of trade (53)

$$\tilde{y}_t = \tilde{n}_t \tag{46}$$

$$\tilde{\tilde{c}}_t = (1-\alpha)\tilde{x}_t \tag{47}$$

$$\tilde{\omega}_t = \tilde{\tilde{c}}_t + \phi\tilde{\tilde{n}}_t \tag{48}$$

$$\tilde{\tilde{\tilde{c}}}_t = \tilde{\omega}_t \tag{49}$$

$$\tilde{\omega}_t = \tilde{\tilde{\tilde{c}}}_t + \phi\tilde{\tilde{\tilde{n}}}_t \tag{50}$$

$$0 = \tilde{\omega}_t + \alpha\tilde{x}_t \tag{51}$$

$$\tilde{y}_t = \alpha\tilde{x}_t + (1-\alpha)\left(\lambda\tilde{\tilde{\tilde{c}}}_t + (1-\lambda)\tilde{\tilde{c}}_t\right) + \alpha\tilde{\tilde{c}}_t \tag{52}$$

$$\pi_t = \pi_{Ht} + \alpha\Delta\tilde{x}_t \tag{53}$$

3.2.3 Optimal Dynamics

I now seek to analyze how the optimal plan, (42), which governs the optimal evolution of the endogenous variables in equations (43) to (53), responds to unexpected disturbances. The focus is primarily on cost-push shocks as these have been predominant in emerging market economies, and arise in practice due to domestic food price volatility and fuel input price volatility (Frankel, 2010). I also analyze productivity shocks. Dynamics are analyzed based on a calibrated version of the framework. Calibration is a challenging task, as the required micro-level data is scarce for EMEs. I thus select parameters from the existing open economy literature, and pair this with extensive sensitivity analysis. The fraction of randomly chosen monopolistic producers that can reset prices, θ, is set at 0.75, which implies an average period of one year between price adjustments, as in Gali and Monacelli (2005).

The household discount factor β equals 0.99, which implies a steady state real interest rate of around four percent. The elasticity of substitution between differentiated monopolistic goods is set at $\varepsilon_P = 6$, which implies a steady state markup of around 20%. The degree of openness, α, is set at 0.5 and the inverse Frisch elasticity of labor supply, ϕ, at 1. These values implies a simple rule indeterminacy threshold, $\lambda^* = \frac{1}{1+\phi(1-\alpha)}$, of around 0.7, which my analysis does not go beyond. With $\alpha = 0.7$, as in the sensitivity analysis, λ^* is around 0.8. The persistence of shocks, ie. ρ_j in the stationary (linearized) autoregressive process $j_t = \rho_j j_{t-1} + s_{j,t}$, where $j_t = \{v_t, a_t, i_t^*\}_{t=0}^{\infty}$, is set at 0.9, consistent with the evidence for emerging market economies provided in Aguiar and Gopinath (2007). In most cases, optimal policy with complete financial market participation, $\lambda = 0$, is contrasted with $\lambda = 0.6$, which is around the EME financial exclusion average (World Bank, 2015).

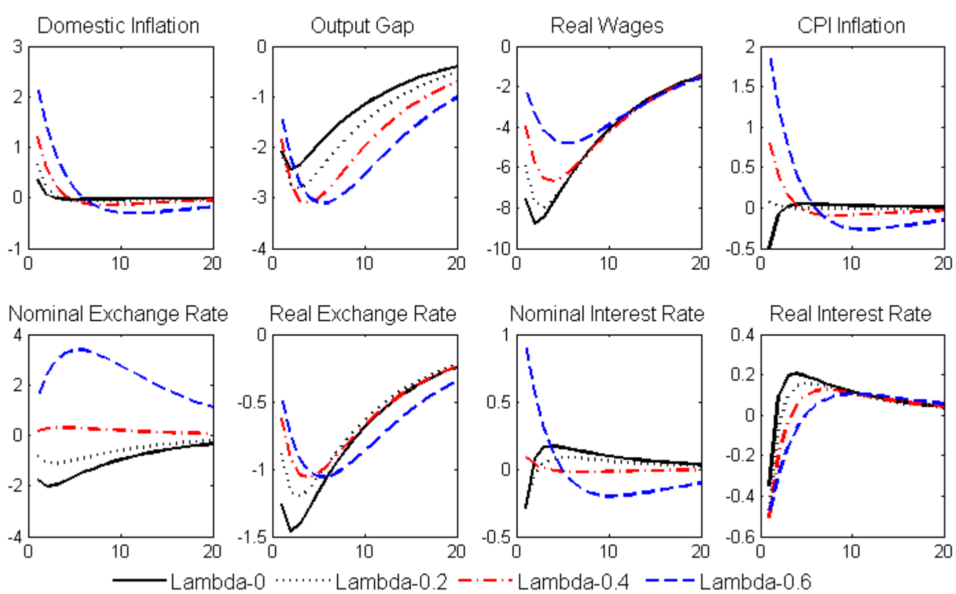

Figure 3: Optimal Monetary Policy and Financial Exclusion

Figure 3 provides dynamics for varying degrees of financial exclusion upon a unit positive cost-push shock. There is an immediate rise in domestic inflation that arises independently from variations in domestic demand. The corresponding jump up in domestic prices leads to a real exchange rate appreciation, and the Central Bank contracts output to control inflation. A recession ensues. Domestic inflation is allowed to rise by more as the fraction of financially-excluded agents increase, as per by the optimal trade-off, (41), embodied in the flexible inflation targeting rule, (42). This erodes the value of domestic currency, putting pressure on the nominal exchange rate to depreciate by more as λ increases. The nominal interest rate, unconstrained in this model, moves in a direction that supports the optimal trade-off. To mitigate the recession, i_t decreases with low λ. It increases, however, when λ is high as this allows the nominal exchange rate to be expected to depreciate by more through the UIP condition, which stabilizes the output gap as the expense of higher domestic inflation.

Note that, overall there is pressure on the nominal exchange rate to depreciate upon the cost-push shock (apart from very low λ where i_t falls, so that domestic asset returns decrease to the extent that, as per UIP, expected nominal appreciation is required to equalize asset returns internationally and hence prevent arbitrage). The higher domestic inflation and greater nominal depreciation as financial exclusion increases account for the lower *real* exchange rate appreciation. The lower real appreciation prevents the consumption of financially-included agents from falling as much, so that producers do not decrease their demand for working hours as much, implying that real wages remain more stable. The lower real appreciation also stabilizes the consumption of financially-excluded agents due to the relatively cheaper imports and relatively higher wages. The real interest rate, $r_t = i_t - E_t \pi_{H,t+1}$, supports these dynamics, and CPI inflation in the economy rises by more as λ increases, due to the higher domestic inflation and lower real exchange rate appreciation.

Result 4. *Monetary policy plays an additional insurance role in the presence of financial exclusion.*

For $\lambda > 0$, optimal monetary policy takes into account the inability to share risk and hence the more volatile consumption of financially-excluded agents. Implementation of the optimal plan, (42), results in disposable income (real wages) being stabilized by more as financial exclusion increases. This can be interpreted as provision of insurance by monetary policy to agents who cannot smooth consumption by themselves, and is a required transmission channel in the efficient plan. Thus, besides macroeconomic stabilization, monetary policy plays an additional insurance role when $\lambda > 0$. The amount of insurance varies depending on the degrees of openness and price stickiness, as shown next.

3.2.4 Sensitivity Analysis

I analyze the implications of the optimal targeting rule (42) for $\alpha = \{0.1, 0.4, 0.7\}$ and $\theta = \{0.7, 0.75, 0.8\}$. The limited ($\lambda = 0.6$) financial inclusion case with varied α is depicted below in Figure 4 (corresponding $\lambda = 0$ case is in Figure 6 in the appendix). The analysis suggests that the amount of insurance provided by monetary policy is increasing in openness and price flexibility.

Openness

When financial inclusion is complete, $\lambda = 0$, the trade-off is independent of openness as changes to real income from exchange rate fluctuations are offset through the proceeds from state-contingent claims. This independence is broken in the presence of financial exclusion, $\lambda > 0$. Here, exchange rate volatility causes equivalent volatility in the consumption of financially-excluded agents (and by a greater amount as $\alpha \to 1$ due to the increased consumption of imports), unlike that of financial asset holders. To mitigate the resulting greater macroeconomic volatility, optimal monetary policy is required to smooth the consumption of hand-to-mouth agents by more. It does so by contracting the output gap by less upon a cost-push shock as λ increases. The supporting nominal interest rate increases by more to induce the greater expected nominal depreciation required for a relative output expansion and higher domestic inflation as per the optimal trade-off, (41). This leads to more muted real appreciation, which stabilizes the consumption of both type of agents as imports are relatively cheaper, and that of financially-excluded agents in particular because real wages fall by less.

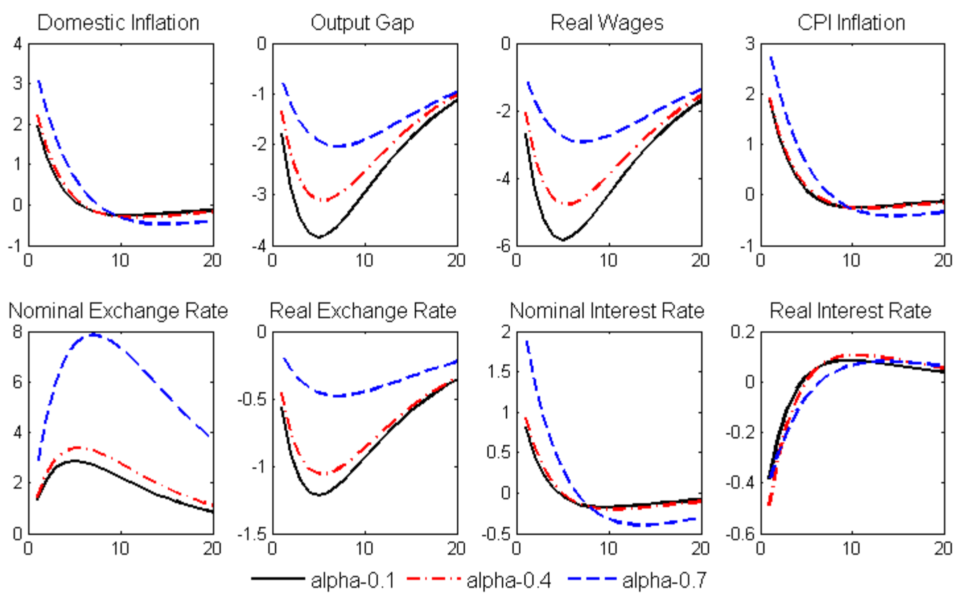

Figure 4: Optimal Monetary Policy with Financial Exclusion, Varying Openness

Nominal Rigidities

Stickier prices, $\theta \to 1$, imply that fewer randomly chosen price-setters re-optimize each period. Thus, upon a cost-push shock, the required downward re-optimization of prices (in anticipation of future high domestic inflation) is hindered. This leads to more domestic inflation volatility, requiring a grater contraction of the output gap as per the optimal plan. The resulting steeper recession dampens labor demand by more. Consequently, lower real wages clear the market, implying that hand-to-mouth consumption decreases by more. Nominal rigidities thus hamper the provision of insurance by optimal monetary policy. These dynamics are found in Figure 7 in the appendix.

4 Nominal Anchor: Peg or Float?

I proceed to analyze the appropriate choice of nominal anchor in an economy with financial exclusion. To do so, simple and implementable monetary rules are ranked in terms of lowest welfare losses away from the optimum. Simple rules as approximations of optimal monetary policy are in particular useful, since the optimum can sometimes be cast as a complicated and unintuitive function of model parameters. Optimal policy is thus often not as transparent and implementable in practice as a simple rule. I will show that targeting the exchange rate is an implementable way to internalize the insurance properties of the optimal plan characterized in the previous section.

Table 1 reports the welfare losses associated with standard active simple rules compared to the benchmark optimal policy, upon trade-off creating cost-push shocks. There are two columns: the first reports welfare losses in the case of complete financial inclusion, $\lambda = 0$, whereas the second does likewise with high financial exclusion, $\lambda = 0.6$. All entries in Table 1 are, as in Gali and Monacelli (2005), percentage units of steady-state consumption and in deviation from the first-best case of optimal monetary policy. It can be seen that while strict CPI Inflation Targeting, $\pi_t = 0$, best approximates the optimal policy when all agents can smooth consumption, a fixed exchange rate, $\Delta e_t = 0$, is least suboptimal when financial exclusion is high.

Simple Rule	Financial Inclusion	Financial Exclusion
Strict CPI IT (CIT)		
$\pi_t = 0$	**7.8**	9.9
Strict domestic IT (DIT)		
$\pi_{H,t} = 0$	12.6	57.1
Fixed exchange rate (PEG)		
$\Delta e_t = 0$	56.0	**0.2**
Flexible CPI IT		
$i = 1.5\pi_t$	21.7	18.2
$i = 1.5\pi_t + 0.5\tilde{y}_t$	1312.4	292.5
Flexible domestic IT		
$i = 1.5\pi_{H,t}$	83.5	45.0
$i = 1.5\pi_{H,t} + 0.5\tilde{y}_t$	1312.4	324.0

Table 1: Welfare Loss Relative to Optimal Policy

Result 5. *A nominal exchange rate peg is least suboptimal with high financial exclusion.*

Table 1 indicates that fixed exchange rates are preferred to flexible exchange rates (all the inflation targeting rules) in the presence of high financial exclusion. The intuition for this result stems from first recalling from section 3.2.3 that with high λ, welfare losses are minimized when optimal policy prevents high real appreciation, which smooths the consumption of financially-excluded agents due to greater real wage and imported good price stabilization. Now, note that a nominal exchange rate peg is the most effective out of all the monetary policy rules in containing real appreciation in the presence of nominal rigidities. Indeed, dynamics under a fixed exchange rate almost completely mirror those under optimal policy, as shown in Figure 5 below, which compares PEG with the optimal targeting rule, (42), CIT (least suboptimal policy when $\lambda = 0$ upon cost-push shocks), and DIT (optimal policy in this model upon productivity shocks). Corresponding volatility numbers are found in Table 2.

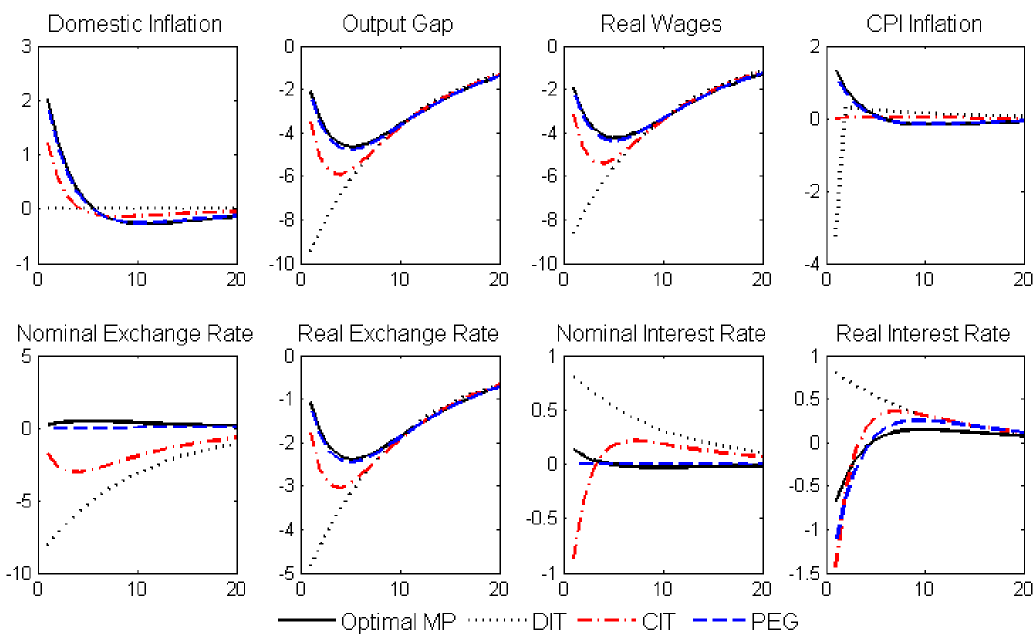

Figure 5: Optimal Policy Versus Simple Rules, Financial Exclusion

	Optimal	DIT	CIT	PEG
Output	227.0	466.3	306.5	236.1
Domestic inflation	7.1	0	2.0	6.2
CPI inflation	2.9	10.9	0	2.2
Real wages	189.5	389.2	255.9	197.0
Real exchange rate	59.9	123.0	80.8	62.3
Nominal depr. rate	0.1	68.3	4.5	0

Table 2: Cyclical Properties of Alternative Policy Regimes, Financial Exclusion

Result 6. *CPI Inflation Targeting is appropriate when all agents are insured through asset markets.*

When financial inclusion is complete, compared to the financial exclusion case, domestic inflation volatility gives rise to greater welfare loss. Thus, optimal policy places greater weight on stabilizing domestic inflation. Now, under a peg, the muted real appreciation implies that marginal costs, and hence domestic inflation, increase by too much. Thus, PEG is highly suboptimal. DIT, on the other hand, leads to extreme output gap volatility (which is still penalized, albeit by less) in fully stabilizing domestic inflation. CIT strikes the appropriate in-between as it results in less output gap volatility than DIT (it stabilizes *both* domestic and imported good inflation, and the output gap responds to *domestic* inflation only as per the targeting rule, (42)), and less domestic inflation volatility than PEG due to greater real appreciation. These dynamics are found in Figure 8 in the appendix, with corresponding volatility numbers in Table 3 below.

	Optimal	DIT	CIT	PEG
Output	149.8	183.9	129.4	102.8
Domestic inflation	0.4	0	1.3	4.3
CPI inflation	0.8	5.9	0	1.5
Real wages	383.4	470.8	331.2	263.3
Real exchange rate	53.9	66.2	46.6	37.0
Nominal depr. rate	9.6	36.8	3.0	0

Table 3: Cyclical Properties of Alternative Policy Regimes, Complete Financial Inclusion

4.1 Role of Openness

I vary the degrees of openness and nominal rigidities, to analyze whether CIT and PEG remain robust. I find that they do. Price stickiness matters to the extent that the simple rules approximate optimal policy better as nominal rigidities lessen ie. $\theta \to 0$ (because the role of optimal policy diminishes with increased price flexibility); however, varying α yields an interesting result.

Result 7. *When $\alpha > 0$, CIT and PEG work better with low degrees of openness. When $\alpha = 0$, this does not hold due to an open economy discontinuity.*

CIT, $\pi_t = 0$, approximates the $\lambda = 0$ optimal monetary policy better, and PEG, $\Delta e_t = 0$, does likewise for the $\lambda = 0.6$ optimal policy, as $\alpha \to 0$. Note, however, that this does *not* imply that $\pi_{H,t} = 0$ and $i_t = 0$ (the closed economy analogs of CIT and PEG) perform well when $\alpha = 0$. In the former case, $\pi_{H,t} = 0$ is achieved at the expense of high output gap volatility, which is suboptimal - CIT does not require this. Furthermore, $i_t = 0$ results in an indeterminate equilibrium for $\alpha = 0$ - there is an open economy discontinuity in this model as was discussed in Result 3 in Section 3.2.2. Intuition for the better CIT and PEG approximations with low $\alpha > 0$ is as follows.

Consider first the $\lambda = 0$ case. It is useful to note first the following facts: (i) lower α implies that more domestic goods are consumed relative to imported goods, (ii) CIT targets both domestic and import price inflation, and (iii) upon a cost-push shock, while relative domestic prices increase, relative import prices *decrease* due to an expenditure-switching effect. Now, as $\alpha \to 0$, CIT implies the stabilization of progressively more *domestic* inflation (which has increased), which requires higher real appreciation (preventing import deflation would, in contrast, require real *depreciation*). This higher real appreciation as α decreases matches that under optimal policy for the complete financial inclusion case. PEG does not provide the required high real appreciation for $\lambda = 0$.

Now, consider the $\lambda = 0.6$ case, where the weight on output gap stabilization increases with α to smooth the consumption of financially-excluded agents. This requires progressively less real appreciation, as discussed in section 3.2.4. With very high α, the optimal real appreciation is required to be so low that although a nominal peg comes closest to engendering this, even it cannot completely mirror the optimal policy (a different simple rule, perhaps targeting the *real* exchange rate itself, might be appropriate here). However, for low ($\alpha = 0.1$) and moderate ($\alpha = 0.4$) degrees of openness, the real appreciation under the peg is appropriate. In contrast, CIT is inefficient for all α with financial exclusion, since it requires suboptimally high real appreciation. The corresponding dynamics for $\lambda = 0$ and $\lambda = 0.6$ are found in Figures 9 and 10 in the appendix.

4.2 Productivity Shocks

Result 8. *A fixed nominal exchange rate in a high financial exclusion economy is appropriate only if cost-push shocks predominate.*

It is useful to understand the circumstances under which a fixed exchange rate provides the most efficient stabilization. As a robustness check, I analyze optimal monetary policy with domestic productivity shocks. I discuss the case with $\lambda = 0.6$, with the analogous $\lambda = 0$ exercise in Gali and Monacelli (2005). In an economy with financial exclusion, I find that the type of shock matters for choice of domestic versus external nominal anchor. As shown in Proposition 3, optimal policy with productivity shocks leads to perfect stabilization and is strict domestic inflation targeting (DIT). This can be seen by setting $\pi_{H,t} = 0$ in the system of equations, (35), (36), and (37), in the absence of cost-push shocks, v_t, so that the output gap is also perfectly stabilized, $\tilde{y}_t = 0$. This policy requires considerable nominal exchange rate volatility, implying that a peg is no longer appropriate.[9] In the presence of financial exclusion, exchange rate stability is appropriate only if cost-push shocks predominate.

5 Conclusion

This study analyzed optimal monetary policy and the choice of nominal anchor in a small emerging market economy with high financial exclusion. The main findings were that, upon cost-push shocks, optimal policy smooths the consumption of financially-excluded agents who cannot do so themselves, and that a fixed nominal exchange rate internalizes the insurance provision properties of the optimal plan. The desirability of exchange rate stability provides a counterpoint to Milton Friedman's longstanding argument for a float. This paper sought to establish benchmark analytical results, and lends itself to some extensions. It would be useful to incorporate another form of nominal rigidity, sticky wages, as well as incomplete financial markets. The latter would imply that international risk-sharing is imperfect, as relevant in practice, and which would be interesting to interact with the form of financial market incompleteness considered in this paper. I leave these extensions to future research.

[9] A productivity shock puts downward pressure on prices and upward pressure on output (but in the absence of optimal policy, sticky price output, y_t, does not increase by as much as efficient output, y_t^e, so that there is downward pressure on the output gap, $\tilde{y}_t = y_t - y_t^e$, too). Unrestricted depreciation of the nominal exchange rate is required to support the transitory increase in domestic inflation and expansion in output required to set $\pi_{H,t} = 0$ and close the output gap.

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Appendix

Equilibrium Determinacy From the IS Equation, $\tilde{y}_t = E_t\tilde{y}_{t+1} - \frac{1}{\Upsilon}\left(i_t - E_t\pi_{Ht+1} - r_t^e\right)$, note that the interest elasticity of output is $-\frac{1}{\Upsilon}$, where $\Upsilon = \frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}$. We have that $-\frac{1}{\Upsilon} < 0$ if $\lambda < \frac{1}{1+\phi(1-\alpha)}$, but $-\frac{1}{\Upsilon} > 0$ if $\lambda \geq \frac{1}{1+\phi(1-\alpha)}$. In the region implied by the latter inequality, active rules give rise to indeterminacy, a phenomenon called the inverted aggregate demand logic (IADL) by Bilbiie (2008). What is the IADL? It is useful to note first that dynamics become more volatile as λ increases, since hand-to-mouth agents have a marginal propensity to consume of 1. These agents spend on output, which thus also becomes more volatile as λ increases, which in turn creates volatility in labor demand, and real wages. Upon a contractionary interest rate surprise that decreases aggregate demand, wages thus fall more as λ increases. This dampens hand-to-mouth consumption, which further dampens real wages and output, and hence asset holder consumption. This effect might initially seem monotonic, so that a monetary contraction would actually have a greater impact as λ increases.

However, due to the specific interaction, with very high λ, of sticky prices with countercyclical profits in the New Keynesian model, there is a critical high λ^* where wages fall by so much (output does not fall as much as wages due to sticky prices), that the corresponding big rise in profits implies a big wealth effect for financially-included agents. Then, despite their initial fall in demand, financially-included agents now demand more output, which leads to an increase in wages and financially-excluded agent demand as well. Thus, an initial monetary contraction leads to an ultimate real expansion with $\lambda \geq \lambda^*$ due to sticky prices and countercyclical profits. The empirical relevancy of the IADL channel and associated implications for equilibrium determinacy with passive rules have been subject to critique (see for eg. Ascari et al. (2011)), and I avoid the IADL region by always setting $\lambda < \lambda^*$ in this paper. Indeed, as λ^* goes as high as 0.8 with a standard range of calibrations, and as the vast majority of EMEs have less than 80% of households excluded from the financial markets, the threshold value for indeterminacy does not impact any of my results or conclusions.

I now discuss how the IADL region implies equilibrium indeterminacy with standard active Taylor Rules. As in the literature, "active" refers specifically to the Taylor Principle being satisfied ie. $\chi > 1$ in any of the following rules: $i_t = \chi\pi_{H,t}$, $i_t = \chi\pi_t$, $i_t = \chi\Delta e_t$. Note that the strict targeting rules, $\pi_{H,t} = 0$, $\pi_t = 0$, and $\Delta e_t = 0$, are special cases of the above with a high enough value for χ (and certainly the Taylor Principle being satisfied). I provide an analytical proof for the first case, $i_t = \chi\pi_{H,t}$, that being in the IADL region, ie. $\lambda \geq \frac{1}{1+\phi(1-\alpha)}$, implies that $\chi > 1$ gives rise to indeterminacy. A numerical check reveals that this holds true for the other similar rules considered. The method of proof is similar to that provided in Bilbiie (2008). I substitute the rule, $i_t = \chi\pi_{H,t}$, in the IS Equation, (35) so that the equilibrium in Proposition 3, (35), (36), and (37), is now a dynamic system in two jump variables, $\pi_{H,t}$ and $\tilde{y}_t$, which can be written, for the $z_t \equiv \left(\tilde{y}_t, \pi_{H,t}\right)'$ vector of endogenous variables and the $u_t \equiv \left(r_t^e, v_t\right)'$ vector of disturbances, as

$$E_t z_{t+1} = Az_t + Bu_t$$

$$\textit{where} \quad A = \begin{bmatrix} 1 + \frac{\xi}{\beta\Upsilon\Lambda} & \frac{1}{\Upsilon}\left(\chi - \frac{1}{\beta}\right) \\ -\frac{1}{\beta}\frac{\xi}{\Lambda} & \frac{1}{\beta} \end{bmatrix} \qquad B = \begin{bmatrix} -\frac{1}{\Upsilon} & \frac{1}{\beta\Upsilon} \\ 0 & -\frac{1}{\beta} \end{bmatrix}$$

As per the Blanchard-Kahn conditions (Blanchard and Kahn, 1980), determinacy requires that both eigenvalues of A are greater than 1, as $\pi_{H,t}$ and $\tilde{y}_t$ are jump variables. This in turn requires that (i) $det\,(A) > 1$, (ii) $det\,(A) - tr\,(A) > -1$, and (iii) $det\,(A) + tr\,(A) < -1$ (Woodford, 2011), where $det\,(A) = \frac{1}{\beta}\left(1 + \frac{\xi}{\Upsilon\chi\Lambda}\right)$ and $tr\,(A) =$

$1 + \frac{\xi}{\beta Y \Lambda} + \frac{1}{\beta}$. The following conditions satisfy these restrictions for a unique, rational expectations equilibrium.

$$\lambda < \lambda^* : \quad \chi \in (1, \infty) \text{ } ie. \text{ } the \text{ } Taylor \text{ } Principle$$
$$\lambda \geq \lambda^* : \quad \chi \in \left[0, min\left\{1, \frac{\Lambda Y(\beta - 1)}{\xi}, -1 - \frac{2\Lambda Y(\beta+1)}{\xi}\right\}\right) \cup \left(max\left\{1, -1 - \frac{2\Lambda Y(\beta+1)}{\xi}\right\}, \infty\right)$$

where $\lambda^* = \frac{1}{1+\phi(1-\alpha)}$ is the threshold value above which the interest elasticity of output becomes positive ie. the IADL region occurs. In the first case above, when $\lambda < \lambda^*$, a standard active rule gives rise to determinacy. In the second case, ie. the IADL region $\lambda \geq \lambda^*$, determinacy only ensues with either a passive rule ($\chi < 1$) or an extremely active rule ($\chi > 30$). To (i) be able to rank standard rules such as $i_t = 1.5\pi_{H,t}$ as well as the stricter rules such as $\pi_{H,t} = 0$ and (ii) avoid the inverted dynamics in the IADL region above λ^* - I always have $\lambda < \lambda^*$ in this paper, so that the equilibrium is determinate under simple rules. This allows for a welfare ranking of standard active simple rules against optimal policy, a primary focus of this paper.

It is worth noting that the simple rule indeterminacy region in an open economy is actually smaller than that in a closed economy (where $\lambda^*|_{\alpha=0} = \frac{1}{1+\phi}$ as in Bilbiie (2008)). This is because contractionary monetary policy in an open economy is generally re-inforced by exchange rate appreciation, which is additionally contractionary. Thus, it takes a higher λ^* to reverse the initial contraction into an expansion. The higher threshold value for indeterminacy is useful as it makes it possible to calibrate λ at a very high number and still ensure a unique, rational expectations equilibrium.

Figures

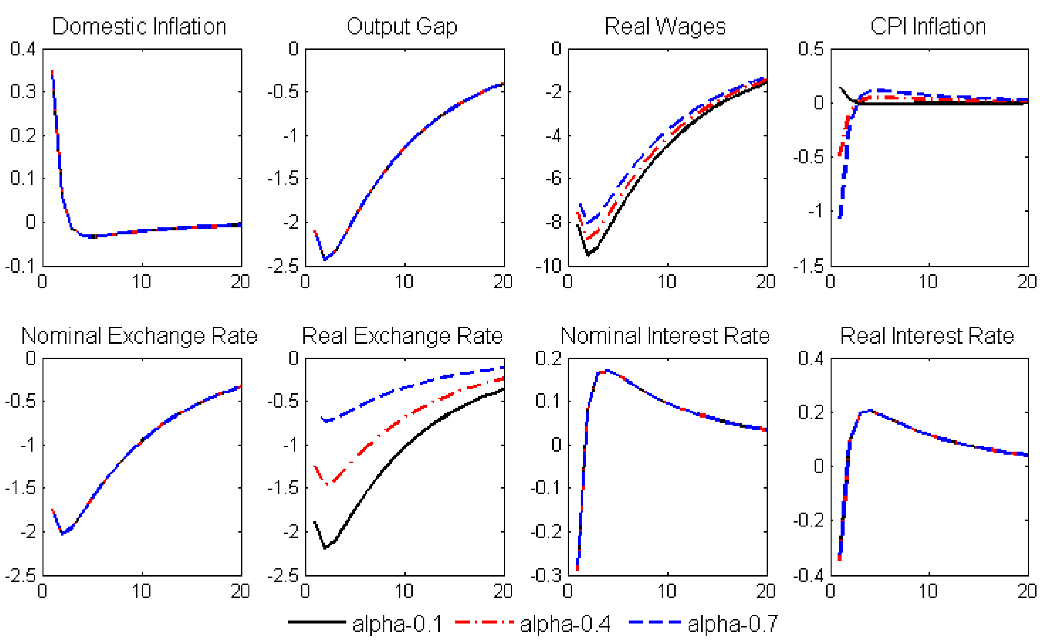

Figure 6: Optimal Monetary Policy with Complete Financial Inclusion: Varying Openness

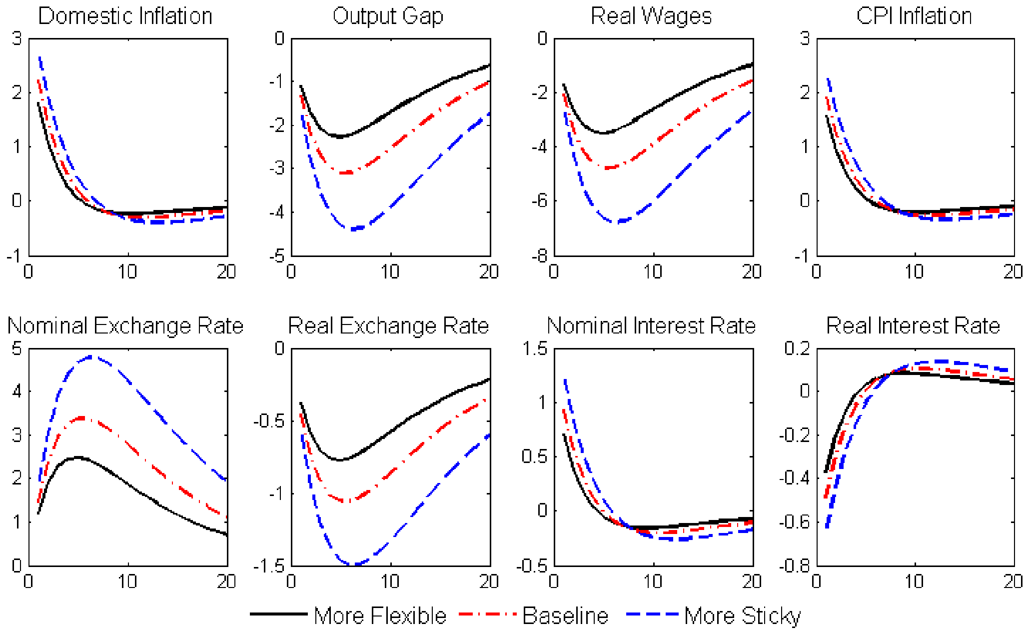

Figure 7: Optimal Monetary Policy with Financial Exclusion: Varying Stickiness

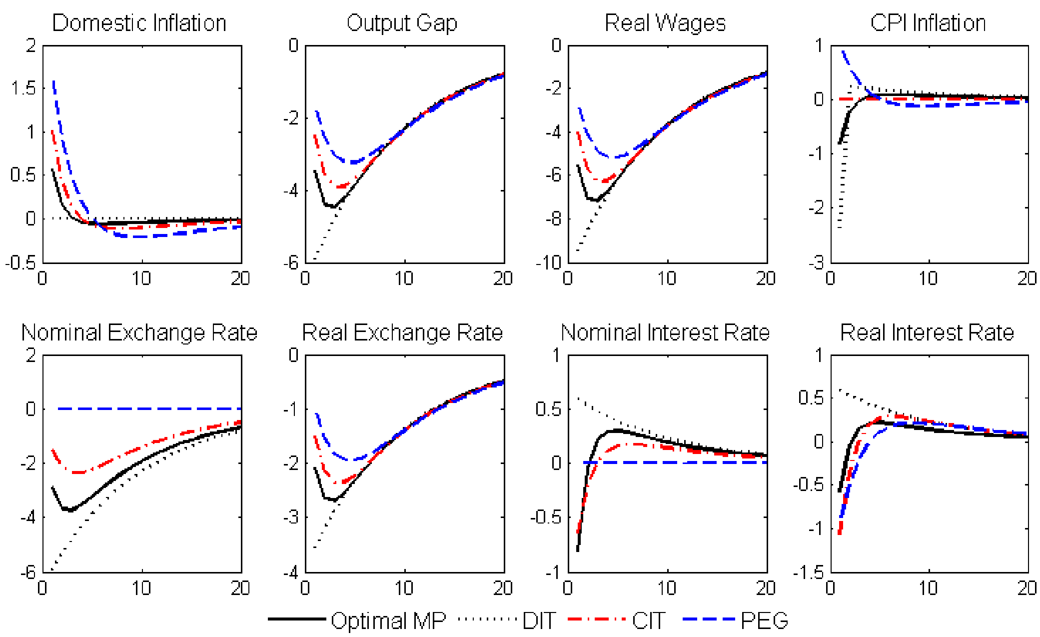

Figure 8: Optimal Policy Versus Simple Rules: Complete Financial Inclusion

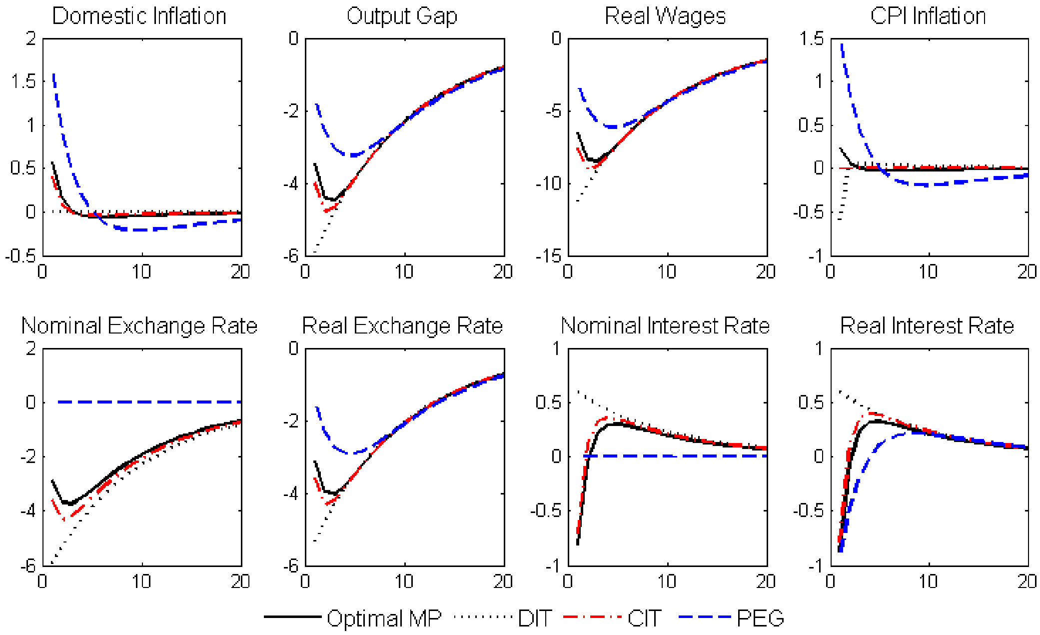

Figure 9: Optimal Policy Versus Simple Rules: Complete Financial Inclusion, Low Openness

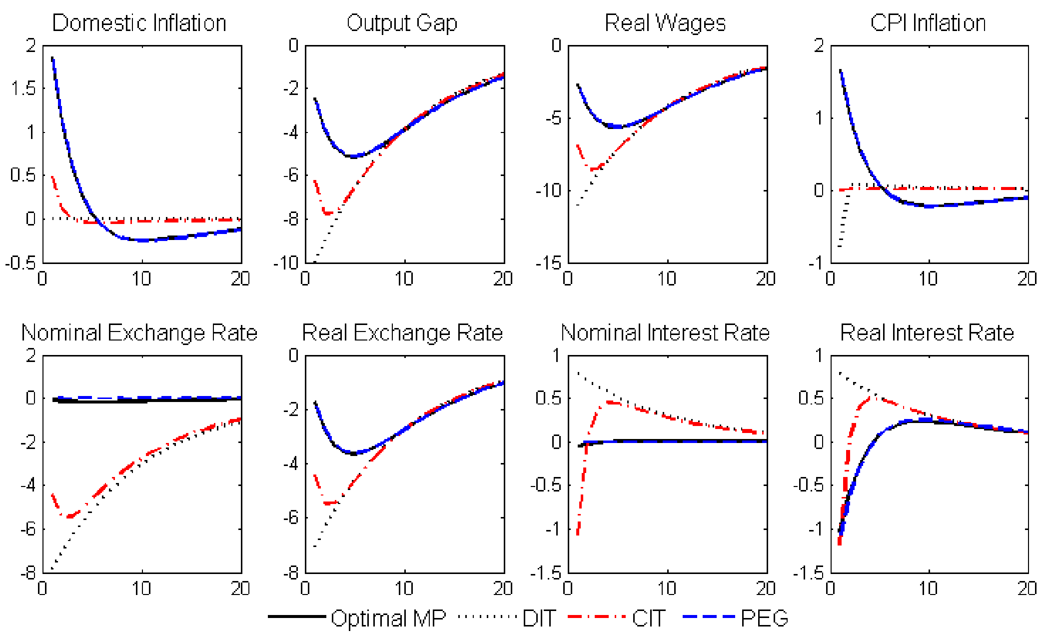

Figure 10: Optimal Policy Versus Simple Rules: Financial Exclusion, Low Openness

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