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and Efficiency in Intermediated Markets**

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Abstract

This paper generalizes Rubinstein and Wolinsky's model of middlemen (intermediation) by incorporating production and search costs, plus more general matching and bargaining. This allows us to study many new issues, including entry, efficiency and dynamics. In the benchmark model, equilibrium exists uniquely, and involves production and intermediation for some parameters but not others. Sometimes intermediation is essential: the market operates iff middlemen are active. If bargaining powers are set correctly equilibrium is efficient; if not there can be too much or too little economic activity. This is novel, compared to the original Rubinstein-Wolinsky model, where equilibrium is always efficient.

1 Introduction

This paper continues the development of theories of middlemen, or intermediaries, going back to Rubinstein and Wolinsky (1987) – hereafter RW. As they said at the time, “Despite the important role played by intermediation in most markets, it is largely ignored by the standard theoretical literature. This is because a study of intermediation requires a basic model that describes explicitly the trade frictions that give rise to the function of intermediation. But this is missing from the standard market models, where the actual process of trading is left unmodeled.” Since then, many people have helped to rectify the situation, contributing to the discussion with various models, but often using search-and-bargaining theory.¹ What makes the models more than a relabeling of, say, textbook search models of labor, goods, or marriage is that they involve three-sided markets – in addition to firms and workers, buyers and sellers, or men and women, they have third parties potentially intermediating between the other two.²

We extend RW on several dimensions, not merely for the sake of generality, but because this allows us to address new substantive issues, including efficiency. Our extensions consist of the following: RW have an endowment economy, while we have production; they do not have search costs, while we do; they assume a special matching process with equal numbers of buyers and sellers, while we use a more general population and matching process; they only

¹We do not review the literature, since that was recently done in Wright and Wong (2014). Surveys by Williamson and Wright (2010) and Nosal and Rocheteau (2011) provide more discussion of work on financial intermediation, in particular, with an emphasis on search. We mention below other papers when they are directly related.

²For labor-market models, see Mortensen and Pissarides (1994) or Pissarides (2000); for goods-market models, see Osborne and Rubinstein (1990), Shi (1995) or Trejos and Wright (1995); for marriage-market models see Burdett and Coles (1997,1999) or Shimer and Smith (2000).

consider the case where buyers and sellers exit the market after they trade, while we allow them to potentially stay in; and they only consider symmetric bargaining, while we allow general bargaining powers, which is especially important for understanding efficiency. Moreover, by taking advantage of advances in search theory over the past 25 years, we provide a more parsimonious presentation of the generalized model, and of RW as a special case.

In terms of results, we first characterize the set of steady-state equilibria for a benchmark model, verifying existence and generic uniqueness.³ Although in principle there are many candidate equilibria, there are basically three distinct outcomes: (i) degenerate equilibria where the market does not open (producers are inactive, middlemen are irrelevant); (ii) equilibria with direct trade between producers and consumers but no intermediation (producers are active, middlemen are not); and (iii) equilibria with direct and intermediated trade (both are active). For some parameters, only a fraction of producers enter the market, as in macro-labor models along the lines of Pissarides (2000). In RW, all producers are always on the market. Whether middlemen, as well as producers, are active depends on parameters, including production and search costs, bargaining power, the matching process, and the probabilities that agents exit the market after trade. In RW, middlemen are active iff they meet consumers faster than producers meet consumers.

We then solve for efficient outcomes. Equilibria are not efficient in general, due to holdup problems, since some costs are sunk when trades occur. Suppose, e.g., all the bargaining power goes to the agent that passes the good on to the next agent – i.e., the producer when he trades with a consumer or

³While uniqueness obtains in our benchmark model, the extension that has producers and consumers exiting the market after trade with some probability can have multiple equilibria for certain parameters, although not for those in the original RW specification.

a middleman, and the middleman when he trades with a consumer. Then there is generally too much entry by producers for reasons discussed below. For arbitrary bargaining powers, there may be too much or too little entry by producers, and there can be too little but not too much intermediation.⁴ However, we show that if bargaining powers are set appropriately, related to Mortensen (1982) and Hosios (1990), equilibrium is efficient. These results are novel compared to the RW model, with no production or search costs and symmetric bargaining, where equilibrium is always efficient.

The rest of the paper involves laying out the details and proving the claims. Section 2 presents the baseline model. Section 3 describes equilibria. Section 4 contains a discussion of extensions and alternative formulations. Section 5 compares efficient and equilibrium outcomes. Section 6 generalizes the model by allowing producers and consumers to exit the market probabilistically. Section 7 concludes.

2 The Model

There are three types of agents, labeled P , M and C for producers, middlemen and consumers, who live forever in continuous time. The measure of type i is n_i with $n_p + n_m + n_c = 1$. There is an indivisible good x that only C values,

⁴Unlike the caveat about uniqueness in fn. 3, the result that there can be too little but not too much intermediation survives in the extension where agents probabilistically exit the market after trade. Still, the result is somewhat model dependent. Li (1998) can get too much or too little intermediation depending on bargaining powers. Her model is different in that agents choose to be either middlemen or producers, so too few of the latter necessarily means too many of the former. Shevchenko (2004) can also get too much or too little intermediation. His model is different in that he allows middlemen to hold multiple units in inventory to study efficiency on the intensive as well as the extensive margin. See also Johri and Leach (2002). One more example is given by Masters (2007,2008). He generally gets too much intermediation, because middlemen in his setup perform no socially useful function – they simply buy low and sell high to consume without producing.

enjoying utility u from consuming one unit. Good x is storable, but only one unit at a time. It is produced by P , who has an entry cost k to participate in the market and another cost c to generate a unit of x for a trading partner. One interpretation is that k is a cost of raw materials and c is a cost to finalize the output. We assume $c + k < u$, as otherwise the market shuts down. If P pays k , we say that he is in the market, looking to trade, in which case he also has a search or storage cost γ_p . While M cannot produce x , and has no desire to consume it, he may acquire it from P with the intent of retrading it to C , in which case M is in the market, with a search or storage cost γ_m . Notice there are no costs for agents looking to acquire x , only for those holding x . Also, there are no production or improvement costs for M , although one can add these, as may be appropriate in applications such as flipping real estate.

There is another good y that is perfectly divisible but nonstorable. Any agent can produce y at unit utility cost. All agents derive utility $U(y)$ from consuming y , in general, although here we assume $U(y) = y$ (how this matters is discussed in Wright and Wong 2014). Therefore, in this exercise, as in RW, one can say there is transferrable utility. Where we generalize their model is that they have $k = c = \gamma_p = \gamma_m = 0$. Adding production and search costs seems obviously relevant for many substantive applications. It is also interesting to have these costs, so that there are nontrivial decisions by producers and middlemen to participate in the market, because this allows us to analyze when there is too little or too much activity in equilibrium. Also, while the only role for intermediation in RW is that M might be able to meet C faster than P can meet C , in this model differences in search costs and other parameters also matter.

The timing is important. For P , costs k and γ_p are sunk when he meets

a potential trading partner, while c is paid only when P delivers the goods. For M , if he acquires x from P , he generally must transfer some y to P , and that as well as γ_m are sunk when M meets C . Sunk costs are interesting in search-and-bargaining models, generally, because they lead to holdup problems that can cause market failures. These problems are often described as the result of imperfect contracting or commitment, and that is accurate here, too. However, compared to models where such imperfections are imposed in an ad hoc fashion, in search theory it is obviously natural to say that it is hard to contract with someone, or commit something to someone, before you meet them. Therefore these are natural models in which to study the efficiency of entry or participation.⁵

Agents meet according to a bilateral random-matching process, where α_{ij} is the Poisson arrival rate at which type i meets j . This implies three identities:

$$n_p\alpha_{pm} = n_m\alpha_{mp}, n_m\alpha_{mc} = n_c\alpha_{cm} \text{ and } n_c\alpha_{cp} = n_p\alpha_{pc}.$$

The first says the measure of type P meeting type M is the same as the measure of type M meeting type P , and similarly for the others. The vector $\alpha = (\alpha_{pm}, \alpha_{mc}, \dots)$ has 6 elements, but the above identities imply $\alpha_{cm}\alpha_{mp}\alpha_{pc} = \alpha_{mc}\alpha_{cp}\alpha_{pm}$, which means that one can choose only 5 independently. In the background one can imagine a population $\mathbf{n} = (n_c, n_m, n_p)$ determining the arrival rates, but we follow RW and take α to be exogenous, since there exists an \mathbf{n} consistent with any α such that $\alpha_{cm}\alpha_{mp}\alpha_{pc} = \alpha_{mc}\alpha_{cp}\alpha_{pm}$. However,

⁵RW also have a holdup problem, since whatever M gives to P is sunk when M meets C . This does not affect efficiency in RW, but can affect the distribution of payoffs. They discuss a “consignment” arrangement, whereby M makes a transfer to P only after trading with C , so it is not sunk when bargaining with C . Of course, this may or not be feasible, depending on the physical environment – e.g., it will not work if M and P cannot reconvene after trading, or if M cannot commit to transfers.

we depart from RW by not focusing exclusively on markets where $\alpha_{cp} = \alpha_{pc}$, $\alpha_{pm} = \alpha_{cm}$ and $\alpha_{mp} = \alpha_{mc}$, the first of which implies $n_c = n_p$ (i.e., the rather special case of equal measures of producers and consumers).

As in RW, middlemen are *recycled* after each trade: any type M agent that chooses to enter the market at $t = 0$ stays in the market forever. For P and C , RW assume that they exit after one trade, to be *cloned* by replicas of themselves to maintain a stationary environment. While it does not matter much whether P and C are cloned or recycled in the simple baseline RW model, it turns out that in our generalized version the latter is much more tractable. To nest both cases, where they are cloned and where they are recycled, we assume that after each trade type $i = P, C$ agents continue in the market with probability ρ_i and exit with probability $1 - \rho_i$. However, for now, as a more tractable benchmark, we assume $\rho_p = \rho_c = 1$, and revisit the general case in Section 6.

Denote the surplus in an ij match by Σ_{ij} . In any ij match, there can be trade if $\Sigma_{ij} \geq 0$, and must be trade if $\Sigma_{ij} > 0$ (see Lemma 2 below). In PC matches, when they trade, P gives x to C for some amount of y , say y_{cp} . Similarly, in MC matches, if M has x gives he it to C for y_{cm} . In PM matches, P cannot give x to M if M already has x , since it is only storable one unit at a time. If M does not have x , P may or may not give it to him; if he does then M gives y_{mp} in return. Our convention for notation is that in Σ_{ij} the subscripts indicate that x goes from i to j , and in y_{ji} they indicate that y goes from j to i . For future reference let $\mathbf{y} = (y_{cp}, y_{mp}, y_{cm})$. Bargaining determines the terms of trade: y_{ij} splits the surplus, where θ_{ij} is the share (bargaining power) of i , and $\theta_{ji} = 1 - \theta_{ij}$. This outcome follows from generalized of Nash (1950) bargaining, Kalai (1977) bargaining, and various other solutions when

$U(y) = y$, although they can give different outcomes when $U''(y) < 0$.⁶

We now analyze behavior. For C it is trivial, since he pays no cost to participate and trades whenever he can (see Lemma 3). The choices for P are π , the probability he enters the market, and σ_p , the probability he searches conditional on holding x ; the choices for M are τ , the probability he tries to enter the market by trading for x , and σ_m , the probability he searches conditional on having x .⁷ Let $\mathbf{p} = (\pi, \sigma_p, \tau, \sigma_m)$ and, since the environment is stationary, assume agents make once-and-for-all decisions at $t = 0$. Thus, P enters with probability π , and if he does then after trading he pays the cost k to remain in the market, while if he does not then he is out forever. Similarly, M decides at $t = 0$ with probability τ to trade for x , and if he does he stays forever, while if he does not then he is out forever. In other words, agents randomize once at $t = 0$, and not in each meeting.

As defined below, an equilibrium determines \mathbf{p} , and hence determines when the market is open (some producers are active), and whether there is intermediation (some middlemen are active). We begin with a few preliminary results.

The first says that M and P would only pay to acquire x if they strictly prefer

⁶Again see Wright and Wong (2014). That paper also shows that there are belief-based (bubble) equilibria in a related model with $U'' < 0$, something that cannot happen with $U'' = 0$ (see Section 4). Another point in that paper is that one ought to resist the temptation to call y_{ij} the price and say i buys x from j , since it makes at least as much sense to call $1/y_{ij}$ the price and say i sells y to j . The argument is that we can only really say who is the buyer and seller in monetary exchange, and one should not call y money, even though people often do in related models – i.e., they use the word money as a sloppy synonym for transferable utility. On reflection, we think that it makes at least as much if not more sense to call x a commodity money: it is a storable asset that P and M use as a medium of exchange to acquire y . Under this interpretation, one can say that Rubinstein and Wolinsky (1987) provide a model of commodity money as a by-product of their analysis of middlemen, the same way that Kiyotaki and Wright (1989) provide a model of middlemen as a by-product of commodity money.

⁷One might anticipate that there are no equilibria where $\pi > 0$ and $\sigma_p = 0$, or $\tau > 0$ and $\sigma_m = 0$. That is true, but it is still important to have σ_p and σ_m in the strategy profile, since an agent who is not willing to pay to get x may or may not try to trade it if, off the equilibrium path, he happened to have it.

to search. This should be obvious for P , since it is costly to enter the market, and hence he will not do so unless he plans to search. Similarly, M would never acquire x unless he plans to search for a trading partner, unless he can get x for free, but P will not give it away for free. This constitutes a proof of Lemma 1. The second result says that trade is mutually agreeable in an PM match whenever the total surplus is positive. The third result says that C always trades with anyone who has x .

Lemma 1 *If $\pi > 0$ then $\sigma_p = 1$. If $\pi > 0$ and $\tau > 0$ then $\sigma_s = 1$.*

Lemma 2 *If $\pi > 0$ then P (strictly) wants to trade with M iff M (strictly) wants to trade with P iff Σ_{pm} is (strictly) positive.*

Lemma 3 *If $\pi > 0$ then C always trades with P , and C always trades with M when M has x .*

Given that P wants to trade with M whenever M wants to trade with P , we can delegate the decision to M . And P always trades with C . Hence, once P is in the market he trades with anyone that is willing and able. To determine who is willing and able, let μ be the fraction of M holding x . Then in any PM match the probability of trade is $\tau - \mu$, since a fraction τ of type M decided at $t = 0$ to accept x , but a fraction μ already have it. The law of motion for μ is

$$\dot{\mu} = (\tau - \mu) \alpha_{mp} \pi \sigma_p - \mu \alpha_{mc}.$$

In the first term, there are $\tau - \mu$ type M that accept x but do not currently have x , they contact P at rate α_{mp} , and the probability is $\pi \sigma_p$ that P is on the market and looking to trade, assuming random matching in the sense that M can meet P even if the latter is not actively on the market.

One way to motivate this is to imagine M calling random P agents on the phone at rate α_{mp} . He may call one that is not in the market or not searching, whence the call goes unanswered. With probability $\pi\sigma_p$ he reaches a P who is active. Note this is not inconsistent with assuming agents with x pay a cost γ_p or γ_m while those looking to acquire it do not – it simply means phone calls are free while storage is costly.⁸ In any case, for the second term in $\dot{\mu}$, there are μ type M agents with x , and they trade whenever they contact C . The SS (steady state) condition $\dot{\mu} = 0$ implies

$$\mu = \frac{\tau\pi\sigma_p\alpha_{mp}}{\pi\sigma_p\alpha_{mp} + \alpha_{mc}}. \quad (1)$$

Let V_c be C 's payoff or value function. Let V_p be P 's value function, given that he has decided to enter the market and search (otherwise his payoff is 0). Let V_0 be M 's value function when he does not have x , given that he has decided to enter the market and trade when he can (otherwise his payoff is 0). Let V_1 be M 's value function when he has x , given that he has decided to search (otherwise his payoff is 0). To develop some intuition, consider first the flow payoff for C ,

$$rV_c = \pi\sigma_p\alpha_{cp}\theta_{cp}\Sigma_{pc} + \mu\sigma_m\alpha_{cm}\theta_{cm}\Sigma_{mc}.$$

The first term says C meets P at rate α_{cp} , and the probability is $\pi\sigma_p$ that P is on the market with goods to trade, in which case C gets a share θ_{cp} of the surplus total Σ_{pc} . The second term is similar.

As in Lagos and Rocheteau (2009), we simplify notation by letting $\eta_{ij} =$

⁸Unlike many search and matching models, our specification does not admit congestion effects. At the suggestion of a referee we note that this simplification can be justified by saying that it allows some new interpretations and additional decisions. So, while slightly nonstandard, the setup is logically consistent, interesting and especially tractable.

$\alpha_{ij}\theta_{ij}$ combine arrival rates and bargaining powers to get

$$rV_p = \eta_{pc}\Sigma_{pc} + (\tau - \mu)\eta_{pm}\Sigma_{pm} - \gamma_p \quad (2)$$

$$rV_0 = \pi\sigma_p\eta_{mp}\Sigma_{pm} \quad (3)$$

$$rV_1 = \eta_{mc}\Sigma_{mc} - \gamma_m \quad (4)$$

$$rV_c = \pi\sigma_p\eta_{cp}\Sigma_{pc} + \mu\sigma_m\eta_{cm}\Sigma_{mc}, \quad (5)$$

the standard DP (dynamic programming) equations. The surpluses are⁹

$$\Sigma_{pc} = u - c + \max\{V_p - k, 0\} - \max\{V_p, 0\} \quad (6)$$

$$\Sigma_{pm} = \max\{V_1, 0\} - \max\{V_0, 0\} - c + \max\{V_p - k, 0\} - \max\{V_p, 0\} \quad (7)$$

$$\Sigma_{mc} = u + \max\{V_0, 0\} - \max\{V_1, 0\}. \quad (8)$$

For future reference, let $\mathbf{V} = (V_p, V_c, V_0, V_1)$ and $\mathbf{\Sigma} = (\Sigma_{pc}, \Sigma_{pm}, \Sigma_{mc})$.

An equilibrium $\mathbf{p} = (\pi, \sigma_p, \tau, \sigma_m)$ must satisfy what we call the BR (best response) conditions. For P , these are:

$$\pi = \begin{cases} 1 & \text{if } V_p > k \\ [0, 1] & \text{if } V_p = k \\ 0 & \text{if } V_p < k \end{cases} \quad \text{and} \quad \sigma_p = \begin{cases} 1 & \text{if } V_p > 0 \\ [0, 1] & \text{if } V_p = 0 \\ 0 & \text{if } V_p < 0 \end{cases} \quad (9)$$

For M , they are:

$$\tau = \begin{cases} 1 & \text{if } \Sigma_{pm} > 0 \\ [0, 1] & \text{if } \Sigma_{pm} = 0 \\ 0 & \text{if } \Sigma_{pm} < 0 \end{cases} \quad \text{and} \quad \sigma_m = \begin{cases} 1 & \text{if } V_1 > 0 \\ [0, 1] & \text{if } V_1 = 0 \\ 0 & \text{if } V_1 < 0 \end{cases} \quad (10)$$

⁹Heuristically, the max operators embody the notion of subgame perfection. Consider Σ_{pc} . If P and C trade, the instantaneous surplus is $u - c$, then P decides whether to pay k to remain in the market, so his continuation value is $\max\{V_p - k, 0\}$; if they do not trade, P decides whether to continue search, so his outside option is $\max\{V_p, 0\}$. In equilibrium, once P decides to enter he is in the market forever, but this way of writing the surplus indicates this is a best response in every subgame. For C the continuation value and outside options are both V_c , so they cancel, which is one reason the analysis is easier when we recycle C .

Definition 1 A (steady-state) equilibrium is a list $\langle \mu, \mathbf{V}, \mathbf{p} \rangle$ such that: μ satisfies the SS condition (1); \mathbf{V} satisfies the DP equations (2)-(5); and \mathbf{p} satisfies the BR conditions (9)-(10).

Given an equilibrium the terms of trade are easily recovered. Assuming $\pi > 0$, C 's surplus when trading directly with P is $u - y_{cp} = \theta_{cp}(u - c - k)$, and so

$$y_{cp} = \theta_{pc}u + \theta_{cp}(c + k). \quad (11)$$

This is a weighted average of C 's gain and P 's cost, including k even though it is sunk, because P has to pay it again to continue in the market. Similarly, assuming $\pi > 0$ and $\tau > 0$, the transfers in wholesale and retail trade are

$$y_{mp} = \theta_{pm}\Delta + \theta_{mp}(c + k) \text{ and } y_{cm} = \theta_{mc}u + \theta_{cm}\Delta, \quad (12)$$

where $\Delta \equiv V_1 - V_0$ is M 's gain from getting (cost to giving) x , which is easily computed from (3)-(4). Finally, the wholesale-retail markup, or spread, $s \equiv y_{cm} - y_{mp}$, is given by¹⁰

$$s = \theta_{mc}u + (\theta_{cm} - \theta_{pm})\Delta - \theta_{mp}(c + k). \quad (13)$$

3 Equilibrium

We now characterize equilibria. There are in principle many candidate equilibrium profiles, but one can rule out those with $\pi > 0$ and $\sigma_p < 1$, plus those with $\tau > 0$ and $\sigma_m < 1$, since it cannot be a BR to pay for x and not search. There

¹⁰Although the terms of trade are interesting, we do not dwell on them since we are more concerned with existence and efficiency. However, if one solves for s , it clearly does not vanish as $r \rightarrow 0$, contrary to RW. In RW, M profits exclusively from the impatience of others when $\alpha_{mc} > \alpha_{pc}$, and as $r \rightarrow 0$ that advantage vanishes. Here M may have other advantages, including costs and bargaining power.

are also candidates with $\pi = 0$, which are relegated to Appendix A, so we can concentrate on nondegenerate cases here. The next result further reduces the set of candidates by establishing that M never randomizes, and while P may randomize, he does so only when M is in the market with probability 1.

Lemma 4 *For generic parameters, in any equilibrium: (i) $\tau \in (0, 1)$ implies $\pi = 0$; and (ii) $\pi \in (0, 1)$ implies $\tau = 1$.*

Proof: For (i), suppose by way of contradiction $\pi > 0$ and $\tau \in (0, 1)$. Then $\Sigma_{pm} = 0$, or $V_1 - V_0 = c + k$. The value functions for M are then given by

$$\begin{aligned} rV_0 &= \pi\eta_{mp} [V_1 - V_0 - (c + k)] - \gamma_m \\ rV_1 &= \eta_{mc} [u - (V_1 - V_0)]. \end{aligned}$$

Solving for $\Delta = V_1 - V_0 = c + k$, $\Sigma_{pm} = 0$ implies $\eta_{mc}u = (r + \eta_{mc})(c + k) - \gamma_m$, which is nongeneric. For (ii), suppose $\pi > 0$ and $\tau < 1$. By (i), $\tau = 0$; then $rV_p = \eta_{pc}\Sigma_{pc} - \gamma_p = \eta_{pc}(u - c - k) - \gamma_p$. For $\pi \in (0, 1)$ we need $V_p = k$, which is nongeneric. ■

Our quest for nondegenerate equilibria is thus reduced to four candidates. There are three where P enters with probability 1: $\mathbf{p} = (1, 1, 0, 0)$, where M does not trade for x and would not search if he had it; $\mathbf{p} = (1, 1, 0, 1)$, where M does not trade for x but would search if he had it; and $\mathbf{p} = (1, 1, 1, 1)$, where M trades for x and searches when he gets it. There is also one candidate where P enters with probability $\pi \in (0, 1)$ and $\tau = 1$. To understand the logic of $\pi \in (0, 1)$, note that for P to be indifferent to entry we need $V_p = k$. As π varies, the probability that M can take x off P 's hands when they meet, $1 - \mu$, adjusts endogenously to make $V_p = k$. We now consider each of these candidates in turn.

1. Equilibrium $\mathbf{p} = (1, 1, 0, 0)$: In this equilibrium, P enters with probability 1, while M neither accepts x nor searches if (off the equilibrium path) he happens to have x . This implies

$$\begin{aligned} rV_p &= \eta_{pc}(u - c - k) - \gamma_p \\ rV_0 &= \eta_{mp}(V_1 - V_0 - c - k) \\ rV_1 &= \eta_{mc}(u - V_1 + V_0) - \gamma_m \\ rV_c &= \eta_{cp}(u - c - k), \end{aligned}$$

where one should interpret V_0 and V_1 as the payoffs to M if he were active, even though he is not active in equilibrium. For P , $\pi = 1$ is a BR iff $V_p \geq k$, which reduces after routine algebra to

$$\gamma_p \leq \tilde{\gamma}_p \equiv \eta_{pc}(u - c - k) - rk. \quad (14)$$

Given $\pi = 1$, $\sigma_p = 1$ is automatic (Lemma 1). For M , consider a deviation where he searches when he has x . The deviation payoff is

$$rV_1^d = \eta_{mc}(u - V_1^d) - \gamma_m.$$

For $\sigma_m = 0$ to be a BR we need $V_1^d \leq 0$, or $\gamma_m \geq \eta_{mc}u$. Given $\sigma_m = 0$, $\tau = 0$ is automatic. Hence $\mathbf{p} = (1, 1, 0, 0)$ is an equilibrium iff $0 \leq \gamma_p \leq \tilde{\gamma}_p$ and $\gamma_m \geq \eta_{mc}u$.

2. Equilibrium $\mathbf{p} = (1, 1, 0, 1)$: For P , the BR condition is again $\gamma_p \leq \tilde{\gamma}$. For M , it is easy to check $\tau = 0$ is a BR iff $\Sigma_{pm} \leq 0$, or

$$\gamma_m \geq \tilde{\gamma}_m \equiv \eta_{mc}(u - c - k) - r(c + k). \quad (15)$$

Also, $\sigma_m = 1$ is a BR iff $\gamma_m \leq \eta_{mc}u$. Hence $\mathbf{p} = (1, 1, 0, 1)$ is an equilibrium iff $\gamma_p \leq \tilde{\gamma}_p$ and $\eta_{mc}u \geq \gamma_m \geq \tilde{\gamma}_m$.

Before moving to other cases, consider Figure 1, where the two equilibria discussed above exist in (γ_p, γ_m) space in the regions to the northwest labeled $(1, 1, 0, 0)$ and $(1, 1, 0, 1)$. Naturally, P is active while M is not when γ_p is small and γ_m is big. If γ_m is very big, M would not search for C even if he had x ; if γ_m is only moderately big M would search for C if he had x , but it is not worth making the transfer to acquire it. To describe what happens for lower γ_m , it is convenient to consider the lines $h_0(\gamma_p)$ and $h_1(\gamma_p)$ in Figure 1. Both are special cases of $\gamma_m = h_\pi(\gamma_p)$, for any $\pi \in [0, 1]$, given by

$$h_\pi(\gamma_p) \equiv \tilde{\gamma}_m + (\tilde{\gamma}_p - \gamma_p) \frac{r + \eta_{mc} + \pi\eta_{mp}}{\eta_{pm} [\tau - \mu(\pi)]}, \quad (16)$$

where $\tilde{\gamma}_m$ is defined in (15) and $\mu = \mu(\pi)$ is now written as a function of π . As π goes from 0 to 1, $h_\pi(\gamma_p)$ rotates around $(\tilde{\gamma}_p, \tilde{\gamma}_m)$ from $h_0(\gamma_p)$ to $h_1(\gamma_p)$.

INSERT FIG 1 ABOUT HERE

3. Equilibrium $\mathbf{p} = (1, 1, 1, 1)$: For $\tau = 1$ we need $\Sigma_{pm} \geq 0$. This reduces to $\gamma_m \leq \tilde{\gamma}_m$, the reverse of (15). For $\pi = 1$ we need $V_p \geq k$, which reduces to $\gamma_m \leq h_1(\gamma_p)$. Hence this equilibrium exists iff $\gamma_m \leq \min\{\tilde{\gamma}_m, h_1(\gamma_p)\}$, as shown in Figure 1.

4. Equilibrium $p = (\pi^e, 1, 1, 1)$ with $\pi^e \in (0, 1)$: One can check M 's BR condition holds iff $\gamma_m \leq \tilde{\gamma}_m$, so it remains only to check $\pi = \pi^e \in (0, 1)$. Substituting μ from (1) into V_p and solving the quadratic equation $V_p = k$ for π , we get

$$\pi^e = \frac{-[\alpha_{mp}(r + \eta_{mc}) + \alpha_{mc}\eta_{mp}] + \sqrt{D}}{2\alpha_{mp}\eta_{mp}}, \quad (17)$$

where $D = [\alpha_{mp}(r + \eta_{mc}) + \alpha_{mc}\eta_{mp}]^2 - 4\alpha_{mp}\alpha_{mc}\eta_{mp}\eta_{pm}A_m/A_p$. Algebra implies $\pi^e \in (0, 1)$ in the region between $h_1(\gamma_p)$ and $h_0(\gamma_p)$ in Figure 1.

5. Degenerate equilibria: Appendix A shows that equilibria with $\pi = 0$ exist in the shaded region of Figure 1. However, there are different equilibria with $\pi = 0$, e.g., where σ_p is either 0 or 1. Appendix A shows that where are in the shaded region determines which degenerate equilibrium exists.

In terms of economics, it is no surprise that for P or M to be active we cannot have γ_p or γ_m too high; the preceding analysis worked out the exact cutoffs. For some parameters, P enters with probability $\pi^e \in (0, 1)$, with μ adjusting endogenously to make $V_p = k$. This is related to discussions of “search externalities” throughout the literature, although in this model, by design, entry does not affect meeting rates, it rather affects μ and hence the probability of trade when P meets M . We also emphasize this: Suppose $\gamma_p > \tilde{\gamma}_p$, as is the case when P has a poor storage technology, a low chance of finding C , or low bargaining power when he does find C . Then intermediation is essential in the sense the market operates iff middlemen are active.¹¹

These results are novel relative to RW, where costs are 0, so P is always active, and M is active iff α_{mc} exceeds α_{pc} . While intermediation can improve welfare in RW, the impact here is more dramatic – sometimes the market opens iff intermediation smooths the way. We summarize as follows:

Proposition 1 *Given $\rho_j = 1$ (everyone recycles), for all values of the other parameters, equilibrium exists and is generically unique, as shown in (γ_p, γ_m) space by Figure 1. For some parameters intermediation is essential.*

¹¹In monetary theory, money is said to be essential if the set of outcomes that can be supported as equilibria is bigger or better with money than without it (e.g., Wallace 2001,2010). For money this is nontrivial because, obviously, it is not essential in standard Arrow-Debreu models. The same is true of intermediation.

4 Alternative Assumptions

Here we mention some extensions, including a different way to describe the results.¹² To begin, note that in addition to preferences (r, u) , arrival rates (α_{ij}) and bargaining powers (θ_{ij}) , the model parameters are given by the vector of costs $\omega = (c, k, \gamma_m, \gamma_p)$. In general, we need all elements of ω to characterize the equilibrium set, but sometimes different equilibria generate the same outcome – e.g., for any (π, σ_p) , both $(\tau, \sigma_m) = (0, 0)$ and $(\tau, \sigma_m) = (0, 1)$ entail no intermediation. If one cares only about outcomes, we claim all that matters is the expected net gain for P from trying to trade with C directly, denoted A_p , and the expected net gain for M from trying to trade with C , denoted A_m , where

$$A_p \equiv \tilde{\gamma}_p - \gamma_p = \eta_{pc}u - \eta_{pc}c - (r + \eta_{pc})k - \gamma_p \quad (18)$$

$$A_m \equiv \tilde{\gamma}_m - \gamma_m = \eta_{mc}u - (r + \eta_{mc})(c + k) - \gamma_m. \quad (19)$$

Appendix B translates the results in Section 3 from (γ_p, γ_m) space into (A_p, A_m) space, as illustrated in Figure 2, which is isomorphic to Figure 1, but is still useful due to the interpretation. First notice that A_p and A_m are bounded above by $\tilde{\gamma}_p$ and $\tilde{\gamma}_m$. Now, since A_p is the net benefit to P of searching for C without using M , $A_p > 0$ implies $\pi = 1$ regardless of M 's decision. Similarly, A_m is the net benefit to M of searching for C , so $\tau = 1$ if $A_m > 0$ and $\tau = 0$ if $A_m < 0$. Hence, outcomes are obvious in three of the four quadrants in Figure 2: (i) $A_p > 0$ and $A_m > 0$ imply $\pi = 1$ and $\tau = 1$;

¹²This is not critical for what follows, and one can move directly to the discussion of efficiency with little loss of continuity, but one message here is that $c = k = 0$ is in a sense without loss of generality. We also show how to extend the analysis to describe what happens out of steady state.

(ii) $A_p > 0$ and $A_m < 0$ imply $\pi = 1$ and $\tau = 0$; (iii) $A_p < 0$ and $A_m < 0$ imply $\pi = 0$ and τ is irrelevant. In the fourth (northwest) quadrant, as we make A_p a bigger negative number for fixed $A_m > 0$, π goes from 1 to $\pi^e \in (0, 1)$ to 0.¹³

INSERT FIG 2 ABOUT HERE

An implication is that there is little loss of generality in setting $c = k = 0$ if we care only about outcomes. By analogy, in labor models firms can have a fixed or flow cost to entering the market, but we do not need both, since all that matters is the total expected discounted cost. To see how this works here, consider two economies with $\bar{\omega} = (\bar{c}, \bar{k}_1, \bar{\gamma}_m, \bar{\gamma}_p)$ and $\hat{\omega} = (\hat{c}, \hat{k}, \hat{\gamma}_m, \hat{\gamma}_p)$. The outcome depends only on

$$\begin{aligned}\bar{A}_m &= \eta_{mc}u - (r + \eta_{mc})(\bar{c} + \bar{k}) - \bar{\gamma}_m \\ \bar{A}_p &= \eta_{pc}u - \eta_{pc}\bar{c} - (r + \eta_{pc})\bar{k} - \bar{\gamma}_p\end{aligned}$$

in the $\bar{\omega}$ economy, and similarly in the $\hat{\omega}$ economy. If we set $\hat{c} = \hat{k} = 0$, then set $\hat{\gamma}_m = (r + \eta_{mc})(\bar{c} + \bar{k}) + \bar{\gamma}_m$ and $\hat{\gamma}_p = \eta_{pc}\bar{c} + (r + \eta_{pc})\bar{k} + \bar{\gamma}_p$, the outcomes in the $\hat{\omega}$ and $\bar{\omega}$ economies are the same. Hence, we can always set $c = k = 0$ and not change outcomes, as long as we adjust the γ 's.¹⁴

Next, consider dynamics. Setting $c = k = 0$ to reduce notation, the DP equation for type P without imposing steady state is

$$\dot{V}_p = rV_p - \eta_{pc}u - \eta_{pm}(\tau - \mu)(V_1 - V_0) + \gamma_p, \quad (20)$$

and similarly for the others. These plus the law of motion for μ define a dy-

¹³It is easy to check $\pi = \pi^e$ occurs between the ray ℓ_0 defined by $A_m = -A_p(r + \eta_{mc})/\eta_{pm}$ and the ray ℓ_1 defined by $A_m = -A_p(r + \eta_{mc} + \eta_{mp})(\alpha_{mc} + \alpha_{mp})/\alpha_{mc}\eta_{pm}$.

¹⁴At least, this is true if we care only about outcomes in terms of π and τ ; the above argument does not say that the two economies will have the same terms of trade \mathbf{y} .

namical system. Given an initial condition μ_0 , an equilibrium is a nonnegative solution to this system that is bounded (more accurately, that does not grow faster than r). In fact, the system is quite simple. First, because type C makes no decisions, ignore them. Second, for type M , at any point in time they choose $\tau = 1$ if $\eta_{mc}u > \gamma_m$ and $\tau = 0$ if $\eta_{mc}u < \gamma_m$, independent of anything else that is going on.

INSERT FIG 3 ABOUT HERE

Thus, the relevant decisions are made by P , although of course these depend on what M is doing. If M is inactive, assuming free disposal, μ jumps to $\mu^s = 0$ and stays there. Then \dot{V}_p is linear with slope $r > 0$, and the only bounded solution is $V_p = V_p^s \forall t$, where $V_p^s = (\eta_{pc}u - \gamma_p)/r$. Hence, when M is inactive: $\eta_{pc}u < \gamma_p \implies V_p^s < 0, \pi = 0 \forall t$; and $\eta_{pc}u > \gamma_p \implies V_p^s > 0, \pi = 1 \forall t$. The outcome is somewhat more interesting when M is active. Since it is obvious the unique steady state is a saddle point, once π is fixed, there can be transitional dynamics as $\mu \rightarrow \mu^s$ but no belief-based (bubble) equilibria. So all we have to do is describe how μ and V_p evolve over time. We break the analysis into cases, depending on parameters and initial conditions.

Suppose first parameters are such that in steady state $\pi^s = 1$. Figure 3 depicts three subcases differing in μ_0 . The top panel is subcase a, defined by $\mu_0 < \mu^s(1)$, which means $V_{p0} > V_p^s(1) > 0$ where we now write $\mu(\pi)$ and $V_p^s(\pi)$ for the steady state given π . Then type P enter at $t = 0$, and stay in, while $\mu \rightarrow \mu^s(1)$ and $V_p \rightarrow V_p^s(1)$ as shown. Subcase b has the opposite initial condition, $\mu_0 > \mu^s(1)$, which means $V_{p0} < V_p^s(1)$. Then two situations can occur. The middle panel is subcase b1 where $V_{p0} > 0$, so P enter at $t = 0$, while $\mu \rightarrow \mu^s(1)$ and $V_p \rightarrow V_p^s(1)$. The bottom panel is subcase b2, where

$V_{p0} < 0$, which means P do not enter at $t = 0$. Since we are supposing $\pi^s = 1$, in this situation $\exists t_1 \in (0, \infty)$ such that $\pi = 0 \forall t < t_1$ and $\pi = 1 \forall t > t_1$. At first, with $\pi = 0$, inventories fall rapidly, making V_p rise until t_1 , at which point P finds it profitable to enter, and then $\mu \rightarrow \mu^s(1)$ and $V_p \rightarrow V_p^s(1)$. The other cases can be analyzed similarly.

Finally, consider an extension, suggested by a referee, to incorporate “occupational choice.” Given a fixed measure n_c , the baseline model takes n_p and n_m as also fixed, but because P and M can choose to be inactive, the ratio of active producers to middlemen is endogenous. But another approach is to let everyone in the $1 - n_c$ set of nonconsumers choose to be either a producer or a middlemen, or be neither and sit out. When we worked out this alternative way to endogenize the PM ratio, the results were similar: there were still three possible outcomes – P inactive; P active but M inactive; or both active – and the analog to Figure 1 looks roughly the same. One interesting difference is this: given $\pi > 0$, the baseline model always has intermediation when $V_0 > 0$, which must be true for small γ_m because M ’s only other option is to sit out; the alternative setup may have $n_m = 0$ even when $\gamma_m = 0$, because for $n_m > 0$ we now not only need $V_0 > 0$, we need $V_0 > V_p$. Since this and some other technical features are rather different, we do not include details.

5 Efficiency

We now consider the planner’s problem, in discrete time, which we find easier and more intuitive, although of course the SS is the same as in continuous time. The state vector is (λ, μ) , where λ is the stock of P ’s in the market with x at the end of a period, and μ as always is the stock of M ’s holding x .

The control vector is (π, ϕ) , where π is the stock of P 's in the market with x at start of the next period, and ϕ denotes the fraction of the $1 - \mu$ type M 's that are currently without x that we instruct to try to trade for x next period. Obviously, (λ, μ) and (π, ϕ) are related to the equilibrium variables, as described below. The planner's problem is¹⁵

$$\begin{aligned} \tilde{W}(\lambda, \mu) = & -(\pi - \lambda)n_pk + \beta\pi n_p[\alpha_{pc}(u - c) - \alpha_{pm}(1 - \mu)\phi c - \gamma_p] \quad (21) \\ & + \beta n_m \mu \alpha_{mc}(\alpha_{mc}u - \gamma_m) + \beta \tilde{W}(\lambda', u'). \end{aligned}$$

The first term on the RHS is the current cost to activating $(\pi - \lambda)n_p$ type P agents that are currently not in the market. The second term is the net benefit from having πn_p type P 's active next period, which includes the following instantaneous payoffs: the net gain $u - c$ to P trading with C , which happens with probability α_{pc} ; the cost $-c$ from P producing for M , which occurs with probability $\alpha_{pm}(1 - \mu)\phi$; and the search cost $-\gamma_p$. Similarly, the third term is net benefit from having $n_m \mu$ type M 's with x . The final term is the continuation value, given the laws of motion

$$\lambda' = \pi [1 - \alpha_{pc} - \alpha_{pm}(1 - \mu)\phi] \quad (22)$$

$$\mu' = \mu(1 - \alpha_{mc}) + \pi \alpha_{mp}(1 - \mu)\phi. \quad (23)$$

Letting $W(\lambda, \mu) = \tilde{W}(\lambda, \mu)/n_p$, and using $\alpha_{mp}n_m = \alpha_{pm}n_p$, rewrite (21) as

$$\begin{aligned} W(\lambda, \mu) = & -(\pi - \lambda)k + \beta\pi[\alpha_{pc}(u - c) - \alpha_{pm}(1 - \mu)\phi c - \gamma_p] \quad (24) \\ & + \beta\mu(\alpha_{pm}/\alpha_{mp})(\alpha_{mc}u - \gamma_m) + \beta W(\lambda', u'). \end{aligned}$$

¹⁵Importantly, we are not imposing steady state and then maximizing welfare; we are solving the dynamic planner's problem and then imposing steady state. The other problem only gives the correct answer in limit as $r \rightarrow 0$.

Let R denote the RHS of (24). Since $\pi, \phi \in [0, 1]$, their optimal values depend on

$$\frac{\partial R}{\partial \pi} \simeq (r + \alpha_{mc} + \phi \alpha_{mp} \pi)(\tilde{\gamma}_p^o - \gamma_p) + (1 - \mu) \phi \alpha_{pm}(\tilde{\gamma}_m^o - \gamma_m) \quad (25)$$

$$\frac{\partial R}{\partial \phi} \simeq \tilde{\gamma}_m^o - \gamma_m, \quad (26)$$

where $a \simeq b$ means that a and b take the same sign, and we define

$$\tilde{\gamma}_m^o \equiv \alpha_{mc} u - (r + \alpha_{mc})(c + k) \quad (27)$$

$$\tilde{\gamma}_p^o \equiv \alpha_{pc}(u - c) - (r + \alpha_{pc})k. \quad (28)$$

Note that $\tilde{\gamma}_p^o$ and $\tilde{\gamma}_m^o$ are closely related to $\tilde{\gamma}_p$ and $\tilde{\gamma}_m$ Section 3: for all values of the other parameters, $\tilde{\gamma}_m = \tilde{\gamma}_m^o$ iff $\theta_{mc} = 1$, and $\tilde{\gamma}_p = \tilde{\gamma}_p^o$ iff $\theta_{pc} = 1$.

Given these results, we arrive at the final simplified versions of (25)-(26), and hence the final answer to the planner's problem,

$$\phi = \begin{cases} 1 \\ [0, 1] \\ 0 \end{cases} \quad \text{if } \tilde{\gamma}_m^o \begin{cases} > \gamma_m \\ = \gamma_m \\ < \gamma_m \end{cases} \quad \text{and } \pi = \begin{cases} 1 \\ [0, 1] \\ 0 \end{cases} \quad \text{if } L^o \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \quad (29)$$

where to conserve space we introduce

$$L^o \equiv \tilde{\gamma}_m^o - \gamma_m + \frac{(r + \alpha_{mc} + \alpha_{mp} \pi)(\pi \alpha_{mp} + \alpha_{mc})}{\alpha_{pm} \alpha_{mc}} (\tilde{\gamma}_p^o - \gamma_p) \quad (30)$$

It is now straightforward to characterize efficient outcomes, as shown in (γ_p, γ_m) space by Figure 4, similar to the equilibrium characterization in Figure 1, except now with $\tilde{\gamma}_j^o$ and h_π^o rather than $\tilde{\gamma}_j$ and h_π .

Proposition 2 *The solution to the planners problem has the following properties in steady state: (i) $\tilde{\gamma}_p^o > \gamma_p$ and $\tilde{\gamma}_m^o > \gamma_m$ implies $\pi = \phi = 1$; (ii) $\tilde{\gamma}_p^o > \gamma_p$*

and $\tilde{\gamma}_m^o < \gamma_m$ implies $\pi = 1$ and $\phi = 0$; (iii) $\tilde{\gamma}_p^o < \gamma_p$ and $\tilde{\gamma}_m^o < \gamma_m$ implies $\pi = 0$, so ϕ is irrelevant; (iv) $\tilde{\gamma}_p^o < \gamma_p$ and $\tilde{\gamma}_m^o > \gamma_m$ implies three subcases: if $L^o < 0$ then $\pi = 0$ and ϕ is irrelevant, if $L^o > 0$ then $\pi = 1$ and $\phi = 1$, and if $L^o = 0$ then $\phi = 1$ and $\pi = \pi^o \in (0, 1)$ is given by

$$\pi^o = \frac{-(r + 2\alpha_{mc}) + \sqrt{r^2 - 4\alpha_{mc}\alpha_{pm}(\tilde{\gamma}_m^o - \gamma_m)/(\tilde{\gamma}_p^o - \gamma_p)}}{2\alpha_{mp}}. \quad (31)$$

INSERT FIG 4 ABOUT HERE

While equilibrium outcomes and efficient outcomes may not coincide, in general, the next result demonstrates that there are conditions on bargaining powers, related to Mortensen (1982) and Hosios (1990), that imply they do coincide.

Proposition 3 *There exists a unique $\theta^o = (\theta_{pc}^o, \theta_{mc}^o, \theta_{pm}^o)$ that implies the equilibrium outcome is efficient for all values of the other parameters, given by $\theta_{pc}^o = \theta_{mc}^o = 1$ and*

$$\theta_{pm}^o = \frac{r + \alpha_{mc} + \pi^o \alpha_{mp}}{r + \alpha_{mc} + 2\pi^o \alpha_{mp}}, \quad (32)$$

where π^o is given in (31).

Proof: The efficient outcome $(\pi, \phi) = (1, 1)$ requires that $\tilde{\gamma}_m^o > \gamma_m$ and $\tilde{\gamma}_p^o > \gamma_p$; the equilibrium outcome $(\pi, \tau) = (1, 1)$ requires that $\tilde{\gamma}_m > \gamma_m$ and $\tilde{\gamma}_p > \gamma_p$. Now $\tilde{\gamma}_m^o > \gamma_m$ implies $\tilde{\gamma}_m > \gamma_m$ for all values of the other parameters iff $\theta_{mc} = 1$, and $\tilde{\gamma}_p^o > \gamma_p$ implies $\tilde{\gamma}_p > \gamma_p$ for all values of the other parameters iff $\theta_{pc} = 1$. When $\theta_{mc} = \theta_{pc} = 1$, $\tilde{\gamma}_m = \tilde{\gamma}_m^o$ and $\tilde{\gamma}_p = \tilde{\gamma}_p^o$. Then from (30) efficient outcome $(\pi, \phi) = (\pi^o, 1)$ requires

$$\tilde{\gamma}_m^o - \gamma_m = -\frac{(r + \alpha_{mc} + \alpha_{mp}\pi^o)(\pi^o\alpha_{mp} + \alpha_{mc})}{\alpha_{pm}\alpha_{mc}}(\tilde{\gamma}_p^o - \gamma_p). \quad (33)$$

The equilibrium outcome $(\pi^e, 1)$, with $\theta_{pc} = \theta_{mc} = 1$, requires

$$\tilde{\gamma}_m^o - \gamma_m = -\frac{(r + \alpha_{mc} + (1 - \theta_{pm})\alpha_{mp}\pi^e)(\pi^e\alpha_{mp} + \alpha_{mc})}{\theta_{pm}\alpha_{pm}\alpha_{mc}}(\tilde{\gamma}_p^o - \gamma_p). \quad (34)$$

Setting (33) to equal (34) and $\pi^e = \pi^o$, we obtain (32). ■

Although efficiency obtains when the θ 's are set just right, for arbitrary parameters, the equilibrium outcomes can be inefficient. In particular,

Proposition 4 *Depending upon parameters: (i) π^e can be too high or too low; (ii) τ^e can be too low but not too high.*

Proof. (i) Suppose $0 < \pi^o < 1$ and $\theta_{pc} = \theta_{mc} = 1$. It is not hard to check from (17) that $\theta_{pm} > \theta_{pm}^o$ implies $\pi^e > \pi^o$ and $\theta_{pm} < \theta_{pm}^o$ implies $\pi^e < \pi^o$. Hence, π^e can be too high or too low.

(ii) Suppose $\tilde{\gamma}_p^o > \gamma_p$ and $\tilde{\gamma}_m^o > \gamma_m$. Then $\phi = 1$. If $0 < \theta_{mc} < 1$, then $\tilde{\gamma}_m^o > \tilde{\gamma}_m$. For θ_{mc} sufficiently small, $\tilde{\gamma}_m < \gamma_m$, which implies $\tau = 0$. Hence, τ^e can be too low. Equilibrium requires that $\tau = 1$ iff $\tilde{\gamma}_m > \gamma_m$; otherwise $\tau = 0$. Efficiency requires that $\phi = 1$ iff $\tilde{\gamma}_m^o > \gamma_m$; otherwise $\phi = 0$. Since $\tilde{\gamma}_m^o \geq \tilde{\gamma}_m$, we have $\phi \geq \tau$. Hence, we would be able to conclude that the equilibrium τ cannot be too high, if we could verify that $\pi^o = 0$ implies $\pi^e = 0$. We need the latter condition because otherwise τ can be too high when $0 < \pi^e < 1$ and $\pi^o = 0$. The result we need, that $\pi^o = 0$ implies $\pi^e = 0$, is true as long as $h_0^o(\gamma_p)$ lies everywhere above $h_0(\gamma_p)$ for all $\gamma_p > \tilde{\gamma}_p^o$ where $h_0(\gamma_p) \geq 0$, as shown in Figure 5 (the graph is drawn assuming $\tilde{\gamma}_p^o > \tilde{\gamma}_p$ and $\tilde{\gamma}_m^o > \tilde{\gamma}_m$ but $\tilde{\gamma}_p^o = \tilde{\gamma}_p$ and $\tilde{\gamma}_m^o = \tilde{\gamma}_m$ has the same qualitative features).

INSERT FIG 5 ABOUT HERE

Since $h_0^o(\gamma_p)$ and $h_0(\gamma_p)$ are linear, it suffices to show the γ_p intercept of $h_0^o(\gamma_p)$ exceeds the γ_p intercept of $h_0(\gamma_p)$. This is equivalent to

$$(1 - \theta_{pc})\alpha_{pc}(u - c - k) \geq [\theta_{mc}\alpha_{mc}(u - c - k) - r(c + k)]\frac{\theta_{pm}\alpha_{pm}}{r + \theta_{mc}\alpha_{mc}} \quad (35)$$

$$-[\alpha_{mc}(u - c - k) - r(c + k)]\frac{\alpha_{pm}}{r + \alpha_{mc}}.$$

We can set $\theta_{pc} = \theta_{pm} = 1$ here, without sacrifice, since if (35) holds for these values it holds for all $\theta_{pc}, \theta_{pm} \in [0, 1)$. When $\theta_{pc} = \theta_{pm} = 1$, (35) simplifies to $\theta_{mc} \leq 1$, which is true. Hence, we have established that $\pi^o = 0$ implies $\pi^e = 0$. As remarked above, this allows us to conclude that τ^e cannot be too high. ■

Heuristically, the intuition for the above results is as follows. We can make τ too low by setting $\theta_{mc} < \theta_{mc}^o$, because this means M is not being sufficiently compensated for his sunk costs when he meets C . We cannot set $\theta_{mc} > \theta_{mc}^o = 1$, however, so we cannot make τ too high. By a similar logic, we can make π too low by setting $\theta_{pm} < \theta_{pm}^o$, but in this case we *can* make π too high by setting $\theta_{pm} > \theta_{pm}^o$, because $\theta_{pm}^o \in (0, 1)$. The reason for $\theta_{pm}^o < 1$ is similar to results in other search-and-bargaining models. When P decides to enter the market, he considers his own costs and benefits, but not those of others. Thus, he ignores the fact that when there are more P 's in the market, μ increases, and this makes it harder for all P 's to trade. The bargaining power that gives efficiency, $\theta_{pm}^o < 1$, is determined so that the socially optimal measure of producers enter.

6 Random Recycling

In this section we reintroduce ρ_j , the probability that type j recycles (stays in the market) after trade, where $\rho_j = 1$ in our baseline specification and $\rho_j = 0$

in the original RW specification. This is relevant for several reasons. First, the analysis presented above is not really a generalization of RW because, although we added general costs, bargaining weights and so on, we also changed ρ_j from 0 to 1. Consideration of $\rho_j = 0$, and a fortiori $\rho_j \in [0, 1]$, delivers a strict generalization of RW. Also, it turns out to be interesting in its own right to understand what happens for different ρ_j . In particular, we want to know how our results on existence, uniqueness and efficiency are affected when we allow $\rho_j < 1$, which means agents randomly continue or exit.

To begin, note that ρ_j affects agents' outside options, and therefore affects the surpluses, as follows:

$$\Sigma_{pc} = u - c + \rho_p \max\{V_p - k, 0\} - \max\{V_p, 0\} - (1 - \rho_c) \max\{V_c, 0\} \quad (36)$$

$$\Sigma_{mp} = -c + \rho_p \max\{V_p - k, 0\} - \max\{V_p, 0\} + \max\{V_1, 0\} - \max\{V_0, 0\} \quad (37)$$

$$\Sigma_{mc} = u - (1 - \rho_c) \max\{V_c, 0\} - \max\{V_1, 0\} + \max\{V_0, 0\} \quad (38)$$

However, other than using the Σ 's in (36)-(38) instead of the special case in (6)-(8), the DP, BR and SS equations are unchanged, as is the definition of equilibrium. In what follows, we characterize equilibrium outcomes with $\rho_j \in [0, 1]$, where to reduce notation we set $c = k = 0$, but one can say that this is without loss in generality given the results in Section 4. Interestingly, there is now a greater variety of outcomes, including more possibilities for mixed-strategies, and sometimes multiplicity.

Before going through each case, to develop some intuition, suppose that γ_m is big enough that we can be sure $\tau = 0$. Then there are effectively two types, P and C , which allows us to illustrate some results easily. The DP equations

are given by

$$\begin{aligned} rV_p &= \eta_{pc} [u - (1 - \rho_p)V_p - (1 - \rho_c)V_c] - \gamma_p \\ rV_c &= \pi\eta_{cp} [u - (1 - \rho_p)V_p - (1 - \rho_c)V_c]. \end{aligned}$$

Now $\pi = 1$ is an equilibrium iff $V_p \geq 0$, which reduces to

$$\gamma_p \leq \hat{\gamma}_p \equiv \frac{\eta_{pc}ur}{r + (1 - \rho_c)\eta_{cp}}.$$

Similarly, $\pi = 0$ is an equilibrium when $\eta_{pc}u \geq \gamma_p$. Then solve $V_p = 0$ for π , and notice $\pi \in (0, 1)$ iff $\hat{\gamma}_p < \gamma_p < \eta_{pc}u$. Hence, conditional on $\tau = 0$, we immediately get existence, generic uniqueness, and for some parameters an interior solution for π , as we found in Section 3.

However, the logic behind $\pi \in (0, 1)$ here is different from the logic with $\rho_j = 1$. With $\rho_j = 1$, we found that $\pi \in (0, 1)$ was only possible when $\tau = 1$, and the equilibrating mechanism was that μ adjusted to make $V_p = k$. Now we can get $\pi \in (0, 1)$ without intermediation, with the terms of trade rather than the probability of trade equilibrating entry. To see this, solve for:

$$y_{cp} = \frac{\Upsilon ru - (1 - \rho_p) [\theta_{cp}r + (1 - \rho_c)\eta_{cp}\pi] \gamma_p}{r [r + (1 - \rho_p)\eta_{cp} + (1 - \rho_c)\eta_{cp}\pi]}$$

where $\Upsilon \equiv \theta_{pc}r + (1 - \rho_p)\eta_{pc} + \theta_{pc}(1 - \rho_c)(1 - \pi)\eta_{cp}$. With $\rho_p < 1$, y_{cp} depends on γ_p ; with $\rho_p = 1$, it does not. Figure 6, drawn for $\rho_p < 1$, shows the following: Starting from a low value, with $\pi = 1$, as γ_p increases V_p initially falls while V_c initially rises, because higher γ_p makes P more keen to trade and this decreases y_{pc} . At $\gamma_p = \hat{\gamma}_p$, V_p hits 0, at which point there emerges a mixed-strategy equilibrium. In this mixed equilibrium, as γ_p rises further, π falls and y_{cp} rises to keep $V_p = 0$.

INSERT FIG 6 ABOUT HERE

For the rest of the candidate equilibria, it turns out there are two scenarios to be considered, shown in Figures 7 and 8. The two scenarios correspond to $\eta_{mc} > \eta_{pc}(1 - \rho_p)$, as in Figure 7, and $\eta_{mc} < \eta_{pc}(1 - \rho_p)$, as in Figure 8. Let us begin with the former case (details are in Appendix C). As one can see, in Figure 7 we still have existence and generic uniqueness, but now there can be three distinct types of mixed-strategy equilibria, $(\pi^e, 1)$, $(\pi^e, 0)$ and $(1, \tau^e)$. The first we encountered in Section 3; the second we discussed just above; and we now discuss the third.

INSERT FIG 7 ABOUT HERE

INSERT FIG 8 ABOUT HERE

There is a region in Figure 7 where the unique equilibrium entails $\tau = \tau^e \in (0, 1)$. This region is bounded above by $F_0(\gamma_p)$ and below by $F_1(\gamma_p)$ and $F_3(\gamma_p)$ (Appendix C). The condition $\eta_{mc} > \eta_{pc}(1 - \rho_p)$ makes $F_1(\gamma_p)$ lie below $F_0(\gamma_p)$ in the relevant range. In particular, for a relatively low γ_p , we have $\tau = 1$ when $\gamma_m \leq F_1(\gamma_p)$, $\tau \in (0, 1)$ when $F_1(\gamma_p) < \gamma_m < F_0(\gamma_p)$, and $\tau = 0$ when $F_0(\gamma_p) < \gamma_m$. Naturally, when γ_m is higher, middlemen are active with a lower probability, which turns the terms of trade in their favor to compensate for higher costs. Figure 8 is similar, except the condition $\eta_{mc} < \eta_{pc}(1 - \rho_p)$ makes $F_1(\gamma_p)$ lie above $F_0(\gamma_p)$ over the relevant range. The equilibrium region for $\tau \in (0, 1)$ in this case is bounded below by $F_0(\gamma_p)$ and above by $F_1(\gamma_p)$ and $F_3(\gamma_p)$. The logic is similar, except now the regions overlap, so there are multiple equilibria. We summarize as follows:

Proposition 5 *For any $\rho_j \in [0, 1]$, for all values of the other parameters, equilibrium exists. If $\eta_{mc} > \eta_{pc}(1 - \rho_p)$ it is generically unique, as shown by*

Figure 7. If $\eta_{mc} < \eta_{pc}(1 - \rho_p)$ there are multiple equilibria for some parameters, as shown by Figure 8.

Multiplicity cannot arise in the baseline model with $\rho_j = 1$ because then we cannot satisfy the condition $\eta_{mc} < \eta_{pc}(1 - \rho_p)$, so we are necessarily in the scenario depicted in Figure 7. When $\rho_p \rightarrow 1$, the regions with $(\pi^e, 0)$ and (π^e, τ^e) disappear, and when $\rho_c \rightarrow 1$ the region with $(\pi^e, 0)$ disappears. Hence, as $\rho_j \rightarrow 1$ the model of course collapses to the benchmark case. In the special case with no search/storage costs, the outcome is especially simple:

Proposition 6 *If $\gamma_p = \gamma_m = 0$ then equilibrium exists, is generically unique, and has $\pi = 1$ for all parameters. It also has $\tau = 1$ if $\eta_{mc} > \eta_{pc}(1 - \rho_p)$ and $\tau = 0$ if $\eta_{mc} < \eta_{pc}(1 - \rho_p)$.*

This is a strict generalization of the original RW result, where τ depends only on $\alpha_{mc} \gtrless \alpha_{pc}$, rather than $\eta_{mc} \gtrless \eta_{pc}(1 - \rho_p)$, because they had $\theta_{ij} = 1/2$ and $\rho_j = 0$. It is good to know that a version of their main result holds when $\gamma_j = 0$, even for general ρ_j 's, α_{ij} 's and θ_{ij} 's. Things are more interesting, however, when $\gamma_j > 0$, because this allows mixed equilibria with $\pi \in (0, 1)$ or $\tau \in (0, 1)$, plus multiple equilibria.¹⁶

Some results are the same for any $\rho_j \in [0, 1]$, including the efficiency results in Section 5. This is because the planner's problem is unaffected by changing ρ_j , since our planner regards incumbent traders and their clones as perfect substitutes. Hence, efficient outcomes are still as shown in Figure 4. We

¹⁶As a special case, if $\gamma_j = 0$ and $\rho_p = 1$, then M is active with probability $\tau = 1$ whenever $\eta_{mc} > 0$. This is because $\rho_p = 1$ implies P has no opportunity cost of trading his output to M , as he can always produce again and continue in the market. This of course uses $c = k = 0$, which we said above was without loss of generality; that is based on results in Section 4, where it was proved that one can always set $c = k = 0$ as long as one resets the γ_s 's, which in general would not lead to $\gamma_j = 0$. In other words, fixing $\gamma_j = 0$ means we cannot also set $c = k = 0$ without loss of generality.

already know π^e can be too high or too low, in general, because we verified this for $\rho_j = 1$. Comparing Figure 4 to Figures 7 and 8, it is still the case that τ can be too low but not too high. Before, with $\rho_1 = 1$, we could get $\tau^e = 0$ and $\tau^o = 1$; now we can also get $\tau^e \in (0, 1)$ and $\tau^o = 1$. In any case, τ^e can be too low but not too high, while π^e can be too low or too high.

7 Conclusion

This project has continued the development of intermediation theory by extending the original Rubinstein-Wolinsky (1987) specification on several dimensions. We verified existence and generic uniqueness in a benchmark case where all agents stay in the market forever. The results are more complicated when consumers and producers continue in the market probabilistically, but the framework is still tractable. An interesting feature compared to RW is that for certain parameters equilibria entail mixed strategies, with some but not all potential entrants participating in the market. What equilibrates participation can be either the terms of trade or the time it takes to trade, which is an attractive feature of a search-based approach. Having participation costs made it interesting to study efficiency. We found that there can be too little or too much production, and there can be too little but not too much intermediation. However, with bargaining powers set appropriately equilibrium is efficient.¹⁷

¹⁷A direction for future research might be to consider directed search, with the terms of trade posted rather than negotiated, which one might conjecture could deliver efficiency endogenously (based on work on labor and other markets by, e.g., Moen 1997, Mortensen and Wright 2001, Shimer 2005 or Eeckhout and Kircher 2010). However, it is not clear how to introduce directed search without compromising some features of a three-sided market, including the feature that C can randomly meet and trade with either P or M . Watanabe (2010) provides one avenue of exploration for middlemen with directed search.

One can use the framework to address many issues in finance, banking, real estate and other areas where intermediation plays a big role. An example of results that we did not have the space to discuss concerns the relative terms of trade in direct, wholesale and retail transactions. Since these obviously depend on bargaining powers, in general, consider the special case of the baseline model with $\theta_{ij} = 1/2$. Then one can show $y_{cm} > y_{cp} > y_{mp}$ (retail exceeds direct exceeds wholesale), at least if $c = k$ are not too big. This is not general, however, and one can show $y_{cp} > y_{cm} > y_{mp}$ if θ_{cp} is small. Another natural case is $\theta = \theta^o$, which means equilibrium is efficient, and implies $y_{cm} = y_{cp} > y_{mp}$. Hence, the efficient outcome is described by having direct and retail transfers from the consumer the same, and above the wholesale transfer from middlemen to producers, reflecting the very real service that intermediation provides in markets with frictions.

Finally, we mention that we have to this point not emphasized that intermediation per se increases production and consumption in these models. Due to good x being storable only one unit of a time, once P produces x , he cannot produce again until he trades. When M takes x off P 's hands, therefore, P produces more often. Although this depends on the technical assumption that inventories are in $\{0, 1\}$, one can also say that it rings true: well-functioning intermediation allows P to tie up fewer resources in marketing and get back more easily to making stuff, something in which he specializes.

Appendix A: Results for Section 3

Here we characterize regions of parameter space where degenerate equilibria exist. Figure 9 shows the results, along with the regions with nondegenerate equilibria discussed in the text.

INSERT FIG 9 ABOUT HERE

1. Equilibrium $\mathbf{p} = (0, 0, 0, 0)$: The BR condition for $\sigma_p = 0$ is $V_p \leq 0$ which reduces to $\eta_{pc}u \leq (\eta_{pc}c + \gamma_p)$. Given $V_p \leq 0$, the BR condition for $\pi = 0$ is not binding. The BR condition for $\sigma_m = 0$ is $V_1 \leq 0$, which reduces to $\eta_{mc}u \leq \gamma_m$. Given $V_1 \leq 0$, the BR condition for $\tau = 0$ is not binding.

2. Equilibrium $\mathbf{p} = (0, 0, 0, 1)$: The BR conditions for $\pi = 0$ and $\sigma_p = 0$ are the same as in $\mathbf{p} = (0, 0, 0, 0)$. The BR condition for $\sigma_m = 1$ is $V_1 \geq 0$, which reduces to $\eta_{mc}u \geq \gamma_m$. Letting

$$f(\gamma_p) \equiv \eta_{mc}u - c(r + \eta_{mc}) - [\eta_{pc}(u - c) - \gamma_p] \frac{r + \eta_{mc}}{r + \eta_{pc}},$$

the BR condition for $\tau = 0$ is $f(\gamma_p) \leq \gamma_m$.

3. Equilibrium $\mathbf{p} = (0, 0, 1, 1)$: The BR condition for $\sigma_p = 0$ is $V_p \leq 0$ which reduces to $\gamma_m \geq g(\gamma_p)$, where

$$g(\gamma_p) \equiv \eta_{mc}u - c(r + \eta_{mc}) + [\eta_{pc}(u - c) - \gamma_p] \frac{r + \eta_{mc}}{\eta_{pm}}$$

Given $V_p \leq 0$, the BR condition for $\pi = 0$ is not binding. The BR condition for $\tau = 1$ is $\Sigma_{pm} \leq 0$, which can be simplified to $\gamma_m \leq f(\gamma_p)$. The BR condition for $\sigma_m = 1$ is $\gamma_m \leq \eta_{mc}u$.

4. Equilibrium $\mathbf{p} = (0, 1, 0, 0)$: The BR condition for $\pi = 0$ is $\gamma_p \geq \eta_{pc}(u - c) - (r + \eta_{pc})k$. The BR condition for $\sigma_p = 1$ is $\gamma_p \leq \eta_{pc}(u - c)$. The BR condition $\tau = 0$ is not binding. The BR condition for $\sigma_m = 0$ is $\gamma_m \geq \eta_{mc}u$.

5. Equilibrium $\mathbf{p} = (0, 1, 0, 1)$: The BR condition for $\pi = 0$ is $\gamma_p \geq \eta_{pc}(u - c) - (r + \eta_{pc})k$. The BR condition for $\sigma_p = 1$ is $\gamma_p \leq \eta_{pc}(u - c)$. The BR condition for $\tau = 0$ is $\gamma_m \geq f(\gamma_p)$: The BR condition for $\sigma_m = 1$ is $\gamma_m \leq \eta_{mc}u$.

6. Equilibrium $\mathbf{p} = (0, 1, 1, 1)$: The BR condition for $\pi = 0$ is $\gamma_m \geq h_0(\gamma_p)$, where $h_0(\gamma_p)$ is defined above. The BR condition for $\sigma_p = 1$ is $\gamma_m \leq g(\gamma_p)$. The BR condition $\tau = 1$ is $\gamma_m \leq f(\gamma_p)$. The BR condition for $\sigma_m = 1$ is $\gamma_m \leq \eta_{mc}u$.

Appendix B: Characterization in Figure 2

1. Outcome $(\pi, \tau) = (1, 0)$: In this case all producers enter, but middlemen are inactive. As shown above, this happens in two equilibria, one with $\sigma_m = 0$ and another with $\sigma_m = 1$, although the outcome is the same because $\tau = 0$ in both cases. From Figure 1, an equilibrium with $(\pi, \tau) = (1, 0)$ exists iff $\gamma_p \leq \tilde{\gamma}_p$ and $\gamma_m \geq \tilde{\gamma}_m$, which from (18) and (19) are equivalent to $A_p \geq 0$ and $A_m \leq 0$.

2. Outcome $(\pi, \tau) = (1, 1)$: Now all producers and middlemen are active. This occurs in the equilibrium that exists when $\gamma_m \leq \tilde{\gamma}_m$ and $\gamma_m \leq h_1(\gamma_p)$, conditions that are equivalent to $A_m \geq 0$ and

$$A_m \geq -A_p \frac{(r + \eta_{mp} + \eta_{mc})(\alpha_{mp} + \alpha_{mc})}{\eta_{pm}\alpha_{mc}},$$

given that $\tau - \mu(1) = \alpha_{mc}/(\alpha_{mp} + \alpha_{mc})$.

3. Outcome $(\pi, \tau) = (\pi^e, 1)$ with $\pi^e \in (0, 1)$: Now some producers and all middlemen are active. This happens in the equilibrium that exists iff $h_1(\gamma_p) \leq \gamma_m \leq h_0(\gamma_p)$, conditions that are equivalent to

$$A_m \leq -A_p \frac{(r + \eta_{mp} + \eta_{mc})(\alpha_{mp} + \alpha_{mc})}{\eta_{pm}\alpha_{mc}} \text{ and } A_m \geq -A_p \frac{r + \eta_{mc}}{\eta_{pm}},$$

since in this case $\tau - \mu(0) = 0$.

4. Outcome $\pi = 0$: This occurs in a degenerate equilibrium, which exists in the complement of set of parameters where the other outcomes occur.

Appendix C: Results for Section 6

We now characterize regions of parameter space where different equilibria exist for the case of general $\rho_j \in [0, 1]$, where to conserve space we focus on (π, τ) , with σ_j implicit.

1. Equilibrium $(\pi, \tau) = (0, 0)$: The BR condition for $\pi = 0$ is $V_p \leq 0$, or $\eta_{pc}u \leq \gamma_p$. The BR condition for $\tau = 0$ is $V_1 \leq (1 - \rho_p)V_p$, which reduces to $F_2(\gamma_p) \leq \gamma_m$, where

$$F_2(\gamma_p) \equiv \frac{ur[\eta_{mc} - \eta_{pc}(1 - \rho_p)] + \gamma_p(1 - \rho_p)(r + \eta_{mc})}{r + \eta_{pc}(1 - \rho_p)}$$

2. Equilibrium $(\pi, \tau) = (0, 1)$: The BR condition for $\pi = 0$ is $H_0(\gamma_p) \leq \gamma_m$, where

$$H_0(\gamma_p) \equiv u\left[\frac{\eta_{pc}}{\eta_{pm}}(r + \eta_{mc}) + \eta_{mc}\right] - \gamma_p\frac{r + \eta_{mc}}{\eta_{pm}}$$

The best response condition for $\tau = 1$ is $F_2(\gamma_p) \geq \gamma_m$.

3. Equilibrium $(\pi, \tau) = (1, 0)$: The BR condition for $\pi = 1$ is $V_p \geq 0$ which reduces to

$$\hat{\gamma}_p \equiv \frac{\eta_{pc}ur}{r + \eta_{cp}(1 - \rho_c)} \geq \gamma_p$$

The best response condition for $\tau = 0$ can be simplified to $\gamma_m \geq F_0(\gamma_p)$, where

$$F_0(\gamma_p) \equiv \frac{ur[\eta_{mc} - \eta_{pc}(1 - \rho_p)] + \gamma_p(1 - \rho_p)[r + \eta_{mc} + \eta_{cp}(1 - \rho_c)]}{r + \eta_{pc}(1 - \rho_p) + \eta_{cp}(1 - \rho_c)}$$

4. Equilibrium $(\pi, \tau) = (1, 1)$: The BR condition for $\pi = 1$ is $H_1(\gamma_p) \geq$

γ_m , where

$$H_1(\gamma_p) \equiv ur[\eta_{pc}(r + \eta_{mc} + \eta_{mp}) + \eta_{mc}\eta_{pm}(1 - \bar{\mu})]D_H^{-1} \\ - \gamma_p\{(r + \eta_{mc} + \eta_{mp})[r + (1 - \rho_c)\eta_{cp}] + (1 - \rho_c)\bar{\mu}\eta_{cm}(r + \eta_{mp})\}D_H^{-1}$$

$D_H = \eta_{pc}(1 - \rho_p)\bar{\mu}\eta_{cm} + (1 - \bar{\mu})\eta_{pm}[r + (1 - \rho_c)(\eta_{cp} + \bar{\mu}\eta_{cm})]$, and $\bar{\mu} = \alpha_{mp}/(\alpha_{mp} + \alpha_{mc})$. The BR condition for $\tau = 1$ is $F_1(\gamma_p) \geq \gamma_m$, where

$$F_1(\gamma_p) \equiv \frac{ur[\eta_{mc} - \eta_{pc}(1 - \rho_p)] + \gamma_p(1 - \rho_p)[r + \eta_{mc} + (\eta_{cp} + \bar{\mu}\eta_{cm})(1 - \rho_c)]}{r + \eta_{pc}(1 - \rho_p) + (\eta_{cp} + \bar{\mu}\eta_{cm})(1 - \rho_c)}$$

5. Equilibrium $(\pi, \tau) = (\pi, 1)$: The BR condition for $\pi \in (0, 1)$ is given by $V_p = 0$, which is

$$ur[\eta_{pc}(r + \eta_{mc} + \pi\eta_{mp}) + \eta_{mc}\eta_{pm}(1 - \mu)] \\ - \gamma_p\{(r + \eta_{mc} + \pi\eta_{mp})[r + (1 - \rho_c)\pi\eta_{cp}] + (1 - \rho_c)\mu\eta_{cm}(r + \pi\eta_{mp})\} \\ = \eta_{pc}(1 - \rho_c)\mu\eta_{cm} + (1 - \mu)\eta_{pm}[r + (1 - \rho_c)(\pi\eta_{cp} + \mu\eta_{cm})]$$

where $\mu = \pi\alpha_{mp}/(\pi\alpha_{mp} + \alpha_{mc})$. When $\pi^e \rightarrow 0$, $\gamma_m \rightarrow H_0(\gamma_p)$, and when $\pi^e \rightarrow 1$, $\gamma_m \rightarrow H_1(\gamma_p)$. Thus, $\pi \in (0, 1)$ when $H_0(\gamma_p) \leq \gamma_m \leq H_1(\gamma_p)$. Given $V_p = 0$, the BR condition for $\tau = 1$ is $V_1 \geq V_0$, which reduces to

$$\frac{\eta_{mc}ur}{r + (1 - \rho_c)(\pi^e\eta_{cp} + \mu(\pi^e)\eta_{cm})} \geq \gamma_m.$$

This is complicated because π^e is a complicated function of parameters. However, it is clear from numerical analysis that it generates a positive relation between γ_m and γ_p , as shown by $F_3(\gamma_p)$ in the diagrams.

6. Equilibrium $(\pi, \tau) = (\pi, 0)$: The BR condition for $\pi \in (0, 1)$ is $V_p = 0$, which solves for

$$\pi^e = \frac{r(\eta_{pc}u - \gamma_p)}{\eta_{cp}(1 - \rho_c)\gamma_p}.$$

This means $\pi \in (0, 1)$ iff $\widehat{\gamma}_p \leq \gamma_p \leq \eta_{pc}u$. Then the BR condition for $\tau = 0$ is $V_1 \leq V_0$, which reduces to $\gamma_p \eta_{mc} / \eta_{pc} \leq \gamma_m$.

7. Equilibrium $(\pi, \tau) = (1, \tau)$: The best response condition for $\tau = (0, 1)$ is given by $V_1 = (1 - \rho_p)V_p$, which can be simplified and solved for τ^e

$$\begin{aligned} \mu^e &= \tau^e \bar{\mu} \\ &= ur[\eta_{mc} - \eta_{pc}(1 - \rho_p)]D_\mu^{-1} + \gamma_p(1 - \rho_p)[r + \eta_{mc} + \eta_{cp}(1 - \rho_c)]D_\mu^{-1} \\ &\quad - \gamma_m[r + \eta_{pc}(1 - \rho_p) + \eta_{cp}(1 - \rho_c)]D_\mu^{-1} \end{aligned}$$

where $D_\mu = \eta_{cm}(1 - \rho_c)[\gamma_m - \gamma_p(1 - \rho_p)]$. Note that the value of τ^e depends on the relative values of η_{mc} and $\eta_{pc}(1 - \rho_p)$; hence, so do the BR conditions. Upon simplification, we have the following: if $\eta_{mc} > \eta_{pc}(1 - \rho_p)$ we need $F_1(\gamma_p) < \gamma_m < F_0(\gamma_p)$ and $\gamma_m > F_3(\gamma_p)$ for this equilibrium; and if $\eta_{mc} < \eta_{pc}(1 - \rho_p)$ we need $F_0(\gamma_p) < \gamma_m < F_1(\gamma_p)$ and $\gamma_m < F_3(\gamma_p)$, where

$$F_3(\gamma_p) \equiv \frac{\gamma_p \{ur[\eta_{mc}(r + \eta_{pc}) - r\eta_{pc}^2\rho_p(1 - \rho_p)] + \gamma_p(1 - \rho_p)[\eta_{mc}(r + \eta_{pc}) + r\eta_{pc}\rho_p]\}}{\eta_{pc}[(r + \eta_{pc})(ru + \gamma_p) - \gamma_p\rho_p\eta_{pc}]}$$

These two cases, $\eta_{mc} > \eta_{pc}(1 - \rho_p)$ and $\eta_{mc} < \eta_{pc}(1 - \rho_p)$, correspond to the scenarios in Figures 7 and 8.

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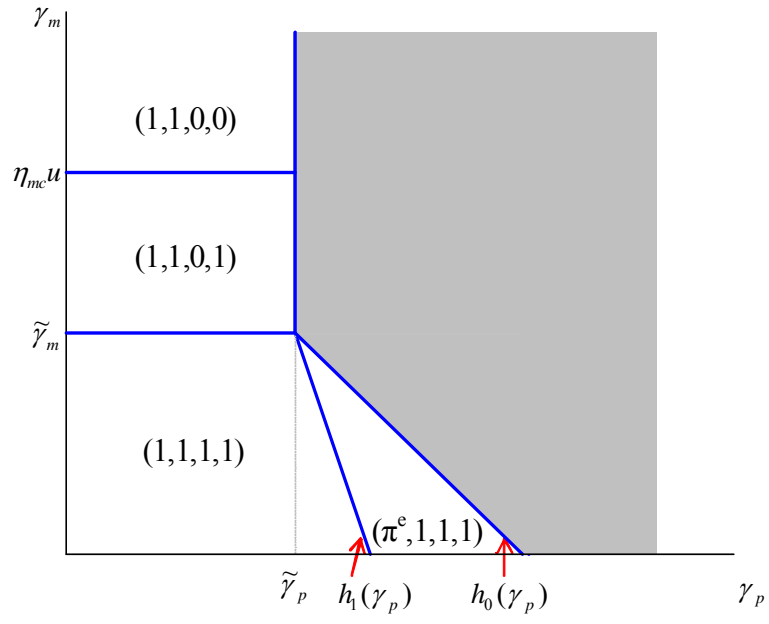


Figure 1: Equilibria in (γ_p, γ_m) Space.

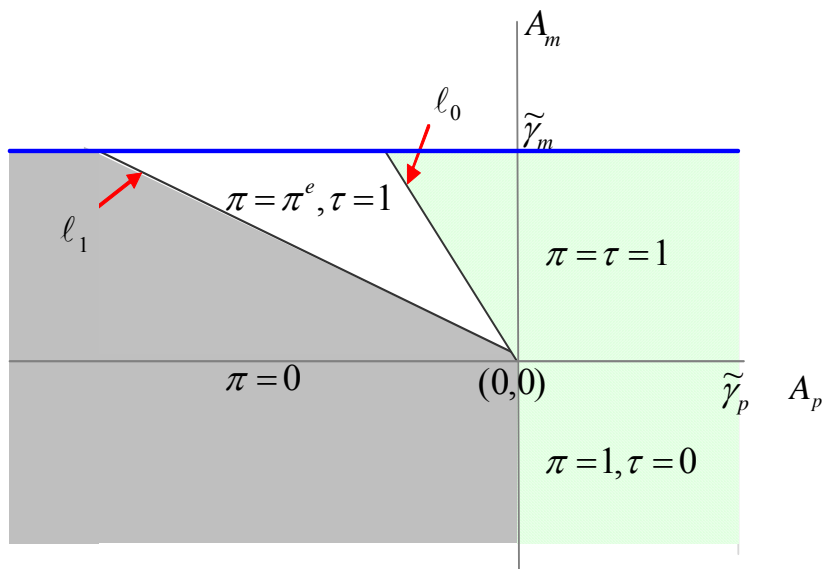


Figure 2: Equilibria in (A_p, A_m) Space.

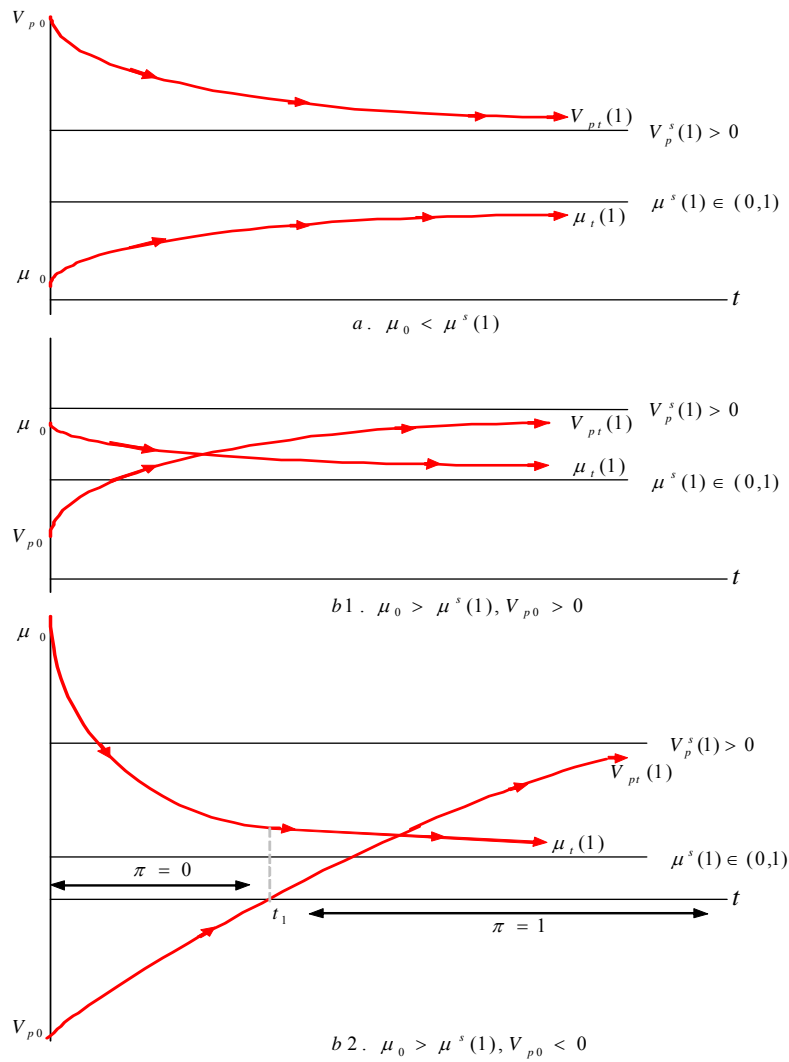


Figure 3: Dynamics and Payoffs for P , $\pi^s = 1$

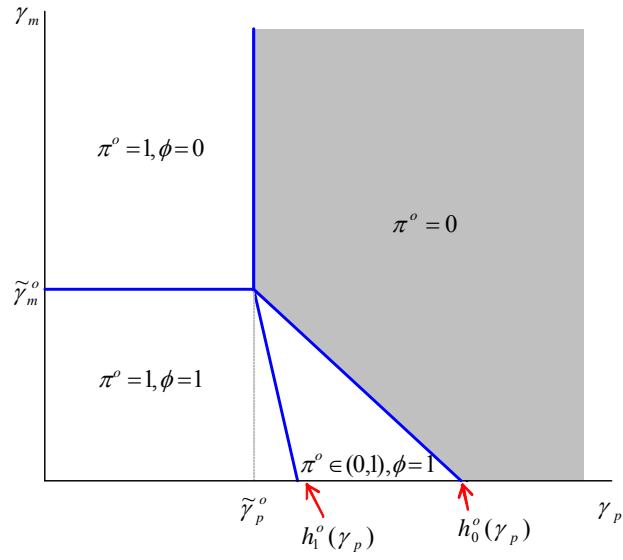


Figure 4: Efficient Outcomes in (γ_p, γ_m) Space

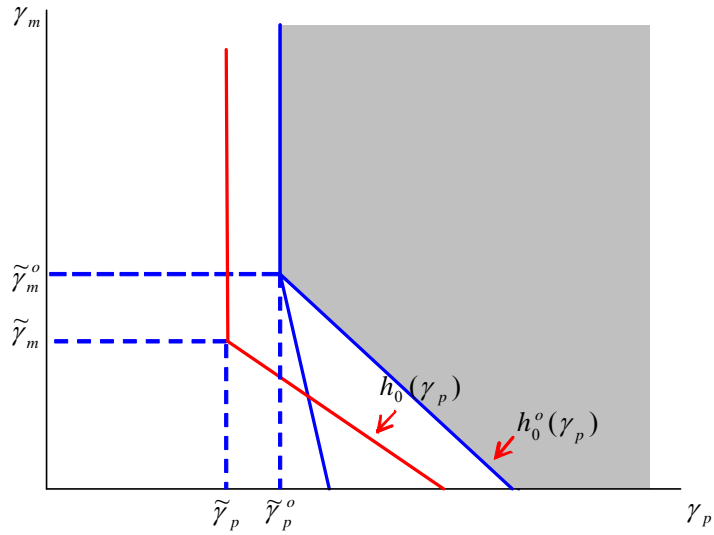


Figure 5: $\pi^o = 0$ Implies $\pi^e = 0$

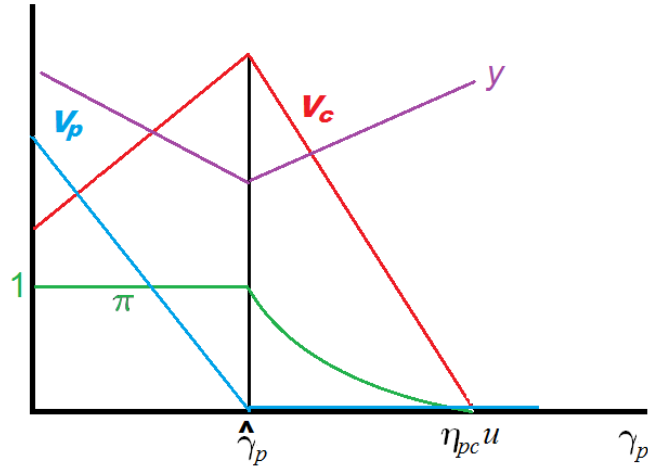


Figure 6: Equilibrium with $\tau = 0$ when $\rho_j < 1$

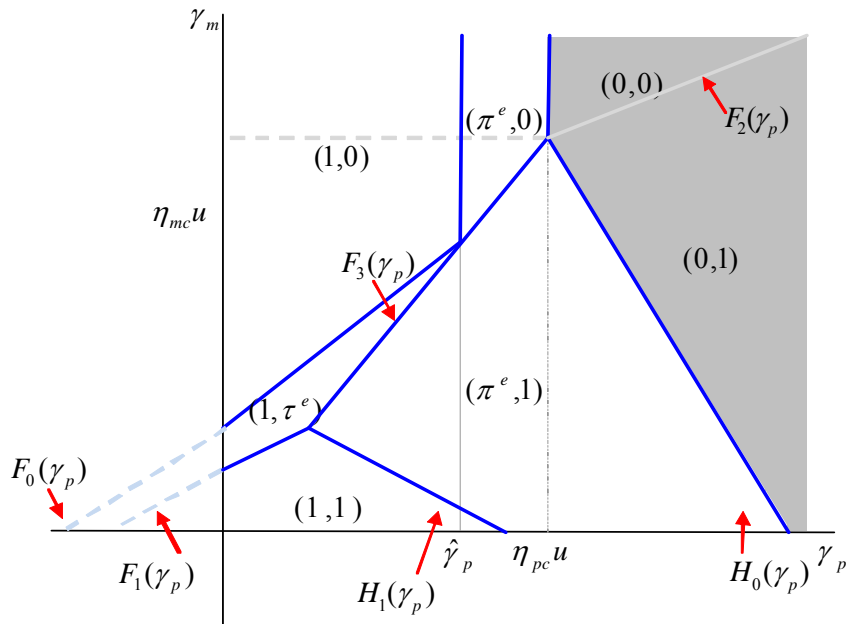


Figure 7: Equilibrium Outcomes for $\rho_j < 1$ and $\eta_{mc} > \eta_{pc}(1 - \rho_p)$

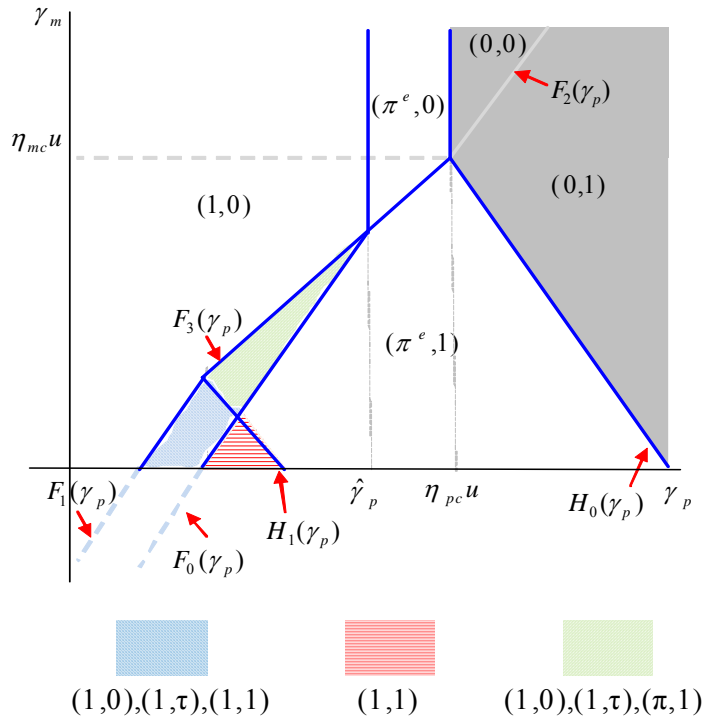


Figure 8: Equilibrium Outcomes for $\rho_j < 1$ and $\eta_{mc} < \eta_{pc}(1 - \rho_p)$

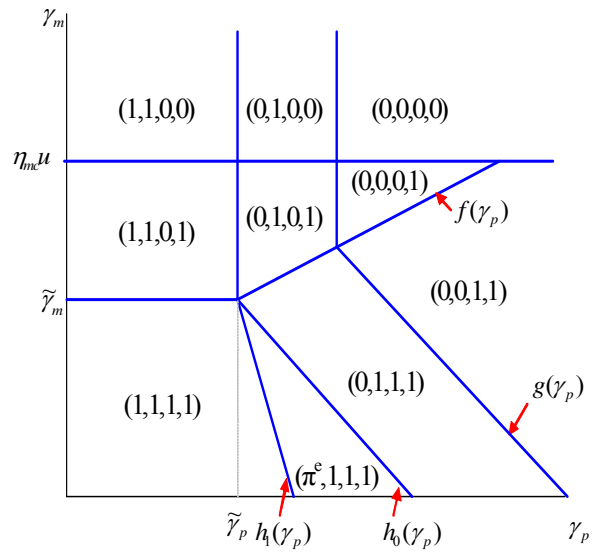


Figure 9: All Equilibria

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