Competition in Large Markets

Jeffrey R. Campbell

REVISED December, 2006
WP 2005-16
Competition in Large Markets

Jeffrey R. Campbell*

December, 2006

Abstract

This paper evaluates the simplifying assumption that producers compete in a large market without substantial strategic interactions using nonparametric regressions of producers’ choices on market size. With such atomistic competition, increasing the number of consumers leaves the distributions of producers’ prices and other choices unchanged. In many models featuring non-trivial strategic considerations, producers’ prices fall as their numbers increase. I examine observations of restaurants’ sales, seating capacities, exit decisions, and prices from 222 U.S. cities. Given factor prices and demographic variables, increasing a city’s size increases restaurants’ average sales and decreases their exit rate and prices. These results suggest that strategic considerations lie at the heart of restaurant pricing and turnover.

JEL Classification: L11, L81

Keywords: Atomistic Competition, Market Size, Free Entry, Exit, Nonparametric Test.

*Federal Reserve Bank of Chicago and NBER. I am grateful to Gadi Barlevy, Meredith Crowley, Hugo Hopenhayn, and Ruilin Zhou for their helpful comments. The National Science Foundation supported my research on this topic through grant SBR-0137048 to the NBER. Please address correspondence to Jeffrey Campbell, Economic Research, Federal Reserve Bank of Chicago, 230 South LaSalle St., Chicago, IL 60604. e-mail: jcampbell@frbchi.org
1 Introduction

Observations of producers’ actions from firm registries or national statistical agencies typically lack an accompanying description of their strategic environments. This unfortunate fact tempts one to assume that producers compete anonymously in a large market, but casual observation nearly always suggests some scope for strategic interaction between firms. This paper quantifies this informal suspicion using nonparametric regressions of producers’ choices on market size. The data come from 222 U.S. cities’ restaurant industries and are reported in the 1992 Census of Retail Trade. Under the null hypothesis of atomistic competition, market size has no impact on these decisions. This result is familiar from highly stylized models of Chamberlinian monopolistic and perfect competition, and this paper proves it in a very general framework without substantial restrictions on the market demand system, producers’ cost functions, or the variables over which they compete. It requires nonparametric regressions to display no influence of market size on producers’ choices given a sufficiently rich set of control variables. In fact, restaurants in larger cities have lower prices, exit less frequently, and have greater sales revenues. Even if one finds atomistic competition implausible ex ante, the theory and regressions together quantify how the strategic environment influences producers’ observable choices. Such quantification is essential for extending the domain of strategically-oriented empirical industrial organization to large markets.

The analysis rests on a nonparametric free-entry model. Potential producers make entry choices and then compete across a possibly large number of variables; such as price and advertising. A producer’s profit depends only on the distribution of its rivals’ actions and not on any particular rival’s choices. This allows the transformation of a free-entry equilibrium for a given market size into one for a market twice as large with double the number of producers and the same distribution of their observable actions. Of course, the distribution of producers’ actions could differ across large and small markets even without substantial strategic interaction if the production technology and consumer tastes systematically change with market size. I eliminate dependence of an individual’s demand and producers’ costs on
market size in the model by assumption; and the regressions control for differences in production possibilities and consumer tastes across U.S. cities with factor prices and demographic measures.

Previous contributions to international trade and industrial organization have recognized the importance of oligopolistic competition in large markets, but this recognition has taxed the desire to work with analytically tractable models. Standard Chamberlinian monopolistic competition cannot capture the idea that increasing market size makes competition “fiercer” by increasing the number of producers, but true oligopoly models with strategic interaction raise difficult issues of dynamic game theory that are not necessarily central to a particular author’s problem. This difficulty has led some authors to use a model with a continuum of producers and goods due to Ottaviano, Tabuchi, and Thisse (2002), in which the elasticity of any given good’s demand decreases with the number of goods offered even though no two producers compete head-to-head. Campbell and Hopenhayn (2005) show that such a model predicts firm size to increase with market size, because the product of the falling markup with firms’ average sales must equal the constant fixed cost of entry. Their empirical results confirm this relationship for a large number of U.S. retail trade industries. Asplund and Nocke (2006) and Nocke (2006) find that an otherwise-standard model of industry dynamics with this specification predicts that producer entry and exit rates increase with market size. Asplund and Nocke’s observations of Swedish hairdressers and Syverson’s (2004) observations of U.S. concrete industries support this conclusion. I find the opposite to be true for U.S. restaurant industries. Apparently, the dynamic aspects of oligopolistic interaction that this specification omits are unimportant for the industries examined by Asplund and Nocke and Syverson, but they substantially lower restaurants’ exit rates in larger markets.

The approach to evaluating competition in large markets I advocate in this paper has one limit worth noting. A model in which oligopolists successfully collude and keep markups at their monopoly level but do not deter entry will replicate the scale invariance of atomistic competition. That is, the test has no power to reject the null in favor of the specific
alternative of collusion with free entry. The empirical results of this paper as well as those of Campbell and Hopenhayn (2005) and Yeap (2005) indicate that this lack of power is not a practical problem for work with U.S. data.

The remainder of this paper proceeds as follows. The next section sets the stage for the analysis with an empirical examination of how restauranteurs’ decisions vary with market size. Section 3 then provides a structural interpretation of these nonparametric results using the general model of atomistic competition. Section 4 relates this paper’s results to those from the relevant literature, and Section 5 offers some concluding remarks.

2 Competition among Restaurants

To motivate this paper’s analysis, consider the U.S. Restaurant industry. The U.S. Census questions the population of restaurants about their sales, cuisine, and pricing decisions every five years when creating the Economic Census. These observations allow researchers to address fundamental questions about the process of business formation, growth, and exit; but they contain only little information about the potential for strategic interactions. This is particularly the case for restaurants in cities, who have a great scope for differentiating themselves by location and cuisine.

The hypothesis that the firms in this data set compete atomistically can greatly simplify its analysis, because each firm’s actions can be cast as the outcome of a single-agent decision problem. This simplification could come at a high price if strategic interaction is a first-order feature of competition, so I desire a simple procedure that can evaluate it before proceeding with a more complicated analysis.

Campbell and Hopenhayn (2005) use a symmetric model of oligopolists with constant marginal cost to build such a procedure. They note that oligopolists’ average sales must rise with market size if their markups fall with additional entry, because they must recover the same fixed cost with a lower markup by selling more. Hence, modelling an industry as a
collection of oligopolies seems promising if we see average sales rising with market size. The
two shortcomings of their procedure are its reliance on a stylized model of competition and
its exclusive focus on producers’ average sales. This paper constructs a very general model
of the null hypothesis which implies that all observable producer decisions are invariant to
market size. The following description of how U.S. restauranteurs’ actions vary with market
size provides this theoretical analysis with a concrete empirical context.

2.1 Data

For this paper, I use observations from the 1992 *Census of Retail Trade* for the same sample of
MSAs examined by Campbell and Hopenhayn (2005). The volume RC92-S-4, “Miscellaneous
Subjects”, reports the number of restaurants operating at any time during 1992 and at the
end of that year. These observations immediately yield one measure of the annual exit rate.
This volume also reports restaurants’ average seating capacities for each MSA, the sales of all
restaurants and of those operating at the end of the year, and the fraction of restaurants with
typical meal prices greater than or equal to $5.00. Although the Census records information
about each restaurant’s cuisine, this information is not disclosed publicly by MSA.\(^1\)

From these observations, we construct four variables of interest. The first summarizes
firms’ pricing decisions. Denote the fraction of restaurants charging a typical meal price of
$5.00 or more with \(S(\$5.00)\), and consider its logistic transformation

\[
L(\$5.00) \equiv \ln(S(\$5.00)/(1 - S(\$5.00))
\]

This is the logarithm of the ratio of “high priced” restaurants’ share of the population to
that of their “low priced” counterparts. Figure 1 plots this variable against the demeaned
logarithm of MSA population. The observations corresponding to the smallest and largest
MSA’s (Enid, OK and Atlanta, GA) are labelled, as are the observations with extreme values

\(^1\)It would be desirable to examine more recent observations. Unfortunately, the Census has not published
MSA level observations of these variables from the two most recent Economic Censuses.
of the log relative market share. The median value of $S(5.00)$ across the sample’s MSA’s is 0.67. The Census reports that only 13 percent of restaurants in Rocky Mount, NC charge $5.00 or more for a meal, and it reports that 96 percent of restaurants charge $5.00 or more in both Longview-Marshall TX and Jackson, MS. Aside from these three outliers, the minimum and maximum values of $S(5.00)$ are 0.32 and 0.92. The correlation between the log relative market share and MSA population equals 0.09.

The second variable of interest measures one aspect of industry dynamics, the exit rate. I constructed this by dividing the number of firms operating at some time of the year but not at the end of the year by the total number of firms to operate in that year. The plot of this against MSA log population in Figure 2 shows a negative correlation. The exit rate for Enid, OK is very close to the maximum observed, 19 percent, while that for Atlanta, GA is close to the median across all MSA’s, 10.3 percent. The correlation between these variables equals $-0.11$.

The other two variables of interest both measure average restaurant size, restaurants’ average revenue and average seating capacity. This average revenue variable differs from that used by Campbell and Hopenhayn only because it excludes restaurants not operating at the end of the year. Figures 3 and 4 plot these variables against MSA population. The strong positive association between MSA population and sales revenue documented by Campbell and Hopenhayn is evident in Figure 3. Figure 4 reveals little correlation between MSA population and average seating capacity.

2.2 Regression Results

Let $Y_i$ denote the value of one of these four measures of restaurateurs’ actions for MSA $i$, and use $S_i$ and $W_i$ to represent that MSA’s population and a vector of control variables that includes relevant factor prices and consumer demographics. The factor prices account for larger cities’ higher cost of commercial space and wages and lower cost of advertising per consumer exposure. The demographic variables control for differences in preferences
associated with income, race, and education that could shift the nature of producers’ products and thereby indirectly influence their observable decisions. These control variables are identical to those used in Campbell and Hopenhayn). The regression of \( Y_i \) on \( \ln S_i \) and \( W_i \) is

\[
Y_i = m(\ln S_i, W_i) + U_i.
\]

Here, \( m(\cdot) \) is not restricted to a particular functional form.\(^2\)

The curse of dimensionality makes the estimation of \( m(\ln S, W) \) infeasible. However, it is still possible to test the hypothesis that its dependence on \( \ln S \) is trivial using estimates of the regression function’s density-weighted average derivatives. These are

\[
\begin{align*}
\delta_S & \equiv E \left[ \frac{\partial m(S, W)}{\partial \ln S} f(\ln S, W) \right] / E[f(\ln S, W)] \\
\delta_W & \equiv E \left[ \frac{\partial m(S, W)}{\partial W} f(\ln S, W) \right] / E[f(\ln S, W)],
\end{align*}
\]

where \( f(\ln S, W) \) is the joint density function of \( \ln S \) and \( W \) across markets and expectations are taken with respect to the same joint density function. Powell, Stock, and Stoker (1989) provide a simple instrumental variables estimator of \( \delta_S \) and \( \delta_W \) which converges to the true parameter values at the parametric rate of \( \sqrt{N} \). If market size does not directly impact producers’ decisions, then \( \delta_S = 0 \).

For the four measures of restaurateurs’ actions, Table 1 reports the estimated values of \( \delta_S \) and \( \delta_W \) along with consistent estimates of their asymptotic standard errors. Before estimation, the elements of \( W \) were scaled by the standard deviation of \( \ln S \), which is 0.86 in this sample. Powell et al.’s (1989) estimator requires a first-stage nonparametric estimation of \( \partial f(\ln S, W) / \partial \ln S \) and \( \partial f(\ln S, W) / \partial W \). The estimates reported here are based on the tenth-order bias-reducing kernel of Bierens (1987) and use a bandwidth equal to 2. To increase the precision of the estimates’ reports, all entries in the table and in the text have been multiplied by 100.

\(^2\)In the case where \( Y_i = \ln(S_i(\$5.00))/(1 - S_i(\$5.00)) \), this specification for the regression function is equivalent to assuming that \( S_i(\$5.00) = e^{m(\ln S_i, W_i) + U_i} / (1 + e^{m(\ln S_i, W_i) + U_i}) \).
The estimate of $\delta_S$ for the regression of $L($5.00) equals $-12.90$ and is statistically significant at the 5 percent level. Thus, restaurants in larger markets charge lower prices given factor costs. To gauge the magnitude of this coefficient, consider an MSA with $S($5.00) at the median level of 0.67. Set all of the elements of $W$ equal to their means and consider increasing $S$ by one standard deviation. If we assume that $\partial m(\ln S, W)/\partial \ln S$ is constant, then such an increase in $\ln S$ decreases $S($5.00) to 0.65. The hypothesis that increasing market size lowers prices permeates empirical industrial organization, but to date only Syverson (Forthcoming) has verified that this is so for a particular industry. This finding that typical meal prices fall with market size complements his results.

The coefficients on two of the factor costs, commercial rent and the retail wage, are positive. They are both statistically significant at the 10 percent level, so the regression confirms the basic intuition that prices rise with factor costs. The third factor price, the cost of purchasing 1,000 advertising exposures in a Sunday newspaper, has a negative coefficient which is statistically significant at the 10 percent level. Perhaps high advertising costs allow producers to segment the market more effectively, thereby raising prices. Certainly, the effect of advertising costs on restaurant prices merits further investigation.

The estimate of $\delta_S$ for the exit rate is also negative, $-0.77$, and statistically significant at the 5 percent level. This implies that doubling $S$ decreases restaurants’ exit rate by 0.53 percentage points. As Campbell and Hopenhayn (2005) document, an increase in $\ln S$ strongly raises restaurants’ average revenue. The estimated coefficient is 4.68, and it is statistically significant at the one-percent level. The final dependent variable is the logarithm of average seats per restaurant. The estimated coefficient is positive, 2.07, but its standard error equals 1.99. Hence, these observations are uninformative about whether the increase in average revenue per restaurant arises from increased capacity utilization or increased average

---

$^3$This estimate differs greatly from that reported by Campbell and Hopenhayn (2005) for the identical regression with nearly the same sample. The discrepancy between the two reflects an error in Campbell and Hopenhayn’s calculations. An erratum to that paper available at http://www.nber.org/~jrc/marketsizematters corrects that error.
capacity. Nevertheless, the estimates in Table 1 clearly indicate that important decisions of restaurateurs vary systematically with market size.

The estimates in Table 1 depend on the particular measure of market size (population) and the bandwidth choice. Table 2 examines the robustness of the estimates of $\delta_S$ to these choices. Its first column reproduces the estimates from the first row of Table 1, and its next two columns report alternative estimates based on measuring market size with geographic population density and the number of housing units. Using either of these alternatives brings the estimate of $\delta_S$ for the regression of $L(5.00)$ closer to zero. It equals $-8.91$ and has a $p$-value of 12 percent with population density, and it equals $-11.93$ with a $p$-value of 5.4 percent using housing units. All other inferences are invariant to changing the measure of market size. The final two columns of Table 2 report estimates based on changing the bandwidth $h$ from its baseline value of 2 to either 1 or 3. Changing the bandwidth moves the estimated standard errors in the opposite direction. Otherwise, the estimates are unaffected. The only inference to change relative to the baseline specification is in the regression of $L(5.00)$. When $h = 1$, the $p$-value for $\delta_S$ equals 10.3 percent.

I have undertaken three other checks of these estimates’ robustness worth mentioning here. First, I have estimated all of the regression equations using ordinary least squares. The estimated coefficients are similar to the nonparametric estimates of $\delta_S$. The only notable change in inference regards the coefficient in the regression of $L(5.00)$. Its estimate drops to $-9.79$, and its $p$-value rises to 6.6 percent. Second, the Census reports the share of restaurants charging less than $7.00 per meal. When I regress $L(7.00)$ on $\ln S$ and $W$, I find no effect of market size on prices. Apparently, the reduction of restaurant prices occurs at the market’s “low end”. Nothing in principle prevents estimating $\delta_S$ using the original values of $S(5.00)$ as a dependent variable. When I do so, the $p$-value for $\delta_S$ rises to 5.9 percent. Finally, I have also constructed analogues of Tables 1 and 2 for a sister industry, Refreshment Places. In that industry, market size has no measurable effect on typical meal prices, but its effects on the other three dependent variables are the same as with Restaurants.
3 A General Model of Atomistic Competition

The results of the previous section clearly conflict with very basic models of atomistic competition. Consider for example Dixit and Stiglitz’s (1977) model of Chamberlinian monopolistic competition. Its free-entry condition implies that each firm’s sales equals the product of the exogenous fixed cost with consumers’ constant elasticity of demand. Doubling the number of consumers leaves producers’ average sales unchanged. In this section, I show that the abstraction from strategic interaction is the sole source of the conflict between such simple models and the data. Their other simplifying features are not to blame.

To do so, I develop the cross-market predictions of atomistic competition in a very general model with no parametric restrictions. So that the analysis is as broadly applicable as possible, I do not present specific conditions to guarantee the existence and uniqueness of a free-entry equilibrium. Instead, the analysis begins with the assumption that an equilibrium exists for a particular market size, and it then constructs an equilibrium with the same observable distribution of producers’ actions for a larger market.

To make following the general model easier, this section begins with a specific example of atomistic competition. It then proceeds to the general model, referring back to the specific example to explain its moving parts.

3.1 A Specific Example

Consider a market for restaurant meals of heterogeneous quality. Production takes place in two stages, entry and competition. In the entry stage, a large number of potential restaurateurs simultaneously decide whether to pay a sunk cost of $i$ to enter the market or to remain inactive at zero cost. After the restaurateurs commit to their entry decisions, each restaurant receives a random endowment of quality, which can equal either the high value $q_H$ with probability $w$ or the low value $q_L$ with the complementary probability.

The competitive stage consists of two periods, early and late. All entrants can operate
with zero fixed costs in the early period, but continuing to the late period requires paying a continuation cost \(i'\). Exit allows a restaurateur to avoid this cost. In both periods, consumers randomly match with restaurants. The market is populated by \(S\) identical consumers, and equal numbers of them match with each restaurant. Restaurateurs simultaneously post their prices, and consumers decide on their purchases. A consumer matched with a restaurant charging a price \(p\) for a meal of quality \(q\) purchases \(d(p/q)\) meals. This demand function is strictly decreasing and concave. Restaurants’ variable cost functions are identical and feature a constant marginal cost of production, \(m\).

A free entry equilibrium consists of a number of entrants, \(N\), quality-contingent pricing decisions for each of the two periods, and quality contingent exit decisions such that each active restaurateur maximizes profit, entry earns a non-negative return, and no inactive potential entrant regrets staying out of the market. It is straightforward to show that this model has a unique free-entry equilibrium. First, consider the restaurants’ pricing decisions, which satisfy the usual inverse-elasticity rule.

\[
\frac{p - m}{p} = \frac{p}{q} d'(p/q) d(p/q)
\]

Because \(d(\cdot)\) is concave, there is a unique price that satisfies this for each quality level. The optimal price increases with the restaurant’s quality.

The assumption of a constant marginal cost implies that a restaurant earns a constant profit per customer. Denote these with \(\pi_L\) and \(\pi_H\) for the low and high quality restaurateurs. Restaurateurs’ exit decisions depend on these profits, the number of entrants, and the cost of continuation. Denote the number of active restaurants in the late period with \(N'\). Restaurateurs’ optimal continuation decisions imply that

\[
N' = \begin{cases} 
N & \text{if } i' \leq (S/N) \times \pi_L, \\
S \frac{\pi_L}{i'} & \text{if } (S/N) \times \pi_L < i' \leq (S/wN) \times \pi_L, \\
wN & \text{if } (S/wN) \times \pi_L < i' \leq (S/wN) \pi_H, \\
S \frac{\pi_H}{i'} & \text{if } (S/wN) \pi_H < i'.
\end{cases}
\]
In the first case all restaurants can profitably produce during the late period. In the second case, low-quality restaurants exit until their continuation value equals zero. In the third case, all low-quality restaurants exit, but all high-quality restaurants continue. In the final case, the continuation cost is high enough so that high-quality restaurants exit until their continuation value equals zero. The equilibrium exit decisions allow the definition of low and high quality restaurants’ values at the beginning of the competitive stage, $V_L(S/N)$ and $V_H(S/N)$. These are both strictly increasing in $S/N$, so there exists a unique value of $N$ that equates the ex-ante value of a new entrant with the entry cost.

Before proceeding to the general model, it is worth highlighting the scale invariance of this free-entry equilibrium. Because the ex-ante value of an entrant depends only on $S/N$, increasing the number of consumers raises the number of entrants proportionately. Restaurants’ optimal prices depend on neither $S$ nor $N$, while increasing both $S$ and $N$ raises $N'$ by the same proportion and leaves the exit rate, $1 - N'/N$, unchanged. Hence, increasing the number of consumers in the market leaves the distributions of all observable producer decisions unchanged.

This specific example is far too stylized for empirical work, but suppose for the moment that it generated the MSA-level observations of restauranteurs’ decisions used in Section 2. If restaurateurs’ marginal costs and consumers’ demand curves depend on a vector of market-specific variables like the factor prices and demographics in $W$, then regressions of restaurants’ exit rate and of the fraction of restaurants with “high” prices on this vector and $\ln S$ would detect no dependence of these market-level summaries of producer actions on market size. In this sense, the specific example yields a testable prediction for cross-market comparisons of producer actions. The fact that the results in Section 2 refute this prediction implies that this very simple model could not have generated the data in hand. The analysis of the general model demonstrates that the conflict arises from the assumption of atomistic competition rather than one of the example’s other simplifying assumptions.
3.2 The General Model

Like the specific example, the general model consists of two stages, entry and competition. In the first stage, a large number of potential entrants simultaneously make their entry decisions. At the same time, entering producers make their product choices. That of a particular entrant is \( x \), and this lies in the set of all possible choices, \( \mathcal{X} \subset \mathbb{R}^k \), where \( k < \infty \). The number of producers that made choice \( x \) is \( F(x) \in \mathbb{N} \), which I call the industry’s entry profile. The example did not make restaurateurs’ product choices explicit, but this can be remedied by assuming that they choose product addresses in \( \mathbb{R} \) and that all consumers match in equal numbers with all offered products.

In the second stage, producers compete to sell their products to the market’s \( S \) consumers. Producers simultaneously choose actions, \( a \in A \subset \mathbb{R}^l \), where \( l < \infty \). Producers’ profits depend on these choices and on realization of a vector of aggregate shocks, \( Z \), which occurs before producers choose actions. An action profile is a function \( A(x; Z, F) \rightarrow A \). If \( F(x') > 0 \), then \( A(x'; Z, F) \) gives the action of a producer that chose \( x' \) at entry. In the example, \( a \) represents a restaurants’ early and late prices and its continuation probability and \( Z \) determines restaurants’ qualities.\(^4\)

For simplicity, we assume that if two or more entrants chose \( x \), they both choose the same post entry action.\(^5\) The total revenues of a producer at \( x' \) that chooses the action \( a' \) when all other producers’ use the action profile \( A(x; Z, F) \) and the entry profile is \( F(x) \) are

---

\(^4\)The specific example relies on idiosyncratic shocks to entrants’ qualities. To use the general model’s aggregate shocks to represent idiosyncratic shocks, assume that \( Z \) is a uniformly distributed location on the unit-circumference circle and that a restaurant has high quality if the clockwise distance between \( x/N \) (interpreted as a location on this circle) and \( Z \) is less than \( w \). A potential entrant is indifferent across all locations on \( [0, N) \) if entrants uniformly distribute themselves on this interval, so such a uniform distribution is an equilibrium outcome that generates the same distribution of high and low quality as in the example.

\(^5\)As in the specific example, an element of \( a \) can represent a mixed strategy over a discrete and finite set of actions; and the revenues and costs specified below can be reinterpreted as expected values. Hence this assumption allows for mixed strategies. However, it does remove asymmetric Nash equilibria from consideration.
$S \times r(a', x'; A, Z, F)$. Here, $S$ denotes the number of consumers and $r(\cdot)$ is the producer’s average revenue per consumer, which does not directly depend on $S$. That producer’s costs are $c(a', x'; A, Z, F, S)$.

The expected post entry profit to a producer choosing $x'$ at entry when it and its competitors follow the action profile $A(x; Z, F)$ are

$$
\pi(x'; A, F, S) \equiv E[S \times r(A(x'; Z, F), x'; A, Z, F) - c(A(x'; Z, F), x'; A, Z, F, S)].
$$

Here, the expectation is taken with respect to the distribution of $Z$. This expectation exists under the assumption that that $r(\cdot)$ and $c(\cdot)$ are uniformly bounded functions of $a$ and $Z$.

For the example, denote the prices charged by a restaurant and the probability that it produces in the late period with $a_1$, $a_2$, and $a_3$. The revenue and cost functions in the case where a single restaurant occupies an address are

$$
r(\cdot) = \frac{a_1}{N} \times d(a_1/q) + a_3 \times \frac{a_2}{N'} \times d(a_2/q), \text{ and}
$$

$$
c(\cdot) = i + m \times \frac{S}{N} d(a_1/q) + a_3 \times \left( i' + m \times \frac{S}{N'} d(a_2/q) \right).
$$

In these expressions, the restaurant’s quality $q$ is the function of $Z$ and $x$ described in Footnote 4.

Define a strategy profile to be an action profile $A(x; Z, F)$ paired with an entry profile $F(x)$ and denote it with $(A, F)$. With this notation in place, the definition of a free-entry equilibrium may proceed.$^6$

**Definition** A strategy profile $(A^*, F^*)$ is a free-entry equilibrium for a market with $S$ consumers if it satisfies the following conditions.

---

$^6$Conventional notation for a dynamic game takes a set of players with names, a strategy space, and payoff functions as primitives. The application of that approach to this model would specify the set of players as an unbounded set of potential entrants with names in $R^k$, the strategy space as $X \times \{A(x; Z, F) \in A\}$, and the payoffs as profit defined above. Because $F(x)$ and $Z$ directly index all subgames, working directly with the strategy profile as defined here simplifies the model’s exposition.
(a) Take any entry profile $F(x)$. If $F(x') > 0$, then for all $a \in A$ and all possible realizations of $Z$,

$$S \times r(a, x'; A, Z, F) - c(a, x'; A, Z, F, S) \leq S \times r(A^*(x'; Z, F), x'; A, Z, F) - c(A^*(x'; Z, F), x'; A, Z, F, S).$$

(b) For all $x' \in X$, $\pi(x'; A, F^* + I \{x = x'\}, S) \leq 0$.

(c) If $F^*(x') > 0$; then $\pi(x'; A^*, F^*, S) \geq 0$, and for all $x'' \in X$

$$\pi(x'; A^*, F^*, S) \geq \pi(x''; A^*, F^* + I \{x = x''\} - I \{x = x'\}, S) - I \{x = x'\}, S)$$

Condition (a) of this definition ensures that the action profile $A^*(x; Z, F)$ forms a Nash equilibrium for all subgames following the entry stage. Condition (b) requires that no further entry is profitable, and condition (c) states that each active producer's entry decision and choice of $x$ is optimal given all other potential entrants' decisions. Together, the definition's three conditions are equivalent to requiring the strategy profile $(A^*, F^*)$ to correspond to a subgame perfect Nash equilibrium with pure strategies in the entry stage.

3.3 Atomistic Competition

At this level of generality, the framework encompasses many models. To specialize it and thereby derive the implications of atomistic competition, we impose the following two conditions. The first condition allows for only trivial strategic interactions between producers when no two of them occupy the same location in $X$, and the second ensures that no such “local oligopolies” will arise in a free-entry equilibrium. Henceforth, I assume that $X$ is a Borel measurable set with positive measure, denote the set of its Borel measurable subsets with $\mathcal{M}$, and use $\mu(M)$ to denote the Borel measure of $M \in \mathcal{M}$.

**Assumption A1 (Atomistic Competition)** Let $(A, F)$ be a strategy profile with $F(x) \leq 1$ and define $M = \{x|F(x) = 1\}$. If $F(x)$ is Borel-measurable, $\mu(M) > 0$, $A(x; Z, F')$ is
Borel-measurable given any shock realization Z and Borel-measurable entry profile F', and F(x') = 1, then the revenues of the producer at x' choosing the action a' satisfy

\[ S \times r (a', x'; A, Z, F) = S \times \rho (a', x'; G (A, Z, F), Z, N_F), \]

where \( N_F \equiv \mu(M) \) is the mass of producers operating and

\[ G (A, Z, F) (a') \equiv \frac{1}{N_F} \int_X I \{ A (x; Z, F) \leq a' \} \times F(x) d\mu(x). \]

Two aspects of Assumption A1 capture the idea that producers compete atomistically. First, a producer’s revenues only depend on its own choices, aggregate shocks, the mass of competing producers, and the empirical distribution of their actions. Second, any one producer has measure zero when computing this distribution, so changing a single producer’s conduct alters no other producer’s revenue. The example revenue function above satisfies Assumption A1, because each producer’s profit depends on its rivals actions only through \( S/N \) and \( S/N' \). A finite-horizon version of Hopenhayn’s (1992) model of perfect competition also satisfies Assumption A1. In any particular industry, the number of producers is obviously countable and not continuous. Models of atomistic competition are of empirical interest because their predictions might fit the data well in spite of the false simplifying assumption of a continuum of producers.

**Assumption A2 (Product Differentiation)** If \( F (x') \geq 2 \) and A satisfies condition (a) of the definition of a free-entry equilibrium, then \( \pi (x'; A, F, S) < 0. \)

Assumption A2 states that competition between producers of identical products is tough enough to guarantee that no more than one producer will occupy any location in \( \mathcal{X} \). Thus, the observed market structure will not contain any “local” oligopolies; and competition is “global” in the sense of Anderson and de Palma (2000). The specific example satisfies this assumption. Any model in which firms’ producing exactly the same product act as Bertrand competitors will satisfy Assumption A2.\(^7\)

\(^7\)A model with price-taking producers of a homogeneous good, such as Hopenhayn’s (1992), could accommodate this assumption by defining a trivial product placement choice \( x \) on the real line and assuming that
3.4 Intrinsic Scale Effects

Thus far, the model’s specification does not rule out direct effects of the scale of the market, measured with either $S$ or $N_F$, on producers’ revenues or costs. For example, the product space might be limited so that entry cannot continue indefinitely. The market shares of producers with particular choices of $x$ might be more or less sensitive to the size of the market, or directly raising $S$ could systematically reduce costs and so encourage entry and production. For all of these reasons, the distribution of producers’ decisions across large and small markets could differ. The following three conditions eliminate them as a theoretical possibility.

Assumption S1 (Invariance of Market Shares)  *The per consumer revenue function $\rho(\cdot)$ is homogeneous of degree -1 in $N_F$.*

This assumption states that doubling the number of producers while holding the distribution of their actions fixed cuts each producer’s revenue in half. In the example, it follows from the uniform random matching of consumers with firms. This assumption is closely related to the independence of irrelevant alternatives: Adding a producer to a market does not change the relative market shares of any two incumbents. The quadratic demand system of Ottaviano, Tabuchi, and Thisse (2002) generally violates Assumption S1, because each consumer’s elasticity of demand depends on the number of varieties for sale. This and similar models hard-wire markups which decline with market size. These arise naturally in true oligopoly models, so excluding them from the definition of atomistic competition is appropriate if one wishes the definition to distinguish substantially between strategic and anonymous competition.\(^8\)

---

\(^8\)Whether or not one agrees with this distinction, the result of Asplund and Nocke (2006) implies that a model which violates only Assumption S1 cannot reproduce the exit rate’s decline with market size documented in Section 2.
Assumption S2 (No Productive Spillovers) For any entry choice \( x' \in X \), action \( a' \in A \), and any two strategy profiles \((A, F), (A^*, F^*)\) and market sizes \( S \) and \( S^* \), if

\[
c(a', x'; A, Z, F, S) < c(a', x'; A^*, Z, F^*, S^*)
\]

then

\[
S \times r(a', x'; A, Z, F) < S^* \times r(a', x'; A^*, Z, F^*)
\]

Assumption S2 implies that it is impossible to hold a producer’s choices of \( x \) and \( a \) fixed, change its competitive environment, and lower that producer’s costs without simultaneously lowering its revenues. Any model in which producers’ costs depend only on their own actions (such as a quantity setting game with no productive spillovers) satisfies this assumption. If the market faces an upward sloping supply curve for some input, as in some versions of Hopenhayn’s (1992) model, then this assumption would be violated. The simple affine technology of the example obviously satisfies Assumption S2.

Assumption S3 (Distinct Observationally-Equivalent Strategy Profiles) For any market size \( S \) and strategy profile \((A, F)\) such that \( F(x) \leq 1 \) for all \( x \in X \), there exists a continuous, one to one, and onto function \( g : X \rightarrow X \) such that if we define \( F^T(x) \equiv F(g^{-1}(x)) \), and \( A^T(x; Z, F^T) = A(g^{-1}(x), Z, F) \) then

(a) \( \forall x \in X, F(x) + F^T(x) \leq 1 \);

(b) if \( F(x') > 0 \), then

\[
S \times r(a', x'; A, Z, F) = S \times r(a', g(x'); A^T, Z, F^T)
\]

and

\[
c(a', x'; A, Z, F, S) = c(a', g(x'); A^T, Z, F^T, S);
\]

(c) \( \forall M \in M, \mu(g^{-1}(M)) = \mu(M) \).
In many models of competition with product differentiation, it is possible to rearrange producers’ locations in $\mathcal{X}$, hold their actions fixed, and leave their payoffs unchanged. Consider two examples of such a rearrangement, moving all producers a short distance to the right in Salop’s (1979) circle model and changing the particular products chosen by entrants in Dixit and Stiglitz’s (1977) model of Chamberlinian monopolistic competition. In both cases, the rearrangement leaves the game played after product placement unaltered. In Assumption S3, conditions (a) and (b) require such a rearrangement to be possible for any given strategy profile. In this section’s simple example, $\mathcal{X}$ is an infinite set of all possible products without any spatial structure or asymmetries in demand or cost, so such a rearrangement is possible. Condition (c) requires $g(x)$ to be measure preserving, so that the rearrangement does not alter the mass of producers. Overall, Assumption S3 asserts that no location in $\mathcal{X}$ has payoff-relevant characteristics that are unique.

### 3.5 Equilibrium

The following two conditions ensure that potential entrants’ expectations about post-entry competition can be well-defined and that a free-entry equilibrium exists.

**Assumption E1 (Existence of Nash Equilibrium)** *For any market size $S$, there exists a strategy profile $A(x, Z, F)$ that satisfies condition (a) of the definition of a free-entry equilibrium.*

**Assumption E2 (Existence of a Measurable Free Entry Equilibrium)** *There exists a market size $S_0 > 0$ with a corresponding free-entry equilibrium $(A_0, F_0)$ such that*

(a) $F_0(x)$ and $A_0(x; Z, F_0)$ are Borel measurable functions of $x$ for any $Z$, and

(b) $A_0(x'; Z, F) = A_0(x'; Z, F_0)$ if $F(x') = F_0(x') = 1$ and $F(x) = F_0(x)$ almost everywhere.
The goal of this section is to show that no well-defined model of atomistic competition can reproduce the dependence of firms’ choices on market size. A model that violates \text{E1} can make no equilibrium prediction for some market size, and a model that violates part (a) of \text{E2} makes no prediction for any market size. In this specific sense, a model that violates either of these assumptions is not well defined. Part (b) of Assumption \text{E2} eliminates the possibility that a positive measure of firms respond to deviations from the equilibrium by a single (measure zero) firm. That is, there is no producer whose actions act as a pure coordination device for the others. Both \text{E1} and \text{E2} are clearly true for this section’s introductory example. Accordingly, I view both assumptions as regularity conditions that any model must satisfy before it could be taken seriously as an explanation for the data.

3.6 Market Size and Producers’ Actions

The specific example and many other models of monopolistic competition satisfy all of the above assumptions. Together, they place sufficient structure on the model to imply the following observational implication.

Proposition  If $S=2^j \times S_0$, where \( j \) is a non-negative integer and $S_0$, $A_0$, and $F_0$ are defined in Assumption \text{E2}, then there exists a free entry equilibrium $(A_j, F_j)$ such that

$$G(A_j, Z, F_j) = G(A_0, Z, F_0)$$

where $G(A, Z, F)$ is as defined in Assumption \text{A1}.

The proposition says that for every measurable equilibrium with a given market size there exists a corresponding equilibrium for a market with twice as many consumers with identical distributions of producers’ actions. Unless there exists more than one free-entry equilibrium and the equilibrium selection rule systematically depends on $S$, there can be no observable relationship between market size and the distribution of any producer choice. The appendix presents the proposition’s proof. Here, I only outline the argument. Consider the
free-entry equilibrium \((A_0, F_0)\) for \(S_0\). We know from Assumption S3 that there is a different but observationally equivalent strategy profile, \((A^T_0, F^T_0)\). Now consider a market with \(S_1 = 2 \times S_0\) and entry profile, \(F_0+F^T_0\). If all producers duplicate the actions they take in the smaller market, then the empirical c.d.f. of producers actions remains unchanged, so Assumptions A1, S1, and S2 imply that each producer’s profit maximizing action remains unchanged. That is, the action profile that duplicates producers’ actions is a Nash equilibrium profile for this larger market size and entry profile. Each producer’s profits remain unchanged, and the profits from producing in an unoccupied location in \(X\) are identical to their value in the original free entry equilibrium with the smaller market size, so conditions (b) and (c) of the definition are satisfied. Each producer’s actions equal those from the original equilibrium, so their empirical distributions are also unchanged as the proposition asserts.\(^9\)

The proposition illustrates that the invariance of producers’ decisions to the market’s size in the specific example extends well beyond its particular assumptions. All of the assumptions excepting A1 are regularity conditions, so I interpret the fact that market size does influence restauranteurs’ observable decisions as a rejection of atomistic competition.

### 3.7 Extensions

The general model is restrictive in two ways that are worth noting. First, it has no role for actions that are taken prior to the realization of \(Z\) that do not directly differentiate firms’ products, such as investments that increase the likelihood of having a high quality restaurant. Adding such pre competition actions to the general model increases its notational burden but does not alter its scale invariance. Second, the use of the Borel integral to form the c.d.f. of producers’ actions in Assumption A1 restricts product placement decisions to be

\(^9\)The proposition’s focus on doubling market size can easily be changed if the assertion that \(g(x)\) is measure preserving in Assumption S3 is replaced with the assumption that for any \(t > 1\), there exists a \(g_t(x)\) satisfying the assumption’s other conditions and which satisfies \(\mu(g_t^{-1}(M)) = t \times \mu(M)\). With this, a parallel argument establishes that a free-entry equilibrium that replicates producers’ decisions exists for any market size greater than \(S_0\).
continuous choices. Scale invariance requires some continuous product placement decision to differentiate firms’ products, but it does not require all product placement decisions to be continuous. Extending the general model to allow for discrete dimensions of firms’ product-placement decisions is straightforward.

3.8 Atomistic and Monopolistic Competition

Before concluding, it is helpful to clarify the relationship between what I have labelled “atomistic competition” with the large theoretical literature on monopolistic competition. For some authors, “monopolistic competition” refers to all imperfect competition among a large number of producers. Models that prominently feature strategic interaction, such as Salop’s (1979) model of spatial competition, then go by the label of “Hotelling-style” monopolistic competition. Models with only trivial strategic considerations, such as Spence’s (1976) are called “Chamberlin-style”.

Hart (1985) and Wolinsky (1986) propose a more exclusive definition of “monopolistic competition” based on four criteria.

(1) there are many firms producing differentiated commodities; (2) each firm is negligible in the sense that it can ignore its impact on, and hence reactions from, other firms; (3) each firm faces a downward sloping demand curve and hence the equilibrium price exceeds marginal cost; (4) free entry results in zero-profit of operating firms (or, at least, of marginal firms). (Wolinsky, 1986, page 493)

These clearly correspond to what others call Chamberlin-style monopolistic competition. Hart and Wolinsky’s first two criteria correspond to Assumptions A2 and A1, and the fourth criterion is implicit in the definition of a free-entry equilibrium. The definition of atomistic competition does not require firms to face downward sloping demand curves, but it clearly allows for that possibility. Hence, models of monopolistic competition (in the sense of Hart and Wolinsky) can usually be written without economically substantial changes to satisfy the
assumptions this paper places on atomistic competition. However, the definition of atomistic competition is broad enough to also encompass models without market power.

4 Related Literature

Structure-conduct-performance studies gave rise to many examinations of competitive outcomes’ dependence on market size. One strand of this literature uses the empirical relationship between market size and the number of competitors to infer how adding competition lowers markups. If doubling market size leads to a less than proportional increase in the number of producers, either per-consumer profits fall with entry or incumbents raise entrants’ fixed costs. Bresnahan and Reiss (1990) apply this approach to concentrated retail automobile markets in isolated towns. Berry and Waldfogel (2006) examine the influence of market size on the number of competitors in a slightly broader sample of MSAs than that used in this paper, and they find that the number of restaurants increases less than proportionally with MSA population.\(^{10}\) The proof of this paper’s proposition makes it clear that the number of producers in atomistically competitive markets is proportional to the number of consumers, so Berry and Waldfogel’s finding reinforces this paper’s empirical conclusion.\(^{11}\)

This paper’s proposition does not stress the relationship between market size and the number of firms under atomistic competition, because a finding that doubling market size less than doubles the number of firms could arise solely from measurement error in market size. Measurement error could make the rejection of this paper’s exclusion restrictions less likely when they are false, but it does not lead directly to their rejection when they are true. In this sense, a test of atomistic competition based on the relationship between noisily

\(^{10}\)See the third and fourth columns of their Table 3.

\(^{11}\)Berry and Waldfogel’s finding also manifests itself in the observations used in the present paper. The estimate of \(\delta_s\) from a nonparametric regression of the number of restaurants’ logarithm on population’s logarithm and the other control variables listed in Table 1 using Campbell and Hopenhayn’s (2005) sample of MSAs equals 0.93, and this is significantly different from one.
measured market size and measures of producer actions is conservative.\footnote{Bresnahan and Reiss (1990) can measure market size accurately because they carefully chose their sample towns. This strategy becomes infeasible when considering competition in large markets in which the definition of the market and industry are themselves somewhat subjective, so prudence requires accounting for possible measurement error.}

This paper derives testable predictions of a free-entry model without the use of parametric assumptions. In this respect, Sutton’s (1991) analysis of models with endogenous sunk costs precedes it. He considers a model of competition in which entrants compete with sunk investments in product quality. The firm with the greatest investment earns a guaranteed minimum market share, regardless of the number of other producers. Sutton shows that these features together imply a nonparametric upper bound on the number of entrants, and he demonstrates that cross-country data from several advertising-intensive food-processing industries satisfy this bound. As the number of consumers grows, the number of entrants remains bounded from above. In this sense, industries that satisfy his model’s assumptions are natural oligopolies. As noted above in Subsection 3.7, it is notationally burdensome but straightforward to add pre-entry investments in quality to this paper’s model. This extension leaves the model’s nonparametric testable implications unaltered. In particular, the number of producers grows linearly with market size. The contrast between that result and Sutton’s highlights the role of endogenous sunk costs in his results: They are necessary but not sufficient for an industry to be a natural oligopoly. Hence, the simple observation that an industry’s producers incur endogenous sunk costs does not imply that its firms are oligopolists. However, tests of the exclusion restrictions from atomistic competition do provide information about the nature of competition.

The analysis of the exit of restaurants places this paper in another vast literature which examines the rate of producer turnover and the reallocation of resources between firms. These papers have focused on differences in firm growth and survival across the life cycle (as in Dunne, Roberts, and Samuelson (1988)) and on the interaction of resource reallocation with the business cycle (as in Davis, Haltiwanger, and Schuh (1996), Campbell (1998), and
Campbell and Lapham (2004)). Analysis of how the pace of resource reallocation varies with local market conditions, similar to that in this paper, is much scarcer. Syverson (2004) shows that ready-mixed concrete producers serving geographically concentrated markets have higher average productivity and less productivity dispersion than their counterparts in more sparsely populated areas, and he interprets this as the result of more intense selection in highly competitive markets. As noted in the introduction, Asplund and Nocke (2006) create a model of such selection by incorporating markups which depend on the number of producers into an otherwise standard model of industry dynamics with Chamberlinian monopolistic competition. They confirm Syverson’s finding by showing that Swedish hairstylists are younger in larger markets. This paper finds the opposite relationship between market size and exit for U.S. restaurants. Together, this paper’s theoretical results and those of Asplund and Nocke imply that dynamic aspects of strategic interaction substantially influence the rate of restaurant turnover.

5 Conclusion

Researchers’ prior beliefs about the usefulness of different modelling approaches influence their investigations of industries. The relative simplicity of atomistic competition models makes them a tempting first choice for the empirical study of competition in large markets. However, those who believe that strategic interaction permeates all producers’ choices have chosen to focus instead on industries with few competitors and relatively well-defined strategic environments. This paper’s results are of use to both sorts of researchers. For those who regularly abstract from strategic interaction, the nonparametric test of atomistic competition can be used to subject this assumption to empirical scrutiny before proceeding to a more involved investigation.\textsuperscript{13} For those unwilling to part from a strategic focus, the regressions indicate the dimensions of the data along which strategic interaction manifests

\textsuperscript{13}For example, Abbring and Campbell (2006) apply this papers’ results to test the assumption of atomistic competition in their structural model of new Texas bars’ growth and exit decisions.
itself quantitatively. This is an essential first step to extending the strategic analysis of competition to large markets.

The application of this paper’s theoretical analysis to observations of U.S. restaurants’ prices, exit rates, sales, and seating capacity indicates that atomistic competition cannot explain how restauranteurs’ key choices depend on market size. A particularly important aspect of this is that exit rates fall with market size. In related work, Yeap (2005) documents that this increase in average size reflects only the decisions of firms owning two or more restaurants. Taken together these findings indicate that better understanding of competition among restaurants in large markets requires confronting restaurateurs’ strategic behavior. Toivanen and Waterson (2005) take an important step in this direction by empirically modelling entry decisions into well defined duopoly fast-food markets. Extending such an analysis to large samples of restaurants without high-quality information about market definitions and strategic interactions is the subject of my current research with Jaap Abbring.

Proof of the Proposition

Clearly, the proposition is true for \( j = 0 \). We now wish to show that it is true for \( j = 1 \). The proposition can then be demonstrated recursively for greater values of \( j \).

Let \( g(x) \) be the function assumed to exist in Assumption \( S3 \). Define the entry profile \( F_1(x) = F_0(x) + F_0^T(x) \), where the latter entry profile is defined as in the statement of Assumption \( S3 \). From Assumption \( A2 \) and the definition of a free-entry equilibrium, we know that \( F_0(x) \in \{0, 1\} \). Therefore, condition (a) of Assumption \( S3 \) ensures that \( F_1(x) \in \{0, 1\} \).

We know from Assumption \( E1 \) that there exists an action profile \( A(x; Z, F) \) that satisfies condition (a) of a free-entry equilibrium’s definition for \( S_1 = 2 \times S_0 \). We now wish to use this and \( A_0(x; Z, F) \) to construct an action profile that forms a candidate free-entry equilibrium when paired with \( F_1 \). For any entry profile \( F(x) \) such that either
(i) $F(x') \geq 2$ for some $x' \in \mathcal{X}$ or

(ii) \( \{x \mid F(x) \neq F_1(x)\} \) is either not measurable or has positive measure,

define \( A_1(x; Z, F) = A(x; Z, F) \).

For any entry profile \( F(x) \in \{0, 1\} \) for which \( F(x) = F_1(x) \) almost everywhere, there exists two measurable sets \( C_p \) and \( C_m \) with \( \mu(C_p) = \mu(C_m) = 0 \) and \( F(x) = F_1(x) + I\{x \in C_p\} - I\{x \in C_m\} \). Define \( F_0(C_p)(x) = F_0(x) + I\{x \in C_p\} \). If \( F(x) = 1 \), then either \( F_0(C_p)(x) = 1 \) or \( F_0(C_p)(x) = 1 \). Therefore, we can define the action profile for these values of \( x \) with

\[
A_1(x; Z, F) = \begin{cases} 
A_0(x; Z, F_0(C_p)) & \text{if } F_0(C_p)(x) = 1, \\
A_0(g^{-1}(x); Z, F_0(C_p)) & \text{otherwise.}
\end{cases}
\]

Because the composition of Borel measurable functions is itself Borel measurable, \( A_1(x; Z, F) \) is a Borel measurable function of \( x \).

The next step is to show that \((A_1, F_1)\) is a free-entry equilibrium. To do so, consider the definition’s three conditions in turn.

**Condition (a)**

Note that by construction \( A_1(x; Z, F) \) satisfies the inequality in condition (a) of a free-entry equilibrium’s definition if it satisfies either (i) or (ii) above. Suppose that \( F(x) \in \{0, 1\} \) and \( F(x) = F_1(x) \) almost everywhere. This implies

\[
G(A_1, Z, F)(a') = \frac{1}{N_F} \int_{\mathcal{X}} I\{A_1(x; Z, F) \leq a'\} F(x) d\mu(x)
= \frac{1}{2} \frac{1}{N_{F_0(C_p)}} \int_{\mathcal{X}} I\{A_0(x; Z, F_0(C_p)) \leq a'\} F_0(C_p)(x) d\mu(x)
+ \frac{1}{2} \frac{1}{N_{F_0^T}} \int_{\mathcal{X}} I\{A_0(g^{-1}(x); Z, F_0(C_p)) \leq a'\} F_0^T(x) d\mu(x)
= G(A_0, Z, F_0(C_p))(a') .
\]

The first equality holds because \( F_1 \) and \( F_0(C_p) + F_0^T(C_p) \) differ by a set of measure zero, and the last equality follows from Proposition 1 in Chapter 15 of Royden (1988).
With this and Assumptions A1 and S1, we can conclude that if $F_0 (C_p) (x) = 1$, then for all $a' \in A$, $S_1 \times \rho (a', x'; G (A_1, Z, F), Z, N_FR) = S_0 \times \rho (a', x'; G (A_0, Z, F_0 (C_p)), Z, N_F_0 (C_p))$

In turn, this and Assumption S2 imply that

$$S_1 \times \rho (a', x'; G (A_1, Z, F), Z, N_FR) - c (a', x'; A_1, Z, F, S_1) = S_0 \times \rho (a', x'; G (A_0, Z, F_0 (C_p)), Z, N_F_0 (C_p)) - c (a', x'; A_0, Z, F_0 (C_p), S_0)$$

The action $A_0 (x'; Z, F_0 (C_p)) = A_1 (x'; Z, F)$ maximizes the right-hand side, so it must also maximize its left-hand side.

Alternatively, if $F_0^\prime (C_p) (x) = 1$, then we can construct a parallel argument to show that $A_1 (x', Z, F)$ maximizes the firm’s profit. Thus $A_1 (x, Z, F)$ satisfies condition (a) of a free-entry equilibrium’s definition.

**Condition (b)**

Next, consider condition (b) of the definition. Extending the notation above, denote $F_1 (x) + \mathbb{1} \{x = x'\}$ with $F_1 (\{x\}) (x)$. If $F_1 (x') = 1$, then the definition of $A_1$ and Assumptions A2 and E1 imply that $\pi (x'; A_1, F_1 (\{x\}), S) \leq 0$. Next, note that if $F_1 (x') = 0$, then we know from above that $G (A_1, Z, F_1 (\{x\})) (a) = G (A_0, Z, F_0 (\{x\})) (a)$, and that $N_{F_1 (\{x\})} = 2 \times N_{F_0 (\{x\})}$. Therefore, Assumptions A1, S1, and S2 and the definition of a free-entry equilibrium imply that $\pi (x'; A_1, F_1 (\{x\}), S) \leq 0$ in this case as well. Hence, condition (b) of the definition is satisfied.

**Condition (c)**

Finally, consider condition (c) of a free-entry equilibrium’s definition. Because

$$G (A_1, Z, F_1) (a) = G (A_0, Z, F_0) (a)$$

and $N_{F_1} = 2 \times N_{F_0}$, so if $F_0 (x') = 1$ then

$$\pi (x'; A_1, F_1, S_1) = \pi (x'; A_0, F_0, S_0) \geq 0.$$
Furthermore, conditions (b) and (c) of Assumption S3 imply that this inequality also applies if $F_0^T(x') = 1$. Therefore, the first inequality in condition (c) of the definition holds good.

The second inequality in this condition holds trivially from Assumption A2 and the definition of $A_1$ if $F_1(x'') = 1$. Suppose instead that $F_1(x'') = 0$ and $F_1(x') = 1$. We know that $F_1(x) + I\{x = x''\} - I\{x = x'\} = F_1(x) + I\{x = x''\}$ almost everywhere. From this and the fact that we have already verified condition (b) of an equilibrium’s definition, we conclude that

$$\pi(x''; A_1, F_1 + I\{x = x''\} - I\{x = x'\}, S_1) \leq 0.$$  

Thus, the second inequality of condition (c) holds and $(A_1, F_1)$ is a free-entry equilibrium.
References


### Table 1: Nonparametric Regression Estimates$^{(i,ii,iii)}$

<table>
<thead>
<tr>
<th>Variable</th>
<th>(\mathcal{L}($5.00))</th>
<th>Exit Rate</th>
<th>Revenue</th>
<th>Seats per Restaurant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>-12.90**</td>
<td>-0.77**</td>
<td>4.68***</td>
<td>2.07</td>
</tr>
<tr>
<td></td>
<td>(6.13)</td>
<td>(0.32)</td>
<td>(1.76)</td>
<td>(1.99)</td>
</tr>
<tr>
<td>Commercial Rent</td>
<td>11.16*</td>
<td>-0.29</td>
<td>1.66</td>
<td>-2.23</td>
</tr>
<tr>
<td></td>
<td>(6.29)</td>
<td>(0.30)</td>
<td>(1.53)</td>
<td>(1.89)</td>
</tr>
<tr>
<td>Retail Wage</td>
<td>17.37**</td>
<td>0.69**</td>
<td>-0.49</td>
<td>-2.56</td>
</tr>
<tr>
<td></td>
<td>(7.12)</td>
<td>(0.35)</td>
<td>(1.56)</td>
<td>(2.18)</td>
</tr>
<tr>
<td>Advertising Cost</td>
<td>-9.00*</td>
<td>-0.30</td>
<td>-1.43</td>
<td>-0.35</td>
</tr>
<tr>
<td></td>
<td>(5.47)</td>
<td>(0.27)</td>
<td>(1.65)</td>
<td>(1.95)</td>
</tr>
<tr>
<td>Income</td>
<td>-1.67</td>
<td>-0.55*</td>
<td>6.17***</td>
<td>4.90**</td>
</tr>
<tr>
<td></td>
<td>(7.03)</td>
<td>(0.33)</td>
<td>(1.63)</td>
<td>(2.39)</td>
</tr>
<tr>
<td>Percent Black</td>
<td>20.89***</td>
<td>0.62**</td>
<td>0.90</td>
<td>-3.95*</td>
</tr>
<tr>
<td></td>
<td>(5.91)</td>
<td>(0.26)</td>
<td>(1.31)</td>
<td>(2.16)</td>
</tr>
<tr>
<td>Percent College</td>
<td>19.70***</td>
<td>-0.09</td>
<td>8.48***</td>
<td>2.50</td>
</tr>
<tr>
<td></td>
<td>(6.22)</td>
<td>(0.33)</td>
<td>(1.28)</td>
<td>(1.83)</td>
</tr>
<tr>
<td>Vehicle Ownership</td>
<td>-2.41</td>
<td>-0.57**</td>
<td>-1.16</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td>(5.08)</td>
<td>(0.28)</td>
<td>(1.58)</td>
<td>(2.00)</td>
</tr>
</tbody>
</table>

Notes: (i) The table reports estimates of density-weighted average derivatives from the regressions of the indicated variables on the regressors listed in the first column. Asymptotic standard errors appear below each estimate in parentheses. The superscripts *, **, and *** indicate statistical significance at the 10, 5, and 1 percent levels. (ii) In the table, \(\mathcal{L}($5.00)\) refers to the logistic transformation of the fraction of restaurants in an MSA with typical meal prices greater than or equal to $5.00. (iii) *All estimates have been multiplied by 100.* See the text for further details.
Table 2: Alternative Regression Estimates$^{(i,iii)}$

<table>
<thead>
<tr>
<th>Market Size Measures</th>
<th>Population</th>
<th>Population Density</th>
<th>Housing Units</th>
<th>Bandwidth Choices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mathbb{L}$(5.00)$^{(ii)}$</td>
<td></td>
<td></td>
<td>$h = 1$</td>
</tr>
<tr>
<td>Exit Rate</td>
<td>-12.90**</td>
<td>-8.91</td>
<td>-11.93*</td>
<td>-12.67</td>
</tr>
<tr>
<td></td>
<td>(6.13)</td>
<td>(5.74)</td>
<td>(6.19)</td>
<td>(7.76)</td>
</tr>
<tr>
<td></td>
<td>-0.77**</td>
<td>-0.89***</td>
<td>-0.75**</td>
<td>-0.94**</td>
</tr>
<tr>
<td></td>
<td>(0.32)</td>
<td>(0.29)</td>
<td>(0.32)</td>
<td>(0.39)</td>
</tr>
<tr>
<td>Log of Restaurants’ Average Revenue</td>
<td>4.68***</td>
<td>4.36**</td>
<td>5.03***</td>
<td>5.76***</td>
</tr>
<tr>
<td></td>
<td>(1.76)</td>
<td>(1.77)</td>
<td>(1.78)</td>
<td>(1.99)</td>
</tr>
<tr>
<td>Log of Average Seats per Restaurant</td>
<td>2.07</td>
<td>2.69</td>
<td>2.45</td>
<td>2.83</td>
</tr>
<tr>
<td></td>
<td>(1.99)</td>
<td>(2.00)</td>
<td>(2.05)</td>
<td>(2.55)</td>
</tr>
</tbody>
</table>

Notes: (i) The table’s entries are estimated density-weighted average derivatives of the indicated variable with respect to the logarithm of the indicated measure of market size. Heteroskedasticity-consistent standard errors appear in parentheses. The superscripts $\star$, $\star\star$, and $\star\star\star$ indicate statistical significance at the 10%, 5%, and 1% levels. (ii) In the table, $\mathbb{L}$(5.00) refers to the logistic transformation of the fraction of restaurants in an MSA with typical meal prices greater than or equal to $5.00. (iii) All estimates have been multiplied by 100. See the text for further details.
Figure 1: Logistic Transformation of the Share of Establishments with High Meal Prices

Note: The figure plots the logistic transformation of the share of establishments with typical meal prices exceeding $5.00 against the demeaned logarithm of MSA population.
Figure 2: Restaurants' Annual Exit Rate in Percentage Points
Figure 3: Logarithm of Restaurants’ Average Revenue
Figure 4: Logarithm of Average Seats per Restaurant
Working Paper Series

A series of research studies on regional economic issues relating to the Seventh Federal Reserve District, and on financial and economic topics.

Outsourcing Business Services and the Role of Central Administrative Offices
Yukako Ono

Strategic Responses to Regulatory Threat in the Credit Card Market*
Victor Stango

The Optimal Mix of Taxes on Money, Consumption and Income
Fiorella De Fiore and Pedro Teles

Expectation Traps and Monetary Policy
Stefania Albanesi, V. V. Chari and Lawrence J. Christiano

Monetary Policy in a Financial Crisis
Lawrence J. Christiano, Christopher Gust and Jorge Roldos

Regulatory Incentives and Consolidation: The Case of Commercial Bank Mergers and the Community Reinvestment Act
Raphael Bostic, Hamid Mehran, Anna Paulson and Marc Saidenberg

Technological Progress and the Geographic Expansion of the Banking Industry
Allen N. Berger and Robert DeYoung

Choosing the Right Parents: Changes in the Intergenerational Transmission of Inequality — Between 1980 and the Early 1990s
David I. Levine and Bhashkar Mazumder

The Immediacy Implications of Exchange Organization
James T. Moser

Maternal Employment and Overweight Children
Patricia M. Anderson, Kristin F. Butcher and Phillip B. Levine

The Costs and Benefits of Moral Suasion: Evidence from the Rescue of Long-Term Capital Management
Craig Furfine

On the Cyclical Behavior of Employment, Unemployment and Labor Force Participation
Marcelo Veracierto

Do Safeguard Tariffs and Antidumping Duties Open or Close Technology Gaps?
Meredith A. Crowley

Technology Shocks Matter
Jonas D. M. Fisher

Money as a Mechanism in a Bewley Economy
Edward J. Green and Ruilin Zhou
Working Paper Series (continued)

Optimal Fiscal and Monetary Policy: Equivalence Results
Isabel Correia, Juan Pablo Nicolini and Pedro Teles

Real Exchange Rate Fluctuations and the Dynamics of Retail Trade Industries on the U.S.-Canada Border
Jeffrey R. Campbell and Beverly Lapham

Bank Procyclicality, Credit Crunches, and Asymmetric Monetary Policy Effects: A Unifying Model
Robert R. Bliss and George G. Kaufman

Location of Headquarter Growth During the 90s
Thomas H. Klier

The Value of Banking Relationships During a Financial Crisis: Evidence from Failures of Japanese Banks
Elijah Brewer III, Hesna Genay, William Curt Hunter and George G. Kaufman

On the Distribution and Dynamics of Health Costs
Eric French and John Bailey Jones

The Effects of Progressive Taxation on Labor Supply when Hours and Wages are Jointly Determined
Daniel Aaronson and Eric French

Inter-industry Contagion and the Competitive Effects of Financial Distress Announcements: Evidence from Commercial Banks and Life Insurance Companies
Elijah Brewer III and William E. Jackson III

State-Contingent Bank Regulation With Unobserved Action and Unobserved Characteristics
David A. Marshall and Edward Simpson Prescott

Local Market Consolidation and Bank Productive Efficiency
Douglas D. Evanoff and Evren Örs

Life-Cycle Dynamics in Industrial Sectors. The Role of Banking Market Structure
Nicola Cetorelli

Private School Location and Neighborhood Characteristics
Lisa Barrow

Teachers and Student Achievement in the Chicago Public High Schools
Daniel Aaronson, Lisa Barrow and William Sander

The Crime of 1873: Back to the Scene
François R. Velde

Trade Structure, Industrial Structure, and International Business Cycles
Marianne Baxter and Michael A. Koupatisas

Estimating the Returns to Community College Schooling for Displaced Workers
Louis Jacobson, Robert LaLonde and Daniel G. Sullivan
A Proposal for Efficiently Resolving Out-of-the-Money Swap Positions at Large Insolvent Banks
George G. Kaufman

Depositor Liquidity and Loss-Sharing in Bank Failure Resolutions
George G. Kaufman

Subordinated Debt and Prompt Corrective Regulatory Action
Douglas D. Evanoff and Larry D. Wall

When is Inter-Transaction Time Informative?
Craig Furfine

Tenure Choice with Location Selection: The Case of Hispanic Neighborhoods in Chicago
Maude Toussaint-Comeau and Sherrie L.W. Rhine

Distinguishing Limited Commitment from Moral Hazard in Models of Growth with Inequality*
Anna L. Paulson and Robert Townsend

Resolving Large Complex Financial Organizations
Robert R. Bliss

The Case of the Missing Productivity Growth: Or, Does information technology explain why productivity accelerated in the United States but not the United Kingdom?
Susanto Basu, John G. Fernald, Nicholas Oulton and Sylaja Srinivasan

Inside-Outside Money Competition
Ramon Marimon, Juan Pablo Nicolini and Pedro Teles

The Importance of Check-Cashing Businesses to the Unbanked: Racial/Ethnic Differences
William H. Greene, Sherrie L.W. Rhine and Maude Toussaint-Comeau

A Firm’s First Year
Jaap H. Abbring and Jeffrey R. Campbell

Market Size Matters
Jeffrey R. Campbell and Hugo A. Hopenhayn

The Cost of Business Cycles under Endogenous Growth
Gadi Barlevy

The Past, Present, and Probable Future for Community Banks
Robert DeYoung, William C. Hunter and Gregory F. Udell

Measuring Productivity Growth in Asia: Do Market Imperfections Matter?
John Fernald and Brent Neiman

Revised Estimates of Intergenerational Income Mobility in the United States
Bhashkar Mazumder
Working Paper Series (continued)

Product Market Evidence on the Employment Effects of the Minimum Wage
Daniel Aaronson and Eric French

Estimating Models of On-the-Job Search using Record Statistics
Gadi Barlevy

Banking Market Conditions and Deposit Interest Rates
Richard J. Rosen

Creating a National State Rainy Day Fund: A Modest Proposal to Improve Future State Fiscal Performance
Richard Mattoon

Managerial Incentive and Financial Contagion
Sujit Chakravorti, Anna Llyina and Subir Lall

Women and the Phillips Curve: Do Women’s and Men’s Labor Market Outcomes Differentially Affect Real Wage Growth and Inflation?
Katharine Anderson, Lisa Barrow and Kristin F. Butcher

Evaluating the Calvo Model of Sticky Prices
Martin Eichenbaum and Jonas D.M. Fisher

The Growing Importance of Family and Community: An Analysis of Changes in the Sibling Correlation in Earnings
Bhashkar Mazumder and David I. Levine

Should We Teach Old Dogs New Tricks? The Impact of Community College Retraining on Older Displaced Workers
Louis Jacobson, Robert J. LaLonde and Daniel Sullivan

Trade Deflection and Trade Depression
Chad P. Brown and Meredith A. Crowley

China and Emerging Asia: Comrades or Competitors?
Alan G. Ahearne, John G. Fernald, Prakash Loungani and John W. Schindler

International Business Cycles Under Fixed and Flexible Exchange Rate Regimes
Michael A. Kouparitsas

Firing Costs and Business Cycle Fluctuations
Marcelo Veracierto

Spatial Organization of Firms
Yukako Ono

Government Equity and Money: John Law’s System in 1720 France
François R. Velde

Deregulation and the Relationship Between Bank CEO Compensation and Risk-Taking
Elijah Brewer III, William Curt Hunter and William E. Jackson III
Working Paper Series (continued)

Compatibility and Pricing with Indirect Network Effects: Evidence from ATMs
Christopher R. Knittel and Victor Stango
WP-03-33

Self-Employment as an Alternative to Unemployment
Ellen R. Rissman
WP-03-34

Where the Headquarters are – Evidence from Large Public Companies 1990-2000
Tyler Diacon and Thomas H. Klier
WP-03-35

Standing Facilities and Interbank Borrowing: Evidence from the Federal Reserve’s New Discount Window
Craig Furline
WP-04-01

Netting, Financial Contracts, and Banks: The Economic Implications
William J. Bergman, Robert R. Bliss, Christian A. Johnson and George G. Kaufman
WP-04-02

Real Effects of Bank Competition
Nicola Cetorelli
WP-04-03

Finance as a Barrier To Entry: Bank Competition and Industry Structure in Local U.S. Markets?
Nicola Cetorelli and Philip E. Strahan
WP-04-04

The Dynamics of Work and Debt
Jeffrey R. Campbell and Zvi Hercowitz
WP-04-05

Fiscal Policy in the Aftermath of 9/11
Jonas Fisher and Martin Eichenbaum
WP-04-06

Merger Momentum and Investor Sentiment: The Stock Market Reaction To Merger Announcements
Richard J. Rosen
WP-04-07

Earnings Inequality and the Business Cycle
Gadi Barlevy and Daniel Tsiddon
WP-04-08

Platform Competition in Two-Sided Markets: The Case of Payment Networks
Sujit Chakravorti and Roberto Roson
WP-04-09

Nominal Debt as a Burden on Monetary Policy
Javier Díaz-Giménez, Giorgia Giovannetti, Ramon Marimon, and Pedro Teles
WP-04-10

On the Timing of Innovation in Stochastic Schumpeterian Growth Models
Gadi Barlevy
WP-04-11

Policy Externalities: How US Antidumping Affects Japanese Exports to the EU
Chad P. Bown and Meredith A. Crowley
WP-04-12

Sibling Similarities, Differences and Economic Inequality
Bhashkar Mazumder
WP-04-13

Determinants of Business Cycle Comovement: A Robust Analysis
Marianne Baxter and Michael A. Kouparitsas
WP-04-14
Working Paper Series (continued)

The Occupational Assimilation of Hispanics in the U.S.: Evidence from Panel Data
Maude Toussaint-Comeau

Reading, Writing, and Raisinet(s): Are School Finances Contributing to Children’s Obesity?
Patricia M. Anderson and Kristin F. Butcher

Learning by Observing: Information Spillovers in the Execution and Valuation of Commercial Bank M&As
Gayle DeLong and Robert DeYoung

Prospects for Immigrant-Native Wealth Assimilation: Evidence from Financial Market Participation
Una Okonkwo Osili and Anna Paulson

Individuals and Institutions: Evidence from International Migrants in the U.S.
Una Okonkwo Osili and Anna Paulson

Are Technology Improvements Contractionary?
Susanto Basu, John Fernald and Miles Kimball

The Minimum Wage, Restaurant Prices and Labor Market Structure
Daniel Aaronson, Eric French and James MacDonald

Betcha can’t acquire just one: merger programs and compensation
Richard J. Rosen

Not Working: Demographic Changes, Policy Changes, and the Distribution of Weeks (Not) Worked
Lisa Barrow and Kristin F. Butcher

The Role of Collateralized Household Debt in Macroeconomic Stabilization
Jeffrey R. Campbell and Zvi Hercowitz

Advertising and Pricing at Multiple-Output Firms: Evidence from U.S. Thrift Institutions
Robert DeYoung and Evren Örs

Monetary Policy with State Contingent Interest Rates
Bernardino Adão, Isabel Correia and Pedro Teles

Comparing location decisions of domestic and foreign auto supplier plants
Thomas Klier, Paul Ma and Daniel P. McMillen

China’s export growth and US trade policy
Chad P. Bown and Meredith A. Crowley

Where do manufacturing firms locate their Headquarters?
J. Vernon Henderson and Yukako Ono

Monetary Policy with Single Instrument Feedback Rules
Bernardino Adão, Isabel Correia and Pedro Teles
Working Paper Series (continued)

Firm-Specific Capital, Nominal Rigidities and the Business Cycle
David Altig, Lawrence J. Christiano, Martin Eichenbaum and Jesper Linde

Do Returns to Schooling Differ by Race and Ethnicity?
Lisa Barrow and Cecilia Elena Rouse

Derivatives and Systemic Risk: Netting, Collateral, and Closeout
Robert R. Bliss and George G. Kaufman

Risk Overhang and Loan Portfolio Decisions
Robert DeYoung, Anne Gron and Andrew Winton

Characterizations in a random record model with a non-identically distributed initial record
Gadi Barlevy and H. N. Nagaraja

Price discovery in a market under stress: the U.S. Treasury market in fall 1998
Craig H. Furfine and Eli M. Remolona

Politics and Efficiency of Separating Capital and Ordinary Government Budgets
Marco Bassetto with Thomas J. Sargent

Rigid Prices: Evidence from U.S. Scanner Data
Jeffrey R. Campbell and Benjamin Eden

Entrepreneurship, Frictions, and Wealth
Marco Cagetti and Mariacristina De Nardi

Wealth inequality: data and models
Marco Cagetti and Mariacristina De Nardi

What Determines Bilateral Trade Flows?
Marianne Baxter and Michael A. Kouparitsas

Intergenerational Economic Mobility in the U.S., 1940 to 2000
Daniel Aaronson and Bhashkar Mazumder

Differential Mortality, Uncertain Medical Expenses, and the Saving of Elderly Singles
Mariacristina De Nardi, Eric French, and John Bailey Jones

Fixed Term Employment Contracts in an Equilibrium Search Model
Fernando Alvarez and Marcelo Veracierto

Causality, Causality, Causality: The View of Education Inputs and Outputs from Economics
Lisa Barrow and Cecilia Elena Rouse
Working Paper Series (continued)

Competition in Large Markets
Jeffrey R. Campbell