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Revised Estimates of Intergenerational Income Mobility in the United States

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Abstract: Solon's (1992) landmark study estimated the intergenerational elasticity (IGE) in income between fathers and sons to be 0.4 or higher. This dramatically changed the consensus view of the U.S. as a highly mobile society. In this comment, I show both analytically and empirically how Solon and others have actually underestimated this parameter by about 30 percent, suggesting that the IGE is actually close to 0.6 and that the U.S. appears to be among the least mobile countries. There are two key measurement issues that lead researchers to underestimate the IGE. First, the use of short-term averages of fathers' earnings is a poor proxy for lifetime economic status due to highly persistent transitory shocks. Second, the variance of transitory fluctuations to earnings varies considerably by age causing a "lifecycle" bias when samples include measures of fathers' earnings when they are especially young or old. In this comment Solon's results are replicated and then re-estimated using a new technique that is able to address these issues using the same PSID sample. The results confirm that the intergenerational elasticity is likely to be around 0.6.

In a highly influential study in the *American Economic Review*, Gary Solon presents compelling evidence that the U.S. exhibits substantially less income mobility than had been previously thought (Solon 1992). Before Solon's article, researchers typically estimated the intergenerational correlation in income between fathers and sons in the U.S. to be 0.2 or less. These studies appeared to confirm the widely held view that the U.S. is an exceptionally mobile society and prompted Gary Becker to conclude that "…low earnings as well as high earnings are not strongly transmitted from fathers to sons…".[1]

Solon demonstrates how these previous estimates were sharply biased downwards by using only a single year of income as a proxy for permanent economic status, and by using non-representative samples. Solon then constructs an intergenerational sample containing as many as five years of income for fathers using the Panel Study of Income Dynamics (PSID) and estimates the intergenerational elasticity (IGE) in income to be "at least 0.4 and possibly higher". In a separate analysis published in the same issue, David Zimmerman finds a similar result using panel data from the National Longitudinal Surveys (NLS).[2]

As a result of their careful analyses of the measurement issues and the use of superior data, these studies led to a rethinking of the degree of intergenerational mobility in the U.S. In particular, it called into question the ideal of America as a highly mobile society. For example, Solon shows that an IGE of 0.4 implies that a son whose father is at the fifth percentile, has only a 0.17 chance of rising above the median.[3] On the other hand, an IGE of 0.4 also implies that on average, 60 percent of earnings differences between two families are eliminated in a generation. So observers might still disagree as to whether we should view the glass as "half-full" or "half-empty".

In this paper, I argue that despite the dramatic improvement over previous work, Solon still underestimates the IGE in the U.S. by about 30 percent or more suggesting that the true value of the parameter is about 0.6. As a point of comparison, recent studies using a similar methodology to Solon

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[1] See Becker (1988).
[2] For the sake of brevity I confine my comment to Solon's work.
[3] See p.404. This examples assumes that long-run economic status in normally distributed in each generation.

have estimated the IGE to be only about 0.2 in Canada and Finland and 0.3 in Germany.[4] Clearly, an IGE

of 0.6 suggests that the U.S. may be exceptional for its relative *lack of mobility*.

Still, how much difference does it make if the IGE is 0.4 or 0.6? To illustrate the implications in

practical terms, consider a family whose earnings are half the mean. If the true IGE is 0.6, then it would

require, *on average*, 5 generations instead of just 3, before the family substantially closed the gap with the

mean.[5] Obviously a difference of 2 generations, or about 50 years, is quite significant and strongly

suggests that the glass is more than half-empty.

If the IGE represents a *causal* relationship, then it also has powerful implications on the

intergenerational impact of government policies.[6] For example, Chay (1995) estimates that the Civil

Rights Act of 1964 reduced the earnings gap between blacks and whites born in the 1920s by about 30

percent.[7] An IGE of 0.6 suggests that the black-white gap for *children* of these families might have been

reduced by as much as 18 percent simply due to the elimination of racially based employment

discrimination for the *previous* generation.[8]

There are two reasons why Solon's study leads to estimates that are too low, and both reasons

reflect problems inherent in using the PSID or the NLS for this type of analysis. First, owing to small

samples and high rates of attrition in panel data, Solon and other researchers are forced to measure

fathers' permanent income using just a few years of data. Solon's sample is reduced to just 290 father-

son pairs when he averages five-years of income for fathers to obtain his highest estimate of 0.41.[9]

However, studies of earnings dynamics suggest that the transitory component to earnings is highly

persistent so that even a five-year average might still provide a rather poor measure of "permanent" or

lifetime economic status. Second, several studies have also shown that there may be substantial

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[4] Even correcting these studies for possible downward bias would not lead to estimates anywhere close to 0.6. See Solon (2002) for a review of international studies. Only studies of the U.K and South Africa have produced estimates greater than 0.4.

[5] In this example, "substantially closing the gap" is defined as reducing the difference from the mean to less than 5 percent.

[6] Whether part of the IGE is *causal* is an open question. See Solon (1999) for a discussion.

[7] Chay finds that the gap in earnings between blacks and whites declined by between 0.25 and 0.35 log points, see Chay (1995) page 94.

[8] This example assumes that the IGE is causal and that it is the same for both blacks and whites.

differences in the variance of transitory fluctuations in earnings by age causing a "lifecycle bias". In particular, the income of fathers who are especially young or especially old, even if averaged over several years, is not likely to produce an accurate proxy for lifetime economic status.

Solon was certainly aware that transitory shocks might be highly correlated but given the state of the literature on earnings dynamics at the time, he preferred not to make any strong assumptions and instead provided bounds for the results. As Solon put it "If the process governing earnings dynamics were known, that knowledge could be exploited to achieve consistent estimation of the intergenerational correlation in long-run earnings…because considerable uncertainty still clouds the current understanding of earnings dynamics and because the data set used in the present study could not possibly resolve the issues, the present study settles for inconsistent estimators and discussing the likely direction of the inconsistency."[10] Recent studies on earnings dynamics using much richer models and significantly better data (e.g. Baker and Solon, 2003; Mazumder 2001a), have made great strides in resolving some of the issues that were unsettled at the time of Solon's study. Given these methodological advances it makes sense to reexamine previous studies on intergenerational mobility.

In this comment I first explore analytically how incorporating serially correlated transitory shocks into Solon's measurement framework affects the analysis. Using a simple model of earnings and incorporating parameter estimates from previous studies on earnings dynamics, I run simulations on the expected bias from using time averages of various lengths as a proxy for lifetime earnings. The results suggest that Solon's estimate of 0.4 based on a five-year average is biased down by just under 30 percent due solely to the persistence of transitory shocks.

I also replicate the results from Solon's article and then re-estimate the IGE on the same sample using a new econometric method, the Heteroskedastic Errors in Variables (HEIV) estimator developed by Sullivan (2001). With this procedure I am able to take into account measurement problems due to both persistent transitory fluctuations to earnings *and lifecycle bias* using data. The HEIV estimator is a two

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[9] Zimmerman's sample is 192 when he uses four-year averages in the NLS.
[10] Solon (1992) footnote 16.

step process. First estimates of the reliability ratio of each data point are needed. I do this by estimating a

highly structured earnings dynamics model using a different dataset containing lifetime earnings histories

drawn from social security earnings records.[11] The parameter estimates from this model are then used to

construct reliability ratios for each observation in Solon's *PSID sample*. In the second step, the HEIV

estimator directly incorporates these estimated reliability ratios to produce an unbiased estimate of the

IGE. The HEIV estimate is actually larger than 0.6.

These results are also consistent with the empirical findings in Mazumder (2001b) which uses a

much larger intergenerational sample containing the social security earnings records of fathers and their

children. In that study the IGE between fathers and sons is estimated to be around 0.4 when using just

four-year averages of fathers' earnings, tracking the findings from earlier studies. However, the estimates

rise to more than 0.6, when 16-year averages of father's earnings are used.

I.   Measurement Issues[12]

The following statistical framework is a useful starting point for the analysis in this section.

$$(1) \qquad y_{0is} = y_{0i} + w_{0it} + v_{0it}$$

$$(2) \qquad y_{1it} = y_{1i} + w_{1is} + v_{1is}$$

$$(3) \qquad w_{0it} = \delta w_{0it-1} + \xi_{it}$$

$$(4) \qquad y_{1i} = \rho y_{0i} + \varepsilon$$

In this setup, y_{0it} represents the father's log earnings in year t, while y_{1is} is the earnings of his son in year

s.[13] Equation 1 breaks down the father's earnings in a particular year into three components: y_{0i}, a

permanent component that reflects the true long-term earnings capacity; w_{0it}, a component that captures

any transitory shocks to earnings; and finally, v_{0it}, a term that captures any errors due simply to

mismeasurement such as an inaccurate report of earnings. Equation 3 models the transitory shock to

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[11] This model and estimation procedure follows Baker and Solon (2003) which used Canadian data.
[12] This section draws from Mazumder (2001b)
[13] For simplicity, earnings are assumed to be measured as deviations from the sample mean and are adjusted for age and age squared.

father's earnings as following a first order autoregressive process where δ represents the autoregressive parameter.[14] Of course, (4) is the key equation relating father's *permanent* earnings to son's *permanent* earnings where ρ represents the intergenerational elasticity (IGE).

The key point of departure from the analysis in Solon (1992) is that I emphasize the persistence of the transitory component, w_{0it}.[15] This is consistent with the long literature on earnings dynamics, which has shown that this persistence is an important phenomenon (e.g. Lillard and Willis, 1978). Of course, in practice researchers do not actually observe permanent earnings, y_{0i}, and so they are unable to estimate (4) but instead, use a T year average of father's earnings as a proxy for y_{0i}. In this case, the estimate of ρ, $\hat{\rho}$, will be biased towards zero by an attenuation factor, λ_T .which is also referred to as the "reliability ratio".

$$(5) \qquad \text{plim } \hat{\rho} \ = \rho\lambda_T,$$

where:

$$\lambda_T = \frac{\sigma^2_{y0}}{\sigma^2_{y0} + \dfrac{1}{T}\alpha\sigma^2_{w0} + \dfrac{1}{T}\sigma^2_{v0}}$$

$$\text{and, } \alpha = 1 + 2\delta\left\{\frac{T - \left[\dfrac{\left(1-\delta^T\right)}{\left(1-\delta\right)}\right]}{T\left(1-\delta\right)}\right\}$$

Intuitively, the reliability ratio gives an estimate of how much "signal" is provided relative to the total variance (signal plus noise). As equation 5 shows, the formula for the reliability ratio is a fairly complicated function of the autocorrelation parameter, δ. In the absence of serial correlation in transitory fluctuations (i.e. $\delta = 0$), the coefficient $\alpha = 1$, and it is clear that averaging lowers the noise relative to the

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[14] The analogous equation for the transitory shocks to son's earnings is not shown. In the regression context, mismeasurement of the dependent variable will not result in bias unless it is correlated with the right hand side variable.

[15] As mentioned earlier, Solon does consider serially correlated errors (see footnote 17 of Solon, 1992), but does not explicitly pursue its implications on the measurement of the IGE when using short-term averages.

signal as Solon argued. With serial correlation, however, the α term creates an offsetting factor and it is less clear whether a short-term average (e.g. $T = 5$), will average away very much of the noise. Indeed, the larger δ is, holding the other parameters constant, the larger the overall attenuation bias will be.[16]

Of course if δ is very large then transitory shocks start to become an important part of one's lifetime earnings stream and ought to be considered part of the "signal". This problem can be addressed by assuming that what is really of interest for the analysis is the relationship between sons' earnings and fathers' *lifetime* earnings. A straightforward and realistic way to define fathers' lifetime earnings is simply the average earnings over the fathers' working years from say age 21 to 65. So now, rather than (4), the equation of interest is:

$$(6) \qquad y_{1i} = \rho \bar{y}_{45,i} + \varepsilon$$

The formula for the reliability ratio for a T-year average using this adjustment is now slightly more complicated than (5) but not too difficult to calculate.[17] It turns out that making this adjustment has only a modest effect on the calculations that follow.

Table 1 presents the results of the simulation using different possible values for the parameters on the reliability ratio for lifetime earnings. A careful reading of some recent studies that decompose the variance of a single year of earnings into permanent versus transitory factors find that only about half the

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[16] Note that σ^2_{w0} and δ are related by $\sigma^2_w = \dfrac{\sigma^2_\xi}{1-\delta^2}$. In the simulations that follow, assumptions are made regarding σ^2_{w0} and δ, while σ^2_ξ adjusts to satisfy this relationship.

[17] The formula for the reliability ratio is now $\lambda^*_{T,s} = \dfrac{\sigma^2_{y0} + \dfrac{1}{T}\displaystyle\sum_{s}^{s+T-1}\dfrac{1}{45}\sum_{t=1}^{45}\delta^{|t-s|}\sigma^2_{w0}}{\sigma^2_{y0} + \dfrac{1}{T}\alpha\sigma^2_{w0} + \dfrac{1}{T}\sigma^2_{v0}}$, where s is the starting age for

the short-term average and α is defined as in (5). The numerator of the reliability ratio now includes the covariance between the transitory shocks in the short-term average and the transitory shocks in the lifetime average. The autoregressive structure of the transitory component implies that the covariance between the transitory components will depend on the distance in time between the years used for the short-term average and the full 45 year average. Dividing the numerator and denominator by the variance in single year earnings, σ^2_{yt} and then using estimates for δ, $\sigma^2_{y0}/\sigma^2_{yt}$, $\sigma^2_{w0}/\sigma^2_{yt}$, and $\sigma^2_{v0,}/\sigma^2_{yt}$, enables one to calculate $\lambda^*_{T,s}$ for all possible values of T and s. In order to get a summary measure of the degree of attenuation bias that is only a function of T, the $\lambda^*_{T,s}$ are averaged over all possible s for a given value of T.

variance is due to the permanent component and that δ is about 0.8.[18] Using the preferred set of

assumptions, the estimate for the reliability ratio when using a five-year average of fathers' earnings is

0.73.[19] This suggests that estimates of the IGE of 0.4 based on five-year averages will be biased down by

about 27 percent. The full path of the reliability ratio as averages are taken over progressively more years

is presented graphically in Figure 1. Here it is clear that an average of something on the order of 15 to 20

years is needed to obtain a reliability ratio close to 0.9.

A possible criticism of this exercise is that the earnings process might be considerably more

complicated than what is considered here. For example, there has been some debate in the literature on

earnings dynamics as to whether there is a significant random walk component to earnings or substantial

heterogeneity in the age-earnings profile of individuals.[20] One might wonder whether the inclusion of

these *permanent* factors in a more structured earnings dynamics model would change the decomposition

of the earnings variance between the permanent and transitory components, or lower the estimates of δ.

Until recently, the data and methodology used has been insufficient to resolve these issues.

However, a recent study by Baker and Solon (2003) using the tax records of an extraordinarily

large sample of over 32,000 *Canadian* men estimates a highly structured, cohort-based, earnings

dynamics model that is able to distinguish all of these effects. Mazumder (2001a), using the identical

model and a similarly large data set on over 23,000 U.S. men, has shown that even this more complete

model still implies that short-term averages of earnings are poor proxies for lifetime economic status.

The reliability ratio for a five-year average of earnings for a typical cohort is estimated to be only 0.68.[21]

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[18] These include Card (1994), Hyslop (2001) and Mazumder (2001a). They also find that the transitory share is about 0.3 and the measurement error share is about 0.2. Solon et al.(1991) find the permanent share to be about 0.55 (Table 3) and argue that the upper bound is 0.7. Their analysis, however, strongly suggests that the estimates of 0.7 from studies in the 1970s and 1980s did not adequately adjust for age.

[19] If the analysis focused on permanent earnings rather than lifetime earnings, the estimate would have been only slightly lower at 0.67.

[20] See Baker and Solon (2003).

[21] This is the implied reliability ratio when earnings are averaged over 5 years for men born in 1943/44 (aged 39 to 40 in 1983). This is the average of column 5 in the top panel of Table 5 of Mazumder (2001a). The results are robust across cohorts.

Several studies have also argued that estimates of the IGE may be sensitive to "lifecycle biases".[22] A stylized fact from studies of earnings dynamics is that the variance of the transitory component follows a pronounced "U-shaped" pattern over the lifecycle.[23] To illustrate this, the estimate of the age profile to the transitory innovation in earnings taken from Mazumder (2001a) is shown in Figure 2. This implies that short-term averages of earnings taken at a time when earnings are considerably noisy may lead to even greater bias in estimates.[24] In the statistical framework shown earlier, σ^2_{w0} would be a function of age rather than constant for all fathers. Obviously the reliability ratio now will not be the same for the whole sample but will differ for each father-son pair. As Figure 2 clearly demonstrates, measures of earnings taken around age 40 are much less noisy than those taken at age 26 and significantly less noisy than those taken when fathers' are in their mid-50s.

In Solon (1992) the mean age of fathers in 1967 is 42.[25] However, this masks a large degree of variation in the sample as the range includes those from age 27 to 68. Therefore, it is likely that many of Solon's observations contain considerably more measurement error than what is implied by just looking at the average age of the sample. This suggests that the 0.73 reliability ratio that accounts for serially correlated transitory shocks actually *understates* the degree to which Solon's estimate might be biased downwards. In any case, an efficient estimation procedure should not just use a single correction factor, but instead take into account the fact that there is heteroskedastic measurement error in averages of fathers' earnings due to lifecycle bias.

II. Re-estimation using the HEIV estimator

I now empirically attempt to quantify the effects of *both* measurement problems on estimates of the IGE using a new econometric method. The basic idea is that rather than simply applying a *single*

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[22] See Jensen (1987) and Grawe (2000).
[23] See Baker and Solon (2003) and Mazumder (2001a).
[24] While most recent studies including Solon's, adjust for the age profile of fathers and sons when running the regression of sons' earnings of fathers' earnings, this does not address the problem. Including age and age squared controls for the lifecycle effects on the *level* of earnings but does not address heteroskedastic measurement error.
[25] See Table 1 of Solon (1992). 1967 is the first of the five years that are averaged.

correction factor for the entire sample an attempt is made to directly address the fact that each observation has a different degree of measurement error due to the differences in the fathers' age range. A very large dataset containing the social security earnings histories of over 20,000 men is used to infer the reliability ratio for each of the fathers in Solon's PSID sample. This information is then incorporated in the most efficient manner possible to derive a new estimate of the IGE. As part of the analysis, Solon's results are almost exactly replicated.

The HEIV estimator

In many economic studies it is known that a right hand side variable is measured with error. If the reliability ratio can be determined, then a simple solution is to divide the regression coefficient by the reliability ratio to scale up the estimate. This "errors in variables" or EIV estimator is commonly estimated by statistical packages such as STATA. However, in many situations there is reason to think that the reliability ratios might *vary* within the sample. A common situation where this occurs is when using individual-level data and an explanatory variable is an average of some characteristic of the population taken at a particular geographic level (e.g. state or county). In this case, the sampling variance for the right hand side variable and the degree of measurement error will vary across individuals depending on the size of the sample for each geographic area.

One approach to address this problem is to average the reliability ratios across the observations, use this as an estimate for an overall reliability ratio and then implement the EIV estimator. While such an approach is consistent, Sullivan (2001) shows that the most efficient estimator will utilize the reliability ratio for each observation. He presents an alternative estimator, the Heteroskedastic Errors-In-Variables (HEIV) estimator that does exactly this. The HEIV estimator is the OLS regression of the dependent variable on the best linear predictor of the right hand side variable. In a case of a single mismeasured variable the best linear predictor is simply the regression of the dependent variable on the right hand side variable multiplied by the observation-specific reliability ratio. The HEIV estimator is

shown to produce significant efficiency gains in Monte Carlo simulations where reliabilities differ across observations.

Earnings Dynamics Model

Of course one cannot implement the HEIV estimator without first calculating the observation-specific reliability ratios through some preliminary procedure. Therefore, I estimate an earnings dynamics model using a different dataset containing the social security earnings histories of over 23,000 men and use the parameter estimates as inputs for implementing the HEIV estimator using Solon's PSID sample.[26] In particular, the analysis will produce reliability ratios for using a five-year average of earnings beginning at age s, as a proxy for permanent earnings. The final output will be a set of reliability ratios that will differ only by age. The model and estimation strategy is based on Baker and Solon (2003) and follows the recent literature on earnings dynamics (e.g. Abowd and Card, 1989; Baker, 1997; Haider, 2001) by exploiting panel data on the autocovariance structure of earnings. I use a minimum distance estimator to identify the parameters of the model by comparing the empirical moments of the data with the moments implied by the model.

What is unique about this analysis is that a highly structured model is estimated on U.S. data that includes a random walk component, heterogeneity in age-earnings profiles, an age-based component to the transitory variance and a serially correlated transitory component. These parameters can be separately identified by virtue of the large sample combined with the cohort-based approach. Incorporating *all* of these components addresses the criticism that the simulations used in the last section were too simplistic.

The data used to estimate the model pools men from the 1984, 1990 and 1996 Surveys of Income and Program Participation (SIPP) who are matched to their social security earnings records (SER) from 1951 to 1998.[27] The time period used for the analysis is 1983 to 1997.[28] The sample includes the 18 two-

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[26] The model and data used here is identical to Mazumder(2001a). The key difference is that the output from the model is used to construct reliability ratios which can then be applied to Solon's PSID sample.

[27] This data is not publicly available and requires a special arrangement with the Census Bureau or the Social Security Administration in order to access the data.

year birth cohorts starting with 1931/32 and ending with 1965/66. The sample is restricted to men who have positive earnings in each year that they are between the ages of 25 and 59 during the sample period.[29] This implies that for 10 of the cohorts earnings information is used for all 15 years, resulting in an autocovariance matrix of earnings that has 120 distinct moments.[30] For the other cohorts where fewer years are available, smaller autocovariance matrices are calculated. Following Baker and Solon, the set of empirical moments from the autocovariance matrix of each of the cohorts is stacked and used for estimation of the model. There are a total of 1660 moments.

Following the standard practice in this literature, the actual model is estimated on log earnings residuals that are adjusted for year effects and for the effects of the age profile on the *level* of earnings. Begin by letting Y_{ibt} represent the log earnings of individual i, in birth cohort b, in year t; μ_{bt} be the cohort-specific mean for that year; and y_{ibt} be the individual-specific deviation from that mean.

$$(7) \qquad Y_{ibt} = \mu_{bt} + y_{ibt}$$

Then y_{ibt} can simply be calculated by subtracting the sample average log earnings for cohort b in year t from the observed earnings, Y_{ibt}. The deviations from the mean are then modeled as follows:

$$(8) \qquad y_{ibt} = p_t[\alpha_{ib} + \beta_{ib}(t-b-26) + u_{ibt}] + \varepsilon_{ibt}$$

where

$$(9) \qquad u_{ibt} = u_{ibt-1} + r_{ibt}$$

$$(10) \qquad \varepsilon_{ibt} = \rho\varepsilon_{ibt-1} + \lambda_t v_{ibt}$$

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[28] Ideally one would not want to use estimates based on data from the 1980s and 1990s to derive reliability ratios for fathers' earnings in the PSID for 1967 to 1971. In practice this is probably not a problem. First, Gottschalk and Moffitt (1994) have shown that the permanent and transitory components of earnings appear to have risen at the same rate from the 1970s to the 1980s. This suggests that the reliability ratio, which is roughly equivalent to the permanent component over the permanent plus transitory component, should not have changed much over this time period. Second, the results of a simple earnings dynamics model using the SIPP-SER data in Mazumder (2001a) yield almost equivalent parameter estimates to those using the PSID (e.g. Card 1994, Hyslop 2001), even though they are from different datasets and cover different time periods.

[29] The restriction of positive years of earnings is standard in studies of earnings dynamics. It is particularly important, however, when using social security earnings data because years of zero earnings might reflect non-coverage rather than unemployment (see Mazumder 2001a).

[30] The autocovariance matrix of earnings calculates the covariance in earnings between all possible pairs of years. This results in a $T \times T$ symmetric matrix with $(T \times (T+1))/2$ distinct elements. Each of these elements is viewed as a sample "average" and is therefore considered a "moment" of the data.

and

$$(11) \qquad \text{var}(v_{ibt}) = \gamma_0 + \gamma_1 (t\text{-}b\text{-}26) + \gamma_2 (t\text{-}b\text{-}26)^2 + \gamma_3(t\text{-}b\text{-}26)^3 + \gamma_4 (t\text{-}b\text{-}26)^4$$

Deviations in log earnings have both a permanent and transitory component. The expression in brackets shown in the right hand side of (8) breaks down the permanent component into three parts. α_{ib}, with variance σ^2_α, is a fixed effect that varies across individuals. The β_{ib} term, with variance σ^2_β, represents heterogeneity in the growth rate of earnings over time and captures the deviation of the individual's idiosyncratic growth rate from his cohort's, after age 26. The u_{ibt} term is a random walk component as shown in (9). Here r_{ibt} is "white noise" with variance σ^2_r.

The transitory component, ε_{ibt}, is modeled in (10) as following a first order autoregressive process. The terms p_t and λ_t are "factor loading" terms on the permanent and transitory components to capture any year-specific changes in the importance of these two components.

The key part of the model that enables the HEIV estimator to address the problem of lifecycle bias is shown in (11). Here, the innovations in the transitory component follows a quartic in experience since age 26, to capture life-cycle effects as shown.

To provide an example of how this decomposition relates to the autocovariance matrix, the model implies that the variance of log earnings for the 1949/50 cohort in 1990 is:

$$(12) \qquad \text{var}(y_{49/50, \, 1990}) = p^2_{1990} (\sigma^2_\alpha + 15^2\sigma^2_\beta + 30\sigma^2_{\alpha\beta} + 15\sigma^2_r) + \rho^2\text{var}(\varepsilon_{i,49/50,1990}) + \lambda^2_{1990}(\gamma_0 + 15\gamma_1$$
$$+ 15^2\gamma_2 + 15^3\gamma_3 + 15^4\gamma_4)$$

The multiples of 15 in the expression arise from the fact that the cohort born in 1949 is 41 years old in 1990, and therefore has 15 years of experience since age 26.[31] The implied parametric structure for each of the 1660 moments is also assembled. I use Equally Weighted Minimum Distance (EWMD) to estimate the model. EWMD chooses the parameters that minimize a distance function between the set of moment

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[31] Following Baker and Solon, the initial transitory variance for each cohort is separately estimated since it makes no sense to impose a common initial variance when the model is designed to differentiate life-cycle effects.

conditions implied by the model with the empirical moments calculated with the actual data.[32] The key

results from this estimation are shown in Table 2. Despite the inclusion of the random-walk term and the

heterogeneous growth parameter, the serial correlation in transitory shocks is still shown to be quite high

at 0.6. [33]

Calculating Reliability Ratios

I begin by assuming that the equation used to estimate the IGE is (4) which relates sons' earnings

to fathers' *permanent* earnings, y_0.[34] I also assume that the fathers' earnings in a given year, y_{0it} can be

decomposed as:

$$(13) \qquad\qquad y_{0it} = a_{it} + \varepsilon_{it}$$

where a_{it} represents the permanent component in year t and ε_{it} denotes the transitory component.[35]

Essentially, (13) can be thought of as simply summarizing the more complicated expression shown in (8).

Now I show how the results of the earnings dynamics model are used to calculate observation-specific

reliability ratios for the PSID sample. For the *sample used to estimate the earnings dynamics model*, the

reliability ratio, $r_{n,sb}$, that arises from using an n year average of earnings, $\bar{y}_{n,sb}$, starting at age s for cohort

b, as a proxy for permanent earnings, y_0, is the following:

$$(14) \qquad\qquad r_{n,sb} = \frac{\mathrm{cov}(\bar{y}_{n,sb}, y_{0i})}{\mathrm{var}(\bar{y}_{n,sb})}$$

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[32] See the appendix of Abowd and Card (1989) for a description of the technique. Evidence from Altonji and Segal (1996) and Clark (1996) suggest that using the theoretically derived optimal weighting matrix can produce serious bias in finite samples. Recent studies therefore, have used the identity matrix as the weighting matrix.

[33] A discussion of the parameter results may be found in Mazumder (2001a).

[34] For simplicity, the analysis here does not calculate reliability ratios for *lifetime* earnings as defined in the last section. The simulation results suggest that this has only a minor effect in understating the reliability ratios.

[35] Note that now the permanent component of earnings, a_t varies with *time*. In order to utilize (4) we must define how fathers' permanent earnings y_0 is related to a_t. To do this, the following formula is used: $y_0 \equiv \frac{1}{\sqrt{T}} \sum_T a_t$. This formula implies that $\sigma_{y0}^2 = \frac{1}{T} \sum_T \sigma_{a_t}^2$, which is convenient for calculating the reliability ratios since the variance of permanent earnings is simply the time average of the variances of the permanent component over the sample period.

To calculate (14) requires estimates of all of the various components from (8) in order to produce the relevant covariances and variances.[36] In order to construct reliability ratios that can be applied to the PSID sample when using five-year time averages of fathers earnings for each starting at age s, n is set equal to 5 and the reliability ratios for five-year averages are averaged across all the relevant cohorts:

$$(15) \qquad r_{5,s} = \sum_b r_{5,sb}$$

The results of this exercise are shown in Table 3. The variability in the reliability ratios is not huge but is large enough to demonstrate the importance of the lifecycle bias. A five year average using fathers' earnings when they are between 26 and 30 produces bias of 50 percent while a time average taken between the ages of 43 and 47 produces bias of about 30 percent –or about what was found when using the simple simulations.

Replicating Solon (1992)

In order to see how the HEIV might alter the results in Solon (1992), the analysis here attempts to construct the identical PSID sample using Solon's exclusion rules. This is not a simple exercise given the complexities of the PSID. A variety of issues arise in trying to create an unbiased intergenerational sample and Solon's detailed analysis of these selection issues is an important contribution to the literature.

Solon's sample size when using five-year averages for father's income, is 290. In the replication exercise undertaken here, a total of 287 pairs are identified. Initially, I start by using the same specification as Solon:

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[36] The details of these calculations are available from the author upon request. The formula varies for each cohort since they may be in the sample for anywhere between 7 and 15 years. As an example, for the cohort born in 1939-40, the reliability ratio for a five-year average starting at age 44 (years 1983 to 1987), the reliability ratio is the following:

$$r_{5,39\text{-}40,44} = \frac{\dfrac{1}{5}\sum_{n=83}^{87}\dfrac{1}{15}\sum_{t=83}^{97}\mathrm{cov}(y_{0in}, y_{0it})}{\dfrac{1}{25}\left(\sum_{n=83}^{87}\mathrm{var}(y_{0in}) + \sum_{n=83}^{87}\mathrm{var}(\varepsilon_{in}) + 2\sum_{n=83}^{87}\sum_{t=n+1}^{87}\mathrm{cov}(y_{0in}, y_{0it}) + 2\sum_{n=83}^{87}\sum_{t=n+1}^{87}\mathrm{cov}(\varepsilon_{in}, \varepsilon_{it})\right)},$$

$$(16) \qquad y_{1i} = \alpha + \rho\, \bar{y}_{0,5i} + \beta_1 Age_{0i} + \beta_2 Age^2_{0i} + \beta_3 Age_{1i} + \beta_4 Age^2_{1i} + \varepsilon_i$$

where $\bar{y}_{0,5i}$, is a five-year average of fathers' earnings and a quadratic in age is included for both the fathers and the sons. The IGE is estimated to be 0.415 compared to Solon's reported result of 0.413 (see rows 1 and 2 of Table 4). Since the earnings dynamics model is used to calculate the observation specific reliability ratios only for individuals between the ages of 26 and 58, the sample of fathers in the PSID is also restricted to this age range. This has the effect of removing 6 fathers who are over the age of 58 and increases the estimate to 0.426 (see row 3).[37] In order to implement the HEIV, it is convenient to use a bivariate regression, so father and son's log earnings are first regressed on age and age squared, and the residuals are used to estimate the IGE. The effect is to lower the estimate to 0.413 as shown in row 4.

Following Sullivan (2001), the HEIV estimator uses OLS to estimate

$$(17) \qquad y^*_{1i} = \alpha + \rho(r_{5,s} * \bar{y}_{0,5i}) + \varepsilon_i$$

where the y^* signifies that the earnings data have been purged of lifecycle effects. Note that the simple time average $\bar{y}^*_{0,5i}$ is replaced with the age-specific reliability ratio multiplied by the time average, $r_{5,s}\bar{y}^*_{0,5i}$. As row 5 of Table 4 shows, using the HEIV procedure sharply increases the estimate of the IGE to 0.620. When viewed in conjunction with the analytical simulations in section 2, these empirical results provide further evidence that the IGE is closer to 0.6 than to 0.4. Mazumder (2001b) also estimates the IGE in earnings to be greater than 0.6 using a large intergenerational sample with the lifetime social security earnings records of fathers and sons. The fact that these three different approaches all reach a similar conclusion provides further evidence that previous studies underestimated the IGE.

It is worth noting that the application of the HEIV estimator also holds promise for future research on intergenerational mobility using small intergenerational samples from the PSID and NLS to address measurement error bias. This technique might also enable cross-sample comparisons where researchers have used different lengths of time averages or different age ranges for the fathers. More

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where n indexes the years over which the average is taken while t indexes the full 15 year sample period.

[37] The youngest age of a father in the PSID sample is 26 so no fathers are removed by the lower boundary.

generally, this approach should be considered for any empirical research where permanent income is considered a key explanatory variable. For example, studies of the effects of family income on college attainment (e.g. Cameron and Heckman, 2001) or children's health outcomes (e.g. Case et al, 2002) typically do not consider measurement error issues related to the age at which parents' income is measured.

III.   Conclusion

Solon's (1992) landmark study on intergenerational mobility presents powerful evidence that the U.S. is not nearly as mobile as previous researchers thought. Using better data and methodology, Solon estimates the intergenerational elasticity (IGE) in earnings to be about 0.4 or higher. Solon argued 0.4 was a lower bound and did not attempt to incorporate the time series properties of earnings into the measurement framework. However, recent studies on earnings dynamics that use larger and richer datasets provide strong evidence that proxies for permanent economic status based on short-term averages of earnings lead to substantial attenuation bias. Several studies have also shown that there is a lifecycle bias in studies of intergenerational mobility that also lead to underestimates of the IGE.

This comment presents an analytical framework that demonstrates that the problem of persistent transitory fluctuations alone is likely to lead researchers to underestimate the IGE by about 30 percent when using a five-year average of fathers' earnings as a proxy for lifetime earnings.

This comment also replicates Solon's analysis and applies a new econometric estimator that addresses the problem of transitory fluctuations and lifecycle bias. The results of the replication are nearly identical to what Solon found. Using a different dataset containing the social security earnings histories of a very large sample of men I estimate an earnings dynamics model and construct reliability ratios for the fathers in Solon's PSID sample. These reliability ratios are used to implement the HEIV estimator. The estimate of the IGE is 0.62 and is consistent with the results from the analytical exercise. Finally, this finding is bolstered by the results in Mazumder (2001b), which uses a large intergenerational sample containing the lifetime earnings histories of fathers and sons derived from social security earnings histories to estimate the IGE. The study finds that four-year averages of fathers' earnings produce results

similar to Solon's but that the use of a 16-year average of fathers' earnings results in an estimate of the IGE greater than 0.6.

The implications of this revised view of intergenerational mobility are quite substantial. If 60 percent of earnings differences in society persists across generations, then it will require many more decades before historical inequities in American society are likely to be alleviated. Such a high degree of persistence also suggests that the recent rise in cross-sectional inequality is likely to remain a feature of the U.S. economy for some time.

In the final analysis, estimates of intergenerational mobility, are most useful as a descriptive statistic, they tell us something about the nature of inequality in the U.S. So far, the literature has not pointed to any particular policy recommendations. Given the rising evidence from studies in other countries it appears that the U.S. may be among the most immobile countries.[38] This comparative view suggests that there might be some important institutional features about the U.S. that create such a high level of persistence of income. Simply measuring this descriptive parameter is only the first step in understanding the economics of intergenerational mobility. The important and difficult task of understanding the underlying mechanisms by which earnings capacity is transmitted from parents to children remains a key area for future research.

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[38] See Solon (2002) for a review of international studies.

References

Abowd, John M and David Card (1989), "On the Covariance Structure of Earnings and Hours Changes" *Econometrica*, 57(2) 411-445

Altonji, Joseph G. and Lewis M. Segal, "Small Sample Bias in GMM Estimation of Covariance Structures," *Journal of Business and Economic Statistics*, 14() 353-366.

Baker, Michael and Gary Solon (2003), "Earnings Dynamics and Inequality Among Canadian Men, 1976-1992: Evidence from Longitudinal Tax Records" *Journal of Labor Economics,* 21(2): 289-322.

Baker, Michael (1997), "Growth-Rate Heterogeneity and the Covariance Structure of Life Cycle Earnings", *Journal of Labor Economics*, 15(2) 338-375

Becker, Gary S. (1988), "Family Economics and Macro Behavior," *American Economic Review*, 78:1-13

Cameron, Stephen V. and James J. Heckman (2001) "The Dynamics of Educational Attainment for Black, Hispanic and White Males" *Journal of Political Economy* 109(3):455-499.

Card, David (1994), "Intertemporal Labor Supply: An Assessment", in Christopher A. Sims (ed.) *Advances in Econometrics, Sixth World Congress*, Vol. 2, Cambridge University Press. Cambridge.

Case, Anne, Darren Lubotsky and Christina Paxson, (2002), "Economic Status and Health in Childhood: The Origins of the Gradient", *American Economic Review*, 92:1308-1334

Chay, Kenneth Y. (1995), "Evaluating the Impact of the 1964 Civil Rights Act on the Economic Status of Black Men Using Censored Longitudinal Earnings Data", Unpublished. Princeton: Department of Economics, Princeton University

Clark, Todd (1996), "Small-Sample Properties of Estimators of Nonlinear Models of Covariance Structure," *Journal of Business and Economic Statistics*, 14() 367-373.

Gordon, Roger H.(1984) "Differences in Earnings and Ability," Garland, New York.

Gottschalk, Peter and Robert A. Moffitt (1994), "The Growth of Earnings Instability in the U.S. Labor Market," *Brookings Papers on Economic Activity*, 217-272

Grawe, Nathan D. (2000), "Lifecycle Bias in the Estimation of Intergenerational Income Persistence," Manuscript, University of Chicago.

Haider, Steven J. (2001), "Earnings Instability and Earnings Inequality of Males in the United States 1967-1991," *Journal of Labor Economics*, 19:799-836

Hyslop, Dean (2001), "Rising U.S. Earnings Inequality and Family Labor Supply: The Covariance Structure of Intrafamily Earnings," *American Economic Review,* 91:755-777.

Jenkins, Stephen (1987), "Snapshots Versus Movies: 'Lifecycle Biases' and the Estimation of Intergenerational Earnings Inheritance," *European Economic Review* 31:1149-1158.

Lillard, Lee A. and Robert J. Willis (1978), "Dynamic Aspects of Earning Mobility," *Econometrica* 46:985-1012

Mazumder, Bhashkar (2001a), "The Mis-Measurement of Permanent Earnings: New Evidence from Social Security Earnings Data," *Federal Reserve Bank of Chicago Working Paper* 2001-24.

Mazumder, Bhashkar (2001b), "Earnings Mobility in the US: A New Look at Intergenerational Inequality," *Federal Reserve Bank of Chicago Working Paper* 2001-18.

Solon, Gary, Mary Corcoran, Roger Gordon and Deborah Laren (1991), "A Longitudinal Analysis of Sibling Correlations in Economic Status," *Journal of Human Resources*, 26:p509-534

Solon, Gary (1992), "Intergenerational Income Mobility in the United States," *American Economic Review*, 82:393-408

Solon, Gary (1999), "Intergenerational Mobility in the Labor Market," *Handbook of Labor Economics, Volume 3A*, Orley C. Ashenfalter and David Card, eds. Elsevier, Amsterdam, North Holland

Solon, Gary (2002), "Cross Country Differences in Earnings Mobility," *Journal of Economic Perspectives,* 16:59-66

Sullivan, Daniel G. (2001), "A Note on the Estimation of Linear Regression Models with Heteroskedastic Measurement Errors," *Federal Reserve Bank of Chicago Working Paper* 2001-23

Zimmerman, David J. (1992), "Regression Toward Mediocrity in Economic Stature," *American Economic Review*, 82:409-429

Table 1: Simulation Results on Attenuation Bias when Using Multiyear Averages

Attenuation Coefficient if….

Number of Years Averaged	$\sigma^2_{w0}/\sigma^2_{y0t} = 0.2$ $\sigma^2_{v0}/\sigma^2_{y0t} = 0.1$		$\sigma^2_{w0}/\sigma^2_{y0t} = 0.3$ $\sigma^2_{v0}/\sigma^2_{y0t} = 0.2$		$\sigma^2_{w0}/\sigma^2_{y0t} = 0.4$ $\sigma^2_{v0}/\sigma^2_{y0t} = 0.1$	
	$d = 0.5$	$d = 0.8$	$d = 0.5$	$d = 0.8$	$d = 0.5$	$d = 0.8$
1	0.641	0.670	0.519	**0.554**	0.526	0.572
2	0.733	0.735	0.630	**0.637**	0.619	0.629
3	0.783	0.767	0.693	**0.680**	0.677	0.662
4	0.817	0.790	0.737	**0.710**	0.720	0.687
5	0.843	0.808	0.772	**0.734**	0.754	0.709
6	0.863	0.823	0.799	**0.754**	0.782	0.728
7	0.879	0.837	0.821	**0.772**	0.806	0.746
8	0.892	0.849	0.840	**0.788**	0.826	0.762
9	0.904	0.859	0.856	**0.802**	0.843	0.777
10	0.913	0.869	0.869	**0.815**	0.857	0.792
11	0.921	0.878	0.881	**0.827**	0.870	0.805
12	0.928	0.887	0.891	**0.839**	0.882	0.817
13	0.935	0.895	0.900	**0.849**	0.892	0.829
14	0.940	0.902	0.908	**0.859**	0.900	0.840
15	0.945	0.908	0.915	**0.868**	0.908	0.850
16	0.949	0.915	0.922	**0.877**	0.916	0.860
17	0.953	0.921	0.927	**0.885**	0.922	0.869
18	0.957	0.926	0.933	**0.892**	0.928	0.877
19	0.960	0.931	0.937	**0.899**	0.934	0.886
20	0.963	0.936	0.942	**0.906**	0.939	0.893
21	0.966	0.940	0.946	**0.912**	0.943	0.901
22	0.968	0.945	0.950	**0.918**	0.947	0.908
23	0.970	0.948	0.953	**0.924**	0.951	0.914
24	0.973	0.952	0.956	**0.929**	0.955	0.920
25	0.974	0.956	0.959	**0.934**	0.958	0.926
26	0.976	0.959	0.962	**0.939**	0.961	0.932
27	0.978	0.962	0.964	**0.943**	0.964	0.937
28	0.980	0.965	0.967	**0.947**	0.967	0.942
29	0.981	0.968	0.969	**0.952**	0.970	0.947
30	0.982	0.971	0.971	**0.955**	0.972	0.952

Note: Simulation is based on the equation shown in footnote 10. In each pair of columns, assumptions are made about the share of the variance in earnings in one year accounted for by transitory fluctuations and measurement error, Within each pair of columns, the auto correlation coefficient is either 0.5 or 0.8. The assumptions based on parameter estimates from Card (1994), Hyslop (2001) and Mazumder (2001a) are shown in bold.

Table 2: Results of Earnings Dynamics Model

Parameter	Description	Estimate	s.e.
	permanent component		
s^2_a	fixed effect	0.165	(0.006)
$s^2_b *1000$	heterogeneous growth	0.002	(0.000)
s^2_r	random walk innovation	0.001	(0.000)
s^2_{ab}	covariance alpha, beta	0.000	(0.000)
	transitory component		
r	autocorrelation coefficient	0.634	(0.025)
g_0	quartic intercept	0.172	(0.033)
g_1	quartic coeff. on age - 26	-0.008	(0.001)
$g_3 *1000$	quartic coeff. on (age - 26)^2	0.018	(0.005)
$g_4 *10000$	quartic coeff. on (age - 26)^3	0.023	(0.008)
$g_5 *10000$	quartic coeff. on (age - 26)^4	0.002	(0.001)
	# of moments	1660	

Note: See text for a description of the data and methodology.
For full set of results see Mazumder (2001a)

Table 3: Estimated Reliability Ratios

Starting Age s	Reliability Ratio r_s
26	0.50
27	0.53
28	0.54
29	0.57
30	0.58
31	0.60
32	0.59
33	0.62
34	0.63
35	0.65
36	0.65
37	0.67
38	0.67
39	0.69
40	0.68
41	0.70
42	0.69
43	0.71
44	0.69
45	0.70
46	0.69
47	0.69
48	0.68
49	0.67
50	0.62
51	0.62
52	0.60
53	0.59
54	0.56
55	0.56
56	0.55
57	0.59
58	0.65

Note: See text for a description of the data and methodology.

Table 4: HEIV estimates using Solon's (1992) PSID Sample

	elasticity	(Std. Error)	N
Solon (1992)	0.413	(0.093)	290
Replication	0.415	(0.095)	287
Fathers Aged <59	0.426	(0.096)	281
Bivariate regression	0.413	(0.094)	281
HEIV estimator*	**0.620**	**(0.222)**	**281**

Notes: Results from Solon (1992) are from Table 2.
*This calculation uses "White corrected" standard errors

Figure 1: Path of Reliability Ratio
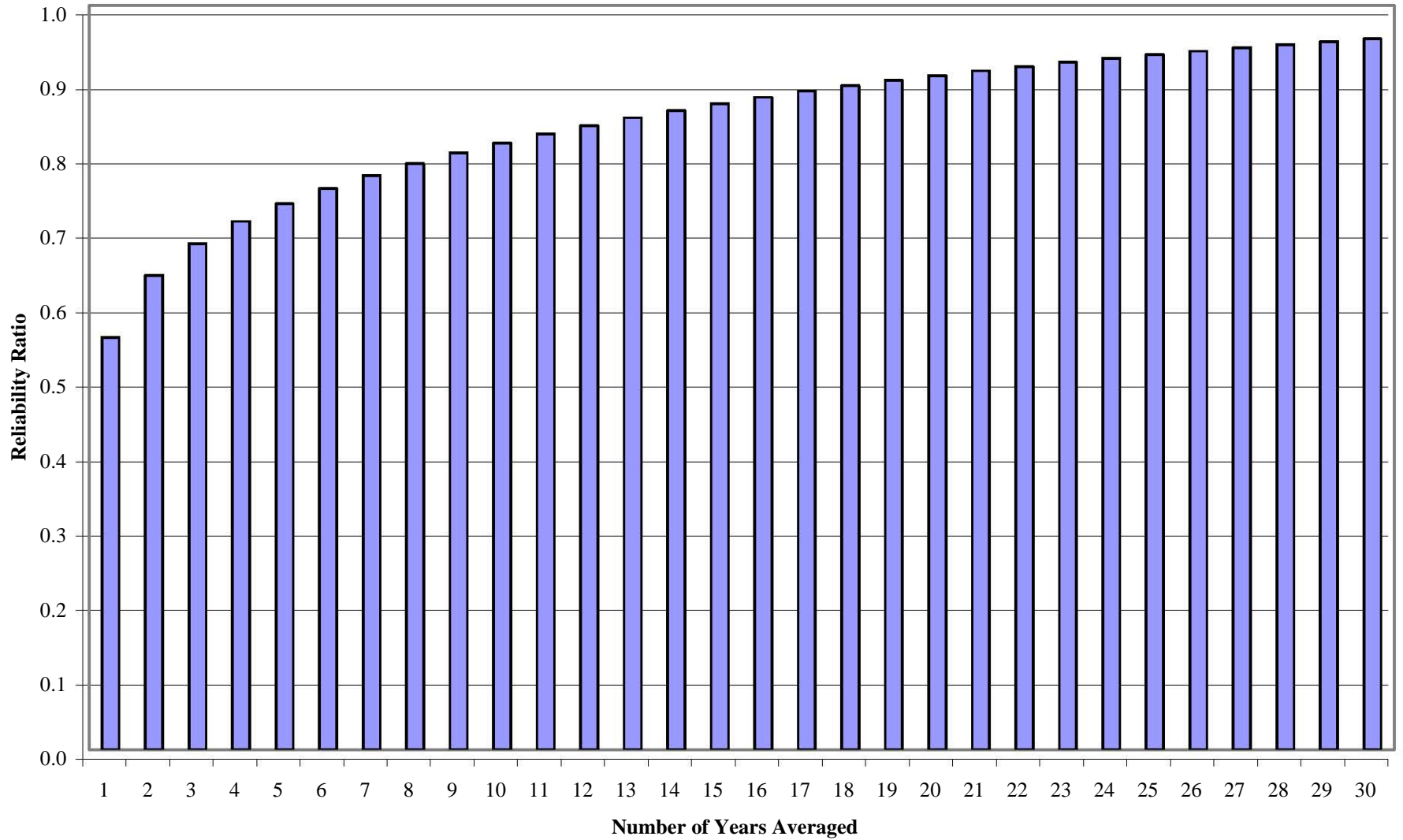
Figure 2: Lifecycle Pattern of Variance of Transitory Innovation
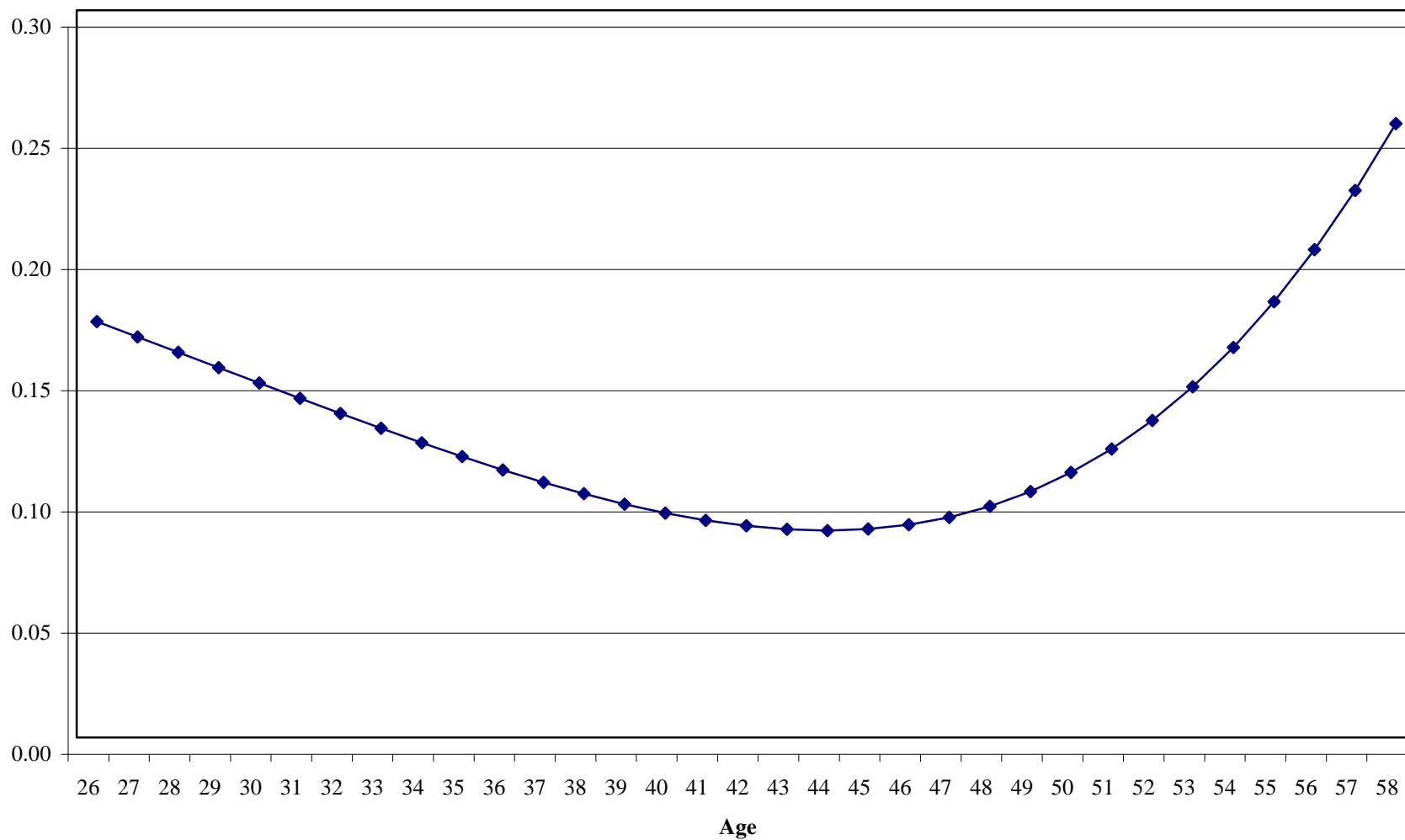
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