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On the Cyclical Behavior of Employment, Unemployment and Labor Force Participation

Marcelo Veracierto

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Abstract: In this paper I evaluate to what extent a real business cycle (RBC) model that incorporates search and home production decisions can simultaneously account for the observed behavior of employment, unemployment and out-of-the-labor-force. This contrasts with the previous RBC literature, which analyzed employment or hours fluctuations either by lumping together unemployment and out-of-the-labor-force into a single non-employment state or by assuming a fixed labor force. Once the three employment states are explicitly introduced I find that the RBC model generates highly counterfactual labor market dynamics.

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1. Introduction

This paper is motivated by a very basic set of facts: that employment varies 60 percent as much as output and is highly procyclical, that unemployment varies 6 times more than output and is countercyclical, and that the labor force varies only 20 percent as much as output and is weakly procyclical. The purpose of this paper is to evaluate to what extent a real business cycle (RBC) model can jointly account for these observations.

Standard RBC models are not designed to address this type of evidence. These models typically lump together unemployment and out-of-the-labor-force into a single nonemployment state and analyze variations in employment and hours worked by either studying a work-leisure decision (e.g. Hansen, 1985, and Prescott, 1986) or a work-home production decision (e.g. Greenwood and Hercowitz, 1991, and Benhabib, Rogerson, and Wright, 1991). Given their assumption of frictionless labor markets, standard RBC models cannot be used to analyze unemployment fluctuations.

In recent years, a number of papers have introduced search frictions into RBC frameworks, some of them (like Andolfatto, 1996, Merz, 1995, 1999, and Den Haan, Ramey and Watson, 2000) using the Mortensen-Pissarides (1994) matching framework, others (like Gomes, Greenwood and Rebelo, 2001) using the Lucas-Prescott (1974) islands framework. A common finding in this literature is that a RBC model that incorporates search frictions can account for salient features of U.S. business cycles and even outperform the standard model in several ways. While this is an important result, none of the above papers attempted to explain the joint behavior of employment, unemployment and out-of-the-labor-force: Merz (1995, 1999) and Gomes, Greenwood and Rebelo (2001) assumed a fixed labor force, while Andolfatto (1996) and Den Haan, Ramey and Watson (2000) lumped together unemployment and out-of-the-labor-force into a single nonemployment state.[1]

Explaining the joint behavior of employment, unemployment and labor force participation is important not only to obtain a better understanding of labor market dynamics, but to test the empirical plausibility of the search and leisure/home-production decisions embodied

[1]Strictly speaking, Den Haan, Ramey and Watson (2000) lumped together unemployment and out-of-the-labor-force workers that claim to want a job.

in a model. Consider, for example, the models by Merz (1995, 1999) or Gomes, Greenwood and Rebelo (2001) that allow agents to search and enjoy leisure while they are unemployed but that restrict them to stay in the labor force. If the main reason why agents become unemployed in those models is to enjoy leisure (i.e. if intertemporal substitution in leisure is the main factor driving employment fluctuations), a significant number of agents would want to leave the labor force in order to enjoy even more leisure if they were given the chance. Thus, most of the flows from employment to unemployment during a recession could end up being flows from employment to out-of-the-labor-force once a labor force participation margin is allowed for, generating highly counterfactual behavior. Lumping together unemployment and out-of-the-labor-force into a single nonemployment state (as in Andolfatto, 1996, and Den Haan, Ramey and Watson, 2000) may hide similar problems.

A first attempt to evaluate this possibility was made by Tripier (2003), who analyzed an efficient RBC version of the Mortensen-Pissarides matching model that makes an explicit distinction between employment, unemployment and out-of-the-labor-force.[2] His main finding was that the model fails to reproduce the countercyclical unemployment rate observed in U.S. data. While this result suggests that RBC models can have serious difficulties in generating empirically reasonable labor market dynamics, it was obtained under two restrictive assumptions: 1) that workers accept the first job-offer that they receive (since all jobs have the same productivity level), and 2) that jobs are destroyed at a constant rate. Given these assumptions, the main mechanism that can give rise to a significant increase in aggregate employment level after a positive productivity shock hits the economy is an increase in labor force participation.[3] Since new market participants must search for a job before they become employed, it is not surprising that this mechanism will give rise to a pro-cyclical

[2] Greenwood, MacDonald and Zhang (1996) also analyzed a RBC model that incorporates the three employment states (but no search frictions). However, their focus was on the cyclical behavior of job creation and job destruction instead of decomposing the cyclical behavior of non-employment into unemployment and out-of-the-labor-force. Other papers that have explicitly modeled the three employment states include Alvarez and Veracierto (2000), Andolfatto and Gomme (1996), Garibaldi and Wasmer (2005), Kim (2004), Moon (2005), Pries and Rogerson (2004) and Veracierto (2007a), but none of them analyzed business cycle dynamics.

[3] In principle, a higher employment level could also be obtained by increasing the number of vacancies posted. However, efficient versions of the Mortensen-Pissarides model fail to generate large fluctuations in vacancies (see Shimer, 2005).

unemployment rate. This paper extends Tripier's analysis by introducing endogenous job-acceptance and job-separation decisions. Incorporating these margins is important because a higher aggregate employment level can now be obtained by increasing the job-acceptance rate or by decreasing the job-separation rate, without having to increase the size of the labor force. Thus, contrary to Tripier's analysis, the theory is given a fair chance at generating a counter-cyclical unemployment rate.

The benchmark model considered is a version of one used by Alvarez and Veracierto (2000), which in turn is based on the Lucas and Prescott (1974) equilibrium search model. Output, which can be consumed or invested, is produced by a large number of islands using capital and labor. Contrary to the deterministic steady state analysis of Alvarez and Veracierto (2000), the islands are subject both to idiosyncratic and aggregate productivity shocks. Once the shocks are observed, agents must decide whether to work in the islands where they are currently located or to leave. The reallocation of workers across islands is subject to search frictions. An important difference with Lucas and Prescott (1974) is that search is undirected, leading to an endogenous average duration of unemployment. Another difference is that an explicit out-of-the-labor-force margin is introduced: Agents are allowed to obtain more home production leaving the labor force than becoming unemployed.

Parameter values are chosen so that the deterministic steady state of the model economy reproduces important observations from the National Income and Product Accounts (NIPA) and some key labor market statistics. Aggregate productivity shocks in turn are selected to match the behavior of measured Solow residuals. Under such parametrization, I find that the model fails to account for the joint behavior of employment, unemployment and out-of-the-labor-force. The search and home production decisions embodied in this version of the neoclassical growth model generate drastically counterfactual behavior, mainly: 1) unemployment fluctuates as much as output while in the data it is six times more variable, 2) unemployment is weakly procyclical while in the data it is strongly countercyclical, 3) employment fluctuates as much as the labor force while in the data it is three times more variable, and 4) the labor force is strongly procyclical while it is weakly procyclical in the actual economy. These results are robust to a wide variety of specifications for the search technology.

Even though the paper fails to account for U.S. observations, it fails in an informative way. The paper finds that the empirical performance of an RBC model can become quite poor once unemployment and endogenous labor force participation are explicitly introduced. Thus, the paper questions the ability of previous RBC models to account for labor market fluctuations. Moreover, the paper suggests that a successful business cycle model, whatever that may end up being, will have to give a much more important role to fluctuations in search decisions than to fluctuations in home production or leisure.

The paper is organized as follows: Section 2 describes the model economy. Section 3 describes a competitive equilibrium. Section 4 parameterizes the model. Section 5 presents the results. Section 6 discusses the role played by worker flows. Finally, Section 7 concludes the paper. An appendix describes the computational algorithm.

2. The benchmark economy

The economy is populated by a representative household constituted by a unit measure of members. Each household member is endowed with one unit of time and has preferences described by:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln c_t + A \left(\frac{h_t^{1-\phi} - 1}{1-\phi} \right) \right] \right\}, \tag{2.1}$$

where c_t is consumption of a market good, h_t is consumption of a home good, $0 < \beta < 1$ is the subjective time discount factor, $\phi > 0$ and $A > 0$. The household serves as a full insurance mechanism, pooling the resources of all its individual members. Thus, all household members obtain an identical consumption bundle (c_t, h_t).

The market good, which can be consumed or invested, is produced by a measure one of spatially separated islands. Each island has a production function given by

$$y_t = e^{(1-\varphi)a_t} z_t l_t^{\gamma} k_t^{\varphi},$$

where y_t is production, l_t is the labor input, k_t is the capital input, z_t is an idiosyncratic productivity shock, a_t is an aggregate productivity level common to all islands, $\gamma > 0$, $\varphi > 0$, and $\gamma + \varphi < 1$.[4] The idiosyncratic productivity shock z_t follows a finite Markov process with transition matrix Q, where $Q(z, z')$ is the probability that $z_{t+1} = z'$ conditional on $z_t = z$. Realizations of z_t are assumed to be independent across islands. The aggregate productivity level evolves according to the following AR(1) process:

$$a_{t+1} = \rho_a a_t + \varepsilon_{t+1}, \tag{2.2}$$

where $0 < \rho_a < 1$ and ε_{t+1} is i.i.d., normally distributed, with variance σ_a^2 and zero mean. Capital is freely mobile across islands but not labor, which is subject to search frictions.

At the beginning of every period household members differ in terms of their physical locations: Some of them are distributed across islands and some others are located outside the islands sector. The household must decide how to allocate its members across different activities.[5] If a household member is initially located in an island, the household has two alternatives: To order him to stay or to order him to leave. If the household member is ordered to stay he produces the market good and starts the following period at that same location. If the household member is ordered to leave he becomes non-employed.

The total number of non-employed household members is given by the sum of all the household members that are ordered to leave and all the household members that are located outside the islands sector at the beginning of the period. The household allocates non-employed members into two mutually exclusive activities: To search or to specialize in home production. If a household member is ordered to search for a new employment opportunity, he arrives to an island at the beginning of the following period with probability p.[6] Since search is undirected, all the household members that arrive to the islands sector become uniformly distributed across all the islands in the economy. If a household member is ordered

[4] The assumption of a fixed factor of production in each island (implicit in the decreasing returns to scale assumption) is made for computational reasons: It guarantees that the labor and capital inputs of all islands remain positive at all times. The linear computational method used in this paper is well designed to handle this case.

[5] This decision is made after all productivity shocks for the current period (both idiosyncratic and aggregate) are observed.

[6] A later specifications of the model will allow this probability to vary endogenously over time.

to specialize in home production, he devotes his full time endowment to that end.

Household members that search or work also contribute to home production, but in a more restricted way. In particular, the total amount of home production obtained by the household is given by

$$h_t = 1 - \pi_U U_t - \pi_N N_t, \tag{2.3}$$

where N_t is the number of household members that work (i.e. that are employed), U_t is the number of household members that search (i.e. that are unemployed), π_N is the fixed length of the workweek, π_U is the fixed amount of time required by the search technology, and $0 < \pi_U \leq \pi_N \leq 1$.[7] Observe that household members generate more home production being out-of-the-labor-force than searching and that they generate more home production searching than being employed.

3. Competitive equilibrium

In order to describe a competitive equilibrium, I will index islands according to their history of idiosyncratic shocks $z^t = (z_0, z_1, ..., z_t)$ and their number of agents available at the beginning of date zero, x_0. Hereon, I will denote the time t distribution of islands across these variables by $q_t(z^t, x_0)$. Observe that $q_{t+1}(z^{t+1}, x_0)$ must satisfy that:

$$q_{t+1}((z^t, z_{t+1}), x_0) = q_t(z^t, x_0) Q(z_t, z_{t+1})$$

for every z^t and z_{t+1}.To simplify notation, I will assume that q_0 has a finite support over the pairs (z_0, x_0).

Since capital if freely movable, there is a single rental rate of capital r_t in the whole economy[8]. On the contrary, because of the search frictions, each type of island (z^t, x_0) has its own wage rate $w_t(z^t, x_0)$. The representative firm in an island of type (z^t, x_0) solves the

[7] Hereon, I will refer to household members that specialize in home production as being "out-of-the-labor-force". The total number of household members that are out-of-the-labor-force is given by $1 - U_t - N_t$.

[8] For convenience, the dependence of all variables on the history of aggregate productivity shocks $a^t = (a_0, a_1, ..., a_t)$ will be suppressed from the notation.

following static maximization problem:

$$\Pi_t\left(z^t, x_0\right) = \max\left\{e^{(1-\varphi)a_t} z_t l_t^\gamma k_t^\varphi - w_t\left(z^t, x_0\right) l_t - r_t k_t\right\}, \tag{3.1}$$

where w_t and r_t are taken as given. Since the representative firm behaves competitively, it equates rental prices to marginal productivities:

$$w_t\left(z^t, x_0\right) = e^{(1-\varphi)a_t} z_t \gamma l_t\left(z^t, x_0\right)^{\gamma-1} k_t\left(z^t, x_0\right)^\varphi, \tag{3.2}$$

$$r_t = e^{(1-\varphi)a_t} z_t l_t\left(z^t, x_0\right)^\gamma \varphi k_t\left(z^t, x_0\right)^{\varphi-1}. \tag{3.3}$$

The total number of household members that the representative household puts to work in islands of type (z^t, x_0) is denoted by $n_t(z^t, x_0)$. This number is constrained by the total number of household members located in islands of type $(z^t x_0)$ at the beginning of the period. At date zero, this constraint takes the simple form

$$n_0(z_0, x_0) \leq x_0 q_0(z_0, x_0), \tag{3.4}$$

where the right hand side is the number of islands of type (z_0, x_0) multiplied by the number of agents present in each of these islands at the beginning of date zero, x_0. In all other dates, the constraint becomes:

$$n_t\left(z^t, x_0\right) \leq n_{t-1}\left(z^{t-1}, x_0\right) Q\left(z_{t-1}, z_t\right) + pU_{t-1} q_t\left(z^t, x_0\right). \tag{3.5}$$

Observe that, in this case, the total number of household members located in islands of type $(z^t x_0)$ at the beginning of the period is given by the sum of two terms. The first term is the total number of household members that were put to work in islands of type (z^{t-1}, x_0) in the previous period and their locations transited from z_{t-1} to z_t. The second term is the total number of household members that were put to search during the previous period U_{t-1}, times the fraction that arrived to the islands sector p, times the fraction that arrived to an island of type (z^t, x_0). Observe that this second term takes into account the fact that search is undirected: Unemployed agents that arrive to the islands sector become uniformly distributed across all the islands in the economy.

The total number of household members that work is given by

$$N_t = \sum_{z^t, x_0} n_t \left(z^t, x_0\right), \tag{3.6}$$

i.e. it is the sum of $n_t\left(z^t, x_0\right)$ across all the different types of islands in the economy.

The problem of the representative household is to maximize the utility function (2.1) subject to equations (2.2), (2.3), (3.4)-(3.6) and the following budget constraint:

$$c_t + K_{t+1} - (1-\delta) K_t \leq r_t K_t + \sum_{z^t, x_0} w_t \left(z^t, x_0\right) n_t \left(z^t, x_0\right) + \sum_{z^t, x_0} \Pi_t \left(z^t, x_0\right) q_t \left(z^t, x_0\right), \tag{3.7}$$

where K_t is the stock of capital owned by the household at the beginning of the period. Equation (3.7) states that the value of consumption and investment cannot exceed total income, which is given by the sum of capital rental income, total wage earnings and total profits. Observe that the last term in equation (3.7) implicitly assumes that the representative household owns one share of each island in the economy. Also observe that the household takes r_t , $w_t\left(z^t, x_0\right)$, $\Pi_t\left(z^t, x_0\right)$, $q_t\left(z^t, x_0\right)$, K_0 and a_0 as given.

The optimal household decisions are easily characterized. The investment decisions must satisfy a standard Euler equation:

$$1 = \beta E_t \left[\frac{c_t}{c_{t+1}} \left(r_{t+1} + 1 - \delta\right)\right]. \tag{3.8}$$

This condition states that the cost of investing in one unit of capital must be equal to its expected discounted return, where date t and date $t+1$ consumption units are valued according to their marginal utilities.

The optimal labor allocation decision can be described in terms of the value $\xi_t(z^t, x_0) \geq 0$ of having a worker in an island of type (z^t, x_0) at the beginning of period t.[9] This value must

[9] Actually, $\xi_0\left(z_0, x_0\right)$ and $\xi_t(z^t, x_0)$ are the Lagrange multipliers of constraints (3.4) and (3.5), respectively.

satisfy the following condition:

$$\begin{aligned}\xi_t(z^t, x_0) \quad \leq \quad & w_t\left(z^t, x_0\right) - \pi_N A\left(1 - \pi_U U_t - \pi_N N_t\right)^{-\phi} c_t \\ & + \beta E_t\left[\frac{c_t}{c_{t+1}} \sum_{z_{t+1}} \xi_{t+1}\left[(z^t, z_{t+1}), x_0\right] Q(z_t, z_{t+1})\right],\end{aligned} \tag{3.9}$$

with equality if $n_t\left(z^t, x_0\right) > 0$. Since $n_t\left(z^t, x_0\right)$ is always positive in equilibrium, equation (3.9) states that $\xi_t(z^t, x_0)$ must be equal to the expected discounted surplus value of having a household member permanently employed in the island instead of permanently specialized in home production.

In addition, the following complementary slackness conditions must hold:

$$\xi_t(z^t, x_0)\left[n_{t-1}\left(z^{t-1}, x_0\right) Q\left(z_{t-1}, z_t\right) + pU_{t-1} q_t\left(z^t, x_0\right) - n_t\left(z^t, x_0\right)\right] = 0, \tag{3.10}$$

$$\xi_0(z_0, x_0)\left[x_0 q_0(z_0, x_0) - n_0\left(z_0, x_0\right)\right] = 0. \tag{3.11}$$

These equations state that whenever the surplus value of having a worker permanently employed in an island is strictly positive, the household puts to work all the workers available in that island at the beginning of the period. The household orders some of its members to leave only when it is indifferent between letting them work in the island and taking them out-of-the-labor-force.

The last decision variable for the household is the number of household members to put to search U_t. Assuming an interior solution, this decision must satisfy the following condition:

$$\pi_U A\left(1 - \pi_U U_t - \pi_N N_t\right)^{-\phi} c_t = p\beta E_t\left[\sum_{z^{t+1}, x_0} \frac{c_t}{c_{t+1}} \xi_{t+1}\left[z^{t+1}, x_0\right] q_{t+1}\left(z^{t+1}, x_0\right)\right] \tag{3.12}$$

The left-hand-side is the amount of home production foregone by putting a household member to search instead of keeping him out-of-the-labor-force. The right-hand-side is the probability that the unemployed worker will arrive to the islands sector p, times the expected discounted value of obtaining one additional household member at some randomly determined island. Observe that this expected value reflects the undirected nature of search.

Because of the search frictions, each type of island (z^t, x_0) has its own labor market clearing condition:

$$\frac{n_t\left(z^t, x_0\right)}{q_t\left(z^t, x_0\right)} = l_t\left(z^t, x_0\right). \tag{3.13}$$

The left-hand-side is the number of household members that the representative household puts to work in each island of type (z^t, x_0). The right-hand-side is the quantity of labor demanded by the representative firm in an island of type (z^t, x_0).

Because capital is fully flexible there is a single market clearing condition for the whole economy:

$$K_t = \sum_{z^t, x_0} k_t\left(z^t, x_0\right) q_t\left(z^t, x_0\right), \tag{3.14}$$

This equation states that the capital supplied by the representative household must be equal to the total quantity demanded by all the islands in the economy.

Finally, the market clearing for the consumption good is given by:

$$c_t + K_{t+1} - (1-\delta) K_t = \sum_{z^t, x_0} e^{(1-\varphi)a_t} z_t l_t(z^t, x_0)^\gamma k_t(z^t, x_0)^\varphi q_t\left(z^t, x_0\right). \tag{3.15}$$

That is, the sum of consumption and investment must be equal to the aggregate output produced by all the islands in the economy.

A competitive equilibrium is a stochastic process $\{c_t,\ K_{t+1},\ n_t,\ U_t,\ N_t,\ h_t,\ \xi_t,\ l_t,\ k_t,\ w_t,\ r_t,\ \Pi_t\ \}_{t=0}^{\infty}$ such that equations (2.2)-(3.15) are satisfied, with a_0, K_0 and q_0 given. Since the economy is convex, the Welfare Theorems hold. Thus a competitive equilibrium can be obtained by solving the social planner's problem, which is to maximize equation (2.1) subject to equations (2.2), (2.3), (3.4), (3.5), (3.6), (3.13), (3.14) and (3.15), taking a_0, K_0 and q_0 as given. The appendix describes the algorithm used to compute the solution to this social planner's problem.

4. Parameterization

This section describes the observations used to calibrate the steady state of a deterministic version of the competitive equilibrium described in the previous section, in which the aggregate productivity shock a_t is set to its unconditional mean of zero. The curvature of home production in the utility function (ϕ) and the time requirements for search and employment (π_U and π_N), will be taken as free parameters in the experiments below. The parameters to be calibrated are β, A, γ, φ, δ, the values for the idiosyncratic productivity shock z, the transition matrix Q, and the parameters determining the driving process for the aggregate productivity shock a. The time period selected for the model is one month. A short time period is called for in order to reproduce the relatively short average duration of unemployment observed in U.S. data.

The stock of capital in the market sector K is identified with business capital, that is, with plant, equipment and inventories. As a result, investment in business capital I is associated in the National Income and Product Accounts with fixed private non-residential investment plus changes in business inventories. Considering that the depreciation rate is related to steady state I and K according to

$$\delta = \frac{I}{K},$$

the average I/K ratio over the period 1967:Q1 to 1999:Q4 gives a monthly depreciation rate $\delta = 0.0066$.

In turn, consumption c is identified with consumption of non-durable goods and services (excluding housing services). Output is then defined as the sum of these consumption and investment measures. The average monthly capital-output ratio K/Y corresponding to the period 1967:Q1 to 1999:Q4 is 25.8.

The interest rate in the model economy is given by

$$1 + i = \frac{1}{\beta}.$$

As a consequence $\beta = 0.9967$ is chosen to reproduce an annual interest rate of 4 percent, roughly the average between the return on equity and the return on treasury bills in the U.S. economy.

The Cobb-Douglas production function and the competitive behavior assumption implies that φ equals the share of capital in output. That is,

$$\left(\frac{1}{\beta} - 1 + \delta\right) \frac{K}{Y} = \varphi.$$

Given the previous values for β, δ, and $\frac{K}{Y}$, it follows that $\varphi = 0.2554$. On the other hand, $\gamma = 0.64$ is selected to reproduce the labor share in National Income.

The idiosyncratic productivity levels z and the transition matrix Q are chosen to approximate (by quadrature methods) the following AR(1) process:

$$\log z_{t+1} = \rho_z \ln z_t + \varepsilon_{t+1}^z,$$

where ε_{t+1}^z is i.i.d., normally distributed, with zero mean and variance σ_z^2.[10] Since the stochastic process for the idiosyncratic productivity shocks is a crucial determinant of the unemployment rate and the average duration of unemployment, ρ_z and σ_z^2 will be selected to reproduce an unemployment rate of 6.2 percent and an average duration of unemployment equal to one quarter, which correspond to U.S. observations. The weight of home production in the utility function A in turn will be selected to reproduce a labor force participation equal to 74 percent (the average ratio between the size of the labor force and total population between 16 and 65 years old). The actual values for ρ_z, σ_z^2 and A will depend on the values for π_U and π_N, which are taken as free parameters.

Finally, using the measure of output described above and a labor share of 0.64, measured Solow residuals are found to be as highly persistent but somewhat more variable than Prescott (1986): the standard deviation of quarterly technology changes is 0.009 instead of 0.0076. As a consequence, $\rho_a = 0.98$ and $\sigma_a^2 = 0.009^2/3$ are chosen here.

Table 1 reports parameter values for all the specifications that will be considered later on.

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[10] Only three values for z will be allowed in the computations. While this may not seem a large number, it leads to a considerable amount of heterogeneity: The support of the invariant distribution of islands across employment levels will be over one thousand points in most of the experiments reported below.

5. Results

In order to evaluate the behavior of the model economy, Table 2 reports quarterly U.S. business cycle statistics. Before any statistics were computed, all time series were logged and detrended using the Hodrick-Prescott filter. The empirical measures for output Y, consumption c, investment I and capital K reported in the table correspond to the measures described in the previous section, and cover the period between 1967:Q1 and 1999:Q4. The table shows some well known facts about U.S. business cycle dynamics: that consumption and capital are less variable than output while investment is much more volatile, and that consumption and investment are strongly procyclical while capital is acyclical. The variability of labor relative to output (0.57) is lower than usual because it refers to employment instead of total hours worked. What is important in Table 2 is the variability of unemployment, which is 6.25 times the variability of output, and the variability of the labor force, which is only 0.20 times the variability of output. While employment is strongly procyclical, labor force participation is only weakly procyclical. On the contrary, unemployment is strongly countercyclical: its correlation with output is -0.83. Note that even though unemployment is a small fraction of the labor force, its behavior is key in generating a much larger variability in employment than in labor force participation.

In what follows I report results for different versions of the model economy. The analysis will relate the paper to the previous literature and show the robustness of the results to different specifications of the search technology. In all cases the free parameter ϕ, which determines the curvature of home production in the utility function, will be chosen to reproduce the standard deviation of employment observed in the U.S. economy. This will give the model the best chances at mimicking observed labor market dynamics. Despite of this, we will see that the model performs quite poorly.

5.1. Home production while unemployed

This section reports the main results of the paper: It shows that the model economy is unable to reproduce the joint behavior of employment, unemployment and labor force participation observed in U.S. data. In order to make the results more transparent, I will assume that all agents that search arrive to the islands sector with probability one, i.e. $p = 1$. Later on, I will relax this assumption.

Since there is no reliable data on the amount of time that unemployed agents spend searching, this section will show results for different values of π_U. The only restriction that I will impose is that total hours spent in market activities $\pi_U U + \pi_N N$ must be equal to 0.33, which is the magnitude commonly used in the RBC literature.

5.1.1. Case $\pi_U = \pi_N$

To start with, let consider the case in which $\pi_U = \pi_N$. This case is important because agents obtain the same amount of home production being unemployed than being employed: The only way that they can obtain additional home production is by leaving the labor force. The first column of Table 3 ("$\pi_U = \pi_N$, flexible labor force") reports the results for this case. The statistics correspond to averages across 100 simulations of 408 periods each (corresponding to the 136 quarters of data). Before computing these statistics, the monthly data generated by the model was aggregated to a quarterly time period and then logged and detrended using the Hodrick-Prescott filter. Comparing the business cycles generated by this version of the model with those of the U.S. economy, we see that consumption fluctuates less than output in both economies but that it is considerably smoother in the model than in the U.S.: its relative volatility is 0.32 instead of 0.57. Investment is about 4.5 times as variable as output in both economies, and it is strongly procyclical in both. Employment fluctuates the same amount in the model as in the data (parameter values were selected to generate this result) and is strongly procyclical in both economies. Thus we see that the model, in principle, has the same ability of reproducing standard business cycle statistics as previous RBC models. However the model fails badly in terms of the labor market dynamics that it generates. There are four main problems: 1) unemployment is only slightly more variable than output while it is six times more volatile than output in the data, 2) unemployment is weakly procyclical while it is strongly countercyclical in the U.S., 3) employment fluctuates as much as the labor force while employment is three times more variable than the labor force in the U.S. economy, and 4) labor force participation is strongly procyclical while it is weakly procyclical in the actual economy.

To shed light on these model statistics Figure 1 shows different impulse responses to a positive aggregate productivity shock equal to one standard deviation.[11] The variables reported are employment (N_t), unemployment (U_t), labor force ($U_t + N_t$), the job acceptance rate ($\frac{Hiring_t}{pU_{t-1}}$), and the job separation rate ($\frac{Firing_t}{N_{t-1}}$). The basic intuition for these responses can be obtained from considering the social planner's problem. When the aggregate shock hits the economy the social planner wants to increase employment as soon as possible. However, on impact this can be achieved only by decreasing the job separation rate or by increasing the job acceptance rate: The number of workers present in the islands sector is initially predetermined. The gains from decreasing the separation rate turn out to be very small. The reason is that, at steady state, most of the job separations already take place in islands with very low idiosyncratic productivity levels. As a consequence, the social planer generates most of the initial increase in aggregate employment through a large increase in the job acceptance rate. It is important to observe that this spike in the job acceptance rate reduces the initial average idiosyncratic productivity of employed workers.

In order to take advantage of the persistent aggregate productivity shock the social planner also decides to increase the size of the labor force. Given that there are no adjustment costs in this margin, the full adjustment takes place as soon as the aggregate shock hits the economy. This has important implications for unemployment. Since workers enter the labor force as searchers, the sudden increase in labor force participation generates a large initial increase in unemployment in spite of the spike in the job acceptance rate.

In subsequent months, the social planner continues to bring workers from unemployment into employment but trying to achieve a more efficient allocation of workers across islands. This is clearly seen during the first month after the aggregate shock: Since the average idiosyncratic productivity of employed workers had decreased significantly during the previous month, the social planner takes corrective actions by sharply reducing the job acceptance rate and by slightly increasing the job separation rate. The social planner can afford to pursue a more efficient allocation of labor because of the large number of workers that he now has arriving to the islands sector. As the effects of the aggregate shock die off, the higher

[11] Observe that the month in which the aggregate productivity shock hits the economy is labelled "month 0" in Figure 1.

desire for efficiency gets reinforced by a lower desired level for aggregate employment.[12] As a result, the social planner keeps reducing the job acceptance rate and keeps raising the job separation rate over time, reallocating workers from low productivity islands to high productivity islands and reducing the size of the labor force. In the long run all variables end-up reverting to their initial steady state levels.[13]

It is important to point out that there are two channels driving the employment fluctuations reported above. The first channel is the standard one: When there is a good aggregate productivity shock it is a bad time to do home production, so the social planner instructs agents to substitute home goods intertemporally and supply more employment. The second channel arises from the timing of the search decisions: When there is a good aggregate shock it is a bad time to search for a good idiosyncratic productivity shock, so the social planner instructs agents to accept employment more easily and leave the islands less frequently. Since the business cycle behavior of employment and labor force participation are almost identical, the results so far suggest that the first channel is the most important: Fluctuations in the decisions to reallocate workers across islands seem to play a small role in aggregate dynamics.

To verify that this is indeed the case I consider a modified version of the economy in which households are subject to a large disutility cost from changing the size of the labor force. This disutility cost does not affect the steady state of the economy but has large effects on the business cycle dynamics. In particular, since additional home production can only be obtained by leaving the labor force, this term kills the intertemporal substitution in home goods as a source of employment fluctuations: Changes in the decisions to reallocate workers across islands (the second channel mentioned above) becomes the only source of employment fluctuations. The second column of Table 3 ("$\pi_U = \pi_N$, fixed labor force") shows the results for this case. Not surprisingly we see that when the labor force is effectively fixed unemployment becomes countercyclical, since recessions are good periods to search for better idiosyncratic shocks. However, we see that this channel is not an important source

[12] Aggregate employment actually peaks two months after the aggregate shock.

[13] This long-run trend cannot be observed in Figure 1 since it shows too few periods.

of employment fluctuations: Compared with the original case (in which both channels are present), the relative standard deviation of employment drops from 57% to 3% while the relative standard deviation of unemployment drops from 147% to 41%. We conclude that intertemporal substitution in home production is, by far, the most important mechanism generating employment fluctuations in the model economy. This will represent a fundamental problem: In all cases considered it will make employment follow labor force participation too closely, generating highly counterfactual business cycle statistics.

5.1.2. Case $\pi_U = 0.01\pi_N$

Let now consider the other extreme: The case in which searching for a job takes only 1% as much time as being employed. Observe that in this case agents obtain almost the same amount of home production being unemployed as being out-of-the-labor-force.[14] As a consequence, the utility weight of home goods A must be increased in order to match the same labor force participation rate. Also, since agents obtain plenty of home production while being unemployed, the variance σ_z^2 and persistence ρ_z need to become much smaller in order to generate the same unemployment rate and average duration of unemployment.

It will be convenient to start the analysis by considering the case of a large disutility cost to changing the labor force. The fourth column in Table 3 ("$\pi_U = 0.01\pi_N$, fixed labor force") shows the results. We see that, when the labor force is effectively fixed, the cyclical behavior of employment generates a highly variable and countercyclical unemployment level.[15] This result was also obtained by papers that introduced search into RBC models, but that fixed the labor force and allowed agents to enjoy leisure while unemployed (e.g. Merz 1995, 1999, Gomes, Greenwood and Rebelo, 2001). However, we'll see that this apparent success relies on a strong home-good intertemporal substitution effect and on the fixed labor force assumption. When the labor force is allowed to change the model will generate implausible dynamics.

[14] The case $\pi_U = 0$ cannot be considered since no agent would be out-of-the-labor-force.

[15] Actually a bit too variable and countercyclical compared to the data: 7.84 versus 6.25 and -0.97 versus -0.83, respectively.

The third column in Table 3 ("$\pi_U = 0.01\pi_N$, flexible labor force") reintroduces the labor force participation margin, i.e. it sets the disutility cost of changing the labor force to zero. In this case, employment becomes as variable as in the data (parameter values were selected to deliver this result) and continues to be strongly procyclical. However, the model displays the same problems as in the previous section: Unemployment fluctuates too little and is weakly procyclical, and the labor force varies as much as employment and is strongly procyclical.

To understand this result, observe that calibrating this version of the model requires that the productivity differences across islands be small (low σ_z^2) and that these differences be quite transitory (low ρ_z). As a result, the social planner becomes quite indifferent about the distribution of agents across islands and the home-good intertemporal substitution effect becomes the only source of employment fluctuations (fluctuations in the reallocation of agents across islands play virtually no role). Given that it is relatively easy to re-employ workers (the arrival rate p is equal to one and all islands look roughly the same) and given that the aggregate productivity shock is highly persistent, during recessions the social planner sends agents outside the labor force (in order to obtain the additional amount of home production) instead of into unemployment. Thus, labor force participation follows the cyclical behavior of employment too closely.

5.2. Low arrival rates

This section explores how the business cycles of the model economy are affected when the arrival rate p is allowed to take values less than one. Table 4 shows results for p equal to 1, 0.75 and 0.50.[16]

Before describing the results it is important to point out that reducing the arrival rate p increases the average duration of unemployment. Also, since a lower value of p makes it more difficult to become re-employed, workers reduce their job separation rate. Thus, in order to match the same unemployment rate and average duration of unemployment as in

[16] Throughout the rest of the paper I will assume that $\pi_U = 0.5\pi_N$. This is between the two extremes considered in the previous section and is the case analyzed by Andolfatto (1996). However, similar results are obtained when $\pi_U = \pi_N$ or $\pi_U = 0.01\pi_N$ are used instead.

the U.S. economy, the persistence ρ_z and the variance σ_z^2 of the idiosyncratic shocks must be recalibrated. In particular, the persistence ρ_z must be lowered and the variance σ_z^2 must be increased quite significantly (see Table 1). The increase in σ_z^2 has an important consequence for business cycle fluctuations: It substantially reduces the variability of the job acceptance rate. The reason is that as the islands become characterized by either very small or very large idiosyncratic productivity levels, the realization of the aggregate productivity shock becomes less relevant for determining which islands workers should join.

Given that the arrival rate is fixed and that the job acceptance rate becomes less responsive to an aggregate shock, obtaining a same increase in aggregate employment now requires a larger increase in the number of agents that search. This in turn requires a larger increase in labor force participation. As a consequence we see in Table 4 that, as the arrival rate p decreases, unemployment and labor force participation become more volatile while the variability of employment remains the same. Also observe that with a lower value of p a smaller fraction of the labor force entrants arrive to the islands sector, but those that arrive are more likely to accept employment right away.[17] Thus, the labor force entrants that do not arrive to the islands sector are soon no longer needed and are quickly sent back to being out-of-the-labor-force. This effect is reflected in Table 4, which shows that unemployment and labor force participation become less procyclical as the arrival rate p decreases.

While the larger variability of unemployment and the lower procyclicality of unemployment and labor force participation may be seen as improvements over the $p = 1$ case, the effects are small and they come at the expense of increasing the volatility of labor force participation (which was already too high). Thus, lowering the arrival rate p does not improve the ability of the model to account for the cyclical behavior of U.S. labor markets.

5.3. Endogenous search intensity

In what follows, three different ways of endogenizing the arrival rate p will be considered. In the first specification p will depend on the amount of goods spent in the search process, in

[17] With a lower arrival rate p the job acceptance rate must be higher in order to match the same average duration of unemployment

the second specification p will depend on the amount of time spent in the search process, and in the third specification p will depend on the amount of effort spent in the search process.

5.3.1. Search requires goods

In this section, the probability p that an unemployed workers arrives to the islands sector is given by

$$p_t = \Omega s_t^{\eta}, \tag{5.1}$$

where s_t is the amount of consumption goods that the worker spends in the search activity and $0 \leq \eta < 1$.

Under this specification, the feasibility condition for the consumption good (3.15) becomes the following:

$$c_t + K_{t+1} - (1-\delta) K_t + I_t^s \leq \int e^{(1-\varphi)a_t} z_t n_t(x,z)^{\gamma} k_t(x,z)^{\varphi} \mu_t(dx,dz), \tag{5.2}$$

where I_t^s is the total amount of consumption goods invested in the search process, i.e.

$$I_t^s = s_t U_t. \tag{5.3}$$

All other feasibility conditions remain unchanged.[18] The implicit assumption in equation (5.3), that each unemployed worker spends the same amount of goods s_t, is justified by the concavity of the search technology.[19]

Table 5 shows the business cycle fluctuations for this modified economy under $\eta = 0$, $\eta = 0.25$ and $\eta = 0.50$. Observe that the $\eta = 0$ case is identical to the economy with fixed

[18] Observe that equations (5.1) and (5.3) imply that the rate at which unemployed workers meet employment opportunities p_t depends positively on the ratio of total search investment I_t^s to total unemployment U_t. This feature is shared by the Mortensen-Pissarides model, in which I_t^s is determined by the total number of vacancies. However, there is a key difference between both models: While in Mortensen-Pissarides the arrival of workers to specific production units can be directed by posting vacancies, in this paper search is undirected. For an islands economy with vacancy posting and matching, see Veracierto (2007b).

[19] To see this, let ζ_t be any distribution of unemployed workers across search intensity levels. Consider an alternative distribution ψ_t that puts all its mass at the search intensity level $\int s d\zeta_t$. From the concavity of equation (5.1) it follows that ψ_t would use the same amount of consumption goods as ζ_t but increase the total number of arrivals to the islands sector.

arrival rate p. The other two cases capture the effects of a time varying arrival rate. In all cases the search productivity parameter Ω is adjusted to generate a steady state arrival rate p equal to 0.75.[20]

As η increases, three different effects take place. The first effect arises from the fact that search becomes costly not only in terms of the foregone home production, but in terms of the consumption goods required by the search process. In response to this additional cost, the social planner decides to increase the job acceptance rate and decrease the job separation rate, lowering the average duration of unemployment and the unemployment rate. Thus, in order to reproduce U.S. observations, the persistence ρ_z and the variance σ_z^2 of the idiosyncratic shocks must be increased. For the same reasons as in the previous section, this reduces the variability of the job acceptance rate over the business cycle, which in turn reduces the variability of employment and unemployment.

The second effect is more straightforward. As η increases, a given increase in search intensity has a larger effect on the arrival rate of agents to the islands sector. As a consequence, employment increases more rapidly (and unemployment decreases more rapidly) after a positive aggregate hits the economy. This makes employment behave more procyclically and unemployment more countercyclically.

The combination of the first two effects can be clearly seen when the preferences are extended to include a large penalty for changing the labor force (i.e. when the labor force is effectively fixed). In this case, Table 5 shows that increasing η reduces the variability of employment and unemployment, and makes employment more procyclical and unemployment more countercyclical.

The third effect arises from a change in the variability of labor force participation. Observe that, during a recession, agents reduce their search intensity in order to avoid paying the search costs. When η is large, this decreases the arrival rate of unemployed agents to the islands sector quite significantly, reducing the benefits of remaining unemployed compared to leaving the labor force (and obtaining more home production). As a consequence, when η increases, part of the flows from employment to unemployment during a recession become

[20] Similar results are obtained when Ω is chosen to generate a steady state arrival rate p equal to 0.50.

flows from employment to out-of-the-labor-force. This makes labor force participation more variable and unemployment more variable and procyclical.

Not only each of the above effects is small, but the third effect works in opposite direction to the previous two. Thus, when the adjustment costs to changing the labor force are removed, i.e. when the labor force becomes fully flexible, Table 5 shows that the effects of increasing η are extremely small. We conclude that introducing a search intensity margin (asuming that search requires goods) does not improve the ability of the model to reproduce U.S. labor market dynamics.

5.3.2. Search requires time

In this section, the probability p that an unemployed workers arrives to the islands sector is given by

$$p = b(s - s_{\min})$$

where $s \geq s_{\min}$ is the amount of time that the worker spends in the search activity, $0 \leq b \leq 1, b' > 0$ and $b'' < 0$.

A straightforward consequence of the strict concavity of b is that the household never chooses to distribute its members across different search intensity levels. To see this, let U_t be the number of household members that search and ζ_t be a distribution of household members across search intensity levels. Observe that the household's total amount of home production is given by

$$h_t = 1 - U_t \int s d\zeta_t - \pi_N N_t, \tag{5.4}$$

and that the total number of arrivals is given by

$$M_t = U_t \int b(s - s_{\min}) d\zeta_t. \tag{5.5}$$

Thus, choosing the same number of searchers U_t but picking an alternative distribution ψ_t that puts all its mass at the search intensity level $\int s d\zeta_t$ would generate the same amount of leisure as before but produce a higher M_t.

Another important property of this search specification is that the optimal search intensity level is constant over time and across states. To see this, substitute equation (5.5) in equation (5.4) and use the fact that the optimal distribution of household members across search intensity levels is degenerate, to obtain that

$$h_t = 1 - M_t \frac{s_t}{b(s_t - s_{\min})} - \pi_N N_t,$$

where s_t is the search intensity level of all household members. Thus, given any desired value for the total arrivals M_t, the household can increase its amount of home production by choosing s_t to maximize:

$$\frac{b(s_t - s_{\min})}{s_t}, \tag{5.6}$$

a choice which is independent of the time and the state.

We conclude that the search specification considered in this section is observationally equivalent to the benchmark specification, in which the arrival probability p is constant and search requires a fixed amount of time π_U: Given arbitrary values for p and π_U, the function b can always be chosen so that the search intensity s that maximizes equation (5.6) satisfies that $p = b(s)$ and $s = \pi_U$. Thus, when search requires time the endogenous search intensity margin does nothing to improve the ability of the model to reproduce U.S. labor market dynamics.

5.3.3. Search requires effort

Similarly to the benchmark economy, this section assumes that search requires a fixed amount of time π_U. However, the probability that a household member arrives to the islands sector is now given by:

$$p_t = \Omega s_t^{\eta}$$

where s_t is the amount of effort that the worker spends in the search activity, $0 \leq \eta < 1$, and $\Omega > 0$. Observe that the assumption that search requires effort is not unreasonable: It is quite plausible that finding a job requires not only time, but putting attention into writing effective resumes, putting the correct attitude in the job interviews, etc.

The representative household values consumption and home production but dislikes to have its members exert search effort. Its utility function is now given by:

$$E\sum_{t=0}^{\infty}\beta^t\left\{\ln c_t + A\left(\frac{h_t^{1-\phi}-1}{1-\phi}\right) - BS_t\right\},$$

where $B > 0$ and $S_t = U_t s_t$ is the total amount of effort exerted by its household members.[21] The functional form with respect to search effort is, admittedly, ad-hoc. However, it is chosen to improve the chances of search intensity affecting business cycle dynamics: With the linear specification, it is very easy for the household to substitute search effort intertemporally and withstand large changes in this variable. Despite of this, it will be shown below that, even in this case, modelling endogenous fluctuations in the arrival rate p_t does not change the basic results.

Before proceeding observe that, contrary to the "search requires time" specification, the arrival rate p_t will generally display fluctuations in this "search requires effort" case. To see this, note that total leisure is still given by equation (2.3) while the total number of arrivals is now given by

$$M_t = U_t \Omega s_t^{\eta}. \tag{5.7}$$

Using equations (2.3) and (5.7), the household's utility function can now be written as follows:

$$E\sum_{t=0}^{\infty}\beta^t\left\{\ln c_t + A\left(\frac{\left[1-\frac{\pi_U}{\Omega}M_t s_t^{-\eta} - \pi_N N_t\right]^{1-\phi}-1}{1-\phi}\right) - B\frac{1}{\Omega}M_t s_t^{1-\eta}\right\},$$

Thus, the optimal choice of search effort s_t will now depend on the choices for M_t and N_t.

In what follows I consider the cases $\eta = 0$, $\eta = 0.25$ and $\eta = 0.50$. The $\eta = 0$ case is identical to the benchmark economy with fixed arrival rate p. The other two cases are meant to capture the effects of a time varying arrival rate. In all cases the productivity parameter Ω is adjusted to generate a steady state arrival rate p equal to 0.75.[22] Table 6 shows the

[21] Similarly to the previous specifications, it is easy to argue that the representative household will always choose a degenerate distribution of household members across search effort levels. That is, all searchers exert the same effort level s_t.

[22] Similar results are obtained when Ω is chosen to generate a steady state arrival rate p equal to 0.50.

results. We see that it makes little difference whether the search intensity costs are in terms of goods or in terms of effort: The results in Table 6 are virtually the same as in Table 5. We conclude that the difficulty of RBC models to account for labor market dynamics is independent of the specification for an endogenous search intensity.

6. Worker flows

Before concluding I would like to provide a few remarks about the role and empirical plausibility of the worker flows underlying the model economy. An apparent drawback of the model is that it does not allow for direct flows from out-of-the-labor-force (OLF) into employment while these flows are quite large in the U.S. economy (e.g. Blanchard and Diamond, 1990). However, if the model time period is short relative to the frequency at which workers are surveyed, positive flows from OLF into employment would be obtained due to time aggregation. Moreover, it wouldn't be difficult to reproduce the magnitude of the flows observed in the data. The reason is that worker flows are actually undetermined in the model economy. The source of the indeterminacy is that all workers become identical once they leave employment: There is nothing pinning down which nonemployed workers leave the labor force and which go into unemployment. Arbitrary rules about which workers go into unemployment can then be used to generate any level of OLF-into-employment flows within a wide range. To be specific, if all searchers are assumed to have been inside the labor force during the previous period, zero flows from OLF into employment would be obtained. On the other extreme, if all searchers are assumed to have been OLF during the previous period, large measured flows from OLF into employment would be obtained after two model periods.

Given this indeterminacy, it is important to observe that the model does not provide a theory of worker flows across labor market states but a theory of how many people are employed, unemployed and OLF in any given period of time. In order to pin down worker flows the model would have to be extended to incorporate heterogenous agents (which would add an additional layer of complication to the analysis). However, since the model already fails to reproduce the behavior of labor market stocks when the flows are allowed to be undetermined, there is little hope that such an extension would improve its business cycle behavior.

Finally, although we have seen that it is not crucial for reproducing worker flows in U.S. data, it is natural to wonder about the consequences of modifying the search technology to allow for direct transitions from OLF into employment. Would it improve the business cycle performance of the model? The answer to this question, however, is negative: The business cycle properties of the model would be worsened. The reason is that by making it easier for people to transit from OLF into employment, there would be even stronger reasons to move OLF (instead of becoming unemployed) when agents separate from their jobs. As a consequence, fluctuations in employment would be even more closely mirrored by fluctuations in OLF, which was already the main reason why the benchmark model failed to reproduce the observed behavior of U.S. labor markets.

7. Conclusions

In this paper I analyzed a RBC model that makes an explicit distinction between unemployment and out-of-the-labor-force. I found that the model has serious difficulties in reproducing the type of labor market dynamics observed in U.S. data. The model delivers four highly counterfactual results: 1) unemployment fluctuates as much as output while in the data it is six times more variable, 2) unemployment is weakly procyclical while in the data it is strongly countercyclical, 3) employment fluctuates as much as the labor force while in the data it is three times more variable, and 4) the labor force is strongly procyclical while it is weakly procyclical in the actual economy. The reason for the poor empirical performance of the model is that most of the employment fluctuations are the result of strong home-goods intertemporal substitution effects: Fluctuations in the decisions to reallocate workers across islands play a minor role. Given that aggregate productivity shocks are highly persistent, that agents obtain more home production being out-of-the-labor-force than being unemployed and that it is relatively easy to find employment, when agents decide to enjoy home goods they choose to leave the labor force instead of becoming unemployed. As a consequence, most of the variations in employment are reflected in fluctuations in labor force participation instead of unemployment, generating counterfactual labor market dynamics.

Despite the failure of the model the paper provides an important lesson. It shows that the empirical performance of a RBC model that relies on persistent aggregate productivity shocks and a high intertemporal elasticity of substitution in home-goods can become quite poor once unemployment and endogenous labor force participation are explicitly introduced. Thus the paper questions the ability of previous RBC models to account for labor market fluctuations. The fact that the model fails under a wide range of specifications for the search technology makes its point stronger.

The key question that the paper has left unanswered is "What type of model would generate empirically reasonable labor market dynamics?". An obvious answer seems to be a RBC model with adjustment costs to labor force participation. In fact, it is straightforward to verify that introducing adjustment costs of this sort to the benchmark model can generate empirically relevant labor market dynamics. The reason is quite simple: When there is a bad aggregate productivity shock and agents are willing to substitute towards home production, the adjustment costs in labor force participation induce agents to become unemployed instead of leaving the labor force. Under large adjustment costs and a high intertemporal substitution in home goods, the model behaves quite similarly to the economy "$\pi_U = 0.01\pi_N$, fixed labor force" in Table 3, generating empirically reasonable labor market dynamics.

However this answer cannot satisfy us. There are no good economic reasons to justify this type of adjustment costs in an RBC model. While there may be many out-of-the-labor-force activities subject to large adjustment costs (such as child rearing), these are not the type of activities that are relevant for understanding fluctuations at business cycle frequencies. What matters is the type of activities that unemployed agents undertake when they are not searching. If a RBC model allows agents to intertemporally substitute these activities very easily, it is not clear why there should be large costs to stop searching (leaving the labor force) and doing more of these same activities. Large adjustment costs in labor force participation and a high intertemporal substitution in home goods appear to be mutually inconsistent assumptions.

These reasons suggest that a successful model, whatever that may end up being, will have to shift the source of employment fluctuations from intertemporal substitution in home goods towards search decisions. If fluctuations in labor force participation are small (because it is difficult to substitute home goods intertemporally) but search decisions respond to aggregate shocks in a significant way, the labor market dynamics thus generated would become much more satisfactory. Finding such a model promises to be a challenging area of research.

One possibility may be to introduce labor market policies that this paper has abstracted from. An unemployment insurance system seems particularly relevant. Since unemployment insurance provides agents additional incentives to remain unemployed, the model would not require a high persistence and variability of idiosyncratic shocks to reproduce the unemployment rate and the average duration of unemployment observed in the U.S. economy, making the job acceptance rate much more responsive to aggregate shocks. In addition, even under the low intertemporal substitution in home goods that would be needed to generate the small observed fluctuations in labor force participation, the shifts from employment to unemployment could become large because agents would be substituting competing sources of income over time (in particular, relatively constant unemployment benefits against time varying wages). While introducing unemployment insurance into the model economy may generate reasonable labor market dynamics, we cannot view this as a plausible explanation. It seems highly unlikely that removing the unemployment insurance system in the U.S. would make unemployment fluctuate procyclically. Moreover, there are several countries that have no unemployment insurance systems but have countercyclical unemployment rates (e.g. Italy, Argentina, etc).

A more promising route would be to introduce a reallocation shock that changes the variance of the idiosyncratic productivity shocks over time. Since the distribution of idiosyncratic shocks is an important determinant of search decisions, this type of shocks can lead to important variations in unemployment that are not the direct consequence of changes in labor force participation. In particular, if the variance of the idiosyncratic shocks is negatively correlated to the aggregate productivity shock, depressions would be accompanied by high incentives to search and unemployment fluctuations could become much larger and countercyclical. In fact, it is not hard to come up with examples where a version of the benchmark economy subject to both aggregate and reallocation shocks generates empirically reasonable labor market dynamics. The challenge will be to verify that this holds under an empirically plausible process for the aggregate and reallocation shocks.

Another promising route is the one proposed by Shimer (2005) and followed by Hall (2005) and Farmer (2004): To introduce wage rigidities in the Mortensen-Pissarides search framework. Shimer suggested that the difficulties of the Mortensen-Pissarides model to account for the large observed fluctuations in unemployment and vacancies are due to the fact that wages are too flexible in that setting. Indeed, Hall (2005) and Farmer (2004) found that introducing rigid or partially adjusting wages can significantly improve the performance of that model. However, the analysis so far has been done under the assumption of a fixed labor force. It would be interesting to evaluate if the success of sticky-wage versions of the Mortensen-Pissarides model is preserved once a labor force participation margin is introduced.

Hagedorn and Manovskii (2007) have recently questioned Shimer's results, showing that an empirically plausible calibration of the Mortensen-Pissarides model is able to reproduce U.S. business cycle fluctuations quite closely. This seems to suggest that introducing a labor force participation margin to their model may be a worthwhile project. However, it is important to observe that Hagedorn and Manovskii's calibration is characterized by two important features: 1) that workers have a very low bargaining power, and 2) that workers value leisure quite significantly. The low bargaining power of labor leads to small wage fluctuations in their model, but because leisure is valued so highly even small decreases in wages are able to generate large inflows into unemployment.[23] While the high value of leisure is a key feature leading to the large unemployment fluctuations in Hagedorn and Manovskii (2007), this same feature is likely to generate a poor empirical performance once a labor force participation margin is introduced. The reason is that if workers were allowed to obtain more of this highly valued leisure being out-of-the-labor-force than being unemployed, a decrease in wages would induce workers to leave the labor force instead of becoming unemployed, generating the same problems as those reported in this paper.

[23] Another consequence of the high value of leisure is that profits are small. As a result, vacancies become very responsive to changes in aggregate productivity.

A. Appendix

This appendix describes the computational algorithm used to solve the social planner's problem, which is to maximize utility (2.1) subject to equations (2.2), (2.3), (3.4), (3.5), (3.6), (3.13), (3.14), and (3.15) taking a_0, K_0 and q_0 as given.[24]

It is straightforward to show that if two islands have identical current productivity levels z_t and identical number of agents available at the beginning of the period $l_{t-1}\left(z^{t-1}, x_0\right) + pU_{t-1}$, that the social planner will assign identical continuation plans to these islands independently of their previous history z^{t-1} and initial x_0.[25] Thus, all the heterogeneity across islands that is relevant to the social planner can be summarized by the distribution μ_t of islands across pairs (x, z), where x is the number of agents available to an island at the beginning of the period and z is its current productivity level.

The social planner's problem in recursive form can then be written as follows:

$$V(a_t, \mu_t, K_t) = \max_{\{l_t, U_t, K_{t+1}\}} \left\{ R\left(a_t, \mu_t, K_t, l_t, U_t, K_{t+1}\right) + \beta E V(a_{t+1}, \mu_{t+1}, K_{t+1}) \right\} \tag{A.1}$$

subject to

$$\mu_{t+1}\left(X', Z'\right) = \int_{\{(x,z): l_t(x,z) + pU_t \in X'\}} Q(z, Z')\mu_t(dx, dz), \tag{A.2}$$

$$l_t(x, z) \leq x, \tag{A.3}$$

$$a_{t+1} = \rho_a a_t + \varepsilon_{t+1}, \tag{A.4}$$

where the return function R describes the utility that can be obtained with the state and decision variables $(a_t, \mu_t, K_t, l_t, U_t, K_{t+1})$.

It is also straightforward to show that the optimal labor allocation rule has the following form:

$$l_t(x, z; a_t, \mu_t, K_t) = \min\left\{ m_t(z; a_t, \mu_t, K_t), x \right\}, \tag{A.5}$$

where $m_t(z; a_t, \mu_t, K_t)$ is an employment threshold independent of x. The (S,s) nature of the

[24] The computational algorithm is closely related to those in Veracierto (2002, 2008).

[25] This is a direct consequence of convexity.

employment allocation rule, together with the assumption of a finite number of values for the idiosyncratic productivity level, imply that the steady state distribution μ^* has a finite support $\mathbf{x}^*$.

In what follows, the jth element of $\mathbf{x}^*$ will be denoted by $\mathbf{x}^*(j)$ and the total number of elements in $\mathbf{x}^*$ will be denoted by J. Moreover, it will be useful to classify the elements of $\mathbf{x}^*$ into two sets: 1) those that correspond to islands that in the previous period let some agents go (set $\mathcal{G}^*$), and 2) those that correspond to islands that in the previous period did not let anybody go (set $\mathcal{L}^*$). That is, for $j = 1, ..., J$:

$$\begin{aligned} j &\in \mathcal{G}^*, \text{ if } \mathbf{x}^*(j) = m^*(z) + pU^* \text{ for some } z \\ j &\in \mathcal{L}^*, \text{ if } \mathbf{x}^*(j) = \mathbf{x}^*(j-1) + pU^* \end{aligned} \tag{A.6}$$

Observe that equation (A.6) implicitly assumes a particular ordering of the elements of $\mathbf{x}^*$.

If the aggregate productivity shock is small enough that m_t and U_t fluctuate in a small neighborhood of their steady state values m^* and U^*, the distribution μ_t will have a finite support $\mathbf{x}_t$ of dimension J (same dimension as $\mathbf{x}^*$) at every date t. Moreover, the support will evolve as follows:

$$\mathbf{x}_{t+1}(j) = \left\{ \begin{array}{c} m_t(z) + pU_t, \text{ if } j \in \mathcal{G}^*, \\ \mathbf{x}_t(j-1) + pU_t, \text{ if } j \in \mathcal{L}^* \end{array} \right\}, \text{ for } j = 1, ..., J, \tag{A.7}$$

where z satisfies that $\mathbf{x}^*(j) = m^*(z) + pU^*$, and $\mathcal{G}^*$ and $\mathcal{L}^*$ are defined by equation (A.6).

This allows to reformulate the social planner's problem as follows:

$$V(a_t, \mathbf{x}_t, K_t) = \max_{\{m_t, U_t, K_{t+1}\}} \left\{ \widetilde{R}(a_t, \mathbf{x}_t, K_t, m_t, U_t, K_{t+1}) + \beta EV(a_{t+1}, \mathbf{x}_{t+1}, K_{t+1}) \right\}, \tag{A.8}$$

subject to (A.4) and (A.7).[26]

The advantages of working with this transformed problem (A.8) instead of the original problem (A.1) are that it has a finite number of variables and that its constraints describe

[26] This reformulation uses the fact that the labor allocation rule l_t is completely determined by the employment thresholds m_t through equation (A.5).

linear laws of motion. Since all the endogenous arguments of $\widetilde{R}$ take positive values in the deterministic steady state, a second order Taylor expansion around the deterministic steady state can be performed to obtain a quadratic return function. This leaves a standard linear-quadratic structure that can be solved using standard techniques. The assumption that a_t, $\mathbf{x}_t$, K_t, m_t, and U_t fluctuate in a sufficiently small neighborhood of their deterministic steady state values is satisfied in all the experiments reported in this paper.

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Table 1

Parameter values

Table	Case	π_U	π_N	ρ_z	σ_z^2	A	ϕ
3	$\pi_U = \pi_N$	0.447	0.447	0.795	0.0223	1.823	0.60
	$\pi_U = 0.01\pi_N$	0.005	0.472	0.553	0.0003	1.852	0.60
4	$p = 1.00$	0.231	0.461	0.786	0.0092	1.829	0.60
	$p = 0.75$	0.231	0.461	0.767	0.0127	1.829	0.60
	$p = 0.50$	0.231	0.461	0.720	0.0251	1.829	0.60
5	$\eta = 0.00$	0.231	0.461	0.767	0.0127	1.829	0.60
	$\eta = 0.25$	0.231	0.461	0.774	0.0181	1.823	0.60
	$\eta = 0.50$	0.231	0.461	0.781	0.0324	1.811	0.60

Table 2

U.S. business cycle statistics

Relative standard deviation (σ_x/σ_Y)	
output	1.00
consumption	0.57
investment	4.28
capital	0.43
employment	0.57
unemployment	6.25
labor force	0.20
productivity	0.63
Correlation with output $(\rho_{x,Y})$	
output	1.00
consumption	0.80
investment	0.91
capital	0.05
employment	0.81
unemployment	-0.83
labor force	0.39
productivity	0.84

Table 3

Home production while unemployed

(Fixed and flexible labor force)

Relative standard deviation (σ_x/σ_Y)				
	$\pi_U = \pi_N$		$\pi_U = 0.01\pi_N$	
	Flex LF	Fixed LF	Flex LF	Fixed LF
output	1.00	1.00	1.00	1.00
consumption	0.32	0.35	0.31	0.32
investment	4.59	4.37	4.63	4.55
capital	0.32	0.31	0.32	0.31
employment	0.57	0.03	0.57	0.51
unemployment	1.47	0.41	1.22	7.84
labor force	0.58	0.0	0.56	0.0
productivity	0.46	0.98	0.45	0.51

Correlation with output $(\rho_{x,Y})$				
	$\pi_U = \pi_N$		$\pi_U = 0.01\pi_N$	
	Flex LF	Fixed LF	Flex LF	Fixed LF
output	1.00	1.00	1.00	1.00
consumption	0.90	0.92	0.89	0.90
investment	0.96	0.99	0.99	0.99
capital	0.17	0.18	0.17	0.17
employment	0.98	0.76	0.98	0.98
unemployment	0.38	-0.76	0.30	-0.97
labor force	0.97	0.0	0.97	0.0
productivity	0.97	1.00	0.97	0.98

Table 4

Low arrival rates

(Flexible labor force)

Relative standard deviation (σ_x/σ_Y)			
	$p = 1.00$	$p = 0.75$	$p = 0.50$
output	1.00	1.00	1.00
consumption	0.32	0.31	0.31
investment	4.61	4.61	4.61
capital	0.32	0.32	0.32
employment	0.56	0.56	0.56
unemployment	1.76	2.14	3.74
labor force	0.57	0.58	0.59
productivity	0.46	0.46	0.46

Correlation with output $(\rho_{x,Y})$			
	$p = 1.00$	$p = 0.75$	$p = 0.50$
output	1.00	1.00	1.00
consumption	0.89	0.89	0.89
investment	0.99	0.99	0.99
capital	0.17	0.17	0.17
employment	0.98	0.98	0.98
unemployment	0.30	0.22	0.12
labor force	0.96	0.95	0.92
productivity	0.97	0.97	0.97

Table 5

Endogenous search intensity: Search requires goods

(Fixed and flexible labor force)

Relative standard deviation (σ_x/σ_Y)						
	Fixed Labor Force			Flexible Labor Force		
	$\eta = 0.00$	$\eta = 0.25$	$\eta = 0.50$	$\eta = 0.00$	$\eta = 0.25$	$\eta = 0.50$
output	1.00	1.00	1.00	1.00	1.00	1.00
consumption	0.35	0.35	0.35	0.31	0.32	0.31
investment	4.39	4.38	4.33	4.61	4.58	4.50
capital	0.30	0.31	0.31	0.32	0.32	0.32
employment	0.09	0.07	0.05	0.56	0.56	0.56
unemployment	1.39	1.08	0.70	2.14	2.18	2.26
labor force	0	0	0	0.58	0.58	0.59
productivity	0.92	0.94	0.96	0.46	0.46	0.46

Correlation with output $(\rho_{x,Y})$						
	Fixed Labor Force			Flexible Labor Force		
	$\eta = 0.00$	$\eta = 0.25$	$\eta = 0.50$	$\eta = 0.00$	$\eta = 0.25$	$\eta = 0.50$
output	1.00	1.00	1.00	1.00	1.00	1.00
consumption	0.92	0.92	0.92	0.89	0.89	0.90
investment	0.99	0.99	0.99	0.99	0.99	0.99
capital	0.18	0.18	0.18	0.17	0.16	0.15
employment	0.84	0.88	0.97	0.98	0.98	0.98
unemployment	-0.84	-0.88	-0.97	0.22	0.25	0.29
labor force	0	0	0	0.95	0.95	0.95
productivity	1.00	1.00	1.00	0.97	0.97	0.97

Table 6

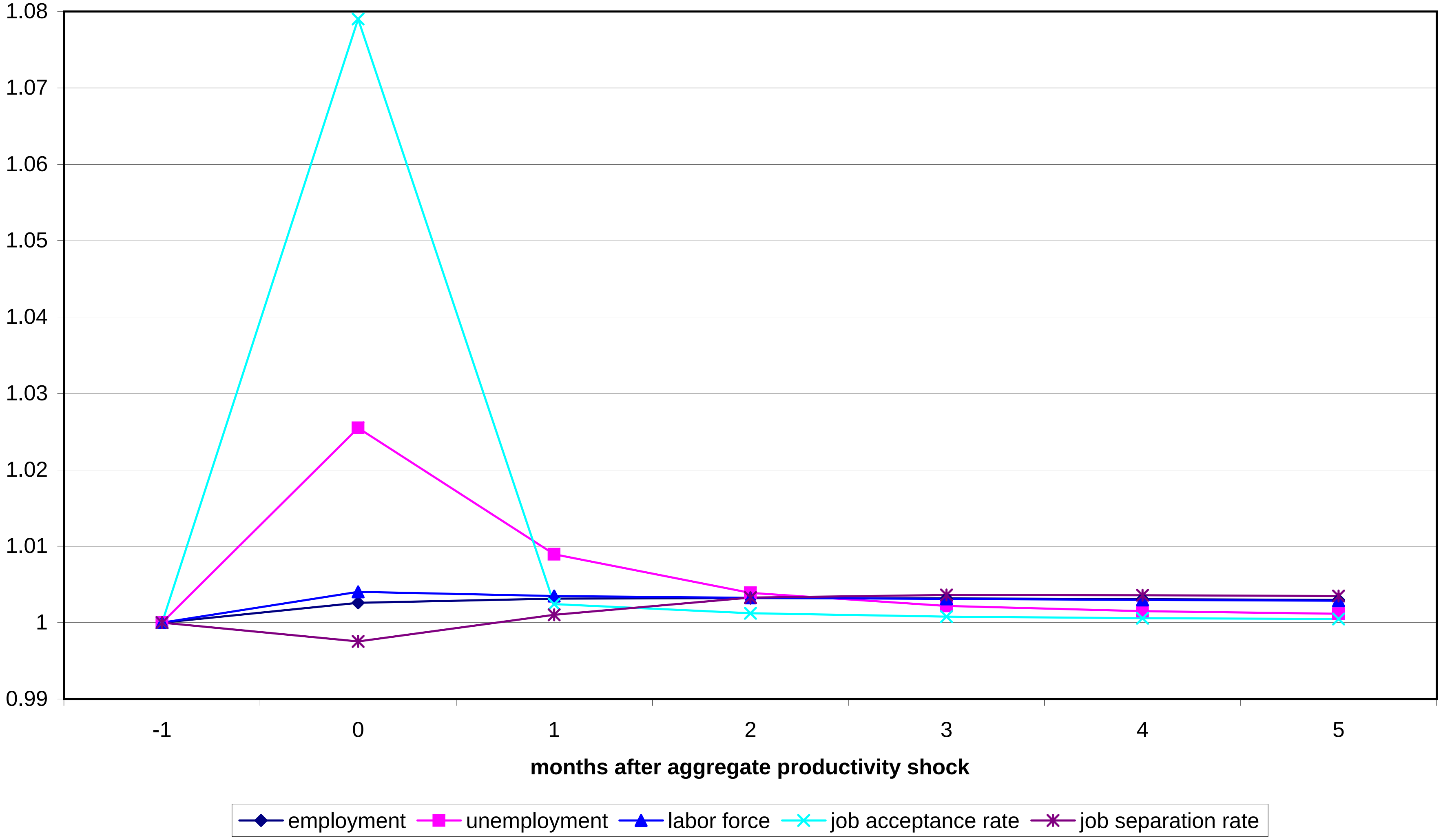
Endogenous search intensity: Search requires effort

(Fixed and flexible labor force)

Relative standard deviation (σ_x/σ_Y)						
	Fixed Labor Force			Flexible Labor Force		
	$\eta = 0.00$	$\eta = 0.25$	$\eta = 0.50$	$\eta = 0.00$	$\eta = 0.25$	$\eta = 0.50$
output	1.00	1.00	1.00	1.00	1.00	1.00
consumption	0.35	0.35	0.35	0.31	0.32	0.32
investment	4.39	4.39	4.37	4.61	4.61	4.60
capital	0.30	0.31	0.31	0.32	0.32	0.32
employment	0.09	0.08	0.06	0.56	0.56	0.56
unemployment	1.39	1.15	0.75	2.14	2.11	3.59
labor force	0	0	0	0.58	0.58	0.59
productivity	0.92	0.93	0.95	0.46	0.46	0.46
Correlation with output $(\rho_{x,Y})$						
	Fixed Labor Force			Flexible Labor Force		
	$\eta = 0.00$	$\eta = 0.25$	$\eta = 0.50$	$\eta = 0.00$	$\eta = 0.25$	$\eta = 0.50$
output	1.00	1.00	1.00	1.00	1.00	1.00
consumption	0.92	0.92	0.92	0.89	0.89	0.90
investment	0.99	0.99	0.99	0.99	0.99	0.99
capital	0.18	0.18	0.18	0.17	0.17	0.17
employment	0.84	0.87	0.98	0.98	0.98	0.98
unemployment	-0.84	-0.88	-0.96	0.22	0.24	0.12
labor force	0	0	0	0.95	0.95	0.92
productivity	1.00	1.00	1.00	0.97	0.97	0.97

Figure 1

Labor market impulse-response functions



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