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The Quest for the Natural Rate: Evidence from a Measure of Labor Market Turbulence

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The Quest for the Natural Rate: Evidence from a Measure of Labor Market Turbulence.*

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Abstract

The natural rate of unemployment changes over time as a result of structural shifts in the distribution of employment across sectors. This paper develops a Kalman filtering approach to measuring these structural shifts or turbulence. The technique is useful in distinguishing those changes in employment shares within an industry that are cyclical from those that are idiosyncratic. A measure of the business cycle is generated that is consistent with alternative measures of the cycle. A time–varying measure of the natural rate is constructed as the rate of unemployment that is consistent with the economy growing at its equilibrium rate, conditional on the level of labor market turbulence. The natural rate of unemployment increased over the seventies in response to increased dispersion in the labor market and has declined steadily since the late eighties. Results indicate that the gap between the actual and natural rates of unemployment enters negatively and significantly in inflation regressions. This result holds across a variety of specifications.

Introduction

Friedman (1968) and Phelps (1968) originated the concept of a "natural rate" of unemployment almost three decades ago and sparked a debate that contin-

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ues today.[1] The basic premise is that when unemployment falls below its natural rate, excess demand for labor produces upward pressure on real wage rates. Conversely, for unemployment above the natural rate, excess labor supply puts downward pressure on real wage rates. The twin benefits of low inflation and low unemployment that has occurred in the late nineties would appear to be the result of a natural rate either around or below the current rate of unemployment. To much discomfort, point estimates of the natural rate are rarely found below 5.5%.[2]

Since Friedman's original insight, the quest for the natural rate has proved to be chimerical. There is little consensus as to how to measure the natural rate nor agreement on the mechanisms linking the natural rate to inflation. After examining the evidence, Blanchard and Katz (1997) concluded that "Economists are a long way from having a good quantitative understanding of the determinants of the natural rate." [3]

Despite such pessimism, King and Watson (1994) document that "there is a pronounced, negative correlation of inflation and unemployment at business cycle frequencies, which is remarkably stable over the post-war period.[4] This result is repeated in Staiger, Stock and Watson (1997), prompting the authors to state that "there is a downward-sloping Phillips curve; it simply is difficult to estimate the level of unemployment at which the curve predicts a constant rate of inflation."[5]After reviewing the evidence, Stiglitz (1997) is convinced that the natural rate concept "is useful as a theory to understand the causes of inflation. It is useful as a general guideline for thinking about macroeconomic policy. And, it is useful as an empirical basis for predicting changes in the inflation rate."

Friedman (1968) was careful to note that the natural rate is not "immutable and unchangeable."[6] Nonetheless, most empirical work assumes either a constant natural rate or one that changes only in a very simple way. For example, Gordon (1982), Weiner (1994), and Tootell (1994) model the natural rate of unemployment as having discrete jump points. More recently, Gordon (1997) acknowledges the

[1]The terms "natural rate of unemployment" and "Non-Accelerating Inflation Rate of Unemployment," or NAIRU, are used interchangeably.

[2]The Congressional Budget Office estimates the NAIRU to be 5.8% in 1996. The council of Economic Advisors estimates the natural rate to be 5.7% in 1996 and Gordon (1997) finds it to be 5.6%.

[3]Page 52.

[4]Page 29.

[5]Page 47.

[6]Page 9.

limitations of this approach in explaining the recent economic environment and proposes an alternative method that models the natural rate as following a random walk. He uses a time–varying parameter regression model to obtain estimates. Staiger, Stock, and Watson (1997) also estimate a time–varying natural rate of unemployment. They model the natural rate as a cubic spline with two knot points at evenly spaced intervals in their sample.

Although these attempts to make the empirical specification more flexible have succeeded in lowering current estimates of the natural rate, they suffer from three serious problems. First, as emphasized in Staiger, et. al (1997), confidence intervals are uncomfortably large. Although Gordon (1997) does not report confidence intervals for his estimates of the natural rate, the problem would appear to exist in his formulation as well. Second, the assumptions of the natural rate following a random walk or the alternative of it being modeled as a cubic spline do not offer any insight into the nature of the change in the NAIRU. The reasons behind the perceived changes in the natural rate are not elucidated. Third, the estimation of a time–varying natural rate in these formulations requires leads in the data to correctly evaluate the natural rate at any given time. For policy–makers who must make decisions in real–time, waiting for additional data to pinpoint the current NAIRU is not a meaningful way to formulate policy.

Friedman (1968) recognized that certain market characteristics such as minimum wage requirements and labor union strength would tend to make the natural rate somewhat higher than it would otherwise be. Furthermore, any activity that would facilitate the flow of workers to employers, such as improved quality and availability of job information, would tend to reduce the natural rate of unemployment. Lilien (1982) attempted to quantify this last point by emphasizing the effect of structural change on the natural rate. According to Lilien, when a sectoral shift occurs in the structure of the economy, whether it be across industries, occupations, or geographical regions, the natural rate of unemployment rises. In the declining sector, workers lose their jobs but cannot find reemployment instantaneously. The displaced worker may lack the necessary skills to be hired in the expanding sector or he may be in the wrong location. Information flows may be imperfect and he may not accurately gauge the distribution of opportunities available to him. In any event, a mismatch arises and a period of increased unemployment ensues even though there is no change in the overall level of economic activity. This increased temporary unemployment can be thought of as structural unemployment and should be reflected in a rise in the natural rate.

In order to test his basic insight that sectoral shifts lead to increases in the natural rate of unemployment, Lilien (1982) first had to measure sectoral shifts. He measures structural change as the dispersion in employment growth across industries and found that dispersion increases markedly during economic downturns. When Lilien included cyclical variables along with this dispersion measure in a regression with the unemployment rate as the dependent variable, he found the coefficient on the dispersion measure to be large and statistically significant. He interpreted this as support for his thesis that increased sectoral shifts raise the natural rate of unemployment, given the level of economic activity.

Abraham and Katz (1986) argued that Lilien's dispersion measure was simply another proxy for the business cycle. They showed that dispersion can increase both because of shifts in the distribution of employment brought about by structural change and because of shifts that occur as a result of normal business cycle activity having a differential impact across industries. This differential impact of the business cycle on industry employment has been well–documented.[7] For example, economic downturns typically coincide with manufacturing's employment share declining and services' share increasing. The increased employment growth dispersion that Lilien documented during cyclical downturns may merely reflect the fact that economic activity was low rather than some fundamental shift in the composition of employment across industries. As a result of the Abraham and Katz critique, Lilien's results were suspect.

Several economists have attempted to refine Lilien's measurement of structural change. Loungani, Rush, and Tave (1990), Brainard and Cutler (1993), and Genay and Loungani (1997) construct a dispersion measure that is based upon stock returns rather than industry employment. The common theme is that by focusing on stock returns which are forward looking over a long time horizon, the effects of temporary cyclical movements are minimized. Taking a different tack, Neumann and Topel (1991) and Rissman (1993) attempt to eliminate short term cyclical fluctuations in employment shares by decomposing changes in employment shares into permanent and temporary movements. Permanent movements are assumed to be attributable to sectoral shifts while temporary movements are believed to be due to the cycle. The dispersion measure then generated from the permanent movements is quite similar to that originally constructed in Lilien (1982) in that permanent dispersion appears to rise when the overall level of economic activity is weak. The main difficulty with this approach is that it relies on future information to determine whether a current movement in an industry's

[7] Mitchell (1927) and Burns and Mitchell (1946) are early examples of this literature. More recent work includes Stock and Watson (1992) and Rissman (1993).

employment share is "permanent" and, therefore, is of little practical use in the policy debate.

The remainder of this paper explores the effect of an alternative measure of labor market dispersion on unemployment and inflation. In Section 1, a Kalman filtering model is presented in which changes in industry employment shares depend upon an unobserved common component thought of as the business cycle, and an idiosyncratic shock to the industry. A turbulence measure is constructed from the estimated industry shocks that is independent of the business cycle. In Section 2 estimates of the natural rate of unemployment are presented. These estimates are constructed directly from unemployment rate regressions as in Lilien (1982), rather than the standard construction of inferring the natural rate from inflation regressions. Finally, the relevance of these constructed natural rate series for the inflation process is assessed in Section 3. Conclusions are contained in Section 4.

1. Industry Employment Growth and the Business Cycle

Employment growth in a given industry reflects both cyclical and industry-specific factors. In obtaining a measure of sectoral shifts, it is important to distinguish between the two. The approach taken here is in the same spirit as Neumann and Topel (1990) and Rissman (1993). However, the procedure does not rely upon ad hoc definitions of permanence to separate shifts in the distribution of employment across industries into those that are structural in origin from those that are cyclical. A model of net employment growth is proposed which explicitly incorporates cyclical movements as well as an idiosyncratic or structural shift. A measure of labor market turbulence is computed as the dispersion in the portion of industry employment growth that is independent of the business cycle, in a way that is similar to Lilien's (1982) original construct. The turbulence measure constructed here is more intuitive and does not suffer from the difficulty in applying it to a policy context in a timely manner.

Let S_{it} be the share of total employment in industry i. Define $y_{it} \equiv \Delta \ln S_{it} = g_{it} - g_t$, where g_{it} and g_t are employment growth in industry i and aggregate employment growth respectively. Figure 1 shows net annualized growth rates (y_{it}) using quarterly data for the following industries: Construction (CON), Mining (MIN), Durable Manufacturing (MFGD), Nondurable Manufacturing (MFGN), Finance, Insurance and Real Estate (FIRE), Government (GOV), Services (SRV), Wholesale Trade (TW), Retail Trade (TR), and Transportation and Public Utilities (TPU) from 1955 through 1997Q2. In looking at these graphs, there are several important points to note. First, the scale differs markedly from industry to industry with, for example, Mining exhibiting relatively stable employment growth punctuated by a few large swings. Other industries, such as Nondurable Manufacturing, and Transportation and Public Utilities show a similar pattern but with more moderate swings. Second, some industries clearly show a trend in employment share, these include a shrinking Durable and Nondurable Manufacturing sector and an expanding Services industry. Third, there is a pronounced cyclical pattern in some industries, most notably in Durable Manufacturing, Construction, Services, Retail Trade, Wholesale Trade, and Finance, Insurance, and Real Estate.

The data shown in Figure 1 suggest three reasons why industry employment growth can differ from the aggregate. First, an industry's employment share may be trending upwards or downwards over long periods of time. Second, the business cycle may cause employment shares to fluctuate over time. Finally, there is an idiosyncratic portion that is industry–specific, an example of which occurred in the mid-seventies in Nondurable Manufacturing.

Let y_{it} have the following specification:

$$y_{it} = a_i + b_i(L)C_t + u_{it} \tag{1.1}$$

where a_i is a constant varying across industries. It is interpreted as the mean net employment growth in industry i. From Figure 1, we expect, for example, this term to be negative in Nondurable Manufacturing and positive in Services. C_t is some common factor that affects the growth in an industry's employment share. This common factor is interpreted here as the business cycle and its construction will be discussed more fully below.[8] u_{it} is the idiosyncratic shock affecting industry net employment growth at time t; and $b_i(L)$ is a polynomial in the lag operator, also referred to as the factor loadings.[9] Specifically,

$$b_i(L) = b_i^0 + b_i^1 L + b_i^2 L^2. \tag{1.2}$$

[8] Rather than modeling C_t as an unobserved component, an alternative specification would include detrended GDP as an explanatory variable instead. The approach taken here remains agnostic about the proper interpretation of C_t and estimates it from the co–movements in employment growth across industries. To the extent that C_t appears correlated with the business cycle, then it can be interpreted as a proxy for the cycle.

[9] In the text the terms net employment growth in industry i and growth of employment share in industry i are used interchangeably.

The polynomial $b_i(L)$ is a flexible but parsimonious way to allow for the business cycle to have a differential impact across industries on net employment growth. It permits the cycle to lead in one industry and lag in another. It also permits the cycle to have a greater impact in one industry than in another.

Since Mitchell (1927) and later Burns and Mitchell (1946) the concept of a business cycle has been defined as "expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle." Thus, the business cycle is essentially unobservable but can be inferred only through its effects on many dimensions simultaneously.[10] In developing a measure of the business cycle, it is assumed that the business cycle component, C_t, is not observable directly. However, its time series properties are restricted to follow an AR(2) specification so that:

$$C_t = \phi_1 C_{t-1} + \phi_2 C_{t-2} + \varepsilon_t. \tag{1.3}$$

The imposition of an AR(2) process generating the business cycle allows for a succinct characterization of Mitchell's description of recessions being followed by expansions.

To completely specify the model it is necessary to give structure to the two types of shocks, u_{it} and ε_t, where u_{it} can be thought of as a sectoral disturbance and ε_t is a business cycle shock. Specifically, it is assumed that the two types of shocks are mean zero, $E(u_{it}), E(\varepsilon_t) = 0$, for all t and i. Furthermore, the shocks are serially uncorrelated, $E(u_{it}u_{it-s}) = E(\varepsilon_t \varepsilon_{t-s}) = 0$ for all i, t, and $s \neq 0$. [11]Nor are the shocks correlated with one another, $E(\varepsilon_t u_{it-s}) = 0$ for all i, t,and s. [12]The shock in one industry is uncorrelated with the shock in another industry, $E(u_{it}u_{jt-s}) = 0$ for all $s, i \neq j$. [13]Finally, each shock has a finite variance, $E(u_{it}^2) = \sigma_i^2$ and $E(\varepsilon_t) = \sigma_\varepsilon^2$.

There are many ways in which this model can be expanded. For example, an additional unobserved component could be introduced that would explain the co–movements across subsets of the I industries. Additionally, the variance of the u_{it}'s could have a cyclical component. These extensions are useful in obtaining a

[10]In fact, this is the concept behind the business cycle expansion and contraction dates published by the National Bureau of Economic Research. Another example is found in Stock and Watson (1992).;

[11]This assumption about the serial correlation properties of the u_{it}'s could be relaxed fairly easily.

[12]This assumption is important for purposes of estimation.

[13]This assumption could be relaxed but with some care so that there are not too many additional parameters to be identified.

richer econometric specification. However, the purpose here is to focus on shifts that are not the result of a common cycle. This objective is met in the current specification. The additional complexity introduced by these extensions does not appear warranted in light of the difficulties with estimating the relatively few parameters in this model.

As currently specified, the model is not identified without additional restrictions.[14] Neither the scale or sign of the business cycle is defined. To see this, suppose that the common cycle C_t is rescaled by multiplying it by some constant β and define $C_t^* = \beta C_t$. Then $C_t^* = \phi_1 C_{t-1}^* + \phi_2 C_{t-2}^* + \varepsilon_t^*$, where $\varepsilon_t^* = \beta\varepsilon_t$ and $\text{var}(\varepsilon_t^*) = \beta^2\sigma_\varepsilon^2$. Restricting σ_ε^2=1 is not sufficient to identify the parameters of the model since setting $\beta = -1$ results in the same likelihood. To address this problem of indeterminate sign, b^1 is set equal to 1 in Durable Manufacturing. This restriction ensures that the estimate of the cycle is positively correlated with other more traditional measures of the business cycle such as real GDP growth.[15] It also allows for a straightforward comparison of the relative responsiveness of a particular industry, where the comparison is made implicitly with the Durable Manufacturing sector.

The model is very much in the spirit of Burns and Mitchell's (1946) idea of comovement. However, here its interpretation is that the cycle represents that portion of net employment growth that is common across the various industries, while permitting the cycle to differ in its impact on industry employment growth in terms of timing, magnitude, and sign through the b_j^i parameters.[16] The Kalman filter can be used to estimate the following: C_t, $u_{it}, \varepsilon_t, a_i$, b_i^j, ϕ_1, ϕ_2, and σ_i for all i and $j = 1, 2, 3$.[17]

Although the data introduced in Figure 1 cover 10 industry categories, in practice the estimation was implemented using only 8 sectors. Both Services and Mining have been omitted, but for different reasons. Mining is a small sector in terms of its share of total employment. However, its employment is also quite volatile over the time period considered due to strikes. Because of its volatility and relatively small magnitude in the total, it was omitted. There is a potential multicolinearity problem that occurs because the sum of the y_{it}'s is approximately

[14] See Harvey's (1989) discussion of common trends.

[15] The estimation also restricts $\sigma_\varepsilon^2 = 1$.

[16] Ideally, each sector should somehow be weighted according to some scheme. The estimation does not currently take this point into consideration. Rather, it treats each sector as being equally important in determining the measure of the business cycle, C_t. Clearly, an improvement would be to treat larger industries differently from smaller sectors.

[17] See Quah and Sargent (1993) for an example.

0. By omitting a second industry, in this case Services, from the estimation, the problem is avoided.

Preliminary results that are not reported here indicate that further restrictions are needed to guarantee that the Hessian is well-behaved. These restrictions include setting $b^1 = 0$ for both Construction and Nondurable Manufacturing. and setting $b^2 = 0$ in Transportation and Public Utilities. These restrictions are the result of the data imprecisely estimating these coefficients rather than any identification issue. Since the unrestricted coefficients were close to 0, the results reported in the text set these parameters equal to 0. Estimation was carried out for the period from 1954:2 to 1997:2. The BFGS algorithm was used in maximizing the likelihood function. After having obtained estimates of the business cycle, C_t, conditional on information prior to time t, a Kalman smoothing technique was applied that uses all available information through 1997:2 to generate smoothed estimates of C_t.[18]

Estimates of the industry parameters a_i, b_i^0, b_i^1, b_i^2, and σ_i are found below in Table 1. Note that the estimation results also include parameter estimates for both Services and Mining even though these two industries were not directly included in the Kalman filtering exercise. These estimates were derived from a secondary procedure. After having estimated the common cycle, C_t, two additional regressions were run essentially treating C_t as a known exogenous variable. Each regression is of the form found in Equation (1.1). The standard errors reported in Table 1 for both Services and Mining are too small in that they do not take into account the uncertainty in the estimates of C_t.

There are several interesting points to note. First, the constant term is significant in all but Construction and Wholesale Trade, indicating that in these two industries there is no discernible long term trend in employment share. The remaining industries exhibit the familiar story of declining employment share in goods–producing industries and the mirrored increasing employment shares in service–producing industries.

Focusing only on the b^0 parameters, service–producing industries tend to be countercyclical while goods–producing industries are procyclical. Within these broad categories, there is a great deal of variation in the magnitude of each industry's responsiveness to changes in economic activity. Relative to Durable Manufacturing, for example Nondurable Manufacturing is not very cyclical, as expected from a glance at Figure 1. Timing also seems to matter. Some industries receive only a small contemporaneous effect from cyclical movements and

[18]The interested reader can find more detail in Chapter 4 of Harvey (1989).

respond to changes in the business cycle only with a lag.

Equation (1.3) is estimated as:

$$\begin{array}{cccccc} C_t = & 1.3995 * C_{t-1} & - & 0.5519 * C_{t-2} & +\varepsilon_t \\ & (0.0890) & & (0.0838) & \end{array}$$

The actual smoothed estimates for C_t are shown in Figure 2 along with its accompanying standard errors.[19] The Kalman filter technique relied only on information about the employment shares in the 8 industries examined. Gross Domestic Product, for example, did not enter into the estimation. Yet the estimates of the cycle, C_t, are remarkably similar in timing to the NBER Expansions and Contractions.[20] C_t typically declines prior to the peak of an NBER expansion and has a turning point *consistently* within one quarter of the dated NBER trough. The 'mini-recession' of the mid '80s shows up clearly. According to this estimation technique, the '90-'91 recession was only a minor event in comparison to prior recessions. In fact this 'recession' does not even qualify as a significant event when compared to the standard error bands. Confirming the results of other economic data, the 'recovery' for this recession was slow to take hold relative to other recoveries.

The downturn in C_t that shows up for the last couple years does not bode well. Although C_t is currently above the expected long term average of 0, its recent decline has been quite sharp. If history sets any precedent, it would indicate that declines of this magnitude are usually followed by contractions. However, it should be noted that there is substantial uncertainty associated with these measures, both because of normal parameter uncertainty and model uncertainty. In addition, the estimation technique implicitly assumes that the nature of the business cycle remains stable over the estimation period. If, for example, Durable Manufacturing has become a less important determinant of the underlying business cycle, the technique will tend to overstate Durable Manufacturing's importance in the more recent past and understate its importance in the earlier part of the sample.

In addition to the estimates of the business cycle generated by the Kalman filter, the idiosyncratic shocks, u_{it}, are also of interest. The estimated u_{it} 's are shown in Figure 3 for the 10 industry categories. Note that the scale varies widely

[19]Standard errors are calculated as discussed in Hamilton (1994), page 398. The Monte Carlo method provides estimates of the mean squared error of the state vector taking into account both the filter uncertainty and the parameter uncertainty.

[20]NBER contractions have been shaded.

with Construction exhibiting the largest shocks and Retail Trade the smallest on average. Transportation and Public Utilities exhibited relatively small disturbances with the exception of a large shock in the early '80s that is coincident with the timing of the PATCO strike. Similarly, Mining has experienced only small disturbances with the exception of a few large deviations. The large swings in the late '70s are related to the strike by the Bituminous Coal Operators Association affecting approximately 160,000 workers. In addition, the relatively large disturbances in Nondurable Manufacturing occurring in 1975 are likely due to the oil price shock's affect on the Petroleum and Chemicals industries.

1.1. Measuring Economic Turbulence

The Kalman filter exercise provides an intuitive technique for measuring structural shifts in the composition of employment where these shifts are constructed so as to be independent of the business cycle. The measure proposed here is in the same spirit as Lilien (1982) in that it focuses on the dispersion in employment growth across broad industry categories. However, Lilien's measure failed to recognize the effects of the business cycle on dispersion. Thus, the measure of sectoral shifts he proposed does not clearly distinguish between movements in employment share that are the result of the business cycle and the sectoral shifts it purports to measure. For purposes of measuring sectoral shifts, it is the portion of dispersion in employment growth that is unrelated to the business cycle that is of importance. In other words, it is the idiosyncratic shock, u_{it}, that reflects the shifts in an industry's employment growth that are orthogonal to the business cycle.

Let $\widehat{\sigma}_t$ be the acyclic measure of dispersion. It is defined as:

$$\widehat{\sigma}_t \equiv \left[\sum_{i=1}^{I} \widehat{S}_{it}\widehat{u}_{it}^2\right]^{1/2} \tag{1.4}$$

where $\widehat{S}_{it}$ is a measure of employment shares in industry i that is constructed to be independent of the cycle, and the $\widehat{u}_{it}$'s are the estimates of the idiosyncratic shock to net employment growth in industry i that are obtained directly from the Kalman filter. The acyclic employment share, $\widehat{S}_{it}$, is constructed from some initial starting condition, $\widehat{S}_{i0}$, and imposing $C_t = 0$ for all $t = 0, ..., T$. In other words, it is what the employment share would have been if the economy had not experienced any cyclical variation but responded only to idiosyncratic shocks and long term trends. Thus, $\widehat{\sigma}_t$ is a measure of the standard deviation of changes in employment shares across industries where these changes are orthogonal to the business cycle.

This turbulence measure, $\widehat{\sigma}_t$,is shown in Figure 4 along with Lilien's original dispersion measure. Note that all 10 industries have been included in the measure with Services and Mining disturbances estimated by auxiliary regressions of the form described above. Compared to Lilien's measure of sectoral shifts, this acyclic measure fluctuates quite a bit more. However, the timing of the peaks is similar to that found in Lilien. It clearly indicates that the early '70s to early '80s was a period of structural change, so that one would expect to see a rise in the natural rate of unemployment over this time period. Here, as well as in Lilien's original measure, increases in turbulence are coincidental with economic contractions. For Lilien, this increase in dispersion during economic downturns is fundamentally linked to his failure to realize that dispersion is itself cyclical. Such is not the case here. To the extent that the business cycle is reflected in co-movements in employment shares, the measure constructed here is independent of these co-movements. Nontheless, the turbulence measure still appears to be highly cyclical. This fact is not necessarily troubling. For example, suppose that the fundamental distrubance is an oil price shock. Such an event has broad implications for the overall economy in which most sectors are affected simultaneously. However, the shock itself will result in some labor reallocation across industries as those that are more oil-price sensitve shrink. The increase in turbulence during this time reflects the structural reallocative effect of this disturbance rather than the cyclical effect.

Since the early '80s, the economy has been remarkably stable in that there have been no large sectoral shifts between and among industries that impinge upon the labor market. The brief increase in dispersion recorded in the early '90s was much smaller in magnitude than that observed in the previous two decades. In fact, the turbulence measure shows that the current state of labor demand is very stable by historical standards with little deviation in employment distribution. Thus, we would expect to see a lower natural rate than would be found in the '70s and '80s and, indeed, even in the earlier part of the decade.

Closer examination of the relative importance of the various sectors in total dispersion provides an interesting insight. Most of the increase in employment dispersion in the early '70s was directly attributable to shocks in Durable Manufacturing, combined with those in Transportation and Public Utilities. In contrast, the increase in turbulence that occurred about the time of the '74-'75 recession was due to industry-specific shocks in Construction and Nondurable Manufacturing, and to a lesser extent disturbances in Durable Manufacturing and Finance, Insurance, and Real Estate. Surprisingly, the most recent increase in turbulence in the early '90s was due primarily to a shift in the Government sector.

Figure 5 shows a scatter plot of the dispersion measure developed above against Lilien's measure of sectoral shifts. Although there does appear to be a positive correlation between the two measures. This correlation is not perfect. In fact there are several periods in which $\hat{\sigma}_t$ is high while Lilien's corresponding measure, σ_L, is low. Conversely, there are many quarters when Lilien's measure is high while $\hat{\sigma}_t$ is low. Such a period appears to have occurred in the mid to late seventies.

2. Estimates of the Natural Rate

The natural rate is typically estimated from a regression of inflation on various explanatory variables, including the unemployment rate and past inflation. The natural rate of unemployment is then calculated as the rate of unemployment that is consistent with stable long run inflation.[21] As noted by Staiger, Stock, and Watson (1997), this technique of constructing estimates of the natural rate from inflation regressions results in imprecise estimates that are not too informative for predicting inflation.

Rather than start with a definition of the natural rate as being defined by the long term inflation–unemployment trade-off, Lilien (1982) instead modeled the underlying determinants of the natural rate. In Lilien's framework the natural rate of unemployment is defined as the rate of unemployment that would occur if there were no cyclical variation affecting the unemployment rate in the long run. In this methodology, it is the unemployment rate that would be ground out by the Walrasian auctioneer noted in Friedman (1968).

Lilien generated alternative estimates of the natural rate of unemployment based upon an unemployment rather than an inflation regression. He modeled the unemployment rate as depending upon cyclical factors and structural shifts that affect the natural rate. The natural rate is then constructed as the rate of unemployment consistent with stable growth in the economy.

Using a different measure of structural shifts, Rissman (1993) showed that such natural rate estimates explained the apparent shifts in the Phillips curve that occurred in the 1970s and 1980s. The methodology is attractive in that a time–

[21] Gordon (1997 and 1982) provide examples of this method.

varying natural rate is a natural outcome of the estimation. Furthermore, unlike other attempts to estimate a time-varying natural rate directly from inflation regressions, the underlying determinants of the natural rate are dealt with more explicitly here.[22] In this formulation the movements in the natural rate are the result of structural shifts, but the underlying reasons for these shifts are not elucidated.

2.1. Modeling the Unemployment Rate:

Assume that the unemployment rate is described by the following regression equation:

$$U_t = \alpha + \beta(L)U_{t-1} + \gamma(L)Z_t + \delta(L)X_t + v_t. \tag{2.1}$$

U_t is the unemployment rate at time t; $\beta(L)$ is a polynomial in the lag operator; Z_t reflects current and lagged cyclical factors; X_t is a vector of variables thought to affect the unemployment rate but that are independent of the cycle. Finally, v_t is an iid error with finite variance.

The analysis presented here uses the Perry-weighted unemployment rate as the dependent variable, where the unemployment rate has been adjusted to reflect shifts in the age, sex, and racial composition of the labor force. Augmented Dickey–Fuller tests were performed with the result that the null hypothesis of a unit root in the unemployment rate could not be rejected at conventional significance levels. This result held for lag lengths up to and including 12 quarters.[23] Consequently, in the following unemployment rate regressions the dependent variable is entered as a first difference.

There are potentially several different measures of the cycle that could be used. The results discussed here include the measure of the business cycle that the Kalman filter exercise generates, C_t. In addition shocks to nonborrowed reserves ($NBRD$) and a quarterly variable constructed similar to Barro's unanticipated money growth ($UAGM$) are also included in the vector Z_t.[24] Included in X_t is a measure of sectoral shifts. Two different measures were evaluated. These include Lilien's original measure (σ_L), updated through 1997Q2, and the measure developed in Section 1.1, $\widehat{\sigma}$.

[22] See Staiger, Stock, and Watson (1997), Gordon (1997), and Weiner (1994) for examples of the former approach.

[23] For example, the Dickey-Fuller test statistic ($\widehat{\tau}_\mu$) with a lag length of 8 is estimated to be -1.8664. This is well above the 10% critical value of -2.58 so that the hypothesis of a unit root cannot be rejected.

[24] The Fed Funds rate shock is constructed as in Christiano, Eichenbaum, and Evans (1996).

Table 2 describes the results for equations estimated without any of the cyclical explanatory variables. Table 3 describes selected results for equations estimated with cyclical explanatory variables. The variable $SMPL$ is a dummy variable taking on the value of 1 for the post 1970 period.

From Table 2, the estimated regressions suggest that the unemployment rate is best described as a random walk with no drift. In general, the unemployment rate regressions that use the Lilien measure of sectoral shifts σ_L have higher adjusted R^2 values than those that use the measure of turbulence $\widehat{\sigma}$ constructed above. There is no compelling reason why the turbulence measure $\widehat{\sigma}$ should be calculated as the square root of the sums of the weighted squared shocks. There are innumerable alternatives. For example, if one believed that the seventies and early eighties were particularly volatile, an alternative measure such as the variance, $\widehat{\sigma}^2$, would be appropriate since it has the effect of increasing the turbulence during this time period relative to others. To test whether this variance measure better describes the unemployment rate, both $\widehat{\sigma}$ and $\widehat{\sigma}^2$ were included both separately and together in the regressions. The results are found in regressions (1), (3), and (5). When entered separately, the contemporaneous effect of an increase in turbulence is to increase the unemployment rate. Lags of both turbulence and its square do not follow any apparent pattern and are largely insignificant. In regression (5), although turbulence still has a positive effect on the unemployment rate, neither turbulence nor its square enters significantly.

A similar analysis is done for the Lilien measure σ_L and its square, σ_L^2. The results of these regressions are found in columns (2), (4), and (6) of Table 2. In contrast to the turbulence measure described above, there does appear to be a difference in the adjusted R^2 depending upon whether σ_L or σ_L^2 is employed, with the latter measure giving a better fit to the data. Interestingly, the contemporaneous effect of structural shifts on the unemployment rate is positive and significant while the lags are negative, significant, and of the same magnitude. This same result is documented in Rissman (1993). When both σ_L and σ_L^2 enter the regression, only the σ_L^2 parameters are significant with contemporaneous structural change increasing the unemployment rate.

Regression results when cyclical variables are included as explanatory variables in the unemployment rate regressions are documented in Table 3. The lag of changes in the unemployment rate is highly significant in each of these regressions. Equation (1) of Table 3 repeats equation (5) of Table 2 but includes the business cycle variable C estimated from the Kalman filter exercise as an explanatory variable. The fit of the equation increases markedly as a result of its inclusion with the $\overline{R}^2$ increasing from 0.4224 to 0.7303. Contemporaneous turbulence tends to increase the unemployment rate significantly and has a significant nonlinear effect. This compares with insignificant parameter estimates on turbulence in the regression where the business cycle measure C is not included in the regression As anticipated, the business cycle variable has a negative impact on changes in the unemployment rate.

The same analysis is repeated using the Lilien sectoral shifts measure with the corresponding regression results found in (3.2) and (2.6)[25] As with the turbulence measure, the inclusion of the cyclical variable C makes a difference in the $\overline{R}^2$ of the regression. The effect of sectoral shifts is less precisely estimated in Regression (3.2) with the sum of lags of σ_L^2 not significantly different from 0. There also appears to be a significant increase in the change in the unemployment rate in the latter part of the sample period once C is included in the analysis.

Columns (3)–(6) examine the separate effect of alternative cyclical measures on changes in the unemployment rate. These measures include shocks to Non-Borrowed Reserves and Unanticipated Money Growth. To summarize, when current and lags of shocks to Non-Borrowed Reserves and Unanticipated Money Growth are included in the regressions without inclusion of the cyclical term C, their cumulative and individual effects on changes in the unemployment rate are insignificantly different from 0 and consequently have little impact on the $\overline{R}^2$ statistics.

Columns (7) through (10) report the regression results for estimation that includes both C and either current and lagged $UAGM$ or $NBRD$. Again, the inclusion of Unanticipated Money Growth has little impact upon the results as shown in columns (7) and (8). Interestingly, when shocks to Non-Borrowed Reserves are included simultaneously with the cyclical measure C, the effect on changes in the unemployment rate is negative and significant.[26] This can be seen from columns (9) and (10).

Finally, regression results that include C, $UAGM$, and $NBRD$ are found in columns (11) and (12). When all three cyclical variables are included in the regressions, $UAGM$ still has little impact upon the results. Contemporaneous turbulence enters significantly and has the theorized sign. The sum of the lags are insignificantly different from 0. In addition the equation appears to be stable in that the intercept term is insignificant throughout the entire period examined. This contrasts with results for Lilien's structural change measure. The intercept

[25] The notation $(a.b)$ refers to Column b of Table a.

[26] This corresponds with the results found in Christiano, Eichenbaum, and Evans (1996)

term is significant and increases in later periods, as shown in column (12).

2.2. Constructing the Natural Rate of Unemployment:

The results of the previous section can now be used to generate estimates of the natural rate of unemployment, U^*. Recall that the Perry-weighted unemployment rate appears to follow a random walk with the following form:

$$\Delta U_t = \alpha + \beta \Delta U_{t-1} + \sum_{i=0}^{2} \gamma_i C_{t-i} + \sum_{i=0}^{p} \delta_i NBRD_{t-i} + \sum_{i=0}^{q} \mu_i \widehat{\sigma}_{t-i} + v_t. \qquad (2.2)$$

The natural rate is the rate of unemployment that would have occurred had the economy been growing at its equilibrium rate, which by definition implies that the business cycle is neutral in its effects on the natural rate. This definition does not imply that the natural rate is itself constant, since other factors such as turbulence cause the amount of search unemployment to vary over time. In the context of Equation (2.2), when the economy is in equilibrium, cyclical variables are set to their long–run values. This implies that the change in the natural rate is constructed as:

$$\Delta \widehat{U}_t^* = \widehat{\alpha} + \widehat{\beta} \Delta \widehat{U}_{t-1}^* + \sum_{i=0}^{q} \widehat{\mu}_i \widehat{\sigma}_{t-i} \qquad (2.3)$$

where the notation '$\widehat{x}$' is the estimated value of x. The change in the natural rate between period t and period 0, $U_t^* - U_0^*$, can be constructed as:

$$\widehat{U_t^* - U_0^*} = \sum_{i=1}^{t} \Delta \widehat{U}_i^*. \qquad (2.4)$$

Figure 6 shows estimates of the deviation of the natural rate from its starting value, $U_t^* - U_0^*$, for three different unemployment regressions such as those found in equation (2.2). The starting period is 1956:4 and $q = 8$ for all three of the estimated models. The solid line reflects natural rate changes for a model in which the effect of labor market turbulence is described by the turbulence measure $\widehat{\sigma}$. The short dashed line shows natural rate changes for a model in which the effect of turbulence is captured by $\widehat{\sigma}^2$. Finally, the last set of estimates shown in the graph is for a model in which both $\widehat{\sigma}$ and $\widehat{\sigma}^2$ are included. The time series pattern is quite similar for all three of the models. The sixties were a relatively stable period in which the natural rate remained fairly constant, although it increased relative to its level in the reference period. In the early seventies the natural rate increased rapidly as a result of the economic turbulence that is captured by the acyclic shocks to changes in industry employment share. By the early eighties, the natural rate had risen to over 4% higher than its level in 1956:4. The subsequent period from the early eighties to the present has been remarkably devoid of structural change. This is reflected in the steady decline in the natural rate over this time period. By 1997:2 the natural rate had declined over 2.5 percentage points from its high in the early eighties, although it was still in excess of its reference level in 1956:4.

Calculating confidence intervals for the natural rate estimates, $\widehat{U_t^* - U_0^*}$, is somewhat complicated due to the cumulative nature of the computation. A bootstrapping methodology provides a way to construct these confidence intervals in a relatively straightforward fashion. A two-step process is employed. First, equation (2.2) is estimated using the available data and estimates of the natural rate $\widehat{U_t^* - U_0^*}$ are constructed. The estimated errors from the regression are used to construct an artificial dataset where shocks are drawn from the distribution of estimated errors with replacement. In the second step, this newly constructed shock series is used to generate artificial data. This data assumes the parameters of equation (2.2) are known constants equal to the estimated coefficients in the first step of the process. Initial values are given by the initial values of the original data. Equation (2.2) is then reestimated using the new artificial data and an alternative estimate of the natural rate is constructed. This process is repeated 2000 times. The 95% confidence interval is constructed as the range of values for $\widehat{U_t^* - U_0^*}$ that occurred in 95% of the estimates.

Figure 7 shows bootstrapped confidence intervals for the natural rate derived from estimates of equation (2.2) in which $\widehat{\sigma}$ and its square are included as explanatory variables. The natural rate is constructed over the period from 1957:1 through 1997:2 with $t = 0$ corresponding to 1956:4. The confidence interval for $\widehat{U_T^* - U_0^*}$ is so large that one cannot say with any degree of certainty whether the natural rate has increased or decreased relative to its value in 1956:4. The unemployment rate is estimated in first differences rather than in levels. To obtain natural rate estimates relative to its starting value at time 0, changes in the natural rate are cumulated. This cumulation implies that, roughly speaking, past errors are added to current errors, giving rise to expanding confidence bands.

The expanding confidence bands does not imply that short term changes in the natural cannot be determined with accuracy. Indeed, changes in the natural rate can be estimated reasonably well over short time horizons. Only when these changes are accumulated over long periods of time is the range uncomfortably large. For example, to assess whether the natural rate has declined relative to its maximum in 1985:2, the same bootstrapping method is used. Estimates of the natural rate relative to its value in 1985:2 are constructed. This estimate along with the relevant 95% confidence intervals are shown in Figure 8.[27] In 1997:2, only 6% of the 2000 bootstrapped estimates showed an increase in the natural rate of unemployment for this model relative to its level in 1985:2; 81% exhibited a greater than 1% decline and 57% exhibited at least a 2% decline. Put another way, the likelihood that the natural rate was lower in 1997:2 than it was in 1985:2 was 94%.

3. The Natural Rate and Inflation

In contrast to the methodology employed here, the NAIRU has most often been inferred by a process which takes as a starting point an expression relating inflation or the change in inflation to the gap between the unemployment rate and the NAIRU. For example, Staiger, Stock, and Watson (1997) begin their analysis of the NAIRU with a regression model such as the following:

$$\Delta\pi_t = b(L)(UR_t - U_t^*) + \gamma X_t + v_t \tag{3.1}$$

where $b(L)$ is a polynomial in the lag operator, U_τ^* is the natural rate at time τ, X_t is a vector of additional variables that includes lags of $\Delta\pi_t$ as well as supply shock variables. Assuming a constant natural rate of $\overline{U}$, the above equation can be rearranged and estimated by OLS. Specifically,

$$\Delta\pi_t = \mu + b(L)UR_t + \gamma X_t + v_t$$

where $\mu = -\sum_i b_i\overline{U}$. The natural rate is simply estimated as:

$$\widehat{\overline{U}} = -\widehat{\mu}/\sum_i \widehat{b}_i$$

where the '$\widehat{x}$' signifies the OLS estimate of x. Various procedures have been proposed to obtain time–varying estimates of the natural rate, again taking as the

[27]The natural rate series is obtained from unemployment regressions employing $\widehat{\sigma}$ as the only explanatory variable describing turbulence. Note that this estimate of the natural rate is in general higher than the other two series constructed and shown in Figure 6.

foundation the relation between inflation and the unemployment gap that is assumed in equation (3.1). Essentially, these methods rely upon a specification for how the natural rate evolves over time. This specification is then incorporated into the estimation of equation (3.1). For example, Staiger, Stock, and Watson (1997) model the natural rate as a cubic spline with two knot points. Gordon (1982), Weiner (1993), and Tootell (1994) model the natural rate as having discrete changes while remaining constant over long periods.

The methodology proposed here is distinct from the traditional analysis in that the natural rate of unemployment has been constructed separately with no reference as to its effect on inflation. In addition, the NAIRU that results from this approach is clearly time–varying. The separation of estimation of the natural rate of unemployment from the inflation process provides an opportunity to evaluate whether the estimates of the NAIRU that are obtained from this approach are consistent with the model of the inflation process proposed in equation (3.1). Let $\widetilde{U}_t \equiv \widehat{U_t^* - U_0^*}$ as constructed above in Section (2.2). Equation (3.1) can be expressed as:

$$\Delta\pi_t = \mu + b(L)(UR_t - \widetilde{U}_t) + \gamma X_t + v_t \tag{3.2}$$

where $\mu = -\sum_i b_i U_0^*$. This regression can be estimated by OLS. Furthermore, as a by–product of this regression, a true time–varying estimate of the natural rate can be constructed as:

$$U_t^* = \widetilde{U}_t + \left(-\widehat{\mu}/\sum_i \widehat{b}_i\right) \tag{3.3}$$

where the expression in parentheses is the estimate of U_0^* that is obtained from the OLS regression of equation (3.2). Note that the NAIRU at time 0 is constructed in a way which is analogous to the estimation of a constant natural rate in the traditional methodology discussed above.

Table 4 below shows results for inflation regressions of the form found in equation (3.2). In addition to the three different estimates of the natural rate, $\widetilde{U}_t$,constructed above, four different measures of inflation have been examined. These measures of inflation include the Personal Consumption Expenditures Deflator (PCE), Personal Consumption Expenditures Less Food and Energy Deflator (PCEX), the Consumer Price Index for Urban Workers (CPI-U), and the Consumer Price Index Less Food and Energy (CPIX).

Table 4.a reports the results in which the change in inflation is the dependent variable and inflation is calculated as a four quarter difference in the log of the PCE Deflator ($\pi_t = \ln(PCE_t) - \ln(PCE_{t-4})$). Similarly, Table 4.b reports re-

gression results for inflation as measured by the growth rate of $PCEX$. Table 4.c and 4.d report results for inflation measured by the growth of $CPI - U$ and $CPIX$ respectively. The GAP series used in the regressions for Columns (1) and (2) on all four tables is based upon unemployment rate regressions in which $\widehat{\sigma}$ measures turbulence. The results reported in Columns (3) and (4) rely upon GAP estimates for which $\widehat{\sigma}^2$ measures turbulence in the unemployment regression. Finally, the results of Columns (5) and (6) show results for which the GAP estimates are derived from unemployment regressions in which both $\widehat{\sigma}$ and $\widehat{\sigma}^2$ have been introduced as separate regressors. Akaike and Schwarz tests were performed to determine the appropriate lag length for both the dependent and independent variables. Generally, four lags of the dependent variable and 5 lags of the GAP tended to adequately fit the data. Odd numbered columns have been estimated without restrictions. Even numbered columns report Shiller estimates in which the coefficients on GAP_{-1} to GAP_{-5} are restricted to be smooth while GAP_0 is left unrestricted.[28]

From Table 4.a results are consistent across different measures of the GAP. This is not surprising given the high correlation between the different $\widetilde{U}$ series. Coefficients on lags of ΔPCE greater than 2 are not significant. For the unrestricted regressions, the GAP tends to decrease changes in inflation. This can be seen from the statistics reported for $\sum GAP$. From column (1) for example, a sustained positive gap between the unemployment rate and the natural rate $\widetilde{U}$ of 1% results in inflation declines as measured by the PCE deflator of 0.1817 quarterly. In the unrestricted regressions, the GAP tended to be significant only for lags of 4 and 5. The Shiller specification forces the coefficient estimates to become more negative and significant for contemporaneous GAP and generally reduces the magnitude of the coefficients on lags of the GAP. It should be noted that the R^2 from these regressions are not particularly high. In fact the R^2 statistics from the Shiller estimates are substantially lower, leading to the possibility that the restrictions imposed are not well–supported in the data. Serial correlation in the residuals does not appear to be a problem. Finally, the estimates of the starting value of the natural rate (U_0^*) are reported. Note that for GAP

[28] Suppose that a second degree polynomial describes the β_i so that $\beta_i = \alpha_0 + \alpha_1 * i + \alpha_2 * i^2$. This specification implies that $\Delta^3\beta_i = 0$. The Shiller method instead assumes that $\Delta^3\beta_i = w_i$ where $w_i \sim iidN(0, \sigma_w^2)$ and Δ is the difference operator. The resulting estimator is a special case of a Ridge estimator. An iterative procedure suggested by Lindley and Smith (1972) is used to determine the value of the weighting parameter, $k = \sigma_v^2/\sigma_w^2$, which is the ratio of the variance of the inflation error to the variance of the smoothness error. Generally, estimates converged after 3 iterations.

estimates that are derived from turbulence measures σ and σ^2 found in columns (5) and (6), the value of the natural rate at time 0 is roughly half a percentage point larger than the other estimates. This is due to the fact that the $\widetilde{U}_t$ series constructed from unemployment rate regressions that have σ and σ^2 entered as separate explanatory variables is lower than either of the other two estimates. The higher estimate of (U_0^*) in this case corrects for this, leaving the levels similar over time.

In Table 4.b the volatile food and energy components have been omitted from the PCE deflator (PCEX). The sum of the GAP coefficients are negative for each of the regressions reported and their magnitudes are marginally, although not significantly larger than those reported for the PCE regressions. In contrast to the PCE results of Table 4.a, the contemporaneous GAP is large, negative, and significantly different from 0. There is no significant change in the R^2 statistics between the restricted and unrestricted regressions leading to the conclusion that the Shiller estimates explain the data reasonably well. The estimates of U_0^* repeat the same pattern as for the PCE regression results. Specifically, the starting value for the NAIRU derived from the GAP estimates obtained from using both the turbulence measure and its square in the unemployment rate regression are higher by about half a percentage point.

Regression results for inflation as measured by the CPI-U are found in Table 4.c. As for the other inflation measures, the sum of the GAP coefficients is negative and significant. The sum is also larger in magnitude than for the PCE deflator measures. The R^2 is larger as well for these equations than for their counterparts in Tables 4.a and 4.b. The result of applying the Shiller ridge estimator to the GAP coefficients is to shift the weights of the coefficients towards the contemporaneous GAP and away from longer lags.

Regression results for inflation as measured by core inflation ($CPIX$) are found in Table 4.d. Again the sum of the GAP coefficients is negative and significantly different from 0. There is not a great deal of difference in magnitudes between the unrestricted and restricted R^2s, so that the Shiller estimates appear to adequately describe the data. The result of the Shiller procedure is to make the coefficient on the contemporaneous GAP measure more negative and significantly different from 0.

The results described above are robust with respect to other variables included in the vector X_t. These other variables include current and lagged values of the Kalman filter measure of the cycle, C_t,and current and lagged shocks to non–borrowed reserves, $NBRD$. Results are unreported here for the sake of brevity.

However, in all cases the GAP was consistently negatively and usually significantly related to changes in inflation.

4. Conclusions

Estimates of the natural rate vary substantially. This appears to be the result of differences in methodology as well as its being estimated only imprecisely. The assumption of a constant natural rate appears to have little support in the data, as many researchers have noted.[29] The evidence reported in previous research is consistent with a stable natural rate over the sixties, an increasing natural rate during the seventies to mid– eighties, and a declining natural rate since the late–eighties. However, when confidence intervals have been reported, it is clear that even these broad characterizations are subject to a great deal of uncertainty.

Attempts to model the natural rate of unemployment as time–varying have focused on using statistical techniques to capture time–variation in a very simple way. The standard approach is to infer the NAIRU from parameter estimates of an inflation or change in inflation regression. Within this framework for example, the natural rate has been modeled as having discrete changes where the timing and magnitude of these changes are estimated from the data. Alternatively, the natural rate has been modeled as a random walk, again relying on the inflation regression for estimation of the parameters. Still another approach has used a cubic spline to describe the natural rate time series. These techniques, while informative, provide little explanation as to why the natural rate has changed over time.

From a policy perspective many of these approaches are also lacking since the econometrician is able to infer that the natural rate has changed only after some period of time has passed. For example, if the natural rate were to move at date t, and it was modeled as changing only at discrete points, the econometrician would require several more periods of data in order to say with any confidence that the natural rate had indeed shifted. A more reliable current estimate of the natural rate of unemployment would be a valuable input in the policy process.

The focus of the work here is to explicitly model how the natural rate changes over time and to do so within a framework that provides a way to obtain current estimates of the natural rate in a timely manner. The natural rate as constructed

[29]Examples include Lilien (1982), Loungani, Rush, and Tave (1990), Brainard and Cutler (1993), Rissman (1993), Tootell (1994), Gordon (1997), Blanchard and Katz (1997), and Staiger, Stock, and Watson (1997),

here is calculated without explicit reference to inflation. This distinguishes it from the traditional approach which relies on inflation to define the natural rate. In fact, the natural rate of unemployment constructed here is determined by an unemployment regression, rather than an inflation regression. The natural rate is assumed to change depending upon the amount of labor market turbulence affecting the distribution of employment across industries. Only after estimates of the gap between the actual unemployment rate and the natural rate have been constructed is inflation expressly considered. There is no *a priori* reason why the natural rate as defined above should be consistent with the traditional view of the natural rate as the non-accelerating inflation rate of unemployment (NAIRU).

The results are encouraging. First, estimates of the natural rate derived from this approach appear reasonable. The natural rate seems to have been relatively flat over the sixties, increased over the seventies to mid- eighties, and declined continuously since then. Unfortunately, standard errors are uncomfortably large when calculating the level of the natural rate. However, changes in the natural rate over shorter time periods are estimated more precisely. This problem of large confidence intervals appears in the traditional approach as well. Second, the estimates of the gap between the actual unemployment rate and the natural rate has the anticipated effect on inflation. Specifically, for a wide variety of natural rate models and inflation measures, increases in the gap tend to lead to declines in inflation changes. Whether these natural rate estimates provide better forecasts of inflation than alternative natural rate models remains open. It is important to note that the purpose of this research is not to obtain the best forecasts of inflation. If this were the goal, then a VAR approach would be the most productive. Instead, the objective is to succinctly capture economic events that plausibly underlie a more structural understanding of the natural rate and to ascertain if such an approach offers some insight into the nature of the inflation process.

Finally, there are presumably many different factors affecting the natural rate. The work here has focused on just one, namely, the effect of sectoral shocks on the distribution of employment across industries. While the way in which turbulence is constructed here does not suffer from some of the criticisms leveled at earlier measures, turbulence may arise from other sectors as well. For example, the natural rate may respond to geographical shocks as well as international restructuring. Incorporating such concepts into models of the natural rate remains to be done.

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Figure 1: Industry Employment Growth Less Aggregate Employment Growth

1955:1 - 1997:2

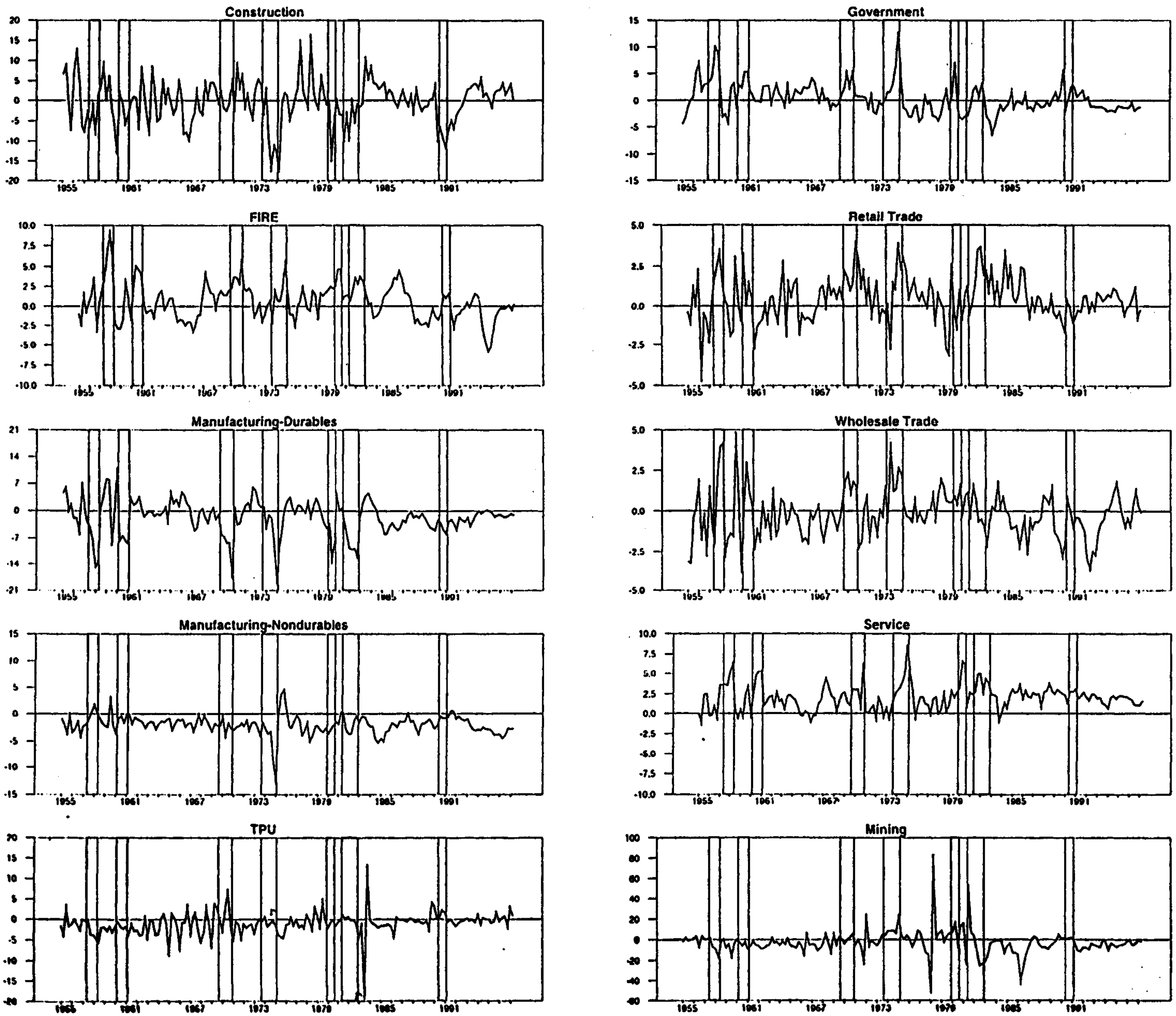


Figure 2: Kalman Filter Estimates of the Business Cycle

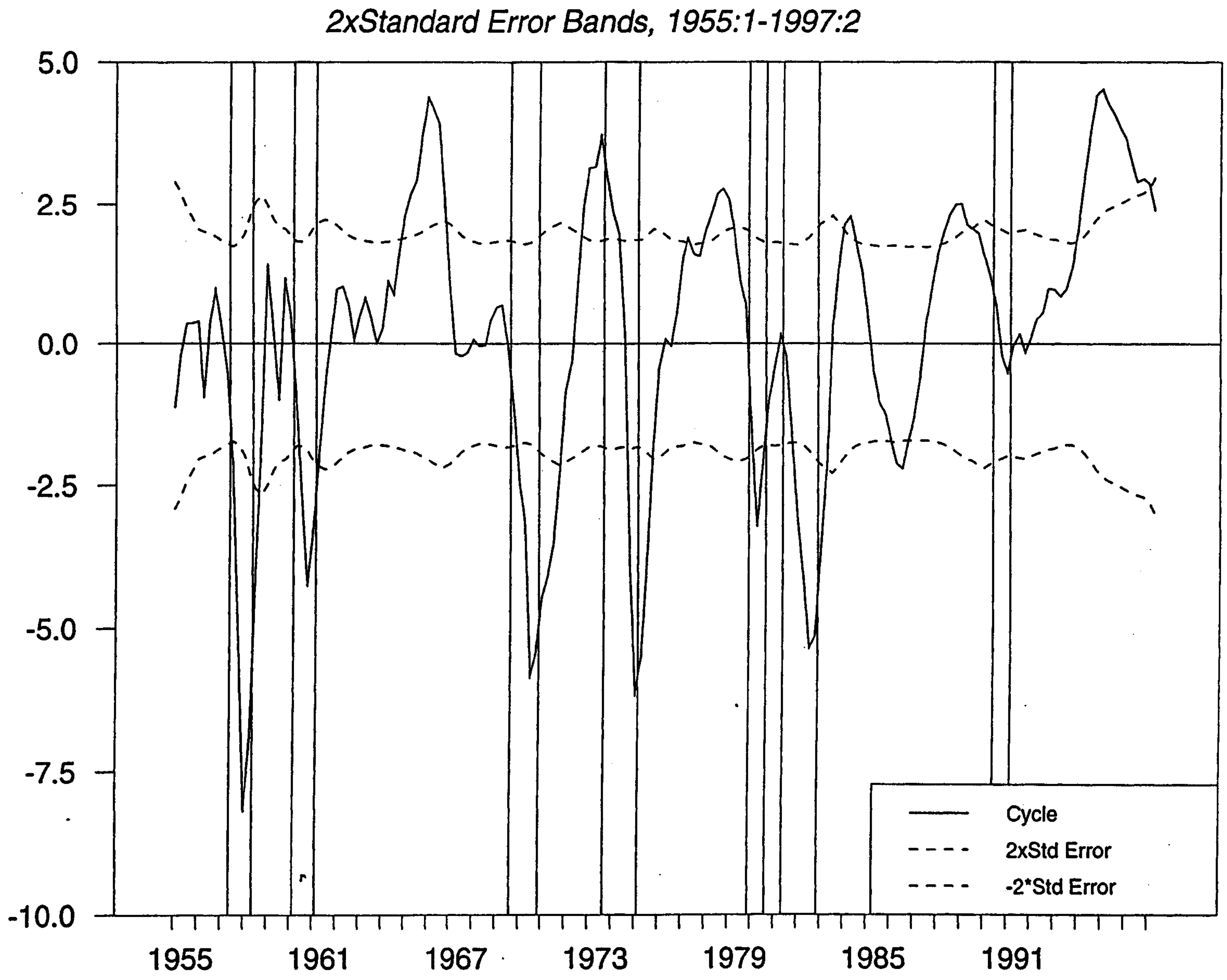


Figure 3: Estimated Industry Shocks

1955:1 - 1997:2

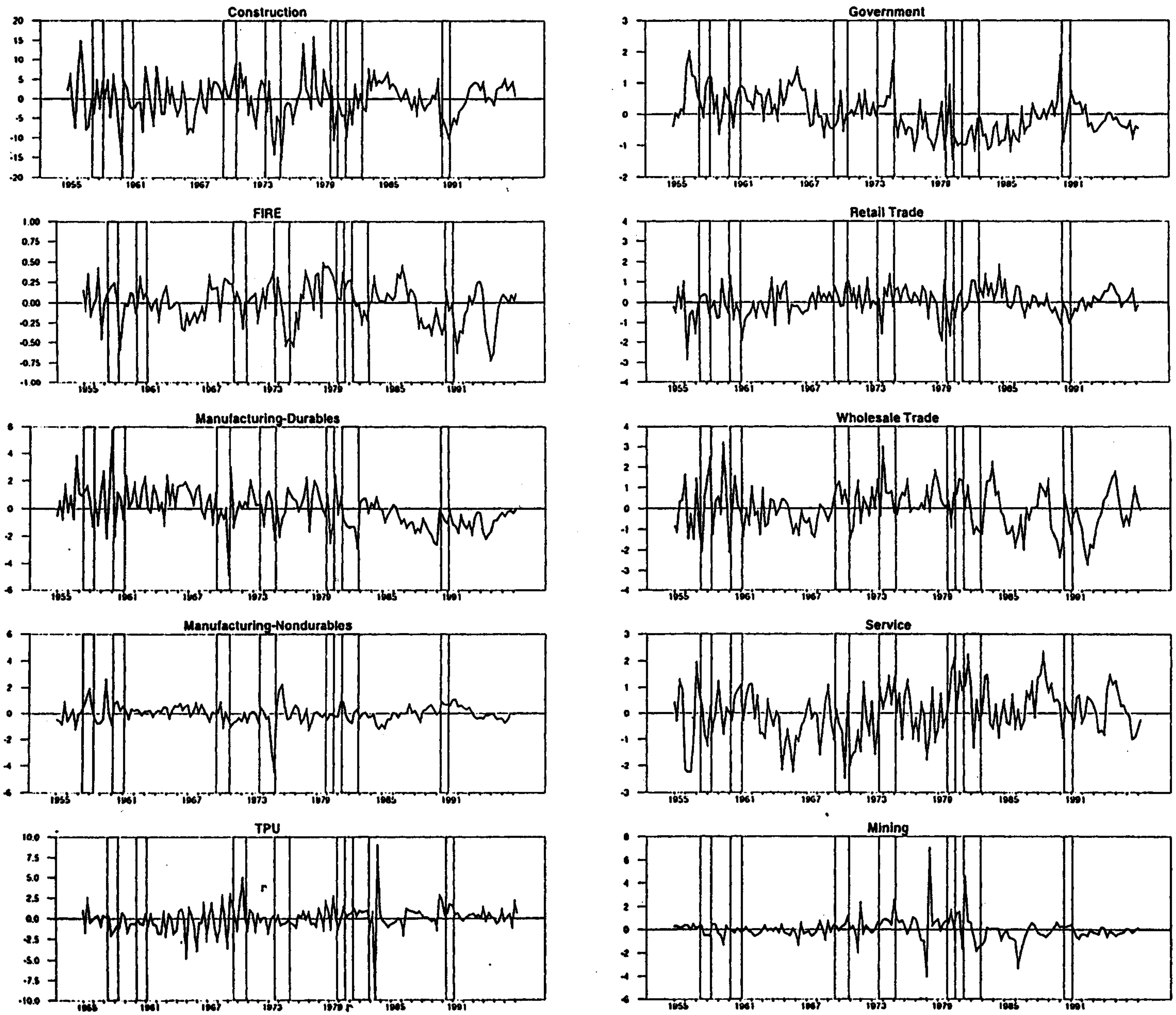
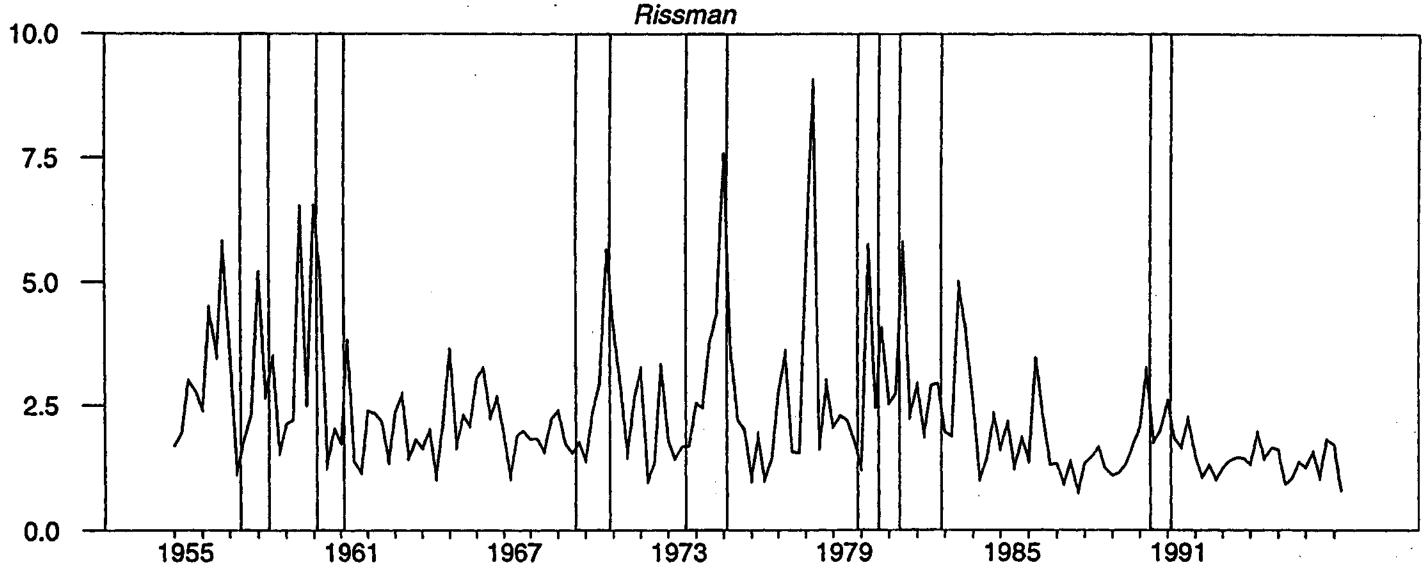


Figure 4: Turbulence Measures



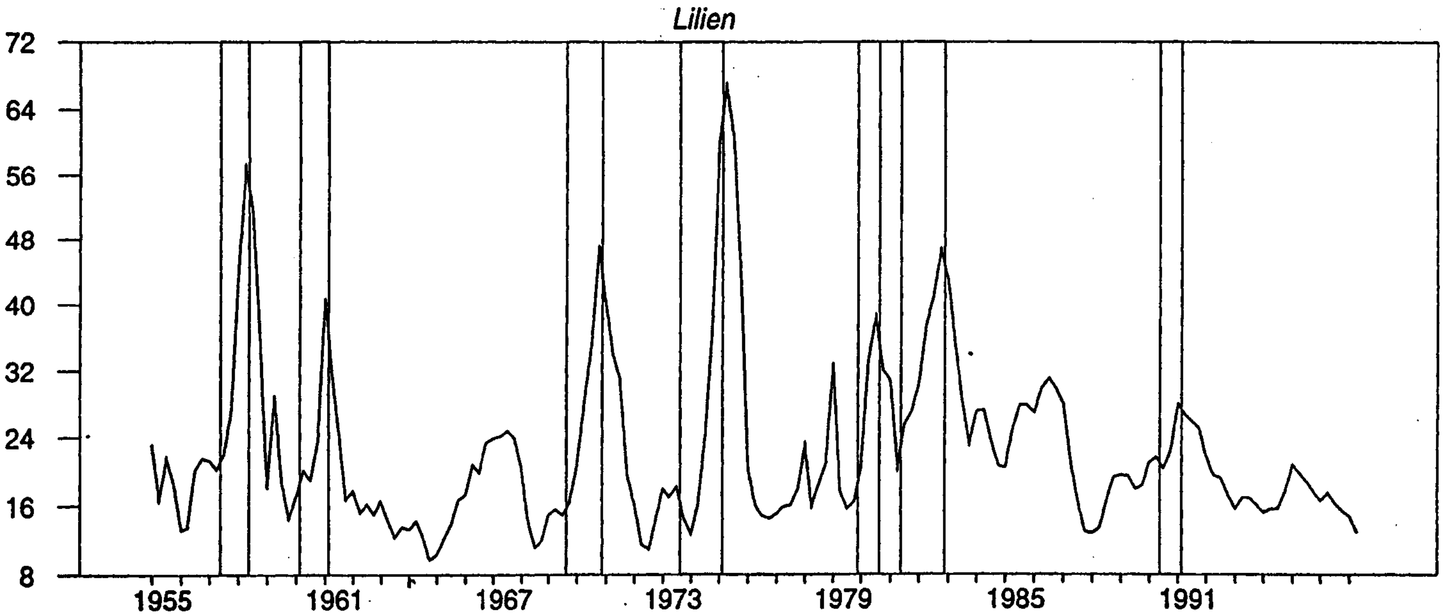


Figure 5: Rissman vs. Lilien

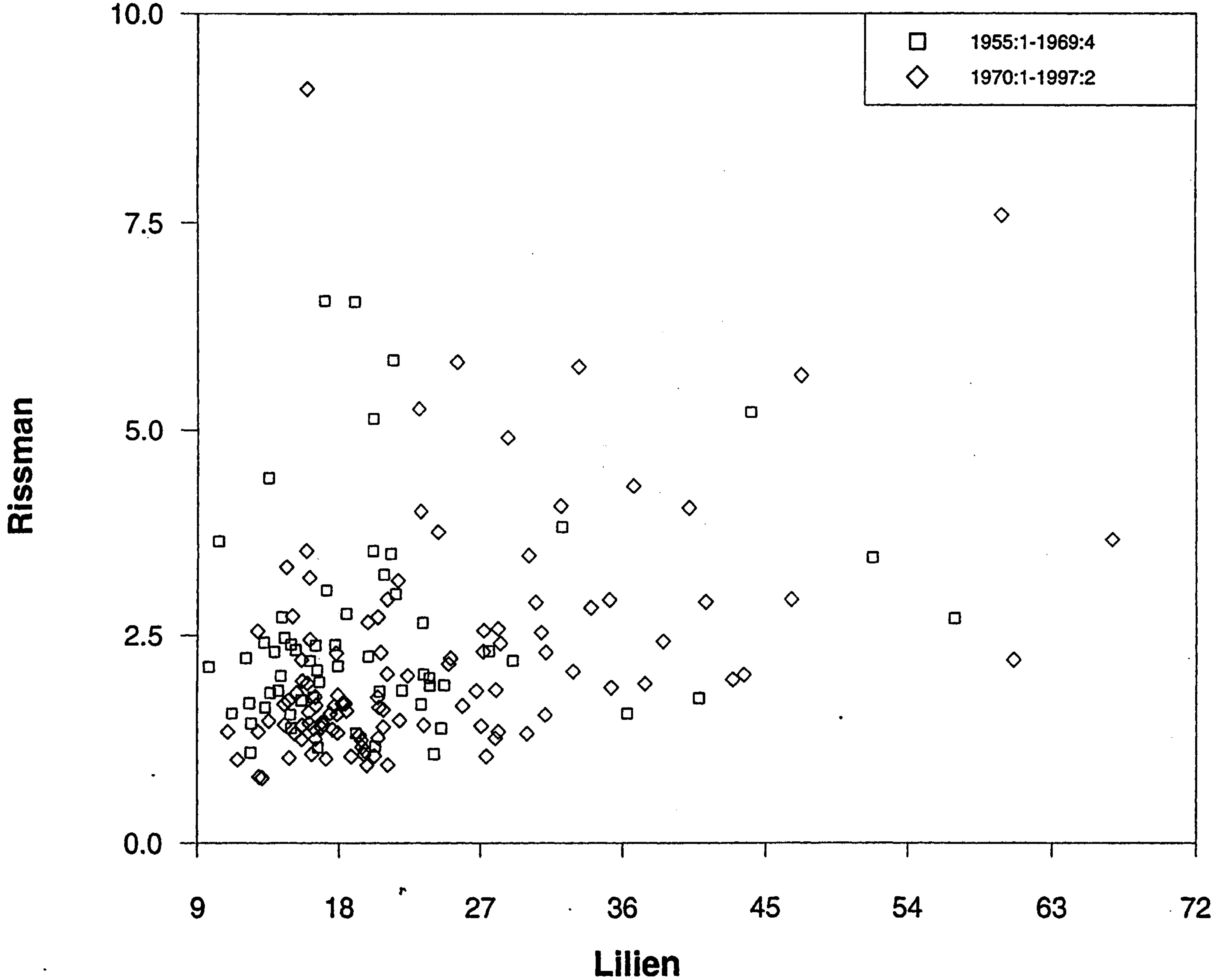


Figure 6: Natural Rate Estimates

1956:4=0

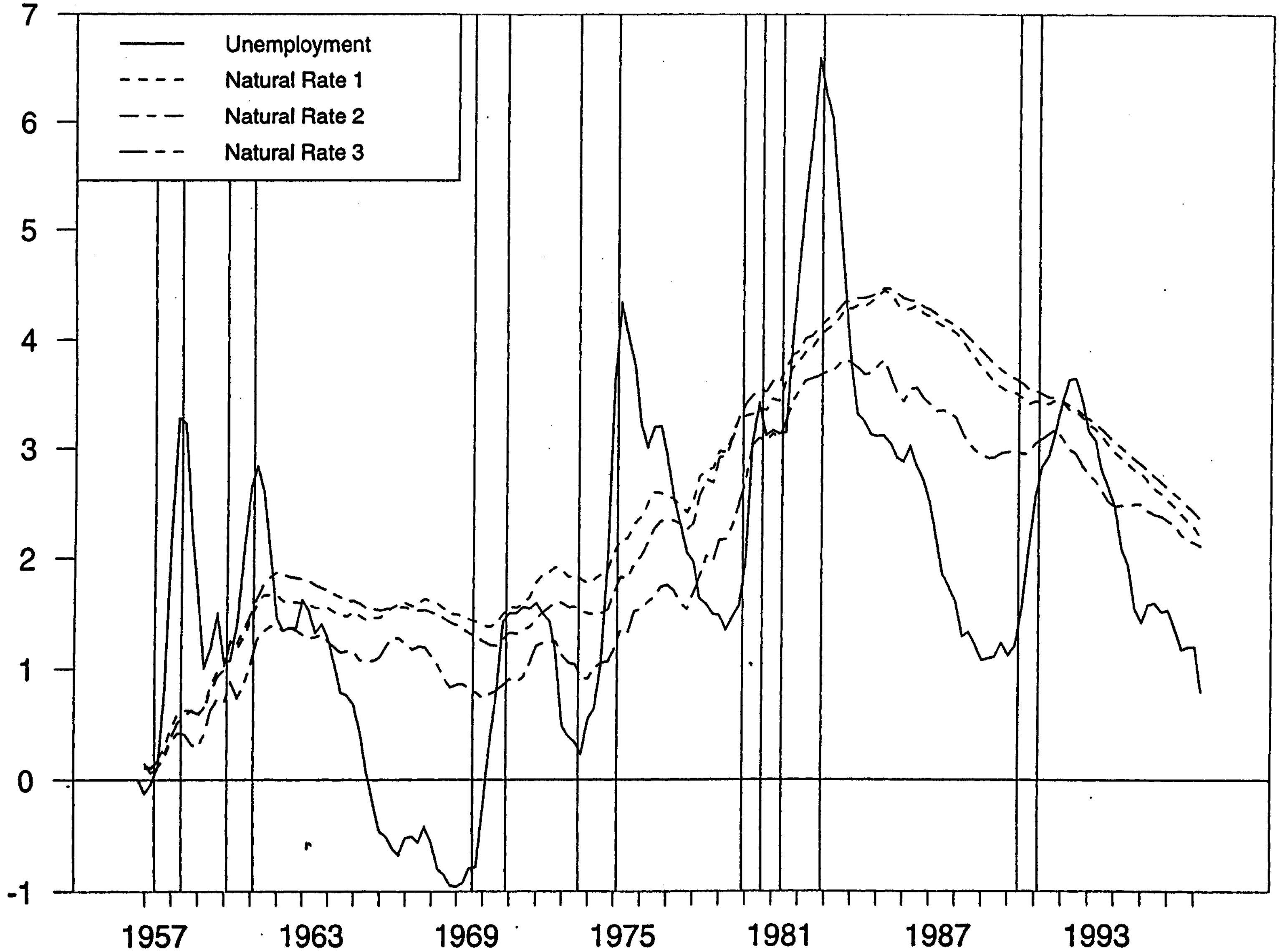


Figure 7: Natural Rate Estimates with Confidence Intervals

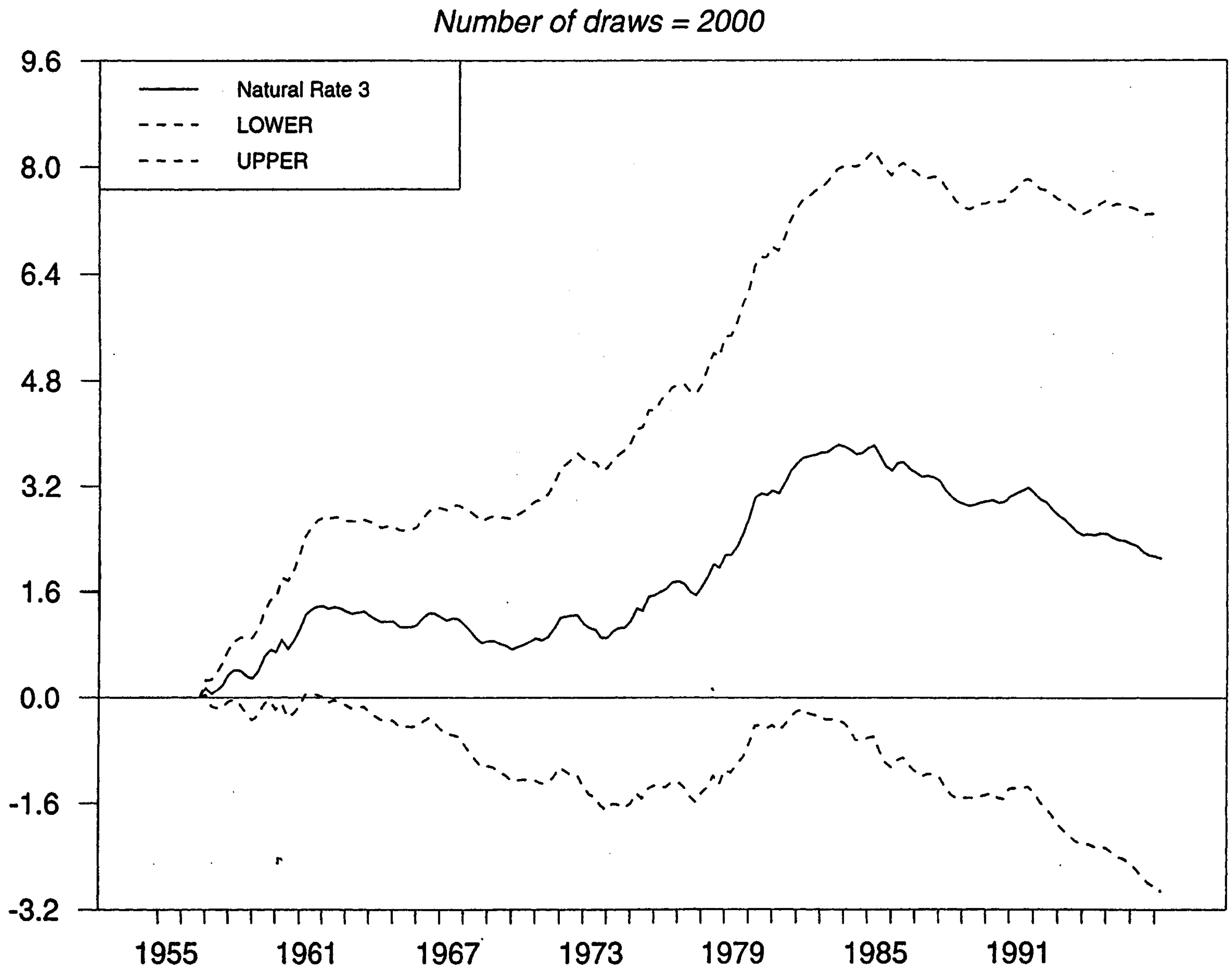


Figure 8: Natural Rate Estimates with Confidence Intervals

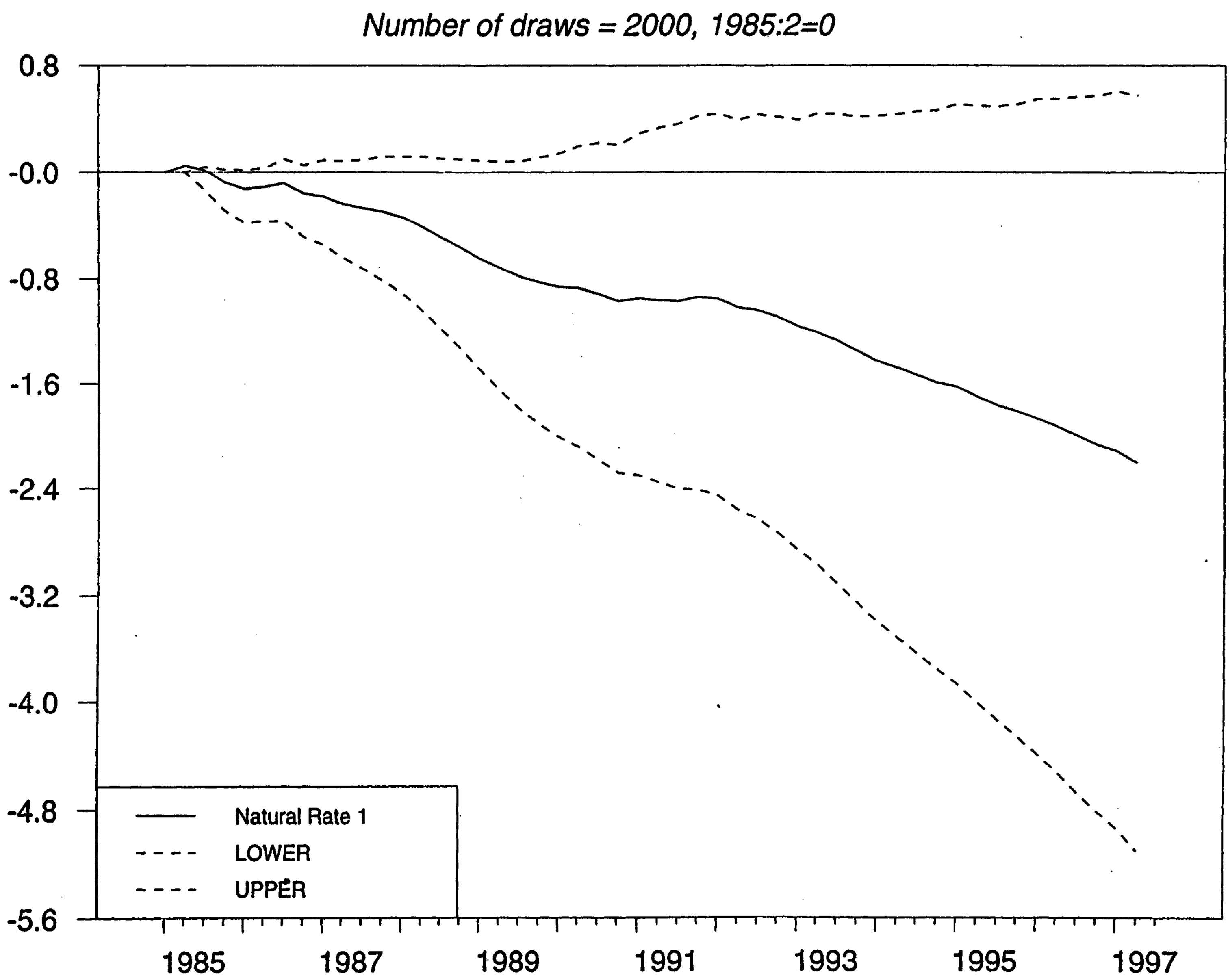


Table 1: Industry parameter estimates from Kalman filtering exercise[†]

	a_i	b_i^0	b_i^1	b_i^2	σ_i
CON	-0.4146 (0.3851)	1.0341*** (0.2277)	0 --	-0.9229*** (0.2339)	5.1308*** (0.2808)
FIR	0.7171*** (0.2539)	-0.6422*** (0.1391)	-0.7054*** (0.2218)	0.8812*** (0.1563)	1.3843*** (0.1334)
MFGD	-1.9048*** (0.3326)	1.0000 --	1.9422*** (0.3073)	-2.3775*** (0.3150)	2.2735*** (0.2025)
MFGN	-1.9971*** (0.1608)	0.1266** (0.0748)	0 --	-0.3320*** (0.0791)	1.6477*** (0.0918)
TPU	-1.0798*** (0.2383)	-0.3862** (0.2000)	0.6323*** (0.1989)	0 --	2.7029*** (0.1472)
GOV	0.4854*** (0.2024)	-0.5071*** (0.1918)	-1.4068*** (0.2999)	-1.2682*** (0.1888)	1.8845*** (0.1262)
TR	0.4617*** (0.1312)	-0.1807* (0.1217)	-0.2427 (0.2034)	0.2446** (0.1256)	1.2772*** (0.0714)
TW	-0.1037 (0.1049)	-0.1711* (0.1309)	-0.3676* (0.2247)	0.5393*** (0.1404)	1.3726*** (0.0778)
SRV	2.1009*** (0.0819)	-0.1260 (0.1031)	-1.0167*** (0.1708)	1.0150*** (0.1003)	1.0710 --
MIN	-2.8618*** (0.9237)	-1.2902 (1.1628)	2.6158 (1.9276)	-0.9995 (1.1321)	12.0898 --

† * Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Table 2: Unemployment Rate Regressions, Dependent variable is ΔWUR.[†]

	1	2	3	4	5	6
c	-0.0619 (0.0921)	0.0228 (0.0774)	-0.0391 (0.0545)	-0.0048 (0.0434)	0.0167 (0.2038)	-0.0181 (0.1561)
SMPL	0.0190 (0.0531)	0.0501 (0.0502)	0.0150 (0.0535)	0.0475 (0.0469)	0.0152 (0.0572)	0.0336 (0.0496)
ΔWUR_{t-1}	0.6357*** (0.0694)	0.5309*** (0.0788)	0.0649*** (0.0680)	0.5631*** (0.0789)	0.6177*** (0.0709)	0.5668*** (0.0796)
σ_t	0.0440** (0.0206)	--	--	--	0.0658 (0.0629)	--
$\Sigma_1^8 \sigma_{t-i}$	-0.0233 (0.0367)	--	--	--	-0.1049 (0.1371)	--
σ_t^2	--	--	0.0046* (0.0024)	--	-0.0029 (0.0074)	--
$\Sigma_1^8 \sigma_{t-i}^2$	--	--	-0.0007 (0.0052)	--	0.0119 (0.0200)	--
$\sigma_{L,t}$	--	0.0294*** (0.0048)	--	--	--	-0.0055 (0.0132)
$\Sigma_1^8 \sigma_{L,t-i}$	--	-0.0323*** (0.0049)	--	--	--	0.0067 (0.0149)
σ_L^2	--	--	--	0.0005*** (0.0001)	--	0.0005*** (0.0002)
$\Sigma_1^8 \sigma_{L,t-i}^2$	--	--	--	-0.0005*** (0.0001)	--	-0.0006*** (0.0002)
R^2	0.4224	0.5800	0.4286	0.6111	0.4388	0.6127
Q(36)	44.6190	55.7376***	45.7257	54.2921**	40.6760	65.6680***
p-value	0.1535	0.0190	0.1285	0.0258	0.2720	0.0018

[†] Eight lags of the sectoral shifts measures are used. Standard errors appear in parentheses below estimates of the parameter. Statistics reported for $x(1 \text{ to } 8) = \Sigma_1^8 \beta_i$ where β_i is the estimated coefficient on x_{t-i}. The statistic *Q(36)* is the Ljung—Box Q statistic using 36 lags of the residual. The marginal significance level of the *Q*-statistic is reported below the *Q*-statistic.

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Table 3: Unemployment Rate Regressions, Dependent variable is ΔWUR.[†]

	1	2	3	4	5	6
c	-0.1159 (0.1710)	0.02310 (0.1553)	0.0029 (0.2054)	-0.0073 (0.1582)	0.0493 (0.2025)	-0.0066 (0.0156)
SMPL	0.0652 (0.0415)	0.0903** (0.0430)	0.0301 (0.0595)	0.0439 (0.0518)	0.0122 (0.0567)	0.0357 (0.0495)
ΔWUR_{t-1}	0.1928*** (0.0744)	0.2792*** (0.0811)	0.6164*** (0.0717)	0.5682*** (0.0806)	0.6243*** (0.0708)	0.5791*** (0.0797)
σ_t	0.0884** (0.0438)	--	0.0755 (0.0645)	--	0.0520 (0.0637)	--
$\Sigma_1^8 \sigma_{t-i}$	-0.0488 (0.1091)	--	-0.1075 (0.1379)	--	-0.1139 (0.1362)	--
σ_t^2	-0.0087* (0.0052)	--	-0.0040* (0.0076)	--	-0.0008 (0.0076)	--
$\Sigma_1^8 \sigma_{t-i}^2$	0.0070 (0.0151)	--	0.0113 (0.0201)	--	0.0131 (0.0198)	--
$\sigma_{L,t}$	--	-0.0101 (0.0109)	--	-0.0061 (0.0134)	--	-0.0056 (0.0132)
$\Sigma_1^8 \sigma_{L,t-i}$	--	-0.0082 (0.0131)	--	0.0060 (0.0151)	--	0.0057 (0.0149)
σ_L^2	--	0.0003* (0.0002)	--	0.0005*** (0.0002)	--	0.0005** (0.0002)
$\Sigma_1^8 \sigma_{L,t-i}^2$	--	-0.0001 (0.0002)	--	-0.0006** (0.0002)	--	-0.0006** (0.0002)
$\Sigma_0^2 C_{t-i}$	-0.0215** (0.0100)	-0.0343** (0.0127)	--	--	--	--
$\Sigma_0^8 NBRD_{t-i}$	--	--	--	--	0.2383 (2.8581)	-1.3595 (2.4321)
$\Sigma_0^8 UAGM_{t-i}$	--	--	-12.7681 (14.7012)	-7.8277 (12.1779)	--	--
R^2	0.7303	0.7444	0.4335	0.6070	0.4491	0.6158
Q(36)	44.3900	61.84**	35.4229	64.0315***	45.1399	77.8831***
p-value	0.1855	0.0047	1.4959	0.0027	0.1413	0.0001

[†] Eight lags of the sectoral shifts measures are used. Standard errors appear in parentheses below estimates of the parameter. Statistics reported for $x(1 \text{ to } 8) = \Sigma_1^8 \beta_i$ where β_i is the estimated coefficient on x_{t-i}. The statistic *Q(36)* is the Ljung—Box Q statistic using 36 lags of the residual. The marginal significance level of the *Q*-statistic is reported below the *Q*-statistic.

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Table 3: Unemployment Rate Regressions, Dependent variable is ΔWUR.[†]

	7	8	9	10	11	12
c	-0.0494 (0.1778)	0.2629* (0.1565)	-0.1673 (0.1681)	0.2277 (0.1528)	-0.0430 (0.1760)	0.2765* (0.1533)
$SMPL$	0.0355 (0.0482)	0.0700 (0.0462)	0.0752* (0.0406)	0.1005** (0.0424)	0.0301 (0.0486)	0.0817* (0.0464)
ΔWUR_{t-1}	0.1849** (0.0750)	0.2568*** (0.0846)	0.1708** (0.0769)	0.2477*** (0.0836)	0.1722** (0.0760)	0.2370*** (0.0855)
σ_t	0.0937** (0.0447)	--	0.0870* (0.0450)	--	0.0837* (0.0454)	--
$\Sigma_1^8 \sigma_{t-i}$	-0.0854 (0.1111)	--	-0.0433 (0.1068)	--	-0.1020 (0.1082)	--
σ_t^2	-0.0097* (0.0053)	--	-0.0073* (0.0054)	--	-0.0077 (0.0054)	--
$\Sigma_1^8 \sigma_{t-i}^2$	0.0118 (0.0154)	--	0.0099 (0.0149)	--	0.0176 (0.0150)	--
$\sigma_{L,t}$	--	-0.0127 (0.0111)	--	-0.0073 (0.0111)	--	-0.0100 (0.0110)
$\Sigma_1^8 \sigma_{L,t-i}$	--	-0.0063 (0.0130)	--	-0.0124 (0.0132)	--	-0.0119 (0.0131)
$\sigma_{L,t}^2$	--	0.0003** (0.0002)	--	0.0003* (0.0002)	--	0.0003* (0.0002)
$\Sigma_1^8 \sigma_{L,t-i}^2$	--	-0.0001 (0.0002)	--	-0.0003** (0.0002)	--	-0.0005** (0.0002)
$\Sigma_0^2 C_{t-i}$	-0.0236** (0.0102)	-0.0370*** (0.0129)	-0.0203** (0.0099)	-0.0361*** (0.0127)	-0.0233** (0.0099)	-0.0372*** (0.0126)
$\Sigma_0^8 NBRD_{t-i}$	--	--	-12.1495*** (3.9703)	-10.1473*** (3.7060)	-12.1418*** (3.9719)	-10.1727*** (3.7425)
$\Sigma_0^8 UAGM_{t-i}$	25.3528 (21.6419)	26.6063 (20.6176)	--	--	35.6795 (22.9826)	26.9380 (22.4162)
R^2	0.7407	0.7525	0.7462	0.7544	0.7622	0.7679
$Q(36)$	46.3546	57.8124**	34.8840	68.7264***	41.0968	63.3247***
p-value	0.1157	0.0120	0.5215	0.0008	0.2571	0.0009

[†] Eight lags of the sectoral shifts measures are used. Standard errors appear in parentheses below estimates of the parameter. Statistics reported for $x(1 \text{ to } 8) = \Sigma_1^8 \beta_i$ where β_i is the estimated coefficient on x_{t-i}. The statistic $Q(36)$ is the Ljung—Box Q statistic using 36 lags of the residual. The marginal significance level of the Q-statistic is reported below the Q-statistic.

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Table 4.a: OLS Inflation Regression Results, Dependent Variable is ΔPCE.[†]

	σ		σ^2		σ and σ^2	
	1	2	3	4	5	6
μ	0.6034* (0.3220)	0.7501** (0.3440)	0.4827 (0.3113)	0.6186* (0.3322)	0.5331 (0.3776)	0.7057* (0.4004)
ΔPCE_{t-1}	-0.3430*** (0.0802)	-0.3052*** (0.0853)	-0.3383*** (0.0804)	-0.2975*** (0.0852)	-0.3299*** (0.0813)	-0.2970*** (0.0853)
ΔPCE_{t-2}	-0.1981** (0.0852)	-0.2964*** (0.0882)	-0.1894** (0.0848)	-0.2834*** (0.0880)	-0.1824** (0.0855)	-0.2761*** (0.0883
ΔPCE_{t-3}	-0.0334 (0.0850)	0.0173 (0.0892)	-0.0198 (0.0848)	0.0282 (0.0890)	-0.0172 (0.0854)	0.0400 (0.0893)
ΔPCE_{t-4}	-0.1234 (0.0796)	-0.0873 (0.0827)	-0.1139 (0.0794)	-0.0804 (0.0829)	-0.0857 (0.0801)	-0.0764 (0.0833)
GAP_t	-0.3837 (0.3612)	-1.0221*** (0.3256)	-0.3932 (0.3621)	-1.0272*** (0.3260)	-0.2846 (0.3738)	-0.9934*** (0.3354)
GAP_{t-1}	-0.8608* (0.7257)	0.8654* (0.4400)	-0.8353 (0.7304)	0.9155** (0.4423)	-1.1339 (0.7498)	0.8521* (0.4521)
GAP_{t-2}	1.3931 (0.8095)	0.0584 (0.1082)	1.4745* (0.8162)	0.0591 (0.1088)	1.8540** (0.8393)	0.0671 (0.1116)
GAP_{t-3}	0.7178 (0.7932)	-0.2975 (0.2077)	0.5969 (0.8048)	-0.3176 (0.2090)	0.2352 (0.8302)	-0.2811 (0.2131
GAP_{t-4}	-2.6082*** (0.7054)	-0.1963 (0.1403)	-2.5460*** (0.7169)	-0.2090 (0.1412)	-2.2451*** (0.7400)	-0.1861 (0.1440
GAP_{t-5}	1.5600*** (0.3528)	0.3692* (0.2123)	1.5558*** (0.3564)	0.3935* (0.2130)	1.4340*** (0.3688)	0.3590 (0.2187
$\Sigma_0^5 GAP_{t-i}$	-0.1817** (0.0892)	-0.2229** (0.0953)	-0.1473* (0.0858)	-0.1853** (0.0915)	-0.1403 (0.0914)	-0.1824* (0.0970
R^2	0.2836	0.1741	0.2778	0.1698	0.2580	0.1562
$Q(36)$	30.5527	36.6890	30.8623	35.4977	34.1835	37.6199
p-value	0.7250	0.4367	0.7113	0.4923	0.5552	0.3949
U^*_0	3.2321	3.3652	3.2758	3.3384	3.8004	3.869

[†] The dependent variable x has been constructed as follows: Let y be the relevant index. Then $x_t = \Delta(\ln y_t - \ln y_{t-1})$, where Δ is the 1 period difference operator. Columns (1), (3), and (5) have been estimated without constraints. Columns (2), (4), and (6) show Shiller estimates of the GAP parameters with β_0 estimated without constraint. Columns (1) and (2) reflect GAP estimates derived from turbulence measure σ. Columns (3) and (4) reflect GAP estimates derived from turbulence measure σ^2. Columns (5) and (6) reflect GAP estimates derived from turbulence measures σ and σ^2. The statistic $Q(36)$ is the Ljung—Box Q statistic using 36 lags of the residual. The marginal significance level of the Q-statistic is reported below the Q-statistic.

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Table 4.b: OLS Inflation Regression Results, Dependent Variable is $\Delta PCEX$.[†]

	σ		σ^2		σ and σ^2	
	1	2	3	4	5	6
μ	0.7278* (0.3882)	0.7452* (0.3871)	0.5997 (0.3772)	0.6126 (0.3761)	0.7729* (0.4537)	0.7954* (0.4521)
ΔPCE_{t-1}	-0.6518*** (0.0849)	-0.6469*** (0.0833)	-0.6467*** (0.0848)	-0.6436*** (0.0832)	-0.6519*** (0.0850)	-0.6453*** (0.0840)
ΔPCE_{t-2}	-0.3680*** (0.0994)	-0.3626*** (0.0989)	-0.3594*** (0.0992)	-0.3541*** (0.0987)	-0.3553*** (0.1004)	-0.3538*** (0.1001)
$\Delta PCEX_{t-3}$	-0.1991** (0.0992)	-0.2083** (0.0982)	-0.1948* (0.0991)	-0.2023** (0.0981)	-0.1924* (0.1004)	-0.1998** (0.0994)
$\Delta PCEX_{t-4}$	-0.1099 (0.0820)	-0.1076 (0.0816)	-0.1061 (0.0820)	-0.1044 (0.0817)	-0.0986 (0.0828)	-0.1016 (0.0823)
GAP_t	-1.0404** (0.4423)	-1.0489*** (0.3793)	-1.0343** (0.4440)	-1.0224*** (0.3813)	-0.9202** (0.4552)	-0.9890** (0.3939)
GAP_{t-1}	1.3603 (0.8883)	1.3729*** (0.5156)	1.4260 (0.8937)	1.3798*** (0.5204)	1.0197 (0.9087)	1.2297** (0.5369)
GAP_{t-2}	-0.0992 (0.9814)	0.0351 (0.1198)	-0.1665 (0.9882)	0.0192 (0.1201)	0.2226** (0.9958)	0.0616* (0.1241)
GAP_{t-3}	-0.1226 (0.0964)	-0.5682** (0.2456)	-0.1621 (0.9720)	-0.5863** (0.2476)	-0.2966 (0.9758)	-0.4747* (0.2555)
GAP_{t-4}	-0.9603 (0.8742)	-0.4365*** (0.1630)	-0.8921 (0.8845)	-0.4363*** (0.1647)	-0.7490*** (0.8913)	-0.3782** (0.1697)
GAP_{t-5}	0.6526 (0.4435)	0.4313* (0.2551)	0.6566 (0.4464)	0.4700 (0.2553)	0.5307 (0.4558)	0.3522 (0.2653)
$\Sigma_0^5 GAP_{t-i}$	-0.2100** (0.1078)	-0.2144** (0.1074)	-0.1724 (0.1042)	-0.1760* (0.1038)	-0.1928* (0.1101)	-0.1984* (0.1096)
R^2	0.3029	0.3010	0.3029	0.2957	0.2896	0.2882
$Q(36)$	47.2819*	45.7535	46.7505	45.3567	47.0920	45.3134
p-value	0.0988	0.1279	0.1082	0.1364	0.1021	0.1374
U^*_0	3.4734	3.4757	3.2758	3.4807	4.0085	4.0091

[†] The dependent variable x has been constructed as follows: Let y be the relevant index. Then $x_t = \Delta(\ln y_t - \ln y_{t-1})$, where Δ is the 1 period difference operator. Columns (1), (3), and (5) have been estimated without constraints. Columns (2), (4), and (6) show Shiller estimates of the GAP parameters with β_0 estimated without constraint. Columns (1) and (2) reflect GAP estimates derived from turbulence measure σ. Columns (3) and (4) reflect GAP estimates derived from turbulence measure σ^2. Columns (5) and (6) reflect GAP estimates derived from turbulence measures σ and σ^2. The statistic $Q(36)$ is the Ljung—Box Q statistic using 36 lags of the residual. The marginal significance level of the Q-statistic is reported below the Q-statistic.

* Significant at the 10% level.

** Significant at the 5% level.

*** Significant at the 1% level.

Table 4.c: OLS Inflation Regression Results, Dependent Variable is $\Delta CPI\text{-}U$.†

	σ		σ^2		σ and σ^2	
	1	2	3	4	5	6
μ	1.0186** (0.4309)	1.1675** (0.4512)	0.8502** (0.4154)	0.9921** (0.4363)	0.9423* (0.5059)	1.1045** (0.5265)
$\Delta CPI\text{-}U_{t-1}$	-0.4034*** (0.0834)	-0.3809*** (0.0866)	-0.3950*** (0.0834)	-0.3693*** (0.0866)	-0.3815*** (0.0840)	-0.3635*** (0.0863)
$\Delta CPI\text{-}U_{t-2}$	-0.3497*** (0.0896)	-0.4432*** (0.0906)	-0.3326*** (0.0890)	-0.4268*** (0.0904)	-0.3383*** (0.0893)	-0.4207*** (0.0900)
$\Delta CPI\text{-}U_{t-3}$	-0.0086** (0.0897)	0.0294 (0.0913)	0.0059 (0.0873)	0.0415 (0.0910)	0.0164 (0.0879)	0.0537 (0.0909)
$\Delta CPI\text{-}U_{t-4}$	-0.1177 (0.0822)	-0.0639 (0.0837)	-0.1104 (0.0815)	-0.0556 (0.0838)	-0.0902 (0.0821)	-0.0535 (0.0840)
GAP_t	-0.5643 (0.4812)	-1.4463*** (0.4332)	-0.5663 (0.4820)	-1.4840*** (0.4346)	-0.5530 (0.4986)	-1.4425*** (0.4454)
GAP_{t-1}	-1.8570* (0.9572)	0.7179 (0.5992)	-1.8824* (0.9647)	0.8350 (0.6043)	-1.8761 (0.9933)	0.7680 (0.6132)
GAP_{t-2}	2.8915*** (1.0579)	0.2380 (0.1488)	3.1251*** (1.0688)	0.2482* (0.1494)	3.1415** (1.1034)	0.2515 (0.1523)
GAP_{t-3}	-0.0775 (1.0708)	-0.0145 (0.2866)	-0.2952 (1.0866)	-0.0541 (0.2892)	-0.5281 (1.1208)	-0.0202 (0.2929)
GAP_{t-4}	-2.0237** (0.9779)	-0.0321 (0.1930)	-1.9782** (0.9912)	-0.0644 (0.1950)	-1.5907 (1.0194)	-0.0398 (0.1973)
GAP_{t-5}	1.3285*** (0.4952)	0.1926 (0.2941)	1.3424*** (0.4983)	0.2250 (0.2957)	1.1628** (0.5145)	0.1995 (0.3020)
$\Sigma_0^5 GAP_{t-i}$	-0.3024** (0.1193)	-0.3444*** (0.1249)	-0.2545** (0.1144)	-0.2943** (0.1201)	-0.2436** (0.1224)	-0.2833** (0.1274)
R^2	0.3707	0.3042	0.3700	0.2993	0.3459	0.2853
$Q(36)$	38.8271	34.3200	39.4421	34.3077	39.6741	35.4220
p-value	0.3435	0.5486	0.3187	0.5492	0.3096	0.4959
U^*_0	3.3690	3.3900	3.3408	3.3710	3.8686	3.8987

† The dependent variable x has been constructed as follows: Let y be the relevant index. Then $x_t = \Delta(\ln y_t - \ln y_{t-4})$, where Δ is the 1 period difference operator. Columns (1), (3), and (5) have been estimated without constraints. Columns (2), (4), and (6) show Shiller estimates of the GAP parameters with β_0 estimated without constraint. Columns (1) and (2) reflect GAP estimates derived from turbulence measure σ. Columns (3) and (4) reflect GAP estimates derived from turbulence measure σ^2. Columns (5) and (6) reflect GAP estimates derived from turbulence measures σ and σ^2. The statistic $Q(36)$ is the Ljung—Box Q statistic using 36 lags of the residual. The marginal significance level of the Q-statistic is reported below the Q-statistic.

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Table 4.d: OLS Inflation Regression Results, Dependent Variable is $\Delta CPIX$.[†]

	σ		σ^2		σ and σ^2	
	1	2	3	4	5	6
μ	1.1775*** (0.4182)	0.2173*** (0.4227)	1.0286** (0.4065)	1.0631** (0.4112)	1.2284** (0.4874)	1.2748** (0.4908)
$\Delta CPIX_{t-1}$	-0.4035*** (0.0875)	-0.3934*** (0.0881)	-0.3924*** (0.0876)	-0.3814*** (0.0881)	-0.3880*** (0.0878)	-0.3745*** (0.0880)
$\Delta CPIX_{t-2}$	-0.2474** (0.0960)	-0.2796*** (0.0957)	-0.2367** (0.0957)	-0.2687*** (0.0956)	-0.2229** (0.0959)	-0.2516*** (0.0954)
$\Delta CPIX_{t-3}$	0.0080 (0.0949)	0.0162 (0.0954)	0.0126 (0.0948)	0.0211 (0.0952)	0.0373 (0.0949)	0.0413 (0.0948)
$\Delta CPIX_{t-4}$	-0.0990 (0.0871)	-0.0773 (0.0865)	-0.0991 (0.0872)	-0.0774 (0.0866)	-0.0801 (0.0872)	-0.0640 (0.0867)
GAP_t	-0.4899 (0.4847)	-0.9496** (0.4330)	-0.4622 (0.4894)	-0.9410** (0.4360)	-0.7076 (0.5003)	-1.1256** (0.4445)
GAP_{t-1}	-1.3132 (0.9402)	0.1645 (0.5960)	-1.3150 (0.9569)	0.2267 (0.6021)	-0.9656 (0.9781)	0.3797 (0.6141)
GAP_{t-2}	2.1988** (1.0178)	0.2159 (0.1545)	2.2930** (1.0397)	0.2045 (0.1548)	2.0511* (1.0613)	0.2766** (0.1573)
GAP_{t-3}	-1.0122 (1.0177)	0.1913 (0.2840)	-1.1789 (1.0394)	0.1523 (0.2871)	-0.8560 (1.0603)	0.1654 (0.2919)
GAP_{t-4}	0.2159 (0.9301)	0.0946 (0.1915)	0.3021 (0.9477)	0.0743 (0.1936)	0.0889 (0.9653)	0.0499 (0.1973)
GAP_{t-5}	0.0573 (0.4782)	-0.0713 (0.2871)	0.0607 (0.4838)	-0.0270 (0.2897)	0.0755 (0.4942)	-0.0674 (0.2953)
$\Sigma_0^5 GAP_{t-i}$	-0.3432*** (0.1157)	-0.3545*** (0.1169)	-0.3004*** (0.1118)	-0.3103*** (0.1131)	-0.3107*** (0.1179)	-0.3241*** (0.1187)
R^2	0.2351	0.2128	0.2258	0.2021	0.2241	0.2069
$Q(36)$	39.8533	36.5482	40.7248	37.1808	37.6271	35.8142
p-value	0.3027	0.4432	0.2703	0.4144	0.3946	0.4774
U^*_0	3.4309	3.4339	3.4241	3.4260	3.9540	3.9334

[†] The dependent variable x has been constructed as follows: Let y be the relevant index. Then $x_t = \Delta(\ln y_t - \ln y_{t-1})$, where Δ is the 1 period difference operator. Columns (1), (3), and (5) have been estimated without constraints. Columns (2), (4), and (6) show Shiller estimates of the GAP parameters with β_0 estimated without constraint. Columns (1) and (2) reflect GAP estimates derived from turbulence measure σ. Columns (3) and (4) reflect GAP estimates derived from turbulence measure σ^2. Columns (5) and (6) reflect GAP estimates derived from turbulence measures σ and σ^2. The statistic $Q(36)$ is the Ljung—Box Q statistic using 36 lags of the residual. The marginal significance level of the Q-statistic is reported below the Q-statistic.

* Significant at the 10% level.
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