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Abstract: This paper evaluates the importance of microeconomic irreversibilities for aggregate dynamics using a general equilibrium approach. To this end a real business cycle model of establishment level dynamics is formulated and analyzed. Investment decisions are subject to irreversibility constraints and consequently, are of the (S,s) variety. This complicates the analysis since the state of the economy is described by an endogenous distribution of agents. The paper develops a computational strategy that makes this class of (S,s) economies fully tractable. Contrary to what the previous literature has suggested, investment irreversibilities are found to have no effects on aggregate business cycle dynamics.

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1. Introduction

The empirical literature has provided substantial evidence of investment irreversibilities at the establishment level. Analyzing the behavior of a large number of manufacturing establishments over time, Caballero, Engel and Haltiwanger [7] determined that establishments are much more inclined to expanding their stock of capital than to reducing it. Ramey and Shapiro [21] presented more direct evidence. Using data from the equipment auction of an aerospace firm, they estimated the wedge between the purchase and resale price for different types of capital. They found that machine tools sell at about 31% of their purchase value, while structural equipment sell at only 5%. These estimates indicate surprisingly large levels of irreversibilities in investment.

Since the early and influential paper by Arrow [2] there has been substantial theoretical microeconomic work on irreversible investment (see Dixit and Pindyck [11] for a survey). A common result is that irreversibilities are extremely important for establishment level dynamics. For example, Abel and Eberly [1] analyzed the problem of a firm facing a resale price of capital which is lower than its purchase price. They showed that the optimal investment decision of the firm is a two-triggers (S,s) policy, and characterized the associated range of inaction as a function of the wedge between the purchase and resale price of capital. They found that even small irreversibilities can have a large impact on the range of inaction, substantially affecting the investment dynamics of the firm.

While the microeconomic consequences of investment irreversibilities are well understood, their macroeconomic implications are not. To evaluate the importance of investment irreversibilities for aggregate dynamics, this paper formulates and analyzes a real business cycle model of establishment level dynamics. The basic framework is analogous to the neoclassical stochastic growth model with indivisible labor analyzed by Hansen [17] and for a particular parametrization the model reduces to his. Output, which can be consumed or invested, is produced by a large number of establishments that use capital and labor as inputs into a decreasing returns to scale production technology. Establishments receive idiosyncratic productivity shocks that determine their expansion, contraction or death. Establishments are also subject to an aggregate productivity shock which generates aggregate fluctuations in the economy. For simplicity, both entry and exit are treated as exogenous.

While labor is assumed to be perfectly mobile across establishments, capital is not. Once capital is in place at an establishment there are costs associated with detaching and moving it. These costs imply that a fraction of the productive services of capital are lost in the process of uninstalling it. This is analogous to the case analyzed by Abel and Eberly [1], where the sale price of capital is lower than its purchase price.

Computing the stochastic general equilibrium dynamics for this (S,s) economy is a challenging task. The difficulty stems from carrying an endogenous distribution of heterogeneous agents as a state variable (which is a highly dimensional object). This paper develops a computational strategy that makes this class of problems fully tractable. The method involves keeping track of long histories of (S,s) thresholds as state variables, instead of the current distribution of agents in the economy. The convenience of this alternative state space is that standard linear-quadratic approximation techniques can be directly applied.[1]

To evaluate the importance of investment irreversibilities for macroeconomic dynamics,

[1]Other papers that analyze stochastic general equilibrium dynamics in (S,s) economies include: Caplin and Leahy [9], Dotsey, King and Wolman [13], Fisher and Hornstein [15], and Thomas [24].

economies with different degrees of irreversibilities are calibrated to U.S. data and their aggregate fluctuations compared. The results are striking. Investment irreversibilities play no important role in aggregate dynamics: economies ranging from fully reversible to completely irreversible investment generate almost identical aggregate business cycle fluctuations. The only way in which investment irreversibilities matter is for establishment level dynamics. In principle, investment irreversibilities could affect aggregate business cycle fluctuations if aggregate productivity shocks were variable enough, but this would require an implausibly large variability of measured solow residuals.

Previous work on the importance of microeconomic irreversibilities for macroeconomic dynamics includes Bertola and Caballero [3, 4], Caballero and Engel [5, 6], and Caballero, Engel and Haltiwanger [7]. None of these studies performs an equilibrium analysis. In some cases, the optimal decision rule of an individual establishment facing an ad-hoc stochastic process for prices is derived; the behavior of a large number of such establishments then aggregated to study aggregate investment dynamics. In other cases, no economic structure is used: even the investment decision rules of establishments are directly assumed. On the contrary, this paper analyzes the general equilibrium dynamics of an explicit economic environment. This is an important methodological innovation. To fully capture the implications of microeconomic irreversibilities for macroeconomic dynamics, a general equilibrium analysis is required.

In fact, the conclusions in this paper are dramatically different from those in the previous literature. In a celebrated paper, Caballero, Engel and Haltiwanger [7] found that non-linear adjustments at the establishment level substantially improve the ability of their aggregate investment equation to keep track of U.S. aggregate investment behavior, specially when aggregate investment is far from its mean. Caballero, Engel and Haltiwanger interpreted this result as evidence that investment irreversibilities have important implications for macroeconomic dynamics. In particular, that irreversibilities are crucial in generating brisk expansions and contractions in aggregate investment. Since this has become the dominant view thereafter, this paper will devote considerable attention to contrasting both methods of analysis. It will be argued that the Caballero-Engel-Haltiwanger approach presents a serious limitation: it assumes that important features of investment decisions are invariant to the adjustment costs that establishments face. As a consequence, it can lead to the wrong answers. In fact, this paper shows that when the CEH method is applied to our model economy, their same conclusions are obtained. But when the analysis is restricted to investment behavior which is fully consistent with the economic environment that establishments face, microeconomic irreversibilities have no effects on aggregate dynamics.

The paper is organized as follows: Section 2 describes the economy, Section 3 discusses the computational method, Section 4 parametrizes the model, Section 5 describes the business cycles of a benchmark economy, Section 6 discusses the importance of investment irreversibilities for business cycle fluctuations, and Section 7 concludes the paper.

2. The model economy

The economy is populated by a continuum of ex-ante identical agents with names in the unit interval. Their preferences are described by the following utility function:

$$E \sum_{t=0}^{\infty} \beta^t \left[\log\left(c_t\right) + v\left(l_t\right)\right] \tag{1}$$

where c_t and l_t are consumption and leisure respectively, and $0 < \beta < 1$ is the subjective time discount factor. Every period agents receive a time endowment equal to ω. Following Rogerson [22] and Hansen [17], it is assumed that there is an institutionally determined workweek of fixed length which is normalized to one, so leisure can only take values ω or ω-1.

Output, which can be consumed or invested, is produced by a large number of establishments. Each establishment uses capital (k) and labor (n) as inputs into a production technology given by:

$$y_t = e^{z_t} s_t k_t^{\theta} n_t^{\gamma} \tag{2}$$

where $\theta + \gamma < 1$, s_t is an idiosyncratic productivity shock and z_t is an aggregate productivity shock common to all establishments. Realizations of the idiosyncratic productivity shock s_t take values in the set $\{0, 1, \lambda\}$ and are independent across establishments. Over time, s_t follows a first order Markov process with transition matrix Π, where $\pi(s, s')$ is the probability that $s_{t+1} = s'$ conditional on $s_t = s$. This process is assumed to be such that: 1) starting from any initial value, with probability one s_t reaches zero in finite time, and 2) once s_t reaches zero, there is zero probability that s_t will receive a positive value in the future. Given these assumptions, it is natural to identify a zero value for the productivity shock with the death of an establishment.[2] The aggregate productivity shock z_t follows a law of motion given by:

$$z_{t+1} = \rho z_t + \varepsilon_{t+1} \tag{3}$$

[2]Given that there are no fixed costs to operate an establishment already created, exit will take place only when the idiosyncratic productivity shock takes a value of zero.

where $0 < \rho < 1$, and ε_t is i.i.d. with variance σ_ε^2 and zero mean.

Labor is perfectly mobile in this economy, but capital is not. On one hand, the amount of capital k_{t+1} in place at an establishment at date $t+1$ must be decided at period t before the realization of s_{t+1} becomes known. On the other hand, investment is partially irreversible: whenever capital is detached from the floor of an establishment it loses a fraction $(1-q)$ of its remaining productive services. To be precise, let $0 < \delta < 1$ be the depreciation rate of capital. In order to increase an establishment's next period stock of capital k_{t+1} above its current level net of depreciation $(1-\delta)k_t$, an investment of $k_{t+1}-(1-\delta)k_t$ units is needed. On the contrary, when an establishment decreases its next period stock of capital k_{t+1} below its current level net of depreciation $(1-\delta)k_t$, the amount of investment goods obtained is only a fraction q of $(1-\delta)k_t - k_{t+1}$. The parameter q is a measure of the degree of the investment irreversibilities in the economy and will play a crucial role in the analysis.

Every period, agents receive an endowment of new establishments which arrive with zero initial capital in place. Initial values for s across new establishments are distributed according to ψ. This exogenous birth of new establishments compensates the ongoing death of existing establishments (as they get absorbed into zero productivity) and results in a constant long run number of establishments.[3]

The presence of idiosyncratic productivity shocks and irreversible investment at the establishment level suggests indexing establishments according to their current productivity shocks s and current stock of capital k. In what follows, a measure x_t over current pro-

[3]Even though the entry and exit decisions of establishments are not endogenously determined in this economy, it seems important to incorporate them at least exogenously. A significant probability of death will probably affect how establishments respond to aggregate productivity shocks in the presence of investment irreversibilities.

ductivity shocks and capital levels will describe the number of establishments of each type at period t.[4] Also, a measurable function n_t will describe the number of workers across establishment types, a measurable function g_{t+1} will describe the next period stock of capital across establishment types, and η_t will denote the fraction of the population that works.

Feasibility constraints consumption as follows:

$$c_t \leq \int \{ e^{z_t} s \; k^{\theta} n_t (k,s)^{\gamma} \quad \cdot \quad [g_{t+1}(k,s) - (1-\delta)k] \;\; Q\, [g_{t+1}(k,s) - (1-\delta)k] \;\; \} dx_t$$
$$+ \quad \int (1-\delta) \; g_t(k,s) \;\; q \; \pi(s,0) \;\; dx_{t-1} \tag{4}$$

where $Q\,()$ is an indicator function that takes value 1 if its argument is positive, and value q (the irreversibility parameter) otherwise. The first term, is the sum of output minus investment across all types of establishments, taking into account the capital losses due to the investment irreversibilities. The second term on the right hand side corresponds to all those establishments that were in operation the previous period and die during the current period (transit to an idiosyncratic shock equal to 0), getting to sell a fraction q of their stock of capital $g_t(k,s)$ net of depreciation.

Similarly, the total number of workers at establishments is constrained not to exceed the fraction of the population that works η_t:

$$\int n_t(k,s) \;\; dx_t \leq \eta_t \tag{5}$$

Finally, the law of motion for the measure x_t must be consistent with the capital decisions

[4]The measure x_t will be defined only over positive productivity levels, i.e. x_t does not keep track of establishments that die.

at the plant level. That is, for every Borel set B:

$$x_{t+1}(B,s') = \int_{(k,s):\ g_{t+1}(k,s)\in B} \pi(s,s')\ dx_t\ +\ \upsilon\,\psi(s')\ \chi(0\in B) \tag{6}$$

where $\chi()$ is an indicator function that takes value 1 if its argument is true, and zero otherwise. In words, the number of establishments that next period have a stock of capital in the set B and a productivity shock s', is given by the sum of two terms: 1) all those establishments that transit from their current shocks to the shock s' and choose a next period stock of capital in the set B, and 2) in the case that $0 \in B$, all new establishments that arrive with an initial productivity shock s' (note that new establishments are born with a zero initial stock of capital).

Following Hansen [17] and Rogerson [22], agents are assumed to trade employment lotteries. These are contracts that specify probabilities of working, and allow agents to perfectly diversify the idiosyncratic risk they face. Since agents are ex-ante identical, they all chose the same lottery. As a consequence, the economy has a representative agent with utility function:

$$E\sum_{t=0}^{\infty}\beta^t\left[\log c_t - \alpha\,\eta_t\right] \tag{7}$$

where $\alpha = v(\omega) - v(\omega - 1)$ (see Hansen [17] and Rogerson [22] for details). Since this is a convex economy with no externalities nor other distortions the competitive equilibrium allocation can be solved by analyzing the Social Planner's problem with equal weights, which is given by maximizing (7) subject to (3), (4), (5), and (6).

3. Computation

This section describes the computational approach. The method is novel and constitutes an important contribution of this paper. However, readers less interested in computational methods and more interested in substantive results can proceed directly to Section 4 with no loss of continuity.

The state of the economy is given by the current aggregate productivity shock z, the current measure x across establishment types, the previous period measure y across establishment types, and the previous period investment decisions d (z_t, x_t, x_{t-1} and g_t respectively in terms of the previous section notation).[5] The Social Planner's Problem can then be described by the following Bellman equation:

$$V(d,x,y,z) = MAX \{ \ln c - \alpha\, \eta + \beta\, E\, V(d',x',y',z') \} \tag{8}$$

subject to

$$c \leq \int \{\, e^{z} s\, k^{\theta} n\,(k,s)^{\gamma} \quad \cdot \quad [g\,(k,s) - (1-\delta)k]\;\; Q\,[g\,(k,s) - (1-\delta)k]\; \} dx$$
$$+ \int (1-\delta)\; d\,(k,s)\;\; q\,\pi\,(s,0)\;\; dy \tag{9}$$

$$\int n\,(k,s)\;\; dx \leq \eta \tag{10}$$

[5]The notation is changed to avoid time subscripts which complicate the recursive formulation of the economy.

$$x'(B, s') = \int_{(k,s):\ g(k,s)\in B} \pi(s, s')\ dx \ + \ \upsilon\, \psi(s')\ \chi(0 \in B) \tag{11}$$

$$d' = g \tag{12}$$

$$y' = x \tag{13}$$

$$z' = \rho z + \varepsilon' \tag{14}$$

where the maximization is over $n()$ and $g()$. Note the high dimensionality of the state space which seems to preclude any possibilities of computing a solution.[6] Below, I will show that this difficulty is only apparent: the problem becomes fully tractable once it is redefined in terms of a convenient set of variables.

To understand the rationale for the transformed problem, it will be convenient to analyze the structure of the problem that establishments face at the competitive equilibrium. The individual state of an establishment is given by its current productivity shock s and its current stock of capital k. The problem of an establishment with individual state (k, s) when the aggregate state is (d, x, y, z) is given by:

[6] The state of the economy could be simplified considerably. Instead of carrying d and y as state variables, $\int(1-\delta)\ d(k,s)\ q\ \pi(s,0)\ dy$ could be used instead. The formulation in (8) is selected since it is more closely related to the computational method employed. In any case, carrying x as a state variable is unavoidable and this is a highly dimensional object.

$$J(k,s;d,x,y,z) \;=\; MAX\ \{\ e^z s\ k^\theta n^\gamma - w\,(d,x,y,z)\,n - [k' - (1-\delta)k]\,Q\,[k' - (1-\delta)\,k]$$

$$+\ E\,[i\,(d,x,y,z;d',x',y',z')\,J\,(k',s';d',x',y',z')] \tag{15}$$

subject to:

$$s' \sim \Pi(s) \tag{16}$$

$$z' = \rho z + \varepsilon' \tag{17}$$

$$(d',x',y') = H\,(d,x,y,z) \tag{18}$$

where $w\,()$ is the equilibrium wage rate, $i\,()$ are the equilibrium prices of Arrow securities, $H\,()$ is the equilibrium law of motion for the aggregate state of the economy, and the maximization is over the scalars n and k'. Note that the decision rule for capital that corresponds to the solution of this Bellman equation is of the (S,s) variety.[7] It is characterized by a pair of lower and upper capital thresholds $a(s)$, $A(s)$ such that:

[7]The optimal k' decision for an establishment that expands ($k' > (1-\delta)k$) is given by:

$$MAX\,\{E\,[i\,(...)\,J\,(k',s',...)] - k'\}$$

Note that the $a(s)$ that solves this problem is independent of k.

Similarly, the optimal k' for an establishment that contracts ($k' < (1-\delta)k$) is given by:

$$MAX\,\{E\,[i\,(...)\,J\,(k',s',...)] - qk'\}$$

The $A(s)$ that solves this problem does not depend on k either. The decision rule (19) is then obtained.

$$
\begin{aligned}
k' &= a(s), && \text{if } (1-\delta)k < a(s) \\
&= A(s), && \text{if } (1-\delta)k > A(s) \\
&= (1-\delta)k, && \text{otherwise}
\end{aligned}
\tag{19}
$$

where the dependence of $a(s)$ and $A(s)$ on the aggregate state of the economy has been suppressed to simplify notation (Figure 1 shows a picture of this decision rule). Note that there is a pair of lower and upper threshold $(a(s), A(s))$ for every possible idiosyncratic productivity shock s. Hereon we will denote (a, A) as being the vector $(a(s), A(s))_{s=1,\lambda}$ across idiosyncratic shocks.

Our strategy will be to keep track of long histories of (a, A) as state variables instead of the actual distributions x and y, and use them to construct approximate distributions for x and y using the law of motion (6).[8] In principle, as we make the length of the history of (a, A) arbitrarily large we would obtain an arbitrarily good approximation for x and y. An important question will be how large to make this length in practice (I will return to this question below). Our solution method will require solving independently for the deterministic steady state of the economy. Appendix A describes how this is performed.

Let $(\underline{a}, \underline{A})$ denote the history of thresholds $\{a_t, A_t\}_{t=1,\ldots,T}$, for some large horizon T, where (a_t, A_t) correspond to the thresholds chosen t periods ago. Also, let (a^c, A^c) be the thresholds corresponding to the current period. Since we know that the optimal decision rules of establishments are of the (S,s) variety, there is no loss of generality in defining the Social Planner's problem directly in terms of choosing the current thresholds (a^c, A^c) and

[8]Note that the previous period thresholds (a, A) define the previous period decision rule d according to (19).

the fraction of people that work η as follows:[9]

$$V(\underline{a}, \underline{A}, z) = MAX\left\{\ln\left[c\left(\underline{a}, \underline{A}, z, a^c, A^c, \eta\right) - \alpha\, \eta + \beta EV(\underline{a}', \underline{A}', z')\right]\right\} \tag{20}$$

subject to:

$$\begin{aligned}
a'_{t+1}(s) &= a_t(s), && \text{for } t = 1, 2, ..., T-1 \text{ and } s = 1, \lambda \qquad (21)\\
a'_1(s) &= a^c(s), && \text{for } s = 1, \lambda\\
A'_{t+1}(s) &= A_t(s), && \text{for } t = 1, 2, ..., T-1 \text{ and } s = 1, \lambda\\
A'_1(s) &= A^c(s), && \text{for } s = 1, \lambda\\
z' &= \rho z + \varepsilon' && \qquad (22)
\end{aligned}$$

where equations (21) update tomorrow's histories given the current threshold choices.

The function $c\left(\underline{a}, \underline{A}, z, a^c, A^c, \eta\right)$ gives the maximum consumption that can be obtained given the history of thresholds $(\underline{a}, \underline{A})$, the current aggregate productivity shock z, the current choices of thresholds (a^c, A^c), and the decision of how many agents to currently put to work η. Formally, $c\left(\underline{a}, \underline{A}, z, a^c, A^c, \eta\right)$ is given by the following static labor allocation problem:

$$c\left(\underline{a}, \underline{A}, z, a^c, A^c, \eta\right) = MAX \int \left\{\, e^z s\; k^\theta n\left(k, s\right)^\gamma \right. \tag{23}$$

$$\left. - \left[g\left(k, s\right) - (1-\delta)k\right]\; Q[g\left(k, s\right) - (1 \;\cdot\cdot\; \delta)k] \,\right\} dx + \int (1-\delta)\; d\left(k, s\right)\; q\; \pi\left(s, 0\right)\; dy$$

[9]Note that problem (20) reduces to the original problem (8) as T goes to infinity.

subject to:

$$\int n(k,s)\ dx \leq \eta \tag{24}$$

where the maximization is with respect to the function $n(k,s)$.

The functions g, x, d, and y in (23) and (24) are determined by $(\underline{a}, \underline{A}, z, a^c, A^c)$ in the following way:

(i) The current investment decision rule g is implied by the current thresholds (a^c, A^c):

$$\begin{aligned} g(k,s) &= a^c(s), && \text{if } (1-\delta)k < a^c(s) \\ &= A^c(s), && \text{if } (1-\delta)k > A^c(s) \\ &= (1-\delta)k, && \text{otherwise} \end{aligned} \tag{25}$$

(ii) The current measure across establishment types x is obtained by initializing this measure T periods ago (x_T) to be the deterministic steady state measure x^*, and updating it recursively by iterating on the law of motion:

$$x_{t-1}(B,s') = \int_{(k,s):\ g_t(k,s)\in B} \pi(s,s')\ dx_t\ +\ v\ \psi(s')\ \chi(0\in B) \tag{26}$$

for $t = T, T-1, ..., 1$. The (approximate) measure x is then given by x_0.

The investment decision rules (g_t) t periods ago (for $t = T, T-1, ..., 1$), which are used in this law of motion are the ones determined by the corresponding thresholds (a_t, A_t) in the history $(\underline{a}, \underline{A})$:

$$\begin{aligned} g_t(k,s) &= a_t(s), && \text{if } (1-\delta)k < a_t(s) \\ &= A_t(s), && \text{if } (1-\delta)k > A_t(s) \\ &= (1-\delta)k, && \text{otherwise} \end{aligned} \tag{27}$$

(iii) The previous period measure across establishment types y, and the previous period decisions over current capital levels across establishment types d are those returned as x_1 and g_1 in (ii).

Note that the Social Planner's problem in equation (20) has linear constraints, and that the deterministic steady state values for the (endogenous) state variables are all strictly positive. We can then perform a quadratic approximation to the return function about the deterministic steady state, leaving us with a standard linear quadratic (L-Q) problem which can be solved by ordinary value function iteration.[10]

Let us now return to the question of how long the history of thresholds $(\underline{a}, \underline{A})$ should be to get a good approximate solution to the original problem (8). It is not difficult to show that there exists a length J for thresholds histories such that solving by L-Q methods the planner's problem (20) corresponding to length J, gives exactly the same solution as solving by L-Q methods the planner's problem (20) corresponding to any other length $T > J$ (Appendix B explains the intuition for this result).[11] It follows that the only approximation error introduced by the solution method stems from the quadratic approximation and not from keeping track of a finite history of thresholds.

4. Parametrization of the model

This section describes the steady state observations used to calibrate the parameters of the model economy. In this section, the irreversibility parameter q will be assumed fixed at

[10] The quadratic approximation is obtained by imposing zero errors of approximation of the return function at the grid points that lie just above and below the steady state grid points computed in Appendix A. This procedure to obtain numerical derivatives follows closely the one described in Kydland and Prescott [16].

[11] A formal proof to this claim is available upon request as a technical appendix. It can also be found in Veracierto [25].

some particular value. Given a fixed q, the rest of the parameters we need to calibrate are $\beta, \theta, \gamma, \delta, \alpha, \upsilon, \psi(1), \lambda$, the transition matrix Π, and the parameters determining the driving process for the aggregate productivity shock: ρ and σ_ϵ^2.

The first issue we must address is what actual measure of capital will our model capital correspond to. Since we are interested in investment irreversibilities at the establishment level it seems natural to abstract from capital components such as land, residential structures and consumer durables. The empirical counterpart for capital was consequently identified with plant and equipment. As a result, investment was associated in the National Income and Product Accounts with non-residential investment. On the other hand, the empirical counterpart for consumption was identified with personal consumption expenditures in non-durable goods and services. Output was then defined to be the sum of these investment and consumption measures. The annual capital-output ratio and the investment-output ratio corresponding to these measures are 1.7 and 0.15 respectively. The depreciation rate δ was selected to be consistent with these two magnitudes.

The annual interest rate was selected to be 4 per cent. This is a compromise between the average real return on equity and the average real return on short-term debt for the period 1889 to 1978 as reported by Mehra and Prescott [18]. The discount factor β was chosen to generate this interest rate at steady state. Given the interest rate i and the depreciation rate δ, the parameter θ was selected to match the capital-output ratio in the U.S. economy. The labor share parameter was in turn selected to replicate a labor share in National Income of 0.64 (this is the standard value used in the business cycle literature). On the other hand, the preference parameter α was picked such that 80% of the population works at steady state (roughly the fraction of the U.S. working age population that is employed).

The transition matrix Π was chosen to be of the following form:

$$\begin{pmatrix} 1 & & \\ \zeta & \phi(1-\zeta) & (1-\phi)(1-\zeta) \\ \zeta & (1-\phi)(1-\zeta) & \phi(1-\zeta) \end{pmatrix} \tag{28}$$

i.e. a process that treats the low and the high productivity shocks symmetrically. The rest of the parameters to calibrate are then $\phi, \zeta, \upsilon, \psi(1)$, and λ. The parameters ζ, ϕ, and λ were selected to reproduce important observations on job creation and job destruction reported in Davis and Haltiwanger [10]. These are: (i) that the average annual job creation rate due to births and the average annual job destruction rate due to deaths are both about 2.35%, (ii) that the average annual job creation rate due to continuing establishments and the average annual job destruction rate due to continuing establishments are both about 7.9%, and (iii) that about 82.3% of the jobs destroyed during a year are still destroyed the following year. The parameter ν determining the number of establishments being created every period was chosen so that the average establishment size in the model economy is about 65 employees, same magnitude as in the data.

Next, we must determine the distribution ψ over initial productivity shocks. If we would allow for a large number of possible idiosyncratic productivity shocks, it would be natural to chose a ψ to reproduce the same size distribution of establishments as in the data.[12] With only two values for the idiosyncratic shocks this approach does not seem restrictive enough since we can pick any two arbitrary employment ranges in the actual size distribution to

[12]In general, allowing for a large number of idiosyncratic shocks would permit to calibrate the model to more realistic establishment level dynamics. However, the associated computational costs would be unbearable (even for computing deterministic steady states).

calibrate to. For this reason I chose to follow the same principle as in the choice of Π and pick $\psi = (0.5, 0.5)$, i.e. a distribution that treats the low and the high productivity shock symmetrically (note that these choices of Π and ψ imply that at steady state there will be as many establishments with the low shock as with the high shock).

Finally, we must determine values for ρ and σ_ε^2. The strategy for selecting values for these parameters was to chose them so that measured Solow residuals in the model economy replicate the behavior of measured Solow residuals in the data. Proportionate changes in measured Solow residual are defined as the proportionate change in aggregate output minus the sum of the proportionate change in labor times the labor share γ, minus the sum of the proportionate change in capital times $(1-\gamma)$. Note that these changes in measured Solow residuals do not coincide with changes in the aggregate productivity variable z in the model (the aggregate production function in the model economy is not a constant returns Cobb-Douglas function in labor and aggregate capital). Using the measure of output described above and a share of labor of 0.64, measured Solow residuals were found to be as highly persistent as in Prescott [20] but the standard deviation of technology changes came up somewhat smaller: 0.0063 instead of the usual 0.0076 value used in the literature. Given a fixed irreversibility parameter q and the rest of the parameters calibrated as above, values for ρ and σ_ε^2 were selected so that measured Solow residuals in the model economy displayed similar persistence and variability as in the data. It happened to be the case that values of $\rho = 0.95$ and $\sigma_\varepsilon^2 = 0.0063^2$ were consistent with these observations in all the experiments reported below.

Parameters values corresponding to economies with several different possible values for q are reported in Table 1.

5. Benchmark aggregate fluctuations

There is considerable uncertainty about the choice of an empirically plausible value for the irreversibility parameter q. Even though a number of empirical papers have documented features of establishment behavior that suggest the presence of investment irreversibilities at the establishment level (e.g. Caballero, Engel and Haltiwanger [7], Doms and Dunne [12]), almost no attempt was made to estimate the magnitude of the irreversibilities that establishments face. A remarkable exception is Ramey and Shapiro [21]. Using data on the equipment auction of an aerospace firm, they estimated the wedge between the purchase price and the resale price for different types of capital. Specifically, they estimated that wedge to be 31% for machine tools and 5% for structural equipment. A difficulty with their estimates is that they correspond to a single firm and cannot be directly extended to the whole economy. In any case, they are indicative of important degrees of investment irreversibilities at the micro level. As a consequence, the economy with $q = 0.5$ will be selected as a benchmark case but later on results will be reported under a variety of values for the irreversibility parameter q.

Table 2 reports summary statistics (standard deviations and correlations with output) for the aggregate fluctuations of the benchmark economy and compares them to those of the actual U.S. economy. Before any statistics were computed, all time series were logged and detrended using the Hodrick-Prescott filter. The statistics reported for the U.S. economy correspond to the output, investment and consumption measures described in the previous section, and refer to the period between 1960:3 and 1993:4. For the artificial economy, time series of length 136 periods (same as in the data) were computed for 100 simulations, the reported statistics being averages across these simulations.

We observe that output fluctuates as much in the model economy as in actual data. Investment is about 5 times more variable than output in the model while it is about 4 times as variable in the U.S. economy. Consumption is less variable than output in both economies (though consumption is less variable in the model than in U.S. data). The aggregate stock of capital varies about the same in both economies. On the other hand, hours variability is only 70% the variability of output in the model economy while they vary as much as output in U.S. data. Productivity fluctuates less in the model economy than in the actual economy. In terms of correlations with output, we see that almost all variables are highly procyclical both in the model and in U.S. data. The only exceptions are capital (which is acyclical both in the model and the actual economy) and productivity (which is highly procyclical in the model while it is acyclical in the data).

We conclude that the benchmark economy is broadly consistent with salient features of U.S. business cycles.

6. Microeconomic irreversibilities and aggregate dynamics

In a celebrated paper Caballero, Engel and Haltiwanger [7] analyzed the importance of microeconomic irreversibilities for macroeconomic dynamics using a non-structural empirical approach. They concluded that irreversibilities play a crucial role in generating brisk expansions and contractions in aggregate investment dynamics. Since this has become the dominant view thereafter, Sections 6.1 and 6.2 will describe their approach in detail and apply it to the benchmark economy. The objective is not only to facilitate comparisons with the previous literature, but to demonstrate the need for a general equilibrium analysis.

Section 6.3 constitutes the core of the paper. It compares the equilibrium business cycles of economies subject to different degrees of investment irreversibilities. Contrary to what the Caballero-Engel-Haltiwanger analysis of Section 6.2 suggests, investment irreversibilities are found to have no effects on aggregate business cycle dynamics.

6.1. The "Caballero-Engel-Haltiwanger" approach

Caballero, Engel and Haltiwanger (hereafter CEH) proposed the following non-structural method of analysis.[13] They defined "desired capital" k_{t+1}^d to be the stock of capital an establishment would like to carry to the following period if its investment irreversibility constraint was momentarily removed during the current period. Correspondingly, they defined "mandated investment" e_t to be:

$$e_t = k_{t+1}^d - (1 - \delta)k_t \tag{29}$$

where k_t is the establishment's stock of capital at date t, and δ is the depreciation rate of capital.

CEH assumed that the investment behavior of establishments could be described by a "hazard function" $H(e)$, which specifies for each possible e the fraction of its mandated investment that an establishment actually undertakes. Letting f_t be the distribution of

[13] This same approach has been used by Caballero, Engel and Haltiwanger [8] to analyze the effects of non-convex adjustment costs in aggregate employment dynamics.

establishments across mandated investments at date t, aggregate investment I_t is given by:

$$I_t = \int e \; H\,(e)\; f_t(e)\; de \tag{30}$$

To empirically implement their model, CEH first had to estimate the desired stock of capital k^d_{t+1} for each establishment and time-period in their sample.[14] This gave CEH an empirical time series for the distribution of cross-sectional mandated investment f_t. A functional form for the hazard function H also had to be specified. CEH chose to work with the following polynomial form:

$$H(e) = \sum_{v=0}^{V} \varphi_v \; e^v \tag{31}$$

Substituting (31) in (30) delivers the following expression for aggregate investment:

$$I_t = \sum_{v=0}^{V} \varphi_v \; M_t^{v+1} \tag{32}$$

where M_t^v is the v-th moment of the distribution of mandated investments f_t.

CEH noticed that when establishments face quadratic costs of adjustment, their optimal investment behavior is described by a constant hazard function. In this case the "linear" model results: aggregate investment depends only on the first moment of the distribution of mandated investment ($V = 0$ in equations 31 and 32). In all other "non-linear" cases, higher moments of the distribution can play an important role in accounting for aggregate investment behavior.

[14] We refer to the original paper for details on how these measures were obtained.

Allowing for a constant, (32) gives rise to the CEH regression equation:

$$I_t = \mu + \sum_{v=0}^{V} \varphi_v \, M_t^{v+1} + \varepsilon_t \tag{33}$$

Using the first five moments of their estimated cross-sectional distribution of mandated investment, CEH fitted this equation to aggregate investment data using ordinary least squares. They found that the hazard function implied by their estimated coefficients φ is highly non-linear. It indicates that establishments undertake small adjustments in response to negative mandated investments and that they respond substantially to positive mandated investments (a behavior consistent with irreversibilities in investment).[15] This finding is extremely important, it suggests that microeconomic irreversibilities exist in the U.S. economy and that the associated non-linear adjustments at the establishment level are consistent with aggregate investment dynamics.

To evaluate the importance of non-linear adjustments for aggregate investment dynamics CEH ran a second regression, this time constraining aggregate investment to depend only on the first moment of the cross-sectional distribution of mandated investments (i.e. setting V to zero in equation 33). CEH considered this linear model to be of particular interest since it corresponds to an economy with quadratic costs of adjustment. CEH believed that comparing the aggregate investment behavior predicted by this linear model with the predictions of the non-linear model would determine the role played by microeconomic irreversibilities in aggregate investment dynamics.

[15] CEH also estimated the hazard function in an alternative and more direct way. They measured $H(e)$ to be the average actual-investment/mandated-investment ratio across all establishments with mandated investment e. They found that this alternative hazard function displays qualitatively similar properties as the one estimated using equation (33).

Analyzing the relative performance of both models, CEH found the following results: 1) the non-linear model kept track of aggregate investment behavior much better than the linear model (the absolute values of the prediction errors were always larger in the linear model than in the non-linear model), and 2) this was specially true at brisk expansions and contractions (the difference between the absolute prediction error of the linear model and the absolute prediction error of the non-linear model was larger at periods when aggregate investment was far from its mean). CEH interpreted these results as evidence that microeconomic irreversibilities are crucial for macroeconomic dynamics. In particular, that irreversibilities play an important role in generating brisk expansions and contractions in aggregate investment.

This way of evaluating the importance of microeconomic irreversibilities for aggregate dynamics presents a serious weakness. The analysis compares the predictions made by the linear model with the predictions made by the non-linear model, conditional on **a same** realization of cross-sectional distributions of mandated investments f_t. However, mandated investments are not invariant to the investment technology that establishments face. If microeconomic irreversibilities were replaced by quadratic costs of adjustment, mandated investments would no longer be the same. As a result, U.S. mandated investments (which were determined to correspond to an economy with investment irreversibilities) cannot be used to evaluate how aggregate investment would behave if establishments faced quadratic costs of adjustment. Sections 6.2 and 6.3 demonstrate that this type of analysis can in fact lead to the wrong answers.

6.2. Caballero, Engel and Haltiwanger visit the benchmark model

This section applies the CEH analysis to the benchmark economy. The objective is twofold. First, to explore wether the benchmark economy is broadly consistent with the CEH empirical findings. Second, to determine what conclusions can be obtained about the importance of investment irreversibilities for aggregate dynamics.

Applying the CEH approach will involve running a similar set of regressions as CEH, but on artificial data generated by the benchmark economy. Before doing so, model-counterparts for the variables defined by CEH must be determined.

Observe that the desired capital of an establishment of type (k, s) is given by $k^d_{t+1} = a(s)$, since this is the stock of capital the establishment would chose if q was set to one during the current period (see footnote 7). As a consequence, its mandated investment is given by $e = a(s) - (1-\delta)k$. The aggregate measure x_t of establishments across idiosyncratic productivity shocks s and capital levels k can then be used to obtain the cumulative distribution of establishments across mandated investments levels in the model economy:[16]

$$F_t(e) = \int_{\{(k,s):\ a(s)-(1-\delta)k\ \leq\ e \text{ and } k>0\}} dx_t$$

The moments M_t^v to be used in the regressions below are those obtained from this cumulative distribution.[17]

[16]Note that in defining this cross-sectional distribution of mandated investments, we have excluded all those establishments that died between periods $t-1$ and t, as well as those establishments that were born between periods $t-1$ and t (and therefore have $k_t = 0$). This is done to parallel CEH, who worked with a balanced panel of establishments. However, similar results are obtained if those establishments are included in the distribution of mandated investments.

[17]In applying the CEH approach to the benchmark economy we are abstracting from any measurement problems. We'll show that the approach can lead to the wrong conclusions even when mandated investments are correctly measured.

To generate artificial data, the empirical realization of Solow residuals for the period 1960:1 to 1993:4 was fed into the model economy.[18] The resulting time series for aggregate investment and the first five moments M_t^v were subsequently used to estimate the CEH investment equation (33) by ordinary least squares. The estimated coefficients gave rise to the hazard function shown in Figure 2.[19] This hazard function displays similar properties as those reported by CEH, i.e. it indicates small adjustments to capital surpluses and large adjustments to capital shortages. This is not surprising since there are substantial investment irreversibilities in the benchmark model economy and consequently, establishments do follow (S,s) decision rules.[20]

Following CEH, the linear model was also estimated. In particular, a second regression allowing aggregate investment to depend only on the first moment of the distribution of mandated investments was fitted to the model-generated data.

Before comparing the predictions of the two statistical models, the time periods from the simulations were sorted in a decreasing order from the largest to the smallest realized absolute deviation of aggregate investment from its mean. Figure 3 plots the difference between the absolute prediction error of the linear model and the absolute prediction error of the nonlinear model, across the sorted time periods.[21] We observe two important patterns.

[18]This constitutes 136 periods of observations. In the experiment, the distribution across establishment types was initialized to be the deterministic steady state distribution. Then, 136 periods were generated from the model economy but only the last 100 periods were considered to be the sample period for the regressions. This was done to minimize the effects of initializing with the deterministic steady state distribution.

[19]The figure shows the hazard function over the range of relevant mandated investment values (i.e. the range of values realized in the experiments).

[20]In the benchmark model economy, the hazard function over the relevant range of mandated investments is equal to one for positive values of e (establishments adjust to $a(s)$ if $(1-\delta)k < a(s)$), and equal to zero for negative values of e (establishments let the capital depreciate if $(1-\delta)k > a(s)$).

It should not surprise the reader that this is the hazard function that corresponds to $q = 0$, since it will be shown in the next section that establishments behave extremely similarly under $q = 0.5$ than under $q = 0$.

[21]The absolute prediction errors were divided by aggregate investment to obtain relative magnitudes.

First, in all periods the difference is positive. Second, the difference is large in the first few periods and then declines towards zero. In other words, the non-linear model predicts the benchmark economy's aggregate investment more accurately and this is specially true when aggregate investment is far from its mean.[22] These are exactly the same findings that CEH encountered in their empirical analysis. We see that the benchmark economy conforms with them quite well.

Based on similar findings, CEH concluded that microeconomic irreversibilities play a crucial role in generating brisk expansions and contractions in aggregate investment. However, the analysis performed provides little economic support for making such type of assessment. While the predictions of the estimated non-linear model (approximately) describe the behavior of aggregate investment in our benchmark economy, it is not clear what investment behavior the estimated linear model is capturing.[23] In principle, it represents some economy with quadratic costs of adjustment. But it would be hard to determine an economy consistent **both** with the constant hazard function of our estimated linear model **and** the mandated investments of our benchmark economy.[24] Even if such economy exists, comparing its aggregate investment dynamics with that of our benchmark economy (as the CEH approach does) is probably not a fruitful exercise: the economies would differ in so many dimensions, that no definite statement could be made about the particular role of microeconomic irreversibilities

[22]The fact that the non-linear model predicts aggregate investment much better than the linear model should not be surprising. The (S,s) decision rules that establishments follow in the benchmark model can be much better approximated by fitting a high order polynomial for the hazard function than a constant function.

[23]Recall that the predictions of the linear model are made conditional on the mandated investments of the benchmark economy.

[24]As has already been mentioned, mandated investments are not invariant to the investment technology that establishments face.

in aggregate dynamics.[25]

Given these difficulties with the CEH approach, Section 6.3 will perform an alternative analysis. It will compare the general equilibrium dynamics of economies subject to different investment irreversibility levels. An important advantage of this approach is that explicit economic environments will be specified and that the aggregate dynamics analyzed will be fully consistent with these environments. As a consequence, the analysis will be able to make precise statements about the role of investment irreversibilities in aggregate dynamics. Interestingly, the conclusions obtained will be in sharp contrast to CEH.

6.3. The effects of micro-irreversibilities in aggregate fluctuations

This section evaluates the effects of irreversibilities on aggregate fluctuations by comparing the equilibrium business cycles of economies subject to different degrees of investment irreversibilities. For this to be a meaningful exercise, the economies must be comparable in important dimensions. We choose them to be identical to the U.S. economy in terms of the long run means and ratios of Section 4, as well as their stochastic processes for measured solow residuals. These are all observations that the real business cycle literature has emphasized as being important for aggregate fluctuations. Controlling for them will help isolate the effects of investment irreversibilities in business cycle dynamics.

Table 3 provides summary statistics for the business cycles of economies with investment irreversibility parameters q's ranging between 1 and 0.[26] The results are striking. Irre-

[25] It must be pointed out that the CEH analysis provides valuable econometric information. It determines the prediction biases that would be obtained from measuring mandated investments correctly in an economy with investment irreversibilities, while misspecifying the hazard function to be a constant function. However, the CEH approach is less useful as an economic analysis.

[26] Parameter values were given in Table 1.

versibilities tend to decrease the variability of output, investment and hours, and increase the variability of consumption. But these differences are surprisingly small. For example, the standard deviation of output decreases monotonically as q goes from 1 to 0 (as one would expect given the adjustment costs introduced), but it goes from 1.41 when $q = 1$ to only 1.39 when $q = 0$. This is a small difference considering that we are moving from the perfectly reversible case to the complete irreversibilities scenario. Overall, the properties of the business cycles generated by all these economies are extremely similar. We conclude that, at least in terms of the standard statistics that the real business cycle literature focuses on, investment irreversibilities play no crucial role for aggregate fluctuations.

Nevertheless, investment irreversibilities could still be important for features of the business cycles not captured by the standard RBC statistics. One possibility is that investment irreversibilities generate brisker expansions and contractions in aggregate investment. Another possibility is that investment irreversibilities create asymmetries in aggregate fluctuations. Figure 4 explores these possibilities: it reports for the economies with $q = 1$ and $q = 0$, the histograms of the deviations of aggregate investment from trend across all realizations. If investment irreversibilities generate brisker expansions or contractions, the histogram under $q = 0$ would have fatter tails than under $q = 1$. On the other hand, if investment irreversibilities generate asymmetries, the histogram under $q = 0$ would be more asymmetric than under $q = 1$. However, Figure 4 shows that these histograms are virtually the same. Investment irreversibilities do not create noticeable asymmetries nor brisk expansions and contractions.[27]

Figure 5 searches for other possible differences in aggregate dynamics created by invest-

[27]The analysis of aggregate output, consumption, and hours worked lead to similar conclusions.

ment irreversibilities. It reports the realizations of aggregate investment which arise from feeding into the economies with $q = 1$ and $q = 0$ the empirical realization of Solow residuals for the period 1960:1 to 1993:4. We observe that the time series for aggregate investment generated by the economy with complete irreversibilities is almost identical to the one generated by the economy with perfectly reversible investment. Figure 6 shows the impulse response functions for output (Y), consumption (c), investment (I), and labor (η) to a one-time aggregate productivity shock of one standard deviation, which correspond to the economies with $q = 1$ and $q = 0$. We also see that they are almost the same. We conclude that investment irreversibilities have no major effects on aggregate business cycle dynamics.

This is a surprising result. Intuition suggests that investment irreversibilities could generate important asymmetries in aggregate fluctuations, as establishments would be much more reluctant to adjust their stock of capital to negative productivity shocks than to positive productivity shocks. To understand our lack of asymmetries result, we must analyze the responses of the model economy to aggregate shocks in further detail.

For the economy with $q = 0.99$, Figure 7 shows the impulse response of the capital support of the distribution x_t to a one-time aggregate productivity shock of one standard deviation, starting from the steady state support (the steady state capital distribution x^* is displayed in Figure 8).[28] We see that in response to a positive shock, the thresholds $a(1)$, $A(1)$ and $a(\lambda)$ increase on impact, continue to increase for a number of periods and eventually decrease, returning gradually to their steady state levels. Instead, the capital

[28]We chose to show the behavior of the economy with $q = 0.99$ over those with a smaller q, since it has a relatively small capital support (simplifying the figures considerably). However, similar patterns can be found in the other economies.

Even though it is not shown, zero always belongs to the support of the capital distribution since new establishments arrive with zero capital in place.

levels pertaining to the range of inaction between $a(1)$ and $A(1)$ are not affected on impact. They follow the same dynamics as the upper threshold $A(1)$ but with a lag, which depends on the number of periods it takes $A(1)$ to depreciate to the corresponding capital level. Observe that the support of the capital distribution responds symmetrically to positive and negative shocks. It is then not surprising that the business cycles generated by these shocks will inherit similar features.[29]

On a first impression, the symmetric response of the capital support to aggregate shocks may appear a necessary consequence of the linear quadratic approximation performed. But this is not generally true. The solution method implies that current capital thresholds (the decision variables) are a linear function of the aggregate shock and the past history of capital thresholds (the state variables). If the driving aggregate productivity shock is symmetrically distributed, of course capital **thresholds** will behave symmetrically. But this does not imply that the capital **support** will behave symmetrically. In fact, it seems safe to conjecture that the symmetry would be lost if aggregate shocks had an empirically implausible large variance.

To be concrete, let consider how the largest point in the capital support would respond to a large negative shock, starting from its steady state value $a^*(\lambda)$. Suppose that the shock is so low that the threshold $a(\lambda)$ decreases on impact below $(1-\delta)a^*(\lambda)$. The highest point in the support would then become $(1-\delta)a^*(\lambda)$, since it would fall in the range of inaction defined by the new value of $a(\lambda)$. What is important to note is that negative shocks of larger magnitude would generate no further effects on impact, since $(1-\delta)a^*(\lambda)$ would still fall in

[29]Strictly speaking, describing the response of the capital support is not enough. The number of establishments at each of these capital levels and idiosyncratic productivity shocks should also be considered. However, at any point in time, the number of establishments at each of these capital levels can be read directly from the corresponding point in the steady state distribution in Figure 8. The reason is that the process for the idiosyncratic shocks is exogenous and the paths illustrated in Figure 7 do not cross.

a range of inaction. On the contrary, there would be no counterpart to this lack of further responsiveness when shocks are positive. If a positive shock drives $a(\lambda)$ above $(1-\delta)a^*(\lambda)$, the highest point in the support would always jump on impact to the new value of $a(\lambda)$. This would be true no matter how large the positive shock is.

Figure 9 illustrates these ideas by showing the impulse responses of the highest point in the capital support to one-time aggregate shocks, ranging from one to twenty standard deviations in magnitude.[30] Let consider the responses in period one to each of these shocks. We see that when shocks are negative, the largest capital level in the support moves to smaller values as the shock becomes larger. However, once the shock reaches fifteen standard deviations, it stops responding to further shocks. On the contrary, when shocks are positive, this capital level always moves to higher values as the shock gets larger. This pattern of response opens interesting possibilities for the creation of asymmetries in aggregate fluctuations. In particular, it suggests that aggregate investment would tend to decrease slowly in response to large negative shocks, and increase sharply in response to large positive shocks.[31]

In view of these arguments, we must consider the lack of asymmetries the theory predicts as arising purely from measurement. Measured solow residuals are not variable enough for investment irreversibilities to create asymmetries in aggregate business cycles: the associated fluctuations in capital thresholds are small compared with the drift introduced by depreci-

[30] This figure is drawn only for heuristic purposes. If shocks were as large as those shown, the linear quadratic approximation performed in the paper would probably be of poor quality.

[31] It should be clear that $a(1)$ would generate similar asymmetries, since it would mimic the behavior of $a(\lambda)$. The analysis would be somewhat more complicated though, since capital levels in the lower portion of the range of inactivity would be affected by large fluctuations in $a(1)$. In particular, these points would collapse into $a(1)$ under sufficiently large increases in $a(1)$, but will not be affected when $a(1)$ decreases. This effect would tend to reinforce the asymmetries described above.

For economies with q's smaller than 0.9 the behavior of $A(1)$ becomes irrelevant, since the ranges of inaction overlap and no establishment has a capital level close to $A(1)$.

ation (actually in none of the simulations reported, the rate of change of thresholds ever exceeded the rate of depreciation).[32]

To complete our analysis, we now consider the importance of irreversibilities for plant level investment dynamics. Figure 10 shows the distribution of plant level gross investment rates (for continuing establishments) across all realizations, under different values for the irreversibility parameter q.[33] We observe that when there are no irreversibilities ($q = 1.0$), a large number of establishments have near-zero net investment (gross investment approximately equals the depreciation of capital).[34] We also observe that a small number of firms make sharp increases and sharp decreases in their stock of capital.[35] On the contrary, when the irreversibility parameter q becomes zero: 1) the number of establishments displaying close-to-zero adjustment is larger, 2) there are no establishments with sizable negative investment rates, and 3) capital increases are not as sharp as under $q = 1.0$. Note that a substantial mass of establishments have zero gross investment, since a large number of plants chose to remain inactive because of the investment irreversibility they face (they lie in ranges of inaction). It is also interesting to note that the histogram of investment rates that arises when $q = 0.95$ is very similar to the one under $q = 0$. This conforms with the finding by Abel and Eberly [1] that small degrees of irreversibilities can matter a lot. Figure 10 shows

[32]This result is related to Dow and Olson [14]. They found that in the real business cycles model of Hansen [17], aggregate irreversibilities play no role since productivity shocks are not variable enough to make the non-negativity constraint in aggregate investment binding. An important difference in this paper is that plant level irreversibilities do bind. However, they bind due to the amount of idiosyncratic risk that establishments face, not because of the level of aggregate uncertainty in the economy. Aggregate productivity shocks play a minor role.

[33]These histograms correspond to the same simulations as those underlying Table 3.

[34]These are establishments that in the absence of aggregate shocks would like to keep the same stock of capital they have. With aggregate fluctuations their investment rates do fluctuate, but the grid in Figure 2 is not fine enough to reflect these movements.

[35]This is due to the fact that only two (positive) idiosyncratic shocks are considered, and that they are very persistent.

that even though irreversibilities play no important role for aggregate fluctuations, they are crucial for establishment level dynamics.

7. Conclusions

Caballero, Engel and Haltiwanger [7] found that non-linear adjustments are crucial for aggregating mandated investments in the U.S. economy into observed aggregate investment dynamics. While their findings represent substantial evidence of investment irreversibilities in the U.S. economy, their analysis is less useful for assessing the importance of investment irreversibilities in business cycle dynamics. To evaluate the effects of investment irreversibilities on aggregate fluctuations, the business cycles of economies subject to different levels of irreversibilities must be analyzed.

An interesting empirical project would be to study how business cycles vary with investment irreversibilities across actual countries, after controlling for variables such as average capital-output ratios, capital shares, investment rates, variability and persistence of solow residuals, etc (i.e. variables that business cycle theory emphasize as being important). However, lack of data on investment irreversibilities makes this exercise unfeasible.

This paper has used economic theory to proxy for this exercise. It has compared the business cycles of artificial economies that look exactly the same as the U.S. economy in terms of their long run observations and solow residuals, but which differ in terms of their investment irreversibility levels. The results were striking: economies ranging from fully reversible to completely irreversible investment generate almost identical business cycle dynamics. The only dimension in which investment irreversibilities matter is for establishment level dynamics.

Our results suggest that macroeconomists can safely abstract from investment irreversibilities when modeling aggregate business cycle dynamics.

A. Appendix

This appendix describes the algorithm used to compute the steady state of the deterministic version of the economy. We will show that the problem is reduced to solving one equation in one unknown (after the relevant substitutions have been made). First, it must be noticed that (similarly to the neoclassical growth model) the steady state interest rate is given by:

$$1 + i = \frac{1}{\beta} \tag{34}$$

Fixing the wage rate at an arbitrary value w, the value of the different types of establishments (as a function of w) can be obtained by solving the following functional equation:

$$\begin{aligned} J(k, s; w) = MAX \quad \{ \quad & s\, k^{\theta} n^{\gamma} - wn - [k' - (1-\delta)k]\, Q\, [k' - (1-\delta)\, k] \\ + \quad & \frac{1}{1+i} \sum_{s'} J\,(k', s'; w)\, \pi\,(s, s') \ \} \end{aligned} \tag{35}$$

The solution to this problem is computed using standard recursive methods. Note that the solution to this problem also gives the decision rules $n(k, s; w)$ and $g(k, s; w)$ as a function of w.

Given a w and the corresponding $g(k, s; w)$, a measure $x(w)$ across productivity shocks and capital levels can be obtained from the law of motion for x:

$$x\left(B, s^{\prime}; w\right)=\int_{(k,s):g(k,s;w)\in B} \pi\left(s, s^{\prime}\right)\ dx(w)+v\ \psi\left(s^{\prime}\right)\ \chi(0 \in B) \tag{36}$$

In practice, this is solved by iterating on this law of motion starting from an arbitrary initial guess for $x(w)$.

Once a $x(w)$ is obtained and given the previous $n(k, s; w)$ and $g(k, s; w)$ found, we can solve for the corresponding consumption $c(w)$ implied by the feasibility condition:

$$\begin{aligned} c(w)=\int s\ k^{\theta} n(k, s; w)^{\gamma} \quad \cdot \quad & [g(k, s; w)-(1-\delta) k]\ Q[g(k, s; w)-(1-\delta) k]\ dx(w) \\ + \quad \int \quad & (1-\delta)\ g(k, s; w)\ q\ \pi(s, 0)\ dx(w) \end{aligned} \tag{37}$$

A wage rate w corresponds to the steady state value if the marginal rate of substitution between consumption and leisure is satisfied, i.e.:

$$c(w)=\frac{w}{\alpha} \tag{38}$$

This is one equation in one unknown and is solved using standard root finding methods.

The actual computer implementation of this algorithm requires working with a finite grid of capital levels. In all experiments reported in the paper, the number of grid points were between 1,000 and 1,800.

B. Appendix

This appendix provides intuition for why it is sufficient to carry a finite history of thresholds when solving the social planner's problem (20) by L-Q methods.

Suppose that it takes exactly J periods for the steady state upper threshold $A^*(1)$ to first depreciate below the steady state lower threshold $a^*(1)$. For simplicity, assume that $A^*(1) < a^*(\lambda)$. Fixing all other thresholds at their steady state values, consider for example the effects of a small change in the upper capital threshold $A(1)$ about its steady state value $A^*(1)$. In what follows it will be argued that within J periods, an establishments that was affected by this small change in $A(1)$ will end up with exactly the same capital level as in the absence of such small perturbation.

There are two possibilities. The first case is if after exactly J periods the establishment has not yet made a transition to $s = \lambda$. In this case, its stock of capital (after J periods) would be given by $(1-\delta)^J A(1)$ (since the establishment would have stayed in the range of inaction corresponding to $s = 1$). Given that $A(1)$ is close to $A^*(1)$, and $(1-\delta)^J A^*(1) < a^*(1)$, it follows from (19) that (after J periods) the establishment will chose its next period capital to be $k' = a^*(1)$ (the same k' the establishment would have chosen without the small perturbation in $A(1)$).

The second case is if the establishment makes a transition to $s = \lambda$ in t periods, where $t < J$. In this case, its capital level would be $(1-\delta)^t A(1)$ at the time of such transition. Since $A(1)$ is close to $A^*(1)$ and $(1-\delta)^t A^*(1) < a^*(\lambda)$, it follows from (19) that the establishment will chose a next period stock of capital $k' = a^*(\lambda)$ at the time of the transition (again, the same k' the establishment would have chosen without the small perturbation in $A(1)$).

Note that if $a(1)$ or $a(\lambda)$ would've differed slightly from their steady state values at the time that these actions take place, the argument would still apply. The establishment would end up within J periods with a stock of capital given by a threshold level determined in a later period, independently of the small perturbation in $A(1)$ made J periods ago. As a consequence no further information is gained from keeping histories more than J periods long, provided that capital thresholds remain in a neighborhood of their steady state values.[36]

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TABLE 1
Parameter Values

	q = 1.0	q = 0.99	q = 0.95	q = 0.90	q = 0.75	q = 0.50	q = 0.00
β	0.99	0.99	0.99	0.99	0.99	0.99	0.99
θ	0.2186	0.2185	0.2179	0.2174	0.2173	0.2172	0.2168
γ	0.64	0.64	0.64	0.64	0.64	0.64	0.64
δ	0.02206	0.02184	0.02163	0.02146	0.02066	0.01914	0.01622
α	0.94	0.94	0.94	0.94	0.94	0.94	0.94
ν	7.257E-5	7.260E-5	7.257E-5	7.273E-5	7.278E-5	7.280E-5	7.287E-5
$\psi(1)$	0.5	0.5	0.5	0.5	0.5	0.5	0.5
λ	1.112	1.127	1.195	1.208	1.210	1.213	1.220
φ	0.92	0.92	0.92	0.92	0.92	0.92	0.92
ζ	0.00593	0.00593	0.00593	0.00593	0.00593	0.00593	0.00593
ρ	0.95	0.95	0.95	0.95	0.95	0.95	0.95
σ_ε^2	0.0063^2	0.0063^2	0.0063^2	0.0063^2	0.0063^2	0.0063^2	0.0063^2

TABLE 2

U.S. and benchmark fluctuations

	U.S. Economy (60:1-93:4)		Benchmark Economy	
	Std. Deviation	Correlation	Std. Deviation	Correlation
Output	1.33	1.00	1.39	1.00
Consumption	0.87	0.91	0.51	0.91
Investment	4.99	0.91	6.83	0.98
Capital	0.63	0.04	0.49	0.08
Hours	1.42	0.85	0.94	0.98
Productivity	0.76	-0.16	0.51	0.92

TABLE 3

Business cycles across economies

Standard Deviations:

	q = 1.0	q = 0.99	q = 0.95	q = 0.90	q = 0.75	q = 0.50	q = 0.00
Output	1.41	1.40	1.39	1.39	1.39	1.39	1.39
Cons.	0.49	0.50	0.51	0.51	0.51	0.51	0.52
Investm.	7.09	7.01	6.89	6.85	6.83	6.83	6.82
Capital	0.51	0.50	0.49	0.49	0.49	0.49	0.48
Hours	0.98	0.97	0.95	0.94	0.94	0.94	0.94
Product.	0.49	0.50	0.51	0.51	0.51	0.51	0.52

Correlations with Output:

	q = 1.0	q = 0.99	q = 0.95	q = 0.90	q = 0.75	q = 0.50	q = 0.00
Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Cons.	0.91	0.91	0.91	0.91	0.91	0.91	0.91
Investm.	0.98	0.98	0.98	0.98	0.98	0.98	0.98
Capital	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Hours	0.98	0.98	0.98	0.98	0.98	0.98	0.98
Product.	0.91	0.91	0.91	0.92	0.92	0.92	0.92

FIGURE 1
Decision Rules

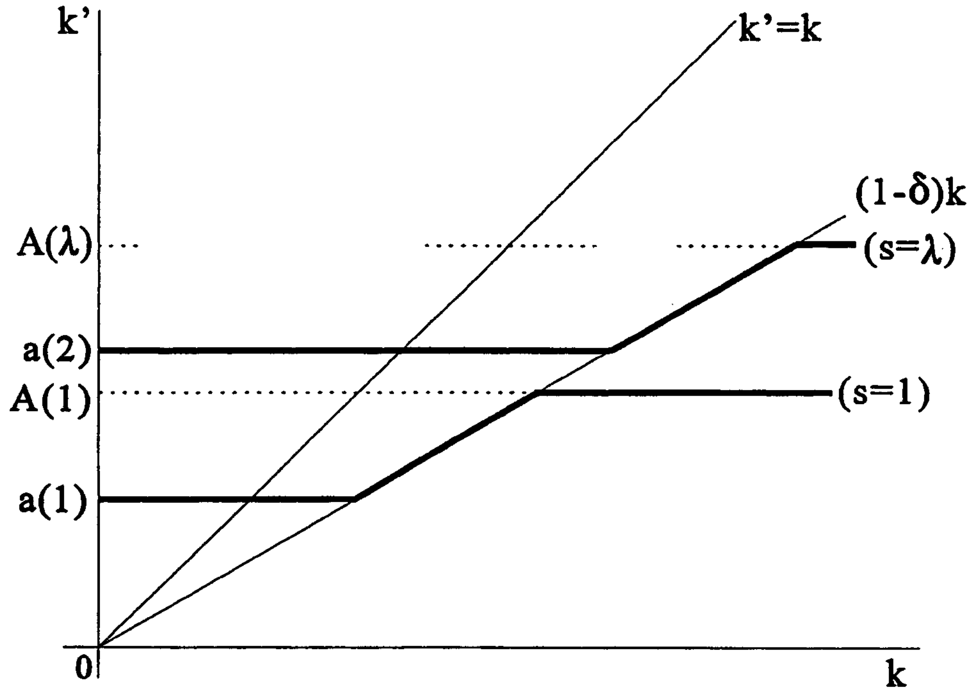


FIGURE 2

Hazard Function

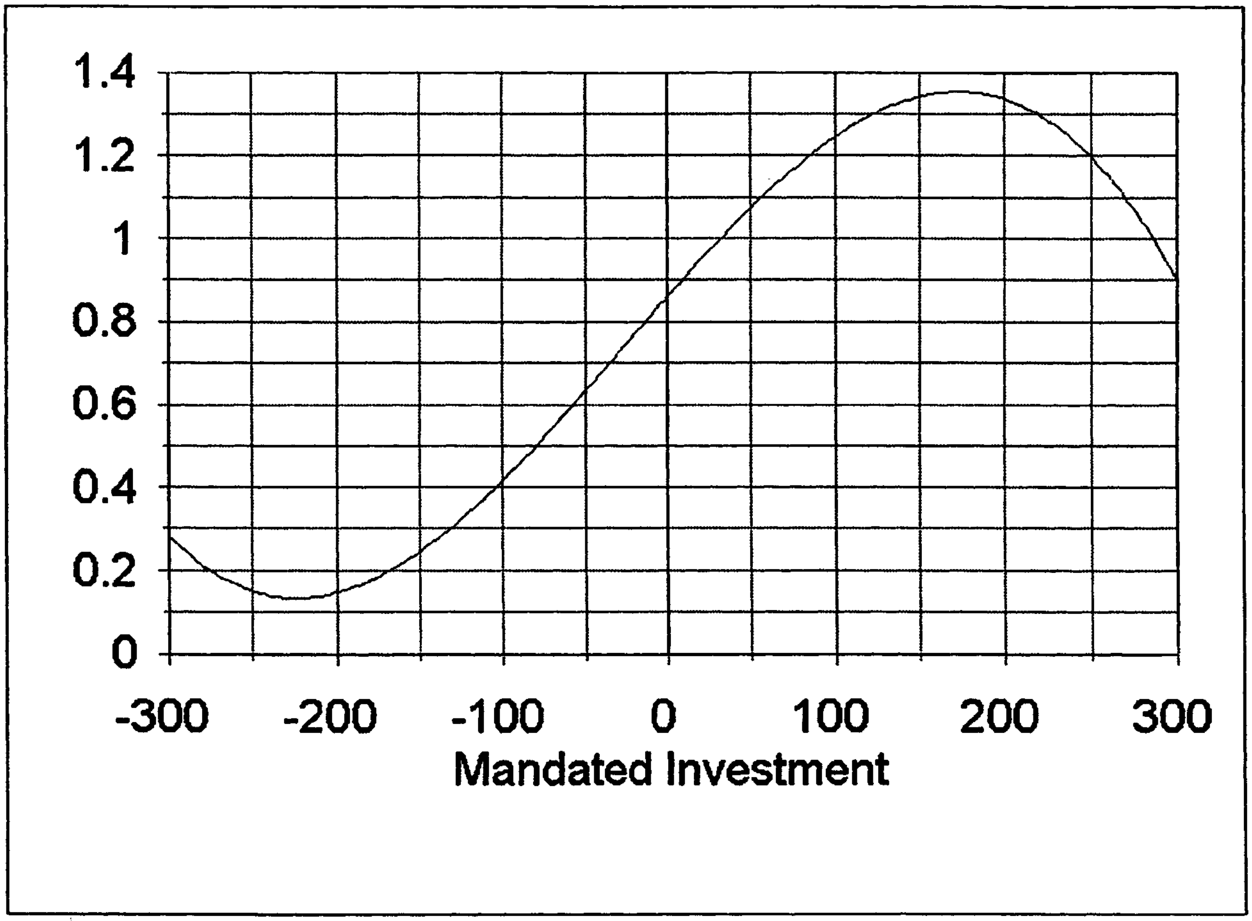


FIGURE 3
Difference Absolute Prediction Errors (Linear vs. Non-linear model)

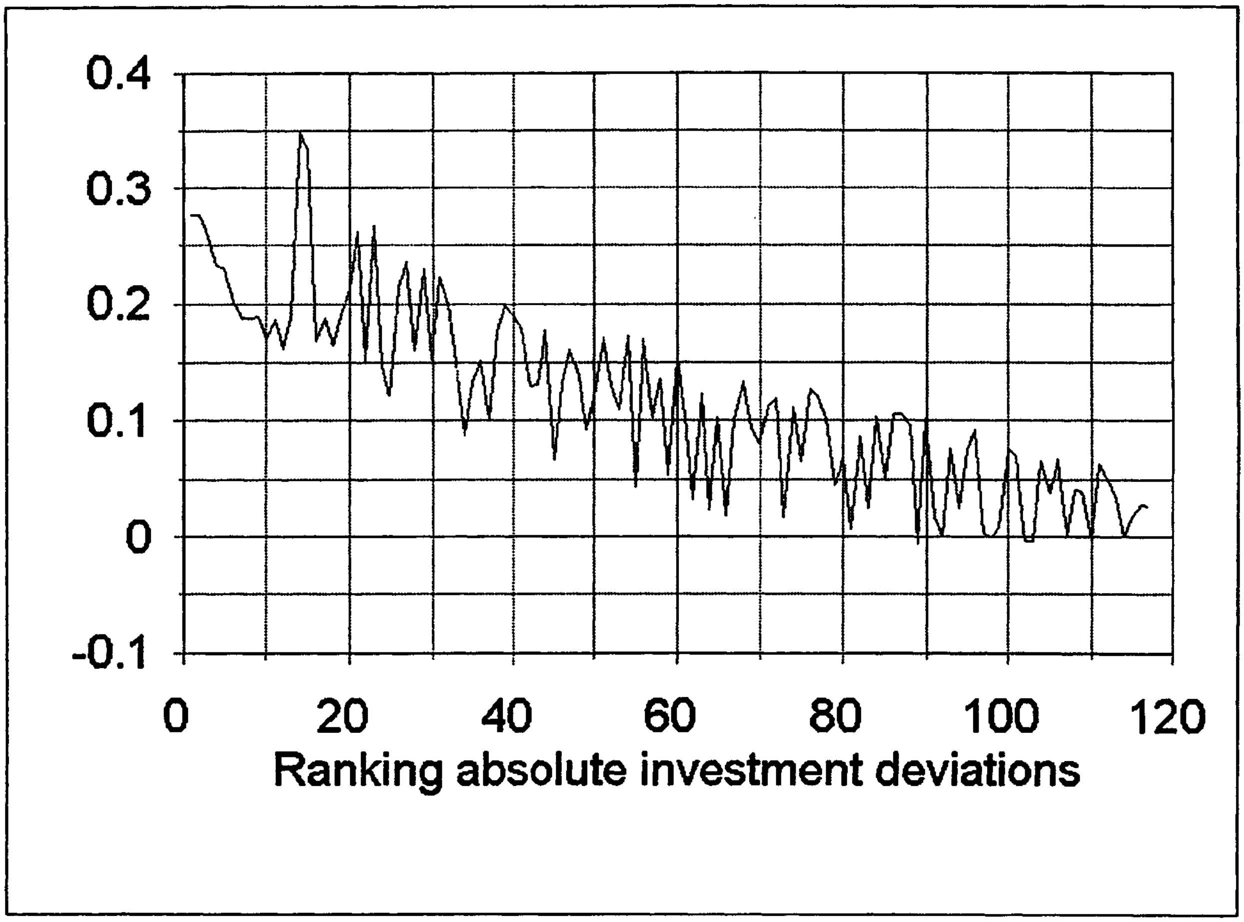
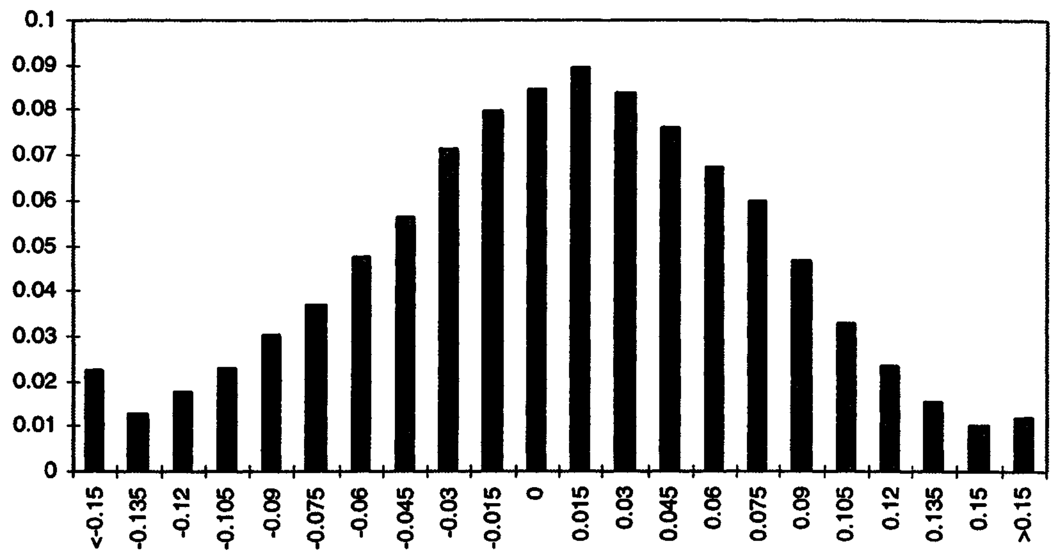


FIGURE 4
Aggregate Investment Deviations from Trend

(q=1.0)



(q=.00)

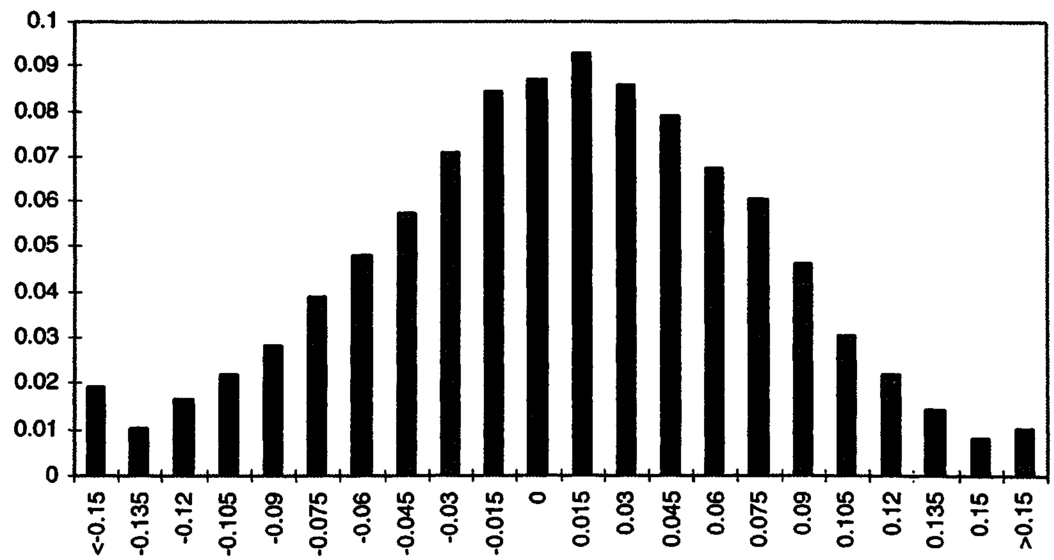


FIGURE 5
Realizations of Aggregate Investment

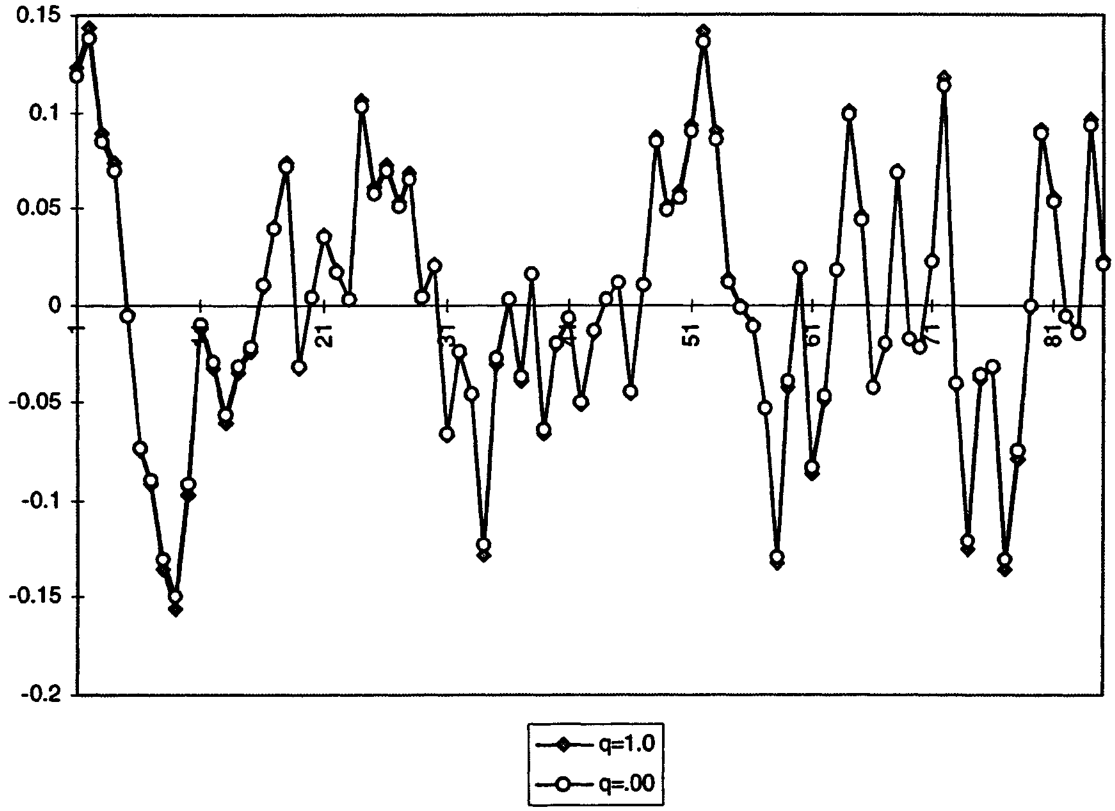
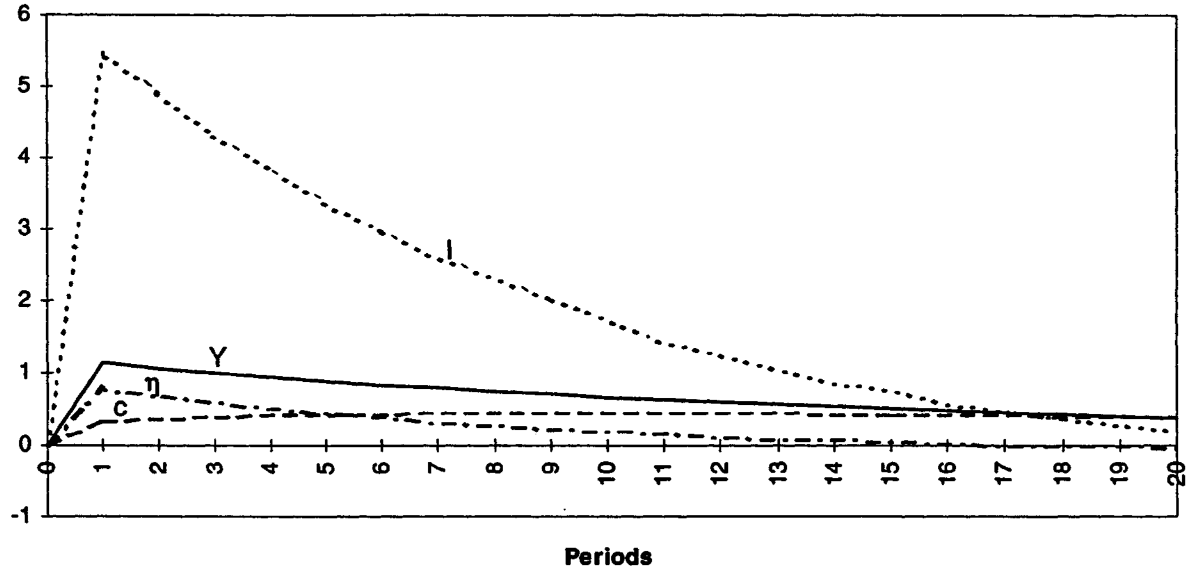


FIGURE 6
Impulse Response Functions

(q=1.0)



(q=.00)

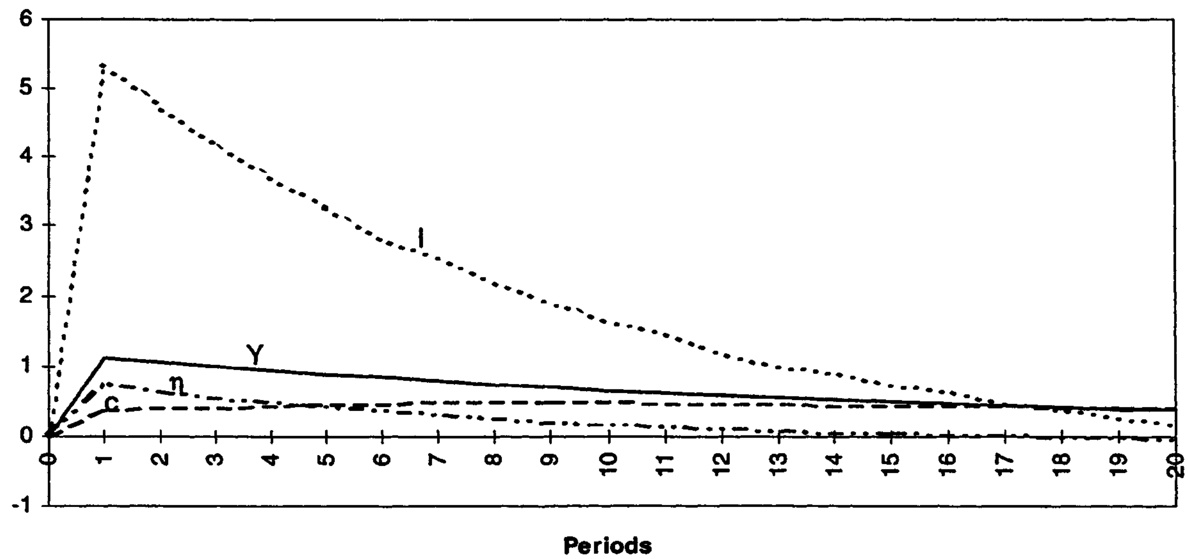


FIGURE 7
Impulse Response-Capital Support

Negative Shock: **Positive Shock:**

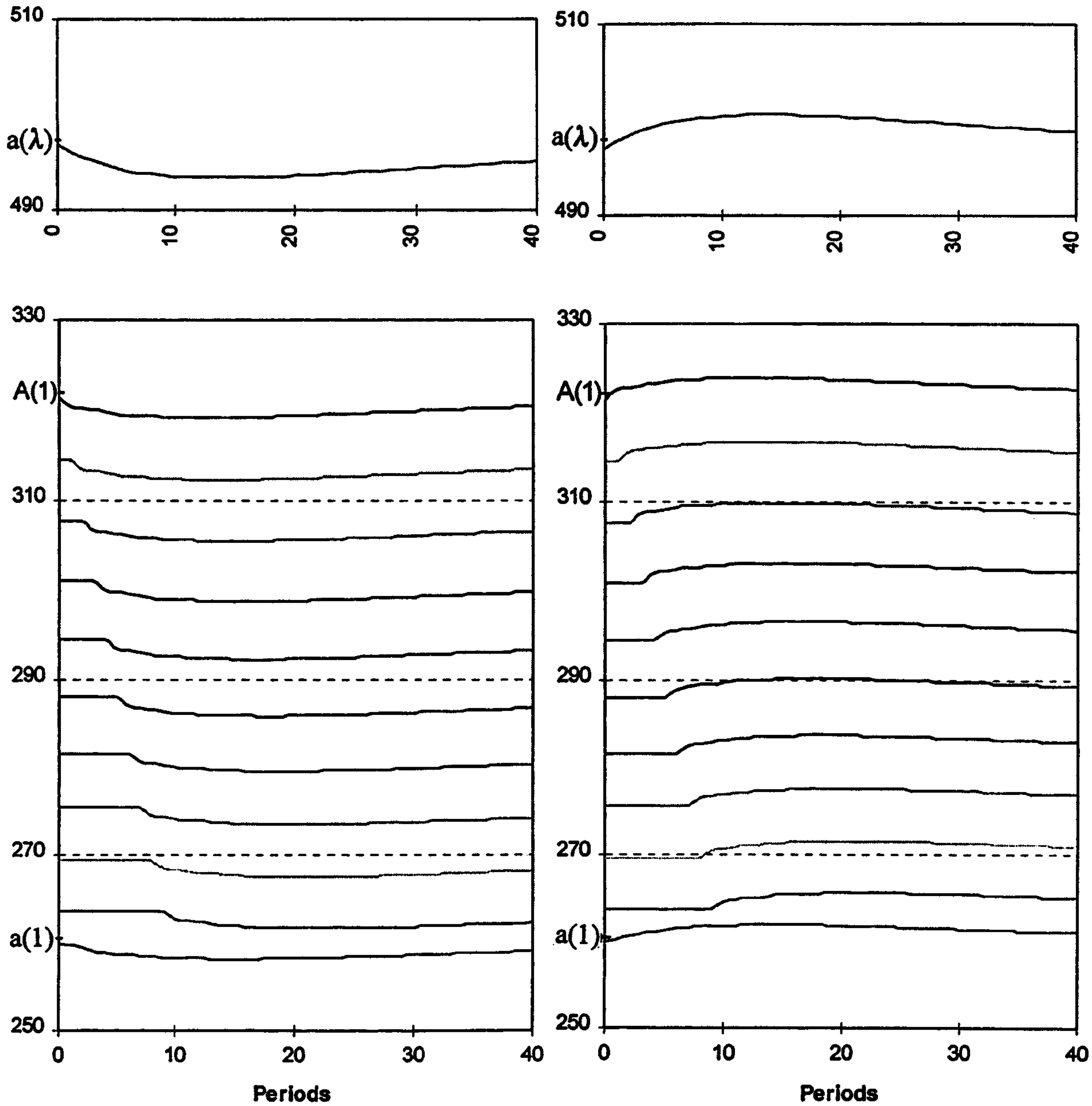


FIGURE 8
Steady State Capital Distribution

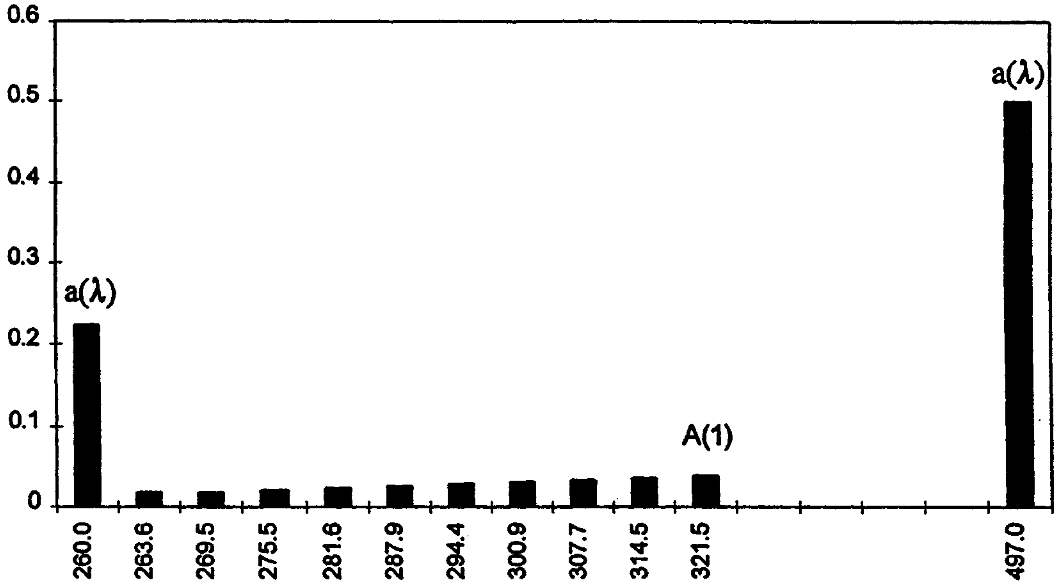
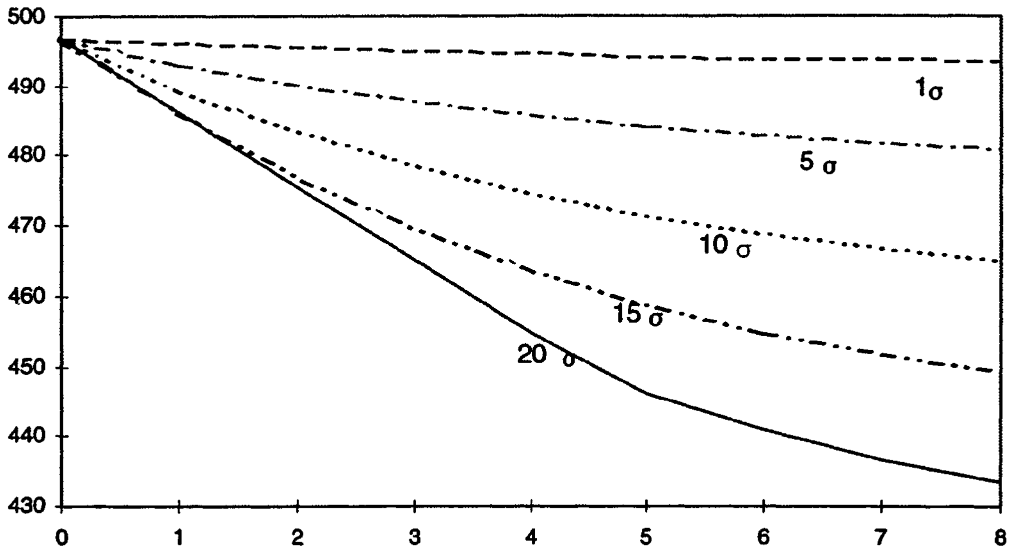


FIGURE 9
Impulse Responses - Highest Point in Support

Negative Shock



Positive Shock

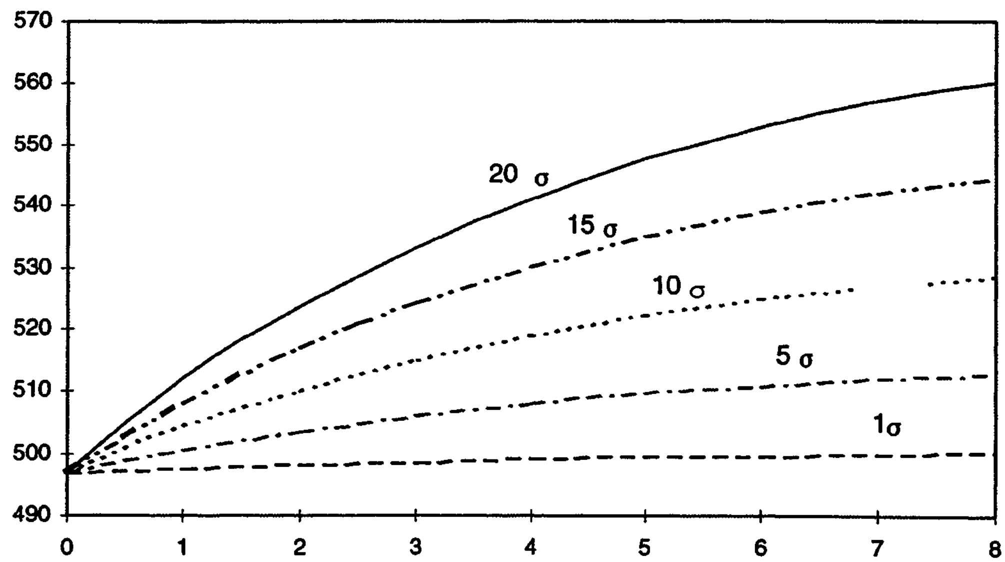
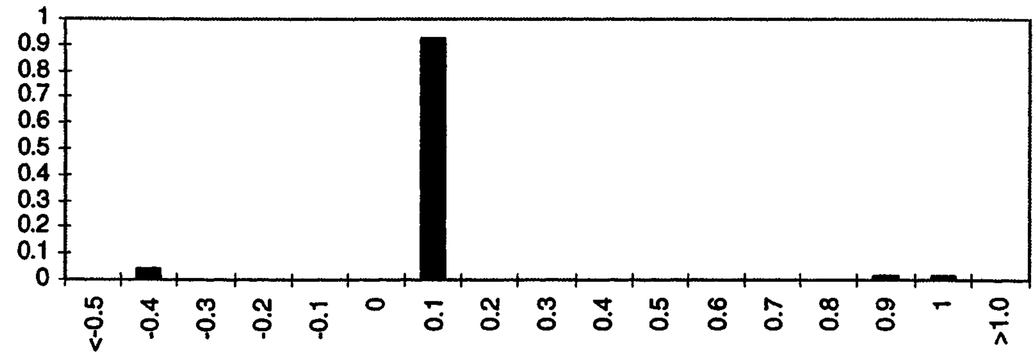
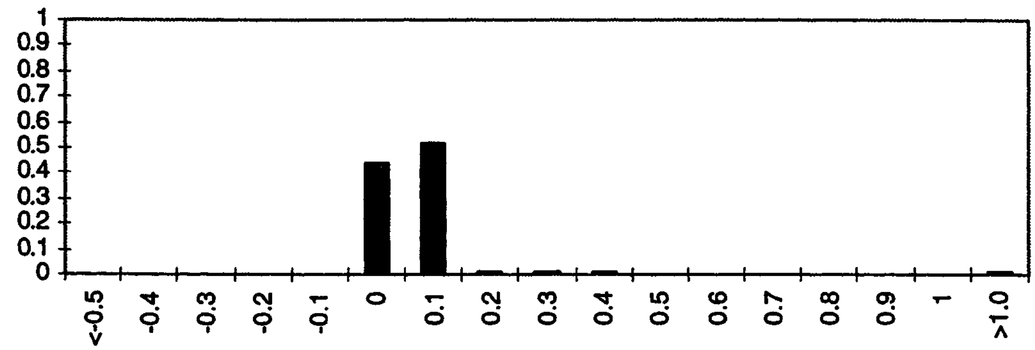


FIGURE 10
Establishment Level Investment Rates

(q=1.0)



(q=.95)



(q=.00)