Working Paper Series

Modeling Money

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Working Papers Series
Macroeconomic Issues
Research Department
Federal Reserve Bank of Chicago
December 1997 (WP-97-17)

FEDERAL RESERVE BANK
OF CHICAGO

Modeling Money*

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Abstract

We develop and implement a limited information diagnostic strategy for assessing the plausibility of monetary business cycle models. Our strategy focuses on a model's ability to reproduce empirical estimates of an actual economy's response to monetary policy shocks. A key input to this diagnostic is a univariate time series representation of the response of money to a shock in monetary policy. We find that a monetary policy shock has only a small contemporaneous effect on the monetary base and M1. Its primary effect is to signal future movements in the money supply. We implement our diagnostic strategy on a limited participation model of money which stresses the importance of credit market frictions in the monetary transmission mechanism.

*This paper was stimulated by conversations with Christopher Sims, to whom we are grateful. We are also grateful to V.V. Chari for discussions, to Chris Gust and Michelle Zaharchuk for helpful comments, and Wendy Edelberg for superlative research assistance. Christiano and Eichenbaum are grateful for the financial support of a grant from the National Science Foundation to the National Bureau of Economic Research. This paper represents the views of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of Chicago or the Federal Reserve System.

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1. Introduction

The past decade has witnessed a resurgence of interest in the monetary transmission mechanism. As a result, we are faced with competing monetary general equilibrium models, each emphasizing the importance of different frictions. One strategy for assessing the empirical plausibility of these models is to compare their predictions for unconditional moments of the data. To do this, one must take an explicit stand on the nature of all shocks to an economy as well as the way that monetary policy responds to them.[1] A second, limited information strategy is to compare the predictions of different models for how the economy responds to just one shock.

To be useful as part of the second strategy, a shock must satisfy two properties. First, different model economies must react differently to the shock. Second, we must know how the actual economy responds to the shock. Monetary policy shocks are a natural candidate for implementing this second strategy.[2] It is well known that different model economies react differently to such a shock.[3] In addition, substantial progress has been made in understanding how a monetary policy shock affects the U.S. economy. Reaching such an understanding is the key objective of the recent Vector Autoregression (VAR) based literature on monetary policy shocks.[4] That literature has not yet converged on a particular set of assumptions for identifying the effects of an exogenous shock to monetary policy. But, there is considerable agreement about the qualitative effects of a monetary policy shock in the sense that inference is robust across a large subset of the identification schemes that have been considered.[5]

The purpose of this paper is to develop a practical strategy for using this information about the effect of a monetary policy shock to evaluate alternative monetary models. Specifically, we develop a simple representation for monetary policy that can be used to assess the empirical plausibility of a model's predictions for how the economy responds to a monetary policy shock. This representation expresses the growth rate of money as a univariate exogenous stochastic process driven solely by current and past shocks to monetary policy. We derive this 'monetary policy shock rule' from the estimated response function of the growth rate of money to a monetary policy shock.

When we measure 'money' by the base or $M1$, we find that the monetary policy shock rule

[1]See for example Ireland (1997), Leeper and Sims (1994) and Kim (1997).

[2]Other recent applications of the second strategy include Gali (1997) who studies the dynamic effects of technology shocks, and Rotemberg and Woodford (1992) and Ramey Shapiro (1997) who study the dynamic effects of shocks to government purchases.

[3]For a recent discussion see Christiano, Eichenbaum and Evans (1997a).

[4]This was also the objective of the early 'unanticipated money' literature associated with Barro (1977) and Mishkin (1983), among others.

[5]See Christiano, Eichenbaum and Evans (1997b) who also dislay some identification schemes which lead to different inferences about the effect of an exogenous shock to monetary policy.

is well approximated by a second order MA process. A key property of this representation is that the contemporaneous effect of a monetary policy shock is small while the lagged effects are much larger. Therefore a shock to monetary policy has a very small impact on the current growth rate of money. Basically it signals that the growth rate of money will be high in the future. Since shocks to monetary policy produce immediate and persistent effects on aggregate economic activity, we infer that empirically plausible models of the monetary transmission mechanism will have to embody important effects of anticipated movements in money.

The previous result depends on how money is measured. When we use $M2$ as our measure of money, the monetary policy shock rule is well-approximated by a first order autoregression. Here a monetary policy shock leads to an immediate increase in the growth rate of money, with smaller increases thereafter. This representation is similar to the representation of monetary policy, often adopted in equilibrium business cycle models.[6] However the interpretation is very different.

We implement our diagnostic strategy on a general equilibrium limited participation model which stresses the persistent liquidity effects of a change in the growth rate of money. When money is measured as the base or $M1$, the model can account for the qualitative response of the interest rate, employment, output and the price level to a shock in U.S. monetary policy. In particular, after a contractionary shock to monetary policy, there is a persistent rise in the interest rate, a delayed, hump shaped contraction in employment and output and a delayed fall in the price level. The model performs less well when money is measured using $M2$.

The basic idea underlying our diagnostic strategy can be summarized as follows. Policymakers often articulate policy in terms of a rule in which the interest rate is 'set' as some systematic function of other endogenous variables like output and inflation. But the only thing the Fed actually directly controls is the money supply. To actually implement an interest rate rule, the growth rate of money has to respond to underlying economic shocks in a particular way. More generally, to implement *any* monetary policy rule, the growth rate of money must (if only implicitly) respond to current and past exogenous shocks in an appropriate way. This is true even when the systematic component of monetary policy is thought of as a relationship between endogenous variables. The literature on monetary policy shocks provides an estimate of the way the growth rate of money actually does respond to a particular shock - a monetary policy shock.

Suppose that an analyst solves a model under the assumption that monetary policy is

[6] See for example Cooley and Hansen (1989,1997), Christiano and Eichenbaum (1992, 1995), Beaudry and Devereux (1995), Farmer (1996), Chari, Kehoe and McGrattan (1996), and Cho and Cooley (1995).

given by the monetary policy shock rule. In addition, suppose that the model has been specified correctly and that it has a linear reduced form. Then, as we show, the dynamic response of the model's variables to a policy shock will be the same as the response of the analog variables in the VAR underlying the estimate of the monetary policy shock rule. This is true even if the monetary policy shock is identified in the VAR assuming a policy rule that is highly reactive to the state of the economy. So, the empirical plausibility of a model can be assessed by comparing the results of an exogenous policy shock in the model to the estimated response of the actual economy to a policy shock.

Other than its simplicity, the key advantage of this diagnostic strategy is as follows. It is often the case that a model economy will have multiple equilibria when policy is represented as a relationship between endogenous variables. Each may be supported by a different rule for the way the growth rate of money responds to fundamental economic shocks. Yet, for any given rule relating the growth rate of money to these shocks, it is often (but not always) the case that there is a unique equilibrium. Under these circumstances, the proposed diagnostic strategy is particularly useful. The monetary policy shock literature tells us which monetary policy shock rule the Fed did adopt and how the economy did respond to a policy shock. These responses can be compared to the unique prediction of the model for what happens after a shock to monetary policy.

The remainder of this paper is organized as follows. In section 2, we discuss the relationship between two representations of monetary policy. In the first, policy is represented as a relationship between endogenous variables. In the second, policy is represented as a relationship between the growth rate of money and the exogenous shocks to the economy. In section 3 we address the issue of uniqueness of equilibrium and how it relates to which representation of monetary policy is used. We do so using a general equilibrium model that is sufficiently simple to permit a closed form analysis of the global set of equilibria. Section 4 derives and discusses empirical estimates of the monetary policy shock rule. In section 5 we apply our diagnostic procedure to a particular monetary business cycle model. We show that in this model there are multiple equilibria for each element in a family of interesting policy rules that take the form of a relationship between endogenous variables. But, in an area around nonstochastic steady state, there is a unique equilibrium when policy is represented as a rule relating the growth rate of money to fundamental economic shocks. The final section presents some concluding remarks.

2. Endogenous Variable and Exogenous Shock Policy Rules

In this section we discuss the relationship between two representations of monetary policy. In the first representation, the growth rate of money responds to current and past exogenous shocks to the economy, i.e., monetary policy is represented by an *exogenous shock policy rule*. In the second representation, policy makers seek to implement a particular relationship between endogenous variables, i.e., they follow an *endogenous variable policy rule*. To support the endogenous variable policy rule as an equilibrium phenomenon, the policy maker must adopt an appropriate exogenous shock policy rule. That is, the growth rate of money must respond to current and past exogenous shocks in an appropriate way. In this section we elaborate on the relationship between these two representations of monetary policy and discuss some of the implications.

We begin with a simple static example that illustrates the relationship between the two types of policy rules. The next section provides additional examples using a general equilibrium model. Suppose there are two endogenous variables in the economy, y and v. Assume that these are related in equilibrium to the monetary policy action, a, and a private economy shock, u, by the relationships:

$$y = \alpha a + \beta u, \;\; v = a + u, \;\; \alpha, \beta \neq 1.$$

In addition, suppose the monetary authority seeks to implement the following endogenous variable policy rule

$$y = \psi v + \varepsilon_{M^{\bullet}},$$

where $\varepsilon_{M^{\bullet}}$ is the monetary policy shock which is assumed to be orthogonal to u. Simple substitution establishes that the exogenous shock policy rule which implements the above relationship is:

$$a = \left(\frac{\psi - \beta}{\alpha - \psi}\right) u + \left(\frac{1}{\alpha - \psi}\right) \varepsilon_{M^{\bullet}}.$$

The monetary policy shock rule which relates the policy action to current and past monetary policy shocks is given by $a = (\alpha - \psi)^{-1} \varepsilon_{M^{\bullet}}$.

A special feature of this example is that there is a one-to-one relationship between the two representations of policy. In general this will not be true. For example, if $v = a^2 + u$, then there may be two exogenous shock policy rules that implement the endogenous variable policy rule and two corresponding equilibria.

To see more generally the relationship between the two representations of policy let $s_t \in S$ denote the time t realization of the exogenous uncertainty in an economy. In general s_t could include the time t realization of shocks to production technologies, private agents' preferences, sunspots, shocks to governments purchases or monetary policy, etc. Let $s^t = (s_0, s_1, ..., s_t)$ denote the history of shocks up to and including time t.

Suppose that the monetary authority sets the time t growth rate of money, X_t, according to the sequence of rules,

$$X_t = X_t(s^t). \tag{2.1}$$

We refer to $X_t(s^t)$ as an exogenous shock policy rule because it relates the time t growth rate of money to current and past realizations of the exogenous shocks to the economy. Let $p_t(s^t)$ and $q_t(s^t)$ denote the state-contingent prices and quantities in this economy. An *exogenous policy equilibrium* is a collection, $p_t(s^t)$, $q_t(s^t)$, $X_t(s^t)$ that satisfies the relevant equilibrium conditions.

Let a bar over a vector denote its history for a given s^t. Thus,

$$\begin{aligned} \bar{p}_t(s^t) &= \left[p_0(s_0), p_1(s^1), p_2(s^2), ..., p_t(s^t)\right] \\ \bar{q}_t(s^t) &= \left[q_0(s_0), q_1(s^1), q_2(s^2), ..., q_t(s^t)\right] \\ \bar{X}_t(s^t) &= \left[X_0(s_0), X_1(s^1), X_2(s^2), ..., X_t(s^t)\right] \end{aligned} \tag{2.2}$$

denote the history of prices, quantities and the growth rate of money in an economy when the history of the exogenous shocks is s^t.

Suppose the monetary authority seeks to enforce a particular relationship between endogenous variables. We represent this relationship by the endogenous variable policy rule:

$$f_t\left(\bar{p}_t(s^t), \bar{q}_t(s^t), \bar{X}_t(s^t), s^t\right) = 0 \text{ for all } t, \text{ all } s^t. \tag{2.3}$$

An example of (2.3) that has received considerable attention in the literature is the Taylor rule. According this rule, the monetary authority makes the interest rate a particular time invariant function of inflation and output. Interestingly, with this specification, money does not appear in the endogenous variable policy rule at all. We consider further examples below.

We define an *endogenous policy equilibrium* as an exogenous policy equilibrium with the additional restriction that (2.3) is satisfied. Since an endogenous policy equilibrium is by construction also an exogenous policy equilibrium, the following two results follow trivially:

RESULT 1: The prices and quantities in an endogenous policy equilibrium are the prices and quantities in an exogenous policy equilibrium.

RESULT 2: The dynamic response functions of prices and quantities to a shock in an endogenous policy equilibrium are the dynamic response functions in the corresponding exogenous policy equilibrium.

The previous results say nothing about uniqueness of equilibrium. For example, corresponding to a given endogenous variable policy rule there may be many equilibria, each of which is supported by a different exogenous shock policy rule. More generally, the equilibrium associated with either a given endogenous variable policy rule or a given exogenous shock policy rule may not be unique. Below we return to this issue and discuss how uniqueness considerations impact on which parametrization of policy is more useful when assessing the empirical plausibility of a model.

3. Multiplicity and the Representation of Monetary Policy

In this section we accomplish three tasks. First, we illustrate the relationship between endogenous and exogenous policy equilibria using a simple general equilibrium model and a particular endogenous variable policy rule. Second, we discuss the possibility of multiple equilibria under the two types of policy rules. By considering alternative parameter values, we construct examples which illustrate various possibilities. In one example, there are multiple endogenous policy equilibria, yet exogenous policy equilibria are always unique. As emphasized in the introduction, this is a situation in which our proposed diagnostic procedure is particularly useful. However, this situation does not arise for all parameter values of our example economy. To illustrate this point, we present an example in which there are multiple exogenous policy equilibria but a unique endogenous policy equilibrium.

3.1. A Simple Monetary Model

Our example economy is populated by three types of agents: a monetary authority, and a representative household and firm. The firm and household interact in a sequence of labor, goods and loan markets as perfect competitors, taking all prices as given. There is no capital, and the monetary authority's only action is to inject or withdraw cash from the economy. Money is introduced by way of a cash in advance constraint on the firm.

At date 0, the representative household's preferences are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right),\ 0<\beta<1,$$

where

$$U(C,N)=\frac{C^{1-\gamma}}{1-\gamma}-\frac{\psi_0}{1+\psi}N^{1+\psi},\ \gamma \geq 0,\ \psi_0, \psi>0. \tag{3.1}$$

Here, C_t and N_t denote time t consumption and hours worked, respectively. In addition, E_0 denotes the time 0 conditional expectation operator. When $\gamma=1$, (3.1) is logarithmic in consumption. The household faces the following cash constraint on its consumption purchases:

$$P_tC_t \leq W_tN_t + Q_t. \tag{3.2}$$

Here, P_t, W_t and Q_t denote the price level, the nominal wage rate, and the quantity of dollars set aside for consumption purchases at time t. The law of motion for the household's money balances is given by:

$$M_{t+1} = [W_tN_t + Q_t - P_tC_t] + R_t[M_t + X_t - Q_t] + D_t, \tag{3.3}$$

for all $t \geq 0$. In (3.3), M_{t+1}, D_t and X_t denote the beginning of period $t+1$ stock of money, time t profits from firms and lump sum transfers from the monetary authority, respectively. With this notation, $M_t + X_t - Q_t$ denotes time t loans to firms, which earn gross interest, R_t. According to (3.3), interest earnings are received at the beginning of time $t+1$. In each period t, the household first sees all variables dated t and earlier. Then, it selects values for C_t, Q_t, N_t and contingency plans for future values of these variables to maximize utility subject to (3.2), (3.3) and $0 \leq Q_t \leq M_t + X_t$, $C_t, 0 \leq N_t \leq N$.[7] The household's intra- and inter-temporal Euler equations are, respectively,

$$-\frac{U_{N,t}}{U_{C,t}} = \frac{W_t}{P_t}, \ \frac{U_{C,t}}{P_t} - R_t\beta E_t\frac{U_{C,t+1}}{P_{t+1}} = 0, \tag{3.4}$$

where U_x denotes the partial derivative of U with respect to x.

Firms operate a simple linear production technology: $C_t = N_t$. At the beginning of the period, they borrow funds, B_t, from the household. They do this because they must finance their wage bill, W_tN_t, before they receive the receipts from production, so that $B_t \geq W_tN_t$. The firm's profits during the period corresponds to its net cash inflow:

$$P_tN_t + B_t - (W_tN_t + R_tB_t). \tag{3.5}$$

The firm chooses $N_t \geq 0$ and $B_t \geq 0$ to maximize this expression, subject to $B_t \geq W_tN_t$.

We are interested in studying two types of equilibria for this economy. The first type is an exogenous policy equilibrium which is a sequence of quantities, prices, interest rates and money growth rates that are consistent with (i) a particular exogenous shock policy rule, (ii) sequential optimality by the household and firm, and (iii) clearing in loan, goods and labor markets. The second type is an endogenous policy equilibrium. This is an exogenous policy equilibrium where a particular endogenous variable policy rule holds.

[7]Note that for the household's problem at time t to be well posed, it must know both the current value of X_t and the future values of X_t contingent upon the future realizations of the shocks to the economy. That is, it must know the exogenous shock policy rule. This illustrates why we had to include the state contingent rule for X in our definition of an endogenous policy equilibrium in the previous section.

3.2. Exogenous Policy Equilibria

To study the equilibria of the simple model economy, it is useful to derive a number of relationships that must hold in any equilibrium. Throughout, we confine ourselves to interior equilibria, i.e., equilibria in which (i) the firm's and household's cash constraints are always binding, and (ii) the other non-negativity constraints never bind. We use the relationships derived in this subsection to state a characterization result for exogenous policy equilibria. We also exploit these relationships in the next subsection.

Optimization by the firm and clearing in the loan and labor markets imply:[8]

$$R_t \geq 1 \text{ and } \frac{W_t}{P_t} R_t = 1. \tag{3.6}$$

The household's cash constraint and the money market clearing condition imply

$$P_t C_t = M_t + X_t = M_{t+1}. \tag{3.7}$$

Combining this with goods market clearing, $C_t = N_t$, as well as (3.4) and (3.6), we obtain:

$$E_t \left\{ \frac{N_{t+1}^{1-\gamma}}{N_t^{1+\psi}} \frac{1+x}{1+x_{t+1}} \right\} = 1, \tag{3.8}$$

where

$$1 + x_t \equiv \frac{M_t + X_t}{M_t}.$$

In (3.8) we have used the normalization, $\psi_0 = \beta / (1+x)$, where x denotes the money growth rate in a nonstochastic steady state. This restriction on ψ_0 implies that employment is unity in a nonstochastic steady state.[9]

Substituting the household's intra-temporal Euler equation into (3.6) we obtain that in any interior equilibrium, R_t must satisfy:

$$R_t = \frac{1+x}{\beta} N_t^{-(\psi+\gamma)}. \tag{3.9}$$

Finally, it must be the case that:[10]

$$M_{t+1} = (1+x_t) M_t, \; P_t = \frac{M_{t+1}}{N_t}, \; W_t = \frac{P_t \beta}{1+x} N_t^{(\psi+\gamma)}, \; Q_t = W_t N_t (R_t - 1). \tag{3.10}$$

[8]To see this, suppose first that $R_t < 1$. Then, by setting N_t to any finite number and $B_t = \infty$, the firm would make infinite profits. But, there is at most a finite supply of funds in this economy, so this is not consistent with loan market clearing. A similar argument establishes that labor market clearing requires $P_t = R_t W_t$.

[9]To see this, evaluate the household's intertemporal euler equation in an interior non-stochastic steady state to obtain $R = (1+x)/\beta$. The result follows by combining the intratemporal Euler equation in (3.4) with (3.6) and solving for nonstochastic steady state N.

[10]The last three expressions make use of the condition $N_t = C_t$. The expression for P_t uses (3.7) and the fact that $M_{t+1} = M_t + X_t$ in equilibrium. The expression for W_t uses the (3.6) and (3.7). Finally, the expression for Q_t is obtained by evaluating (3.2) at equality and imposing (3.6).

It is easily verified that the previous results imply the following characterization result for exogenous policy equilibria. Suppose we have a stochastic process for X_t and a strictly positive, bounded stochastic process, $\{N_t\}$, which satisfies (3.8) and generates an R_t process via (3.9) which satisfies $R_t \geq 1$. Then the given N_t, X_t and R_t stochastic processes, together with the implied prices and quantities defined in (3.10), constitute an exogenous policy equilibrium.

3.3. Endogenous Policy Equilibria

In this subsection we provide a characterization result for endogenous policy equilibria in our example economy. We do this for a particular endogenous variable policy rule in which the growth rate of money is countercyclical:

$$1 + x_t = (1 + x)N_t^{-\nu} \exp(\varepsilon_{M^s,t}), \ \nu > 0. \tag{3.11}$$

Here, N_t refers to aggregate employment and $\varepsilon_{M^s,t}$ is the monetary policy shock, which is independently distributed over time, with mean zero. We assume $-\infty < \tilde{\varepsilon} \leq \varepsilon_{M^s,t} \leq \bar{\varepsilon} < \infty$.

Substituting out for $1+x_{t+1}$ in (3.8) using the endogenous variable policy rule, and taking into account the restriction on ψ_0 we obtain:

$$E_t \left\{ \frac{N_{t+1}^{1-\gamma+\nu}}{N_t^{1+\psi}} \exp(-\varepsilon_{M^s,t+1}) \right\} = 1. \tag{3.12}$$

This can be equivalently represented as

$$\frac{N_{t+1}^{1-\gamma+\nu}}{N_t^{1+\psi}} \exp(-\varepsilon_{M^s,t+1}) = \omega_{t+1}, \tag{3.13}$$

where ω_{t+1} is any stochastic process belonging to the following set:

$$\Omega = \{\{\omega_t\} : \omega_{t+1} \geq 0, \ E_t\omega_{t+1} = 1\}. \tag{3.14}$$

The sequences, $\{N_t; t \geq 0\}$, that satisfy (3.13) have the following representation:

$$N_t = N_0^{\left(\frac{1+\psi}{1-\gamma+\nu}\right)^t} \prod_{i=0}^{t-1} \left\{ [\exp(\varepsilon_{M^s,t-i})\, \omega_{t-i}]^{\frac{1}{1-\gamma+\nu}} \right\}^{\left(\frac{1+\psi}{1-\gamma+\nu}\right)^i} \tag{3.15}$$

Associated with the levels of employment that are consistent with (3.15), there is an exogenous representation of monetary policy, obtained by combining (3.13) with (3.11):

$$\begin{aligned} &\log(1 + x_t) = (1 - \varphi)\log(1 + x) + \varphi \log(1 + x_{t-1}) \\ &+ \tfrac{1-\gamma}{1-\gamma+\nu}\varepsilon_{M^s,t} - \tfrac{\nu}{1-\gamma+\nu}\log(\omega_t) - \varphi\varepsilon_{M^s,t-1}, \ t > 0 \end{aligned} \tag{3.16}$$

$$\log(1 + x_0) = \log(1 + x) - \nu \log(N_0) + \varepsilon_{M^s,0}, \ t = 0, \tag{3.17}$$

where

$$\varphi = \frac{1+\psi}{1-\gamma+\nu}.$$

We can now state a characterization result for endogenous policy equilibria:

If:

(i) a stochastic process, $\{N_t\}$, satisfies (3.15) for some $\{\omega_t\} \in \Omega$ and for some N_0;

(ii) $R_t \geq 1$ in (3.9) for all possible realizations of $\{\varepsilon_{M^s,t}\}$ and of $\{\omega_t\}$;

(iii) N_t is positive and bounded above for all possible realizations of $\{\varepsilon_{M^s,t}\}$ and $\{\omega_t\}$.

Then, the following objects: the given N_t and R_t processes, the prices and quantities defined in (3.10) and the exogenous shock policy rule, (3.16)-(3.17), constitute an endogenous policy equilibrium.

Proof It is easily verified that the objects identified in the proposition satisfy the endogenous variable policy rule, (3.11). In addition, they also satisfy the characterization result for exogenous policy equilibria. Taken together, these two conditions guarantee the result sought. For details, see the Appendix.

Note that the previous proposition points to a large class of equilibria. It includes 'fundamental equilibria', in which ω_t is an exact function of $\varepsilon_{M^s,t}$ and 'sunspot equilibria' in which ω_t is a function of other random variables as well.

Proposition 1 makes it possible for us to be precise about the observational equivalence result discussed in the introduction. An endogenous policy equilibrium corresponds to a particular exogenous policy equilibrium, one in which the policy rule is given by (3.16) and (3.17). If we observe a data record corresponding to a particular endogenous policy equilibrium, there is no meaningful way to distinguish between the propositions that the policy authority's rule is (3.11) or (3.16)-(3.17).

It is worth emphasizing that this observational equivalence result pertains to a feature of a particular endogenous policy equilibrium. A separate, related issue concerns the uniqueness of equilibrium. As the proposition suggests, and the examples in the next subsection prove, there may be many endogenous policy equilibria corresponding to a particular endogenous variable policy rule. The exogenous shock policy rule corresponding to each such equilibrium may itself map into a set of exogenous policy equilibria.

3.4. Illustrating the Relationship Between the Equilibria

In this subsection we illustrate, via a series of examples, the relationship between the two types of equilibria for the model economy. Our first example illustrates how an exogenous shock policy rule supports an endogenous policy equilibrium. The remaining examples illustrates the various possibilities with respect to multiplicity. It is possible for there to be many endogenous policy equilibria while exogenous policy equilibria are unique. But the opposite can also happen.

Example 2 identifies a configuration of parameter values for which there are many endogenous policy equilibria. The exogenous shock policy rule associated with each equilibrium is identified. Example 3 identifies a subset of the parameter values identified in Example 2 with the following two properties: (i) there are many endogenous policy equilibria and (ii) the exogenous shock policy rule associated with each of these equilibria maps into a unique exogenous policy equilibrium. As we have emphasized, this is the situation in which the diagnostic strategy studied in this paper is particularly useful. Example 4 shows that the opposite situation is also possible. Specifically, we display an endogenous variable policy rule for which: (i) there is a unique endogenous policy equilibrium and (ii) the associated exogenous shock policy rule maps into many exogenous policy equilibria.

Example 1 Consider the following candidate fundamental equilibrium:

$$\omega_t = \frac{\exp(-\varepsilon_{M^s,t})}{A},\ N_0 = A^{\frac{1}{\gamma+\psi-\nu}},\ N_t = N_0,\ t > 0,$$

where $A \equiv E\exp(-\varepsilon_{M^s,t})$, so that $\{\omega_t\} \in \Omega$ in (3.14). Here E denotes the unconditional expectations operator. According to (3.9), this constant sequence of N_t's is associated with an interest rate strictly greater than one if

$$A < \left(\frac{1+x}{\beta}\right)^{\frac{\gamma+\psi-\nu}{\psi+\gamma}}.$$

The conditions of the characterization result are trivially satisfied, and so we have an endogenous policy equilibrium. The exogenous shock policy rule in this equilibrium is (3.11) with N_t replaced by the constant, $A^{\frac{1}{\gamma+\psi-\nu}}$:

$$1 + x_t = (1+x)A^{\frac{-\nu}{\gamma+\psi-\nu}}\exp\left(\varepsilon_{M^s,t}\right).$$

The set of exogenous policy equilibria associated with this exogenous shock policy rule contains at least the equilibrium in which $N_t = A^{\frac{1}{\gamma+\psi-\nu}}$ for all t.

If

$$\left|\frac{1+\psi}{1-\gamma+\nu}\right| < 1, \tag{3.18}$$

then there are many endogenous policy equilibria. To see this, consider the following example:

Example 2 Consider those $\{\omega_t\} \in \Omega$ which satisfy $0 < \tilde{\omega} \leq \omega_t \leq \bar{\omega} < \infty$. Let $\tilde{N}$ and $\bar{N}$ be the smaller and larger, respectively, of the following two numbers:

$$[\exp(\tilde{\varepsilon})\tilde{\omega}]^{\frac{1}{\nu-(\gamma+\psi)}}, [\exp(\bar{\varepsilon})\bar{\omega}]^{\frac{1}{\nu-(\gamma+\psi)}}.$$

Then, it is easily verified that, given (3.18), $[\tilde{N}\ ,\bar{N}]$ form an ergodic set for N_t. That is, solutions to (3.15) satisfy:

$$\text{if } 0 < \tilde{N} \leq N_0 \leq \bar{N} < \infty, \text{ then } \tilde{N} \leq N_t \leq \bar{N}.$$

Recall from (3.9) that equilibrium R_t is monotonically decreasing in N_t. Therefore, if the bounds on ω_t are consistent with $(1+x)\bar{N}^{-(\psi+\gamma)}/\beta \geq 1$, then the stochastic process associated with any $N_0 \in [\tilde{N}, \bar{N}]$ is necessarily associated with a positive interest rate. The rest of the conditions of the characterization result are satisfied, and so we have an endogenous policy equilibrium.

To verify that the set of equilibria mentioned here is non-empty, consider the case $\gamma = \psi = 1$, $x = .04$, $\beta = 1.03^{-.25}$, $\nu = 3$, $\tilde{\varepsilon} = -0.02$, $\bar{\varepsilon} = 0.02$, and $\omega_t = 1 + 0.01\varepsilon_{M^*,t}$. Then $\tilde{N} = 0.98$ and $\bar{N} = 1.02$. Also, the gross rate of interest associated with these two levels of employment is 1.0909 and 1.0062, respectively. Since the rate of interest is monotonic in N, we conclude that we have an equilibrium for each choice of $N_0 \in [\tilde{N}, \bar{N}]$.

Note from the presence of N_0 and $\{\omega_t\}$ in (3.16) and (3.17) that for each endogenous policy equilibrium, there corresponds a different exogenous shock policy rule. According to the observational equivalence proposition, each exogenous shock policy rule (where N_0 is treated as a fixed parameter by the monetary authority, and $\{\omega_t\}$ is treated like a sequence of shocks analogous to $\{\varepsilon_{M^*,t}\}$) has associated with it at least the equilibrium that was used to construct it in the first place.

Before proceeding we provide some intuition for the multiplicity of endogenous policy equilibria in the previous example. We do this by discussing the indeterminacy of the deterministic steady state equilibrium of the economy when (3.18) holds. Consider the deterministic steady state equilibrium in which employment is constant. When $0 < (1+\psi)/(1-\gamma+\nu) < 1$, then there is another deterministic equilibrium in which employment is low in the current period and gradually returns to the steady state from below. To see why this is also an equilibrium, note that the anticipation that next period's employment is low creates the expectation that next period's money growth will be high. That in turn, for anticipated inflation effect reasons, raises the current period rate of interest. By increasing the tax on firms who must finance the wage bill, the higher current rate of interest produces lower employment in the current period. The lower employment anticipated for the next period is justified by lower anticipated employment in the subsequent period, which raises next period's rate of interest, and so on. For the reasons outlined in Woodford (1986), the fact that there exist deterministic equilibria of this type guarantees that there also exist sunspot equilibria.

The following example draws attention to the possibility that an arbitrary exogenous shock policy rule has a unique exogenous policy equilibrium, even though the countercyclical monetary policy rule produces multiple equilibria.

Example 3 Consider the case $\gamma = 1$. It is easily verified that if the authorities implement an arbitrary exogenous shock policy rule, then the unique solution to (3.8) is:

$$N_t = \left\{ E_t \left[\frac{1+x}{1+x_{t+1}} \right] \right\}^{\frac{1}{1+\psi}}. \tag{3.19}$$

If the exogenous shock policy rule is given by (3.16)-(3.17) (with N_0 treated as a parameter), then the unique equilibrium employment process implied by (3.19) is the one associated with the endogenous variable policy rule, (3.11), and the equilibrium selected by N_0 and $\{\omega_t\}$.

As the next example shows, the results of Example 3 are not general. In particular, Example 4 illustrates a situation in which an exogenous shock policy rule is associated with multiple equilibria but an endogenous variable policy rule is associated with a unique equilibrium.

Example 4 Consider the exogenous shock policy rule given by (3.11) with $\nu = 0$.[11] As before, (3.18) is the key condition guaranteeing the existence of multiple equilibria. The equation is satisfied, for example, if $\psi = 1$, $\gamma = 4$. From (3.9), it is clear that an interest rate policy produces a unique equilibrium. For a given rate of interest, R_t, there is only one level of employment consistent with equilibrium.

4. The U.S. Monetary Policy Shock Rule

In the introduction we referred to the portion of the exogenous shock policy rule relating the growth rate of money to current and past monetary policy shocks as the 'monetary

[11]This is basically an example discussed in Farmer (1993) and analyzed further in Mathene (1996).

policy shock rule'. In this section we estimate parsimonious time series representations of the monetary policy shock rule using postwar U.S. data. To do this, we must first identify the policy shocks themselves. We do so using the methods developed in Christiano, Eichenbaum and Evans (1996) (CEE) who assume an endogenous variable policy rule that takes the form of an interest rate targeting rule.[12] Second, we discuss our proposed diagnostic procedure. Third, we discuss our method for parsimoniously parameterizing the estimated response of money growth to an exogenous monetary policy shock. Fourth, we present the result of implementing this method.

4.1. Estimates of the Monetary Policy Shock Rule

As in CEE, we assume that the monetary authority implements an endogenous variable policy rule of the form:

$$R_t = f(\Omega_t) + \varepsilon_{M^s,t}. \tag{4.1}$$

Here R_t is the federal funds rate, f is a linear function, Ω_t summarizes the information set available to the monetary authority when it sets its target for R_t, and $\varepsilon_{M^s,t}$ is a serially uncorrelated shock that is orthogonal to the elements of Ω_t. We refer to this orthogonality condition as the 'recursiveness assumption'. As emphasized above, we think of (4.1) as a relationship that the monetary authority sustains in equilibrium by appropriately manipulating the money supply. To do so, it must make the money supply respond in the right way to current and past shocks, including $\varepsilon_{M^s,t}$. This response pattern is what we refer to as the exogenous shock policy rule.

CEE assume that the information set, Ω_t, includes current and past observations on commodity prices, the aggregate price level and aggregate output. In addition, Ω_t includes lagged values of the federal funds rate, total reserves, nonborrowed reserves and money. We repeat the analysis for three measures of money: $M1$, $M2$ and the monetary base, MB. Apart from commodity prices, the rate of interest, and money, all variables are measured in log-levels. Commodity prices are measured in rates of change, the rate of interest is in nominal levels, and money is measured in log-difference form.

[12]In an earlier version of this paper we showed that the results of our analysis are robust to working with the identifying assumptions in Sims and Zha (1995).

The recursiveness assumption on $\varepsilon_{M^s,t}$ implies that the monetary policy shock can be estimated by the fitted residuals from an ordinary least squares regression of R_t on the elements of Ω_t. The response of any variable, say y, to the monetary policy shock, can then be estimated by the coefficients in an ordinary least squares regression with y as the dependent variable and various lags of the monetary policy shock as the right-hand variables. This is asymptotically equivalent to a procedure based on vector autoregressions described below.

Consider the following representation for the vector of variables, Z_t:

$$B_0 Z_t = \kappa_Z + B(L) Z_{t-1} + \varepsilon_t, \tag{4.2}$$

where L denotes the lag operator, and $B(L) = B_1 + B_2 L + ... + B_q L^{q-1}$. Multiplying both sides of (4.2) by B_0^{-1} we obtain the VAR for Z_t. For convenience, we assume that system (4.2) has been ordered so that the endogenous variable policy rule, (4.1), is the last equation. In addition, we assume that the seven variables in Z_t have been ordered as follows:

$$Z_t = \begin{pmatrix} \bar{Z}_t \\ \Delta \log(M_t) \end{pmatrix}, \tag{4.3}$$

where $\Delta \log(M_t) \equiv \log(M_{t+1}/M_t)$, and M_t denotes the beginning-of-period stock of money. The vector, $\bar{Z}_t$ denotes the other variables in the CEE system.

CEE's point estimates imply

$$\det\left(I - B_0^{-1} B(z) z\right) = 0 \;\Rightarrow |z| > 1 \tag{4.4}$$

where $|\cdot|$ denotes the absolute value operator. The moving average representation corresponding to (4.2) is given by:

$$Z_t = \pi_0 + \pi(L)\varepsilon_t, \;\; \pi(L) = \sum_{i=1}^{\infty} \pi_i L^{i-1}. \tag{4.5}$$

Condition (4.4) guarantees that $\{\pi_i\}$ forms a square summable sequence. We write the last row of (4.5) as follows:

$$\Delta \log(M_t) = \pi_0^2 + \pi^{21}(L)\varepsilon_t^1 + \pi^{22}(L)\varepsilon_{M^s,t}, \tag{4.6}$$

where π_0, $\pi(L)$,and ε_t are partitioned conformably with the definition of Z_t:

$$\pi(L) = \begin{bmatrix} \pi^{11}(L) & \pi^{12}(L) \\ \pi^{21}(L) & \pi^{22}(L) \end{bmatrix}, \ \pi_0 = \begin{bmatrix} \pi_0^1 \\ \pi_0^2 \end{bmatrix}, \ \varepsilon_t = \begin{pmatrix} \varepsilon_t^1 \\ \varepsilon_{M^*,t} \end{pmatrix}.$$

Relation (4.6) is the exogenous shock policy rule of the monetary authority. The expressions, $\pi^{11}(L)$ and $\pi^{21}(L)$ characterize the dynamic response of $\bar{Z}_t$ and $\Delta\log(M_t)$ to the nonmonetary policy shocks, ε_t^1. The expressions, $\pi^{12}(L)$ and $\pi^{22}(L)$ characterize the dynamic response of $\bar{Z}_t$ and $\Delta\log(M_t)$ to a policy shock, $\varepsilon_{M^*,t}$. CEE's assumptions only permit them to identify $\pi^{12}(L)$ and $\pi^{22}(L)$. The coefficients in $\pi^{11}(L)$ and $\pi^{21}(L)$ have no interpretation because the components of ε_t^1 are unspecified combinations of nonmonetary policy shocks to the economy. However, CEE's identifying assumptions do imply that $\varepsilon_{M^*,t}$ is orthogonal to ε_t^1. This orthogonality allows one to study the economic effects of $\varepsilon_{M^*,t}$, abstracting from movements in ε_t^1.

As we emphasized in example 3 of the previous section, there could in principle be many exogenous shock policy rules that support the endogenous variable policy rule (4.1). If so, these would give rise to different dynamic responses of the economy to a policy shock. But CEE's identifying assumptions allow them to estimate which monetary policy shock rule the monetary authority rule did adopt to support (4.1) and how the economy did respond to a monetary policy shock.

Figures 1–3 displays the estimated effect of a contractionary monetary policy shock on the elements of Z_t. These are obtained setting $q = 4$ and the data sample period, 1965Q3-1995Q2. Each figure also reports the response of the log level of money implied by the response of $\Delta\log(M_t)$ to a monetary policy shock. The solid lines in the middle of the graphs correspond to the point estimates of the dynamic response functions implied by the estimated VARs. The outer solid lines denote a 95% confidence interval for the dynamic response functions.[13] The dashed lines refer to results to be discussed below. Figures 1, 2 and 3 present results when money is measured as MB, $M1$ and $M2$, respectively.

[13]The 95 percent confidence intervals for the CEE impulse response functions were computed using the following bootstrap Monte Carlo procedure. We first constructed 1000 time series on the vector Z_t, each of length T, where T denotes the number of observations in our data sample. Let $\{\hat{u}_t\}_{t=1}^T$ denote the vector of residuals from the estimated VAR. We constructed 1,000 sets of new time series of residuals, $\{\hat{u}_t(j)\}_{t=1}^T$, $j = 1, ..., 1000$. The t^{th} element of $\{\hat{u}_t(j)\}_{t=1}^T$ was selected by drawing randomly, with replacement, from the set of fitted residual vectors, $\{\hat{u}_t\}_{t=1}^T$. For each $\{\hat{u}_t(j)\}_{t=1}^T$, we constructed a synthetic time series of Z_t, denoted $\{Z_t(j)\}_{t=1}^T$, using the estimated VAR and the historical initial conditions on Z_t. Second, we re-estimated our VAR using $\{Z_t(j)\}_{t=1}^T$ and the historical initial conditions, and calculated the implied impulse

A number of results here are worth noting. First, regardless of which measure of money we use, a contractionary monetary policy shock (rise in ε_{M^s}) results in a relatively quick, hump-shaped decline in output, a persistent rise in the federal funds rate and a one-and-a-half year delayed decline in the aggregate price level. Second, the estimated monetary policy shock rule for the growth rate in MB, $M1$ and $M2$ is given in the third panel of the second column of figures 1, 2 and 3, respectively. Notice that the contemporaneous reduction in money associated with a contractionary shock is greatest when money is measured as $M2$, and smaller when money is measured as $M1$ and MB. Indeed, when money is measured as MB, the point estimate actually shows a *rise* in the impact period of a monetary contraction, though the rise is not statistically different from zero. In all cases, nonborrowed reserves exhibit an immediate negative response, while the response of total reserves resembles that of MB. So according to our estimates, the short run effect of a Fed policy action is to alter the composition of reserves, in terms of borrowed and nonborrowed components, but not the total quantity. The basic effect of a contractionary policy shock is to signal that total reserves, MB and $M1$ will fall in the *future*.[14] Yet, the policy shock is associated with a persistent decline in output that begins relatively quickly. This suggests that the primary mechanism by which a monetary policy shock impacts on the economy is by its effect on anticipated future money. It is unlikely that models which emphasize only the effects of unanticipated money can account for the dynamic response of the economy to a monetary policy shock.

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response functions for $j = 1, ..., 1000$. For each fixed lag, we calculated the 25^{th} lowest and 975^{th} highest value of the corresponding impulse response coefficient across all 1000 synthetic impulse response functions. The boundaries of the confidence intervals in the figures correspond to a graph of these coefficients. The solid line reports our point estimate of the impulse response function. For the most part, these point estimates are quite similar to the mean value of the simulated impulse response functions.

[14]The institutional structure of the Federal Reserve allows it to credibly signal its intention to contract total bank reserves without actually affecting total reserves immediately. When a Fed open market operation reduces the nonborrowed reserves of banks, it simultaneously creates conditions which, under the Fed's rules, make it possible for banks to enter the discount window and make up the reduction in nonborrowed reserves by an increase in borrowed reserves. However, under the rules governing discount window borrowing, such borrowed reserves must be returned in a relatively short period of time. Interest rates rise even in the absence of contemporaneous effects on total reserves because borrowed and nonborrowed reserves are not perfect substitutes: borrowing funds at the discount window reduces a bank's option to do so again in the future, and such funds have to be returned in a short period of time.

4.2. A Diagnostic Procedure

To assess the empirical plausibility of a model's implications for a monetary policy shock, we can proceed as follows. First, characterize monetary policy in a theoretical model by the monetary policy shock rule:

$$\Delta \log(M_t) = \pi_0^2 + \pi^{22}(L)\varepsilon_{M^s,t}. \tag{4.7}$$

Second, study the theoretical model's implications for the dynamic response of the nonmonetary variables to a monetary policy shock. Third, compare these responses to their empirical counterparts. These are characterized by the coefficients in $\pi^{12}(L)$. Section 2 established that the two sets of responses would be identical if the model were true, abstracting from sampling uncertainty and the various linearity assumptions in the VAR analysis.

In this section, we establish the analog result for the VAR. Namely, we show that $\pi^{12}(L)$, which characterizes the dynamic response function of the nonmonetary variables in the system to a policy shock, is invariant to replacing the endogenous policy rule, (4.1), underlying the VAR system, with (4.7).

In the previous subsection we discussed the representation for Z_t, (4.2), which incorporates the endogenous variable policy rule (4.1). We now consider the following alternative representation obtained by replacing the endogenous variable policy rule, with the exogenous shock policy rule, (4.6). We show that this new system, which we call the *exogenous policy system,* is equivalent to (4.2).

In the exogenous policy system, Z_t is determined according to (4.6) and:

$$B_0^{11}\bar{Z}_t + B_0^{12}\Delta\log(M_t) = \kappa_Z^1 + B^{11}(L)\bar{Z}_{t-1} + B^{12}(L)\Delta\log(M_{t-1}) + \varepsilon_t^1 \tag{4.8}$$

where

$$B_0 = \begin{bmatrix} B_0^{11} & B_0^{12} \\ B_0^{21} & B_0^{22} \end{bmatrix},\ B(L) = \begin{bmatrix} B^{11}(L) & B^{12}(L) \\ B^{21}(L) & B^{22}(L) \end{bmatrix},\ \kappa_Z = \begin{pmatrix} \kappa_Z^1 \\ \kappa_Z^2 \end{pmatrix},$$

These objects have been partitioned in a way that is consistent with the partitioning of Z_t.

The original system, (4.2), and the exogenous policy system, are identical in the following sense. We say that a solution to a difference equation like (4.2) is a sequence, $Z \equiv \{Z_t\}_{t=-\infty}^{\infty}$ that satisfies (4.2) for a given sequence $\varepsilon \equiv \{\varepsilon_t\}_{t=-\infty}^{\infty}$ and which has the property that Z_t is expressible as a linear, square summable function of ε_{t-s}, $s \geq 0$, for all t. The coefficients in this function are the impulse response function of Z_t to the shocks. Given our assumptions on (4.2) and condition (4.4), the solution to (4.2) exists and is uniquely given by (4.5). It follows that for a given sequence, ε, the solution to (4.2) also solves the system defined by (4.6) and (4.8). That is, the impulse response functions of (4.2) are also impulse response functions for the exogenous policy system.

Now suppose that we are only interested in the dynamic response of the system to a monetary policy shock. Linearity of the system and orthogonality of the monetary policy shock with ε_t^1 means that we can work with the version of the exogenous policy system in which the growth rate of money has the univariate representation, (4.7). The dynamic response of $\bar{Z}_t$ to a monetary policy shock in this version of the exogenous policy system is exactly the same as it is in the original system which embeds the endogenous variable policy rule, (4.1).

Mechanically, it might appear that switching between the exogenous and endogenous formulations of monetary policy in the VAR would be in violation of the Lucas critique. In general, changing the policy rule would change the coefficients in (4.8) and $\pi^{12}(L)$. However the point of section 2 is that there is no substantive difference between the exogenous and endogenous formulations of policy. Therefore the Lucas critique does not apply here.

The type of considerations discussed in section 2 raise another possibility that we need to confront: the exogenous shock policy rule, (4.6), may be consistent with many dynamic response functions of $\bar{Z}_t$ to a monetary policy shock. As suggested by the reasoning in section 2, this can happen even if the dynamic response of $\bar{Z}_t$ is unique when policy is characterized by the endogenous variable policy rule. To see this consider the following example in which B_0^{11} is singular even though B_0 is invertible.

Example 5 Suppose $\bar{Z}_t$ and ε_t^1 are scalars and

$$B_0 = \begin{bmatrix} 0 & -1 \\ 1 & d \end{bmatrix},\ B(L) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ \kappa_Z = 0.$$

In this system, the dynamic response of Z_t to the shocks is uniquely given by (4.5) with

$$\pi(L) = B_0^{-1} = \begin{bmatrix} d & 1 \\ -1 & 0 \end{bmatrix}. \tag{4.9}$$

Note, however, that the exogenous policy system, (4.6) and (4.8), is:

$$-\Delta \log(M_t) = \varepsilon_t^1$$

$$\Delta \log(M_t) = -\varepsilon_t^1.$$

Evidently, this system has multiple solutions for Z_t: literally, any sequence, $\{\bar{Z}_t\}$, will solve (4.6) and (4.8). One of the solutions is the unique solution associated with the endogenous policy system.

The situation in example 5 is the counterpart to example 4 in section 2, where the system with an exogenous policy representation has many solutions, but there exists an endogenous policy representation for which the system has a unique solution. In our empirical analysis we never encountered a non-invertible B_0^{11}.

4.3. Approximating the Monetary Policy Shock Rule

One way to implement the proposed diagnostic procedure is to adopt the point estimate of $\pi^{22}(L)$ in (4.7) as the specification of monetary policy in a theoretical model. The problem with this strategy is that $\pi^{22}(L)$ involves a large number of parameters, so working with the estimated monetary policy shock would be quite cumbersome. Some approximation is clearly desirable. For example, one could work with a first order autoregressive approximation to $\pi^{22}(L)$, so that the monetary policy shock rule takes the form:

$$\Delta \log(M_t) = \tilde{\pi}^{22}(L)\varepsilon_{M^s,t}, \tag{4.10}$$

with

$$\tilde{\pi}^{22}(L) = \frac{\rho_0}{1 - \rho L},\ \rho_0 = \pi^{22}(0). \tag{4.11}$$

Alternatively, one could adopt a finite-ordered moving average representation:

$$\tilde{\pi}^{22}(L) = \gamma_0 + \gamma_1 L + \gamma_2 L^2 + ... + \gamma^k L^k, \tag{4.12}$$

for a small value of k. Below we provide examples of each parameterization. As we show there is a potential pitfall to be avoided when selecting a $\tilde{\pi}^{22}(L)$ that approximates $\pi^{22}(L)$.

We consider two related criteria for $\tilde{\pi}^{22}(L)$ to be a good approximation to $\pi^{22}(L)$:

(i) the parameterization of $\tilde{\pi}^{22}(L)$ should not be rejected by the data,

(ii) the dynamic response functions of the non-monetary variables to a policy shock should not be qualitatively different when $\pi^{22}(L)$ is replaced by $\tilde{\pi}^{22}(L)$.

There are in principle at least two approaches to finding a parameterization of $\tilde{\pi}^{22}(L)$ which satisfies (i) and (ii). The first approach involves re-estimating (4.2) with the null hypothesis, (4.11) or (4.12), imposed. This approach respects the assumed functional form of the endogenous variable policy rule, (4.1). However, we suspect that the set of parameterizations of B_0 and $B(L)$ that literally satisfy (4.11) or (4.12) is small and uninteresting.

We adopt a different approach instead, which finds an approximation, $\tilde{\pi}^{22}(L)$, to $\pi^{22}(L)$ in two steps. First, we identify a parameterization of $\pi^{22}(L)$ that fails to be rejected, so that criterion (i) above is satisfied. Second, we evaluate whether the resulting approximation satisfies (ii). We discuss these two steps in turn.

We began by testing the null hypothesis, (4.11) with $\rho = 0.3$ and 0.5, using a Wald-type test. The specific null hypothesis is that the i^{th} lag response of money growth to a monetary policy shock is $\hat{\rho}_0(\rho)^i$ for $i = 1, 2, 3, ...\kappa$, where $\widehat{\rho}_0$ is the estimated contemporaneous impact of a policy shock and $\kappa = 4$. Probability values for our test statistics are reported in Table 1.[15] Consider first our results for $M2$. Notice that the $AR(1)$ hypothesis with $\rho = 0.5$ cannot be rejected. There is substantially more evidence against the $AR(1)$ hypothesis for $\rho = 0.3$. We conclude that $AR(1)$ specification with $\rho = .5$ is a reasonable statistical approximation to $\pi^{22}(L)$ when money is measured is as $M2$.

In contrast, there is substantial evidence against the $AR(1)$ specification for $M1$ and MB. This is true whether ρ is equal to 0.3 or 0.5. Both specifications are rejected for both of the narrow monetary aggregates at the 5% levels. For MB, both specifications are rejected at the 1% level.

Figures 1-3 suggest that the impulse responses of the narrower monetary aggregates are

[15]The details of the test are as follows. Let a_i denote the coefficient on L^i in $\pi^{22}(L)$, $i = 0, ..., \kappa$. Let $a = [a_1, ..., a_\kappa]'$. We used a standard bootstrap methodology to simulate the estimated system (4.2) R = 1,000 times, giving us R random draws, $a^1, ..., a^R$. Define

$$V = \frac{1}{R}\sum_{r=1}^{R}(a^r - \bar{a})(a^r - \bar{a})',\ \bar{a} = \frac{1}{R}\sum_{r=1}^{R}a^r.$$

Let $\tilde{a}$ denote the value of a under the null hypothesis, i.e., $\tilde{a} = [\rho\hat{a}_0, \rho^2\hat{a}_0, \rho^3\hat{a}_0, ..., \rho^\kappa\hat{a}_0]'$, with $\rho = 0.5$. Let

$$\begin{aligned} \tilde{s} &= (\tilde{a} - \bar{a})'V^{-1}(\tilde{a} - \bar{a}), \\ s^r &= (a^r - \bar{a})'V^{-1}(a^r - \bar{a}),\ r = 1, ..., R. \end{aligned}$$

The entries in Table 1 report the percent of s^r's exceeding $\tilde{s}$. The analysis was also performed with $\bar{a}$ replaced everywhere (including the definition of V) by $\hat{a}$, the empirical point estimate of a.

better characterized by (4.12), with $k = 2$. One such specification, obtained by equating $\tilde{\pi}^{22}(L)$ to a version of $\pi^{22}(L)$, in which the coefficients on powers of L greater than 2 are set to zero, passes the Wald test. We adopt a slight variant of this specification because of a possible pitfall in selecting an approximation to $\pi^{22}(L)$ that satisfies criterion (ii).

To understand the nature of this pitfall, note that (4.7) and (4.8) imply that, in the exogenous policy system, the dynamic response of $\bar{Z}_t$ to a policy shock, is given by the coefficients of $\pi^{12}(L)$,

$$\pi^{12}(L) = \left[I - \left(B_0^{11}\right)^{-1} B^{11}(L)L\right]^{-1} \left(B_0^{11}\right)^{-1} \left[B^{12}(L)L - B_0^{12}\right] \pi^{22}(L). \tag{4.13}$$

The dynamic response of $\bar{Z}_t$ to a policy shock in the approximate system is given by (4.13) with $\pi^{22}(L)$ replaced by $\tilde{\pi}^{22}(L)$.

As it turns out, it is possible to perturb $\pi^{22}(L)$ in directions that would cause the implied π^{12} to be explosive for even the smallest perturbation. This can happen if there is an explosive root in the first term after the equality in (4.13), i.e.,

$$\det\left[I - \left(B_0^{11}\right)^{-1} B^{11}(\bar{z})\bar{z}\right] = 0 \text{ for some } \bar{z},\ |\bar{z}| < 1. \tag{4.14}$$

Suppose also that $\pi^{22}(\bar{z}) = 0$. In this case, the response of $\bar{Z}_t$ to a policy shock is damped, because the explosive root in the first term after the equality is cancelled by an explosive root moving average representation for money. However, even the slightest adjustment in the coefficients of $\pi^{22}(L)$ which alters the root at $\bar{z}$ will cause $\pi^{12}(L)$ to become explosive. Since this turns out to be an important example in practice, the Appendix contains an example of this case.

Condition (4.14) is not sufficient for the type of extreme sensitivity to alterations in π^{22} that we just discussed. If an explosive root in the first term after the equality in (4.13) is cancelled by zeros in the same location in the term in square brackets just prior to π^{22} in that expression, then there may not be large sensitivity to perturbations in $\pi^{22}(L)$.

With these considerations in mind, we explored the eigenvalue properties of the estimated versions of (4.2). Regardless of which measure of money we use, all 28 zeros of $\det\left[I - (B_0)^{-1} B(z)z\right] = 0$ are stable, so that $\pi(L)$ in (4.5) is damped. However, π^{22} has explosive eigenvalues when money is measured by MB or $M1$. In particular, $\pi^{22}(-1/2.7767) =$

0 when money is measured by $M1$ and $\pi^{22}(1/5.8630) = 0$ when money is measured by MB. (Here, the roots have been rounded). Apparently the data does not 'want' explosive roots in $\pi^{12}(L)$. So the parameter estimates place an explosive eigenvalue in $\det\left[I - (B_0^{11})^{-1} B^{11}(L)L\right]$ to precisely cancel the explosive eigenvalue in π^{22}.

That the moving average representations of $M1$ and MB in terms of policy shocks, π^{22}, have explosive roots is not surprising, in view of Figures 1-2. In each case, a policy shock has little impact on the growth rate of money, but then has larger effects one or two period thereafter. For example, in the case of $M1$, the impact effect on money growth is -0.08, the lag one effect is -0.26, the lag two effect is -0.11, and the higher order effects are all roughly zero. Then,

$$\tilde{\pi}^{22}(L) = -0.08 - 0.26L - 0.11L^2 = -0.11\left(\frac{1}{2.75} - L\right)\left(\frac{1}{.5} - L\right). \qquad (4.15)$$

Even though the explosive root in $\tilde{\pi}^{22}$ is very close to the explosive root in π^{22} (compare 2.75 and 2.7767), the two roots are far enough apart to fail any reasonable version of criterion (ii) above. In particular, the dynamic response function of $\bar{Z}_t$ to a policy shock in the exogenous policy system formed from (4.8) and (4.10) with $\tilde{\pi}^{22}$ set as in (4.15) is extremely different from the analog response function in the original system.

With these considerations in mind, we obtained candidate monetary policy shock rules for MB and $M1$ by adjusting parameter values so that $\tilde{\pi}^{22}$ has precisely the moving average root needed to prevent explosiveness in π^{12}. The strategy we used can be illustrated in the case emphasized in (4.15). Replace the $\tilde{\pi}^{22}$ defined there by

$$-0.11\left(\frac{1}{2.7767} - L\right)\left(\frac{1}{.5} - L\right) = -.0792 - .2596L - 0.11L^2,$$

after rounding. Clearly, this is a very small change from (4.15).[16]

Proceeding in this way, we constructed two sets of representations for $\tilde{\pi}^{22}(L)$, corresponding to the two measures of money, MB and $M1$. These are summarized in Table 2. To investigate the statistical plausibility of these representations for $\pi^{22}(L)$, as well as the analog $MA(1)$ representations, we implemented the Wald test discussed above. Probability values for our test statistics are reported in Table 2. Notice that there is substantial evidence

[16]It should be clear that there is no $\tilde{\pi}^{22}$ of the form (4.11) that produces a stable π^{12} in the case of MB and $M1$. This is because such a $\tilde{\pi}^{22}$ has no zeros to cancel the explosive root, $\bar{z}$, in (4.14).

against the $MA(1)$ representation for MB. In contrast there is very little evidence against the $MA(2)$ representations for either MB or $M1$.

We now show that our candidate approximations to $\pi^{22}(L)$ satisfy criterion (ii) discussed above. Specifically, the dashed lines in Figure 1-3 display the responses to a monetary shock in the exogenous policy system formed from (4.8) and (4.10) with $\tilde{\pi}^{22}$ constructed as indicated above. In all cases, the responses closely resemble the responses implied by our point estimates. For MB and $M1$ the first few lags' responses are particularly close, since $\tilde{\pi}^{22}$ and π^{22} are virtually identical for lags 0, 1, and 2.[17]

In sum, our parametrization of the monetary policy shock rule is given by

$$\Delta M_{2t} = .5\Delta M_{2t-1} + \varepsilon_{M^s,t} \tag{4.16}$$

$$\Delta M_{1t} = .0792\varepsilon_{M^s,t} + .2596\varepsilon_{M^s,t-1} + 0.11\varepsilon_{M^s,t-2} \tag{4.17}$$

$$\Delta MB_t = -.0303\varepsilon_{M^s,t} + .1324\varepsilon_{M^s,t-1} + .1144\varepsilon_{M^s,t-2}. \tag{4.18}$$

for $M2$, $M1$ and MB, respectively. The negative sign on the first term in the equation for ΔMB_t represents the small, insignificant rise in the growth rate of MB after a contractionary policy shock.

5. Assessing the Performance of a Particular Dynamic Monetary Model

In this section we apply the diagnostic procedure discussed in the previous section. We do so using a particular model which is a combination of the models in Christiano and Eichenbaum (1992) and CEE (1997a). Both models incorporate credit market imperfections in the form of the limited participation assumption introduced by Lucas (1990). According to this assumption, households do not adjust the cash they send to financial intermediaries immediately following a monetary policy shock. As a consequence, an open market operation disproportionately affects the reserves available to financial intermediaries. CEE (1997a) analyze the contemporaneous effects of a monetary policy shock in this type of model assuming a particular form of the utility function (zero income effects on leisure) and monopolistic

[17]At very high lags, the exogenous policy system eventually explodes since it is not possible to precisely cancel an explosive root in a computer.

competition in the goods market. In their model, a shock to the growth rate of money has a large contemporaneous effect on aggregate output and a small effect on the price level. To assure that these effects persist over time we assume, as in Christiano and Eichenbaum (1992), that there are costs of adjusting the amount of cash sent to the goods market.

We now describe the model, which is a version of the one analyzed in section 2, modified to include capital accumulation, adjustment costs and the limited participation constraint. There are four types of agents: households, firms, financial intermediaries, and a monetary authority. We consider each of these in turn.

5.1. Households

At date t, a representative household selects $\{C_{t+j}, N_{t+j}, Q_{t+j}, M_{t+j+1}, K_{t+j+1}\}_{j=0}^{\infty}$ to optimize expected discounted utility subject to a particular cash constraint and an asset evolution constraint. The variable K_t denotes the beginning of period t stock of capital. The other variables are as defined earlier in the paper. We think of $M_t - Q_t$ as the amount of money that the household sends to financial intermediaries at time t. The household takes as given the gross interest rate on money sent to financial intermediaries, R_t, the rental rate on capital, r_t, the wage rate, W_t, and the price of consumption goods, P_t. The household's choices of $C_t, N_t, M_{t+1}, K_{t+1}$ are constrained to be functions of variables dated t and earlier. Consistent with the limited participation assumption, the choice of Q_t is constrained to be a function of variables dated $t-1$ and earlier, as well as K_t and M_t. The household's preferences are:

$$E_{-1}\sum_{t=0}^{\infty}\beta^t U\left(C_t, N_t, H_t\right),$$

where

$$U(C,N,H) = \left[C - \psi_0\frac{(N+H)^{(1+\psi)}}{1+\psi}\right]^{(1-\sigma)}/(1-\sigma). \tag{5.1}$$

Here, H_t, is the time t cost, denominated in units of labor, of adjusting Q_t. These are given by:

$$H_t = H\left(\frac{Q_t}{Q_{t-1}}\right) = d\left\{\exp\left[c\left(\frac{Q_t}{Q_{t-1}} - 1 - x\right)\right] + \exp\left[-c\left(\frac{Q_t}{Q_{t-1}} - 1 - x\right)\right] - 2\right\}, \tag{5.2}$$

where x is the net growth rate of Q_t and ΔM_{t+1} in nonstochastic steady state. With this specification, the nonstochastic steady state of the model is the same as when there are no adjustment costs ($c = d = 0$). The utility function in (5.1) reflects our assumption that the income effect on leisure is zero.[18]

The household's cash constraint is:

$$P_t\left(C_t + K_{t+1} - (1-\delta)K_t\right) \leq Q_t + W_t N_t. \tag{5.3}$$

The household's stock of money evolves according to:

$$M_{t+1} \leq R_t(M_t - Q_t) + R_t X_t + D_t + r_t K_t + \left[Q_t + W_t N_t - P_t\left(C_t + K_{t+1} - (1-\delta)K_t\right),\right] \tag{5.4}$$

where D_t denotes profits received from firms and $R_t X_t$ reflects lump sum profits received from the financial intermediary. Households must respect the constraint $0 \leq Q_t \leq M_{t-1}$. The fact that capital rental income, $r_t K_t$, appears in (5.4), but not in the cash constraint (5.3) indicates that this payment is received at the end of the period.

5.2. Firms

At time t, a final consumption good, Y_t, is produced by a perfectly competitive, representative firm. The firm does so by combining a continuum of intermediate goods, indexed by $i \in (0,1)$, using the technology

$$Y_t = \left[\int_0^1 Y_{it}{}^{\frac{1}{\mu}} di\right]^{\mu} \tag{5.5}$$

where $1 \leq \mu < \infty$, and Y_{it} denotes the time t input of intermediate good i. Let P_{it} denote the time t price of intermediate good i. Profit maximization and (5.5) imply that

$$P_t = \left[\int_0^1 P_{it}^{\frac{1}{1-\mu}} di\right]^{(1-\mu)}. \tag{5.6}$$

Intermediate good i is produced by a monopolist who uses the following technology:

$$Y_{it} = \begin{cases} K_{it}^{\alpha} N_{it}^{1-\alpha} - \phi & \text{if } K_{it}^{\alpha} N_{it}^{1-\alpha} \geq \phi \\ 0, & \text{otherwise} \end{cases} \tag{5.7}$$

where $0 < \alpha < 1$. Here, N_{it} and K_{it} denote time t labor and capital used to produce the i^{th} intermediate good. The parameter ϕ denotes a fixed cost of production. We rule out entry and exit into the production of intermediate good i.

[18]See *CEE* (1997a) for a discussion of how to reconcile this utility function with balanced growth.

Intermediate good firms rent capital and labor in perfectly competitive factor markets. Economic profits are distributed to the firms' owner, the representative household, at the beginning of time period $t+1$. Firms must pay workers in advance of production. As a result, firms need to borrow their wage bill, $W_t N_{it}$, from the financial intermediary at the beginning of the period. Repayment occurs at the end of time period t at the gross interest rate, R_t. Optimization on the part of intermediate good firms implies that they set their price as a fixed markup, μ, over marginal cost.

5.3. The Financial Intermediary and the Monetary Authority

A representative financial intermediary receives funds of $M_t - Q_t$ from the household, as well as lump sum injections from the monetary authority, X_t. It lends these funds to firms who must finance their wage bill. Profits, $R_t X_t$, are distributed to the financial intermediary's owner, the representative household, at the end time period t. The market clearing condition for loans is given by,

$$W_t N_t = M_t - Q_t + X_t.$$

where N_t denotes aggregate hours worked.

In our diagnostic procedure, we assume the monetary policy rule is given by either (4.16) or (4.17). We adopt a standard sequence-of-markets equilibrium concept for this model.[19]

5.4. Parameter Values

We choose parameter values as follows. First, ψ_0 is picked to guarantee that in nonstochastic steady state, employment is unity. We choose a value for ϕ so that economic profits are zero in nonstochastic steady state:

$$\phi = \left(\frac{\mu - 1}{\mu}\right) K^{\alpha}.$$

Here K denotes the nonstochastic steady state stock of capital. In addition we assume that,

$$\alpha = 0.36,\ \beta = 1.03^{-.25},\ \psi = 0.40,\ \mu = 1.40,\ x = 0.01,\ \delta = 0.02.$$

These parameter values imply,

$$1/\psi = 2.5,\ \phi = 0.99,\ \psi_0 = 1.54.$$

[19] Further details of the model and the linearized method used to solve it are spelled out in a technical appendix, available on request.

When there are no adjustment costs, the elasticity of labor supply is equal to $1/\psi$. The assumed value of this elasticity, 2.5, is high relative to standard estimates from the micro literature. The markup of 40 percent ($\mu = 1.4$) is also at the high end of the range used in the literature.[20] As explained in $CEE(1997a)$, the high markup and labor supply elasticities play an important role in enabling the model to produce a large output and small price response from a monetary policy shock.

The adjustment cost parameter values that we used are:

$$c = d = 2.$$

These were chosen for illustrative purposes only. The key features of these values is that they imply relatively small adjustment costs. To see this suppose we maintain the limited participation assumption but assume that the household solves its optimization problem as if $c = d = 0$. Next calculate the dynamic response of Q_t to a monetary policy shock, assuming the system is initially in nonstochastic steady state. Then calculate the implied time path of H_t by evaluating (5.2) at this sequence of Q_t's. The resulting values of H_t are a measure of the adjustment costs which the household is trying to avoid when it solves its actual optimization problem. We performed these calculations assuming the monetary policy shock rule is given by (4.16) and (4.17). In both cases, the maximal value of H_t/L_t, which occurs one period after the policy shock, is less than .06; two periods after the shock this ratio is less than .003, and thereafter it is virtually zero. Of course, along the equilibrium path of the economy, the actual adjustment costs incurred by the household are much smaller. As we show below, these small adjustment costs are sufficient to generate substantial persistent effects from a monetary policy shock.

The steady state properties of the fully parameterized model are summarized by:

$$K = 32.15,\ Y = 2.49,\ K/Y = 12.91,\ C = 1.85,\ C/Y = 0.74,\ \delta K/Y = 0.26,\ Q/M = 0.38.$$

In addition, the annualized value of R_t in nonstochastic steady state is equal to 1.072.

[20]See the discussion in Rotemberg and Woodford (1995) who use a markup of this size.

5.5. Results with an Example Endogenous Variable Policy Rule

In the introduction we motivated our diagnostic procedure with the observation that model economies often have multiple equilibria when policy is represented by an endogenous variable policy rule. We illustrate this possibility using the model economy just described. Consider the class of rules given by

$$\frac{R_t}{R} = \left(\frac{Y_t}{Y}\right)^{a_1} \left(\frac{\Psi_t}{\Psi}\right)^{a_2} exp(\varepsilon_{M^s,t}). \tag{5.8}$$

Here Ψ_t denotes the time t inflation rate, P_t/P_{t-1}, $\varepsilon_{M^s,t}$ is a time t policy shock, while Ψ and R denote the nonstochastic steady state values of the inflation rate and the nominal interest rate, respectively. With $a_1, a_2 > 0$, relation (5.8) captures key features of what is sometimes referred to as the Taylor rule (see Taylor (1993)). In particular it embodies the notion that the monetary authority wishes to raise the nominal interest rate when either output or inflation is high relative to some target, here assumed to equal the nonstochastic steady state value of these variables. We find that with $0 \leq a_i \leq 2$, $i = 1, 2$, the nonstochastic steady state equilibrium of the model is indeterminate. That is, for the deterministic version of the model there are multiple equilibria in which prices and quantities converge to their nonstochastic steady state values. We verified this local multiplicity result by standard methods which focus on the eigenvalues of a particular matrix.[21] The same methods were used to verify that the exogenous policy equilibrium is locally unique.

We illustrate the nature of the multiplicity by focusing on the case $a_i = 0$, $i = 1, 2$, in which case (5.8) reduces to a simple interest rate pegging rule. It is well known that the indeterminacy of the nonstochastic steady state implies the existence of a continuum of stochastic equilibria, including sun spot equilibria (see Woodford (1986)). Here, we confine ourselves to studying the two minimum state variable equilibria.[22] Figure 4 displays the dynamic response of different model variables to a 0.01 shock in $\varepsilon_{M^s,t}$ (i.e., a 100 basis point shock to R_t) in the two equilibria. The key differences pertain to the dynamic responses of money and the price level. In both equilibria, the contemporaneous response of the money

[21] We considered all possible combinations of a_1 and a_2 between 0 and 2, moving in increments of 0.1.

[22] By a minimum state variable equilibrium we mean one in which the date t equilibrium prices and quantities are represented as functions of the smallest set of predetermined variables need to characterize date t preferences, technology and monetary policy.

stock and the price level are the same: the money stock falls by 0.20 percent and the price level hardly changes. Thereafter, the money stock and prices behave quite differently. In one equilibrium, the money stock essentially returns to its unshocked path immediately, while the price level remains basically unchanged. In the other equilibrium, the money stock falls part of the way to its unshocked path, after which it rises until eventually it remains 0.34 percent above is unshocked path. The price level smoothly asymptotes to a level that is 0.34 percent above its unshocked path.

The two response functions of money correspond to two monetary policy shock rules, each of which supports the endogenous variable policy rule. As we just saw, each corresponds to a different exogenous policy equilibrium.

5.6. Results with the Estimated Monetary Policy Shock Rules

Figure 5 reports the dynamic response of selected variables in our model to an expansionary monetary policy shock for the AR and MA monetary policy shock rules, (4.16) and (4.17), respectively. In both cases, we normalize the magnitude of the shock so that the long run effect of a policy shock is to raise the stock of money by one percent. We begin by discussing the AR case. A number of results are worth noting here. First, output rises immediately by more than one half of a percent, and then slowly returns to its steady state value. The persistence of the effect is such that output has not returned to its steady state value even four years after the shock. Employment responds to the policy shock much like output. Second, the dynamic response functions of output and employment do not display the hump shape response that the estimated response functions exhibit (see section 4). Third, employment and output rise immediately in response to the policy shock. This is inconsistent with the estimated response functions discussed in section 4. Fourth, both consumption and investment initially rise in response to the policy shock before slowly returning to their steady state levels. Fifth, the price level initially falls slightly in response to the monetary policy shock, before slowly rising to its new steady state value. Reflecting the persistent rise in output, the price level has not completely adjusted even four years after the shock. Paralleling these price level movements, the rate of inflation initially falls, then reaches its peak value of 5.25%, after which it slowly returns to its nonstochastic steady state value.

Finally, note that R_t initially falls from about 7.2 percent (annual rate) to 5.30 percent, and then slowly returns to its steady state level. One way to summarize the size of the initial liquidity effect in the model is to compute the ratio of the average of the first six responses in R_t to the average of the first six responses in M_t. The resulting statistic, which we denote by R/M, summarizes the average percentage point change in R_t from a policy - induced one percentage point change in M_t. In the model, R/M is roughly equal to 151. Performing the analog calculations using the estimated impulse response functions for $M2$, we obtain a value of R/M equal to 258.

Next consider the effects of a monetary policy shock in the MA case. In many ways the qualitative response of the system is similar to the AR case. There are however a number of important differences. First, as in the AR case, output rises by more than the initial (small) rise in the money stock, so the price level drops by a small amount. But as the money stock continues to rise, so too does output. The latter reaches its peak response only in the second period after the quarter in which the policy shock occurs. As a result, both output and employment exhibit a clear hump shape response to the policy shock, much like in the data. Second, the initial output and price effects of a policy shock are small, consistent with the estimated response functions discussed in section 4. Finally note that inflation behaves in a similar manner in the AR and MA cases, and a policy shock continues to generate a persistent decline in R_t, with the value of R/M being roughly equal to 142. Performing the analog calculations using the estimated impulse response functions for $M1$, we obtain a value of R/M equal to 217.

We conclude that which representation of monetary policy we use makes a noticeable difference in the model dynamics. Our pure MA model, in which the effects of a monetary policy shock are delayed, appears to work best. This representation for money enables the model to replicate the hump-shaped response of output and employment to money, while preserving a strong and persistent effect on output and an initially muted effect on the price level. Finally, it is worth emphasizing that the absence of an initial price or output response is consistent with the identifying assumptions of our empirical analysis.[23] Thus, converting from the pure AR to the pure MA representation of monetary policy improves the fit between

[23]See Rotemberg and Woodford (1997) for another equilibrium model which has this property.

the model and the data.

6. Conclusion

This paper developed a practical strategy for using information about the effects of a monetary policy shock to evaluate alternative business cycle models. Specifically, we use the data to estimate simple exogenous representations for the growth rate of money that can be used to assess the empirical plausibility of a model's predictions for how the economy responds to a monetary policy shock. These representations are consistent with the widespread view that monetary policy is highly reactive to the state of the economy. We use these representations to assess the plausibility of a particular model which stresses the role of credit market frictions in the monetary transmission mechanism.

We conclude by noting that the paper did not discuss formal statistical tests of the hypothesis that model based and estimated impulse response functions are drawn from the same data generating mechanism. There are a variety of ways to test such a hypothesis. One straightforward way is to simultaneously estimate, via generalized method of moments procedures, the parameters of the model and the coefficients of the VAR. The latter embodies the endogenous variable policy rule used to identify policy shocks in the data. One could then test the null hypothesis that the first k coefficients of the model and VAR based impulse response functions are the same in population using a variant of the Wald type statistic discussed in Christiano and Eichenbaum (1992a). We leave this task to future work.

7. Appendix

7.1. Proof of Proposition 1

Following is a proof to Proposition 1:

That the endogenous variable policy rule, (3.11), is satisfied by the objects identified in the proposition is verified simply by reversing the steps used to derive (3.16)-(3.17). To see this note first that given the definition of N_t in (3.15), (3.16) can be written,

$$\log(1+x_t) = (1-\varphi)\log(1+x) - \nu\left(\log(N_t) - \varphi\log(N_{t-1})\right) + \varepsilon_{M^s,t} - \varphi\varepsilon_{M^s,t-1},$$

or, collecting terms,

$$\log(1+x_t) = \log(1+x)-\nu\log(N_t)+\varepsilon_{M^s,t}-\varphi\left(\log(1+x)-\nu\log(N_{t-1})+\varepsilon_{M^s,t-1}-\log(1+x_{t-1})\right).$$

Consider the previous expression for $t=1$. The term in parentheses is zero by (3.17). Moving forward in time recursively, one then obtains that (3.11) holds at each date.

We now establish that the objects identified in the proposition also constitute an exogenous policy equilibrium. Condition (iii) is sufficient to guarantee that the household's transversality condition is satisfied. That the household's first order condition, (3.8), is satisfied is verified as follows:

$$E_t\left\{\frac{N_{t+1}^{1-\gamma}}{N_t^{1+\psi}}\frac{1+x}{1+x_{t+1}}\right\} = \left\{\left[\frac{N_{t+1}^{1-\gamma+\nu}}{N_t^{1+\psi}}\exp(-\varepsilon_{M^s,t+1})\right]\left[N_{t+1}^{-\nu}\exp(\varepsilon_{M^s,t+1})\frac{1+x}{1+x_{t+1}}\right]\right\} = 1,$$

by (3.11) and (3.12). That the household's intra-temporal Euler equation is satisfied is guaranteed by the construction of the nominal wage in (3.10). That the household's intra-temporal Euler equation holds is guaranteed by the construction of the nominal wage in (3.10). That the household's inter-temporal Euler equation holds is guaranteed by the construction of R_t in (3.9), the fact the intra-temporal Euler equation holds and the fact that (3.8) holds. The construction of C_t, Q_t and P_t in (3.10) guarantee that the resource constraint, the household's cash constraint and the loan market clearing condition hold. That the household's law of motion for money, (3.3), holds is guaranteed by the fact that firm profits are zero, the cash constraint holds with equality, and the loan market clearing condition holds.

7.2. A Pitfall in Approximating the Estimated Monetary Policy Shock Rule: An Example

In section 4.3 we discussed the possibility that even the slightest adjustment in the coefficients of the estimated monetary policy rule can cause non monetary variables to explode in response to a policy shock. Here we provide an illustration of this possibility using the notation of that section. The perturbation of $\pi^{22}(L)$ which we consider alters the root at $\bar{z}$ and causes $\pi^{12}(L)$ to become explosive.

Suppose $\kappa_Z = 0$, $B_0 = I$, $B(L) = B = P\Lambda P^{-1}$,

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix},\ P = \begin{bmatrix} 1 & 1 \\ a & a+1 \end{bmatrix},\ B = \begin{bmatrix} \lambda_1 + (\lambda_1-\lambda_2)a & \lambda_2-\lambda_1 \\ a(a+1)(\lambda_1-\lambda_2) & \lambda_2 + a(\lambda_2-\lambda_1) \end{bmatrix},$$

$$\pi(L) = \frac{\begin{bmatrix} 1 - B^{22}L & B^{12}L \\ B^{21}L & 1 - B^{11}L \end{bmatrix}}{(1 - \lambda_1 L)(1 - \lambda_2 L)}.$$

If $|\lambda_i| < 1$, $i = 1, 2$, then the response of Z_t to $\varepsilon_{M^{\bullet},t}$ is damped (as is the response to ε_t^1.) Suppose that $B^{11} > 1$ (as in $\lambda_1 = 0.2$, $\lambda_2 = 0.1$, $a = 10$). Now consider the associated exogenous policy system:

$$\begin{aligned} \bar{Z}_t &= B^{11}\bar{Z}_{t-1} + B^{12}\Delta log(M_{t-1}) \\ \Delta log(M_t) &= \pi^{22}(L)\varepsilon_{M^{\bullet},t}, \end{aligned}$$

where we ignore the non-monetary policy shocks. In this system:

$$\bar{Z}_t = \frac{B^{12}L}{1 - B^{11}L}\frac{1 - B^{11}L}{(1 - \lambda_1 L)(1 - \lambda_2 L)}\varepsilon_{M^{\bullet},t}. \tag{7.1}$$

Note that for the dynamic response function of $\bar{Z}_t$ to be damped requires cancellation of the explosive polynomial, $1 - B^{11}L$. This is ensured by $\pi^{22}(L)$, because it has a zero at $(B^{11})^{-1}$. Any perturbation of $\pi^{22}(L)$ which changes the location of this zero by even the smallest amount results in an explosive dynamic response function for $\bar{Z}_t$.

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Table 1: Probability Values for AR Wald Tests

$H_0 : a_i = \hat{\rho}_0 \times .\rho^i,\ i = 1, 2, 3, 4.$

$a_i \sim i^{th}$ lag response of $\Delta \log(M_{t+1}/M_t)$ to $\varepsilon_{M^s,t}$

$\hat{\rho}_0 \sim$ estimated impact on $\Delta \log(M_{t+1}/M_t)$ of $\varepsilon_{M^s,t}$

	CEE	
Measure of Money	$\rho = .3$	$\rho = .5$
$M2$	13.9	41.9
$M1$	0.6	1.7
MB	0.2	0.2

Table 2: Probability Values for MA Wald Tests

	CEE	
	MA(1)	MA(2)
M1	42.0	94.3
MB	5.4	76.3

Fig. 1: CEE with M = MB

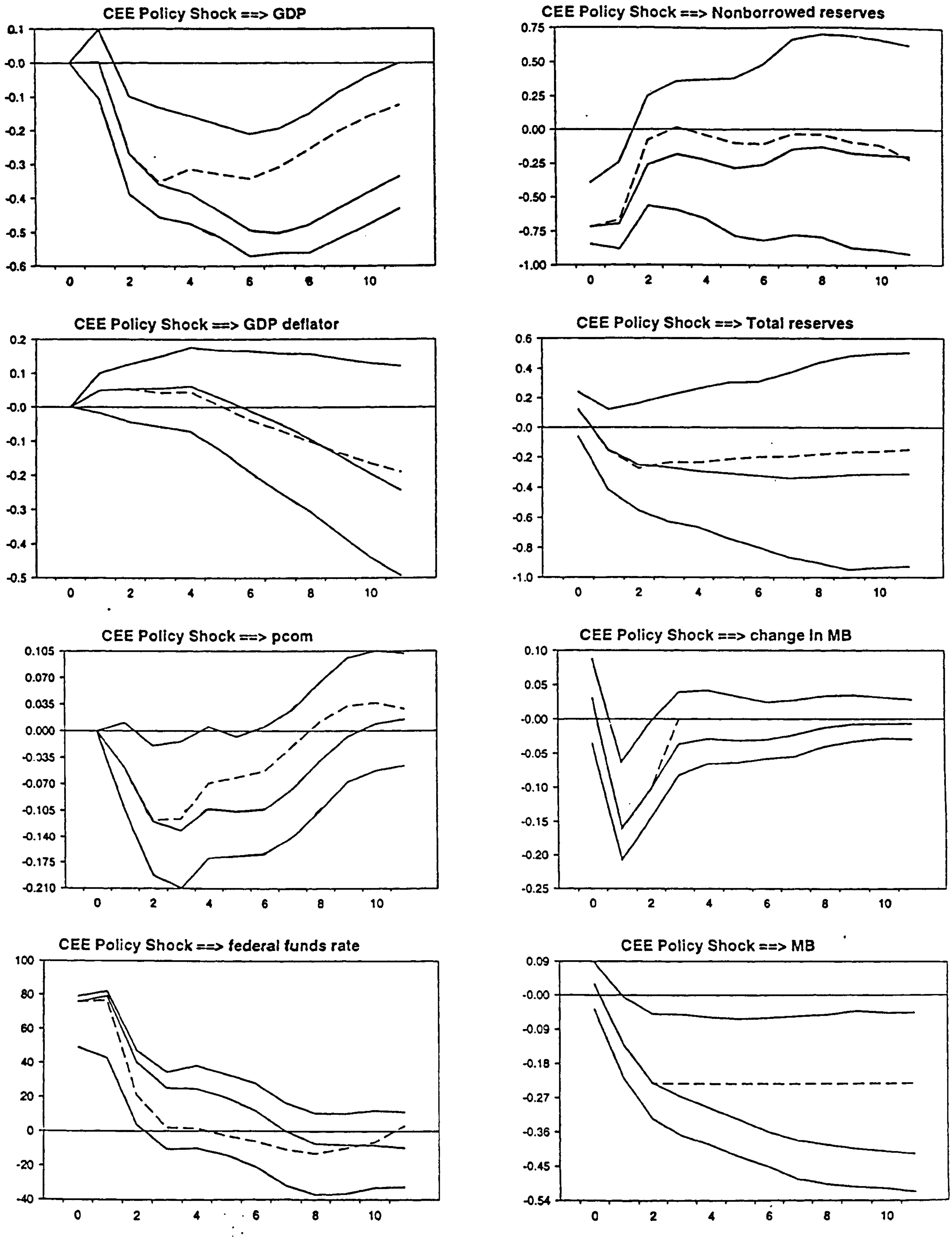


Fig. 2: CEE with M = M1

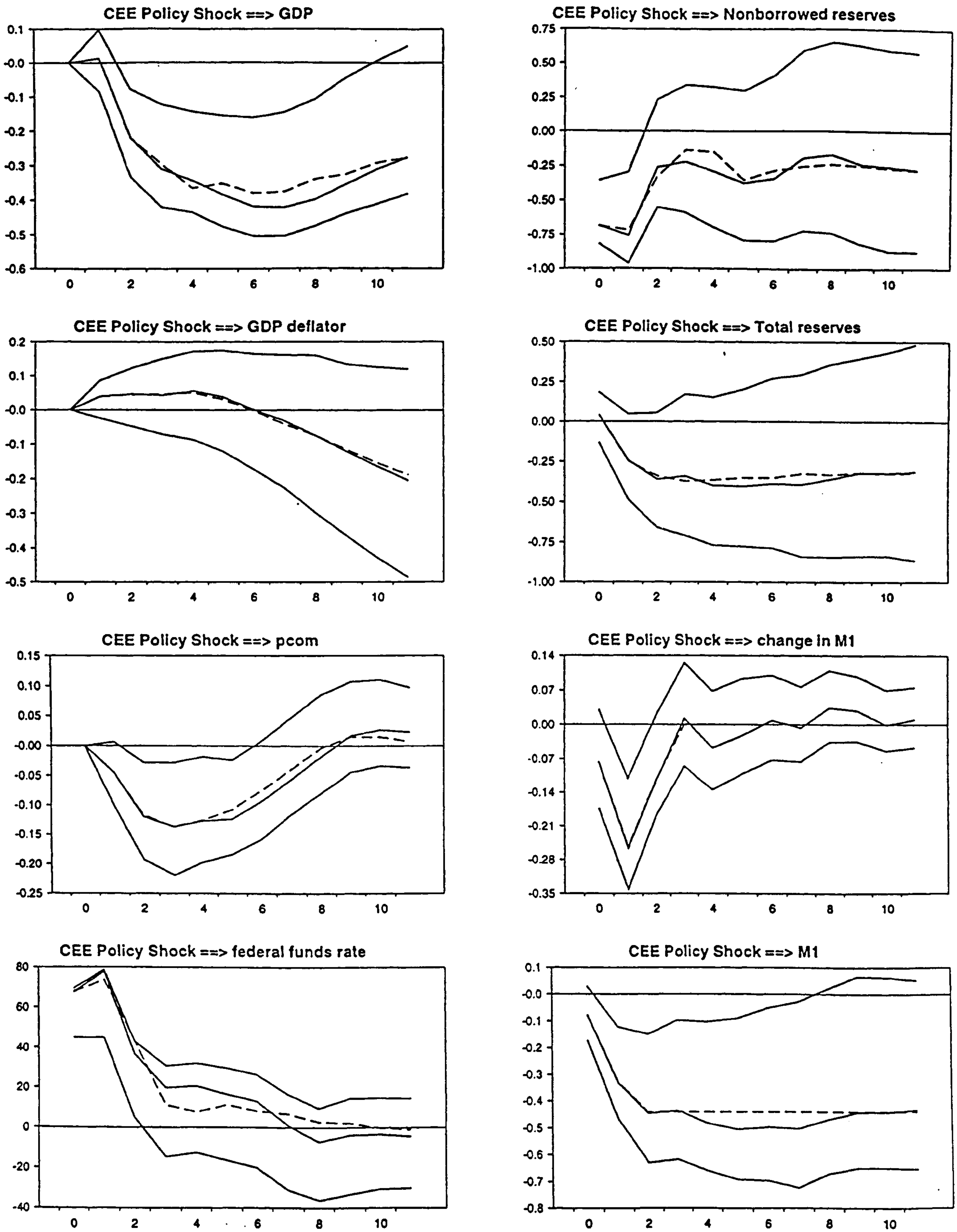
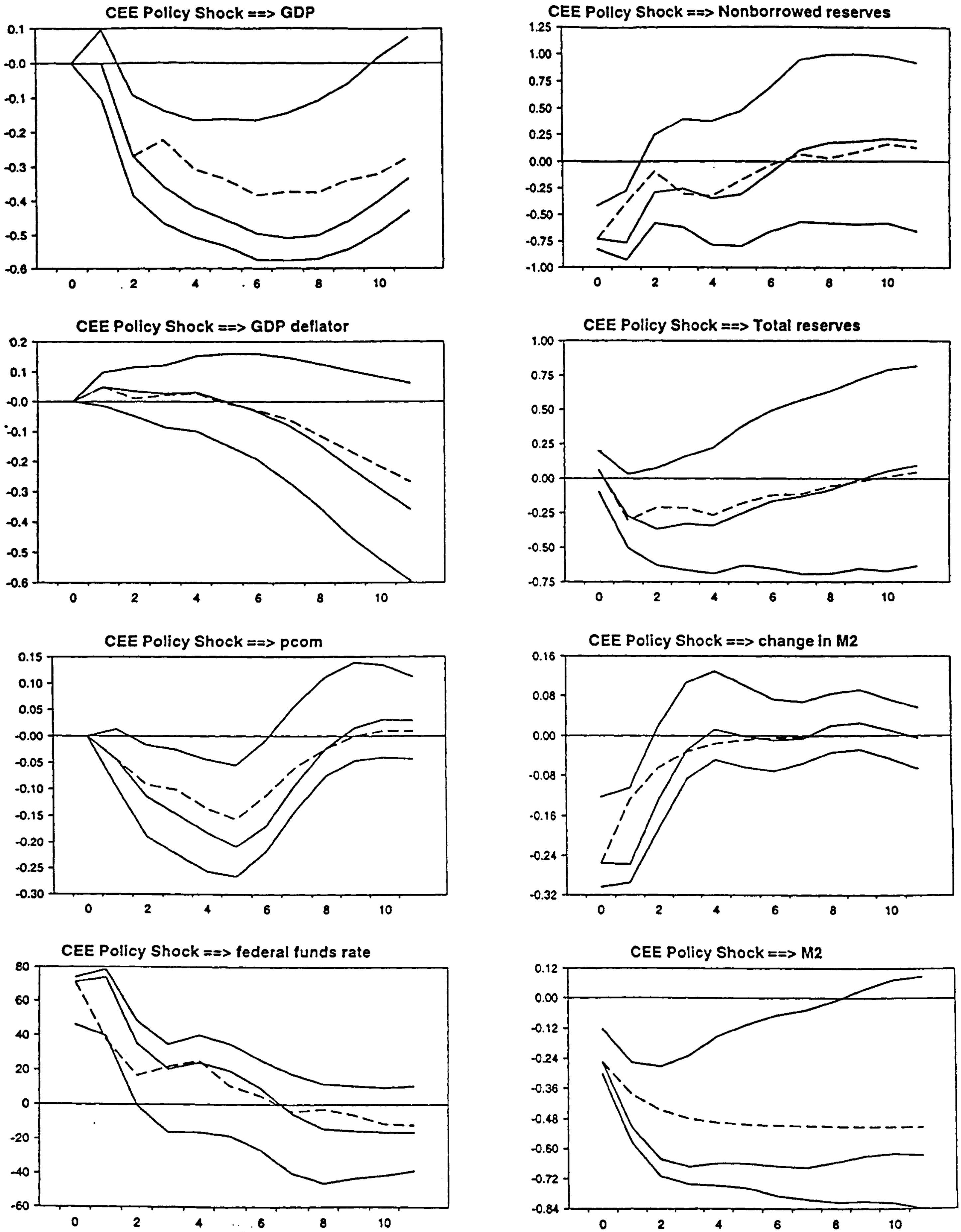
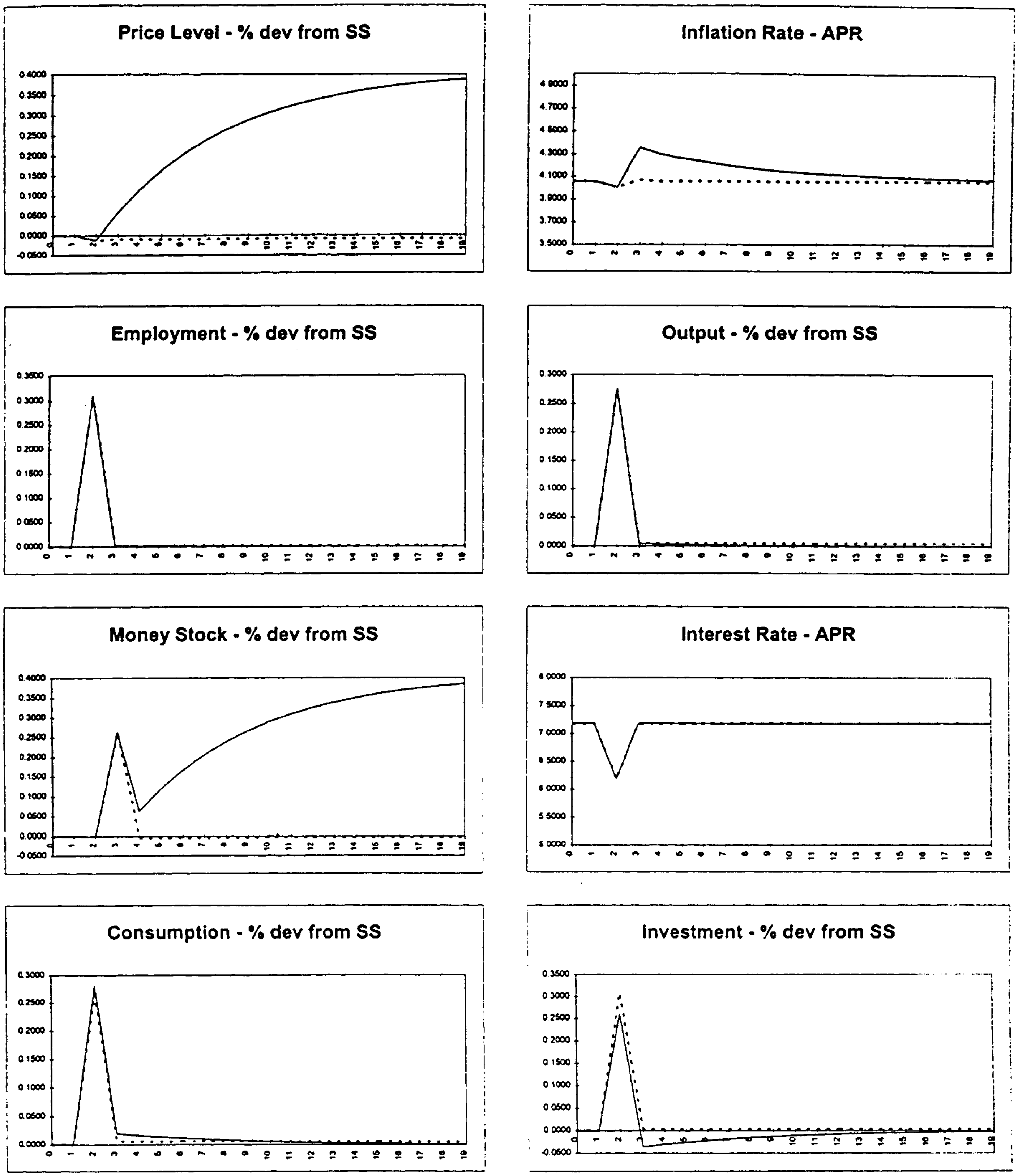


Fig. 3: CEE with M = M2



Response to an Interest Rate Shock for Two Different Equilibria



Equilibrium 1 ————

Equilibrium 2 - - - - - - -

Response to a Money Growth Rate Shock