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Changes in Trading Activity Following Stock Splits and Their Impact on Volatility and the Adverse Information Component of the Bid-Ask Spread

ABSTRACT

This paper examines the changes in trading activity around stock splits, and its impact on both the volatility and the bid-ask spread. After a stock split, there is a significant increase in the volatility and the spread, even after controlling for the effects of microstructure biases like price discreteness and bid-ask bounce. The change in the number of trades is positively related to the change in total volatility, as well as to the temporary and permanent components of volatility. This suggests that the change in trading activity is associated with both informed and noise traders. The change in the number of trades is also negatively related to the change in the total spread, as well as the adverse information content of the spread. Firms that are successful in attracting a large number of additional trades to their stock experience a smaller increase in spreads. These results suggest that a crucial determinant of the liquidity changes experienced by a firm after a stock split is the success of the split in attracting new trades to the security.

Changes in Trading Activity Following Stock Splits and their Impact on Volatility and the Adverse Information Component of the Bid-Ask Spread.

1. Introduction

We examine the impact of trading activity and market microstructure on the volatility and the adverse information component of the bid-ask spread around stock splits. Previous studies of stock splits document an increase in both the volatility and the proportional bid-ask spread after a split. However, they do not provide a satisfactory explanation for changes in these characteristics. In this study, we first examine the relationship between changes in volatility and changes in trading activity. This analysis is motivated by Jones, Kaul and Lipson (1994), who find that volatility is primarily and positively related to the number of trades. We also examine the relationship between changes in trading activity and the components of the bid-ask spread.

Our analyses also provides insights about the managerial objective of enhancing the liquidity of the stock by splitting it. Splits per se do not alter either the cash flows of the firm or the claims of the security holders. Yet, in any given year, about 10% of the firms split their stock. Surveys of corporate managers by Baker and Gallagher (1980) and Baker and Powell (1993) reveal that the two most important reasons given by managers for undertaking a split are to bring the stock price into a better trading range and to improve its liquidity. Managers believe that the lower stock price makes it possible for wealth constrained "small" traders to purchase round lots. Baker and Powell argue that the managerial view of enhanced liquidity is this increase in the diversity and number of shareholders. Lamoureux and Poon (1987) and Maloney and Mulherin (1992) document an increase in the number of shareholders after the stock split, and their evidence is therefore consistent with the managerial motivations for stock splits.

On the other hand, studies have also found that after a split, there is also an increase in the proportional bid ask spread (Copeland (1979) and Conroy, Harris, and Benet (1990)), a decrease in the split-adjusted trading volume (Copeland and Lamoureux and Poon), an increase in brokerage fees (Copeland), and an increase in the volatility of the stock's returns (Ohlson and Penman (1985) and Dubofsky (1991)). Based on these measures, liquidity appears to decrease. Taken together, the evidence suggests that by splitting the stock, managers achieve the objective of increasing shareholder diversity but the split does not, on average, lead to an improvement in traditional measures of liquidity. However, these studies do not examine the implications of the change in shareholder diversity for trading activity, and the consequent changes in volatility and the bid-ask spread.

We investigate two potential explanations for the increase in volatility following a split. First, Ohlson and Penman (1985), Dravid (1988), and Dubofsky (1991) argue that part of the increase in volatility could be attributed to microstructure biases. Particularly, both bid-ask bounce and price discreteness induce an upward bias in volatility estimates based on transaction prices, and this bias is exacerbated after the split due to the lower share prices. We avoid the bias due to bid-ask bounce by using returns based on bid-bid prices (see Kaul and Nimalendran (1990)). The correction for price discreteness follows the model in Ball (1988). We document that while bid-ask bounce and price discreteness do inflate volatility estimates, the stock's volatility increases after the split even after we correct for these biases.

The second explanation for the increase in volatility is an increase in trading activity. Jones, Kaul, and Lipson (1994) find that volatility is primarily and positively related to the number of trades. Thus, if the number of trades increases after the split, then this would increase the volatility. There are several theoretical reasons why the number of trades might increase after a split. Black (1986) argues that noise traders prefer low priced stocks to high priced stocks. If they do, then the lower per share price after a split would attract noise traders and the resulting increase in the number of trades would increase the volatility. The increase in noise trading would be consistent with the managerial objective of increasing shareholder diversity. Brennan and Hughes (1991) argue that the lower per share price after the split might give analysts the incentive to collect more information on firms. They provide evidence that the number of analysts following a firm increases after the firm announces a stock split. This suggests the presence of a larger number of informed traders in the security after the split, and again, the resulting increase in the number of trades would lead to an increase in the volatility. It is also possible that the levels of both noise and informed traders increase after a split. In Admati and Pfleiderer's (1988) model of strategic trading with costly information acquisition, the number of informed traders is determined endogenously. A higher number of noise traders would result in a higher number of informed traders as well. Thus, if lower share prices after a split attract noise traders, the level of informed traders would increase endogenously.

Note that all three models above imply an increase in the number of trades but differ in their implications about the mix of traders (noise versus informed). Our analysis shows that the number of trades does increase after the split. Further, we find a significant positive relationship between the change in the number of trades and the change in the volatility. However, since we cannot directly observe the mix of traders, we have to rely on indirect methods to provide relative impacts of noise and information on volatility changes. We employ two approaches. The first approach uses multi-period volatility and variance ratios (the ratio of multi-period to single-period volatilities). In the second approach, we use insider trading intensity as a proxy for informed trading in a cross-sectional regression model. Our analysis of multi-day volatility and variance ratios indicates that a significant part of the increase in the volatility is in the component that is due to pricing errors, and that there is also an increase in the permanent component of the volatility. In the cross-sectional regression framework, we find that changes in the total volatility, as well as changes in the transient and permanent components of the volatility are positively related to changes in the number of trades. This relationship is consistent with an increase in both noise and informed trading after the split.

The models of Black (1986), Brennan and Hughes (1991), and Admati and Pfleiderer (1988) have implications for the adverse information component of the bid-ask spread as well. An increase in noise trading after a split (as in Black) will decrease the adverse information component, while an increase in informed trading (as in Brennan and Hughes) will increase this component of the spread. Admati and Pfleiderer argue that an increase in the number of noise traders would also endogenously increase the number of informed traders. The impact of this change in trading activity would depend on whether the signals received by the informed traders are close substitutes or complements.[1] When informed traders get identical signals, Admati and Pfleiderer show that competition among traders would decrease the adverse information in the

[1] Evidence to support the hypothesis that splits convey private information to the market is presented in Brennan and Copeland (1988) and in McNichols and Dravid (1990).

market. However, if informed traders get diverse signals, then the effect on the adverse information depends on the precision of the signal. If the signal is sufficiently imprecise, then adverse information would decrease due to competition between informed traders, even if the number of informed traders increases. Thus, by relating the change in the adverse information component of the spread to measures of trading activity, we can draw inferences about the impact of splits on liquidity. While we find that, on average, both the proportional spread and the adverse information component of the spread increase after the stock split, these changes are less pronounced the larger the increase in the number of trades. This negative relationship implies that spread changes are either driven by noise traders or by informed traders with substantially similar signals (or some combination thereof).

The evidence sheds light on several aspects puzzle regarding stock splits. First, we establish that the observed increase in volatility and spreads cannot be attributed solely to statistical problems (like price discreteness and bid-ask bounce). Consistent with the findings of Jones, Kaul and Lipson, we also establish a positive relationship between the change in volatility and the change in the number of trades. The fact that this relationship holds true for both the transitory and permanent components of volatility suggests that both informed and noise traders are contributing to the observed increase in the number of trades. Finally, we establish that while spreads (total, as well as the adverse information component) increase from before to after a split, firms that experience a substantial increase in the number of trades have a smaller increase in spreads relative to other firms. This suggests that, all else kept the same, attracting additional traders to the security enhances the liquidity of the stock. The puzzle that still remains, of course, is (i) what distinguishes firms in terms of their ability to attract additional traders to their stock, and (ii) why a split unconditionally worsens liquidity. These remain interesting areas for future research.

The rest of the paper is organized as follows. In section 2, we describe the data and the sample characteristics. In Section 3, we present our analysis of volatility changes after stock splits. Changes in the adverse information of the spread are discussed in Section 4. Our conclusions and a summary are presented in the last section.

2. Data and Sample Characteristics

2.1 Data

The initial sample consists of NASDAQ-NMS firms that announced stock splits and are listed in the CRSP 1990 data base. We confine our sample to NASDAQ-NMS firms because CRSP provides bid-ask spreads (inside quotes), daily trading volume, and number of trades presently only for these firms. We further restrict our sample to announcements of splits during the period January, 1983 to December, 1990. This restriction is imposed because transaction prices and bid-ask spreads for NMS securities are available in the CRSP data base on a regular basis only after November, 1982. There are 980 splits announced by 739 firms meeting the above screens.

Pre-split microstructure variables for the sample (such as volatility, spreads, trading intensity, etc.) are estimated over the 180 day period ending 21 days before the announcement of the split. The post-split characteristics are estimated over the 180 day period beginning from 21 trading days after the stock first trades ex-split. We exclude the period from 20 days before the announcement to 20 days after the stock trades ex-split to avoid any contamination due to information effects around the announcement day and the transient microstructure effects around the ex-split date. We require that all relevant data items be available during the pre-split and the post-split estimation periods. Finally, we exclude from our initial sample all observations that have either a stock split or a stock dividend within 400 days of each other. This screen ensures that our estimation period data are not contaminated by events similar to the ones examined in this study. The final sample consists of 366 stock splits announced by 341 firms.

In our sample of 366 split announcements, 147 are 3-for-2 splits (a split factor of 0.5) and 138 are 2-for-1 (a split factor of 1.0). Sixty three announcements involve a split factor of less than 3-for-2, while eighteen are splits greater than 2-for-1. Previous studies have suggested that there is a difference between the motives of firms issuing small versus large stock splits. Elgers and Murray (1985) document that small splits (with split factor < 25%) are associated with smaller pre-split prices and smaller market value firms than large splits. Further, they also suggest that small split factors may be motivated by a desire to signal optimistic expectations, while larger split factors are motivated by liquidity reasons. Baker and Powell (1992) find a significant difference between the preferred trading ranges for the small (< 2-for-1) versus large (≥ 2-for-1) splits. Since there appears to be differences between the motives for small versus large splits, we partition our sample into two groups: a small split factor group of 3-for-2 splits and smaller, and a large split factor group of 2-for-1 splits and greater.

2.2 Sample Characteristics

In Table 1A we report sample characteristics for some selected variables (market value, number of shares, price, and number of market makers) for the entire sample, and for the two sub-samples based on the split-factor. We find that there is a significant difference in the median pre-split market values of the equities of the firms in the two sub-samples. The median value of equity in the small split factor sub-sample is $85 million compared to $182 million for the large split factor sub-sample. This difference in firm size is driven by the higher pre-split share price for the large split factor sub-sample ($37.7) compared to the small split factor sample ($21.8), because the difference in the median pre-split outstanding shares is not statistically significant. We also find that there is no statistical difference in the number of market makers between the two groups in the pre- and post-split periods. Since we find significant differences in some firm characteristics between the two groups, we analyze the market microstructure variables also by groups.

[Insert Tables 1A and 1B here]

In Table 1B, we report statistics for microstructure variables and measures of trading activity. The table reports pre- and post-split means and medians for the proportional bid-ask spread, the daily volatility based on transaction prices, the average daily number of trades, the average volume turnover, the average volume turnover per trade, and the average number of insider trades. The statistics are also presented for the ratio of post-split to pre-split values for these variables. Our objective is to confirm for our sample what existing studies have documented. First, the median values of both the pre- and post-split proportional spreads are higher for the small split factor sub-sample than for the large split factor sub-sample. This is consistent with a smaller market value and lower pre-split share prices for the small split factor sub-sample compared to the large split factor sub-sample. Next, we find a significant increase in the total spread after the stock split for the total sample as well as for each sub-sample. This increase in the post-split spreads is consistent with the findings of Copeland (1979) and Conroy, Harris, and Benet (1990). The median ratio of the post-split to the pre-split proportional spread is 1.08 for the small split factor sub-sample, and 1.45 for the large split factor sub-sample. While both these values are significantly greater than 1.0, the median change in the spread is obviously much greater for the large split factor sub-sample.

We also document a significant increase in the post-split volatility relative to the pre-split volatility for the total sample and also for each sub-sample. For the total sample the median increase in volatility based on closing transaction prices is 70%. For the small split factor sub-sample, the median increase in volatility is 42%, while for the large split factor sub-sample, it is 119%. Ohlson and Penman (1985), who also use closing transaction prices for their volatility estimates, report an increase of 63% to 83% in the volatility after the split, and their results are comparable to ours.

In addition to spreads and volatility, we also examine changes in trading activity. We find a significant increase in the number of trades after the split for both groups. The median increase for the small split factor sub-sample is 12% compared to 37% for the large split factor sub-sample. However, the median volume turnover (defined as the ratio of the transaction volume to the number of outstanding shares) increases by only 7% for the small split factor group and decreases by 7% for the large split factor group, with the decrease for the latter group being statistically insignificant. The increase in number of trades is consistent with the findings by

Lamoureux and Poon (1987), and the effect on volume is consistent with Murray (1985) and Lakonishok and Lev (1987), who find that splits do not appear to exert a permanent effect on volume. More interestingly, we find that the turnover volume per trade is significantly smaller in the post-split period compared to the pre-split period for both groups. The median reduction in the order size is 18% for the small split factor stocks and 35% for the large split factor stocks. The smaller order size is consistent with an increase in noise traders who are likely to trade smaller quantities.

Finally, we report statistics on the average daily number of insider trades for our sample. Seyhun (1988) finds evidence consistent with the argument that corporate insiders generally trade on private information. This suggests that measures of trading activity by insiders can be used as a reasonable proxy for informed trading. However, insiders may also trade for liquidity reasons. But it is not obvious why their liquidity motivations for trading in their firms' stock would be altered during the 180 day intervals before the announcement and after the ex-day of the split. If insiders withhold such trading prior to the split in anticipation of the announcement, then such trades are likely to be executed soon after the split is announced. In our study, we measure the post-split insider trading activity beginning twenty days after the ex-split date. Thus, it is unlikely that the execution of any such backlog of trades will have a significant effect on our measures of informed trading activity by this group of investors.

For the sample of 366 firm events, we obtained insider transactions from the Securities and Exchange Commission's Ownership Reporting System (ORS) cumulative file (which covers the period 1980-1991). For this study, we classify all officers and directors and others who have substantial ownership in the firm as insiders. In addition, any transaction that was classified as an open market purchase/sale, private purchase/sale, and exercise of options was considered as valid transactions. The time period used for measuring insider trading activity in the pre-split and post-split is identical to that used for all other variables. In our sample, while the mean ratio of the post-split insider transactions to the corresponding pre-split value is significantly greater than 1.0, the median is not. This difference arises due to skewness of the data.

The observed changes in the microstructure variables are consistent with previous empirical findings. In the next section, we test the effect of microstructure biases and trading activity on changes in the daily volatility.

3. Changes in Volatility Following Stock Splits

3.1 Effects of Microstructure Biases on Volatility Estimates

Ohlson and Penman (1985), and Dravid (1988) suggest that the increased bid-ask spread and the larger effect of price discreteness on lower priced stocks may account for part of the increase in volatility that is observed after the split. Dubofsky (1991) finds that, for a sample of AMEX listed stocks, there is no statistically significant increase in the weekly return volatility after the split. Since volatility estimates based on weekly returns are less affected by bid-ask errors and price discreteness, Dubofsky argues that measurement errors created by the bid-ask errors and price discreteness could partially explain the increase in the return volatility of daily returns. In this section, we first test this microstructure explanation for the observed increase in the volatility using our sample of stock splits.

3.*1.a* *Bias Due to Bid-Ask Bounce*

Roll (1984) shows that in an efficient market, if the probability of the transaction price being at the bid or the ask is equally likely, then, using transactions prices to estimate the true volatility of the stock returns would induce spurious volatility equal to $s^2/2$, where s is the percentage bid-ask spread. Kaul and Nimalendran (1990) show that for a portfolio of small market value NASDAQ-NMS firms, this spurious volatility could be as high as 50% of the underlying true volatility. Even for the largest firms, this proportion could, on average, be as high as 23%. For our study, this bias could be particularly significant if the bid-ask spread increases after a split. However, given the availability of bid and ask prices for the NASDAQ-NMS firms, we can avoid the bid-ask spread bias by estimating volatilities based on returns computed using bid-bid prices.[2]

In Table 2 we report statistics for sample variances estimated using daily returns in the pre- and post-split periods. The estimate based on transaction price returns is denoted by σ^2_T, and that based on bid-to-bid returns is denoted by σ^2_B. Since the sampling distribution of the estimated variances are highly skewed and kurtotic, inferences based on the sample mean under the assumption that the underlying distribution is normal could be misleading. Therefore, we report only the sample medians before and after the split, and robust non-parametric test statistics for the differences in the sample medians (note that sample means also give the same results).

[Insert Table 2 here]

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[2] The CRSP data base gives closing bid and closing ask prices in addition to the closing transactions price. We construct return series based on bid prices by adjusting for dividends and distributions on ex-days.

For the total sample, the median ratio of the post-split volatility to the pre-split volatility, based on transaction price returns, is 1.70, indicating a 70% increase in the volatility. However, there is an 80% increase in the volatility based on bid-to-bid returns. Both of these increases are significant at the 1% level. Since the latter measure avoids the bias due to the bid-ask bounce, our results suggest that bid-ask bounce alone cannot explain the increase in volatility. An increase in the bid-ask bounce corrected volatility is also observed for the two sub-samples based on the split factor. For the small split factor sub-sample, this volatility increases by 41%, while for the large split factor sub-sample, the increase is 124%.

To estimate the bias in the transaction return volatility, we compare the median values of σ_T^2 with those of σ_B^2 for each sample, in both the pre- and post-split periods. For the total sample, the pre-split transaction return volatility is about 95% higher than the corresponding bid-bid return volatility, indicating a significantly large bias due to the bid-ask bounce. Likewise, in the post-split period, the bias due to the bid-ask bounce is about 84%. These biases are also observed for each sub-sample. For the small split factor sub-sample, the biases are 90% and 89% in the two periods respectively, while those for the large split factor sub-sample are 91% and 71% in the pre- and post-split periods respectively.

Since previous studies have also documented an increase in the transaction return volatility after stock splits, it would be informative to estimate how much of this increase is due to the bid-ask bounce. Note that the increase in the spread after the split will exacerbate the bias due to the bid-ask bounce. For the total sample, the median σ_T^2 increases by 3.17×10^{-4}. On the other hand, the bid-bid return volatility increases by 1.91×10^{-4}, which is about 60% of the increase in σ_T^2. Therefore, about 40% of the increase in the transaction return volatility can be attributed to the bid-ask bounce. Similarly, about 47% and 36% of the increase in σ_T^2 for the small and large split factor sub-samples respectively can be attributed to the bid-ask bounce.

3.1.b Bias Due to Price Discreteness

Gottlieb and Kalay (1985) and Ball (1988) examine the effect of price discreteness on the inflation in the volatility estimates. Ball shows that if stock prices follow a Geometric Brownian motion with an instantaneous true underlying variance σ^2, and price P, then the bias induced by price discreteness can be approximated by $d^2/6P^2$, where d is equal to the minimum price change (typically, \$0.125). We apply this correction to the volatility measure to obtain an unbiased estimator. The estimator, $\hat{\sigma}^2_{B,D}$, is computed using Equation A-1 in Appendix A, and the results are presented in the last row of Table 2.

The median ratio of the post-split volatility corrected for both the bid-ask bounce and price discreteness is 1.81 in the total sample, and this estimate is significantly greater than 1.0 at the 1% level. Thus, even after correcting for price discreteness, there is a significant increase in the volatility after the split. A significant increase in the bias corrected volatility is also observed for each of the two sub-samples, although we observe a much larger increase for the firms with a large split factor (118% versus 42%).

To estimate the bias due to price discreteness, we compare the volatility corrected for both the bid-ask bounce and price discreteness ($\hat{\sigma}^2_{B,D}$) with the volatility corrected for the bid-ask bounce alone (σ^2_B). For the total sample, price discreteness inflates this volatility estimate by 4.7% in the pre-split period, and by 3.9% in the post-split period. This bias is relatively small compared to the bid-ask bias, and further, it has a negligible effect on the change in volatility. For the small split factor sub-sample, the biases in the pre- and post-split periods are 5.4% and 5.7% respectively, while those for the large split factor sub-sample are 0.4% and 7.8% in the two periods.

The preceding analysis suggests that the bid-ask bounce introduces a substantial bias in estimates of volatility based on transaction prices, while the bias due to price discreteness is negligible. More importantly, even after correcting for these biases, we find a significant increase in the volatility after the split. Thus, microstructure biases alone cannot account for the previously documented increase in the volatility after stock splits. Further, firms that execute large splits experience a much larger increase in the bias-corrected volatility than do firms that execute small stock splits.

3.2 Effect of Changes in Trading Activity on Changes in Volatility

As argued earlier, an increase in either noise traders or informed traders (or both) would lead to an increase in the volatility of the stock. To examine the effect of the change in the trader mix after the split on the change in the volatility estimates, we need to estimate changes in trader types after stock splits. Since it is not possible to directly identify the type of traders, we use two approaches. First, we analyze stock return dynamics and market microstructure variables to infer the effect of changes in the types of traders. This allows us to decompose the change in the volatility into changes in the permanent (information driven) component and the transient (noise driven) component. In the second approach, we relate changes in volatility (and its components) to changes in trading activity using proxy variables for noise and informed trading.

3.2.a Changes in Permanent Volatility

French and Roll (1986) argue that, if the effects due to noise trading (i.e. pricing errors) are subsequently corrected, then the volatility based on longer period returns would reflect the permanent component. Thus we estimate the volatility based on multi-day returns. Due to the limited number of observations in each estimation period, we use overlapping data and the estimator in Lo and MacKinlay (1988). This estimator is given by Equation A-2 in Appendix A, and corrects for the effects of both the bid-ask bounce and price-discreteness on volatility.

In Table 3, we report these multi-period volatility estimates for cumulating intervals up to 30 days for the total sample and for sub-samples based on the split factor. For the total sample, we find that the median 30-day return volatility increases by 57% after the split, and the increase is statistically significant at the 1% level. This indicates that a significant component of the increase in volatility is permanent. The one day return volatility however, increases by 81% for this sample (see Table 2). The larger increase in the one-day volatility relative to the increase in the 30-day volatility suggests that there is also a large component of the increase in volatility that is transient and attributable to noise. Similar results are obtained for the sub-samples based on the split factor. However, for the small split factor group, the increase in the permanent component of the volatility (based on 30-day returns) is only 32% compared to 81% for the large split factor group.

[Insert Table 3 here]

These results indicate that for our sample, there is a substantial increase in volatility that is permanent, in addition to a significant increase that is transient. Further, our finding that there is a significant increase in the multi-day volatility are in contrast to those of Dubofsky (1991). He finds that, for a sample of firms listed on AMEX that executed stock splits, there is no significant change in the volatility based on weekly returns.

3.2.b *Changes in Volatility due to Noise Trading*

An alternative metric to determine the relative contribution of noise trading to the total volatility of a security's returns is the variance ratio, defined as the ratio of the variance based on k-period returns to k times the variance based on one-period returns. The presence of noise trading would induce negative autocorrelation in the returns, thereby reducing the variance based on multi-period returns. The variance based on one period returns would be unaffected by this negative autocorrelation if it takes more than one period for the mis-pricing to be corrected. Consequently, French and Roll (1986) argue that one minus the variance ratio reflects the fraction of the one period volatility that can be attributed to noise.

Lo and MacKinlay (1988) show that the variance ratio can be written as a weighted sum of the autocorrelations:

$$VR(k) = 1 + \sum_{j=1}^{k-1} \frac{2(k-j)}{k} \hat{\rho}_j \qquad (1)$$

where $\hat{\rho}_j$ denotes the estimate of the j^{th} order autocorrelation of daily returns. If all the autocorrelations are due only to mis-pricing errors that are subsequently corrected, then, as suggested by French and Roll, (1-VR) would estimate the effect of noise trading on volatility. However, Kaul and Nimalendran (1990) document that for NASDAQ-NMS firms, the average autocorrelation of returns based on bid-bid prices at lag one is 0.15, and this is much larger than the negative autocorrelations at higher lags.

In Table 4 we report the median autocorrelations up to lag 10 for our sample. We find positive autocorrelations at lags one and two that are similar to the numbers reported by previous researchers. These positive autocorrelations at low lags lead to variance ratios that are greater than one, and gives infeasible estimates for the effects of noise trading. To mitigate the effects due to large positive autocorrelations at short lags, we define one period of time as being three days. Our long run measure of variance is computed over 30 days, i.e. 10 periods. Estimates of the variance ratios based on these returns provide feasible estimates for the effects of noise.

[Insert Table 4 here]

Sample statistics for the variance ratios are reported in Table 5. These variance ratios are computed using the estimator given by Equation A-3 in Appendix A. The median differences in the ratios (computed as the median of the matched difference in the post-split and pre-split variance ratios) are significantly negative for the entire sample and also for the two sub-samples based on the split factor. These ratios suggest that the fraction of volatility that can be attributed to noise trading is higher after the split relative to the pre-split level for all three samples. Further, since the total one-day volatility is also increasing for these groups, it suggests that the volatility due to noise trading after the split is substantially higher relative to the pre-split level.

[Insert Table 5 here]

These estimates of the variance ratios, along with the bias-corrected estimates of the pre- and post-split one-day volatilities (reported in Table 2) can be used to estimate the fraction of the increase in the volatility that is due to an increase in noise trading. Since both the variance ratio estimates and the volatility estimates have been purged of microstructure biases, we can assume that any remaining mis-pricing errors are caused by noise traders. For the total sample, the median variance ratio is 0.94 before the split, and, from Table 2, the median pre-split one-day volatility is $2.75\text{x}10^{-4}$. Thus, the volatility due to noise is $0.165\text{x}10^{-4}$ before the split. Similarly, after the split, the median variance ratio is 0.79, the total volatility is $4.61\text{x}10^{-4}$, and the volatility due to noise is $0.968\text{x}10^{-4}$. Thus, about 43% of the increase in the total volatility of $1.86\text{x}10^{-4}$ is due to the increase in noise volatility, with the remaining being attributable to an increase in the permanent component. Similar results are obtained for the sub-samples based on the split factor. For the small split factor sub-sample, about 41% of the increase in the total volatility is due to noise, and for the large split factor sub-sample, approximately about 40% of the increase in the total volatility can be attributed as such.

The above results suggest that there is a significant increase in *both* the noise and the permanent components of the volatility after the split. Further, as indicated in Table 1B, the total number of trades increases, on average, after the split. Note that the total number of trades consists of trades executed by both noise and informed traders. Thus, an increase in both components of the volatility, coupled with an increase in the total number of trades, is consistent with the argument that after a stock split, there is an increase in the level of both noise and informed trading. In the next section, we investigate this further using a cross-sectional regression framework.

3.2.c Cross-sectional Analysis of Volatility Changes

Trading activity can be measured by either the number of transactions or the size of the trade (i.e. the turnover volume). Earlier studies have documented a positive relationship between volatility and trading volume (see Karpoff (1987) for a review). However, Jones, Kaul, and Lipson (1994) conclude that it is the number of transactions per se, and not their size, that generates volatility. That is, the effect of trade size is subsumed in the number of transactions. Given their conclusions, we use changes in the number of trades as our measure of changes in trading activity.[3]

If we assume that insider transactions are information driven, then we can use the number of trades executed by insiders as a proxy for informed trading. The change in the number of insider transactions after the split (from the pre-split values) would then proxy for the change in informed trading activity. By netting out the insider trades from the total number of trades, we can use the change in the net number of trades as a proxy for the change in noise trading.[4]

Using these proxy variables, we investigate the relationship between changes in trading

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[3] Jones, Kaul, and Lipson include both the number of trades and volume in their cross-sectional analysis of volatility. In our study, we are interested in the change in trading activity. We find that in our sample, the <u>change</u> in the number of trades is highly correlated with the <u>change</u> in turnover volume: the correlation coefficient between these two variables is 0.79. Thus, including both measures of changes in trading activity results in the usual problems associated with multi-collinearity in the independent variables.

[4] However, if insider transactions are motivated by liquidity concerns, then the change in trades would not allow us to estimate the change in informed trading. Moreover, since the number of insider trades is a small fraction of the total number of trades (see Table 1B), the effect of their trades on volatility would be subsumed in our proxy for noise trading.

activity and changes in the bias-corrected total volatility, the change in the long term volatility, and the noise component of the volatility. In our first model, the dependent variable is the change in the bias-corrected total volatility of the stock. Specifically, the model is:

$$\ln\left(\frac{\sigma^2_{B,D,2}}{\sigma^2_{B,D,1}}\right)_i = \alpha_0 + \alpha_1 \ln\left(\frac{NNT_2}{NNT_1}\right)_i + \alpha_2 \ln\left(\frac{NIT_2}{NIT_1}\right)_i + \alpha_3\left(1+SFAC_i\right) + \alpha_4 \ln\left(MVAL_i\right) + \varepsilon_i \tag{2}$$

where $\sigma^2_{B,D,j}$ is the bias corrected volatility in period j (j equals 1 for the pre-split period and 2 for the post split period), NNT_j is the number of trades in period j, net of insider trades, NIT_j is the number of insider trades in period j, SFAC is the announced split factor, and MVAL is the market value of the splitting firm's equity, measured two days before the announcement of the split.

The data presented in Table 2 indicates that the change in the bias-corrected volatility is much higher for the sub-sample with the large split factors (SFAC $\geq$ 1.0). To the extent that the magnitude of the split factor conveys information to the market (see McNichols and Dravid (1990)), this would affect the volatility as well, through its effect on the permanent component of the volatility. To control for the effect of this signal on volatility changes, we include the split factor as an additional independent variable in our regressions. Finally, inclusion of MVAL allows us to control for omitted variables which are correlated with the size of the firm.

The estimates of the model parameters in Equation (2) are presented in the first row of Table 6. Our estimate of the coefficient on the change in the net number of trades (α_1) is significantly positive. Thus, an increase in the number of noise transactions increases the volatility of the stock after the split. The coefficient on our measure of informed trading (α_2) is insignificantly different from zero at all conventional levels. There are two possible explanations for this observation. First, this coefficient would be insignificant if there is no significant change in the number of insider transactions after the split. The statistics in Table 1B indicate that the median change is indeed zero. Alternatively, if insider transactions are a poor proxy for informed trading, and in fact are motivated by liquidity concerns, then their effect will be subsumed in the measure of the change in net trades.

[Insert Table 6 here]

The coefficient on the split factor is also significantly positive. Firms which employ a larger split factor experience a greater is the change in the total volatility. While this finding is consistent with the statistics presented in Table 2, additional insights about the role of the size of the split factor can be obtained by examining the change in the components of the volatility. If splits convey private information to the market, as suggested by the signaling hypothesis of splits, then the change in the permanent component of the volatility would be positively related to the magnitude of the split factor, and the change in the noise component of the volatility would be unrelated to the split factor. We further investigate this below.

Finally, the coefficient on MVAL is significantly positive. If large firms have low volatility to begin with, even a small increase in the volatility would result in a large percentage change. This would manifest in a positive relationship between firm size and volatility changes.[5,6]

[5] When we partition our sample into two sub-samples based on the pre-split firm size (MVAL), we find that the mean pre-split bias-corrected volatilities for the small and large firm sub-samples are 0.000461 and 0.000342 respectively. The mean ratios of the post-split to pre-split volatility for these two sub-samples are 2.35

In our second model, the dependent variable is the change in the long term volatility of the stock. Specifically, we estimate the following model:

$$\ln\left(\frac{\Sigma_2^2}{\Sigma_1^2}\right)_i = \alpha_0 + \alpha_1 \ln\left(\frac{NNT_2}{NNT_1}\right)_i + \alpha_2 \ln\left(\frac{NIT_2}{NIT_1}\right)_i + \alpha_3\left(1 + SFAC_i\right) + \alpha_4 \ln\left(MVAL_i\right) + \varepsilon_i \qquad (3)$$

where Σ_j^2 is the volatility based on 30-day returns in period j (j equals 1 for the pre-split period and 2 for the post split period) and the other variables are as defined earlier. Estimates of the parameters of this model are presented in the second row of Table 6. There is a significant positive relationship between the change in the long term volatility and the change in the net number of trades. The change in the long term volatility is again unrelated to the change in insider trades. A possible reason for this is that the change in insider trades is a poor proxy for informed trading.

Further, the coefficient on the split factor (α_3) is significant and positive. Moreover, the estimate of this coefficient (and the associated t-statistic) is greater than that in Equation 2. This is consistent with the hypothesis that splits are signals of private information about the firm. The effect of this release of information on the total volatility is primarily driven by its effect on the permanent component of the volatility. Further evidence to support this argument is provided in the model relating the change in the noise component of the volatility to trading activity.

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and 3.43 respectively.

6 We also estimate the model in Equation (2), as well as all subsequent cross-sectional regression models, without the term involving our measure of firm size (MVAL). In all cases, our parameter estimates are virtually identical to those obtained with the inclusion of the MVAL term as an independent variable. Thus, our conclusions do not depend on the inclusion or omission of this variable, and we report the results for the estimates obtained by inclusion of this term.

The last row of Table 6 reports our estimates of the following model:

$$\ln\left(\frac{\sigma^2_{B,D,2}}{\sigma^2_{B,D,1}}\right)_i = \alpha_0 + \alpha_1 \ln\left(\frac{NNT_2}{NNT_1}\right)_i + \alpha_2 \ln\left(\frac{NIT_2}{NIT_1}\right)_i + \alpha_3 (1 + SFAC_i) + \alpha_4 \ln(MVAL_i) + \alpha_5 \ln\left(\frac{\Sigma^2_2}{\Sigma^2_1}\right)_i + \varepsilon_i \tag{4}$$

In this model, since we control for the change in the permanent component of the volatility on the right hand side, the dependent variable then measures the change in the noise component of the volatility. Once again, the estimate of α_1 is significantly positive, while the estimate of α_2 is insignificantly differenc from zero. Thus, the change in the noise component of the volatility is significantly positively related to the change in the net number of trades, but not to the change in insider trades. Interestingly, the coefficient on the split factor variable (α_3) is also insignificant and provides supporting evidence for the argument presented earlier, that splits convey private information to the market. Since the noise component of the volatility would be unrelated to private information released through the split, the change in this component of the volatility would be unrelated to the size of the announced split factor.

Taken together, our results suggest the following. Stock splits, on average, result in an increase in trading activity, and this in turn leads to an increase in the volatility even after we control for microstructure biases in the estimated volatility. There is an increase in both the permanent as well as the transient component of the volatility, and the increase in trading activity positively affects both of these components. These results allow us to extend the conclusions drawn by Jones, Kaul, and Lipson (1994) about the relationship between the level of the volatility and the level of the number of transactions. Changes in these variables are also positively correlated in the case of stock splits. Finally, the change in the permanent component of volatility is positively related to the split factor, but the change in the noise component is not. This is consistent with the signaling hypothesis for stock splits.

4. Changes in Bid-Ask Spreads Following Stock Splits

In this section, we investigate changes in bid-ask spreads following stock splits. The descriptive statistics presented in Table 1B indicate that for the total sample, the proportional spread increases after the split by an average of 32% and that this increase is even higher for firms that employ a large split factor. We examine two possible reasons for this increase. First, an upward bias in post-split absolute spreads, caused by price discreteness, could result in an increase in the post-split proportional spread. Discreteness in stock prices leads to minimum absolute spreads of 12.5 cents, and increments in the spreads are also forced to be in steps of 12.5 cents.[7] Second, the increase in the proportional spread could be due to an increase in one or more components of the bid-ask spread. Specifically, an increase in the information asymmetry in the market would lead to an increase in the adverse information component of the spread, thereby leading to an increase in both the absolute spread as well as the proportional spread.

4.1 Effect of Price Discreteness on Spreads

Suppose a stock has a pre-split share price of $50 and a pre-split bid-ask spread of $1.375

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7 Unlike the NYSE, NASD does not have a minimum spread policy. However, NASDAQ is designed to process spreads of 1/32nd for stocks priced under $10 and 1/8th for stocks priced above $10. On the consolidated tapes (CTA), trades in NASDAQ stocks under $10 are rounded to 1/16th (Source: Market 2000 Study by the SEC).

(i.e. a proportional spread of 2.75%). After a 2:1 stock split, the share price would drop to $25. In order to keep the proportional spread constant, the absolute spread after the split would have to be $0.6875. If the spread is constrained to be in multiple of eighths, the post-split spread would be adjusted upwards to the nearest eighths. This results in a 'target' post-split absolute spread of $0.75. If the post-split spread is indeed $0.75, the proportional spread would be $0.75/$25 or 3% after the split, indicating a 9.1% increase in the proportional spread. In this case, all of the increase in the proportional spread can be attributed to price discreteness. If the post-split spread is set at $0.875 (i.e. a proportional spread of 3.5%), the observed increase in the proportional spread would be 27.27%. If we control for price discreteness, the spread increases from the target spread of $0.75 to the actual spread of $0.875, representing a 16.67% increase.

The preceding example illustrates how discreteness in absolute spreads can inflate the observed increase in proportional spreads. We employ the adjustment procedure illustrated in this example to our sample of stock splits. Specifically, let ABS_j represent the absolute spread in period j (j = 1 for the pre-split period and j=2 for the post split period). Let ΔS represent the ratio of ABS_2 to $[ABS_1/(1 + SFAC)]$, where SFAC is the announced split factor. Thus, ΔS measures the uncorrected change in the spread. The target post-split spread, TS_2, is computed as $[ABS_1/(1 + SFAC)]$ rounded up (if necessary) to the next highest multiple of $0.125. Then the ratio $[ABS_2/TS_2]$, denoted by ΔS^* represents the change in the absolute spread after correcting for price discreteness. If price discreteness is the only reason for the observed increase in spreads, then ΔS^* should equal 1.0. On the other hand, if ΔS^* is greater than 1, then price discreteness alone cannot explain the observed increase in spreads.

To estimate ΔS^*, we use the median absolute spread in the pre and post-split periods for each stock. In Table 7, we report the cross-sectional mean and median values of ΔS and ΔS^* for the total sample as well as for the sub-samples grouped by the split factor. For the total sample, we find that the median increase in the absolute spread is a significant 50% before correcting for price discreteness, and 33% after the correction. Thus, price discreteness alone cannot account for the observed increase in spreads following stock splits. The statistics for the sub-samples based on the split factor lead to similar conclusions for each sub-sample. We also note that the increase in spreads is greater for the large split factor sub-sample, both before and after correcting for price discreteness.

[Insert Table 7 here]

4.2 Changes in the Adverse Selection Component of the Spread

Since price discreteness alone cannot account for the observed increase in spreads, we focus on the components of the proportional spread to gain insights about the reasons for the observed increase. In particular, we decompose the proportional spread into its order processing and adverse information components, using the methodology in George, Kaul, and Nimalendran (1991). The total spread in general consists of three components: order processing, adverse information, and inventory cost. In George, Kaul, and Nimalendran, the part of the inventory cost component that decays within a day across a number of transactions is included in the order processing component, while that which does not decay within a day is included in the adverse selection component [see Jegadeesh and Subrahmanyam (1993)]. However, Stoll (1989) has found that the inventory cost component is a small fraction of the total spread (less than 10%). Madhavan and Smidt (1991) also find that inventory effects are economically and statistically insignificant. Given these results, we focus only on the order processing and adverse information components in our study.

The methodology in George, Kaul, and Nimalendran depends on using the difference in returns based on transaction prices and returns based on bid-to-bid prices to purge the bias due to changing expected returns and partial adjustment. In addition, by taking the difference between the two returns, the effects due to the unanticipated component of returns (which are a large fraction of the error) are eliminated. This substantially increases the efficiency of the estimates.

Let $R^T_{i,t}$ and $R^B_{i,t}$ represent the returns based on the closing transaction price and the closing bid price of firm i at time t respectively. Define $R^D_{i,t} = R^T_{i,t} - R^B_{i,t}$ as the difference in these returns. George, Kaul, and Nimalendran show that if S_i is the quoted spread, and π_i is the fraction of the quoted spread due to order processing costs (and 1- π_i is the fraction due to adverse selection costs, then

$$C_i = 2\sqrt{-[\text{Cov}(R^D_{i,t}, R^D_{i,t-1})]} = \pi_i S_i \qquad \textbf{(5)}$$

If we assume that for a group of stocks the fraction of the quoted spread that is due to order processing costs (π_i) is constant and equal to π, then we can use the following cross-sectional model to estimate π:

$$C_i = \pi_0 + \pi S_i + \varepsilon_i,\ i = 1, \ldots, N \qquad \textbf{(6)}$$

We first estimate Equation (6) separately for the pre-split and the post-split periods for the total sample, as well as for the sub-samples based on the split factor. In this estimation, S_i is the average proportional spread. For each sample, this procedure provides us with estimates of π in the pre-split and the post-split periods. These estimates of π are then used to compute the order processing costs as a percentage of the share price ($= \pi S_i$) and the adverse information costs ($= (1 - \pi)S_i$).

Table 8 reports the average values of the proportional spread, as well as of the order processing and adverse information components of the spread. These values are reported for the pre- and the post-split periods, and for the total sample as well as for the sub-samples based on the magnitude of the split factor. The last column in Table 8 reports the difference between the pre- and post-split values.

[Insert Table 8 here]

In the total sample, the proportional spread increase by 0.513, and this increase is roughly evenly divided into the increases in the order processing and the adverse information component of the spread. For the small split factor sub-sample, virtually all of the increase in the proportional spread is due to the increase in the adverse information component, with the order processing component remaining statistically unchanged from the pre-split level. By contrast, there is a relatively large increase in the proportional spread, and about 69% of this increase is due to an increase in the order processing component.

The increase in both the total proportional spread and the adverse information component of the spread suggest that unconditionally, liquidity worsens after a stock split. Moreover, the increase in the adverse information component also suggests an increase in the information asymmetry in the market. This is consistent with an increase in informed trading activity (as in Brennan and Hughes (1991)). However, Admati and Pfleiderer (1988) argue that an increase in noise trading would endogenously increase the number of informed traders, and the net impact of this change in trading activity would depend on whether the signals received by the informed traders are close substitutes or complements. If the signals received are diverse and sufficiently precise, information asymmetry would increase even if the number of noise traders in the market increases. Thus, a net increase in the adverse information component is not inconsistent with an increase in the level of noise trading after the split. We examine below the relationship between changes in trading activity and changes in both the total proportional spread and the adverse information component.

4.3 *Effect of Changes in Trading Activity on Changes in Spreads*

To examine the relationship between changes in the proportional spread and changes in trading activity, we estimate the following cross-sectional model using ordinary least squares regression:

$$\ln\left(\frac{S_2}{S_1}\right)_i = \alpha_0 + \alpha_1 \ln\left(\frac{NNT_2}{NNT_1}\right)_i + \alpha_2 \ln\left(\frac{NIT_2}{NIT_1}\right)_i + \alpha_3(1 + SFAC_i) + \alpha_4 \ln(MVAL_i) + \varepsilon_i \qquad (7)$$

where S_j is the proportional spread in period j (j = 1 for the pre-split period and 2 for the post-split period) and the other variables are as defined earlier. Estimates of the parameters of this model are presented in Table 9. In the first row of Table 9, we present our estimates using both measures of changes in trading activity. In the second and third rows, we use the change in the net number of trades and the change in insider trades as the measure of change in trading activity, respectively.

[Insert Table 9 here]

The results presented in Table 9 indicate that changes in proportional spreads are inversely related to changes in the net number of trades, regardless of whether we control for the change in insider trades. Our estimates of α_1 are significantly less than zero in the first two rows of Table 9. Thus, while unconditionally, the proportional spread increases after the split, this effect is less pronounced the larger the increase in the net number of trades.

Curiously, the change in the proportional spread is unrelated to the change in insider trades when the change in net trades is also used as an explanatory variable, but inversely related to it when the change in insider trades is the only measure of the change in trading activity. The estimate of α_2 is insignificant in the first row of Table 9, but significantly less than zero in the last row. This suggests that the effect of the change in insider trades on proportional spreads is subsumed in the effect of the change in non-insider trades. There are two possible reasons why this might be the case. First, insider trades are relatively small in number when compared to non-insider trades (see Table 1B). Second, changes in insider trades may be motivated by liquidity reasons rather than being information driven. If so, an increase in insider trades after the split would tend to attenuate the increase in spreads. This inverse relationship is observed in the negative estimate of α_2 in the last row of Table 9. If changes in insider trades were information driven, then we would expect this estimate to be positive. We investigate this further by examining the relationship between changes in the adverse information component of the spread and our measures of changes in trading activity.

Finally, we note that the estimates of the coefficients on the split factor, as well as those on the firm size variable are significantly positive in all three versions of Equation (7) in Table 9. Firms that employ a larger split factor experience a greater increase in the proportional spread, as do large firms.

Changes in trading activity would also have an impact on the adverse selection component of the spread. An increase in noise trading would reduce this component of the spread, while an increase in informed trading would tend to increase this component. To examine the relationship between the change in the adverse information component of the spread and our measures of trading activity, we estimate the following model:

$$\ln\left(\frac{S_2}{S_1}\right)_i = \alpha_0 + \beta_0 \ln\left(\frac{COV_2}{COV_1}\right)_i + \alpha_1 \ln\left(\frac{NNT_2}{NNT_1}\right)_i + \alpha_2 \ln\left(\frac{NIT_2}{NIT_1}\right)_i + \alpha_3(1 + SFAC_i) + \alpha_4 \ln(MVAL_i) + \varepsilon_i \tag{8}$$

where COV_j is defined as the auto-covariance of $R^D_{i,t}$ in period j, and the other variables are as defined earlier. From Equation (6), this allows us to control for the change in the order processing component of the spread. Since the dependent variable in Equation (8) is the change in the total proportional spread, and we control for the change in the order processing costs on the right hand side, Equation (8) allows us to estimate the change in the adverse information component of the spread.

Table 10 reports our estimates of the parameters of Equation (8). As in Table 9, we estimate three versions of the model. The change in the adverse information component of the spread is inversely related to the change in the net number of trades (the estimate of α_1 is significantly negative in the first two rows of Table 9). This is consistent with an increase in noise trading after the split. Interestingly, all of our estimates of the coefficient on the change in insider trades (α_I) are significantly negative. If changes in insider trades are motivated by information, a large increase in these trades would result in a proportionately large increase in the adverse information component of the spread. Consequently, we would expect these coefficients to be positive. Our results suggest that this is not the case.

[Insert Table 10 here]

Taken together, our analysis of the changes in spreads following stock splits suggest the following. The observed increase in the proportional spread is not solely due to price discreteness. Rather, we observe an increase in both the order processing as and the adverse information component of the spread. While this indicates that liquidity worsens after the split, the increase in spreads is inversely related to changes in trades. Since we find that on average, the number of trades increases after the split, the decline in liquidity is less pronounced the larger the increase in the number of trades.

5. Conclusion

This paper has examined the impact of the change in trading activity surrounding a stock split on the liquidity of the stock. The results establish that the increase in volatility and spreads cannot be attributed solely to microstructure biases like price discreteness and bid-ask bounce. There is an increase in the number of trades around the split, and the larger the increase in the number of trades, the larger is the change in volatility and the smaller is the change in spreads (total, as well as adverse information component). Since the change in the number of trades affects both the transient and permanent components of volatility, the increased trading activity cannot be attributed solely to either an increase in informed trading or an increase in noise trading. The finding that increased trades lead to lower spreads is consistent either with these trades being predominantly noise motivated, or with the informed traders having very similar information (so that competition between them reduces spreads).

These findings suggest that any analysis of the impact of stock splits on traditional measures of liquidity (like volatility and spreads) must first examine why different firms seem to be more or less successful in attracting additional trades to their security. The subsequent consequences for liquidity then seem to be consistent with existing theories on the way in which a change in trading activity affects liquidity. An analysis of these causes for a change in the number of trades represents an interesting direction for future research.

APPENDIX A

A.1 Estimator of the volatility corrected for price discreteness

Ball (1988) shows that if stock prices follow a Geometric Brownian motion with an instantaneous true underlying variance σ^2 and price P, then the bias induced by price discreteness can be approximated as $d^2/6P^2$, where d is equal to 1/8. This approximation is valid for values of $d/\sigma P$ less than 2.50 (see Ball (1988), Table III). To correct for this bias, we need to estimate $1/P^2$. Since there is a price trend in the pre-split period, we use the average of $1/P_B^2$ from the estimation periods instead of one over the average of price squared. From Jensen's inequality, since $E(1/P^2) > 1/E(P^2)$, the estimated bias would bias it towards an upper value.

For the sample of firms in this study, the average value of $d/\sigma P$ is 0.34 and 99.9% of the estimates are less than 2.50 (based on an estimate of s using bid-to-bid prices and an estimate of P using the average bid price in the estimation period). Hence, Ball's approximation should be valid for our sample. We correct for the bias due to price discreteness by deflating the volatility estimate using bid-to-bid prices as follows:

$$\sigma_{B,D}^2 = \hat{\sigma}_B^2 - \frac{d^2}{6(T_2 - T_1)} \sum_{t=T_1}^{t=T_2} \frac{1}{P_{B,t}^2} \qquad \textbf{(A.1)}$$

where $\sigma_{B,D}^2$ is the volatility corrected for price discreteness, $\hat{\sigma}_B^2$ is the volatility estimated using bid-to-bid prices, $(T_2 - T_1)$ is the range of the estimation period, and $P_{B,t}$ is the bid price at time t ($T_1 \leq t \leq T_2$).

B.2 Estimator of multi-period volatility

We use the following estimator of the k-period volatility, based on Lo and MacKinlay (1988):

$$\hat{\sigma}_c^2(k) = \frac{1}{m\left(1-\frac{k}{n}\right)} \sum_{t=1}^{n-k+1} \left(R_{B,t}^k - \hat{\mu}_k\right)^2 - \frac{d^2}{6T}\sum_{t=1}^{T}\frac{1}{P_{B,t}^2} \qquad \text{(A.2)}$$

where $\hat{\sigma}_c^2(k)$ = estimate of k-period volatility based on bid-to-bid prices and with correction for price discreteness (the second term corrects for discreteness),

m = actual number of overlapping k-period observations,

n = number of one-period (daily) observations,

R_B^k = k-period return using overlapping one period returns based on bid-bid prices,

μ^k = the sample mean of overlapping k-period returns,

T = number of daily observations (= 180), and

$P_{B,t}$ = bid price on day t.

B.3 Estimator of the variance ratio

The variance ratio is defined as the ratio of the k-period volatility to k times the one-period volatility. The volatility estimates are corrected for the biases due to the bid-ask bounce and price discreteness. For each firm i, the variance ratio is given by:

$$VR(k) = \frac{Var(R_B^k) - \frac{d^2}{6P^2}}{k * \left[Var(R_B^1) - \frac{d^2}{6P^2} \right]} + \frac{2}{T-1} \sum_{j=1}^{k-1} \left(\frac{k-j}{k} \right) \qquad \textbf{(A.3)}$$

where $Var(R_B^k)$ is the k-period variance based on bid-to-bid returns. The quantity $[d^2/6P^2]$ in the RHS of the above equation is the adjustment for the bias die to price discreteness. The final term in the RHS of above corrects for the small sample bias in the expected value of the autocorrelation. Even if the returns are uncorrelated, the expected value of the autocorrelation is biased by -1/(T-1) [See Kendall and Stuart (1977)].

TABLE 1A

Sample Characteristics for 366 Stock Split Announcements made by NASDAQ-NMS Firms between January 1983 and December 1990, for the Total Sample, and for Sub-samples Classified by the Split Factor.‡

Variable	Total Sample		Small Split Factor Sub-Sample (SFAC ≤ 0.5)		Large Split Factor Sub-Sample (SFAC ≥ 1.0)		p-value⁺⁺
	Mean	Median	Mean	Median	Mean	Median	
N	366		210		156		
SFAC	0.72	0.50	0.41	0.50	1.14	1.00	
MVAL	259	123	176	85	371	182	< 0.001
NSHR	7.36	4.38	6.52	4.10	8.50	4.93	0.110
P_1^{AV}	25.80	22.80	19.65	17.51	34.14	31.61	< 0.001
P_1^B	30.50	27.50	23.30	21.80	40.30	37.70	< 0.001
P_2^{AV}	17.70	16.50	16.50	15.50	19.30	18.0	0.001
P_2^B	18.20	17.70	16.90	15.90	19.90	19.0	< 0.001
$NMMK_1$	7.80	6.00	7.30	6.00	8.40	7.0	0.12
$NMMK_2$	7.00	6.00	6.70	6.00	7.50	6.0	0.32

‡ N is the number of observations in each sub-sample; SFAC is the announced split factor; MVAL is the market value of equity (in $ millions), measured two days before the announcement of the split; NSHR is the number of outstanding shares, in millions, as of two days before the announcement of the split; P_1^{AV} and P_2^{AV} are the average bid prices in the pre-split and the post-split estimation periods, respectively; P_1^B and P_2^B are the closing bid prices two days before the announcement of the split and two days after the ex-split day, respectively; and $NMMK_1$ and $NMMK_2$ are the average number of market makers in the pre-split and post-split periods.

⁺⁺ p-Value is for the Wilcoxon Sign Rank Sum test of differences in medians between the two sub-samples.

TABLE 1B

Changes in Samples Estimates of the Proportional Spread, Transaction Price Based Daily Volatility, and Measures of Trading Activity for 366 Stock Split Announcements Made by NASDAQ-NMS firms between January 1983 and December 1990.

Variable, Subscript 1 = Pre-Split, Subscript 2 = Post-Split		Total Sample		SFAC ≤ 0.5		SFAC ≥ 1.0	
		Mean	Median	Mean	Median	Mean	Median
Sample Size (N)		366		210		156	
Proportional Spread	S_1	3.11	2.32	3.36	2.68	2.78	1.77
	S_2	3.62	2.79	3.64	3.09	3.60	2.48
	S_2 / S_1	1.32**	1.23††	1.20**	1.08††	1.49**	1.45††
Daily Return Volatility Based on Closing Transaction Prices (x 10^4)	σ_1^2	7.73	5.63	8.29	5.95	6.97	4.56
	σ_2^2	12.42	8.80	11.38	8.37	13.83	9.67
	σ_2^2 / σ_1^2	2.25**	1.70††	1.98**	1.42††	2.61**	2.19††
Average Number of Daily Trades	NT_1	21.56	9.37	15.88	7.76	29.21	10.96
	NT_2	32.33	12.00	20.77	9.95	47.90	17.56
	NT_2 / NT_1	1.53**	1.28††	1.39**	1.12††	1.71**	1.37††
Average Volume Turnover (x 10^3)	VT_1	3.40	2.50	3.24	2.52	3.69	2.49
	VT_2	3.80	2.30	3.81	2.41	3.90	2.13
	VT_2 / VT_1	1.19**	1.00	1.24**	1.07††	1.11	0.93
Average Volume Turnover per Trade (x 10^4)	$(VT / NT)_1$	3.19	2.64	3.45	3.00	2.83	2.09
	$(VT / NT)_2$	2.53	1.92	2.81	2.44	1.89	1.41
	$(VT / NT)_2 / (VT / NT)_1$	0.83**	0.77††	0.93*	0.82††	0.68**	0.65††
Average Number of Insider Trades (x 10^2)	NIT_1	11.98	6.11	10.21	5.56	14.37	6.67
	NIT_2	11.76	6.67	10.26	5.83	13.77	7.22
	NIT_2 / NIT_1	2.12**	1.00	2.21††	1.00	2.01††	1.00

†† (†) The median ratio is significantly different from 1 at the 1% (5%) level based on the Wilcoxon Sign Rank Test.

** (*) The mean ratio is statistically different from 1 at the 1% (5%) level using a simple t-test.

TABLE 2

Median Estimates of Daily Return Volatilities Before and After a Split, Based on Transaction Prices, Bid Prices, and Bid Prices Adjusted for Price Discreteness for the Total Sample and for the Sub-Samples by Split Factor.

For each firm, the pre- and post-split volatility estimates are based on 180 daily returns in the two estimation periods respectively. Median estimates reported in the table are based on the cross-sectional estimates for each sample. All reported median values are scaled by a factor of 10^4. $\hat{\sigma}^2_T$ is the volatility estimate based on transaction price returns, $\hat{\sigma}^2_B$ is the volatility estimate based on bid-bid returns, and $\hat{\sigma}^2_{B,D}$ is the volatility estimate corrected for both the bid-ask bounce and price discreteness. It is computed as follows:

$$\hat{\sigma}^2_{B,D} = \hat{\sigma}^2_B - \frac{d^2}{(T_2 - T_1)} \sum_{t=T_1}^{t=T_2} \frac{1}{P^2_{B,T}}$$

where $(T_2 - T_1)$ is the range of the estimation period, and P_{Bt} is the bid price on day t.

	Total Sample, N = 366				SFAC ≤ 0.5, N = 210				SFAC ≥ 1.0, N = 156			
	Pre-Split	Post-Split	Difference (p-value)†	Ratio (p)††	Pre-Split	Post-Split	Difference (p)†	Ratio (p)††	Pre-Split	Post-Split	Difference (p)†	Ratio (p)††
$\hat{\sigma}^2_T$	5.63	8.80	2.90 (<0.001)	1.70 (<0.001)	5.95	8.37	1.74 (<0.001)	1.42 (<0.001)	4.56	9.67	4.66 (<0.001)	2.19 (<0.001)
$\hat{\sigma}^2_B$	2.88	4.79	1.72 (<0.001)	1.80 (<0.001)	3 14	4.43	0.99 (<0.001)	1.41 (<0.001)	2.39	5.66	2.65 (<0.001)	2.24 (<0.001)
$\hat{\sigma}^2_{B,D}$	2.75	4.61	1.63 (<0.001)	1.81 (<0.001)	2.98	4.19	1.00 (<.001)	1.42 (<0.001)	2.38	5.25	2.61 (<0.001)	2.18 (<0.001)

† p-value for two tail test based on the Wilcoxon matched pair sign rank test for H_0: Median Difference = 0.
†† p-value for two tail test based on the Wilcoxon matched pair sign rank test for H_0: Median Ratio = 1.

TABLE 3

Median Estimates of Multi-Period Return Volatilities in the 180 Day Pre-Split and Post-Split Periods, for the Total Sample and for Sub-Samples Classified by the Split Factor.

The estimator used is

$$\sigma_c^2(k) = \frac{1}{m(1-k/n)} \sum_{t=1}^{n-k+1} \left(R_B^k - \hat{\mu}_k\right)^2 - \frac{d^2}{6T} \sum_{t=1}^{T} \frac{1}{P_{B,t}^2}$$

where, m is the actual number of overlapping k-period observations, n is the number of one period (daily) observations, R_t^k is the k-period return using overlapping one period returns based on bid-bid prices, μ^k is the sample mean of the overlapping k-period returns, and P_{Bt} is the bid price on day t.

Number of Days	Pre-Split Volatility (x 10^4)	Post-Split Volatility (x 10^4)	Median Difference (x 10^4)	Median Post-Split Volatility/Pre-Split Volatility
Total Sample, N = 366				
5	18.38	28.43	9.26**	1.62††
10	37.29	57.36	18.11**	1.58††
20	68.43	102.74	28.20**	1.54††
30	95.76	139.17	34.33**	1.57††
SFAC ≤ 0.5, N = 210				
5	19.06	27.19	5.69**	1.37††
10	38.09	53.17	10.65**	1.32††
20	75.79	97.96	12.11**	1.29††
30	102.10	126.52	11.87**	1.32††
SFAC ≥ 1.0, N = 156				
5	16.76	32.33	12.85**	2.00††
10	36.29	64.71	25.04**	2.08††
20	61.51	111.56	47.74**	1.96††
30	77.24	146.57	55.65**	1.81††

* (**) Indicates a rejection of the null hypothesis that the median difference is zero based on the Wilcoxon Matched Pair Sign Rank Test, at the 5 % (1%) level for a two-tail test.

† (††) Indicates a rejection of the null hypothesis that the median ratio is one based on the Wilcoxon Sign Rank Test, at the 5% (1%) level for a two-tail test.

TABLE 4

Median Auto-Correlations of Returns ($\times 10^2$) Based on Bid Prices for the Total Sample, and for the Sub-Samples Classified by the Split Factor.

	Total Sample, N=366			SFAC ≤ 0.5, N= 210			SAFC ≥ 1.0, N=156		
Lag	Pre-Split	Post-Split	Difference (p-value)†	Pre-Split	Post-Split	Difference (p-value)†	Pre-Split	Post-Split	Difference (p-value)†
1	15.40	10.98	-3.19 (<.001)	14.57	11.45	- 2.30 (0.094)	16.60	10.77	- 4.85 (<0.001)
2	3.38	1.95	-1.16 (0.011)	2.46	3.81	0.60 (0.670)	4.52	0.24	- 3.31 (<0.001)
3	1.21	-1.00	-1.89 (<0.001)	1.25	- 0.45	- 1.06 (0.111)	1.10	- 1.31	- 3.95 (<0.001)
4	0.07	1.13	1.05 (0.12)	- 0.33	1.19	1.28 (0.277)	0.45	1.11	1.01 (0.286)
5	0.30	0.72	0.55 (0.62)	0.65	0.15	0.189 (0.716)	- 0.97	1.04	1.08 (0.239)
6	-0.30	0.05	0.56 (0.40)	- 0.01	- 0.11	- 0.55 (0.552)	- 1.21	0.77	2.23 (0.048)
7	-1.10	-0.97	-0.26 (0.62)	- 1.08	- 0.92	- 0.18 (0.950)	- 1.28	- 1.12	- 0.26 (0.410)
8	-1.15	-1.26	-0.21 (0.35)	- 1.14	- 0.62	0.11 (0.602)	- 1.16	- 2.54	- 0.66 (0.407)
9	-1.38	-2.08	-0.73 (0.11)	- 2.18	- 1.56	0.24 (0.866)	- 0.09	- 2.24	- 2.41 (0.008)
10	-0.85	-0.51	0.22 (0.99)	- 1.10	- 0.14	0.40 (0.394)	- 0.48	- 1.29	-0.53 (0.384)

† p-value based on the Wilcoxon matched pairs sign rank test. H_0 : Median difference = 0.

TABLE 5

Median Estimates of Variance Ratios During the 180 Day Pre-Split Period and the 180 Day Post-Split Period, For the Total Sample and for Sub-Samples Classified by the Split Factor.

The estimator for the variance ratio is

$$VR = \frac{Var(R_B^k) - \frac{d^2}{6P^2}}{k \times \left(Var(R_B^1) - \frac{d^2}{6P^2} \right)} + \frac{2}{T-1} \sum_{j=1}^{k-1} \left(\frac{k-j}{k} \right)$$

where, $Var(R_B^1)$ is the 1-period (3-day) day return variance based on bid-to-bid returns, and $Var(R_B^k)$ is the k-period (30-day) return variance based on bid-bid prices, and k is equal to 10. The quantity $[d/6P^2]$ is the adjustment for the bias due to price discreetness, where d is equal to 1/8 th., and P is the price of the stock. The final term in the equation corrects for the small sample bias in the expected value of the autocorrelation, and T is the number of one-period returns used.

	Pre-Split	Post-Split	Difference
Total (N = 366)	0.94	0.79	-0.13**
SFAC ≤ 0.5 (N=210)	0.95	0.88	-0.12**
SFAC ≥ 1.0 (N=156)	0.93	0.75	-0.15**

** Indicates a rejection of the null hypothesis that the median difference is zero based on the Wilcoxon Matched Pair Sign Rank Test, at the 1 percent level.

TABLE 6

Ordinary Least Squares Estimates of Model of the Determinants of the Change in the Bias Corrected Volatility Following Stock Splits.†

Dependent Variable	Intercept	$\ln\left(\frac{NNT_2}{NNT_1}\right)$	$\ln\left(\frac{NIT_2}{NIT_1}\right)$	(1 + SFAC)	ln(MVAL)	$\ln\left(\frac{\Sigma_2^2}{\Sigma_1^2}\right)$	Adjusted R^2
$\ln\left(\frac{\sigma^2_{B,D,2}}{\sigma^2_{B,D,1}}\right)$	-0.892 (-1.68)*	0.393 (4.62)***	0.028 (0.53)	0.173 (1.75)*	0.089 (2.02)***		0.08
$\ln\left(\frac{\Sigma_2^2}{\Sigma_1^2}\right)$	-1.136 (-2.07)**	0.428 (4.25)***	-0.22 (-0.38)	0.198 (1.66)***	0.086 (1.84)**		0.08
$\ln\left(\frac{\sigma^2_{B,D,2}}{\sigma^2_{B,D,1}}\right)$	-0.196 (-0.53)	0.117 (1.73)*	0.042 (1.11)	0.051 (0.65)	0.035 (1.11)	0.633 (18.25)***	0.52

$\sigma^2_{B,D,j}$ is the daily volatility in period j, corrected for the bias due to both the bid-ask bounce and price discreteness (j = 1 for the pre-split period and 2 for the post-split period); Σ_j^2 is the volatility based on 30-day returns in period j; NNT_j is the number of trades in period j, net of insider trades; NIT_j is the number of insider trades in period j; SFAC is the announced split factor; and MVAL is the market value of the splitting firm's equity, measured two days before the announcement of the split.

† t-statistics are in parenthesis. * indicates significance at the 10% level, ** at the 5% level and *** at the 1% level.

TABLE 7

Effect of Price Discreteness on Absolute Spreads Around Stock Splits for the Total Sample and for Sub-Samples Classified by the Split Factor.

The table reports the cross-sectional mean and median values for the ratio of the median post-split spread to the median pre-split spread (ΔS), and also for the ratio of the median post-split spread to the median target spread in the post-split period (ΔS^*). For each firm i,

$$\Delta S = \frac{ABS_{2,i}}{ABS_{1,i}} \text{ and } \Delta S^* = \frac{ABS_{2,i}}{TS_{2,i}}$$

where $ABS_{j,i}$ is the median absolute spread in period j (j = 1 for the pre-split period and 2 for the post-split period), $TS_{2,i} = \{ABS_{1,i}/(1+SFAC_i)\}$ rounded up to the next highest eighths, and $SFAC_i$ is the split factor.

		Total Sample (N = 366)	SFAC ≤ 0.5 (N = 210)	SFAC ≥ 1.0 (N = 156)
ΔS	Mean (p-value)†	1.55 (< 0.001)	1.36 (< 0.001)	1.80 (< 0.001)
	Median (p-value)††	1.50 (< 0.001)	1.50 (< 0.001)	2.00 (< 0.001)
ΔS^*	Mean (p-value)†	1.53 (< 0.001)	1.42 (< 0.001)	1.69 (< 0.001)
	Median (p-value)††	1.33 (< 0.001)	1.33 (< 0.001)	2.00 (< 0.001)

† p-values are for the t-test that the sample mean is equal to one.
†† p-values are for the Wilcoxon Signed Rank test that the sample median is equal to one.

TABLE 8

Changes in the Average Proportional Spread and its Order Processing and Adverse Information Components Following Stock Splits.

		Pre-Split	Post-Split	Difference
Total Sample	Spread	3.111	3.624	0.513***
	Order Processing	1.683	1.947	0.264***
	Adverse Information	1.428	1.677	0.249***
Small Split Factor Sub-Sample (SFAC ≤ 0.5)	Spread	3.360	3.638	0.278***
	Order Processing	1.783	1.765	-0.018
	Adverse Information	1.577	1.873	0.296***
Large Split Factor Sub-Sample (SFAC ≥ 1.0)	Spread	2.777	3.605	0.828***
	Order Processing	1.498	2.067	0.569***
	Adverse Information	1.279	1.538	0.259***

TABLE 9

Ordinary Least Squares Estimates of Models of the Determinants of the Change in the Proportional Spread Following Stock Splits.†

Dependent Variable	Intercept	$\ln\left(\frac{NNT_2}{NNT_1}\right)$	$\ln\left(\frac{NIT_2}{NIT_1}\right)$	(1 + SFAC)	ln(MVAL)	Adjusted R^2
$\ln\left(\frac{S_2}{S_1}\right)$	-1.338 (-7.49)***	-0.429 (-13.06)***	-0.017 (-0.88)	0.239 (6.16)***	0.105 (6.90)***	0.39
$\ln\left(\frac{S_2}{S_1}\right)$	-1.355 (-7.64)***	-0.435 (-13.57)***		0.240 (6.20)***	0.107 (7.01)***	0.39
$\ln\left(\frac{S_2}{S_1}\right)$	-0.923 (-4.59)***		-0.070 (-2.82)***	0.139 (2.19)**	0.076 (4.03)***	0.10

S_j is the proportional spread in period j (j = 1 for the pre-split period and 2 for the post-split period); NNT_j is the number of trades in period j, net of insider trades ; NIT_j is the number of insider trades in period j; SFAC is the announced split factor; and MVAL is the market value of the splitting firm's equity, measured two days before the announcement of the split.

† t-statistics are in parenthesis. * indicates significance at the 10% level, ** at the 5% level and *** at the 1% level.

TABLE 10

Ordinary Least Squares Estimates of Models of the Determinants of the Change in the Adverse Information Component of the Bid-Ask Spread Following Stock Splits.

Dependent Variable	Intercept	$\ln\left(\frac{COV_2}{COV_1}\right)$	$\ln\left(\frac{NNT_2}{NNT_1}\right)$	$\ln\left(\frac{NIT_2}{NIT_1}\right)$	(1 + SFAC)	ln(MVAL)	Adjusted R^2
$\ln\left(\frac{S_2}{S_1}\right)$	-0.486 (-5.26)***	0.800 (33.21)***	-0.171 (-9.47)***	-0.024 (-2.49)***	-0.015 (-0.73)	0.049 (6.32)***	0.85
$\ln\left(\frac{S_2}{S_1}\right)$	-0.511 (-5 53)***	0.799 (32.92)***	-0.180 (-10.12)***		-0.013 (-0.62)	0.051 (6.54)***	0.85
$\ln\left(\frac{S_2}{S_1}\right)$	-0.247 (-2.49)***	0.898 (36.99)***		-0.042 (-4.04)***	-0.079 (-3.60)***	0.033 (3.84)***	0.81

S_j is the proportional spread in period j (j = 1 for the pre-split period and 2 for the post-split period); COV_j is the covariance between R_t^D and R_{t-1}^D, where R_t^D is the difference between the transaction-price based return and the return based on bid-to-bid prices; NNT_j is the number of trades in period j, net of insider trades; NIT_j is the number of insider trades in period j; SFAC is the announced split factor; and MVAL is the market value of the splitting firm's equity, measured two days before the announcement of the split.

† t-statistics are in parenthesis. * indicates significance at the 10% level, ** at the 5% level and *** at the 1% level.

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