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The Equity Premium Puzzle and the Risk-Free Rate Puzzle at Long Horizons †

Kent Daniel* and David Marshall**

Abstract

The failure of consumption based asset pricing models to match the stochastic properties of the equity premium and the risk-free rate has been attributed by some authors to frictions, transaction costs or durability. However, such frictions would primarily affect the higher frequency data components: consumption-based pricing models that concentrate on long-horizon returns should be more successful.

We consider three consumption-based models of the asset-pricing kernel: time-separable utility, and the models of Abel (1990) and Constantinides (1990). We estimate a vector ARCH model that includes the pricing kernel and the equity return, and use the fitted model to assess the model's implications for the equity premium and for the risk-free rate. We find that time-separable preferences fail at all horizons, and none of the models perform well at the quarterly horizon. When consumption is measured as nondurables plus services, the Abel and the Constantinides models show modest improvement at the one- and two-year horizon. However, when consumption is measured either as expenditures on nondurables or as total consumption purchases, versions the Abel and the Constantinides match the mean and the variance of the observed equity premium at the two-year horizon, capture a good deal of the time-variation of the equity premium in post-war data, and have more success matching the first and second moments of the observed risk-free rate. A major unresolved issue is to understand why the measured consumption services series performs so poorly in these models.

† The views expressed in this paper are strictly those of the author. They do not necessarily represent the position of the Federal Reserve Bank of Chicago, or the Federal Reserve System. We thank Steve Cecchetti, Larry Christiano, John Cochrane, George Constantinides, Larry Epstein and Lars Hansen for helpful discussions, and we acknowledge Glenn McAfee for superlative research assistance. Daniel gratefully acknowledges research support from the the Center for Research in Security Prices (CRSP) at the University of Chicago. Some results in this paper were previously circulated in a working paper entitled, "Consumption-Based Modeling of Long-Horizon Returns."

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1 Introduction

Most research on consumption-based asset pricing focuses on short-horizon returns. The starting point is the familiar intertemporal Euler equation

$$1 = E_t \left[m_{t+\tau}^{\tau} r_{t+\tau}^{\tau} \right] \tag{1.1}$$

where $r_{t+\tau}^{\tau}$ denotes the gross real cumulative equity return from date t to date $t+\tau$ and $m_{t+\tau}^{\tau}$ denotes the intertemporal marginal rate of substitution (IMRS) between wealth at date t and wealth at date $t+\tau$. A model of preferences is then posited that delivers m_t^{τ} as a function of aggregate consumption. Implications of equation (1.1) are tested for return horizons τ equal to one month or one quarter. Typically, the tests reject the model, often decisively.

Of particular interest are the implications of (1.1) for the equity premium and the risk-free rate. Let rf_t^{τ} denote the gross real risk-free return between dates t and $t+\tau$. Equation (1.1) implies the following characterizations of the equity premium and the risk-free rate:

$$\frac{E_t r_{t+\tau}^{\tau} - rf_t^{\tau}}{rf_t^{\tau}} = -cov_t \left(m_{t+\tau}^{\tau}, r_{t+\tau}^{\tau} \right) \tag{1.2}$$

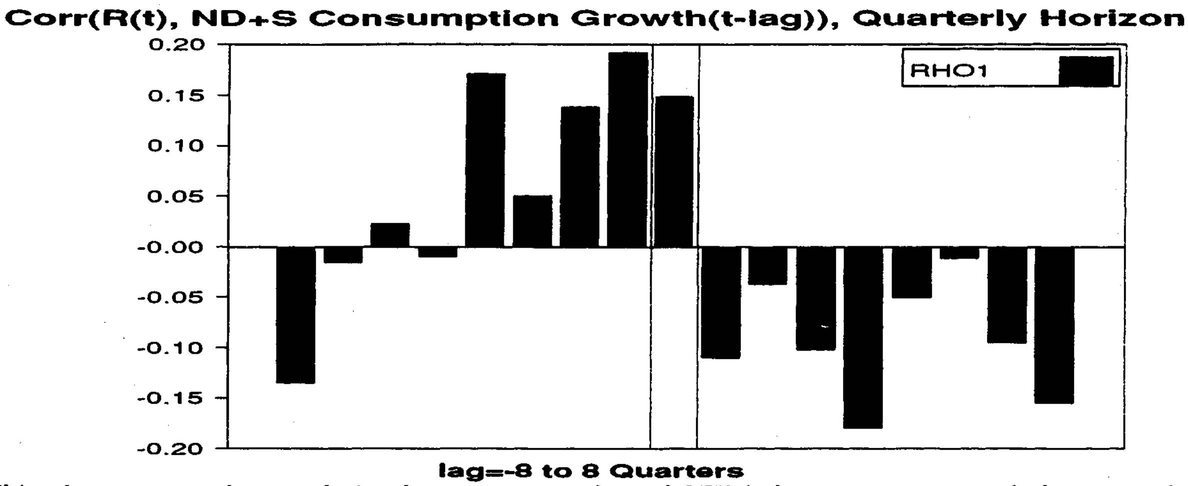
$$rf_t^{\tau} = \frac{1}{E_t m_{t+\tau}^{\tau}} \tag{1.3}$$

The *equity premium puzzle*, as defined by Mehra and Prescott (1985), is the claim that the mean of $\frac{E_t r_{t+\tau}^{\tau} - rf_t^{\tau}}{rf_t^{\tau}}$ is much bigger than $-E\left[cov_t\left(m_{t+\tau}^{\tau}, r_{t+\tau}^{\tau}\right)\right]$, for plausible consumption-based models of $m_{t+\tau}^{\tau}$. In the context of equation (1.2) we can also define the *predictability puzzle* as the claim that, for plausible consumption-based models of $m_{t+\tau}^{\tau}$, there is insufficient time-variation in $cov_t\left(m_{t+\tau}^{\tau}, r_{t+\tau}^{\tau}\right)$ to explain the observed variability of $\frac{E_t r_{t+\tau}^{\tau} - rf_t^{\tau}}{rf_t^{\tau}}$.[1] Finally, the *risk-free rate puzzle* is the claim that, for consumption-based models of m_t^{τ} that come within striking distance of resolving the equity premium puzzle, both the mean[2] and the variance[3]

[1] The predictability of excess returns to equities and to other financial assets is discussed in Fama and French (1988), Bekaert and Hodrick (1992), Cutler, Poterba, and Summers (1991), and Froot (1990), among others. These observations are anomalous according to the traditional random-walk characterization of market efficiency. They could be explained if, in equilibrium, risk premiums required by investors vary through time. Equation (1.2) is a precise characterization of the needed variation in risk premiums, in terms of the time-series properties of m_t^{τ}.

[2] See Weil (1989) and Cochrane and Hansen (1992).

[3] See the discussion in Cochrane and Hansen (1992, p.137).

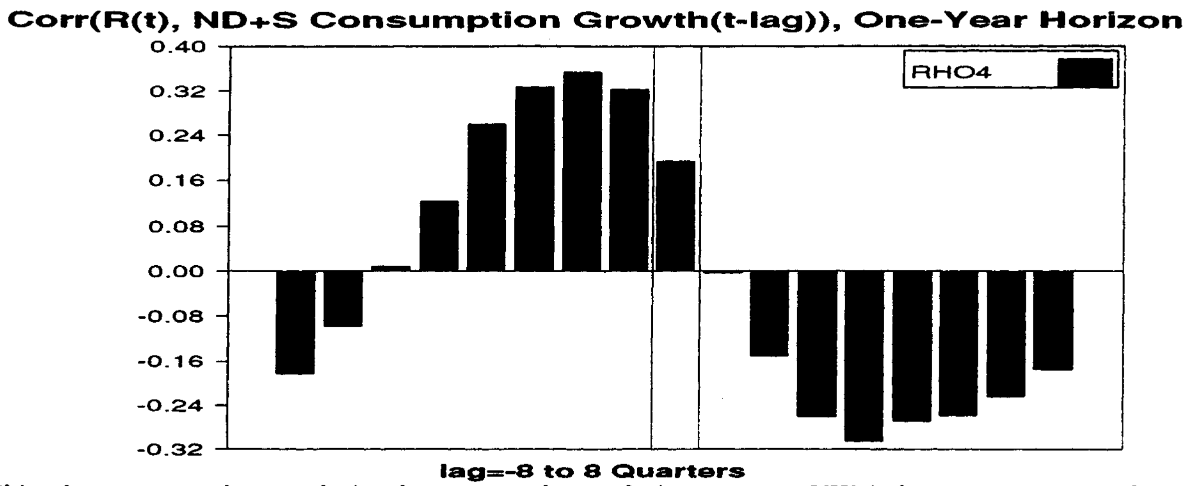


This plot presents the correlation between quarterly real VW index returns at t and the quarterly growth rate of real non-durable and services consumption at $(t - lag)$ for lag = -8 through +8 quarters. The vertical lines in the center of the graph indicate the contemporaneous correlation.

Figure 1.1: **Correlation between Real Quarterly VW Index Returns at t and Real Quarterly Non-Durable and Services Consumption Growth**

of $\frac{1}{E_t m_{t+\tau}^{\tau}}$ are too high to match the corresponding moments of rf_t^{τ}. Cecchetti, Lam and Mark (1993) estimate a representative-agent, time-separable model where consumption and dividends are governed by a bivariate model Markov switching model, and find that while they can match the first moments of the equity and risk-free return data, they cannot match both the first and second moments.

The equity premium puzzle and the predictability puzzle are both reflections of a more fundamental problem: the low correlation between consumption-growth and equity returns at short horizons. Consider Figure 1.1, which displays correlations between real quarterly stock returns quarterly and quarterly consumption growth at various leads and lags. (In this figure, consumption is measured as purchases of nondurables plus services.) The contemporaneous correlation between quarterly returns and quarterly consumption growth is small (less than 0.15), and the largest correlation at any lead/lag (when returns lead consumption growth by two quarters) is less than 0.2. Cochrane and Hansen (1992) call this low correlation between the return on market proxies and consumption growth the "correlation puzzle." A number of factors have been proposed to account for the low correlations between stock returns and aggregate consumption growth at short-horizons, including uninsurable cross-sectional



This plot presents the correlation between real cumulative one-year VW index returns at t and the one-year growth rate of real non-durable and services consumption at $(t - lag)$ for lag = -8 through +8 quarters. The vertical lines in the center of the graph indicate the contemporaneous correlation

Figure 1.2: **Correlation between Real One-Year VW Index Returns at *t* and Real One-Year Non-Durable and Services Consumption Growth**

heterogeneity,[4] fixed costs of adjusting consumption,[5] costs of portfolio adjustment,[6] and even small deviations from perfect rationality.[7]

While these factors could substantially affect the co-movements of asset returns and aggregate consumption at high frequencies, they should be less disruptive to the theory at longer horizons. Figure 1.2 suggests that there may be merit in this argument. Figure 1.2 is analogous to Figure 1.1, except that the correlations in Figure 1.2 are between cumulative stock returns over a *one year* horizon and *one year* consumption growth (i.e., c_{t+4}/c_t, where the timing interval is one quarter). Figure 1.2 suggests that Cochrane and Hansen's (1992) correlation puzzle is less pronounced for one-year cumulative returns. While the contemporaneous correlation between consumption growth and returns at the one-year horizon is only slightly higher than that shown in Figure 1.1 for the one-quarter horizon, the maximal correlation over all leads and lags is much higher. In particular, the correlation between one-year consumption growth and one-year returns lagged by two quarters is 0.35, almost twice as high as the maximal

[4]See, e.g., Constantinides and Duffie (1992).

[5]Grossman and Laroque (1990), Marshall (1994), Marshall and Parekh (1994).

[6]Luttmer (1995), He and Modest (1995).

[7]Cochrane (1989).

correlation found between quarterly returns and quarterly consumption growth.[8]

These results suggest that consumption-based models of the equity premium may have more success if they focus on longer horizon returns. Specifically, if the higher *unconditional correlations* displayed in Figure 1.2 imply higher *conditional covariances* between returns and consumption-growth, then the equation (1.2) should provide a better fit to observed data as the horizon increases.[9] However, it is unclear how the longer horizon horizon will affect the risk-free rate. In fact, Cochrane and Hansen (1992) find that lengthening the horizon actually exacerbates the risk-free rate puzzle.[10]

In this paper, we explore these questions directly. For several models of m_t^τ, we estimate a model of the vector process $(r_{t+\tau}^\tau, m_{t+\tau}^\tau)$ that allows for time-varying conditional second moments. We use this model to generate estimates of the conditional moments $E_t r_{t+\tau}^\tau$, $E_t m_{t+\tau}^\tau$, and $cov_t\left(m_{t+\tau}^\tau, r_{t+\tau}^\tau\right)$, and we then use these estimates to evaluate (1.2) and (1.3) for investment horizons τ ranging from three months through three years.

Most empirical studies of consumption-based pricing measure consumption as purchases of nondurables plus services, as measured by the Bureau of Economic Analysis. We see two potential problems with this measure of consumption. First, the BEA's data series on consumer services includes the imputed rental value of owner-occupied housing. The quantity of these housing services consumed is roughly proportional to the stock of housing. Given the substantial transaction costs in the residential housing market, it is unlikely that changes in investment opportunities induce agents to vary their consumption of housing services to the extent predicted by the consumption-based paradigm.[11] Second, it is not clear that the omission of consumer durables is appropriate when studying longer-horizon returns. If the durable good depreciates quickly enough, the distinction between durables purchases and the durables stock becomes less important as the horizon lengthens. For example, if the half-life of the durable good were about the same as the decision interval, fluctuations in durables purchases

[8]The frequency-domain analysis in Daniel and Marshall (1995) delivers a similar result.

[9]Some form of time-nonseparability must be incorporated to account for the two-quarter lag in the maximal correlation: if utility is time-nonseparable, lagged consumption and conditional moments of future consumption will enter into the contemporaneous $m_{t+\tau}^\tau$ process.

[10]However, Cecchetti, Lam and Mark (1994) use annual equity and bond returns from 1890 to 1987, and find that, taking account of sampling variability, volatility bounds are satisfied.

[11]This insight is modeled formally by Grossman and Laroque (1990).

would capture a good deal of the variability in the durables stock. To omit durables in a such a model may potentially represent as big a specification error as the inclusion of durables purchases.

These considerations suggest that our results may be sensitive to the way consumption is measured. We start by using the conventional measure of consumption as purchases of nondurables plus services. We then perform our analysis using only nondurables purchases as our measure of consumption. Finally, as a somewhat crude check on the importance of durables, we also replicate our results with consumption measured by total consumption expenditures.

According to our empirical results, none of the consumption-based models fit the equity premium or the risk-free rate at the quarterly horizon, regardless of the way consumption is measured. When consumption is measured by nondurables plus services, there is a some improvement in the ability of the Abel (1990) and Constantinides (1990) models to match moments of the equity premium when the horizon is lengthened to one or two years, but the models still fail to capture the time-series properties of the observed equity premium series. In contrast, if we discard the data on consumer services, and measure consumption as purchases of nondurables, both the Abel (1990) and Constantinides (1990) models perform remarkably well at the two-year horizon. In particular, versions of these models replicate both the mean and the standard deviation of the observed equity premium, and the theoretical equity premium series generated by these models shows some ability to track the observed equity premium through time. We obtain similar results when consumption is measured by total consumption purchases (including purchases of durables). In our tests of the risk-free rate equation (1.3), we find that the Abel (1990) and Constantinides (1990) models provide a better fit to the mean of the risk-free rate as the horizon is lengthened, but only if we measure consumption either as nondurables or as total consumption expenditures. Interestingly, the excessive variability of the risk-free rate implied by these models at the quarterly horizon completely disappears when the horizon is set between two and three years. We conclude that consumption-based models can match important features of the observed equity premium and risk-free rate processes at longer horizons, provided that data on consumer services is excluded. The critical question to be resolved is why the BEA's measure of consumer services performs so poorly as an input into these models.

The remainder of the paper is organized as follows: Section 2 describes the models of preferences we use for the remainder of the paper. Section 3 describes the time-series model we use to investigate (1.2) and (1.3). Section 4 presents our empirical results, and section 5 summarizes.

2 Models of Preferences

2.1 Time Separable Power Utility

The most widely-studied (and widely-rejected) preference specification in the consumption-based pricing literature is time-separable power utility. In this specification, agents solve the following maximization problem:

$$\max_{\{c_{t+j}\}_{j=0}^{\infty}} U \equiv E_t \sum_{j=0}^{\infty} \beta^j \frac{c_{t+j}^{1-\gamma}}{1-\gamma} \tag{2.1}$$

subject to the usual budget constraint. The τ-period IMRS is:

$$m_t^{\tau} = \beta^{\tau} \left(\frac{c_{t+\tau}}{c_t} \right)^{-\gamma}. \tag{2.2}$$

Cochrane and Hansen (1992) find that the performance of time-separable utility actually *deteriorates* as the horizon lengthens. The problem is that aggregate consumption is a (stochastically) growing series. In the time-separable model, agents seek to transfer some of the high future consumption to the present by borrowing. A counterfactually high risk-free rate is needed to discourage this borrowing. (Recall that net borrowing must equal zero in equilibrium.) In principle, this effect could be countered by a strong precautionary motive for saving: agents may wish to insure against the possibility of consumption downturns. However, the probability of a consumption downturn gets smaller as the horizon lengthens: Cochrane and Hansen (1992) note that there is no five-year period in post-war US data over which aggregate consumption declines. As a result, the time-separable model predicts a lower precautionary demand for savings, and a higher equilibrium risk-free rate, as the horizon lengthens.

What is needed, then, is a reason why the precautionary motive for saving remains strong at longer horizons. One possible reason is that agent's within-period utility-of-consumption changes through time. In particular, suppose agents seek protection, not against an absolute decline in consumption, but against a decline in consumption *relative to some reference point*, where the reference point itself grows at the same rate that consumption grows. In such a model, the precautionary motive for saving would not become attenuated as the horizon grows. Preference specifications with this property include Constantinides' (1990) habit-formation preferences and Abel's (1990) "catching-up-with-the-Joneses" preferences. In the following section, we formalize the Abel (1990) and Constantinides (1990) preference specifications.

2.2 Abel's (1990) "Catching-Up-With-the-Joneses" Preferences

Let $\bar{c}_t$ denote the per-capita consumption at date t. The agent solves

$$\max_{\{c_{t+j}\}_{j=0}^{\infty}} U \equiv E_t \sum_{j=0}^{\infty} \beta^j \frac{(c_{t+j} - h_{t+j})^{1-\gamma}}{1-\gamma} \tag{2.3}$$

subject to the usual budget constraint, where

$$h_t \equiv \frac{\eta(1-\delta)}{\delta - \delta^{m+1}} \sum_{i=1}^{m} \delta^i \bar{c}_{t-i}, \ \eta > 0, \ 0 < \delta < 1. \tag{2.4}$$

The interpretation is that agents compare their consumption to the consumption of their neighbors (the "Joneses") in the recent past. In the formal model, the neighbors' consumption is represented by $\bar{c}_t$, and agents have a subsistence point equal to η times a weighted average of the per-capita consumption levels over the past m periods. Notice that agents treat h_t as exogenous: the marginal utility of a fixed level of consumption inherits the upward trend in $\bar{c}_t$, but agents cannot alter the h_t process by their own actions. (Of course, in equilibrium $c_t = \bar{c}_t$.)

With Abel preferences,

$$m_{t+\tau}^{\tau} = \beta^{\tau} \frac{(c_{t+\tau} - h_{t+\tau})^{-\gamma}}{(c_t - h_t)^{-\gamma}}. \tag{2.5}$$

Let the value function $V(W, h)$ be defined as the maximum value of the objective function that can be attained given initial wealth W and an initial subsistence point h. We define the coefficient of relative risk aversion (denoted RRA_t) by

$$RRA_t \equiv -W \frac{V_{WW}(W, h)}{V_W(W, h)}. \tag{2.6}$$

With the time-separable preferences described in section 2.1, $RRA_t = \gamma$, for all t. With Abel preferences,

$$RRA_t = \gamma \frac{c_t}{c_t - h_t} \tag{2.7}$$

so the coefficient of relative risk aversion is time-varying, and everywhere exceeds γ. The model parameters we use are $\eta = 0.8$, $\delta = 0.7$, $n = 8$, $\beta = 1$. We vary γ between 2 and 14.

2.3 Constantinides (1990) Habit-Formation Preferences

Constantinides (1990) models agents as maximizing an objective function of the same form as (2.3) with the following alternative specification for h_t:

$$h_t \equiv \frac{\eta(1-\delta)}{\delta - \delta^{m+1}} \sum_{i=1}^{m} \delta^i c_{t-i},\ \eta > 0,\ 0 < \delta < 1. \tag{2.8}$$

The difference between (2.4) and (2.8) is that in (2.8) the stochastic subsistence point h_t is a function of the agent's own consumption c_t, rather than the per-capita consumption. The marginal rate of substitution is now

$$m_{t+\tau}^{\tau} = \beta^{\tau} \frac{MU_{t+\tau}}{MU_t} \tag{2.9}$$

where the marginal utility of consumption MU_t is defined by

$$MU_t \equiv (c_t - h_t)^{-\gamma} - \frac{\eta(1-\delta)}{\delta - \delta^{m+1}} \sum_{i=1}^{m} (\beta\delta)^i E_t \left[c_{t+i} - h_{t+i})\right]^{-\gamma} \tag{2.10}$$

Terms involving conditional expectations appear in equation (2.10) because agents consider the effect of their current consumption on future values of h_t. These conditional expectations must be computed when we construct m_t^{τ}. We do this as follows. First, define the variable D_t by:

$$D_t \equiv 1 - \frac{\eta(1-\delta)}{\delta - \delta^{m+1}} \frac{\sum_{i=1}^{m} (\beta\delta)^i \left[c_{t-m+i} - h_{t-m+i})\right]^{1-\gamma}}{\left(c_{t-m} - h_{t-m}\right)^{-\gamma}}. \tag{2.11}$$

The variable D_t behaves as a stationary stochastic process. Equations (2.9) and (2.11) imply that, in the Constantinides model,

$$m_{t+\tau}^{\tau} = \beta^{\tau} \frac{\left(c_{t+\tau} - h_{t+\tau}\right) E_{t+\tau} D_{t+\tau+m}}{\left(c_t - h_t\right) E_t D_{t+m}} \tag{2.12}$$

Since D_t is stationary, we can fit an autoregressive time-series model for this variable: we use the fitted values as our estimate of $E_t D_{t+m}$. For most models, the likelihood ratio statistics testing n lags against $n-1$ lags in the autoregression for D_t (for n between 1 and 5) favor four lags. We estimate a fourth-order autoregression in D_t, and project the fitted regression m periods into the future. We estimated $\{m_t^\tau\}$ only for values of γ that do not imply negative marginal utilities (as defined by equation (2.10)) for any observations.[12] As with the Abel model, we set $\eta = 0.8$, $\delta = 0.7$, $n = 8$, $\beta = 1$.

We consider Constantinides preferences separately from Abel preferences for two reasons. First, many empirical applications of time-nonseparable preferences use the Abel model, rather than implementing true habit-formation, because the Abel model is much easier to solve: there is no need to compute the conditional expectation terms in (2.10).[13] It is of interest, therefore, to see whether the Abel model does function as a good empirical proxy for the less-tractable Constantinides model. A second reason is that habit formation preferences do not accentuate risk aversion in the way that Abel's preferences do.[14] In response to a wealth shock at date t, the agent with habit-formation preferences adjusts her state-contingent plans for future consumption so as to optimally adjust h_{t+i}, $i = 1, ..., m$. This attenuates the impact of a given wealth shock on the objective function, as compared to the Abel specification. In particular, higher η does not increase RRA_t as much as in (2.7). While RRA_t cannot be computed analytically for our model of habit-formation, Constantinides (1990) and Constantinides and Ferson (1991) obtain closed-form solutions for RRA_t in the context of a simpler model. They show that, for preference parameters similar to ours, the mean coefficient of relative risk aversion is not too far above γ.[15]

3 A Vector ARCH Model of Conditional Covariances

[12]When consumption is measured as nondurables-plus-services, the maximum usable value of γ is 12. With nondurable consumption and total consumption expenditures, the maximum value of γ is 9 and 11, respectively.

[13]For example, Campbell and Cochrane (1995) is entitled, "By Force of Habit". However, these authors model the habit stock as a function of the per-capita consumption process, so the model is actually a variant of the Abel model.

[14]This point is extensively discussed by Constantinides (1990), and Ferson and Constantinides (1991). We thank Larry Christiano for pointing this out to us.

[15]Boldrin, Christiano, and Fisher (1995) report a similar result.

3.1 The Basic Set-Up

In this section, we describe the time-series model we use to evaluate (1.2) and (1.3). Since there is no observable asset with a risk-free real payoff over a multi-year horizon, we examine the implications of (1.1) for nominal returns. Let P_t denotes the price level at date t, and let $R^{\tau}_{t+\tau}$ denote the nominal cumulative equity return from date t to date $t+\tau$ (so $R^{\tau}_{t+\tau} \equiv r^{\tau}_{t+\tau}\left[\frac{P_{t+\tau}}{P_t}\right]$). Equation (1.1) then implies:

$$1 = E_t\left[M^{\tau}_{t+\tau} R^{\tau}_{t+\tau}\right] \tag{3.1}$$

where $M^{\tau}_{t+\tau}$ is the marginal-rate-of-substitution in nominal wealth between t and $t+\tau$:

$$M^{\tau}_{t+\tau} \equiv m^{\tau}_{t+\tau}\left[\frac{P_t}{P_{t+\tau}}\right]. \tag{3.2}$$

Let RF^{τ}_t denote the risk-free nominal return from t to $t+\tau$. The observable analogue to RF^{τ}_t is the return on a τ-period zero-coupon dollar bond. Equation (3.1) implies the analogue s to (1.2) and (1.3):

$$\frac{E_t R^{\tau}_{t+\tau} - RF^{\tau}_t}{RF^{\tau}_t} = -cov_t\left(M^{\tau}_{t+\tau},\, R^{\tau}_{t+\tau}\right) \tag{3.3}$$

$$RF^{\tau}_t = \frac{1}{E_t M^{\tau}_{t+\tau}} \tag{3.4}$$

To test (3.3), we need a model of the conditional first moment of R^{τ}_t and of the conditional second moments of the joint $\{M^{\tau}_{t+\tau},\, R^{\tau}_{t+\tau}\}$ process. We use the following vector ARCH model. Let X_t denote an $(N-2)\times 1$ vector of variables that is useful in predicting $\{M^{\tau}_{t+\tau},\, R^{\tau}_{t+\tau}\}$, and let $Y_t' \equiv (R^{\tau}_t, M^{\tau}_t, X_t')$. We assume that the $(N\times 1)$-dimensional process Y_t follows a vector autoregression:

$$Y_{t+1} = A_0 + A_1 Y_t + A_2 Y_{t-1} + \; ... + A_p Y_{t-p+1} + u_{t+1} \tag{3.5}$$

where A_0 is an $N\times 1$ vector of constants, A_i, $i = 1, ..., p$ are $N\times N$ matrices, and

$$u_{t+1} = L_{t+1} v_{t+1}\ ,\ v_{t+1} \sim i.i.d.\mathcal{N}(0, I),$$

and L_{t+1} is a lower triangular matrix such that

$$L_{t+1}L'_{t+1} \equiv H_{t+1} = E_t\left(u_{t+1}u'_{t+1}\right). \tag{3.6}$$

We now must specify the law-of-motion for H_{t+1}. We use the following notation: For any $N \times N$ symmetric matrix Σ let $vec(\Sigma)$ stack the distinct elements of Σ into a $\frac{N(N+1)}{2} \times 1$ vector. Following the ARCH approach of Engle (1982), we assume that $vec(H_{t+1})$ can be approximated by a linear function of squared residuals dated t and earlier (i.e., elements of the matrices $u_t u_t'$, $u_{t-1}u_{t-1}'$, . . ., $u_{t-q}u_{t-q}'$). That is,

$$vec(H_{t+1}) = B_0 + B_1 vec(u_t u_t') + B_2 vec(u_{t-1}u_{t-1}') + \dots + B_q vec(u_{t-q+1}u_{t-q+1}') \tag{3.7}$$

where B_0 is an $N(N+1)/2 \times 1$ vector of constants, and B_i, $i = 1, \dots, q$, are $N(N+1)/2 \times N(N+1)/2$ matrices. The parameters in (3.7) can be estimated by fitting the regression

$$vec(u_{t+1}u_{t+1}') = B_0 + B_1 vec(u_t u_t') + B_2 vec(u_{t-1}u_{t-1}') + \dots + B_q vec(u_{t-q+1}u_{t-q+1}') + w_{t+1} \tag{3.8}$$

where w_{t+1} is an *i.i.d* $N(N+1)/2 \times 1$ vector process. The linear model (3.5) - (3.7) allows for easy computation of the τ-step-ahead conditional first and second moments of Y_t: Let us write (3.5) in first-order "companion" form by defining $\mathcal{Y}_t' \equiv \left(Y_t', Y_{t-1}', \dots, Y_{t-p+1}'\right)$, $\mathcal{U}_t' \equiv \left(u_t, 0_{(N\times N)}, \dots, 0_{(N\times N)}\right)$, and $\mathcal{A}_0' \equiv \left(A_0', 0_{(N\times N)}, \dots, 0_{(N\times N)}\right)$. We define the $Np \times Np$ coefficient matrix $\mathcal{A}$ by:

$$\mathcal{A} \equiv \begin{bmatrix} A_1 & A_2 & \cdots & A_{p-1} & A_p \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}.$$

Equation (3.5) can now be written

$$\mathcal{Y}_{t+1} = \mathcal{A}_0 + \mathcal{A}Y_t + \mathcal{U}_{t+1}$$

so

$$E_t \mathcal{Y}_{t+\tau} = (I - \mathcal{A}^\tau)(I - \mathcal{A})^{-1}\mathcal{A}_0 + \mathcal{A}^\tau \mathcal{Y}_t \tag{3.9}$$

and

$$var_t \mathcal{Y}_{t+\tau} = \sum_{i=0}^{\tau-1} \mathcal{A}^i E_t \left[\mathcal{U}_{t+\tau-i}\mathcal{U}_{t+\tau-i}'\right] \left[\mathcal{A}^i\right]' \tag{3.10}$$

In (3.10), $E_t\left[\mathcal{U}_{t+\tau-i}\mathcal{U}'_{t+\tau-i}\right]$ is computed from (3.8): If we use the notation that $B(L) \equiv \sum_{k=1}^{q} B_k L^k$, (3.8) implies

$$E_t vec\left[u_{t+j}u'_{t+j}\right] = B_0 \sum_{k=0}^{j-1}[B(L)]^k + [B(L)]^j\, vec[u_t u'_t]. \tag{3.11}$$

Equations (3.9) and (3.10) are used to evaluate the conditional moments in (3.3) and (3.4): $E_t R^{\tau}_{t+\tau}$ and $E_t M^{\tau}_{t+\tau}$ are the first and second elements of the vector $E_t\mathcal{Y}_{t+\tau}$ in (3.9), and $cov_t\left(M^{\tau}_{t+\tau},\, R^{\tau}_{t+\tau}\right)$ is the $(2,1)^{th}$ element of the matrix $var_t\mathcal{Y}_{t+\tau}$ in (3.10). Notice that, in principle, $cov_t\left(M^{\tau}_{t+\tau},\, R^{\tau}_{t+\tau}\right)$ depends on *all* elements of the matrices $E_t\left[u_{t+\tau-i}u'_{t+\tau-i}\right], i = 1, ..., \tau - 1$.

We use the linear model (3.5) - (3.8) because it provides the straightforward analytic expressions (3.9) and (3.10) for $E_t R^{\tau}_{t+\tau}$, $E_t M^{\tau}_{t+\tau}$, and $cov_t\left(M^{\tau}_{t+\tau},\, R^{\tau}_{t+\tau}\right)$. However, the linear model is not without drawbacks. First, it tends to generate a large number of free parameters. For example, if no ex-ante restrictions were placed on the matrices $B_1, ..., B_q$, (that is, if equation (3.8) were treated as an unrestricted VAR in the $\frac{N(N+1)}{2}$ elements of $u_{t+1}u'_{t+1}$), there would be a total of $\frac{3N+N^2}{2} + pN^2 + q\left[\frac{N(N+1)}{2}\right]^2$ free parameters to be estimated This number grows at rate N^4. It is easy to see that modest values of N, p, and q can give severe degrees-of-freedom problems.

Given this problem of parameter proliferation, we experimented with rather ruthless zero-restrictions on the coefficient matrices $B_1, ..., B_q$. We arrived at the following specification: First, we exclude all cross-terms (of the form $u_{i,t-k}u_{j,t-k}, k = 0, 1, ..., q-1, i \neq j$) from the right-hand side of (3.8). Second, in those equations of (3.8) where the dependent variable is a squared residual of the form $u^2_{i,t+1}$, only own lagged dependent variables (i.e., $u^2_{i,t}$, $u^2_{i,t-1}$, ..., $u^2_{i,t-q+1}$) are used as regressors. Third, where the dependent variable is a cross-term of the form $u_{i,t+1}u_{j,t+1}, i \neq j$, only lagged squared residuals $u^2_{i,t}, u^2_{j,t}$ $u^2_{i,t-1}, u^2_{j,t-1}$ ..., $u^2_{i,t-q+1}, u^2_{j,t-q+1}$ are used as regressors. (That is, $u^2_{k,t-n}, k \neq i, j$ is never used as an explanatory variable for $u_{i,t+1}u_{j,t+1}$.) These restrictions were loosely patterned after the constant-correlation model, which also excludes cross-terms as explanatory variables and only uses lagged dependent variables as explanatory variables in the squared-residual equations. These restrictions reduce the number of free parameters to $\frac{3N+N^2}{2} + (p+q)\,N^2$, a number which grows at rate N^2. In our empirical work, we set $N = 4$, $p = 1$, and $q = 8$, so the total number of free parameters is 158.

A second drawback of our linear model is that it does not guarantee positive-definiteness of the $var_t Y_{t+\tau}$ matrix. Positive-definiteness is a nonlinear restriction, so multivariate models with time-varying second moments that impose positive-definiteness necessarily must introduce nonlinearities either into the model structure or into the estimation procedure. These sorts of nonlinearities substantially increase the computational burden in estimating and solving the model. For example, a widely-used multivariate model that guarantees positive-definite conditional covariance matrices is the constant-correlation model of Bollerslev (1990). This model is not suitable for our purposes, since, for $i > 1$, the elements of the matrix $\mathcal{U}_{t+i}\mathcal{U}'_{t+i}$ are *nonlinear* functions of the innovations $w_{t+1}, w_{t+2}, ..., w_{t+i}$. As a result, the matrices $E_t\left[\mathcal{U}_{t+\tau-i}\mathcal{U}'_{t+\tau-i}\right]$ in equation (3.10) cannot be computed as a linear projection, as in equation (3.11). For $\tau > 1$, computing these conditional covariance matrices would require integrating out the innovations $w_{t+1}, w_{t+2}, ..., w_{t+\tau-i}$, a computationally burdensome task. An alternative way of imposing positive-definiteness is the diagonal GARCH model of Bollerslev, et. al (1988). This model delivers a linear model of the general form (3.8), but guarantees positive-definiteness by imposing a nonlinear restriction on the coefficient matrices $B_1, B_2, ..., B_q$. As such, this model must be estimated using nonlinear techniques, such as maximum likelihood. Due to the large number of parameters in the models we use, nonlinear estimation would be extremely burdensome. Furthermore, it is not clear how much our inference is distorted by our failure to impose positive-definiteness as a restriction. For these reasons, we estimate (3.5) and (3.8) by OLS. We report the number of violations of positive-definiteness for each model studied, and we use the number of such violations as a check for model mis-specification.

3.2 Implementation of the Model

We include two predictor variables in X_t: the term spread and the default spread.[16] The inflation rate $\frac{P_{t+\tau}}{P_t}$ is constructed from the deflator associated with the consumption series being used. For each model of m_t^τ, vector process $\{Y_t\}$ is constructed, the first vector autoregression

[16]We initially included the dividend yield on the CRSP value-weighted portfolio as a third predictor variable in X_t. We found, however, that for most preference specifications the dividend yield was insignificant in the equations for M_{t+1}^τ and R_{t+1}^τ, according to standard F-tests. This result is consistent with Fama and French (1989). In the interest of parsimony, we therefore exclude the dividend yield from X_t. (Recall that the number of free parameters grows at rate N^2.)

(3.5) is estimated by OLS, vector process $\{vec\,(u_t u_t')\}$ is constructed from the residuals of (3.5), and the second regression (3.8) is estimated by OLS.

We used the multivariate Schwartz and Akaike Information Criteria to determine the appropriate order p of the first VAR, equation (3.5). In most models, these criteria favored a single lag, so we set $p = 1$ for all models. It is unclear how relevant these information-based criteria are for the second regression (3.8), since the elements of $vec\,(u_t u_t')$are generated from a smaller number of distinct information sources. Instead, we use a more informal procedure to choose the order q of regression (3.8). We seek to maximize the variability of $cov_t\,(M_{t+\tau}^{\tau},\, R_{t+\tau}^{\tau})$ while keeping the number of non-positive-definite estimates for $var_t Y_{t+\tau}$ low. For most models, we found that $q = 8$ worked well according to this standard. For each model, Table 3.1 reports the number of times that our proxy for $var_t Y_{t+\tau}$ failed to be a positive-definite matrix. Failures of positive-definiteness are distressingly frequent at the quarterly horizon ($\tau = 1$), but are infrequent or nonexistent at longer horizons. These results could be interpreted as evidence of misspecification at when $\tau = 1$: our linear time-series model (3.5) - (3.8) may simply be inappropriate for modeling conditional second moments of the $\{r_t^{\tau}, m_t^{\tau}\}$ process for very small τ's. Alternatively, the problem may be that the true eigenvalues of $var_t Y_{t+1}$ are very close to zero at the quarterly horizon. When taking a linear approximation to $var_t Y_{t+1}$, it would not be surprising that the smallest eigenvalue of the approximate covariance matrices frequently falls below zero.[17]

Using the estimated values for parameters$\{A_1, B_j, j = 1, ..., 8\}$, we construct the equity-premium series EP_t^{τ}

$$EP_t^{\tau} \equiv \frac{E_t R_{t+\tau}^{\tau} - RF_t^{\tau}}{RF_t^{\tau}}, \tag{3.12}$$

the "theoretical equity-premium" series implied by the particular model, which we denote $\widetilde{EP}_t$:

$$\widetilde{EP}_t^{\tau} \equiv -cov_t\,(M_{t+\tau}^{\tau},\, R_{t+\tau}^{\tau})\,,$$

and the "theoretical risk-free rate" series, which we denote $\widetilde{RF}_t$:

$$\frac{1}{E_t M_{t+\tau}^{\tau}} \equiv \widetilde{RF}_t^{\tau} \tag{3.13}$$

[17]In support of this interpretation, we note that when positive-definiteness fails, there is usually only one negative eigenvalue, and its absolute value is usually several orders of magnitude smaller than the other three eigenvalues of the conditional covariance matrix estimate.

4 Results

4.1 Implications for the Equity Premium

In this section we examine the implications of the models for equation (3.3). In order to make our results comparable across different time horizons, we compute annualized continuously-compounded equity premiums, denoted $EP_t^{\tau A}$ and $\widetilde{EP}_t^{\tau A}$ as follows:[18]

$$EP_t^{\tau A} \equiv \frac{4}{\tau} \log \left[EP_t^{\tau} + 1 \right] \tag{4.1}$$

$$\widetilde{EP}_t^{\tau A} \equiv \frac{4}{\tau} \log \left[\widetilde{EP}_t^{\tau} + 1 \right] \tag{4.2}$$

If a model of m_t^τ and the time-series model (3.5) - (3.8) together described the data perfectly, we would find $EP_t^{\tau A} = \widetilde{EP}_t^{\tau A}$ for every date t. No one would expect such an outcome even for a successful model. Rather, we wish to see whether, for any of the pricing models, $\widetilde{EP}_t^{\tau A}$ approximates some of the key properties of $EP_t^{\tau A}$. In particular, we ask whether the following hold:

$$mean \left[EP_t^{\tau A} \right] \approx mean \left[\widetilde{EP}_t^{\tau A} \right] \tag{4.3}$$

$$var \left[EP_t^{\tau A} \right] \approx var \left[\widetilde{EP}_t^{\tau A} \right] \tag{4.4}$$

$$corr \left(EP_t^{\tau A}, \widetilde{EP}_t^{\tau A} \right) >> 0 \tag{4.5}$$

4.1.1 Consumption Measured by Nondurables Plus Services

Table 4.1 summarizes our results for (4.3) and (4.4) when we measure consumption by purchases of nondurables plus services. The first line of the table gives our estimate of the mean and the variance of the equity premium $EP_t^{\tau A}$ at horizons equal to one quarter, one year, two years, and three years. The remainder of the table gives the corresponding moments

[18] The annualization in (4.1) is appropriate, since $EP_t + 1 = E_t \frac{R_{t+\tau}^\tau}{RF_t^\tau}$, where both R_t^τ and $RF_{t+\tau}^\tau$ are gross rates of return. Also, recall that the horizon τ is in units of quarter-years.

of the theoretical equity premium $\widetilde{EP}_t^{\tau A}$ implied by the three preference models. The numbers in parentheses are asymptotic p-values testing whether $(mean[EP_t^{\tau A}] - mean[\widetilde{EP}_t^{A}])$ and $(var[EP_t^{\tau A}] - var[\widetilde{EP}_t^{A}])$ are significantly different from zero. In the case of the means, we use the test statistic

$$Z_{mean} \equiv \frac{\frac{1}{T}\sum_{t=1}^{T}\left(EP_t^{\tau A} - \widetilde{EP}_t^{\tau A}\right)}{std\left[\frac{1}{T}\sum_{t=1}^{T}\left(EP_t^{\tau A} - \widetilde{EP}_t^{\tau A}\right)\right]} \tag{4.6}$$

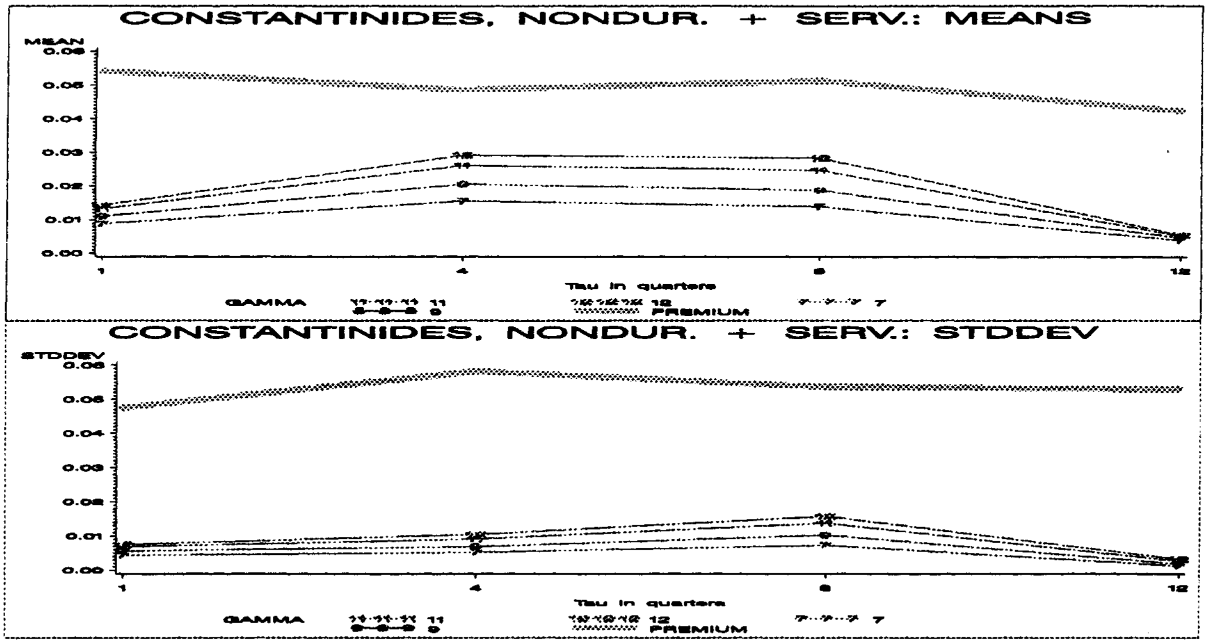
In the case of the variances, we use the test statistic

$$Z_{var} \equiv \frac{\frac{1}{T}\sum_{t=1}^{T}\left([EP_t^{\tau A} - \mu]^2 - \left[\widetilde{EP}_t^{\tau A} - \widetilde{\mu}\right]^2\right)}{std\left[\frac{1}{T}\sum_{t=1}^{T}\left([EP_t^{\tau A} - \mu]^2 - \left[\widetilde{EP}_t^{\tau A} - \widetilde{\mu}\right]^2\right)\right]} \tag{4.7}$$

where μ and $\widetilde{\mu}$ are constants set equal to the sample means of $EP_t^{\tau A}$ and $\widetilde{EP}_t^{\tau A}$, respectively. Under the hypothesis that $E\left[EP_t^{\tau A}\right] = E\left[\widetilde{EP}_t^{\tau A}\right]$, statistic Z_{mean} is asymptotically distributed as a standard normal variate; similarly, if $var\left[EP_t^{\tau A}\right] = var\left[\widetilde{EP}_t^{\tau A}\right]$, statistic Z_{var} is asymptotically standard normal. We compute the standard deviations in the denominators of (4.6) and (4.7) using 12 Newey-West lags for the quarterly, yearly, and two-year horizons, and 16 lags for the three-year horizon.[19] Note that we treat $EP_t^{\tau A}$ and $\widetilde{EP}_t^{A}$ as known data series, not generated series, so the uncertainty in estimating the VAR parameters in 3.5 and 3.8 is not taken into consideration. As a result the standard deviations used in constructing Z_{mean} and Z_{var} are understated.

The time-separable model exhibits both the equity-premium puzzle and the predictability puzzle at all horizons. The observed annualized equity premiums have means between 4.5%

[19]We want the number of Newey-West lags to equal the maximum of the appropriate lag-lengths for $EP_t^{\tau A}$ and for $\widetilde{EP}_t^{A}$. According to equation (3.8), $\widetilde{EP}_t^{A}$ is a function of eight lagged regressors, each of which is serially correlated, so the appropriate lag-length for this variable is at least 9. The τ-horizon equity premium involves τ overlapping observations, so the appropriate equity-premium lag-length is at least $\tau + 1$. We then experimented lag-lengths above $max\,(9, \tau + 1)$ until there were no large changes in the standard deviations.



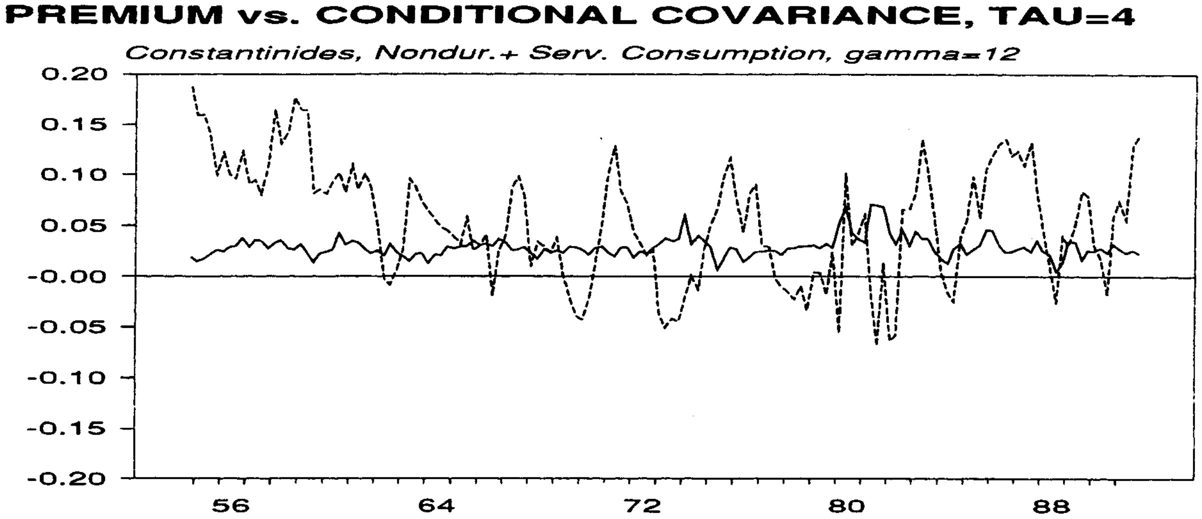
This figure displays means (top panel) and standard deviations (bottom panel) of the annualized equity premium $EP_t^{\tau A}$ and the annualized theoretical equity premium $\widetilde{EP}_t^{\tau A}$ implied by the Constantinides model with $\gamma = 7$ (lowest line), 9, 11, and 12 (highest line), and with consumption measured by expenditures on nondurables plus services. The standard deviations are all scaled by $(\frac{\tau}{4})^{\frac{1}{2}}$ as in (4.4), to facilitate comparison across horizons. The horizons are $\tau = 1$, 4, 8, and 12 quarters.

Figure 4.1: **Equity Premium vs. Theoretical Equity Premium: Constantinides Model, Nondurables Plus Services**

and 6.5%. The time-separable model has difficulty generating a mean equity premium in excess of one percent for any horizon. Furthermore, the equity-premium variance generated by the model is an order of magnitude too small at all horizons.

Let us now turn to the Abel and Constantinides models. Figure 4.1 plots the results reported in Table 4.1 for the Constantinides model. (The pattern for the Abel model is similar.) The upper panel plots the mean of $EP_t^{\tau A}$ (heavy lines) at the four horizons,along with the mean of $\widetilde{EP}_t^{\tau A}$ for four different values of γ. The lower panel in each figure displays the analogous plots for the standard deviations of EP_t^{A} and $\widetilde{EP}_t^{A}$, scaled by $(\frac{\tau}{4})^{\frac{1}{2}}$.[20] The

[20]In equation (4.4), we multiply by $(\frac{\tau}{4})^{\frac{1}{2}}$ to counteract the effect of annualization on the standard deviation of $EP_t^{\tau A}$. If $\log[EP_t^{\tau} + 1]$were the sum of τ i.i.d. random processes, then $std(EP_t^{\tau A})$ would decline at rate $\tau^{\frac{1}{2}}$ as τ increases, but $(\frac{\tau}{4})^{\frac{1}{2}}\, std(EP_t^{\tau A})$ would be constant. While definition (3.12) of EP_t^{τ} does not imply that $\log[EP_t^{\tau} + 1]$ is determined in precisely this way, we find that, in practice, $(\frac{\tau}{4})^{\frac{1}{2}}\, std(EP_t^{\tau A})$ is approximately constant in τ.



This figure displays time-series plots of the annualized equity premium $EP_t^{\tau A}$ (dashed lines) and the annualized theoretical equity premium $\widetilde{EP}_t^{\tau A}$ (solid lines) implied by the Constantinides model with $\gamma = 12$. Consumption is measured by expenditures on nondurables plus services, and the horizon $\tau = 4$ quarters.

Figure 4.2: **Equity Premium vs. Theoretical Equity Premium (Constantinides Model) Using Nondurables Plus Services: Time-Series Plots**

Constantinides model does not fare much better than the time-separable model at the shortest and longest horizons: the means of $\widetilde{EP}_t^A$ are less than 2% when $\tau = 1$ and $\tau = 12$, even with extremely high risk-aversion. According to Table 4.1, these point estimates are significantly below the mean equity premium. However, the model performs somewhat better at the one-year horizon: when the Constantinides model is implemented using one-year returns with $\gamma =$ 12, the mean of $\widetilde{EP}_t^A$ is about 3%, which is somewhat closer to the mean value of EP_t^A than was obtained using quarterly returns. According to Table 4.1, equality of the means of $\widetilde{EP}_t^A$ and EP_t^A is not rejected at the 5% significance level. The point estimate for the standard deviation is substantially below that of the observed equity premium. However, equality of the variances cannot be rejected at the 1% level for the one-year horizon, and cannot be rejected at any conventional significance level at the two year horizon. Similar results obtain for the Abel model with $\gamma = 14$.

While the Constantinides model shows some improved ability to fit conditions (4.3) and (4.4) at the one-year horizon, it fails to satisfy equation (4.5). Figure 4.2 plots the time-series for EP_t^{4A} (dotted lines) and $\widetilde{EP}_t^{4A}$ (solid lines) generated by the Constantinides model with $\gamma = 12$. If (3.3) held exactly, the two series would be identical. As can be seen from the figure, the two series appear to be totally unrelated. As a more formal test, we regress EP_t^{4A} on the $\widetilde{EP}_t^{4A}$ series implied by the Constantinides model, along with the linear and quadratic trends.[21] In order to account for the high serial persistence in these series, we compute standard errors using 12 Newey-West lags. For both models, the slope coefficient on $\widetilde{EP}_t^{4A}$ is insignificantly different from zero (p-value of .43), and the point estimate is actually negative.[22] This regression evidence confirms the visual impression of Figure 4.2.

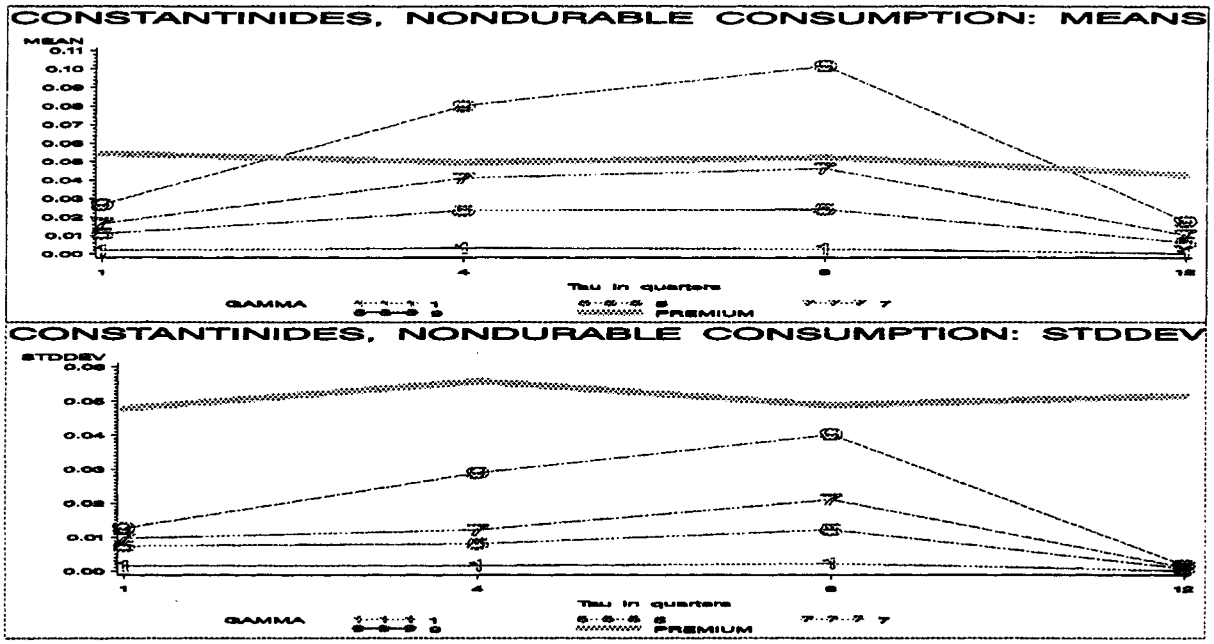
To summarize, when consumption is measured as nondurables plus services: (1) there is a modest improvement in the models' ability to match the mean and variance of the equity premium if the horizon is lengthened from one quarter to one year, but (2) time-series variation in the observed equity premium is explained by the model to any significant degree. This would appear to be a rather disappointing result for consumption-based pricing models. In the next section, we consider whether this result is due to the use of nondurables plus services as our measure of consumption.

4.1.2 Alternative Specifications of Consumption

In the previous section measure consumption as consumer expenditures on nondurable goods plus consumer services, as measured by the Bureau of Economic Analysis. This is the consumption series used in most consumption-based asset pricing research since the original work of Hansen and Singleton (1982). As discussed in the introduction, we believe that our results may not be robust to alternative specifications of consumption. To look at this possibility, we first replicate our analysis with consumption measured by nondurables only. Table 4.2 is analogous to Table 4.1, except that consumption is measured by purchases of nondurable consumption goods. (That is, the services component is omitted from the consumption data.)

[21]We include the trend terms to accommodate the slight "U" shape in EP_t^{4A}. This pattern is due almost entirely to the secular rise in the nominal two-year risk free rate over this period. (Nominal equity returns do not display any pronounced trend in post-war data.) Both the linear and quadratic trend terms enter significantly. As in Table 4.1, we do not take into consideration the fact that we are using generated regressors, so the standard errors are understated.

[22]The Abel model yields similar results. When we regress EP_t^{4A} on the $\widetilde{EP}_t^{4A}$ series implied by the "best" Abel model ($\gamma = 14$), along with the linear and quadratic trends, the coefficient on $\widetilde{EP}_t^{4A}$ is negative, and insignificantly different from zero.

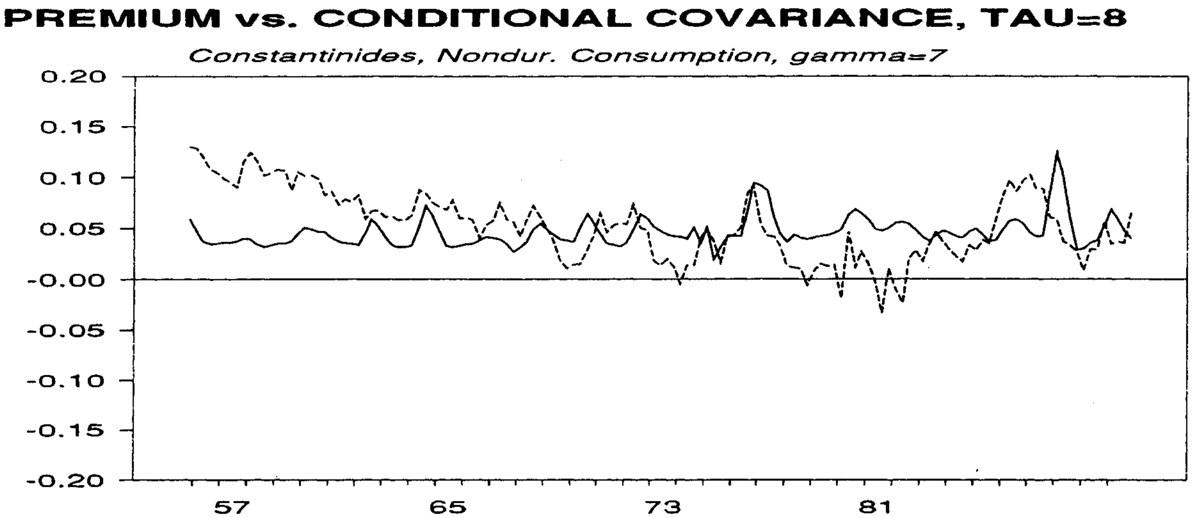


This figure displays means (top panel) and standard deviations (bottom panel) of the annualized equity premium $EP_t^{\tau A}$ and the annualized theoretical equity premium $\widetilde{EP}_t^{\tau A}$ implied by the Constantinides model with $\gamma = 1$ (lowest line), 5, 7, and 9 (highest line), and with consumption measured by purchases of consumer nondurables. The standard deviations are all scaled by $(\frac{\tau}{4})^{\frac{1}{2}}$ as in (4.4), to facilitate comparison across horizons. The horizons are $\tau = 1$, 4, 8, and 12 quarters.

Figure 4.3: **Equity Premium vs. Theoretical Equity Premium: Constantinides Model, Nondurables**

The results for the time-separable model improve somewhat at the highest levels of risk aversion for the one- and two year horizons. With the Abel and Constantinides models, however, the difference in model performance is striking. Figure 4.3 plots the point estimates reported in Table 4.2 for the Constantinides model. Note that the theoretical equity premium matches the mean equity premium observed in the data at both the one- and two-year horizons with $\gamma = 7$. (Comparable results obtain in the Abel model with $\gamma = 10$.)[23] According to the p-values in Table 4.2, the means of the theoretical equity premiums for these models are insignificantly different from the means of the observed equity premiums at any conventional significance level The variances are insignificantly different at the two-year horizon. Figure 4.4 plots the EP_t^{8A}

[23]In the Abel model, these results require extremely high risk aversion: $\gamma = 10$ implies a mean coefficient of relative risk aversion of approximately 49. In contrast, the level of risk-aversion implied by $\gamma = 7$ in the Constantinides model is less extreme. As noted in section 2.3, the steady-state coefficient of relative risk aversion in the Constantinides model is not too much higher than γ.



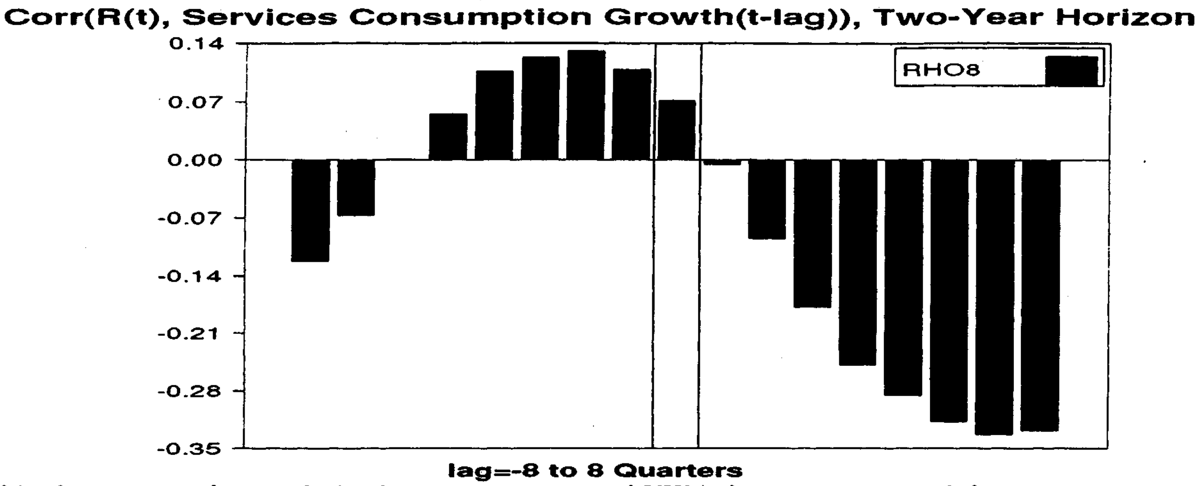
This figure displays time-series plots of the annualized equity premium $EP_t^{\tau A}$ (dashed lines) and the annualized theoretical equity premium $\widetilde{EP}_t^{\tau A}$ (solid lines) implied by the Constantinides model with $\gamma = 7$. Consumption is measured by expenditures on consumer nondurables, and the horizon $\tau = 8$ quarters.

Figure 4.4: **Equity Premium vs. Theoretical Equity Premium: Time-Series Plots, Nondurable Consumption**

against the $\widetilde{EP}_t^{8A}$ series implied by the Constantinides model with $\gamma = 7$. Unlike Figure 4.2, these plots appear to display clear (albeit imperfect) co-movement between the theoretical and observed equity premium series. The main discrepancy is that the theoretical equity premium $\widetilde{EP}_t^{\tau A}$ does not capture the secular decline in the observed equity premium from 1954 through 1980: the theoretical series is too low in the 1950's and too high in the early 1980's. However, our construct for the theoretical premium does appear to capture some of the cyclical fluctuation in the equity premium: note equity-premium peaks in 1956, 1965, and 1976-77, as well as the sharp fall-offs in 1977-78 and 1988. On the whole, Figure 4.4 provides some evidence that consumption-based models can generate time-varying risk premiums appropriate to the observed data. When EP_t^{8A} is regressed on this $\widetilde{EP}_t^{8A}$ process, along with a linear and quadratic time-trend, the coefficient on $\widetilde{EP}_t^{\tau A}$ is 0.364, with a standard error of 0.161 (significant at the 5% marginal significance level).[24]

We conclude from this evidence that the rather tepid performance of our consumption-based

[24]When the comparable regression is performed for the Abel model with $\gamma = 10$, the coefficient is 0.313, with a standard error of 0.129. Again, the coefficient is significantly positive.

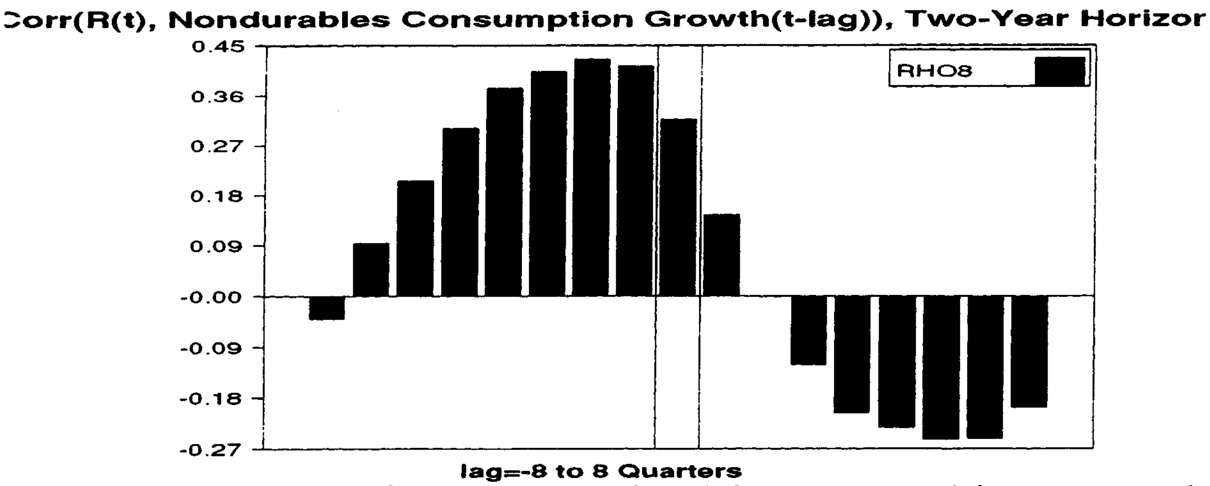


This plot presents the correlation between two-year real VW index returns at t and the two-year growth rate of real and services consumption at $(t - lag)$ for lag = -8 through +8 quarters. The vertical lines in the center of the graph indicate the contemporaneous correlation

Figure 4.5: **Correlation between Real Two-Year VW Index Returns at t and Real Two-Year Services Consumption Growth**

models with the standard measure of consumption (at least at one-and two-year horizons) is due to the consumption-services component of the consumption measure. This should not be all that surprising, since the consumption services data are less well-correlated with equity returns than are the data on purchases of nondurables. Consider Figures 4.5 and 4.6, which plot the correlations between two-year equity returns and the two-year growth rates of consumption services and consumption of nondurables, respectively. According to Figure 4.5, the maximum contemporaneous correlation between the growth of consumer services and real equity returns at any of the horizons tested is 0.09 (at the one-year horizon). The maximum correlation at any lead or lag is 0.22 (also for the one-year horizon, when growth of consumer services leads returns by two quarters). In contrast, the contemporaneous correlation between nondurables growth the equity return series is 0.26 for the one-year horizon (0.30 for the two-year horizon), and the maximum correlation (again, when consumption-growth has a two-quarter lead) is above .40. Evidently, the data on consumer services provided by the BEA has little explanatory power for equity returns.

We conclude this section by replicating the analysis with consumption measured by total consumption purchases. For completeness, we include results for horizons ranging from one quarter to three years. As discussed above, this measure of consumption is inappropriate

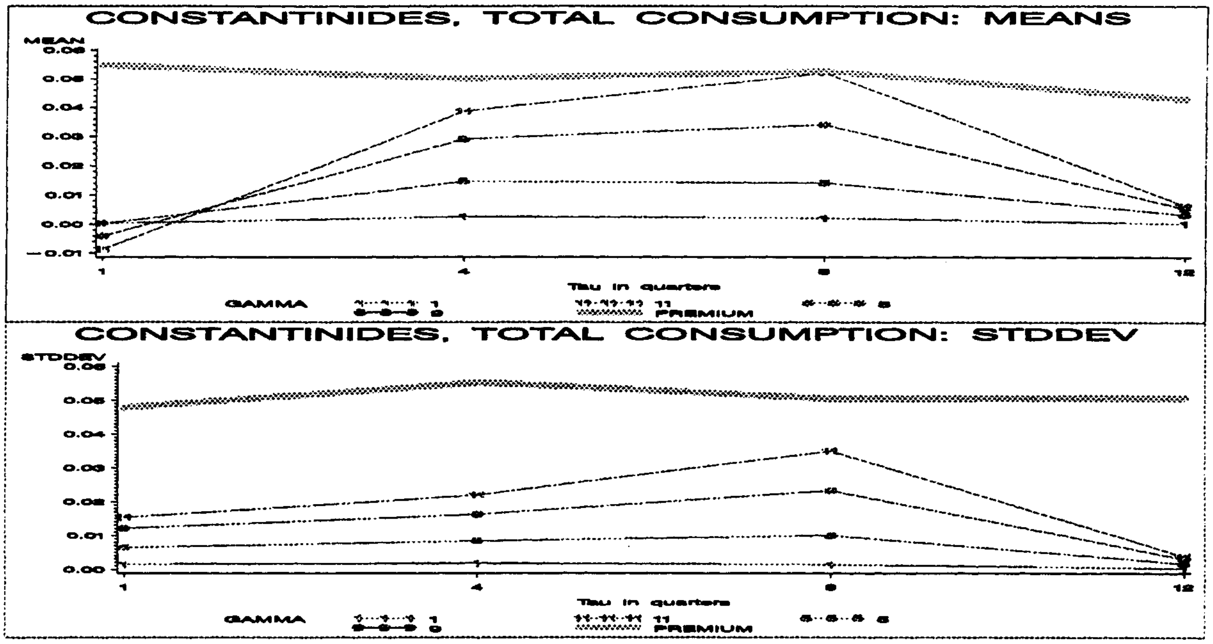


This plot presents the correlation between two-year real VW index returns at t and the two-year growth rate of real and services consumption at $(t - lag)$ for lag = -8 through +8 quarters. The vertical lines in the center of the graph indicate the contemporaneous correlation

Figure 4.6: **Correlation between Real Two-Year VW Index Returns at t and Real Two-Year Services Consumption Growth**

for short horizons, but may give some indication of the role of durables in longer horizon returns. These results are displayed in Table 4.3, which is analogous to Table 4.2, and in Figure 4.7, which is analogous to Figure 4.3. The results look remarkably like the results when only nondurables purchases are used. The Constantinides model comes quite close to replicating both the mean and the variance of the equity premium at the two-year horizon when $\gamma = 11$; The Abel model behaves similarly when $\gamma = 14$. Table 4.3 shows that these results are statistically significant. For both of these models, at both the one- and two-year horizons, neither the mean nor the variance of $\widetilde{EP}_t^A$ is significantly different from the corresponding moments of EP_t^A.

As is the case when only nondurables are used, the time-series properties of $EP_t^{\tau A}$ (dotted lines) are replicated to some extent by the behavior of $\widetilde{EP}_t^{\tau A}$ in these models. Figure 4.8 is analogous to Figure 4.4. Again, the theoretical equity premium series generated by the Constantinides model cannot replicate the long-run secular movement of the observed equity premium, but does appear to mimic the cyclical movements. Unlike the models that use only nondurable consumption, the patterns in Figure 4.8 do not emerge as significant when tested using formal statistical methods. When we regress EP_t^{8A} on $\widetilde{EP}_t^{8A}$ from either model, along with the linear and quadratic trends, the coefficients on $\widetilde{EP}_t^{\tau A}$ are positive, but insignificant



This figure displays means (top panel) and standard deviations (bottom panel) of the annualized equity premium $EP_t^{\tau A}$ and the annualized theoretical equity premium $\widetilde{EP}_t^{\tau A}$ implied by the Constantinides model with $\gamma = 1$ (lowest line), 5, 9, and 11 (highest line), and with consumption measured by total consumption expenditures. The standard deviations are all scaled by $\left(\frac{\tau}{4}\right)^{\frac{1}{2}}$ as in (4.4), to facilitate comparison across horizons. The horizons are $\tau = 1$, 4, 8, and 12 quarters.

Figure 4.7: **Equity Premium vs. Theoretical Equity Premium: Constantinides Model, Total Consumption**

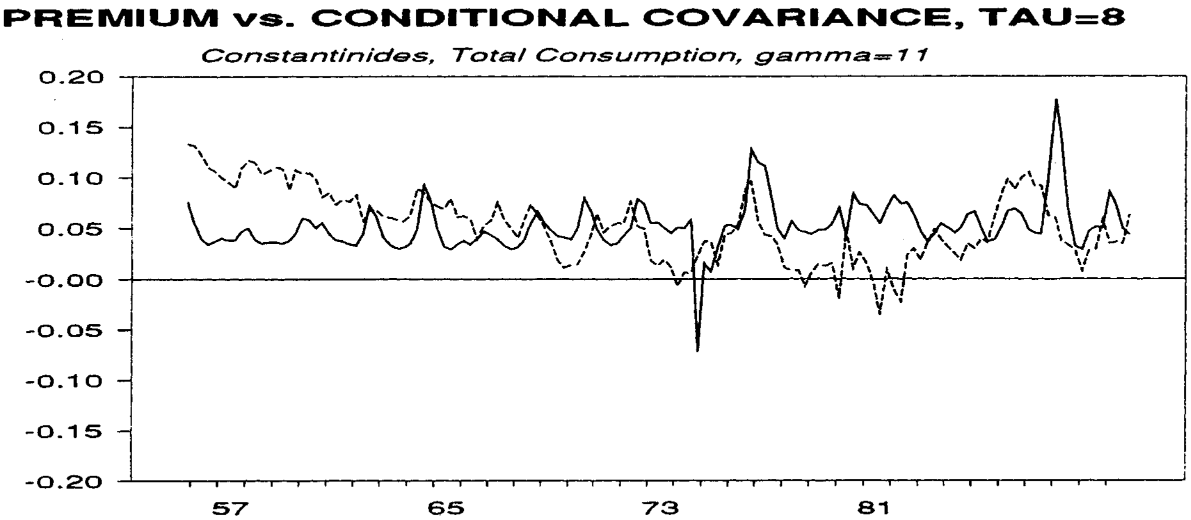
at the 10% marginal significance level.[25]

4.2 Implications for the Risk-Free Rate

We now test equation (3.4) for the models studied in the previous section. We ask whether the means and standard deviations of $\widetilde{RF}_t^{\tau}$ match those of RF_t^{τ}, and whether these two series have substantial positive correlation. As with the equity premium, we annualize by setting

$$RF_t^{\tau A} \equiv \frac{4}{\tau} \log\left(RF_t^{\tau}\right)$$

[25]For the Constantinides model, the estimated coefficient on $\widetilde{EP}_t^{8A}$ is 0.137 with a standard error of 0.105. The same regression for the Abel model yields a coefficient estimate of 0.137 with a standard error of 0.104. As before, we compute standard errors using 12 Newey-West lags.



This figure displays time-series plots of the annualized equity premium $EP_t^{\tau A}$ (dashed lines) and the annualized theoretical equity premium $\widetilde{EP}_t^{\tau A}$ (solid lines) implied by the Constantinides model with $\gamma = 11$. Consumption is measured by total consumption expenditures, and the horizon $\tau = 8$ quarters.

Figure 4.8: **Equity Premium vs. Theoretical Equity Premium: Time-Series Plots**

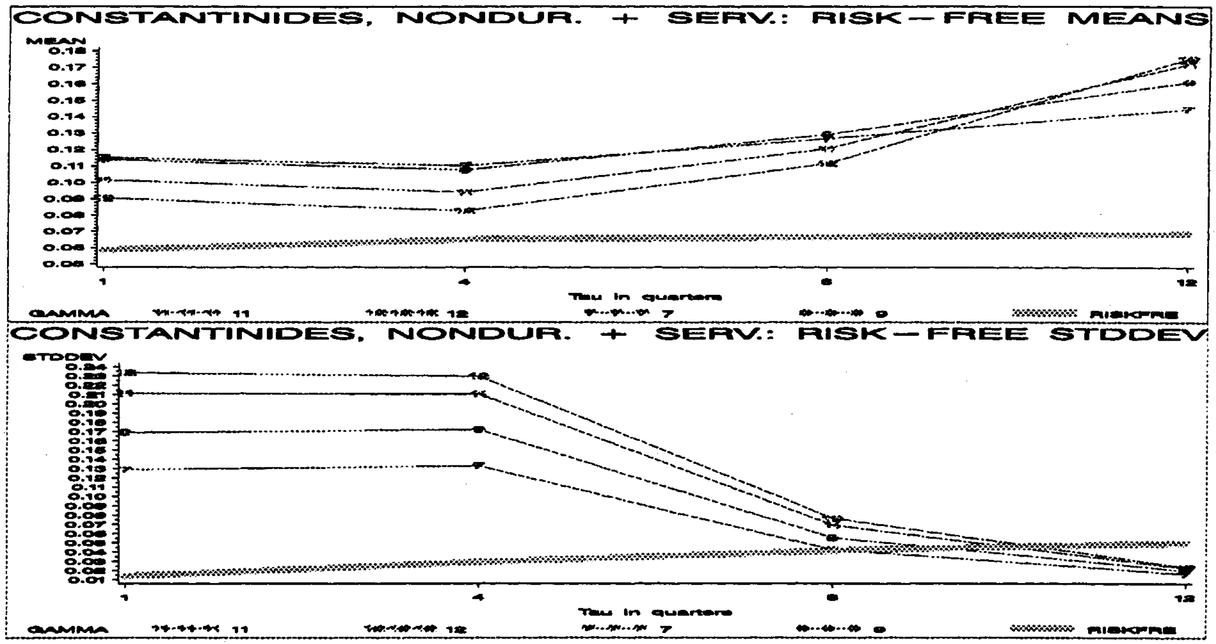
$$\widetilde{RF}_t^{\tau A} \equiv \frac{4}{\tau} \log \left(\widetilde{RF}_t^{\tau} \right)$$

4.2.1 Consumption Measured by Nondurables Plus Services

Table 4.4 displays our results for the risk-free rate when consumption is measured by expenditures on nondurables plus services. The first line of the table gives our estimate of the mean and the variance of the observed nominal risk- free rate at the four horizons. The remainder of the table gives the corresponding moments of the theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ implied by our three preference models. The numbers in parentheses are asymptotic p-values testing whether $(mean[RF_t^{\tau A}] - mean[\widetilde{RF}_t^{A}])$ and $(var[RF_t^{\tau A}] - var[\widetilde{RF}_t^{A}])$ are significantly different from zero.[26]

Our results for the time-separable model clearly illustrate the risk-free rate puzzles: For all γ's up to 50, all horizons, and both measures of consumption, the means and variances of the risk-free rates implied by the models vastly exceed the values observed in the data. The Abel

[26]As in Table 4.1, we construct the p-values using 12 Newey-West lags for the quarterly, yearly, and two-year horizons, and 16 lags for the three-year horizon. The fact that $\widetilde{RF}_t^{A}$ is a generated series is not taken into consideration in computing the p-values.



The top panel plots the mean of the annualized risk-free rate $RF_t^{\tau A}$ at the four horizons $\tau = 1, 4, 8,$ and 12 quarters (heavy line), along with the means of the annualized theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ for the Constantinides model with $\gamma = 7$, 9, 11, and 12. The bottom panel plot $(\frac{\tau}{4})^{\frac{1}{2}}$ times the standard deviation of $RF_t^{\tau A}$ at the four horizons (heavy line) against the corresponding statistic for $\widetilde{RF}_t^{\tau A}$ for the four values of γ. Consumption is measured by expenditures on nondurables plus services.

Figure 4.9: **Risk-Free Rate vs. Theoretical Risk-Free Rate: Constantinides Model, Nondurables Plus Services Consumption**

and Constantinides models do not perform much better at matching the mean risk-free rate. In Figure 4.9, we plot the point estimates from Table 4.4 for the Constantinides model. (The patterns for the Abel model are similar.) The top panels in Figure 4.9 plots the mean of $RF_t^{\tau A}$ (heavy lines) at the four horizons, along with the means of $\widetilde{RF}_t^{\tau A}$ for the Constantinides model using four values of γ. The bottom panels plot $(\frac{\tau}{4})^{\frac{1}{2}}$ times the standard deviation of $RF_t^{\tau A}$ against the corresponding statistic for $\widetilde{RF}_t^{\tau A}$. As demonstrated by Figure 4.9, the model fails to match the mean of the risk-free rate (although the failure is less dramatic than with the time-separable model). As with the time-separable model, lengthening the horizon increases the mean theoretical risk-free rate counterfactually.

While these models have difficulty matching the mean of the risk-free rate at any horizon, the excessive volatility of the theoretical risk-free rate does become less of a problem at longer horizons with the Abel and Constantinides models. This is of considerable interest, since these models have been criticized for implying counterfactually high interest-rate variability at short horizons. Figure 4.9 shows that, for all values of γ studied, the variance of $\widetilde{RF}_t^{\tau A}$ actually approximates the variance of the observed $RF_t^{\tau A}$ for a horizon τ somewhere between two and three years.

It would appear, then, that the problem of excessive variability in the risk-free rate implied by time-nonseparable models is primarily a short-horizon problem. The intuition behind this result is that the variability of $\widetilde{RF}_t^{\tau A}$ is determined by the variability of $\frac{1}{\tau}\left(E_t M_{t+\tau}^{\tau}\right)^{-1}$. (See equation (3.4).) Let us use the decomposition

$$M_{t+\tau}^{\tau} = \prod_{i=1}^{\tau} M_{t+i}^{1}.$$

For purposes of exposition, let us ignore Jensen's inequality, and consider

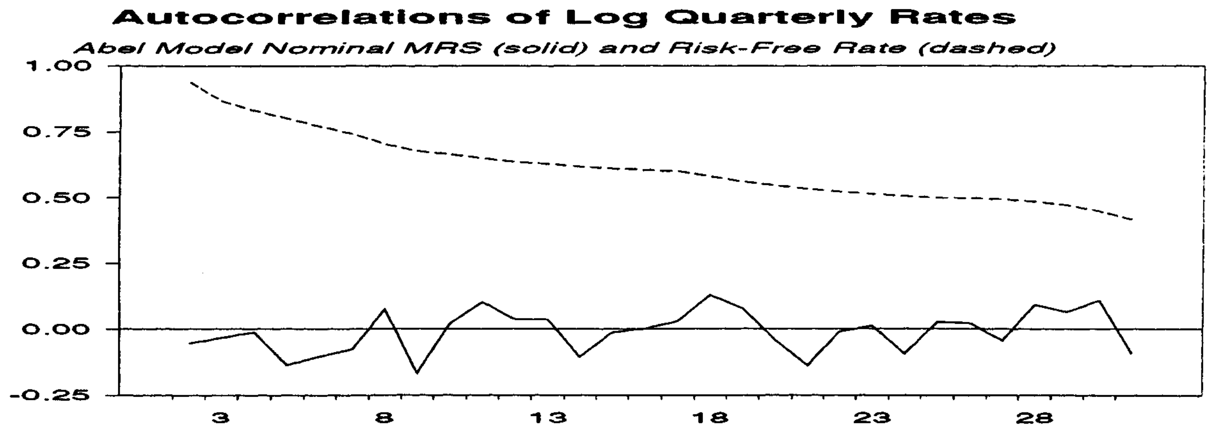
$$\frac{1}{\tau} E_t\left[\log(M_{t+\tau}^{\tau})^{-1}\right] = \frac{-1}{\tau}\sum_{i=1}^{\tau} E_t\left[\log(M_{t+i}^{1})\right]. \tag{4.8}$$

According to (4.8), the variability of $\frac{1}{\tau}E_t\left[\log(M_{t+\tau}^{\tau})^{-1}\right]$ is determined by the *predictability* of M_{t+i}^{1}, the one-period marginal rate of substitution prevailing i periods in the future. If M_{t+i}^{1} is highly predictable (for example, if $log(M_{t+i}^{1})$ were a random walk), then the variability of $E_t\left[\log(M_{t+i}^{1})\right]$ would not decline substantially as τ gets big. If, on the other hand, M_{t+i}^{1} displays rapid mean-reversion, then we would find that $E_t\left[\log(M_{t+i}^{1})\right] \cong E\left[\log(M_{t+i}^{1})\right]$ for moderately large values of i. In that case, most of the terms in the right-hand side of (4.8) would be approximately non stochastic, and the variance of $\frac{1}{\tau}E_t\left[\log(M_{t+\tau}^{\tau})\right]$ would drop off rapidly as τ increases. A similar intuition holds for the observed risk-free rate RF_t^{τ}. Let us assume, for purposes of exposition, that long interest rates satisfy the following version of the expectations hypothesis:

$$\log RF_{t+\tau}^{\tau A} = \frac{1}{\tau}\sum_{i=1}^{\tau} E_t\left[\log(RF_{t+i}^{1})\right]. \tag{4.9}$$

According to (4.9), the variability in $RF_{t+\tau}^{\tau A}$ is determined by the predictability of the future one-period interest rates RF_{t+i}^{1}. The patterns displayed in the bottom panel of Figure 4.9 can be explained if, as i increases, the predictability of RF_{t+i}^{1} attenuates more slowly than the predictability of M_{t+i}^{1}.

In Figure 4.10, we compare the i-step ahead predictability of $log\,(RF_t^1)$ with the i-step ahead predictability of the $log\,(M_t^1)$ series implied by the Abel model. (The log transformation makes



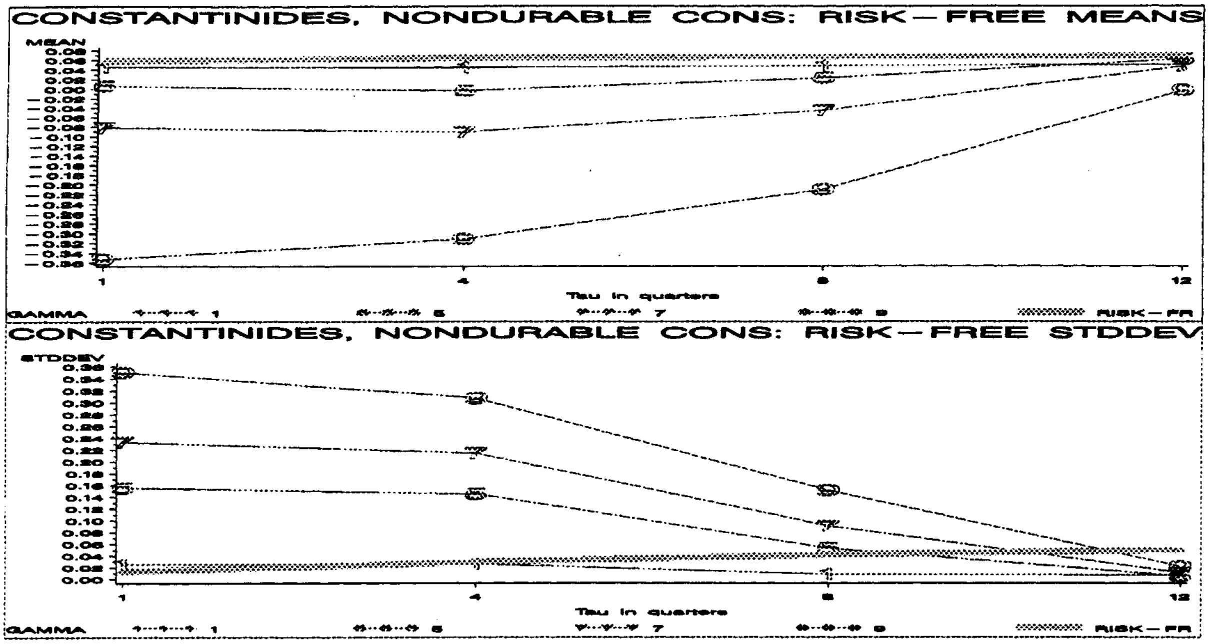
This figure plots the first thirty autocorrelations of the log nominal quarterly marginal rate of substitution $log\left(M_t^1\right)$ in the Abel model, and of the observed log quarterly risk-free rate $log\left(RF_t^1\right)$ (dashed line). In computing the marginal rates of substitution, we measure consumption as expenditures on nondurables plus services.

Figure 4.10: **Autocorrelations of the Risk-Free Rate and of the Marginal Rate of Substitution (Abel Model)**

the autocorrelations invariant to γ.) For each series, we display the first thirty autocorrelations. Notice that the autocorrelations for $log\left(RF_t^1\right)$ die out much more slowly than for $log\left(M_t^1\right)$. While $log\left(RF_t^1\right)$ behaves as a near-random walk, $log\left(M_t^1\right)$ displays no long-run predictability. These results explain why $var\left(\widetilde{RF}_t^{\tau A}\right)$ falls so much more rapidly than $var\left(\widetilde{RF}_t^{\tau A}\right)$ as τ is lengthened from one quarter to twelve quarters.

4.2.2 Alternative Specifications of Consumption

In tables 4.5 - 4.6, and in Figures 4.11 - 4.12, we display implications of the models for the risk-free rate when consumption is measured either as expenditures on nondurables or as total consumption expenditures. The qualitative behavior of $var\left(\widetilde{RF}_t^{\tau A}\right)$ is similar to the case of nondurables plus services: For all models, this variance is decreasing in τ, but this effect is far more pronounced in the time-nonseparable models than with time-separable preferences. However, the behavior of $mean\left(\widetilde{RF}_t^{\tau A}\right)$ is rather different. With either Abel or Constantinides preferences, $mean\left(\widetilde{RF}_t^{\tau A}\right)$ is much lower with either nondurable consumption or total consumption than when the standard nondurables-plus-services measure is used. At

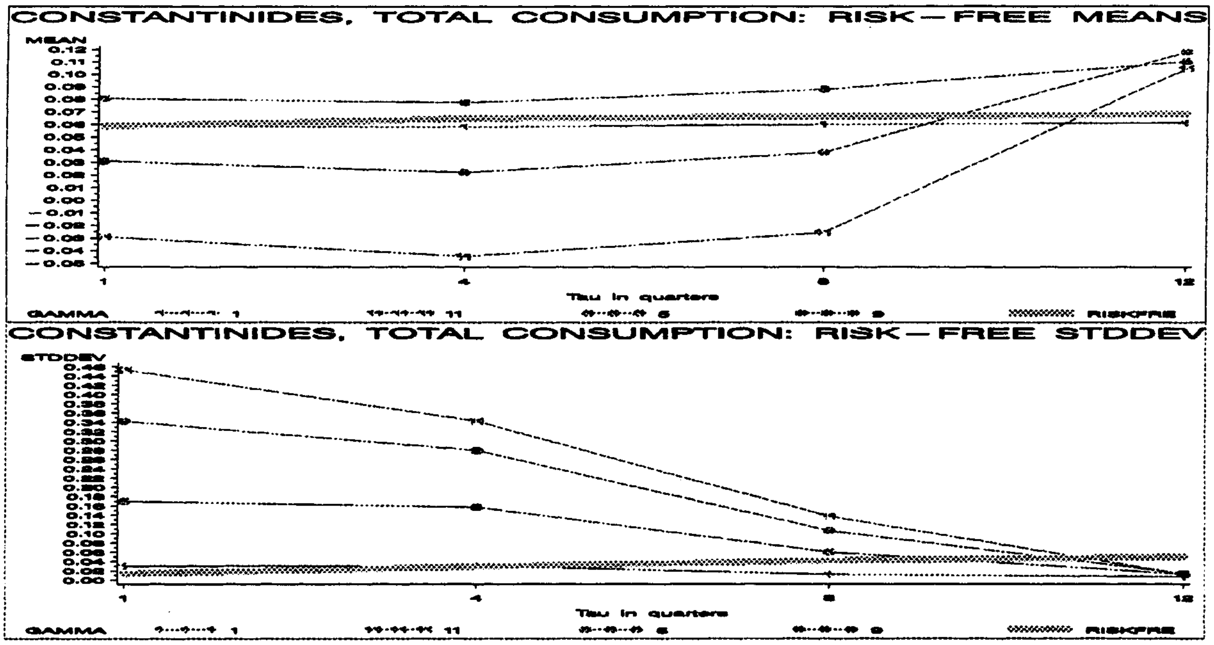


These figures display means and standard deviations of the annualized risk-free rate $RF_t^{\tau A}$ and the annualized theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ implied by the Constantinides model with $\gamma = 1, 5, 7$, and 9, and with consumption measured by consumption expenditures on nondurables. The standard deviations are all scaled by $\left(\frac{\tau}{4}\right)^{\frac{1}{2}}$ as in (4.4), to facilitate comparison across horizons. The horizons are $\tau = 1, 4$, 8, and 12 quarters.

Figure 4.11: **Risk-Free Rate vs. Theoretical Risk-Free Rate: Constantinides Model, Nondurable Consumption**

the quarterly horizon, this mean is actually *below* the mean of the observed $RF_t^{\tau A}$ series for most specifications, and is extremely negative for the higher γ's. As with nondurable-plus-services consumption, $mean\left(\widetilde{RF}_t^{\tau A}\right)$ tends to increase with the horizon τ, approaching $mean\left(RF_t^{\tau A}\right)$ at a horizon between two and three years.

The explanation for these results is that $mean\left(\widetilde{RF}_t^{\tau A}\right)$ is increasing in the mean growth rate of consumption (higher consumption growth increases the incentive of agents to borrow from the future) and decreasing in the variability of the consumption growth rate (higher variability accentuates the precautionary motive for saving). The mean quarterly growth rate of expenditures on nondurables from 1947 to 1994 is 0.26%, and the standard deviation of this growth rate is 0.80%. In contrast, the mean growth rate for nondurables plus services over this period is 0.45%, with a standard deviation of only 0.56%, so both effects tend to reduce $mean\left(\widetilde{RF}_t^{\tau A}\right)$ when expenditures on services are eliminated from the consumption measure.



These figures display means and standard deviations of the annualized risk-free rate $RF_t^{\tau A}$ and the annualized theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ implied by the Constantinides model with $\gamma = 1, 5, 9$, and 11, and with consumption measured by expenditures on nondurables plus services. four versions of the Constantinides model with consumption measured by total consumption expenditures. The standard deviations are all scaled by $\left(\frac{\tau}{4}\right)^{\frac{1}{2}}$ as in (4.4), to facilitate comparison across horizons. The horizons are $\tau = 1, 4, 8$, and 12 quarters.

Figure 4.12: **Risk-Free Rate vs. Theoretical Risk-Free Rate: Constantinides Model, Total Consumption**

With total consumption expenditures, the mean growth rate is 0.49%, with a standard deviation of 0.77%, so the reduction in $mean\left(\widetilde{RF}_t^{\tau A}\right)$ when this series is used is due solely to the enhanced precautionary demand.

While the alternative measures of consumption allow the models to match the mean and standard deviation of the risk-free rate for a horizons between two and three years, the models do not capture any of the time-series variation in the observed risk-free rate. As in section 4.1, we regress the observed risk- free rate $RF_t^{\tau A}$ on the theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ along with a constant and linear and quadratic time-trends. Our point estimates for the coefficient on $\widetilde{RF}_t^{\tau A}$ are in all cases negative; the estimates are insignificant for the two- year horizon, but are significantly negative for the three year horizon. We conclude that these models do not succeed in replicating the time-series movement of the nominal risk-free rate.

5 Conclusions

In this paper, we ask whether consumption-based models are better able to match observed equity premiums and risk-free rates as the horizon lengthens. We find that time separable utility fails at all horizons with all measures of consumption. When we follow the prevailing literature and measure consumption as expenditures on consumer nondurables plus services, the Abel and Constantinides models display a marginal improvement in fit when the horizon is lengthened to one year. However, significant features of the data are not captured by the models. In contrast, when consumption is measured either as expenditures on nondurables or as total consumption purchases, versions the Abel and the Constantinides can can match the mean and the variance of the observed equity premium at the two-year horizon, and can capture a good deal of the time-variation of the equity premium in post-war data. In addition, these measures of consumption allow the models more success matching the first and second moments of the observed risk-free rate. The time-variation in the risk-free rate is not captured by any of the models at any horizon.

These results are intruiging. They suggest that the equity premium and risk-free rate puzzles are less puzzling for one-year cumulative equity returns than for quarterly returns, and can be substantially resolved for two-year returns. However, this conclusion emerges only if we throw out the BEA series for expenditures on consumer services, or if we include expenditures on durables in the measure of consumption. If one is convinced that measured expenditures on nondurables plus services is the correct measure of consumption, then this result would simply constitute a rejection of the consumption-based pricing paradigm. Alternatively, our results could be evidence that consumer services are separable from nondurables in agents' preference orderings. As described in the introduction, however, we are inclined to view our results as indicating that the BEA data on consumer services is flawed, and that these data do not provide a good empirical analogue of the consumption concept appropriate to these models. We regard this issue as an important topic for future research.

The improvement in performance when total consumption expenditures is used suggests that the durables component of consumption is important for longer-horizon returns. A more careful treatment would be desirable, in which the service flow from consumer durables is explicitly modeled.[27] Finally, a major puzzle is why all models dramatically fail to match the equity-premium at the three-year horizon. Market frictions could disrupt the linkage between asset returns and the consumption-based pricing kernel at short horizons. It is not clear what economic model would similarly disrupt this linkage at the very long horizons.

[27]Eichenbaum and Hansen (1990) model preferences over durables in this way, but they only consider the one-month investment horizon.

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Appendix

A Construction of the Data

The total quarterly real non-durable, durable, services, and total consumption series, the deflators for each of the three components series, and the population series (GPOP) were extracted from CITIBASE for the 1947:1-1994:1 period.

Monthly VW index returns were obtained from CRSP, and were cumulated to obtain quarterly returns. One month T-Bill returns were taken from the CRSP RISKFREE file. One, two, and three year nominally risk-free rates were computed as the returns to one-, two-, and three-year zero-coupon bonds, computed from the Fama-Bliss data in CRSP.

The default spread, term spread, and dividend yield are calculated following Fama and French (1989), using data supplied by Roger Ibbotsen.

Table 3.1: **Number of non-positive-definite approximate covariance matrices generated by the linear model (3.5) and (3.8)**

For each model, this gives the number of observations for which we obtained non-positive-definite estimates for the matrix H_{t+1}, defined in 3.6. The columns labeled "Nondur + Serv." are for the models where consumption is measured as expenditures on consumer nondurables plus services; the columns labeled "Nondurables" are for the models where consumption is measured as nondurable consumption expenditures; the columns labeled "Total" are for the models where consumption is measured as total consumption expenditures. In the Constantinides model, the maximum values of γ for which all marginal utilities were positive equaled 12 for nondurables plus services, 9 for nondurable consumption, and 11 for total consumption.

	Time-Separable Preferences											
	Nondur. + Serv.				Nondurables				Total			
	Horizon (Yrs.)				Horizon (Yrs)				Horizon (Yrs)			
γ	0.25	1	2	3	0.25	1	2	3	0.25	1	2	3
20	43	1	0	0	46	1	1	0	71	3	0	0
30	44	1	0	0	47	3	2	0	64	1	0	0
40	48	2	0	0	47	5	1	0	63	2	0	0
50	52	2	0	0	44	5	1	0	63	1	0	0
	Abel Preferences											
	Nondur. + Serv.				Nondurables				Total			
	Horizon (Yrs.)				Horizon (Yrs)				Horizon (Yrs)			
γ	0.25	1	2	3	0.25	1	2	3	0.25	1	2	3
2	49	1	0	0	46	1	0	0	59	1	0	0
6	48	1	0	0	46	2	2	0	67	0	0	0
10	47	1	0	0	49	5	2	0	67	0	0	0
14	47	1	0	0	51	5	0	0	65	1	0	0
	Constantinides Preferences											
	Nondur. + Serv.				Nondurables				Total			
	Horizon (Yrs.)				Horizon (Yrs)				Horizon (Yrs)			
γ	0.25	1	2	3	0.25	1	2	3	0.25	1	2	3
1	49	2	0	0	53	1	0	0	57	5	0	0
5	50	1	0	0	57	1	0	0	63	1	0	0
7	53	0	0	0	55	1	0	0	63	3	0	0
9	54	0	0	0	52	2	0	0	64	2	0	0
11	56	0	0	0					66	5	0	0
12	56	1	2	0								

Table 4.1: **Tests of Equity-Premium Model: Nondurables + Services**

This table displays means and variances of $EP_t^{\tau A}$, along with the corresponding moments of the theoretical equity premium $\widetilde{EP}_t^{\tau A}$ implied by the time-separable model, the Abel (1990) model, and the Constantinides (1990) model. Each model is evaluated at four values for the curvature parameter γ, with consumption measured as expenditures on nondurables plus services. The numbers in parentheses are asymptotic p-values testing whether the means and variance of $EP_t^{\tau A}$ equal the corresponding moments of $\widetilde{EP}_t^{\tau A}$. Specifically, the numbers in parentheses the two-sided p-values for the standard normal distribution evaluated at Z_{mean} (for the columns labeled "mean") and Z_{var} (for the columns labeled "var"), as defined in equations (4.6) and (4.7)

	HORIZON							
	quarterly		1-year		2-year		3-year	
γ	mean	var	mean	var	mean	var	mean	var
	Estimated Equity Premium							
	0.0621	0.0075	0.0556	0.0036	0.0551	0.0015	0.0478	0.0011
	Time-Separable Model, Nondurables + Services							
20	0.0037	0.00002	0.0063	0.00001	0.0043	0.00000	0.0010	0.00000
	(0.0000)	(0.0000)	(0.0000)	(0.0243)	(0.0000)	(0.2569)	(0.0000)	(0.3616)
30	0.0051	0.00005	0.0084	0.00002	0.0049	0.00000	0.0009	0.00000
	(0.0001)	(0.0000)	(0.0000)	(0.0247)	(0.0000)	(0.2571)	(0.0000)	(0.3616)
40	0.0063	0.00010	0.0100	0.00002	0.0050	0.00000	0.0007	0.00000
	(0.0001)	(0.0000)	(0.0001)	(0.0241)	(0.0000)	(0.2573)	(0.0000)	(0.3616)
50	0.0072	0.00010	0.0111	0.00003	0.0049	0.00000	0.0006	0.00000
	(0.0001)	(0.0000)	(0.0001)	(0.0256)	(0.0000)	(0.2573)	(0.0000)	(0.3616)
	Abel Model, Nondurables + Services							
2	0.0023	0.00000	0.0043	0.00000	0.0032	0.00000	0.0010	0.00000
	(0.0003)	(0.0002)	(0.0000)	(0.0342)	(0.0000)	(0.2300)	(0.0000)	(0.3076)
6	0.0069	0.00008	0.0130	0.00003	0.0101	0.00001	0.0032	0.00000
	(0.0008)	(0.0002)	(0.0002)	(0.0352)	(0.0000)	(0.2361)	(0.0000)	(0.3081)
10	0.0109	0.00020	0.0215	0.00008	0.0170	0.00005	0.0044	0.00000
	(0.0019)	(0.0002)	(0.0026)	(0.0371)	(0.0003)	(0.2512)	(0.0000)	(0.3087)
14	0.0144	0.00040	0.0310	0.00018	0.0255	0.00012	0.0049	0.00000
	(0.0036)	(0.0002)	(0.0250)	(0.0415)	(0.0054)	(0.2832)	(0.0000)	(0.3093)
	Constantinides Model, Nondurables + Services							
7	0.0087	0.00008	0.0155	0.00003	0.0139	0.00003	0.0039	0.00000
	(0.0018)	(0.0001)	(0.0005)	(0.0209)	(0.0000)	(0.2457)	(0.0000)	(0.2813)
9	0.0109	0.00014	0.0203	0.00005	0.0187	0.00005	0.0046	0.00000
	(0.0028)	(0.0001)	(0.0025)	(0.0223)	(0.0002)	(0.2612)	(0.0000)	(0.2816)
11	0.0130	0.00020	0.0257	0.00008	0.0246	0.00009	0.0052	0.00000
	(0.0043)	(0.0002)	(0.0123)	(0.0248)	(0.0028)	(0.2867)	(0.0000)	(0.2821)
12	0.0140	0.00024	0.0288	0.00011	0.0282	0.00012	0.0055	0.00000
	(0.0052)	(0.0002)	(0.0267)	(0.0269)	(0.0103)	(0.3053)	(0.0000)	(0.2825)

Table 4.2: **Tests of Equity-Premium Model: Nondurable Consumption**

This table displays means and variances of $EP_t^{\tau A}$, along with the corresponding moments of the theoretical equity premium $\widetilde{EP}_t^{\tau A}$ implied by the time-separable model, the Abel (1990) model, and the Constantinides (1990) model. Each model is evaluated at four values for the curvature parameter γ, with consumption measured as expenditures on nondurables services.

The numbers in parentheses are asymptotic p-values testing whether the means and variance of $EP_t^{\tau A}$ equal the corresponding moments of $\widetilde{EP}_t^{\tau A}$: Specifically, the numbers in parentheses the two-sided p-values for the standard normal distribution evaluated at Z_{mean} (for the columns labeled "mean") and Z_{var} (for the columns labeled "var"), as defined in equations (4.6) and (4.7)

	HORIZON							
	quarterly		1-year		2-year		3-year	
γ	mean	var	mean	var	mean	var	mean	var
	Estimated Equity Premium							
	0.0621	0.0075	0.0556	0.0036	0.0551	0.0015	0.0478	0.0011
	Time-Separable Model, Nondurable Consumption							
20	0.0045	0.00007	0.0107	0.00002	0.0105	0.00002	0.0030	0.00000
	(0.0001)	(0.0000)	(0.0000)	(0.0150)	(0.0000)	(0.2737)	(0.0000)	(0.3456)
30	0.0062	0.00015	0.0173	0.00004	0.0161	0.00004	0.0040	0.00000
	(0.0001)	(0.0000)	(0.0004)	(0.0157)	(0.0000)	(0.2848)	(0.0000)	(0.3458)
40	0.0076	0.00026	0.0250	0.00009	0.0236	0.00008	0.0049	0.00000
	(0.0001)	(0.0000)	(0.0044)	(0.0172)	(0.0009)	(0.3056)	(0.0000)	(0.3459)
50	0.0084	0.00039	0.0348	0.00017	0.0351	0.00017	0.0058	0.00000
	(0.0002)	(0.0000)	(0.0490)	(0.0200)	(0.0353)	(0.3525)	(0.0000)	(0.3460)
	Abel Model, Nondurable Consumption							
2	0.0033	0.00002	0.0066	0.00000	0.0054	0.00000	0.0011	0.00000
	(0.0004)	(0.0001)	(0.0000)	(0.0323)	(0.0000)	(0.2399)	(0.0000)	(0.3105)
6	0.0096	0.00014	0.0225	0.00011	0.0221	0.00007	0.0058	0.00000
	(0.0014)	(0.0001)	(0.0038)	(0.0373)	(0.0017)	(0.2712)	(0.0000)	(0.3112)
10	0.0151	0.00039	0.0435	0.00043	0.0523	0.00033	0.0109	0.00000
	(0.0041)	(0.0001)	(0.2509)	(0.0779)	(0.7616)	(0.4036)	(0.0004)	(0.3129)
14	0.0198	0.00086	0.0766	0.00145	0.1199	0.00294	0.0178	0.00000
	(0.0095)	(0.0002)	(0.2418)	(0.3493)	(0.0000)	(0.8962)	(0.0044)	(0.3166)
	Constantinides Model, Nondurable Consumption							
1	0.0021	0.00001	0.0033	0.00000	0.0027	0.00000	0.0002	0.00000
	(0.0006)	(0.0002)	(0.0000)	(0.0110)	(0.0000)	(0.1971)	(0.0000)	(0.2693)
5	0.0116	0.00031	0.0239	0.00008	0.0241	0.00008	0.0063	0.00000
	(0.0035)	(0.0002)	(0.0165)	(0.0118)	(0.0092)	(0.2342)	(0.0002)	(0.2698)
7	0.0173	0.00067	0.0414	0.00016	0.0462	0.00022	0.0102	0.00000
	(0.0094)	(0.0004)	(0.3481)	(0.0149)	(0.6804)	(0.3215)	(0.0009)	(0.2706)
9	0.0277	0.00143	0.0803	0.00082	0.1016	0.00081	0.0177	0.00000
	(0.0472)	(0.0015)	(0.0322)	(0.0603)	(0.0000)	(0.7086)	(0.0120)	(0.2723)

Table 4.3: **Tests of Equity-Premium Model: Total Consumption Expenditures**
This table displays means and variances of $EP_t^{\tau A}$, along with the corresponding moments of the theoretical equity premium $\widetilde{EP}_t^{\tau A}$ implied by the time-separable model, the Abel (1990) model, and the Constantinides (1990) model. Each model is evaluated at four values for the curvature parameter γ, with consumption measured as total consumption expenditures. The numbers in parentheses are asymptotic p-values testing whether the means and variance of $EP_t^{\tau A}$ equal the corresponding moments of $\widetilde{EP}_t^{\tau A}$: Specifically, the numbers in parentheses the two-sided p-values for the standard normal distribution evaluated at Z_{mean} (for the columns labeled "mean") and Z_{var} (for the columns labeled "var"), as defined in equations (4.6) and (4.7)

	HORIZON							
	quarterly		1-year		2-year		3-year	
γ	mean	var	mean	var	mean	var	mean	var
	Estimated Equity Premium							
	0.0621	0.0075	0.0556	0.0036	0.0551	0.0015	0.0478	0.0011
	Time-Separable Model, Total Consumption							
20	0.0001	0.00006	0.0074	0.00002	0.0048	0.00000	0.0013	0.00000
	(0.0000)	(0.0000)	(0.0000)	(0.0340)	(0.0000)	(0.2649)	(0.0000)	(0.3377)
30	0.0010	0.00013	0.0095	0.00003	0.0058	0.00001	0.0013	0.00000
	(0.0000)	(0.0000)	(0.0000)	(0.0352)	(0.0000)	(0.2667)	(0.0000)	(0.3379)
40	0.0029	0.00024	0.0109	0.00005	0.0065	0.00002	0.0013	0.00000
	(0.0000)	(0.0000)	(0.0000)	(0.0366)	(0.0000)	(0.2695)	(0.0000)	(0.3381)
50	0.0057	0.00040	0.0121	0.00006	0.0071	0.00003	0.0012	0.00000
	(0.0002)	(0.0001)	(0.0001)	(0.0382)	(0.0000)	(0.2723)	(0.0000)	(0.3381)
	Abel Model, Total Consumption							
2	0.0005	0.00003	0.0056	0.00000	0.0039	0.00000	0.0012	0.00000
	(0.0002)	(0.0004)	(0.0000)	(0.0347)	(0.0000)	(0.2236)	(0.0000)	(0.3075)
6	0.0001	0.00020	0.0171	0.00006	0.0135	0.00004	0.0038	0.00000
	(0.0002)	(0.0004)	(0.0008)	(0.0399)	(0.0000)	(0.2405)	(0.0000)	(0.3086)
10	0.0034	0.00059	0.0298	0.00017	0.0283	0.00021	0.0061	0.00000
	(0.0001)	(0.0006)	(0.0264)	(0.0520)	(0.0090)	(0.3178)	(0.0000)	(0.3107)
14	0.0110	0.00126	0.0465	0.00048	0.0561	0.00082	0.0087	0.00000
	(0.0000)	(0.0014)	(0.3917)	(0.1084)	(0.9786)	(0.6614)	(0.0002)	(0.3147)
	Constantinides Model, Total Consumption							
1	0.0003	0.00000	0.0026	0.00000	0.0022	0.00000	0.0003	0.00000
	(0.0002)	(0.0002)	(0.0000)	(0.0202)	(0.0000)	(0.2122)	(0.0000)	(0.2699)
5	0.0000	0.00016	0.0146	0.00007	0.0142	0.00005	0.0034	0.00000
	(0.0002)	(0.0002)	(0.0006)	(0.0240)	(0.0000)	(0.2354)	(0.0000)	(0.2709)
9	0.0046	0.00055	0.0287	0.00026	0.0339	0.00027	0.0055	0.00000
	(0.0001)	(0.0003)	(0.0392)	(0.0422)	(0.0486)	(0.3628)	(0.0000)	(0.2734)
11	0.0094	0.00088	0.0381	0.00047	0.0520	0.00061	0.0065	0.00000
	(0.0000)	(0.0004)	(0.2159)	(0.0750)	(0.9417)	(0.5963)	(0.0001)	(0.2752)

Table 4.4: **Tests of Riskfree Rate Model: Nondurables + Services**

This table displays means and variances of the nominal risk-free rate $RF_t^{\tau A}$, along with the corresponding moments of the theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ implied by the time-separable model, the Abel (1990) model, and the Constantinides (1990) model. Each model is evaluated at four values for the curvature parameter γ, with consumption measured as expenditures on nondurables plus services. The numbers in parentheses are asymptotic p-values testing whether the means and variance of $RF_t^{\tau A}$ equal the corresponding moments of $\widetilde{RF}_t^{\tau A}$: Specifically, the numbers in parentheses the two-sided p-values for the standard normal distribution evaluated analogously to Z_{mean} (for the columns labeled "mean") and Z_{var} (for the columns labeled "var"), as defined in equations (4.6) and (4.7)

	HORIZON							
	quarterly		1-year		2-year		3-year	
γ	mean	var	mean	var	mean	var	mean	var
	Observed Risk-Free Rate							
	0.0541	0.00084	0.0608	0.00096	0.0630	0.00097	0.0664	0.00088
	Time-Separable Model, Nondurables + Services Consumption							
20	0.4089	0.0253	0.3970	0.0184	0.3949	0.0061	0.4055	0.0026
	(0.0001)	(0.2155)	(0.0000)	(0.2160)	(0.0000)	(0.6061)	(0.0000)	(0.8592)
30	0.5783	0.0617	0.5546	0.0450	0.5425	0.0139	0.5585	0.0053
	(0.0000)	(0.1680)	(0.0000)	(0.1619)	(0.0000)	(0.5164)	(0.0000)	(0.8073)
40	0.7388	0.1184	0.7001	0.0868	0.6685	0.0239	0.6899	0.0076
	(0.0000)	(0.1406)	(0.0000)	(0.1319)	(0.0000)	(0.4661)	(0.0000)	(0.7941)
50	0.8905	0.1999	0.8348	0.1503	0.7713	0.0354	0.7977	0.0087
	(0.0000)	(0.1209)	(0.0000)	(0.1111)	(0.0000)	(0.4297)	(0.0000)	(0.8000)
	Abel Model, Nondurables + Services Consumption							
2	0.0793	0.0043	0.0782	0.0009	0.0766	0.0001	0.0790	0.0000
	(0.0120)	(0.0031)	(0.0394)	(0.9453)	(0.0999)	(0.4508)	(0.1354)	(0.5159)
6	0.1252	0.0445	0.1220	0.0099	0.1242	0.0006	0.1368	0.0000
	(0.0041)	(0.0000)	(0.0001)	(0.0320)	(0.0000)	(0.7987)	(0.0000)	(0.5513)
10	0.1352	0.1354	0.1307	0.0293	0.1437	0.0018	0.1757	0.0001
	(0.0573)	(0.0000)	(0.0063)	(0.0024)	(0.0000)	(0.7089)	(0.0000)	(0.6315)
14	0.1094	0.2936	0.1039	0.0585	0.1343	0.0036	0.1958	0.0002
	(0.3815)	(0.0000)	(0.2362)	(0.0001)	(0.0000)	(0.3752)	(0.0000)	(0.7249)
	Constantinides Model, Nondurables + Services Consumption							
7	0.1285	0.0633	0.1157	0.0178	0.1264	0.0009	0.1446	0.0000
	(0.0163)	(0.0000)	(0.0077)	(0.0043)	(0.0000)	(0.9827)	(0.0000)	(0.6540)
9	0.1312	0.1090	0.1148	0.0299	0.1290	0.0015	0.1612	0.0001
	(0.0549)	(0.0000)	(0.0430)	(0.0010)	(0.0000)	(0.7203)	(0.0000)	(0.6955)
11	0.1226	0.1703	0.1030	0.0449	0.1205	0.0024	0.1722	0.0002
	(0.1703)	(0.0000)	(0.2069)	(0.0002)	(0.0000)	(0.4813)	(0.0000)	(0.7298)
12	0.1135	0.2078	0.0926	0.0534	0.1116	0.0029	0.1754	0.0002
	(0.2827)	(0.0000)	(0.3963)	(0.0000)	(0.0000)	(0.3604)	(0.0000)	(0.7423)

Table 4.5: **Tests of Risk-Free Rate Model: Nondurable Consumption**

This table displays means and variances of the nominal risk-free rate $RF_t^{\tau A}$, along with the corresponding moments of the theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ implied by the time-separable model, the Abel (1990) model, and the Constantinides (1990) model. Each model is evaluated at four values for the curvature parameter γ, with consumption measured as expenditures on nondurables. The numbers in parentheses are asymptotic p-values testing whether the means and variance of $RF_t^{\tau A}$ equal the corresponding moments of $\widetilde{RF}_t^{\tau A}$: Specifically, the numbers in parentheses the two-sided p-values for the standard normal distribution evaluated analogously to Z_{mean} (for the columns labeled "mean") and Z_{var} (for the columns labeled "var"), as defined in equations (4.6) and (4.7)

	HORIZON							
	quarterly		1-year		2-year		3-year	
γ	mean	var	mean	var	mean	var	mean	var
	Observed Risk-Free Rate							
	0.0541	0.00084	0.0608	0.00096	0.0630	0.00097	0.0664	0.00088
	Time-Separable Model, Nondurable Consumption							
20	0.2250	0.0424	0.2062	0.0250	0.2100	0.0054	0.2159	0.0016
	(0.0001)	(0.0003)	(0.0000)	(0.0051)	(0.0000)	(0.2570)	(0.0000)	(0.8393)
30	0.2873	0.1039	0.2500	0.0631	0.2324	0.0128	0.2646	0.0033
	(0.0000)	(0.0001)	(0.0000)	(0.0024)	(0.0000)	(0.1133)	(0.0000)	(0.6770)
40	0.3292	0.2001	0.2675	0.1283	0.2280	0.0234	0.2870	0.0045
	(0.0000)	(0.0000)	(0.0000)	(0.0030)	(0.0000)	(0.0572)	(0.0000)	(0.5999)
50	0.3508	0.3395	0.2407	0.1876	0.1907	0.0400	0.2849	0.0046
	(0.0002)	(0.0000)	(0.0004)	(0.0002)	(0.0000)	(0.0300)	(0.0000)	(0.5692)
	Abel Model, Nondurable Consumption							
2	0.0554	0.0127	0.0524	0.0016	0.0524	0.0001	0.0563	0.0000
	(0.9097)	(0.0000)	(0.3302)	(0.5888)	(0.1937)	(0.4896)	(0.3312)	(0.5201)
6	0.0259	0.1308	0.0201	0.0197	0.0344	0.0017	0.0678	0.0000
	(0.3806)	(0.0000)	(0.0316)	(0.0000)	(0.0034)	(0.5994)	(0.7912)	(0.5169)
10	-0.0893	0.4030	-0.0876	0.0652	-0.0551	0.0074	0.0488	0.00000
	(0.0153)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0017)	(0.7188)	(0.5270)
14	-0.2905	0.9323	-0.2656	0.1537	-0.2273	0.0237	0.0027	0.00000
	(0.0003)	(0.0009)	(0.0000)	(0.0000)	(0.0000)	(0.0098)	(0.0000)	(0.5498)
	Constantinides Model, Nondurable Consumption							
1	0.0489	0.0025	0.0462	0.0007	0.0480	0.0000	0.0500	0.0000
	(0.3889)	(0.0633)	(0.0210)	(0.8545)	(0.0200)	(0.5030)	(0.0461)	(0.5738)
5	0.0211	0.0921	0.0040	0.0217	0.0226	0.0014	0.0618	0.0000
	(0.1881)	(0.0000)	(0.0016)	(0.0000)	(0.0000)	(0.6653)	(0.4573)	(0.5600)
7	-0.0618	0.2071	-0.0791	0.0477	-0.0452	0.0041	0.0470	0.0000
	(0.0021)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0563)	(0.0141)	(0.5662)
9	-0.3320	0.4651	-0.2929	0.0998	-0.2104	0.0114	-0.0031	0.0002
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.1001)	(0.0000)	(0.6362)

Table 4.6: **Tests of Risk-Free Rate Model: Total Consumption Expenditures**
This table displays means and variances of the nominal risk-free rate $RF_t^{\tau A}$, along with the corresponding moments of the theoretical risk-free rate $\widetilde{RF}_t^{\tau A}$ implied by the time-separable model, the Abel (1990) model, and the Constantinides (1990) model. Each model is evaluated at four values for the curvature parameter γ, with consumption measured as total consumption expenditures. The numbers in parentheses are asymptotic p-values testing whether the means and variance of $RF_t^{\tau A}$ equal the corresponding moments of $\widetilde{RF}_t^{\tau A}$: Specifically, the numbers in parentheses the two-sided p-values for the standard normal distribution evaluated analogously to Z_{mean} (for the columns labeled "mean") and Z_{var} (for the columns labeled "var"), as defined in equations (4.6) and (4.7)

	HORIZON							
	quarterly		1-year		2-year		3-year	
γ	mean	var	mean	var	mean	var	mean	var
	Observed Risk-Free Rate							
	0.0541	0.00084	0.0608	0.00096	0.0630	0.00097	0.0664	0.00088
	Time-Separable Model, Total Consumption							
20	0.4176	0.0601	0.3934	0.0329	0.3906	0.0091	0.3942	0.0034
	(0.0000)	(0.0360)	(0.0000)	(0.1160)	(0.0000)	(0.5175)	(0.0000)	(0.8049)
30	0.5776	0.1503	0.5287	0.0744	0.5125	0.0187	0.5191	0.0069
	(0.0000)	(0.0211)	(0.0000)	(0.0916)	(0.0000)	(0.4504)	(0.0000)	(0.7461)
40	0.7189	0.2991	0.6336	0.1291	0.5967	0.0281	0.6095	0.0097
	(0.0000)	(0.0132)	(0.0000)	(0.0784)	(0.0000)	(0.4160)	(0.0000)	(0.7316)
50	0.8421	0.5302	0.7070	0.1961	0.6459	0.0353	0.6679	0.0106
	(0.0000)	(0.0085)	(0.0000)	(0.0720)	(0.0000)	(0.3934)	(0.0000)	(0.7436)
	Abel Model, Total Consumption							
2	0.0745	0.0157	0.0729	0.0023	0.0728	0.0001	0.0777	0.0000
	(0.1552)	(0.0000)	(0.2225)	(0.3384)	(0.2395)	(0.4924)	(0.1858)	(0.5196)
6	0.0833	0.1658	0.0801	0.0202	0.0929	0.0012	0.1212	0.0000
	(0.4863)	(0.0002)	(0.3724)	(0.0002)	(0.0020)	(0.8911)	(0.0000)	(0.5433)
10	0.0147	0.6241	0.0029	0.0430	0.0465	0.0029	0.1262	0.0000
	(0.6104)	(0.0101)	(0.0546)	(0.0000)	(0.1528)	(0.2025)	(0.0000)	(0.5780)
14	-0.2080	0.9726	-0.1695	0.0595	-0.0751	0.0060	0.0949	0.0001
	(0.0642)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0255)	(0.0027)	(0.6167)
	Constantinides Model, Total Consumption							
1	0.0589	0.0035	0.0583	0.0010	0.0602	0.0001	0.0616	0.0000
	(0.5283)	(0.0050)	(0.6193)	(0.9352)	(0.4034)	(0.5000)	(0.4374)	(0.5647)
5	0.0807	0.1095	0.0775	0.0252	0.0880	0.0018	0.1102	0.0001
	(0.2581)	(0.0000)	(0.3748)	(0.0003)	(0.0280)	(0.5846)	(0.0000)	(0.5995)
9	0.0311	0.4438	0.0221	0.0796	0.0382	0.0057	0.1179	0.0001
	(0.9376)	(0.0000)	(0.5190)	(0.0000)	(0.0481)	(0.0237)	(0.0000)	(0.6038)
11	-0.0381	0.7769	-0.0446	0.1198	-0.0256	0.0094	0.1046	0.0001
	(0.6183)	(0.0000)	(0.0908)	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.5908)