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Tobin's q and Asset Returns: Implications for Business Cycle Analysis

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ABSTRACT

The marginal cost of plant capacity, measured by the price of equity, is significantly procyclical. Yet, the price of a major intermediate input into expanding plant capacity, investment goods, is countercyclical. The ratio of these prices is Tobin's q. Following convention, we interpret the fact that Tobin's q differs from unity at all, as reflecting that there are diminishing returns to expanding plant capacity by installing investment goods ("adjustment costs"). However, the phenomenon that interests us is not just that Tobin's q differs from unity, but also that its numerator and denominator have such different cyclical properties. We interpret the sign switch in their covariation with output as reflecting the interaction of our adjustment cost specification with the operation of two shocks: one which affects the demand for equity and another which shifts the technology for producing investment goods. The adjustment costs cause the two prices to respond differently to these two shocks, and this is why it is possible to choose the shock variances to reproduce the sign switch.

These model features are incorporated into a modified version of a model analyzed in Boldrin, Christiano and Fisher (1995). That model incorporates assumptions designed to help account for the observed mean return on risk free and risky assets. We find that the various modifications not only account for the sign switch, but they also continue to account for the salient features of mean asset returns.

We turn to the business cycle implications of our model. The model does as well as standard models with respect to conventional business cycle measures of volatility and comovement with output, and on one dimension the model significantly dominates standard models. The factors that help it account for prices and rates of return on assets also help it account for the fact that employment across a broad range of sectors moves together over the cycle.

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1 Introduction

The price of a marginal unit of plant capacity, as measured by the price of equity, is significantly procyclical. Yet, the price of a major input into expanding plant capacity, new investment goods, is countercyclical. In this paper we provide a quantitative account for this *sign switch phenomenon*. We do so in a model that does at least as well as previous models in accounting for key features of (a) asset returns and (b) the business cycle. The features of asset returns that we have in mind include the observed low average return on risk free assets and the high average return on equity. The features of business cycles we have in mind include standard measures of comovement and volatility as well as measures of persistence.

To capture (a) and (b), we build on the recent model by Boldrin, Christiano and Fisher (1995) (BCF). In particular, we assume consumption and investment goods are produced by distinct production technologies and that there are limitations on the mobility of factors of production across sectors. In addition, we adopt the habit persistence specification of preferences proposed in Constantinides (1990) and Sundaresan (1989).

We show that the sign switch phenomenon can be accounted for by a combination of separate shocks to the two production technologies and the "adjustment cost" model of investment analyzed in Lucas and Prescott (1971) and the references they cite. According to this model, the more quickly a firm attempts to incorporate new investment goods into an existing plant, the less effective those goods are on the margin at expanding plant capacity.

After verifying that our model can account for the sign switch phenomenon and that it continues to account for mean asset returns as in BCF, we examine its business cycle characteristics. We find that, not only does the model do about as well as standard models on the business cycle statistics usually emphasized, but the model actually makes a step forward on one particularly important business cycle fact. Perhaps the *defining* characteristic of business cycles is comovement: activity across a broad range of sectors moves up and down together over the business cycle (see Lucas (1981, p.217) and Sargent (1979, p.215).) Standard real business cycle models are consistent with this fact in that they imply that the outputs of the consumption sector and the investment goods sector are both procyclical.

However, we report evidence suggesting that employment across these sectors is procyclical as well.[1] Standard real business cycle models are inconsistent with this evidence. They have the property that consumption is smoothed over the cycle: in a boom, consumption rises relatively little, as the improvement in technology is partially offset by a reallocation of factors of production out of consumption and into investment goods sectors. The opposite occurs in a recession. This is why standard models have the dubious implication that hours worked in the production of consumption goods is countercyclical. Although this is a feature of most real business cycle models, it is not a feature of ours.

The following section provides a brief, nontechnical overview of the analysis. After that, we document the empirical properties of equity prices and prices and quantities of investment goods. Then, we formally describe our model and present the quantitative analysis. Finally, we present concluding remarks.

2 Overview of the Analysis

In what follows we first discuss the cyclical properties of investment prices, and then we go on to explain how our model accounts for these properties. We then discuss the sign switch phenomenon. Finally, we discuss the business cycle implications of the model.

Investment Goods Prices

The time series behavior of the price of an important component of investment goods, producers' durable equipment, has recently been documented and analyzed by Greenwood, Hercowitz and Krusell (1992). They show that the price deflator of this good, as measured by Gordon (1990), is countercyclical. In addition, they document that, starting in particular in the 1980s, this price index exhibits a downward trend. These trend and cyclical characteristics are a feature of household durable goods too. Together, these two components account for about 65 percent of the value of overall investment activity. The remaining components of investment—investment in structures and residential investment—also exhibit a downward trend in their price starting in the 1980s, but they do not display the same significant

[1] See also Benhabib, Rogerson and Wright (1991, ftn.14), Murphy, Schleifer and Vishny (1989).

countercyclicality. As a result, the price index of overall investment activity is only slightly countercyclical.

We follow Greenwood, Hercowitz and Krusell (1992) in interpreting these features of the price data as reflecting that investment dynamics are driven by both demand and supply shocks. Demand shocks are modeled as arising from a technology shock that is common across investment and consumption goods sectors. A boom triggered by this kind of shock generates a relative shift in demand towards investment goods for consumption-smoothing reasons and so produces a rise in their price. Supply shocks are modeled as arising from a disturbance that is specific to the technology for producing investment goods. Innovations in this shock generate a negative covariance between the price of investment goods and output. We parameterize the variances of our two shocks so that the model reproduces the observed weak countercyclicality of investment goods prices and also reproduces the estimated variance of the aggregate Solow residual reported in Christiano (1988).

We accommodate the trend in the relative price of investment goods by the assumption that the investment-specific technology shock is a random walk with a positive drift. The implication that disturbances from this source are permanent is consistent with the notion that they represent shocks in the rate of arrival of innovations. At the same time, we posit that the economy-wide shock is transitory and has no trend. This is the only shock affecting the sector producing consumption goods, and its transitory nature captures the notion that the only disturbances to that technology are shocks to the weather, or natural disasters, or perhaps even labor disputes.[2] The drift in the investment-specific technology shock is set to reproduce the observed rate of growth in consumption. The single autoregressive parameter in the stationary economy-wide shock is selected to reproduce the slight negative autocorrelation in the growth rate of the measured Solow residual reported in Christiano (1988).

[2]In effect, the model takes the position that we know how to bake bread or serve a hearty meal about as well as we did centuries ago. Permanent shifts in the technology for producing consumption goods are viewed as being embodied in capital.

The Sign Switch

To see how adjustment costs help account for the sign switch, it is useful to first consider the benchmark case in which there are no adjustment costs. Then, the price of equity—we identify this with the marginal cost of new plant capacity—and the marginal cost of new investment goods are identical, i.e., Tobin's q is identically equal to unity. Thus, in the absence of adjustment costs, the weak countercyclicality of investment goods prices would be shared by equity prices and there would be no sign switch.

Under adjustment costs, the price of equity becomes procyclical. This is because adjustment costs have the effect of (i) reducing the response of equity prices to investment-specific technology shocks (these shocks make equity prices countercyclical), and (ii) enhancing their response to aggregate shocks (a force for procyclicality). The reason for (i) is that with adjustment costs, an investment-specific technology shock triggers two opposing effects on equity prices. On the one hand, the fall in the price of investment goods exerts a downward pressure on the price of equity. On the other hand, the higher level of investment is associated with a drop in the marginal effectiveness with which investment goods enhance new plant capacity, and this exerts upward pressure on the price of equity. The reason for (ii) is that with adjustment costs, an aggregate technology shock triggers two reinforcing effects on equity prices. This type of shock generates rises in both the price and quantity of investment goods. The rise in the price generates upward pressure on equity prices, and under adjustment costs the rise in investment does too.

Our adjustment cost formulation is controlled by a single curvature parameter. We set this parameter so that the model reproduces the observed positive correlation between equity prices and output.

Other Model Implications

We have enough free parameters so that our model can exactly capture the sign switch phenomenon. To test the model, we examine other implications. First, our adjustment cost formulation generates an elasticity of investment to Tobin's q, an object for which there exist empirical estimates. We compare our model's implications with these estimates.

Second, the progress that our model makes on the comovement puzzle - the fact that sectoral employment moves together over the business cycle - reflects two assumptions. (i) Following BCF, we assume factors of production must be allocated prior to the realization of the current period shocks. This assumption is intended to capture the various real-world frictions that make it difficult for workers to quickly leave a job in one sector and start work in another. In our model, it is simply not possible to immediately shift factors of production across sectors in the period of a shock. (ii) There is also little incentive to shift labor resources out of the consumption goods sector in the periods after a positive technology shock. Employment in the production of consumption goods rises in the periods after a favorable shock in the investment goods sector because the associated wealth effect makes consumption goods more valuable. Employment also rises in the periods after an aggregate shock. The expansion in the supply of consumption goods in the period of the shock has the effect of raising the value of consumption goods in subsequent periods because of the effects of habit persistence in the utility function. The transient nature of the aggregate shock then ensures that employment must be high to satisfy that desire. Employment in the production of consumption goods is procyclical because it is procyclical relative to each of the shocks in the model.[3]

Third, we document that our model implies low risk aversion on the part of households. As in BCF, steady state relative risk aversion with respect to a bet on wealth is restricted to unity. In addition, our model can account for the observed equity premium by assuming a very small degree of relative risk aversion with respect to bets on consumption. However, the ability of the model to account for the equity premium with low relative risk aversion with respect to consumption reflects some seemingly counterfactual implications for equilibrium consumption growth. It implies consumption growth is negatively autocorrelated—in the data it is positively autocorrelated—and it overstates the innovation in consumption.

To understand how these implications of the model help account for its success in explaining the mean equity premium, it is useful to repeat an observation in BCF. They argue

[3]Another feature of our model environment helps account for (ii); namely, our assumption that utility is linear in leisure. Within a certain class of utility functions, this assumption maximizes the likelihood of positive comovement of sectoral employment. We discuss this issue in detail below.

that the key to getting an equity premium in a model like ours lies in generating an equilibrium process for the capital gains component of the return on equity which has the "right" pattern. In practice this translates into the requirement that (i) households have a strong desire to buy assets when consumption is high and to sell assets when consumption is low; and (ii) the nature of the technology has the effect of frustrating these desires. Habit persistence in preferences delivers (i) and the limitations on the mobility of resources deliver (ii). Another way to enhance (i) is to construct a model environment in which equilibrium consumption growth is negatively autocorrelated. This feature particularly enhances the motive to smooth consumption and, thus, to buy assets when there is a positive innovation in consumption.

3 Price Data

In this section, we present our analysis of the dynamic properties of postwar U.S. data on share prices and the price of new investment goods. Our results are that the price of aggregate investment goods is slightly countercyclical and displays a downward trend, particularly beginning in the 1980s. The cyclical behavior of the price of equity differs sharply from that of investment. Equity prices are significantly procyclical.

3.1 New Investment Goods Prices

We study the components of U.S. investment reported in the National Income and Product Accounts (NIPA). We also consider the annual price indexes for consumer durables and for business equipment reported in Gordon (1990) for the period 1947-1983. Investment price indices were divided by the implicit price deflator for household consumption of nondurables and services and then logged prior to analysis. We now consider the trend and business cycle characteristics of these data.[4]

Trend

[4]For a related discussion, see Fisher (1994a).

Figure 1 graphs the price data, together with their Hodrick-Prescott (HP) trend, for the period 1947QI-1995QI. Our broadest measure of investment is the NIPA measure of business fixed investment plus consumer durables. Note from Figure 1a that the associated implicit price deflator displays roughly no trend until the 1980s, whereupon it takes a sharp turn down. In interpreting this, note the difference between the Gordon price series for household durables and business equipment and the associated implicit price deflators from the NIPA. Gordon's are the series that fall sharply throughout the sample in Figures 1b and 1g. He argues that the difference reflects the failure of the NIPA data to properly take into account quality improvement in consumer and producer durables. This suggests that, despite the apparent lack of trend prior to the 1980s in Figure 1a, investment good prices probably were falling then. The behavior of the price of residential investment (Figure 1e) and of structures investment (Figure 1f) suggests, though, that the fall in aggregate investment prices probably was slower before the 1980s than after.

Figure 2 displays the ratio of the various categories of investment to Gross Domestic Product. In each case, the solid line depicts expenditure shares, that is, the numbers are formed as a ratio of nominal investment to nominal GDP. The dashed line depicts the ratio in real terms. Note in Figure 2a that the ratio in value terms of our broad measure of investment is roughly stationary, while the ratio in real terms trends up from about 21 percent of GDP in the early 1950s to about 27 percent of GDP now. Thus, the fall in the price of investment goods in the 1980s has been offset by a simultaneous increase in real output. This is also a feature of components of investment, for example consumer durables (Figure 2b) and business equipment in the 1980s (Figure 2g). Investment in structures appears to be an exception, with quantity not rising by enough to offset the reduced price in the 1980s (Figure 2f). Figures 2h and 2i indicate that the sum of private consumption of nondurables and services, and government purchases, expressed as a ratio to total output, is roughly stationary. However, the share of the components does not appear stationary.

We infer from Figures 1 and 2 that, to a first approximation, the aggregate data display balanced growth in expenditure share terms, but that the quantity of investment goods grows

more rapidly than the quantity of consumption goods.[5] This abstracts from other important features of the data, including the significant upward trend in the price index of important components of investment prior to the 1980s.

Business Cycles

Now consider the business cycle properties of the price data. Figure 3 displays the deviations of the (logged) prices from their HP trend (solid line) together with the associated deviations for log GDP (dashes). Casual inspection confirms the Greenwood, Hercowitz and Krusell (1992) finding that the price of equipment is strongly countercyclical. Figure 4 displays the associated cross correlation functions and associated plus and minus two standard deviation error bands. First, note that the contemporaneous correlation is negative, though not significantly so, for our broad measure of investment (see Figure 4a). Durables are countercyclical—though significantly so only when correlated with output one quarter in the past—but the correlation between fixed investment prices and output is not significant. This reflects the very different cyclical behavior of equipment versus structures and residential investment. Equipment is significantly countercyclical, whereas residential investment is strongly procyclical and structures are acyclical.

The data suggest that there are interesting differences in the business cycle properties of the components of investment. Further analysis of these differences is beyond the scope of this paper.[6] Our model recognizes only one form of investment, and we calibrate it based on our point estimate for the correlation between the price of aggregate investment and output, which is -0.15.

3.2 Stock Prices

We consider the cyclical behavior of the S & P 500, Dow Jones and New York Stock Exchange stock price indexes for various industries, as supplied in Citibase.[7] In each case, the price

[5]This is consistent with the results of the formal statistical analysis presented in Eichenbaum and Hansen (1990) and Fisher (1994b).

[6]Implications of the procyclicality in residential investment price deflator are explored in Fisher (1995).

[7]These indexes are best thought of as the product of price and quantity. We assume that most of their business cycle variation reflects variations in price.

index was deflated by the same implicit price deflator for consumption of nondurables and services used to deflate the price indexes of new investment. All data were logged and HP filtered prior to analysis. Table 1 reports the volatilities of the price data, divided by the volatility of output, which is roughly 1.8 percent. In addition, the correlations between the price indexes and the cyclical part of U.S. GDP are also reported. Note that, with two exceptions, these correlations are significantly positive. The two exceptions are the S&P 500 data for the transportation and utilities industries. The dynamic correlations with output are presented in Figure 5. Note that the largest correlations are between the stock price and next quarter's GDP. These correlations are almost all near 0.5.

4 Model Economy

This section presents our model economy. There is a single representative household and two production sectors. One produces the consumption good, and the other the investment good. There are two technology shocks: a logarithmic random walk shifts the production function for investment goods, and a stationary first order autoregressive shock shifts both production functions. Households and firms are competitive.

In what follows we first present the household problem and a discussion of risk aversion. Then we consider the problem of the firm and equilibrium. We also discuss various features of the equilibrium of the model.

4.1 Household Problem

Prior to the realization of the date t random variables, the household evaluates consumption and leisure henceforth according to

$$(1) \quad E_{t-1}\sum_{j=0}^{\infty}\beta^j\left[\log(C_{t+j}-X_{t+j})+\eta(1-H_{c,t+j}-H_{i,t+j})\right],$$

where η is a positive scalar, $H_{c,t}, H_{i,t}$ denote employment in the consumption and investment goods-producing industries, and X_t denotes the habit stock, which is assumed to evolve according to

$$(2) \quad X_{t+1} = hX_t + bC_t.$$

Here, E_t is the expectation operator conditioned on all variables dated t and earlier. Also, C_t denotes consumption, and we specify that utility is linear in leisure following Rogerson (1988) and Hansen (1985).

The household budget constraint is

$$(3) \qquad C_t + S_t^c + S_t^i + B_t$$
$$\leq (1 + r_{ct}^e)S_{t-1}^c + (1 + r_{it}^e)S_{t-1}^i + (1 + r_{t-1}^f)B_{t-1} + w_t^c H_{c,t} + w_t^i H_{i,t},$$

for $t = 0, 1, ...$ Here, S_t^x denotes date t purchases of shares of equity in industry x, for $x = c, i$, and B_t denotes purchases of risk-free debt. The rate of return on equity purchased in period t, $(1 + r_{x,t+1}^e)$, depends upon the period $t + 1$ state of nature, while the rate of return on debt, $(1 + r_t^f)$, depends on the date t state of nature. Also, w_t^x denotes the wage rate in industry x, which is a function of the date t state of nature.

The household's date t state variables are $S_{t-1}^c, S_{t-1}^i, B_{t-1}, X_t$. In addition, the household knows the values of all prices and rates of return for each date and state of nature. The household's problem at time t is to select values for its time t choice variables, $H_{c,t}, H_{i,t}, C_t, S_t^c, S_t^i, B_t$. We capture the notion that there is a degree of precommitment in the labor supply decision by imposing a particular information constraint on the variables. In particular, we require that households choose $H_{c,t}, H_{i,t}$ prior to the realization of the date t state of nature, while the remaining choice variables are selected afterward. We refer to this restriction on the allocation of work effort as the *limited labor mobility* assumption. The household's objective is to maximize (1) subject to (2)–(3) and the condition that the future choice variables satisfy the same information constraints. The household's intratemporal first order necessary conditions for labor are

$$(4) \quad E_{t-1} w_t^x \Lambda_{c,t} = \eta, \; x = c, i.$$

Its intertemporal first order conditions are

$$(5) \quad E_t p_{c,t+1}(1 + r_{x,t+1}^e) = 1 = E_t p_{c,t+1}(1 + r_t^f),$$

for $x = c, i$, where

$$(6) \quad p_{c,t+1} = \frac{\beta \Lambda_{c,t+1}}{\Lambda_{c,t}},$$

and $\Lambda_{c,t}$ is the derivative of (1) with respect to C_t, when E_{t-1} is replaced by E_t. The variable $p_{c,t+1}$ is the value, in date t consumption units, of a unit of date $t+1$ consumption indexed by state of nature and scaled by the conditional probability of that state of nature.[8]

4.2 Risk Aversion

Evaluations of models of asset prices often focus on the implications for risk aversion. One measure of risk aversion is the amount a person is willing to pay to avoid an unanticipated gamble. Two types of gamble are of interest: "gambles on wealth" and "gambles on consumption." These are differentiated according to whether agents can use credit markets to mitigate the effects of the outcome of the gamble. With a gamble on wealth, agents have full access to credit markets in the period of the gamble. Constantinides (1990) argues that habit persistence agents have little aversion to gambles like this because they have a relatively painless way of dealing with the state of the world in which they lose. The fall in the present value of consumption that must occur with the loss of a bet on wealth can be accommodated by reducing consumption slowly so that the habit stock has a chance to fall. By specifying β to be close to unity and formulating habit persistence in terms of the logarithm, the steady state level of relative risk aversion in wealth is unity in our model (for further discussion, see BCF.)

A gamble on consumption has the property that agents have no access to credit markets in the period of the gamble. As a result, the full amount of a loss or gain must be absorbed by current consumption. Agents then have full access to credit markets in the periods after the gamble.

We suspect that risk aversion over consumption gambles is harder to measure (or introspect on) than risk aversion over wealth gambles. Still, it is useful to define a measure of risk aversion over consumption gambles precisely so that we can report on this aspect of the model in the results section. For tractability, we define this concept of risk aversion relative to a slightly simpler environment, in which hours worked is fixed, the rate of return on sav-

[8] In particular, let $g_t(s')$ denote the date t conditional probability that state of nature s' will be realized in period $t+1$. Then $g_t(s')p_{c,t+1}(s')$ is the value—denominated in date t consumption units—of a unit of consumption in date $t+1$, sate of nature s'. Here, $p_{c,t+1}(s')$ is the value of $p_{c,t+1}$in state of nature s'.

ing is constant, and there is no uncertainty. Thus, suppose a household has the following preferences:

$$(7) \quad \sum_{t=0}^{\infty} \beta^t \log(C_t - X_t),$$

where the habit stock evolves as before. At date 0, the household has a given stock of wealth, W_0 $(= S_{-1} + B_{-1})$, and habit, X_0, and seeks to optimize (7) subject to the following intertemporal budget constraint:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t C_t = (1+r)W_0.$$

The solution to this problem is characterized by

$$(C_t - X_t) = Q\gamma^t, \ \gamma = \beta(1+r),$$

where

$$(8) \quad Q(W_0, X_0) = \frac{(\frac{\gamma}{\beta} - \gamma)\left\{\left[\frac{\gamma}{\beta} - (h+b)\right]W_0 - X_0\right\}}{(\frac{\gamma}{\beta} - h)}.$$

Therefore, the value function for this problem is, apart from an additive constant,

$$(9) \quad v(W_0, X_0) = \frac{\log(Q)}{1-\beta}.$$

For further details, see BCF.

Let $C_0, C_1, ...,$ be the solution to this problem. Now, suppose the household is confronted with the following gamble: it is given μC_0 units of consumption goods with probability $1/2$ and must give up μC_0 consumption goods also with probability $1/2$. We measure relative risk aversion in consumption, RRA_c, by the fraction, ν, of C_0 the household is willing to sacrifice with probability one in order to avoid this gamble. That is, ν solves

$$\begin{aligned} &\log\left(C_0(1-\nu) - X_0\right) + \beta v\left(W_1, hX_0 + bC_0(1-\nu)\right) \\ (10) \quad &= \frac{1}{2}\left\{\log\left(C_0(1-\mu) - X_0\right) + \beta v\left(W_1, hX_0 + bC_0(1-\mu)\right)\right. \\ &\left. + \log\left(C_0(1+\mu) - X_0\right) + \beta v\left(W_1, hX_0 + bC_0(1+\mu)\right)\right\}. \end{aligned}$$

Here, $W_1 = (1+r)W_0 - C_0$ and is unaffected by the outcome of the gamble. We solve this problem on a steady state growth path by factoring C_0 from (10) and setting X_0/C_0 and W_0/C_0 to their steady state values. Evidently, the measure of risk aversion we use is a function of μ.

4.3 Firms

There are consumption goods producing and investment goods producing firms. Each has a one period planning horizon. Whatever physical capital the firm uses in production in period $t+1$ must be put into place by the end of period t. This capital is produced by combining previously installed capital with new investment goods. To finance the purchase of these inputs, the firm issues debt and equity in period t. There are separate equity markets for the two types of firms, and the two types of equity command different, competitively determined, state-contingent rates of return. Since the competitive rate of return on debt is known at the time it is issued, in equilibrium there can be only one rate of return for that financial instrument. When period $t+1$ occurs, the firm observes the state of nature and, hence, the prevailing wage rate. It then enters competitive labor markets to hire that amount of labor which maximizes cash flow in that state of nature. The firm's cash flow is the value of its production, plus its undepreciated stock of capital, net of expenses. The firm's objective at date t is to maximize the date t value of cash flow at $t+1$, summed across all possible states of nature.

There are several prices relevant to the firm's capital decisions. There are the prices of the raw materials used in period t to produce end of period t capital—i.e., the price of new investment goods and of previously installed capital. Also, there is the price (actually, marginal cost) of end of period t capital. This is, in general, different from the date $t+1$ price of previously installed capital because the latter reflects the realized state of nature in period $t+1$. In sum, these prices are

- $P_{k_x,t} \sim$ price of previously installed capital in sector $x = i, c$.
- $P_{i,t} \sim$ price of new investment goods.
- $P_{k'_x,t} \sim$ price of newly produced capital in sector $x = i, c$, available for production in $t+1$.

Each of these prices is taken as parametric by the firm. We now provide a formal statement of the firm problem in each sector.

The technology for producing consumption goods in $t+1$ is

(11) $C_{t+1} \leq K_{c,t+1}^{\alpha}(\exp(\theta_{t+1})H_{c,t+1})^{1-\alpha}$,

where θ_{t+1} is a covariance stationary shock to technology:

(12) $\theta_{t+1} = \rho\theta_t + \varepsilon_{t+1}$,

$0 < \rho < 1$. The technology for producing new investment goods in $t+1$ is

(13) $I_{c,t+1} + I_{i,t+1} \leq V_{t+1}K_{i,t+1}^{\alpha}(\exp(\theta_{t+1})H_{i,t+1})^{1-\alpha}$

where

(14) $V_{t+1} = \exp(\bar{\mu} + \mu_{t+1})V_t$.

Here, ε_t and μ_t are zero-mean, random variables which are independent of each other and over time and which have standard deviations σ_ε and σ_μ, respectively. The linear rate of transformation between $I_{c,t+1}$ and $I_{i,t+1}$ implicit in (13) guarantees that, in equilibrium, the prices of new investment goods for the consumption and investment goods sectors are equalized.

The technology for producing end of period t capital, $K_{x,t+1}$, for industry x is

(15) $K_{x,t+1} \leq Q^x(y,z)$,

where

(16) $Q^x(y,z) = \left[a_1 y^{\psi} + a_2 z^{\psi}\right]^{1/\psi}$,

for $x = c, i$ and $\psi \leq 1$. In (15)–(16), y denotes previously installed capital and z denotes new investment goods. When $\psi = 1$, (15) corresponds to the conventional linear capital accumulation equation. When $\psi < 1$, then the marginal product of new investment goods is decreasing in the flow of investment. The technology described in (15)–(16) is a special case of the adjustment cost formulation posited in Lucas and Prescott (1971) and in the references they cite. We choose the constants, $a_1 > 0$ and $a_2 > 0$, to guarantee $Q_1^x = Q_2^x = 1$ in nonstochastic steady state. This has the effect of making the nonstochastic steady state properties identical to what they are when $\psi = 1$, regardless of the actual value of ψ. Also,

it has the effect of forcing Tobin's q to be unity on a steady state growth path. Finally, as discussed further below, it has the effect of normalizing Tobin's q at unity when $\psi = 1$.[9]

The financing constraint faced by a firm in industry x is

$$(17)\quad P_{k_x,t}y + P_{i,t}z \leq S_{x,t} + B_{x,t},$$

and its period $t+1$ cash flow constraint is

$$(18)\quad \pi^x_{t+1} = Y_{x,t+1} + (1-\delta)Q^x(y,z)\,P_{k_x,t+1} - w^x_{t+1}H_{x,t+1} - (1+r^e_{x,t+1})S^x_t - (1+r^f_t)B_t \geq 0.$$

Here, $Y_{x,t+1}$ is the firm's gross output, given by (11) or (13), measured in date $t+1$ consumption units.[10] Also, we assume that if Q^x is the amount of capital used by the firm during period $t+1$, then $(1-\delta)Q^x$ remains at the end of the period, when it is made available for sale. The firm's profit function is the value of π^x_{t+1} denominated in units of the date t consumption good, summed across all possible data $t+1$ states of nature: $E_t p_{c,t+1}\pi^x_{t+1}$. The variable, $p_{c,t+1}$, is given by (6) in equilibrium and is viewed as parametric by the firm. Other variables viewed as exogenous by the firm are $P_{k_x,t+1}, w^x_{t+1}, r^e_{x,t+1}, r^f_t$ for $x = i, c$. The $x = i$ firm also makes use of $P_{i,t+1}$.

The firm's objective is to find $S_{x,t}, B_{x,t}, z, y, H_{x,t+1}$ to solve

$$(19)\quad \max_{S_{x,t},B_{x,t},z,y} E_t p_{c,t+1} \max_{H_{x,t+1}} \pi^x_{t+1},$$

subject to the relevant production technology and (17)–(18).

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[9]The formulas for a_1 and a_2 are

$$a_1 = \left[(1-\delta)\exp(-\bar{\mu}/(1-\alpha))\right]^{1-\psi},$$
$$a_2 = \left[1 - \left((1-\delta)\exp(-\bar{\mu}/(1-\alpha))\right)^{\psi}\right]^{1-\psi}.$$

[10]Thus, for the consumption industry,

$$Y_{c,t+1} = Q^c(y,z)^{\alpha}\left(\exp(\theta_{t+1})H_{c,t+1}\right)^{1-\alpha},$$

and for the investment goods industry,

$$Y_{i,t+1} = P_{i,t+1}V_{t+1}Q^c(y,z)^{\alpha}\left(\exp(\theta_{t+1})H_{i,t+1}\right)^{1-\alpha}.$$

There are a variety of useful ways to write the efficiency conditions associated with this problem. The first order condition for hours worked is

(20) $mpl_{x,t+1} = w_t^x$,

where $mpl_{x,t+1}$ denotes the marginal product of labor, denoted in period $t+1$ consumption units. Let the marginal value to the firm of an extra unit of $K_{x,t+1}$ be denoted by $V_{x,t}$,

(21) $V_{x,t} = E_t p_{c,t+1} \left[mpk_{x,t+1} + (1-\delta) P_{k_x,t+1} \right]$,

where $mpk_{x,t+1}$ denotes the marginal product of capital, denoted in period $t+1$ consumption units. The first order conditions associated with $z, y, S_{x,t}, B_{x,t}$, are

(22) $V_{x,t} Q_{2,t}^x = P_{i,t}\lambda$, $V_{x,t} Q_{1,t}^x = P_{k_x,t}\lambda$, $E_t p_{c,t+1}(1 + r_{x,t+1}^e) = E_t p_{c,t+1}(1 + r_t^f) = \lambda$,

where $\lambda > 0$ is the multiplier on the constraint (17), and $Q_{i,t}^x$ is the partial derivative of Q^x with respect to its i^{th} argument $i = 1, 2$.

Let the marginal cost of producing $K_{x,t+1}$ by a firm in industry x be denoted by $P_{k_x',t}$. It is readily established that

(23) $P_{k_x',t} = \dfrac{P_{i,t}}{Q_{2,t}^x} = \dfrac{P_{k_x,t}}{Q_{1,t}^x}$.

Household optimization ensures, via (5) and (22), that $\lambda = 1$ in equilibrium. This, together with (23) implies

(24) $V_{x,t} = P_{k_x',t}$, $x = i, c$,

i.e., the marginal value of end-of-period t capital is equated to its marginal cost.

4.4 Equilibrium

We adopt the normalization that the number of firms of each type and the number of households is one, and we assume that all agents of each type behave identically. A sequence-of-markets equilibrium is then defined in the usual way. Market clearing implies that, in a symmetric equilibrium, the demand for previously installed industry x capital in period t, denoted above by z, equals the supply, $(1-\delta)K_{x,t}$. Similarly, the demand for period t new investment goods by industry x, denoted by y, is $I_{x,t}$.

We proceed now to discuss various features of the equilibrium, including the sign switch, equity premium, Tobin's q, and comovement of employment.

4.4.1 The Sign Switch

We now endeavor to provide insight into how it is that our model can account for the sign switch observations: the fact that the price of equity—which we identify with $P_{k'_x,t}$—is procyclical, while the price of new investment goods, $P_{i,t}$, is slightly countercyclical. Consider $P_{i,t}$ first. Investment productivity shocks alone create a negative covariance between $P_{i,t}$ and output, and shocks to aggregate productivity by themselves create a positive covariance between $P_{i,t}$ and output. Thus, it should be no surprise that we can select relative magnitudes for σ_ε and σ_μ so that the model generates a slightly countercyclical $P_{i,t}$.

Now consider $P_{k'_x,t}$. If $Q^x_{2,t} \equiv 1$, as in the conventional formulation without adjustment costs (i.e., $\psi = 1$), then obviously $P_{k'_x,t} = P_{i,t}$, and there is no way to account for the sign switch. However, when $\psi < 1$, then there is a wedge between these two prices. The wedge has the effect of reducing the impact on $P_{k'_x,t}$ of investment-specific technology shocks and of increasing the impact of aggregate technology shocks. Consider a positive investment shock first. Not surprisingly, in our computational experiments we find that this generates a fall in equilibrium $P_{i,t}$ and an increase in $I_{x,t}$. The first relation in (23) indicates that this triggers two offsetting effects on $P_{k'_x,t}$. The fall in $P_{i,t}$ has the effect of driving $P_{k'_x,t}$ down, but the rise in $I_{x,t}$ has the opposite effect, by driving $Q^x_{2,t}$ down. In view of this, it is not surprising that $P_{k'_x,t}$ falls proportionally less than does $P_{i,t}$ after an investment technology shock. Consider now a positive shock to aggregate technology. This triggers an increased demand for capital for consumption-smoothing reasons. Not surprisingly, this results in a rise in $P_{i,t}$ and also a rise in $I_{x,t}$. By reducing $Q^x_{2,t}$, the rise in $I_{x,t}$ has the effect of driving $P_{k'_x,t}$ up proportionally more than the rise in $P_{i,t}$. By reducing the impact on $P_{k'_x,t}$ of investment shocks, adjustment costs in effect reduce the source of countercyclicality in $P_{k'_x}$. This is why the model predicts that this variable is procyclical.

In our quantitative analysis, we study an aggregate price index, which we obtain by combining our two equity prices as follows:

$$P_{k',t} = \frac{K_{c,t+1}}{K_{t+1}} P_{k'_c,t} + \frac{K_{i,t+1}}{K_{t+1}} P_{k'_i,t},$$

where $K_{t+1} = K_{c,t+1} + K_{i,t+1}$.

4.4.2 Tobin's q

Tobin's q, the ratio of the marginal value to the firm of $K_{x,t+1}$ divided by the marginal cost of a new investment good, is

$$(25)\quad q_t^x \equiv \frac{V_{x,t}}{P_{i,t}} = \frac{1}{Q_{2,t}^x} = \frac{1}{a_2}\left[a_1\left(\frac{(1-\delta)K_{x,t}}{I_{x,t}}\right)^{\psi} + a_2\right]^{\frac{\psi-1}{\psi}},$$

which is unity when $\psi = 1$, since $a_2 = 1$ in that case. The sign switch phenomenon can be stated in terms of the elements of Tobin's q: the numerator is procyclical, while the denominator is countercyclical.

4.4.3 The Equity Premium

To discuss the equity premium, it is convenient to first obtain an expression for the rate of return on equity. Linear homogeneity guarantees that, in equilibrium, maximized profits (19) are zero. The cash flow constraint (18) then guarantees $\pi_{t+1}^x = 0$ in each date and state of nature. Using this and (20)–(24), one gets the following equilibrium condition after some algebra:

$$(26)\quad 1 + r_{x,t+1}^e = \frac{mpk_{x,t+1} + (1-\delta)P_{k_x,t+1}}{P_{k_x',t}}(1+\gamma_t^x) - (1+r_t^f)\gamma_t^x.$$

Here, $\gamma_t^x = B_t^x/S_t^x$ denotes the firm's debt to equity ratio. The household's intertemporal Euler equation, $E_t p_{c,t+1}(1+r_{t+1}^e) = 1$, implies $E_t p_{c,t+1} E_t(1+r_{t+1}^e) = 1 - Cov_t(p_{c,t+1}, 1+r_{t+1}^e)$ or, using (5) and (26)

$$(27)\quad \frac{E_t(1+r_{x,t+1}^e)}{1+r_t^f} - 1 = -Cov_t\left(p_{c,t+1}, \frac{mpk_{x,t+1} + (1-\delta)P_{k_x,t+1}}{P_{k_x',t}}\right)(1+\gamma_t^x),$$

where the object on the left of the equality is the date t premium on equity in industry x and is, approximately, $E_t r_{x,t+1}^e - r_t^f$. BCF argue that a key channel by which a change in model specification impacts upon the equity premium operates via its impact on the equilibrium stochastic process for capital gains, $P_{k_x,t+1}/P_{k_x',t}$. The alternative channels, which operate via changes in the stochastic processes for $p_{c,t+1}$ and $mpk_{x,t+1}/P_{k_x',t}$, exert very little direct effect on the conditional covariance. BCF stress that the combination of habit persistence preferences and limited factor mobility are effective in producing the sort of stochastic process for $P_{k_x,t+1}/P_{k_x',t}$ that results in a sizeable equity premium.

As is well known, γ_t^x is indeterminate in a model like ours. Equilibrium is consistent with any state-date contingent pattern for γ_t^x, although the equilibrium quantity allocations in the model are unique. To make the analysis interesting, we must therefore fix γ_t^x exogenously. We do so by setting $\gamma_t^c = \gamma_t^i = \gamma$. A numerical value is assigned to γ in the next section.

We define the overall return on equity as r_{t+1}^e:

$$(28)\quad r_{t+1}^e = \frac{P_{k_c',t}K_{c,t+1}}{\tilde{K}_{t+1}} r_{c,t+1}^e + \frac{P_{k_i',t}K_{i,t+1}}{\tilde{K}_{t+1}} r_{i,t+1}^e,$$

where $\tilde{K}_{t+1} = P_{k_c',t}K_{c,t+1} + P_{k_i',t}K_{i,t+1}$.[11]

4.4.4 Comovement

To understand our model's implications for comovement, it is useful to consider the benchmark case where $b = h = 0$ and the utility of leisure is a power function, separable from consumption. In this case, equilibrium in the labor market associated with the consumption good sector implies via (11), (20), and the appropriate analog of (4)

$$(29)\quad \nu E_{t-1}\frac{1}{C_t}\frac{C_t}{H_{c,t}} = E_{t-1}(1 - H_{c,t} - H_{i,t})^{-\xi},$$

where $\nu > 0$ and $\xi \geq 0$. The specification in (1) corresponds to $\xi = 0$. Because the employment decision is made prior to the realization of the date t shocks

$$(30)\quad H_{c,t} = \nu(1 - H_{c,t} - H_{i,t})^{\xi}.$$

It is easily verified that this equation must hold even when the limited labor mobility assumption is dropped so that the date t labor decision is contingent upon the date t exogenous shocks. Thus, without habit persistence, getting comovement in labor is impossible, with or without limited labor mobility: if $\xi > 0$, then $H_{i,t}$ and $H_{c,t}$ must move in opposite directions. The case $\xi = 1$ is also inconsistent with comovement, since employment in the consumption sector is predicted to be constant. Still, relative to preferences based on alternative values of ξ, $\xi = 1$ appears to be the most favorable to comovement.

In our quantitative results below, which are based on $\xi = 1$, we find that to get comovement, habit persistence and limited labor mobility are both required.

[11] This is just $\left[(1 + r_{c,t+1}^e)S_{c,t} + (1 + r_{i,t+1}^e)S_{i,t}\right]/(S_{c,t} + S_{i,t})$, after making use of the firms' first order conditions and our restrictions on the debt to equity ratio.

5 Assigning Values to the Parameters

In this section, we explain how we assigned values to our model's parameters. As a preliminary evaluation of the model, we report on its trend properties and on its implications for Tobin's q.

There are 11 model parameters, three preference parameters, and four each of technology and the exogenous shocks:

$$(31) \qquad \begin{array}{l} \beta, h, b, \\ \alpha, \psi, \delta, \gamma, \\ \rho, \sigma_\epsilon, \bar{\mu}, \sigma_\mu. \end{array}$$

We find it convenient to consider 5 parameters, $\beta, \alpha, \delta, \gamma, \mu$, and the remaining 6, h, b, ρ, σ_ϵ, $\bar{\mu}$, σ_μ, separately. Loosely, the first set controls the steady state properties of the model, while the second set controls the business cycle and asset pricing properties.

5.1 Parameters Controlling Steady State

We set

$$(32) \quad \beta = 0.99999, \alpha = 0.36, \delta = 0.021, \gamma = 2/3, \bar{\mu}\alpha/(1-\alpha) = 0.004.$$

The indicated value of β was selected to maximize the model's ability to account for the observed low risk free rate. The value of α was chosen so that the model's implication for the share of GDP earned by capital coincides with an empirical estimate of that quantity based on data for the 1970s and 1980s taken from the NIPA, as reported in Christiano (1988, ftn.3). However, as emphasized there, this is the midpoint of a relatively large range of values for α, determined by the details of how one measures capital income in the NIPA. Also, Christiano and Eichenbaum (1992, p.441) report that the sample average of $[1-(K_{t+1}-I_t)/K_t]$ is 0.021. By setting $\delta = 0.021$ in the model, this empirical sample average is reproduced along the model's nonstochastic steady state growth path.[12] The value of γ was selected to match

[12]Christiano and Eichenbaum (1992) also report an estimate of α. However, their estimate exploits properties of the structure of their model, which are not shared by our model.

the corresponding empirical estimate of the debt to equity ratio reported in Benninga and Protopapadakis (1990). The linear form of preferences for the representative agent was chosen to enhance the model's implication for the volatility of labor. Finally $\bar{\mu}\alpha/(1-\alpha)$ is the steady state growth rate of consumption in the model, and $\bar{\mu}$ was selected so that it coincides with the corresponding sample average reported in Christiano and Eichenbaum (1992, p.441). The parameter η just controls scale, and we set it to 1.

The properties of the model along a steady state growth path are reported in Table 2 (see the column marked "Calibrated"). A corresponding set of estimates for the U.S. economy is reported for the entire postwar period and for the period starting the 1980s. We report results for two sample periods because of the evidence described earlier, which suggests the official estimates may underestimate the trend fall in the price of investment goods before the 1980s.

Two empirical measures of consumption are reported, C and $\tilde{C}$, with the latter including government purchases. Given our level of abstraction, it probably makes sense to identify consumption in the model with the sum of household and government consumption.[13] With this measure of consumption, the model evidently understates consumption's expenditure share in output, and correspondingly overstates investment, by about 12 percent of output. To some extent, this mismatch between model and data reflects that our empirical measure of government consumption includes government investment. From the perspective of the model, it makes more sense to include this in our measure of investment. The evidence suggests that this consideration would not entirely close the gap between the model and data. Government gross fixed capital formation (including military) has taken a declining share of GDP. It peaked at about 7.5 percent of output at around the time of the Korean war and has been falling steadily since then. In the decade after 1975, the ratio stabilized at about 3.5 percent of output.[14] So, at best, these considerations can account for only a part of the discrepancy between the empirical and model expenditure shares. Given the imprecision

[13] As is well known, this interpretation is formally rationalized by the assumption that private and public consumption are perfect substitutes. Under these circumstances, innovations in government consumption would be mirrored by equal reductions in private consumption. Interestingly, the gross features of postwar U.S. data appear consistent with this view (compare Figures 2h and 2i.)

[14] This is based on an analysis of the government investment data studied in Christiano (1988).

in the estimated value of α, there is probably room for reducing it in order to improve the model's implications for expenditure shares. This is consistent with the information in the column marked "$\alpha = 0.28$" which reports results for a lower value of α that is within the range of estimates reported in Christiano (1988). In the analysis in the next section, we report results for this reduced value of α. However, we do not comment on them because they correspond closely to the results based on $\alpha = 0.36$.

Now consider the growth statistics in the Table 2. Consistent with the model, variables measured in consumption units grow less rapidly than does real investment. However, the difference in growth rates based on the entire postwar period is not as great as the model predicts. For the model to capture this, we would need to introduce growth into our aggregate technology shock too. Note how different, however, the period since the 1980s is. There is a sharp decline in the growth rate of the investment price deflator and a corresponding sharp rise in the growth of real investment (see Figures 1 and 2.) With such a relatively short period there is, of course, a danger of confounding trend and business cycle movements. Still, the reduction in the price trend spans two business cycles (see Figure 1). Our model's assumption that all growth originates in the investment sector is not a bad approximation to the experience of the past decade.

5.2 Parameters Controlling Business Cycles and Asset Pricing

The remaining parameters are $h, b, \psi, \rho, \sigma_\epsilon, \sigma_\mu$. Values for these six parameters were set based on the following six moments of the data:

(33) $\rho(Y, P_i) = -0.15,\ \rho(Y, P_{k'}) = 0.30,\ \rho(\Delta Solow) = -0.10,\ \sigma(\Delta Solow) = 0.018,$

and

(34) $r^e - r^f = 6.63,\ r^f = 1.19.$

Here, $\rho(x, y)$ denotes the correlation between the logged, HP filtered variable x and the similarly filtered variable, y; $\rho(x)$ denotes the first order autocorrelation of the untransformed variable, x; and $\sigma(x)$ denotes the corresponding standard deviation. Also, Y denotes aggregate GDP, measured in base year prices, and $\Delta Solow$ denotes the logarithmic first difference

of the Solow residual, computed using a simple aggregate production function.[15] The first two statistics in (33) characterize the sign switch. The two statistics in (34) are taken from Cecchetti, Lam, and Mark (1993) (CLM).

Conditional on a set of feasible values for h, b, values for $\psi, \rho, \sigma_\varepsilon, \sigma_\mu$ were selected so that the model exactly reproduces the four statistics in (33). The model's implication for these statistics was computed by Monte Carlo simulation. In simulated data sets, time series on the growth rate of the Solow residual, $\Delta Solow$, were computed using the same algorithm used in the data. Thus, an aggregate production function was used for this calculation, even though there does not exist an aggregate production function relationship between aggregate inputs and aggregate outputs in our model.

We can define a mapping from feasible h, b to $\nu = [(r^e - r^f), r^f]'$ as follows. For given h, b, first compute the four parameters, $\psi, \rho, \sigma_\varepsilon, \sigma_\mu$, as described above. With the model now fully parameterized, its implied value of ν was computed by Monte Carlo simulation. In particular, we simulated 500 artificial datasets, each of length 120 observations. In each data set we computed the sample average of the annualized risk free rate and the equity premium on annualized equity returns. The model's implied value of ν was approximated by the mean of these 500 sample averages. Denote this mapping by $\nu = f(h, b)$. We define the set of feasible h, b as the set of points in the unit box having the property that $C_t \leq X_t$ and $\Lambda_{c,t} \leq 0$ are never observed in the Monte Carlo simulations used to evaluate f.

We chose feasible values of h, b so that the model's implied ν is as close as possible to $\hat{\nu}_T$, the sample estimates of ν provided in CLM. Our distance metric is $\mathcal{L}(b, h)$, where

$$(35) \quad \mathcal{L}(b, h) = [\hat{\nu}_T - f(b, h)]' \hat{V}_T^{-1} [\hat{\nu}_T - f(b, h)].$$

Also, the 2×2 matrix $\hat{V}_T$ is the CLM estimate of the underlying sampling variance in $\hat{\nu}_T$. Let

$$(36) \quad J = \mathcal{L}(\hat{b}_T, \hat{h}_T),$$

[15]The aggregate Solow residual is

$$Z_t = \frac{Y_t}{K_t^\alpha (H_t)^{1-\alpha}},$$

where $Y_t = C_t + I_{c,t} + I_{i,t}$, $K_t = K_{c,t} + K_{i,t}$, $H_t = H_{i,t} + H_{c,t}$. In section 6.2 below, we explain our rationale for interpreting these measures of Y_t and K_t as the "base" year measures of output and capital.

where $\hat{b}_T, \hat{h}_T$ minimizes $\mathcal{L}(b,h)$ over the feasible values of b, h. In practice, we could not find values of b, h which set $J = 0$.

This procedure for determining the parameters in effect treats the statistics in (33) as though they were known with perfect certainty. Presumably, a procedure which took into account the sampling uncertainty in (33) would "sacrifice" a bit on hitting the elements in (33) that are estimated the least precisely, in exchange for doing better on (34). We have not explored such statistical estimation procedures.

We obtained the following results:

(37) $\hat{b} = 0.55,\ \hat{h} = 0.0,\ J = 4.23.$

The corresponding estimates of $\psi, \rho, \sigma_\varepsilon, \sigma_\mu$ are[16]

(38) $\hat{\psi} = 0.40, \hat{\rho} = 0.52, \hat{\sigma}_\varepsilon = 0.017, \hat{\sigma}_\mu = 0.028.$

To see how f and $\mathcal{L}$ vary with b and h, consider Figure 6. It displays the empirical equity premium/risk free rate combination in (33) and 5 and 1 percent confidence intervals about this point based on the estimates of CLM. In addition, there are four lines with stars. Each line corresponds to a particular value of h, as indicated. Starting from the lower left, the stars correspond to $b = 0.4$ to $b = 0.6$, in increments of 0.025. For $h = 0.2$ and 0.3, not all values of b up to 0.6 were feasible. The optimal point, $b = 0.55, h = 0.00$, is also indicated in the figure. As the figure makes clear, increases in h sharply increase the risk free rate, and that is the reason why $h = 0$ at the optimum.

It is interesting to compare these results to those in BCF. That paper uses the same estimation strategy, but comes up with different estimates for b and h: 0.35 and 0.40 respectively. These differences reflect differences in the specification of the models: in BCH there is only an aggregate technology shock, which is specified to be a random walk with drift, and leisure enters log-linearly in utility, rather than linearly. In one respect, the two sets of estimates of b and h are similar. Ours imply a steady state ratio of the habit stock to consumption equal to 0.55, whereas the BCF estimates imply a value of 0.58. Not surprisingly, we report below that the implications for steady state risk aversion in consumption are also quite similar. The implications for steady state risk aversion in wealth are identical.

[16]The implied values of a_1 and a_2 are 0.98 and 0.12, respectively.

Though both sets of estimates imply roughly the same magnitude for the habit stock, their difference lies in how sensitive the habit stock is to recent consumption. In BCF's estimates the habit stock is relatively insensitive, whereas in our estimates the habit stock is very sensitive to recent consumption. We suspect that this is an important part of the explanation for the difference in estimation results.

An important finding in BCF is that the magnitude of the equity premium is decreasing in the autocorrelation of consumption growth. The BCF estimation procedure appears to have exploited this fact by selecting a positive value of h in order to produce negative autocorrelation in equilibrium consumption growth. With a positive value of h, the surge in consumption in the period of a shock leaves the habit stock relatively unaffected in the subsequent period. As a result, the value of consumption in that period is not particularly high and so households cut back their consumption from the high level in the previous period. This reduction is what produces the negative autocorrelation in consumption growth in the BCF model.

In our model, there are other sources of negative persistence in consumption growth, and so the estimation strategy has less need to manipulate b and h to accomplish this. Our aggregate technology shock is stationary, and the estimation strategy chooses a low value for its autocorrelation in order to reproduce the negative autocorrelation in the growth rate of the Solow residual. Negative autocorrelation in equilibrium consumption growth is a consequence of this.

To help evaluate our parameter estimates, we computed the implied elasticity of investment with respect to Tobin's q.[17] That quantity is 1.66 in our model. For comparison, Abel (1980) reports estimates of this quantity that range from 0.27 to 0.52. Relative to Abel's estimates, we have understated the degree of adjustment costs (i.e., overstated ψ).

[17]The elasticity of investment in industry x with respect to Tobin's q is, using (25)

$$\frac{d\log I_{x,t}}{d\log q_t^x} = \frac{1+\frac{a_2}{a_1}\left(\frac{I_{x,t}}{(1-\delta)K_{x,t}}\right)^{\psi}}{1-\psi}.$$

6 Quantitative Results

We quantify the mechanisms in our model that enable it to account for the salient features of asset prices and returns (Table 3). We then go on to examine our model's implications for business fluctuations and for risk aversion (Tables 4–6). We compare the model's business cycle implications with the corresponding empirical evidence. We show, for example, that employment across a wide variety of sectors is strongly procyclical. This similarity is particularly striking because the trends in these sectors are very different (Figures 7–8). Though the model does not replicate the diversity in trends, it does replicate the procyclicality of employment across sectors.

6.1 Financial Markets

Table 3 presents various statistics which capture the implications of our model for financial variables. The column marked "calibrated" reports results for the model calibrated in the previous section. The columns to the right of that report results based on various perturbations of the calibrated model, obtained by Monte Carlo simulation. The first column presents the corresponding sample estimates.

The Sign Switch

Consider the phenomenon that the model was specifically designed to address, the sign switch. That it exactly reproduces the statistics we use to characterize that phenomenon is not surprising—the parameter values were picked in part to accomplish just that. Table 3 is constructed to help assess the role played in accounting for the sign switch by two model features: the assumption of adjustment costs in the installation of investment goods, and the multiple shock assumption. (The intuition about how these factors are supposed to work is reviewed in the overview section above.)

The column marked "$\psi = 0.9$" is suggestive of what happens when adjustment costs in the investment function are shut down. In this case, the wedge between the price of equity and the price of investment goods is essentially eliminated. As a result, both have roughly the same correlation with output. That correlation turns out to be nearly zero, because

the two shocks in the model have roughly offsetting effects, in terms of their impact on the cyclicality of these prices. To gauge the role played by the multiple shock assumption, consider the column marked "$\sigma_\mu = 0$" so that the investment-specific technology shock is set to zero. In this case there are only sources of procyclicality in the two prices, and so it is not surprising that there is a strongly positive correlation between equity prices and output, and between the price of investment goods and output.

Further insight into our model's account for the sign switch may be obtained from Figure 9. This figure displays the response of the model variables to a one-standard deviation innovation in the aggregate technology shock (solid line) and in the investment-specific shock (dashed line). Consider the response to the aggregate shock. This response produces a sharp rise in both the investment goods price, P_i, and the price of equity, $P_{k'}$. Consistent with the intuition in the overview and the discussion in the model section, the jump in the equity price exceeds that in the price of investment goods. Now consider the response to an investment-specific technology shock. As anticipated by our earlier discussion, Figure 9h shows that $P_{k'}$ falls relatively little, by comparison with P_i.

Investment adjustment costs have the effect of muting the response of the price of equity to investment shocks and amplifying their response to aggregate shocks. This is why the price of equity is more procyclical than the price of investment goods, and is at the heart of our model's account of the sign switch phenomenon.

Although the model accounts well for the cyclical comovement with output of investment and equity prices, it does not account well for the magnitude of their cyclical volatility. In the data, the standard deviation of equity prices is a little below 10 percent, while the standard deviation of investment good prices is a little above 1 percent. In the model, these two prices have roughly the same standard deviation, equal to the midpoint between the two empirical standard deviations. Interestingly, the Shiller (1981) "excess volatility" puzzle stands here. Despite its (counterfactually, as we will see) high volatility in interest rates, the model still cannot account for the observed high volatility of stock prices.

The Equity Premium and Risk Free Rate

For convenience, our model's implications for the mean return on assets reported in Figure 6 are reproduced in Table 3. The table also shows that the equity premium is reduced by a factor of 10 by eliminating habit persistence in preferences (see the column, "$b = h = 0$"), or by dropping the limited labor mobility assumption, that the labor supply decision to each sector is determined prior to the realization of the current period shock (see the column, "Full Labor"). This is consistent with BCF's conclusion that limitations on factor mobility and habit persistence can produce an equity premium in a business cycle model. By contrast, the model's ability to account for the risk free rate is, if anything, hurt by habit persistence and limited labor mobility.

Other features that are important for the model's ability to replicate the equity premium are the persistence of the aggregate technology shock and the standard deviation of its innovation. By contrast, the investment specific shock has essentially nothing to do with the equity premium (see the "$\sigma_\mu = 0$" column). This latter is not surprising, in view of (27). Abstracting from the (relatively small) impact of future consumption on the marginal utility of present consumption, the equity premium would be zero if there were only investment specific shocks. This is because there is no contemporaneous impact on consumption from these shocks.

Regarding the aggregate shock, consider the impact of reducing the standard deviation in the innovation in aggregate technology, σ_ϵ, from its value in the calibrated model to 0.0085 (see the column marked "$\sigma_\epsilon = 0.85\%$, $b = 0.55$"). This value of σ_ϵ is of independent interest because it equates the standard deviation of the innovation in equilibrium consumption, $(1-\alpha)\sigma_\epsilon$, with an estimate of the corresponding empirical magnitude.[18] The drop in the value of σ_ϵ causes the equity premium to fall to 1.45 percent, and it also results in a fall in the risk free rate to 1.93 percent.

The equity premium is decreasing in the persistence of the aggregate shock, ρ. The intuition for this is explored extensively in BCF and is based on the reasoning associated

[18] A regression of per capita consumption growth on one lag of itself with estimation period 1947:2–1995:1 produces a fitted residual with standard error 0.0054. When lagged per capita GDP growth is also included in the regression, the standard error drops to 0.0051. The data used for these regressions were taken from Citibase. Consumption is measured as consumption of nondurables and services (GCNQ+GCSQ), and the population variable is QPOP.

with the permanent income hypothesis. When ρ is small, the innovation in consumption due to an aggregate shock is temporary, producing a large increase in the demand for new capital for consumption-smoothing reasons, and this in turn generates a big rise in $P_{k'}$. The resulting high capital gain is the reason equity is a bad hedge against risk, and this is what underlies the high equity premium.

Table 3 shows that the model does less well on the second moment properties of asset returns. In particular, it overstates by a factor of two the volatility of the equity premium and the return on equity. It overstates by a factor of three the volatility of the risk free rate.[19]

6.2 Business Cycle Implications

Table 4 presents the business cycle implications of our model. The format of that table corresponds to that of Table 3. The results in the "Data" column are based on data from the NIPA, which are computed using base year prices. To put our simulated output and investment data on a comparable basis, we measure them in "base" year prices too. The base year in our simulations is the initial observation, when the state of investment-specific technology, V_t, is set to unity, and the model is assumed to be on a steady state growth path, so that the relative price of investment goods is one. Thus, base year output in the model is the simple sum of the physical quantity of consumption and investment goods produced. Similarly, base year investment is the quantity of investment goods.

Standard Business Cycle Statistics

The first 7 rows of Table 4 report the model's performance in relation to standard business cycle statistics. Its performance is roughly comparable to that of standard models. It shares a problem with standard models in that it understates the volatility of hours worked.

[19]We are investigating ways of accounting for the low volatility in the risk free rate. One way, inspired by Campbell and Cochrane (1995), is to make assumptions which have the implication that when equilibrium consumption drops close to habit, the uncertainty in future consumption relative to habit rises. These two phenomena exert opposing effects on the risk free rate. If they cancel, as in Campbell and Cochrane (1995), then the risk free rate is constant.

The table suggests both shocks play an important role in the model's business cycle implications. If the investment-specific shock is too small ("$\sigma_\mu = 0$"), then the relative volatility of consumption is too high. If the aggregate shock is too small, then the relative volatility of consumption is too low (see "$\sigma_\epsilon = 0.0085$"). Not surprisingly, adjustment costs reduce the volatility of hours worked (see "$\psi = 0.9$"). The limited labor mobility assumption increases the relative volatility of consumption. Again, this is not surprising.

Comovement

We begin by characterizing the salient characteristics of employment comovement over the cycle. Doing so is complicated to some extent by the fact that some sectors produce both consumption and investment goods, and so the available employment data to not come neatly categorized according to whether they correspond to H_i or H_c. To overcome this complication, we report results for 10 different measures of hours worked. Among these there are some that clearly correspond primarily to H_c (for example, "food and kindred products"), and others seem clearly related to H_i ("construction").

Figure 7 reports the logged data and the associated HP filter trends. What is perhaps most striking about these data is the *lack* of uniformity: some trend up and some trend down and one ("food and kindred products," Fig. 7g) even does both. A sharply different picture emerges when one considers the deviations of these data from their HP trend. These are reported in Figure 8. Also reported in Figure 8 is the deviation of logged GDP from its HP trend. The picture that emerges in Figure 8 is one of great similarity among the variables: all comove positively with GDP. They differ in terms of amplitude, but there is little visual evidence of a phase shift. Table 5 reports the correlations with output, and the relative volatility to output, and associated standard errors. In all cases the correlations are large, positive, and statistically significant.

Table 4 reports the model's implications for comovement (see $\rho(Y, H_c)$ and $\rho(Y, H_i)$.) For ease of comparison, we report the empirical correlation for construction (H_i) and for food and kindred products (H_c). Note that the model implies employment in both sectors is procyclical, with the degree of procyclicality being less strong in the consumption sector.

To gain insight into how the model manages to deliver this comovement, consider the response of variables to the aggregate shock, reported in Figure 9. Note that consumption rises by nearly 1.1 percent in the period of the shock. In a standard model, the impact of an aggregate shock on consumption would be much lower, as workers and capital are predicted to instantaneously switch out of the consumption sector and into the investment sector.[20]

Thus, the specification of technology in our model prevents a countercyclical response of hours worked in the consumption sector in the period of the shock. The transient nature of the technology shock, together with habit persistence, ensures that the employment response in the consumption sector remains strong and positive in subsequent periods. Habit persistence implies that because consumption was high in the period of the shock, the value of consumption is high in the periods after. Our estimate, $h = 0$, plays an important role here. The transience of the shock implies that to supply consumption in those periods requires high labor effort. And this is exactly what happens according to Figure 9f.

Now consider the response to an investment shock. The response of employment in the consumption sector to this shock is positive—presumably reflecting a wealth effect—but very close to zero. The fact that hours worked in the consumption sector is procyclical relative to both shocks guarantees that our model is able to account for the comovement phenomenon.

To see what happens in our model when these restrictions on intersectoral factor mobility are relaxed, consider the "Full Labor" column in Table 4. That column reports the dynamic implications of a model parameterized just like our calibrated model, with the only exception that employment responds flexibly to the shocks. Note that now employment in the consumption sector is countercyclical. Evidently, to get comovement, habit persistence is not enough, and the limited labor mobility assumption is needed too. In section 3 we showed that with the limited mobility assumption only, and not habit persistence, comovement is not possible either. That is, to get comovement in our framework, *both* habit persistence and the limit labor mobility assumption are required.

Several features of our framework have played an important role in delivering our comovement result. These include the transitory nature of the aggregate technology shock, our small

[20]To be concrete, bakery equipment and bakers in the "food and kindred products" sector are predicted to transform instantaneously into bulldozers and bulldozer drivers in the "construction sector."

estimated value of h, and, as explained in the previous section, the linear specification of utility in leisure. These considerations help explain why BCF do not find comovement. Their aggregate technology shock is a random walk, their estimated value of h is positive, and they adopt a log-linear specification of utility in leisure.

Persistence

One indicator that our calibrated model introduces persistence is that its implied growth rate of the Solow residual has autocorrelation -0.1, while equilibrium output growth has autocorrelation 0.02 (see Table 4). Christiano (1988) shows that a standard business cycle which reproduces a pattern like this in the Solow residual implies first order autocorrelation in output growth equal to roughly -0.1. Another indicator of endogenous persistence can be seen in the parameterization, $\sigma_\epsilon = 0.0085$, $b = 0.65$. In this case, the growth rate of the Solow residual is essentially uncorrelated over time. Yet, the autocorrelation in output growth is 0.11 (see Table 4).

6.3 Implications for Risk Aversion

It is useful, for purposes of assessing the plausibility of our model, to document its implications for RRA_c, as defined in (10). This is done in Table 6, which reports the values of ν ($\times 100$) associated with various values of μ and the various values of b we consider, along a steady state growth path. We report risk aversion for our calibrated model and the various perturbations on it studied above. In addition, for comparison, we report consumption risk aversion implied by the parameterization considered in Constantinides (1990) and for the production and exchange models studied in BCF.

Note that for our calibrated model ($b = 0.55$), a household would be willing to pay 2.5 percent of one period's consumption in order to avoid a fair bet on 10 percent of that consumption. This is a low level of risk aversion, particularly by comparison with the levels of risk aversion required in other studies that seek to account for the equity premium. Moreover, our model's implication for the risk free rate and equity premium is very close to the empirical values of these variables (recall Figure 6.) So, it is natural to investigate how

it is that our model manages to do this, with so little risk aversion. The explanation lies in two apparently counterfactual implications of the model.

Why is RRA_c so Low in the Calibrated Model?

First, note from Table 4 that our model implies the autocorrelation of consumption growth is -0.14, whereas the corresponding empirical estimate is 0.19. This implication of the model reflects the importance of the aggregate shock, the transitory nature of which enhances the model's ability to account for the equity premium for a given level of RRA_c. The transitory nature of the aggregate shock to technology implies that households have a strong smoothing motive when there is a positive innovation. To investigate the quantitative importance of these considerations, we examined a version of the model in which $\rho = 0.99$, so that the aggregate technology shock is almost a random walk. Now, consumption growth is also virtually a random walk. Predictably, we found that this change reduces the equity premium to 3.46 percent (see Table 3, $b = 0.55$, $\rho = 0.99$.) To offset this, we raised b to 0.60. According to Table 6, with this specification of utility, households are willing to give up 2.9 percent of a period's consumption to avoid a 10 percent gamble, up only a little from 2.5 when $b = 0.55$. Although this is a higher level of risk aversion, it is perhaps not beyond the realm of empirical plausibility. Interestingly, this version of our model preserves the basic features of the calibrated model: its ability to account for the sign switch, comovement, and the basic features of the business cycle.

The second reason our calibrated model can account for the observed equity premium with so little risk aversion is that it overpredicts the innovations in consumption. In the model, the standard deviation in the innovation in consumption is just $(1-\alpha)\sigma_\varepsilon \simeq .011$, for reasons discussed above. But, in the data this quantity is about one-half of one percent, 0.005. Not surprisingly, this feature of the model also enhances its ability to account for the equity premium. To investigate how important it is, we studied a version of the model with $(1-\alpha)\sigma_\varepsilon = .0054$ (see the indicated column in Tables 3 and 4). Not surprisingly, the equity premium is reduced substantially with this change, down to 1.45 percent. We then increased b to 0.65, and this returned the equity premium up to 4.10 percent. Interestingly, we still can account qualitatively for the sign switch and comovement observations. This value of b

implies higher risk aversion in consumption (see Table 4), but presumably not a level that economists will find implausibly high. Note that this change also raises the autocorrelation of consumption growth to nearly zero. This reflects that increasing b enhances the motive to smooth consumption.

What Level of RRA_c Does it Take?

Thus, relatively small increases in the model's implication for risk aversion in consumption can move it in the direction of being more consistent with the consumption data. But we have not moved the model all the way. What sort of risk aversion in consumption would that imply? The answer is in BCF. They study a pure exchange economy in which equilibrium consumption growth is modelled based on U.S. consumption data. They account for the equity premium with $b = 0.58$ and $h = 0.30$. According to Table 6, with these parameters, a household is willing to give up 6.7 percent of consumption to avoid a fair, 10 percent gamble. Some will perhaps view this as a high degree of risk aversion. Does this mean that, necessarily, to account for the observed equity premium, high RRA_c is required? The answer may be yes. But, there are at least three reasons to think that the answer might actually be *no*. All of these reasons build, in different ways, on the notion that the information observed by the economic analyst and that observed by households differ in some way.

First, from the analysis above, it is clear that the details of the consumption process matter a lot for determining how much RRA_c is required to account for the equity premium. Yet, there is little confidence in the quality of this data (see Wilcox (1992).) Gibbons (1989) cites this low quality as a reason for ignoring consumption altogether in evaluating asset pricing models. The range of uncertainty about the consumption data when these quality considerations are integrated with the usual sampling uncertainty may include parameterizations of consumption which permit accounting for the observed equity premium with low RRA_c.[21]

[21]One indicator of data uncertainty is the fact that the first order autocorrelation in the growth rate of BCF's consumption data is 0.34. This reflects that their measure of consumption and their sample period differ from ours. This high level of consumption autocorrelation underlies their estimate of RRA_c. Presumably, they would have reported a lower RRA_c, had they used our data set.

Second, suppose all the features of the univariate stochastic process underlying the consumption data were known accurately. Quah (1990) has shown that, even a process in which the univariate representation is a first order autoregression in growth rates with positive AR(1) parameter is consistent with an unobserved components representation in which transitory shocks play a very large role. The analysis in this paper has exhibited various empirical considerations that make such representations plausible, although the calibrated model fails to reproduce crucial features of the type of statistical environment contemplated in Quah (1990). This is because the univariate representation of equilibrium consumption exhibits negative persistence in its growth rate. The parameterization, $\sigma_\varepsilon = 0.0085$, $b = 0.65$, does exhibit features of Quah's environment: in this case, the univariate representation of consumption resembles a random walk because its growth rate is nearly uncorrelated over time; yet the innovation in consumption entirely reflects its transitory component (see Table 4). In the statistical environment like the one studied by Quah, as long as agents observe the two underlying components driving consumption, their demand for equity may be driven in an important way by the transitory component, possibly leading to a large premium on equity. We are currently exploring this possibility further.

Third, as is well known, various transformations are applied to the data, which are likely to have the implication that measured consumption displays more persistence than the actual consumption choices made by agents. The fact that the data are aggregated over time is perhaps the prime example of this possibility.[22] Thus, agents could be living in an environment with relatively little persistence in consumption, which could be reflected in a high equity premium (like in our $\sigma_\varepsilon = 0.0085$, $b = 0.65$ model), even though published data exhibit substantial persistence due to time aggregation. For a quantitative investigation of this idea in a closely related context, a discussion of the "Deaton paradox" for consumption, see Christiano (1989).

[22] Another possibility is seasonal adjustment, which is thought by some to have the effect of smoothing the data. However, some preliminary analysis suggests this may not be an important part of the explanation of the gap between our model and the data. We generated 1,000 observations from the calibrated model. The first order autocorrelation of consumption growth in this data is -0.0773. We then applied the version of the Census X-11 seasonal adjustment procedure implemented in RATS' EZ-X11 program to seasonally adjust the data (we used the "multiplicative adjustment" option). The first order autocorrelation of the growth rate of the seasonally adjusted artificial consumption data is -0.0382. We concluded that the smoothing implicit in seasonal adjustment does very little to increase first order autocorrelation in growth rates.

To summarize this discussion of risk aversion, our model can account for the mean equity premium and risk free rate with low risk aversion. On the one hand, we argued that the model's apparent counterfactual implications for consumption played an important role in this result. On the other hand, the model's implications are not *very* far off. Moreover, we discussed a variety of considerations that could in principle reconcile the model with the data. These considerations make us optimistic that a model can be found that accounts for the key first moments of asset prices without having counterfactual implications for consumption.

7 Concluding Remarks

A number of researchers have argued that asset pricing data contain useful information for macroeconomists. For example, the early work of Hall (1978) and Hansen and Singleton (1982,1983) showed how to use asset pricing data to test implications of equilibrium models and estimate their parameters. Data on asset prices are not only useful for evaluating models, but also for providing guidance about how to further develop them. In view of these considerations, it is surprising that business cycle researchers have made relatively little use of asset pricing data. In this and a previous paper with Michele Boldrin, we took a few steps in this direction.

In BCF, we explored modifications in a standard business cycle model that could account for the observed high equity premium and low risk free rate. In this paper, we were interested in understanding an observation that we initially found puzzling: equity prices are procyclical, while investment prices are (weakly) countercyclical. Although the literature on Tobin's q prepares one for the possibility that these two prices are not identical, we were nevertheless surprised find that their business cycle dynamics are so very different.

In this paper we incorporated the features proposed in BCF to account for key aspects of the first moment properties of asset returns, together with additional features designed to account for the business cycle properties of asset prices. After establishing that there is a parameterization of our model that can account for the price and rates of return on assets, we turned to see what this parameterization implies for business cycles and for risk aversion.

We find that the model does at least as well as standard business cycle models in ac-

counting for conventional business cycles facts. On two dimensions, we find that the model actually represents a step forward relative to the standard business cycle model. First, as in BCF, we find that the modifications designed to account for mean asset returns help confer an internal propagation mechanism to the model. Second, in our model environment, the modifications also allow the model to be consistent with the fact that employment is procyclical across a broad range of sectors.

The basic features that we use to account for the asset pricing phenomena are habit persistence preferences and limitations on the ability to quickly move factors of production both cross-sectionally and intertemporally. These same limitations, by slowing the economy's ability to respond to shocks, have the effect of introducing persistence. At the same time, limitations on intersectoral mobility, coupled with habit persistence, have the effect of making employment across sectors move up and down together over the cycle.

The results in this paper and in BCF support the notion that the same frictions needed to account for the salient features of asset prices and returns are also useful in understanding the salient features of business cycles.

A The Planner's Problem

The quantities in a competitive equilibrium of our model can be computed by solving the following planner problem: maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \{\ln(C_t - X_t) + \eta(T - H_{c,t} - H_{i,t})$$

$$+\Lambda_{x,t}\left[X_{t+1} - hX_t - bC_t\right]$$

$$+\Lambda_{c,t}\left[K_{c,t}^{\alpha}(\exp(\theta_t)H_{c,t})^{1-\alpha} - C_t\right]$$

$$+\Lambda_{i,t}\left[V_t K_{i,t}^{\alpha}(\exp(\theta_t)H_{i,t})^{1-\alpha} - I_{c,t} - I_{i,t}\right]$$

$$+\Omega_{c,t}\left[Q^c\left((1-\delta)K_{c,t}, I_{c,t}\right) - K_{c,t+1}\right]$$

$$+\Omega_{i,t}\left[Q^i\left((1-\delta)K_{i,t}, I_{i,t}\right) - K_{i,t+1}\right]\Big\}$$

subject to

$$\theta_t = \rho\theta_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

$$V_t = \exp(\bar{\mu} + \mu_t)V_{t-1}, \quad \mu_t \sim N(0, \sigma_\mu^2)$$

Here, $\Lambda_{y,t}$, $y = x, c, i$ and $\Omega_{y,t}$, $y = c, i$ are Lagrange multipliers. The planner is assumed to choose $H_{c,t}$ and $H_{i,t}$ prior to the realization of the date t state of nature, while the remaining choice variables (including the multipliers) are selected afterward.

To solve the planner problem we first have to transform it into its stationary form. The variables in the transformed problem are $c_t, k_{c,t+1}, k_{i,t+1}, x_{t+1}, i_{c,t}, i_{i,t}, \lambda_{c,t}, \lambda_{i,t}, \lambda_{x,t}, \omega_{c,t}$ and $\omega_{i,t}$. These are defined as follows (not all variables are transformed in the same way):

$$c_t = \frac{C_t}{V_t^{\alpha/(1-\alpha)}}, \quad k_{c,t+1} = \frac{K_{c,t+1}}{V_t^{1/(1-\alpha)}}, \quad k_{i,t+1} = \frac{K_{i,t+1}}{V_t^{1/(1-\alpha)}}, \quad x_{t+1} = \frac{X_{t+1}}{V_t^{\alpha/(1-\alpha)}}$$

$$i_{c,t} = \frac{I_{c,t}}{V_t^{1/(1-\alpha)}}, \quad i_{i,t} = \frac{I_{i,t}}{V_t^{1/(1-\alpha)}},$$

$$\lambda_{c,t} = V_t^{\alpha/(1-\alpha)}\Lambda_{c,t}, \quad \lambda_{i,t} = V_t^{1/(1-\alpha)}\Lambda_{i,t}, \quad \lambda_{x,t} = V_t^{\alpha/(1-\alpha)}\Lambda_{x,t}$$

$$\omega_{c,t} = V_t^{1/(1-\alpha)}\Omega_{c,t}, \quad \omega_{i,t} = V_t^{1/(1-\alpha)}\Omega_{i,t}$$

In what follows it will also be helpful to use the following:

$$y_{c,t} = \frac{Y_{c,t}}{V_t^{\alpha/(1-\alpha)}}, \quad y_{i,t} = \frac{Y_{i,t}}{V_t^{1/(1-\alpha)}},$$

where $Y_{c,t} = K_{c,t}^{\alpha}(\exp(\theta_t)H_{c,t})^{1-\alpha}$ and $Y_{i,t} = V_t K_{i,t}^{\alpha}(\exp(\theta_t)H_{i,t})^{1-\alpha}$.

The first order conditions for an interior solution to the stationary version of the planner problem can be rearranged to form the following system of equations:

(39) $$\lambda_{x,t} - \beta E\left[\exp(-\hat{\mu}_{t+1})(\lambda_{x,t+1}h + \frac{1}{c_{t+1} - \exp(-\hat{\mu}_{t+1})x_{t+1}}\middle|\Omega_t\right] = 0$$

$$\frac{\lambda_{i,t}}{Q_{2,t}^c} - \beta E\left[\left(\frac{1}{c_{t+1} - \exp(-\hat{\mu}_{t+1})x_{t+1}} - b\lambda_{x,t+1}\right)\alpha\frac{y_{c,t+1}}{k_{c,t+1}}\right.$$

(40) $$\left. + \exp(-\tilde{\mu}_{t+1})(1-\delta)\frac{\lambda_{i,t+1}}{Q_{2,t+1}^c}Q_{1,t+1}^c\middle|\Omega_t\right] = 0$$

(41) $$\frac{\lambda_{i,t}}{Q_{2,t}^i} - \beta E\left[\lambda_{i,t+1}\left(\alpha\frac{y_{i,t+1}}{k_{i,t+1}} + \exp(-\tilde{\mu}_{t+1})(1-\delta)\frac{\lambda_{i,t+1}}{Q_{2,t+1}^i}Q_{1,t+1}^i\right)\middle|\Omega_t\right] = 0$$

(42) $$E\left[\left(\frac{1}{c_t - \exp(-\hat{\mu}_t)x_t} - b\lambda_{x,t}\right)(1-\alpha)\frac{y_{c,t}}{H_{c,t}} - \eta\middle|\Omega_t^*\right] = 0$$

(43) $$E\left[\lambda_{i,t}(1-\alpha)\frac{y_{i,t}}{H_{i,t}} - \eta\middle|\Omega_t^*\right] = 0$$

(44) $$\frac{x_{t+1} - h\exp(-\hat{\mu}_t)x_t}{b} - y_{c,t} = 0$$

(45) $$i_{c,t} + i_{i,t} - y_{i,t} = 0$$

In these expressions Ω_t denotes the information set that includes realizations of the technology shocks up to and including time t, and Ω_t^* denotes the information set identical to Ω_t except that the time t realizations of the technology shocks are excluded. Also, $Q_{j,t}^x$, $j = 1, 2$ denote partial derivatives of Q_t^x with respect to its first and second arguments, respectively, evaluated at the time t values of the arguments, for $x = c, i$. Finally, $\hat{\mu}_t = \alpha\mu_t/(1-\alpha)$ and $\tilde{\mu}_{t+1} = \hat{\mu}_t/\alpha$.

Making use of the resource constraints implicit in the formulation of the planner problem, we can collapse the model to the following seven equations

$$E\left[v_j(k_{c,t}, k_{c,t+1}, k_{c,t+2}, k_{i,t}, k_{i,t+1}, k_{i,t+2}, x_t, x_{t+1}, x_{t+2}, H_{c,t}, H_{c,t+1}, H_{i,t}, H_{i,t+1},\right.$$

$$\left.\lambda_{x,t}, \lambda_{x,t+1}, \lambda_{i,t}, \lambda_{i,t+1}, \theta_t, \theta_{t+1}, \mu_t, \mu_{t+1})\middle|\, \Omega_t\right] = 0, \quad j = 1, 2, 3, 6, 7;$$

$$E\left[v_j(k_{c,t}, k_{c,t+1}, k_{c,t+2}, k_{i,t}, k_{i,t+1}, k_{i,t+2}, x_t, x_{t+1}, x_{t+2}, H_{c,t}, H_{c,t+1}, H_{i,t}, H_{i,t+1},\right.$$

$$\left.\lambda_{x,t}, \lambda_{x,t+1}, \lambda_{i,t}, \lambda_{i,t+1}, \theta_t, \theta_{t+1}, \mu_t, \mu_{t+1})\middle|\, \Omega_t^*\right] = 0, \quad j = 4, 5.$$

Here $v_j(\cdot)$, $j = 1, 2, 3, 6, 7$ are equations (39), (40), (41), (44), and (45) respectively, and $v_j(\cdot)$, $j = 4, 5$ are equations (42) and (43), respectively. These equations can be solved using the methods described in Christiano, Fisher and Valdivia (1995).

We can use the multipliers from a solution to the planner problem to compute the relative prices studied in the main text. First, the relative price of the new investment good is given by

$$P_{i,t} \equiv \frac{\Lambda_{i,t}}{\Lambda_{c,t}} = \frac{\lambda_{i,t}}{\lambda_{c,t}} \frac{1}{V_t}.$$

Second, the prices for $K_{c,t+1}$ and $K_{i,t+1}$ are

$$P_{k'_c,t} \equiv \frac{\Omega_{c,t}}{\Lambda_{c,t}} = \frac{\omega_{c,t}}{\lambda_{c,t}} \frac{1}{V_t}, \quad P_{k'_i,t} \equiv \frac{\Omega_{i,t}}{\Lambda_{c,t}} = \frac{\omega_{i,t}}{\lambda_{c,t}} \frac{1}{V_t},$$

respectively. Third, the prices for installed capital are

$$P_{k_c,t} \equiv \frac{Q^c_{1,t}\Omega_{c,t}}{\Lambda_{c,t}} = \frac{\omega_{c,t}}{\lambda_{c,t}} \frac{Q^c_{1,t}}{V_t}, \quad P_{k_i,t} \equiv \frac{Q^i_{1,t}\Omega_{i,t}}{\Lambda_{C,t}} = \frac{\omega_{i,t}}{\lambda_{c,t}} \frac{Q^i_{1,t}}{V_t}.$$

Notice that each of these prices will trend downward if V_t has a positive trend. To use these formulas we require expressions for $\lambda_{c,t}, \omega_{c,t}$ and $\omega_{i,t}$. These can be computed from the first order conditions from the planner problem. The expressions derived in this way are given by

$$\lambda_{c,t} = \frac{1}{y_{c,t} - \exp(\hat{\mu}_t)x_t} - b\lambda_{x,t},$$

$$\omega_{c,t} = \frac{\lambda_{i,t}}{Q^c_{2,t}}, \text{ and}$$

$$\omega_{i,t} = \frac{\lambda_{i,t}}{Q^i_{2,t}}.$$

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Table 1: Business Cycle Properties of Stock Prices

Industry	Second Moment Statistics S&P 500		Dow Jones		NYSE	
	$\frac{\sigma_p}{\sigma_y}$	$corr(p,y)$	$\frac{\sigma_p}{\sigma_y}$	$corr(p,y)$	$\frac{\sigma_p}{\sigma_y}$	$corr(p,y)$
Composite	5.14	0.36	5.22	0.30	5.20	0.37
	(0.53)	(0.09)	(0.55)	(0.10)	(0.53)	(0.09)
Capital Goods	6.06	0.39			na	na
	(0.56)	(0.09)				
Utilities	4.66	0.21			6.82	0.30
	(0.64)	(0.11)			(1.40)	(0.09)
Finance	6.42	0.30			6.73	0.38
	(1.03)	(0.12)			(0.93)	(0.10)
Industrial	5.33	0.35			5.62	0.35
	(0.55)	(0.09)			(0.75)	(0.12)
Transportation	5.82	0.19			7.45	0.36
	(0.79)	(0.15)			(1.08)	(0.10)

Notes: (i) Data source—CITIBASE. The sample period is 1947I–1995I for NYSE Composite, Dow Jones, and 1966I–1995I for the other NYSE variables. The sample period is 1947I–1995I for all but two of the S&P 500 data series. It is 1970I–1995I for S&P 500 finance and transportation. Data on gross domestic product (GDP) cover the period 1947I–1995I. Stock price data are deflated by implicit price deflator for consumption of nondurables and services.

(ii) Statistics—All data were logged, and then Hodrick-Prescott filtered prior to analysis. σ_p denotes the standard deviation of the (detrended) stock price, σ_y denotes the standard deviation of output, and $corr(p,y)$ denotes the correlation between p and y. Numbers in parentheses denote the standard errors of σ_p/σ_y and $corr(p,y)$, computed as in Christiano and Eichenbaum (1992). For estimation of the relevant zero-frequency spectral density a Bartlett window, truncated at lag four, was used.

Table 2: Sample First Moments, Macroeconomic Variables

	Data		Model	
	1947:I-1995:I	1983:I-1995:I	Calibrated	$\alpha = 0.28$
$\frac{C}{Y^*}$	0.56	0.59	0.64	0.72
$\frac{\tilde{C}}{Y^*}$	0.75	0.76		
$\frac{P_i I}{Y^*}$	0.24	0.24	0.36	0.28
$\frac{P_k K}{Y^*}$	10.6		11.3	9.1
ΔY^*	1.85	1.46	1.60	1.60
ΔC	1.84	1.68	1.60	1.60
$\Delta \tilde{C}$	1.86	1.58		
ΔI	2.35	3.75	4.44	4.44
$\Delta P_i I$	1.81	1.11	1.60	1.60
ΔP_i	−0.57	−2.54	−2.84	−2.84

Notes: (i) Data from "Data" column are taken from Citibase, except the capital-output ratio, which was taken from Christiano (1988) and covers the period 1956:III–1984:I. Data in "Calibrated Model" column are properties of the nonstochastic steady state. $\Delta x \sim$ first difference of log of otherwise untransformed variable.

(ii) U.S. data—$C \sim$ consumption of nondurables (GCNQ) and services (GCSQ); $\tilde{C} \sim$ consumption of nondurables and services, plus government purchases (GGEQ); $I \sim$ business fixed investment (GIFQ) plus household durable purchases (GCDQ). The preceding variables are measured in 1987 dollars. $Y^* \sim$ current dollar Gross Domestic Product (GDP) divided by implicit price deflator for consumption of nondurables and services $((GCN + GCS)/(GCNQ + GCSQ))$; $P_i I \sim$ current dollar business fixed investment (GIF) plus current dollar durable purchases (GCD), divided by implicit price deflator for consumption of nondurables and services; $P_i \sim$ business fixed investment plus household durable goods deflator ((GCD+GIF)/(GCDQ+GIFQ)), divided by implicit price deflator for consumption of nondurables and services. Growth rate results are for per capita variables, obtained by scaling by the population between 16 and 64 years old (PAN17-PAN19). The growth rate results extend only to the period 1995:I, reflecting the availability of the population data in our version of Citibase. All mnemonics correspond to variable names in Citibase.

(iii) Calibrated model data—$Y^* \sim C + P_i I$.

Table 3: Prices and Rates of Return

					$\sigma_\varepsilon = 0.85\%$					$\rho = 0.99$		
Statistic	Data	Calibrated	Full Labor	$b = h = 0$	$b = 0.55$	$b = 0.65$	$\sigma_\mu = 0$	$\psi = 0.9$	$\rho = 0$	$b = 0.55$	$b = 0.60$	$\alpha = 0.28$
r^e	7.82	8.56	2.24	1.96	3.35	6.66	8.46	8.95	11.1	5.47	8.18	7.98
		(0.08)	(0.02)	(0.01)	(0.03)	(0.06)	(0.08)	(0.08)	(0.1)	(0.06)	(0.08)	(0.08)
r^f	1.19	2.78	1.65	1.64	1.93	2.56	2.77	2.80	3.76	2.01	2.43	2.26
	(0.81)	(0.09)	(0.03)	(0.03)	(0.05)	(0.07)	(0.08)	(0.09)	(0.10)	(0.06)	(0.07)	(0.08)
$r^e - r^f$	6.63	5.78	0.58	0.32	1.45	4.10	5.69	6.15	7.31	3.46	5.75	5.72
	(1.78)	(0.15)	(0.04)	(0.03)	(0.07)	(0.12)	(0.15)	(0.15)	(0.17)	(0.11)	(0.15)	(0.15)
σ_{r^e}	19.53	41.2	11.3	8.67	13.8	34.5	40.9	42.7	49.4	29.7	39.8	40.9
		(0.2)	(0.04)	(0.03)	(0.1)	(0.1)	(0.2)	(0.2)	(0.3)	(0.1)	(0.2)	(0.2)
σ_{r^f}	5.27	18.2	3.58	2.42	9.02	16.0	18.2	18.2	23.9	11.8	16.3	18.1
	(0.74)	(0.1)	(0.01)	(0.01)	(0.03)	(4e-2)	(0.1)	(0.1)	(0.1)	(0.04)	(0.1)	(0.1)
$\sigma_{r^e - r^f}$	19.02	36.4	10.7	8.28	17.4	30.1	36.0	38.1	42.3	27.0	35.9	36.1
	(1.73)	(0.2)	(0.03)	(0.02)	(0.1)	(0.1)	(0.1)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)
σ_{P_i}	1.11	5.16	3.22	3.11	3.73	4.85	4.11	5.25	5.65	4.31	5.10	5.18
		(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
$\sigma_{P_{k'}}$	9.31	4.77	1.69	1.37	2.50	4.07	4.69	5.13	5.39	3.53	4.52	4.77
		(0.01)	(0.01)	(4e-3)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)
$\rho(Y, P_i)$	-0.15	-0.15	-0.78	-0.75	-0.64	-0.42	0.77	-0.09	-0.07	-0.38	-0.28	-0.08
		(0.01)	(3e-3)	(3e-3)	(4e-4)	(4e-3)	(3e-4)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$\rho(Y, P_{k'})$	0.30	0.30	-0.06	-0.08	-0.08	0.04	0.82	0.00	0.33	0.18	0.18	0.37
		(4e-3)	(0.01)	(0.01)	(4e-4)	(4e-3)	(2e-4)	(0.01)	(4e-3)	(4e-3)	(4e-3)	(4e-3)

Notes: (i) The "Data" column contains estimates of the mean and standard deviation of the risk-free return and the equity premium, with standard errors in parentheses over the period 1892-1987 for U.S. data. These estimates are taken from Cecchetti, Lam and Mark (1993). These authors do not report the analogous values for the return to equity; (ii) $\rho(x, y)$ denote the correlation between variables x and y; (iii) Except for the correlations, all statistics are in percent terms; (iv) Rates of return are annualized; (v) Results for the models are based on 500 replications of sample size 120, and Monte Carlo standard errors are reported in parentheses. The latter are the standard deviation, across replications, of the associated statistics, divided by $\sqrt{500}$; (vi) Prices and output in the model and the data are logged and HP filtered prior to analysis, rates of return are not filtered. P_i is measured by the ratio of 1987 dollar business fixed investment plus consumption of durables to the implicit price deflator for consumption of nondurables and services. $\sigma_{P_{k'}}$, $\rho(Y, P_{k'})$ are taken from Table 1, and the fact that $\sigma_Y = 1.79$ percent.

Table 4: Business Cycle Statistics

					$\sigma_\varepsilon = 0.0085$					$\rho = 0.99$		
Statistic	Data	Calibrated	Full Labor	$b = h = 0$	$b = 0.55$	$b = 0.65$	$\sigma_\mu = 0$	$\psi = 0.9$	$\rho = 0$	$b = 0.55$	$b = 0.60$	$\alpha = 0.28$
σ_Y	1.79	2.16	2.09	2.14	1.90	1.90	1.19	2.65	2.09	2.32	2.32	1.92
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(4e-3)	(0.02)	(0.01)	(0.02)	(0.02)	(0.01)
σ_C/σ_Y	0.47	0.56	0.27	0.53	0.33	0.34	0.99	0.46	0.54	0.61	0.61	0.63
		(4e-3)	(2e-3)	(3e-3)	(1e-3)	(3e-3)	(2e-4)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)
σ_I/σ_Y	2.91	1.87	2.05	1.89	2.06	2.06	1.01	1.95	1.92	1.80	1.80	2.14
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(2e-4)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
σ_H/σ_Y	0.82	0.20	0.29	0.15	0.18	0.19	0.26	0.44	0.29	0.14	0.14	0.20
		(1e-3)	(2e-3)	(4e-4)	(1e-3)	(1e-3)	(2e-4)	(1e-3)	(1e-3)	(4e-4)	(4e-4)	(1e-3)
$\rho(Y, C)$	0.78	0.55	0.43	0.53	0.31	0.32	1.00	0.48	0.49	0.62	0.62	0.63
		(4e-3)	(0.01)	(4e-3)	(0.01)	(0.01)	(3e-6)	(0.01)	(4e-3)	(0.01)	(0.01)	(4e-3)
$\rho(Y, I)$	0.70	0.95	0.98	0.95	0.97	0.97	1.00	0.96	0.95	0.94	0.95	0.91
		(1e-3)	(3e-4)	(1e-3)	(4e-4)	(4e-4)	(4e-6)	(1e-3)	(1e-3)	(1e-3)	(1e-3)	(1e-3)
$\rho(Y, H)$	0.81	0.63	0.14	0.69	0.72	0.68	0.48	0.80	0.38	0.75	0.75	0.60
		(3e-3)	(0.01)	(3e-3)	(3e-3)	(3e-3)	(3e-3)	(2e-3)	(4e-3)	(2e-3)	(3e-3)	(3e-3)
$\rho(Y, H_C)$	0.52	0.23	-0.38	NA	0.13	0.13	0.43	0.31	0.07	0.16	0.13	0.27
		(0.01)	(4e-3)	(NA)	(0.01)	(0.01)	(3e-3)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$\rho(Y, H_i)$	0.79	0.70	0.93	0.69	0.75	0.75	0.53	0.81	0.67	0.75	0.75	0.68
		(3e-3)	(2e-3)	(3e-3)	(3e-3)	(3e-3)	(3e-3)	(2e-3)	(3e-3)	(2e-3)	(3e-3)	(3e-3)
$\rho(\Delta Y)$	0.36	0.02	-0.05	-0.003	0.10	0.11	-0.12	0.26	-0.13	0.12	0.12	0.01
		(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)
$\rho(\Delta C)$	0.19	-0.14	0.16	-0.23	-0.12	-0.09	-0.15	-0.14	-0.25	0.01	0.01	-0.14
		(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)
$\rho(\Delta S)$	-0.10	-0.10	-0.13	-0.10	-0.03	-0.03	-0.23	-0.04	-0.29	0.01	0.01	-0.12
		(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)	(4e-3)
$\sigma(\Delta S)$	1.80	1.80	1.92	1.80	1.45	1.45	1.24	1.80	2.04	1.69	1.70	1.64
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(4e-3)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)

Notes: (i) Figures in the "Data" column are based on U.S. data, covering the period 1947:1-1995:1, taken from Citibase. Consumption is measured by consumption of nondurables and services, GCNQ+GCSQ, measured in 1987 dollars, divided by GPOP, a not-seasonally-adjusted measure of population (including armed forces overseas). Investment is business fixed investment, GIFQ, plus consumption of durable goods, GCDQ, measured in 1987 dollars, and scaled by GPOP. H_c is measured by LWH20X, which is employment in the industry in Table 5 that looks most (to us) like a consumption goods industry. Similarly, H_i is measured by LWHCX in Table 5; (ii) With the exception of the correlations and the relative volatilities, all the statistics are reported in percentage terms; (iii) Results for the model are based on 500 replications of sample size 120, and Monte Carlo standard errors are reported in parentheses; (iv) $\rho(x)$ means the first order autocorrelation of the variable x, $\rho(x,y)$ means the correlation between variables x and y, and $\sigma(x)$. means the standard deviation of x. Δx means the first difference of the log of (otherwise untransformed) x. $\rho(\Delta S)$ denotes the first order autocorrelation of the growth rate of the model implied Solow residual, and $\sigma(\Delta s)$ denotes the standard deviation of the growth rate of the model implied Solow residual. (v) Variables without Δ have been logged and HP filtered prior to analysis; (vi) the entry NA (Not Applicable) signifies that the indicated number is not defined. The discussion in Section 4.4.4 indicates that when $b = h = 0$, then H_c is a constant.

Table 5: Business Cycle Properties of Hours Worked

Hours worked, H_t	σ_H/σ_Y	$corr(H_t, Y_{t-\tau})$						
		$\tau=-3$	$\tau=-2$	$\tau=-1$	$\tau=0$	$\tau=1$	$\tau=2$	$\tau=3$
1.Household	0.82	0.18	0.40	0.65	0.81	0.81	0.69	0.51
(lhours)	0.06	0.09	0.08	0.05	0.03	0.05	0.07	0.09
2.Total private	1.18	0.22	0.46	0.71	0.90	0.93	0.84	0.70
(lwhx)	0.05	0.11	0.09	0.05	0.03	0.05	0.08	0.11
2.a. Goods producing	2.26	0.26	0.49	0.73	0.92	0.93	0.81	0.65
(lwhgx)	0.10	0.11	0.08	0.05	0.02	0.05	0.08	0.11
Construction	3.18	0.25	0.45	0.64	0.79	0.83	0.77	0.69
(lwhcx)	0.23	0.11	0.09	0.07	0.05	0.07	0.10	0.12
Manufacturing:								
Durable goods	2.96	0.26	0.48	0.72	0.91	0.92	0.82	0.64
(lwhdx)	0.14	0.11	0.09	0.06	0.02	0.04	0.08	0.10
Non durable goods	1.43	0.35	0.55	0.75	0.86	0.77	0.56	0.33
(lwhnx)	0.18	0.10	0.08	0.05	0.03	0.05	0.09	0.11
Food and kindred products	0.69	0.10	0.25	0.41	0.52	0.52	0.46	0.39
(lwh20x)	0.08	0.12	0.12	0.13	0.13	0.12	0.12	0.14
Apparel and textiles	1.94	0.40	0.56	0.69	0.73	0.60	0.35	0.08
(lwh23x)	0.25	0.08	0.08	0.07	0.06	0.07	0.10	0.11
2.b. Service producing	0.66	0.15	0.38	0.61	0.80	0.85	0.82	0.73
(lwhpx)	0.05	0.11	0.10	0.07	0.05	0.06	0.08	0.11
Retail trade	0.82	0.51	0.65	0.79	0.88	0.81	0.66	0.47
(lwhrx)	0.05	0.11	0.09	0.06	0.03	0.06	0.09	0.11

Notes: (i) $H \sim$ measure of hours worked, taken from Citibase, with mnemonic indicated in parentheses in column 1.

$Y_t \sim$ GDP in 1987 dollars, from Citibase. Sample period—all data are quarterly, and with the exception of lhours, they cover the period 1964:I–1995:I. Lhours covers the period 1947:I–1993:IV. Numbers in the second row of each block are standard deviations, computed using the procedure described in note (ii) to Table 1.

(ii) Sources and definitions—data from 1 $\sim$ household survey, manhours employed per week; data from 2 $\sim$ establishment survey, indexes of aggregate weekly hours of production or nonsupervisory workers on private nonagricultural payrolls by industry.

Table 6. Measures of Risk Aversion in Consumption.

Model	b	h	x	μ 1%	μ 10%	μ 20%
Calibrated	0.55	0.0	0.55	0.02	2.5	9.8
BCF Calibrated	0.35	0.40	0.58	0.03	2.7	9.7
Perturbed, Calibrated	0.60	0.0	0.60	0.03	2.9	10.3
Perturbed, Calibrated	0.65	0.0	0.65	0.04	3.6	11.9
Perturbed, Calibrated	0.80	0.0	0.80	0.12	6.7	18.6
BCF, Exchange Economy	0.58	0.30	0.82	0.16	7.3	12.4

Notes: Entries in the last three columns are the value of $100 \times \nu$ associated with the indicated row value of b and the column value of μ. The value of ν solves (4.10). The model column indicates what motivated the particular parameterization considered.

Fig. 1a: Price, Fixed Investment Plus Consumer Durables

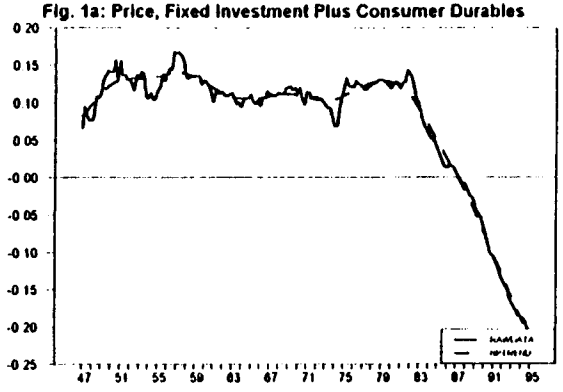

Fig. 1b: Price, Durable Goods Investment

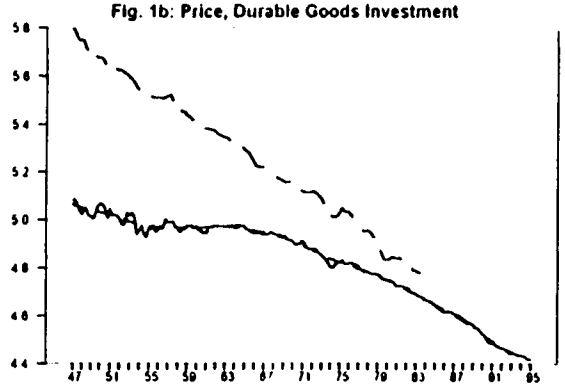

Fig 1c: Price, Fixed Investment

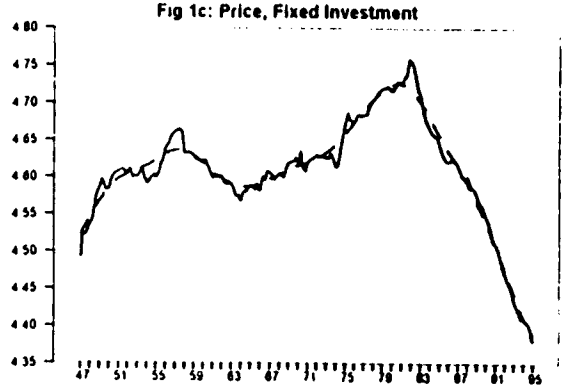

Fig. 1d: Price, Nonresidential Investment

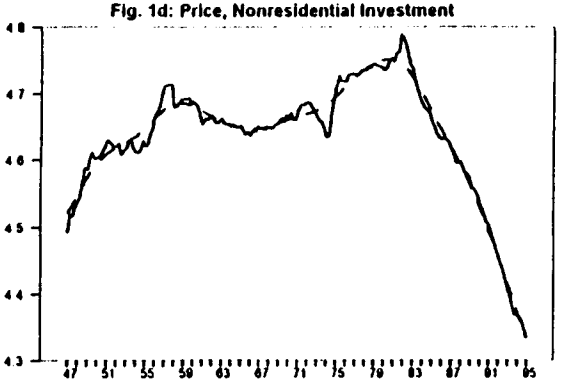

Fig 1e: Price, Residential Investment

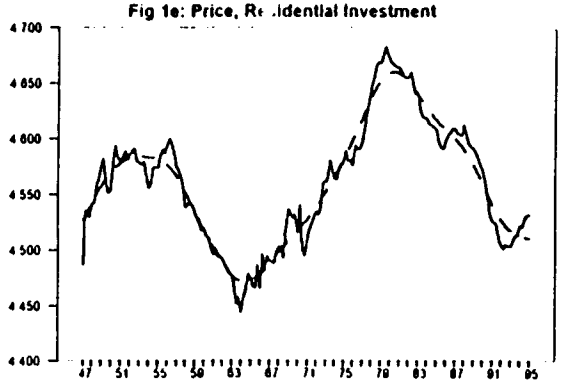

Fig. 1f: Price, Structures Investment

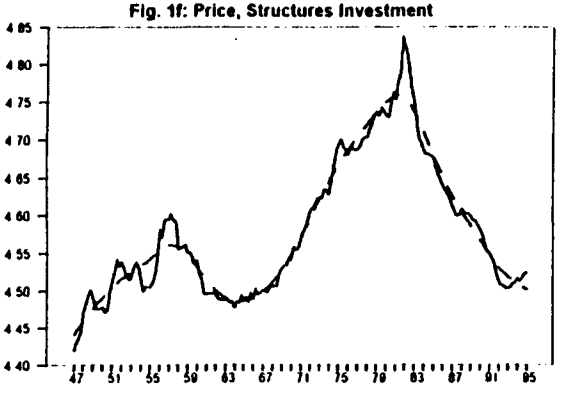

Fig. 1g: Price, Equipment Investment

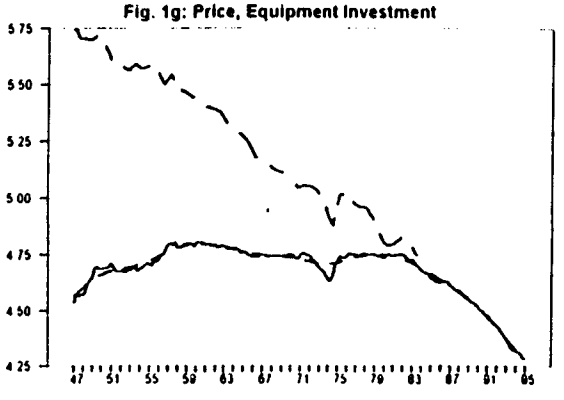

Fig. 2a: Fixed Investment Plus Consumer Durables

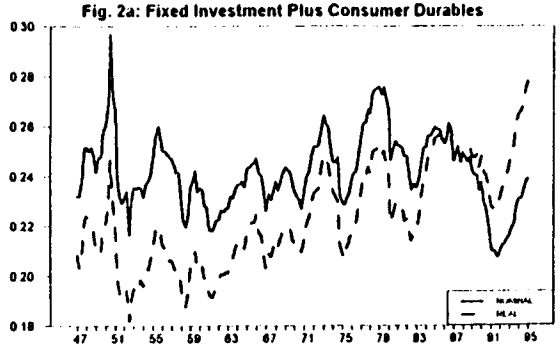

Fig. 2b: Consumer Durables

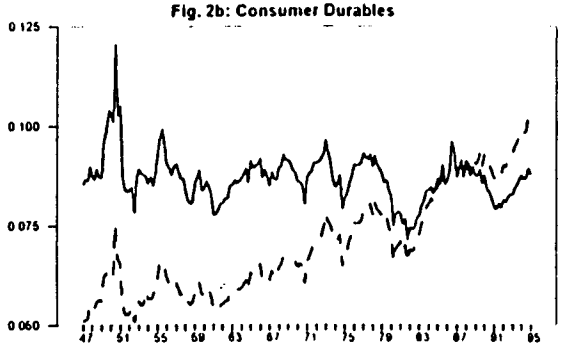

Fig. 2c: Fixed Investment

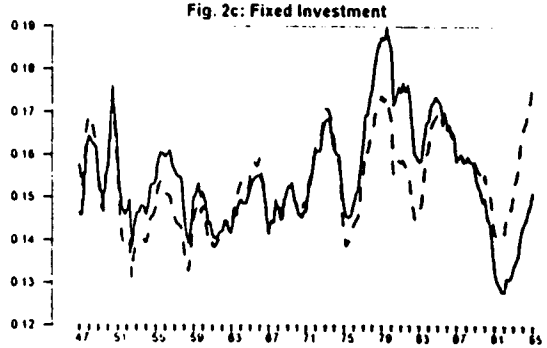

Fig. 2d: Non Residential Investment

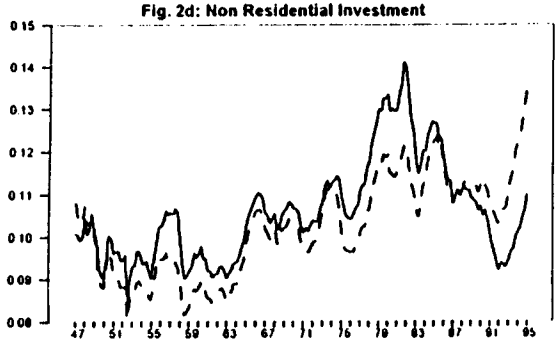

Fig. 2e: Residential Investment

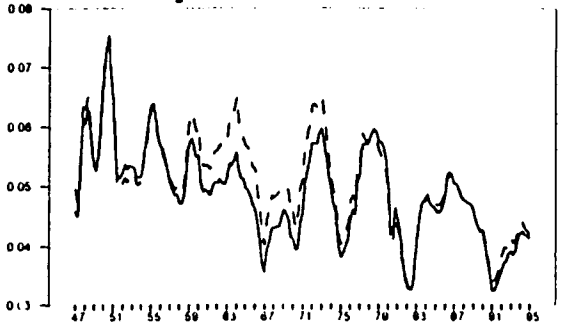

Fig. 2h: Consumption of Nondurables and Services

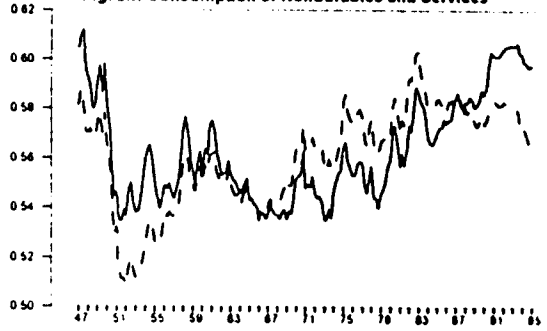

Fig. 2f: Structures Investment

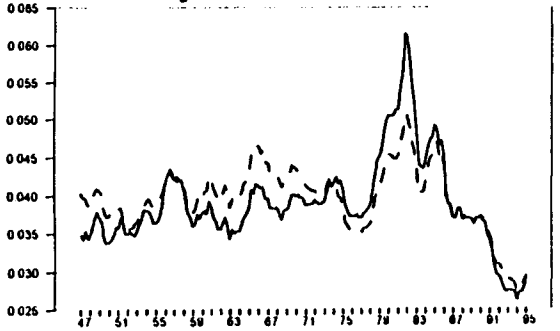

Fig. 2g: Equipment Investment

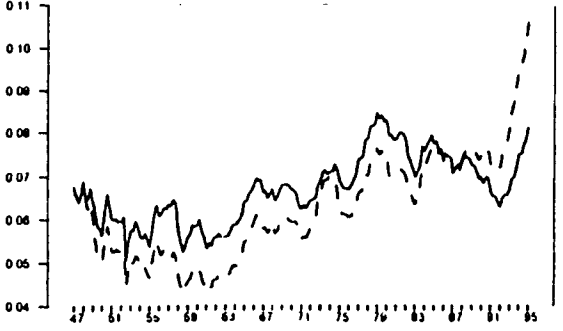

Fig. 2i: Government Purchases

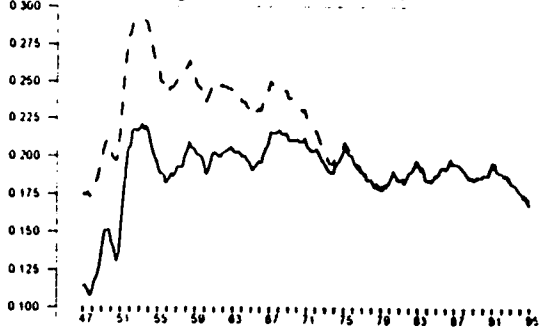

Fig. 3a: Fixed Investment Plus Durables vs GDP

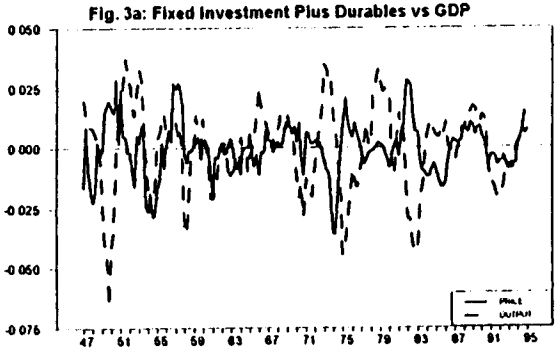

Fig. 3b: Consumer Durables vs GDP

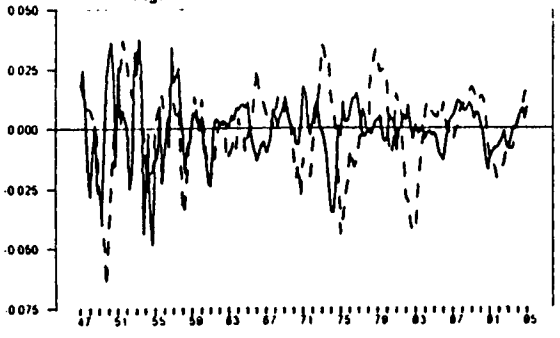

Fig. 3c: Fixed Investment vs GDP

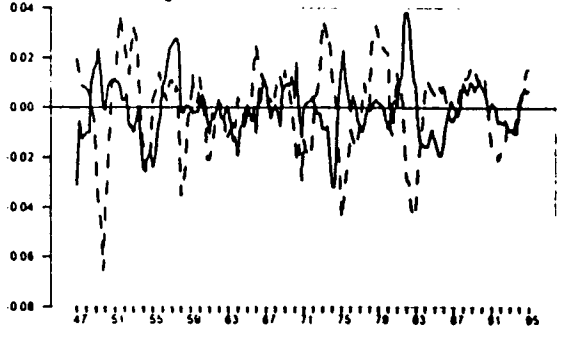

Fig. 3d: Non Residential Investment vs GDP

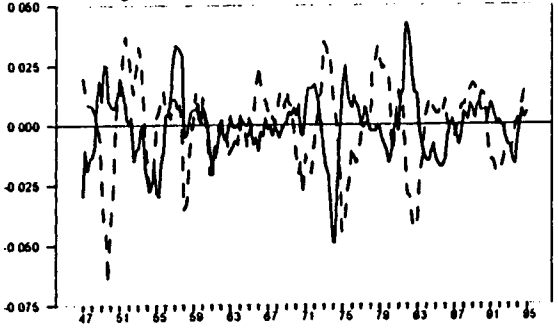

Fig. 3e: Residential Investment vs GDP

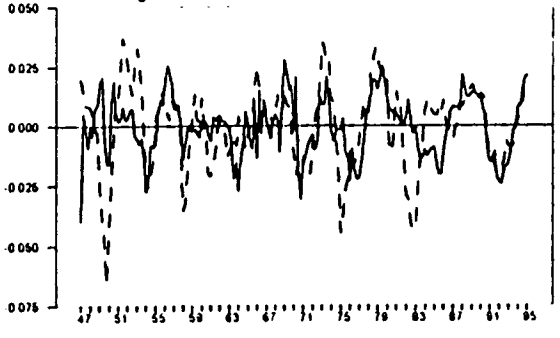

Fig. 3f: Structures Investment vs GDP

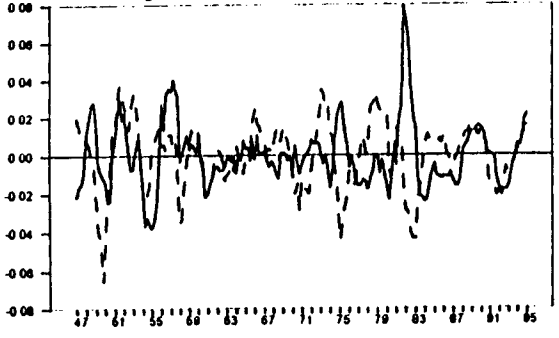

Fig. 3g: Equipment Investment vs GDP

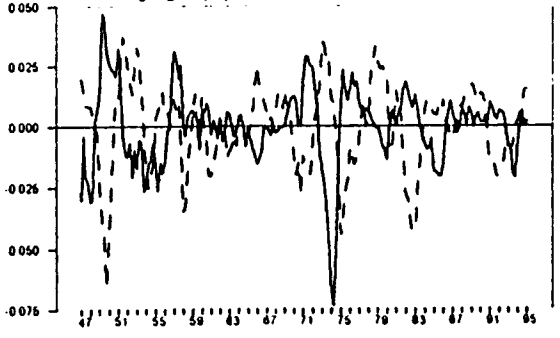

Figure 4: Correlation Between Quarterly Price of Investment at *t* and Output at *t-k*
Investment price and output data logged and detrended by HP filter

Fig. 4a: Fixed investment and durable goods

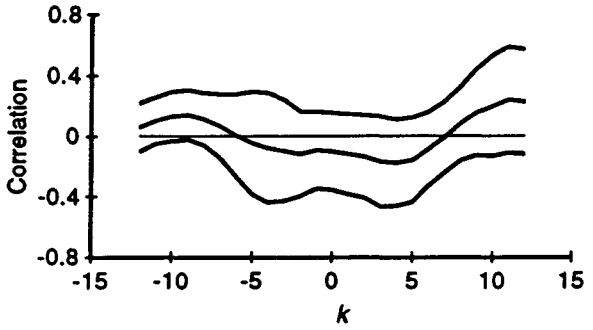

Fig. 4b: Household durable investment

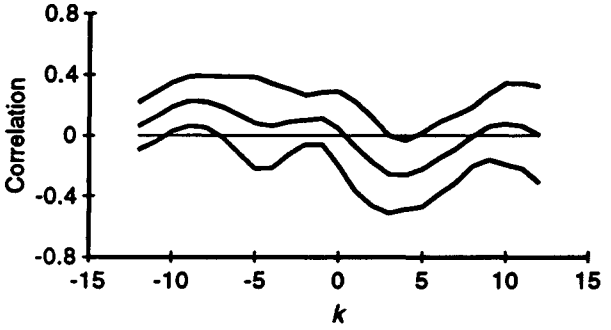

Fig. 4c: Fixed investment

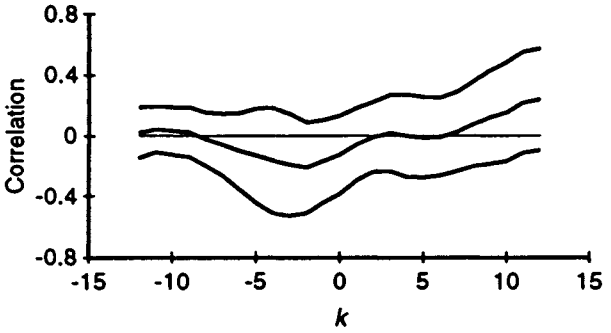

Fig. 4d: Nonresidential investment

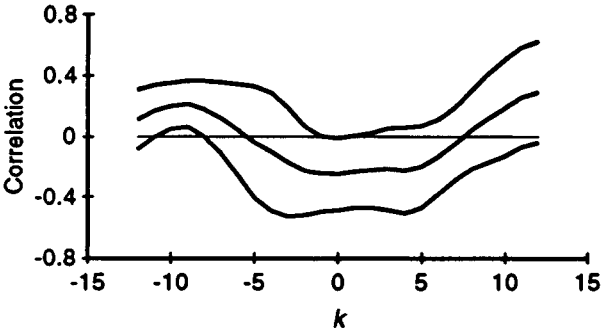

Fig. 4e: Residential investment

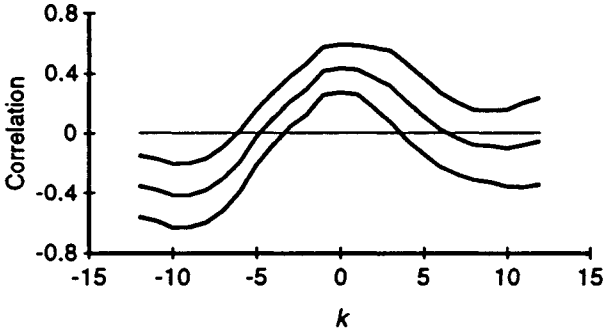

Fig. 4f: Structures investment

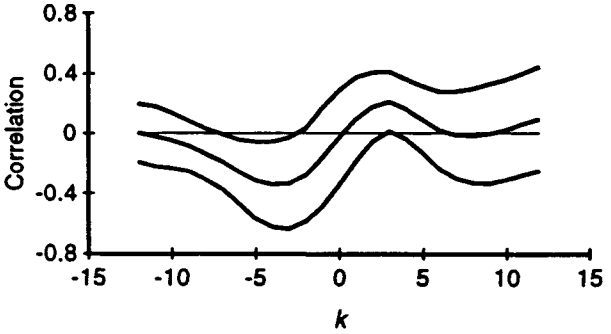

Fig. 4g: NIPA equipment investment

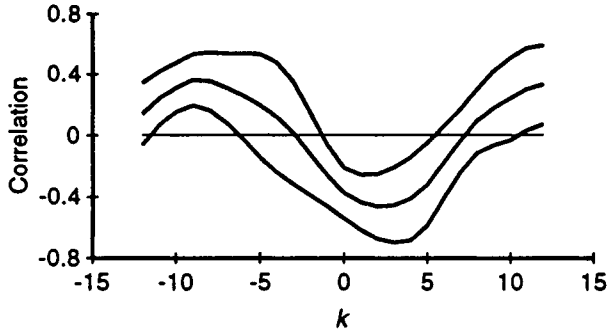

Figure 5: Correlation Between Stock Price Measure at *t* and Output at *t-k*
Price and output data logged and detrended by HP filter

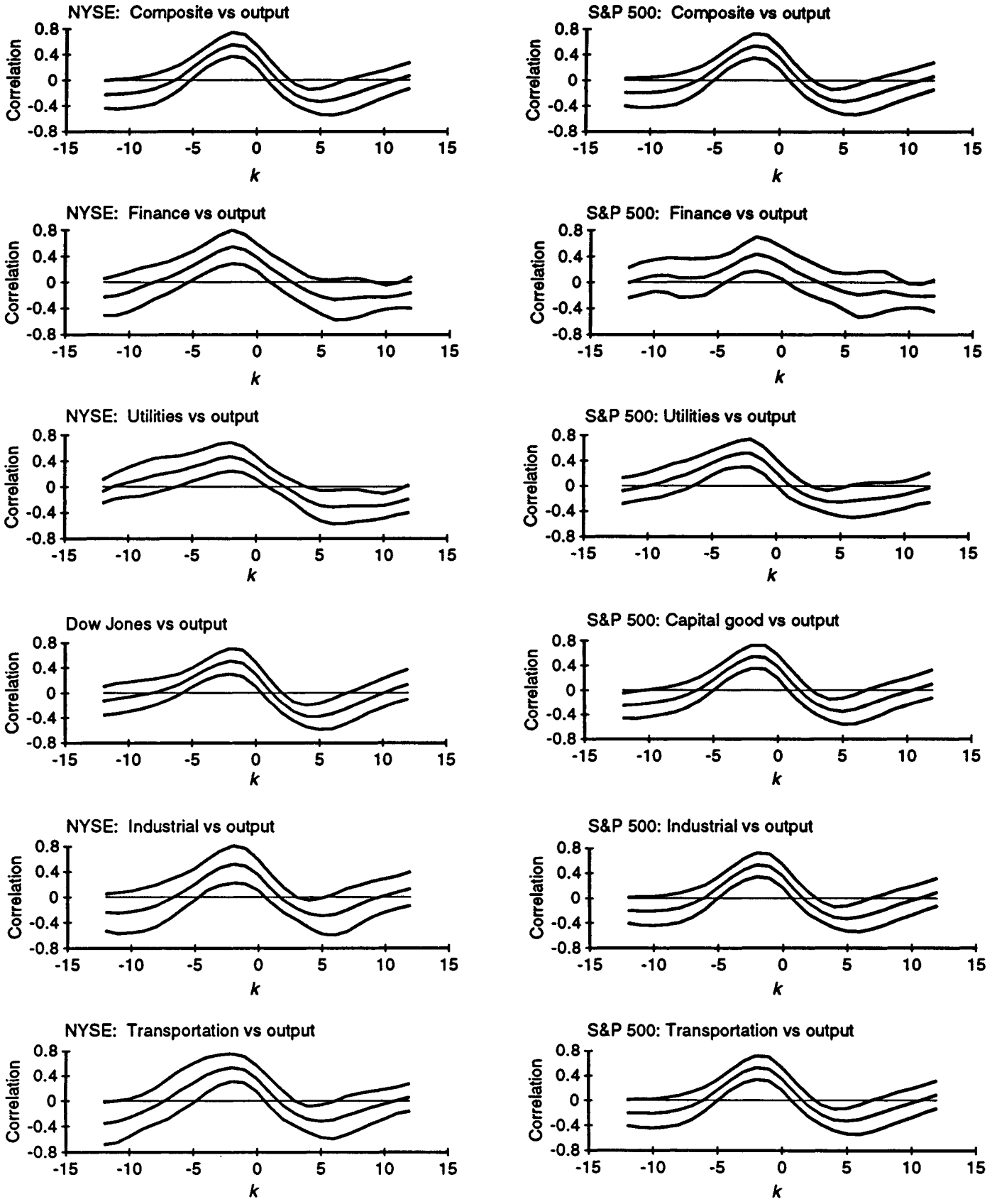

Figure 6: Assigning Values to Preference Parameters, *b* and *h*

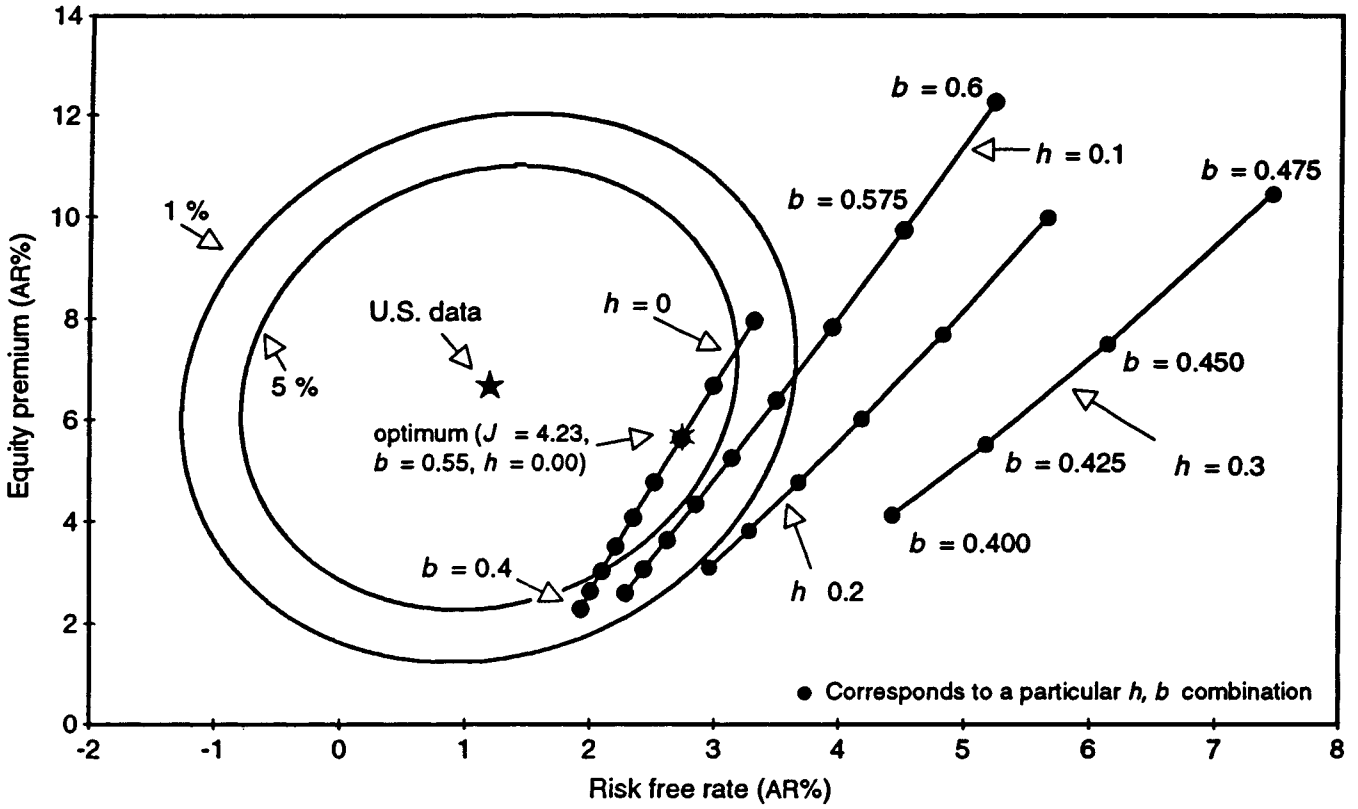

Fig. 7a: Household hours versus trend

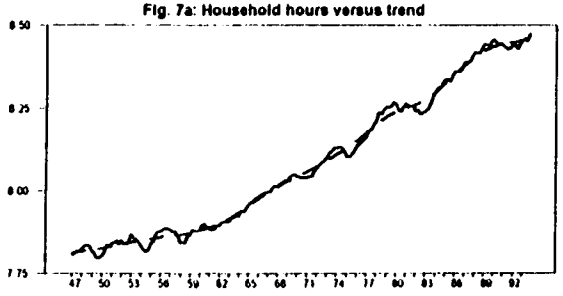

Fig. 7b: Total Private hours versus trend

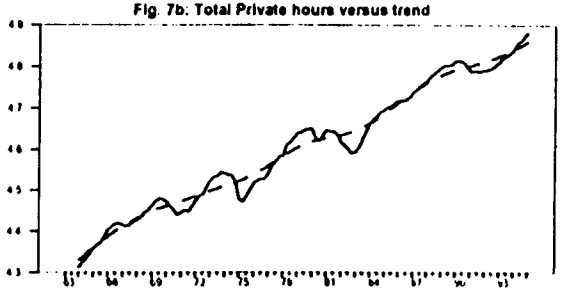

Fig. 7c: Goods producing hours versus trend

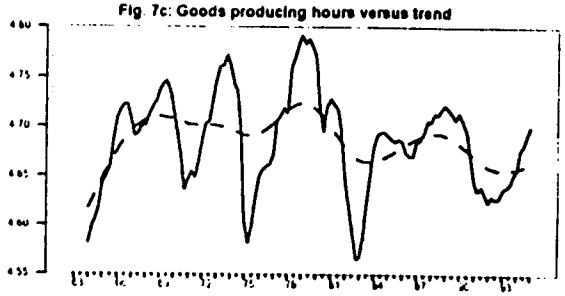

Fig. 7d: Construction hours versus trend

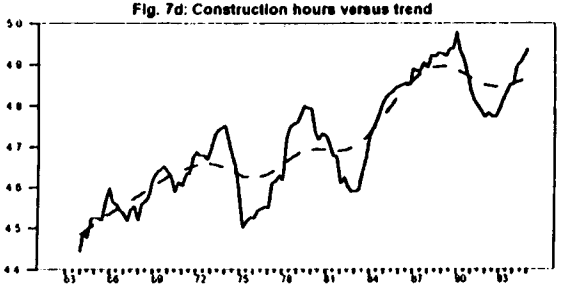

Fig. 7e: Durable goods hours versus trend

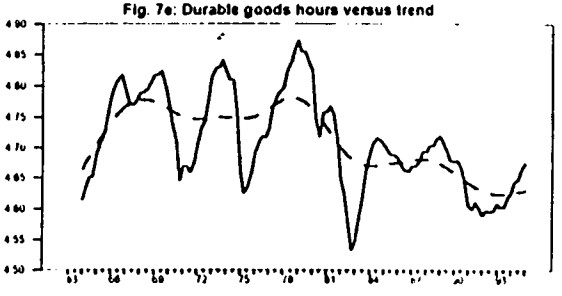

Fig. 7f: Non durable goods hours versus trend

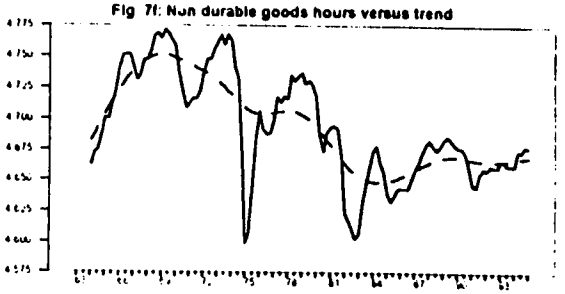

Fig. 7g: Food and kindred product hours versus trend

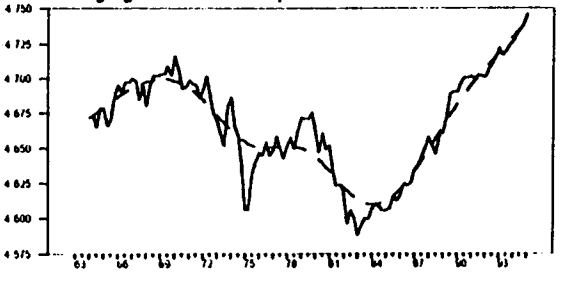

Fig. 7h: Apparel and textile hours versus trend

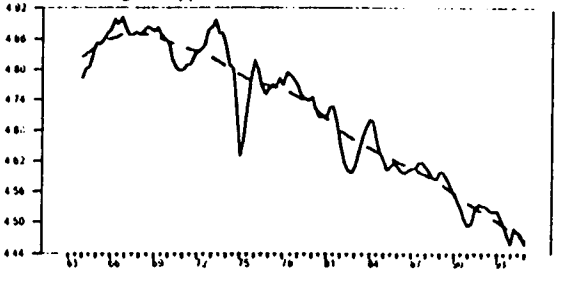

Fig. 7i: Service-producing hours versus trend

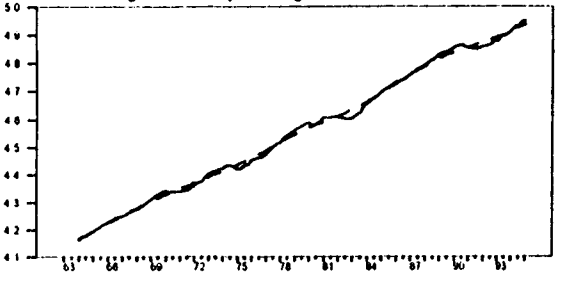

Fig. 7j: Retail trade versus trend

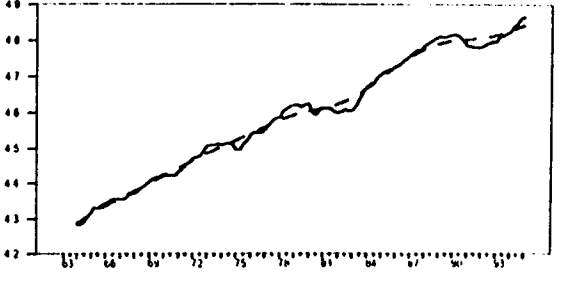

Fig. 8a: Household hours versus output

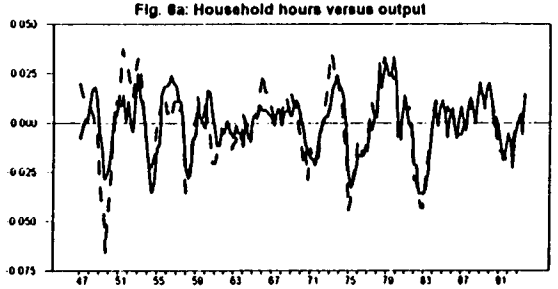

Fig. 8b: Total Private hours versus output

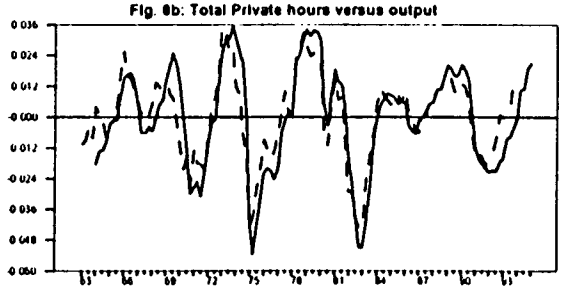

Fig. 8c: Goods producing hours versus output

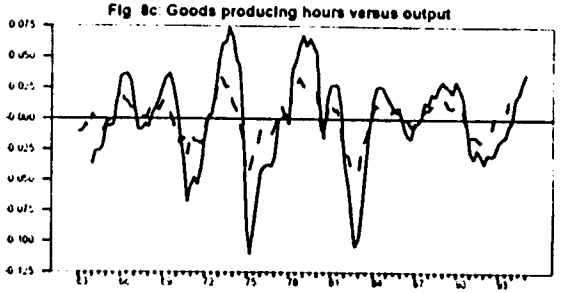

Fig. 8d: Construction hours versus output

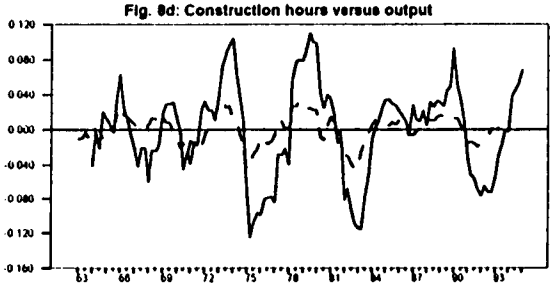

Fig. 8e: Durable goods hours versus output

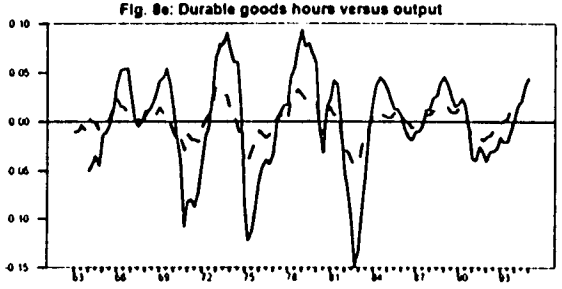

Fig. 8f: Non durable goods hours versus output

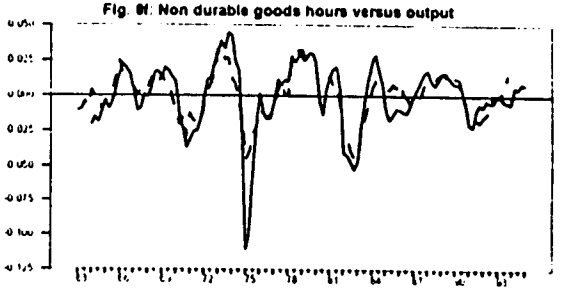

Fig. 8g: Food and kindred product hours versus output

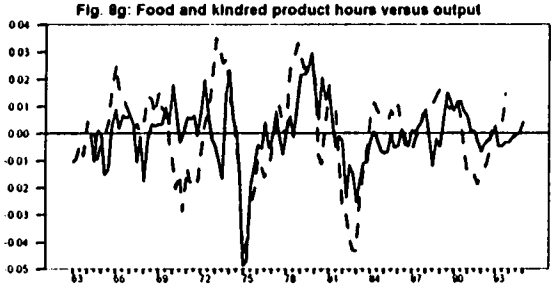

Fig. 8h: Apparel and textile hours versus output

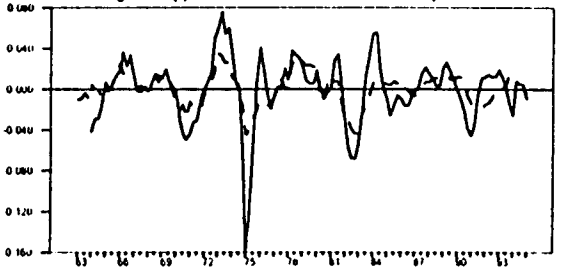

Fig. 8i: Service-producing hours versus output

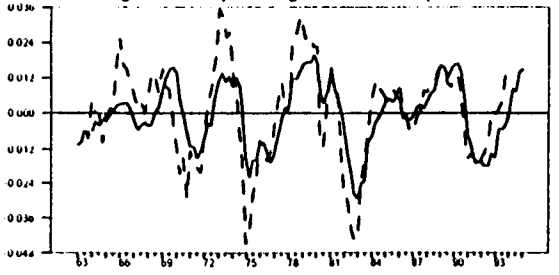

Fig. 8j: Retail trade versus output

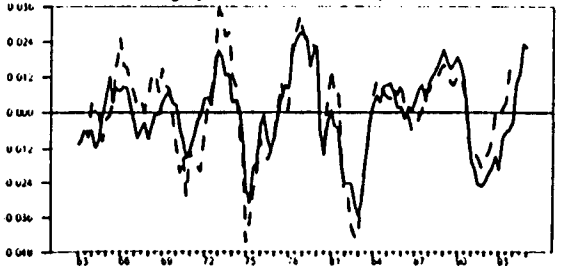

Figure 9: Impulse Response Function for Calibrated Model

Fig. 9a: Response of C

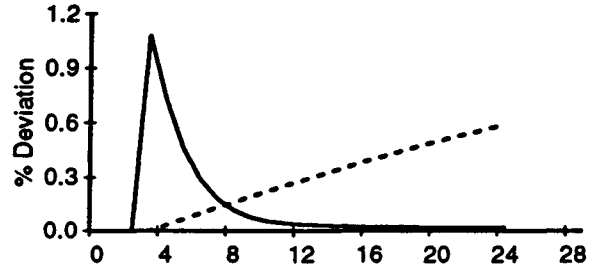

Fig. 9b: Response of I

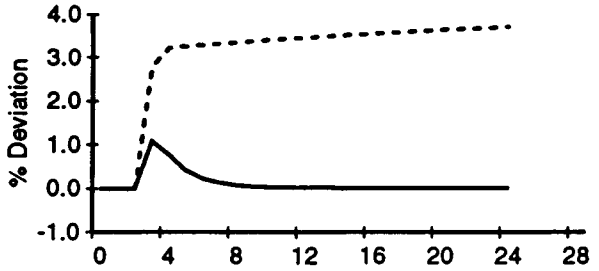

Fig. 9c: Response of h

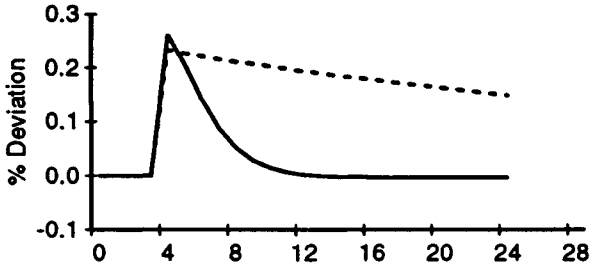

Fig. 9d: Response of I_c

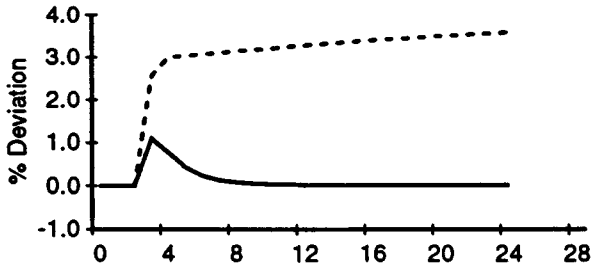

Fig.9e: Response of I_i

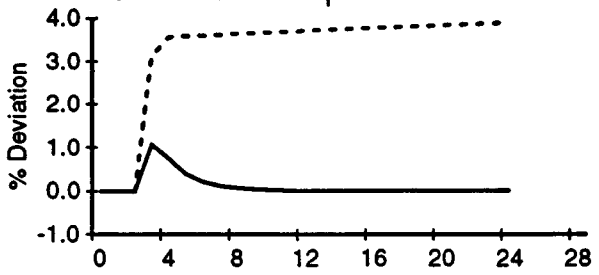

Fig. 9f Response of h_c

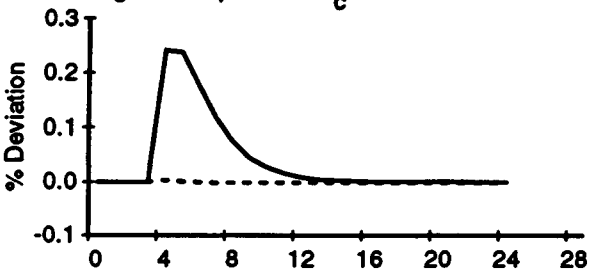

Fig. 9g: Response of P_i

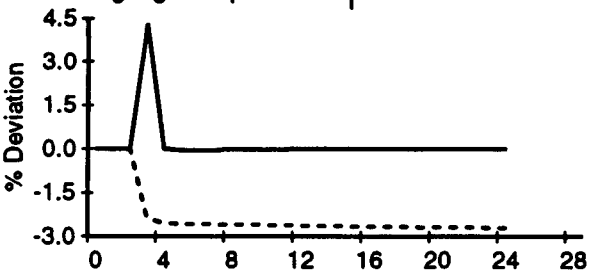

Fig. 9h: Response of P_k

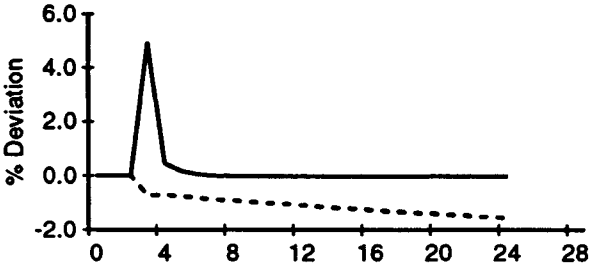

Fig.9i: Response of h_i