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Overtime, effort and the propagation of business cycle shocks

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Abstract

This paper presents and estimates a variant of Hansen and Sargent's (1988) real business cycle model with straight time and overtime. The model presented has only one latent variable, the state of technology, yet it does as good a job propagating and magnifying shocks as labor hoarding models which incorporate unobserved effort. This paper also finds that the implied effort series of labor hoarding models displays a high coherence with U.S. overtime data at business cycle frequencies. This supports the view that effort is procyclical.

1 Introduction

This paper estimates a dynamic general equilibrium real business cycle model of the U.S. economy incorporating straight time and overtime. This model is a hybrid of Hansen and Sargent's (1988) model with straight time and overtime and Burnside, Eichenbaum and Rebelo's (1993) labor hoarding model. This model is studied along with an estimated version of Burnside, Eichenbaum and Rebelo's model. The two models are analyzed to answer two questions. First, how do different assumptions about labor market rigidities effect the real business cycle model's ability to propagate and magnify shocks? Second, does the unobservable time series effort implied by the labor hoarding models make sense?

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A well-known and often repeated criticism of standard real business cycle (RBC) models is their dependence on implausibly large aggregate technology shocks to account for the variability in output.[1] Indeed Cochrane (1994) concludes "we haven't found large, identifiable, exogenous shocks to account for the bulk of output fluctuations." Another way of phrasing this criticism is that standard real business cycle models possess weak propagation mechanisms. That is, if RBC models could plausibly magnify and spread over time the effect of the shocks, these models would be less dependent on large aggregate shocks to produce sufficiently volatile time series. Consequently a challenge to business cycle theorists is to construct models that amplify shocks and realistically propagate shocks over the business cycle.

This challenge has not gone unanswered; Burnside and Eichenbaum (1994) studies two potential channels of propagation: time-varying effort and time-varying capital utilization. Their model has the ability to substantially magnify and propagate shocks. Motivated by Burnside and Eichenbaum's success, the current paper puts capital utilization aside for the moment and focuses attention on the implications for shock amplification and propagation inherent in the time-varying effort assumption. Consequently the relevant model is Burnside, Eichenbaum and Rebelo's (1993) labor hoarding model although the issues addressed are from Burnside and Eichenbaum (1994). The current paper studies in detail the propagation mechanism of the labor hoarding channel and proposes a complementary channel – the differentiation between straight time and overtime.

The model presented here builds on the model of Hansen and Sargent. Like Hansen and Sargent this model incorporates a version of Lucas's (1970) instantaneous production function in which straight time and overtime are not perfect substitutes. However, it differs from Hansen and Sargent's model in two ways. First, a government sector and a government shock are added to the model. Second, firms must commit to the number of workers they will employ before observing any shocks to the economy; once the shocks are observed, firms can adjust the number of employees working overtime. In other words, the cost that firms face in adjusting the contemporaneous number of workers after observing the shocks is infinite.

The model presented can also be viewed as a variation on the model of Burnside,

[1]See for example Summers (1986), McCallum (1989) and Eichenbaum (1991).

Eichenbaum and Rebelo (BER).[2] In both models, firms must commit to the number of workers employed before observing any shocks to the economy. In BER's model, firms can adjust the work effort after observing the shocks. In the model presented here, firms can adjust the number of persons working overtime after observing the shocks. The advantage to the model presented here is that unlike effort which cannot be measured, overtime employment is observed and measured.

Using economically interpretable parameter values obtained by maximum likelihood, the model presented here has the ability to magnify and propagate shocks over the business cycle as well as BER's labor hoarding model. This success is achieved under the constraint that all variables in this model, with the exception of technology, are observed. Moreover, the model does as well as BER's model in matching the first- and second-moment properties of the data. This result demonstrates that theorists can construct models which embody quantitatively important propagation and amplification mechanisms without using the unobservable variable effort.

This paper also analyzes the relationship between effort and overtime. It finds that the time series on effort implied by Burnside, Eichenbaum and Rebelo's labor hoarding model displays a high coherence with U.S. overtime employment data at business cycle frequencies. The result is consistent with the belief that overtime and effort move together over the cycle.

The remainder of this paper is organized as follows. In the second section the model is presented. In the third section the log LQ approximation technique for solving the model is described, and the method for estimating the parameter values of the model is discussed. The data are reported in the fourth section. In the fifth section, empirical results are calculated and assessed. In the final section some concluding remarks are made.

2 The economy

This section presents a variant of Hansen and Sargent's (1988) straight time and overtime model modified to incorporate a government sector and the precommitment of total employment.

[2]Other models in which employment must be set before observing the shocks include Christiano (1988) and Sbordone (1993).

Consider an economy with a continuum on [0,1] of identical infinitely lived agents who have preferences over consumption of a single nondurable good, C_t, and leisure, L_t. They maximize:

$$E\sum_{t=0}^{\infty}\beta^t[\log C_t + v\log L_t], \tag{1}$$

where $0 < \beta < 1$ is their subjective discount factor and v is strictly greater than zero.

Agents are endowed with T units of time each period which can be divided between labor and leisure; consequently $0 < L_t \leq T$. Furthermore L_t is restricted to take only one of three values. Let h_1 and h_2 be the lengths of a 'straight time shift' and an 'overtime shift' respectively. Let $0 < h_1 < h_1 + h_2 < T$. Therefore:

$$L_t = \begin{cases} T & \text{if the agent is unemployed} \\ T - h_1 & \text{if the agent works only the straight time shift} \\ T - h_1 - h_2 & \text{if the agent works both the straight time and overtime shifts} \end{cases}$$

Proceeding as in Hansen (1985) and Rogerson (1988) lotteries are employed to convexify the commodity space. Assume π_{1t} and π_{2t} are the probability of working just a straight time shift and the probability of working both straight time and overtime shifts, respectively. Hence $1 - \pi_{1t} - \pi_{2t}$ is the probability of being unemployed. Agents choose over labor probabilities to maximize their expected single period utility:

$$\pi_{1t}[\log C_t + v\log(T-h_1)] + \pi_{2t}[\log C_t + v\log(T-h_1-h_2)] + (1-\pi_{1t}-\pi_{2t})[\log C_t + v\log(T)]. \tag{2}$$

Define N_{2t} to be the fraction of agents who work both shifts (overtime employment) and N_{1t} to be the fraction of agents who work the straight shift (total employment). Hence N_{1t} equals $\pi_{1t} + \pi_{2t}$, N_{2t} equals π_{2t} and the agent's utility function, (1), can be written:

$$E\sum_{t=0}^{\infty}\beta^t[\log C_t - a_1(N_{1t} - N_{2t}) - a_2 N_{2t} - a_0(1 - N_{1t})], \tag{3}$$

where $a_0 = -v\log(T)$, $a_1 = -v\log(T - h_1)$ and $a_2 = -v\log(T - h_1 - h_2)$. This preference specification is from Hansen and Sargent.

Aggregate output, Q_t, is produced by an instantaneous Cobb-Douglas production technology such that at any instant, τ, during period t the rate of output per unit time is

$$Q(t+\tau) = \exp(\lambda_{t+\tau})(\gamma^{t+\tau})^{\alpha} K_{t+\tau}^{1-\alpha} N_{t+\tau}^{\alpha},$$

where $K_{t+\tau}$ is the capital stock, $\lambda_{t+\tau}$ is the state of technology, and $N_{t+\tau}$ is the labor force; γ represents the growth rate of exogenous labor-augmenting technological progress. It is assumed that technology and capital are constant throughout the period. Hence aggregate output produced over the two shifts during period t is:

$$Q_t = \exp(\lambda_t)(\gamma^t)^\alpha K_t^{1-\alpha}[h_1 N_{1t}^\alpha + h_2 N_{2t}^\alpha]. \tag{4}$$

Labor's share, α, is restricted such that $0 < \alpha < 1$.

Total output is allocated each period to private consumption, C_t, government consumption, G_t, and investment, I_t:

$$C_t + G_t + I_t \leq Q_t. \tag{5}$$

Productive capital depreciates each period at the rate $0 < \delta \leq 1$, so:

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{6}$$

Technology has the following law of motion

$$\lambda_{t+1} = \mu_\lambda + \rho_\lambda \lambda_t + \sigma_\lambda w_{\lambda t+1}, \tag{7}$$

where $0 < \rho_\lambda < 1$ and $\{w_{\lambda t}\}$ is a sequence of i.i.d. normally distributed random variables with mean zero and variance one.

Government consumption evolves according to

$$G_t = \gamma^t \exp(g_t), \tag{8}$$

such that g_t follows the first order autoregression

$$g_{t+1} = \mu_g + \rho_g g_t + \sigma_g w_{gt+1}, \tag{9}$$

where $0 < \rho_g < 1$. $\{w_{gt}\}$ is a sequence of i.i.d. normally distributed random variables with mean zero and variance one and is orthogonal to innovations in technology.

The timing of this economy differs from the one presented in Hansen and Sargent. In this model N_{1t} must be chosen before, instead of after, (λ_t, G_t) are known; N_{2t} and K_{1t+1} are chosen after observing the shocks. Formally, let the initial information set, I_0, be generated by the set of initial conditions, $\{\lambda_0, G_0, K_0, N_{10}\}$. Let the information set

I_t be generated by $\{\lambda_0, G_0, K_0, N_{10}\}$ and $\{(w_{\lambda s}, w_{gs}) : s = 1, 2, ...t\}$, so I_t consists of all measurable functions of $\{\lambda_0, G_0, K_0, N_{10}\} \cup \{(w_{\lambda s}, w_{gs}) : s = 1, 2, ...t\}$. Hence $\{w_{\lambda t}, w_{gt}\}_{t=0}^{\infty}$, is a conditionally homoskedastic martingale difference sequence adapted to the sequence of information sets, $\{I_t\}_{t=0}^{\infty}$. In the model presented here, the set $\{K_{t+1}, N_{1t+1}, N_{2t}\}$ is restricted to reside in the space of measurable functions of I_t. That is, the set of stochastic processes $\{K_{t+1}, N_{1t+1}, N_{2t}\}_{t=0}^{\infty}$ resides in the space L_0^2 given by:

$$L_0^2 = \{y_t : y_t \text{ is in } I_t \text{ for } t = 0, 1, 2, ..., \text{ and } E\Sigma_{t=0}^{\infty}\beta^t y_t^2 | I_0 < \infty\}. \tag{10}$$

This condition requires that decisions made at time t depend only on information available at time t.

If markets are complete the decentralized competitive equilibrium corresponds to the solution of a social planning problem. In this case, the social planning problem is to choose a set of stochastic processes $\{K_{t+1}, N_{1t+1}, N_{2t}\}_{t=0}^{\infty}$ to maximize (3) subject to (4) - (9) given initial conditions $\{\lambda_0, G_0, K_0, N_{10}\}$. Moreover $\{K_{t+1}, N_{1t+1}, N_{2t}\}_{t=0}^{\infty}$ is required to reside in the space L_0^2 defined by (10). This definition of the social planning problem completes the description of the model; this model is referred to as the labor precommitment model.

For comparison purposes, a version of Burnside, Eichenbaum and Rebelo's labor hoarding model is presented. In their model, firms commit to the number of workers employed before observing any shocks to the economy; after observing the shocks, firms can adjust the work effort of their employees. Higher work effort increases output but lowers the agents' utility. Formally, in the labor hoarding model the social planner chooses a set of stochastic processes $\{K_{t+1}, N_{1t+1}, e_t\}_{t=0}^{\infty}$ to maximize

$$E\sum_{t=0}^{\infty}\beta^t[N_{1t}[\log C_t + v\log(T - \xi - e_t h_1)] + (1 - N_{1t})[\log C_t + v\log(T)]] \tag{11}$$

subject to

$$Q_t = \exp(\lambda_t)(\gamma^t)^{\alpha} K_t^{1-\alpha} h_1 [e_t N_{1t}]^{\alpha}, \tag{12}$$

and (5) - (9) where $\{K_{t+1}, N_{1t+1}, e_t\}_{t=0}^{\infty}$ is in the space L_0^2 given by (10). In this model N_{1t} is the fraction of agents who work a single shift of length h_1. Two additional variables are introduced in BER's model. The date t level of employee work effort is denoted by e_t; ξ is a fixed cost of going to work. The only difference between this model and the one presented in BER is that the production function in (12) is instantaneous. This model is referred to as the labor hoarding model.

Due to the nonlinearity of both social planner problems, it is not possible to obtain an analytic solution to either of these models. Hence Christiano's (1988) logarithmic modification of Kydland and Prescott's (1982) linear-quadratic approximation is used to obtain an approximate solution to the social planner's problem. In the next section this approximation procedure is discussed, and the estimation technique of the parameter values is presented.

3 Computing an equilibrium

In order to solve the social planning problems discussed above, the following procedure is employed. For each model, the economy is transformed into one which possesses a steady-state; then a log-linear-quadratic approximation of the transformed economy is made. Decision rules for the log-linear-quadratic economy are computed. With these decision rules in hand, the parameter values of the approximate economy are estimated using maximum likelihood. Using the obtained parameter values and decision rules, equilibrium allocations are computed. To keep the presentation simple only the labor precommitment model is discussed; however the analogous steps are used to solve the labor hoarding model as well.

In the labor precommitment model the endogenous variables do not converge to a non-stochastic steady state but to a steady state growth path. Therefore the economy is transformed so that it has a steady state. Define the following:

$$\begin{aligned} k_t &= \log(K_t/\gamma^t) \quad & c_t = \log(C_t/\gamma^t) \\ i_t &= \log(I_t/\gamma^t) \quad & q_t = \log(Q_t/\gamma^t) \\ n_{1t} &= \log(N_{1t}) \quad & n_{2t} = \log(N_{2t}) \end{aligned}$$

For the remainder of the paper, lower case letters denote logged and detrended variables.

The social planner's problem can be written:

$$\max E \sum_{t=0}^{\infty} \beta^t [c_t - a_1(\exp(n_{1t}) - \exp(n_{2t})) - a_2 \exp(n_{2t}) - a_0(1 - \exp(n_{1t})) + \gamma^t] \tag{13}$$

subject to:

$$\exp(c_t) + \exp(g_t) + \gamma \exp(k_{t+1}) - (1-\delta)\exp(k_t) = \exp(q_t) \tag{14}$$

$$\exp(q_t) = \exp(\lambda_t)\exp(k_t)^{1-\alpha}[h_1\exp(n_{1t})^\alpha + h_2\exp(n_{2t})^\alpha] \tag{15}$$

and (7)-(9). Since the last term in the planner's objective function is beyond the planner's control it is dropped for the remainder of the analysis.

The model is solved using the approximation technique employed by Christiano (1988). Define the state vector, $x_t = (1, \lambda_t, g_t, k_t, n_{1t})$, the control vector, $u_t = (n_{1t+1}, n_{2t}, k_{t+1})$, and the shock vector, $w_t = (w_{\lambda t}, w_{gt})$. Let $\theta = (\alpha, \beta, \delta, \gamma, \mu_\lambda, \mu_g, \rho_\lambda, \rho_g, h_1, h_2, T, v, \sigma_\lambda, \sigma_g)$ be a vector of parameters. Finally, let $z_t = [x_t', u_t']$.

The approximation technique proceeds as follows. First, substitute (14) and (15) into (13) so the utility function is now a nonlinear function of the parameters and the state and control variables. Call this function $r(z_t, \theta)$. Hence the social planning problem can be written as:

$$\max_{u_t} E\sum_{t=0}^{\infty}\beta^t r(z_t, \theta)$$

subject to:

$$\begin{bmatrix} 1 \\ \lambda_{t+1} \\ g_{t+1} \\ k_{t+1} \\ n_{1t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \mu_\lambda & \rho_\lambda & 0 & 0 & 0 \\ \mu_g & 0 & \rho_g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda_t \\ g_t \\ k_t \\ n_{1t} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} n_{1t+1} \\ n_{2t} \\ k_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \sigma_w & 0 \\ 0 & \sigma_g \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w_{\lambda t+1} \\ w_{gt+1} \end{bmatrix}.$$

This constraint can be written shorthand as:

$$x_{t+1} = \mathcal{A}x_t + \mathcal{B}u_t + \mathcal{C}w_{t+1}. \tag{16}$$

Second, compute the steady state values of the state and control variables; call this vector $\bar{z}$. Third, take a second order Taylor series approximation to $r(z_t, \theta)$ around the steady state to form a matrix $\mathcal{M}$ such that:

$$\mathcal{M} = e(r(\bar{z},\theta) - \frac{\partial r(\bar{z},\theta)'}{\partial z}\bar{z} + \frac{1}{2}\bar{z}'\frac{\partial^2 r(\bar{z},\theta)}{\partial z^2}\bar{z})e' + \frac{1}{2}(e\frac{\partial r(\bar{z},\theta)'}{\partial z} + \frac{\partial r(\bar{z},\theta)}{\partial z}e' - e\bar{z}'\frac{\partial^2 r(\bar{z},\theta)}{\partial z^2} - \frac{\partial^2 r(\bar{z},\theta)}{\partial z^2}\bar{z}e' + \frac{\partial^2 r(\bar{z},\theta)}{\partial z^2})$$

where e is a selector matrix – a vector of zeros with a one in the element corresponding to the constant term. Notice that $\mathcal{M}$ can be written:

$$\mathcal{M} = \begin{bmatrix} \mathcal{S} & \mathcal{W} \\ \mathcal{W}' & \mathcal{Q} \end{bmatrix}.$$

Fourth and finally, substitute $z_t'\mathcal{M}z_t$ for $r(z_t,\theta)$. Hence the social planner's problem is now approximated by the following linear-quadratic problem.

$$\max_{u_t} E_0 \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} x_t \\ u_t \end{bmatrix}' \begin{bmatrix} \mathcal{S} & \mathcal{W} \\ \mathcal{W}' & \mathcal{Q} \end{bmatrix} \begin{bmatrix} x_t \\ u_t \end{bmatrix}$$

subject to (16). This approximated problem is a discounted optimal linear regulator problem, and its solution is obtained in a straightforward manner.

The solution to the above optimal linear regulator problem yields the following time invariant decision rules:

$$u_t = -\mathcal{F}x_t. \tag{17}$$

The matrix $\mathcal{F}$ is the solution to following matrix Riccati equation:

$$\mathcal{F} = (\mathcal{Q} + \beta\mathcal{B}'\mathcal{P}\mathcal{B})^{-1}(\beta\mathcal{B}'\mathcal{P}\mathcal{A} + \mathcal{W}') \tag{18}$$

for $\mathcal{P}$ satisfying

$$\mathcal{P} = \mathcal{S} + \beta\mathcal{A}'\mathcal{P}\mathcal{A} - (\mathcal{W} + \beta\mathcal{A}'\mathcal{P}\mathcal{B})(\mathcal{Q} + \beta\mathcal{B}'\mathcal{P}\mathcal{B})^{-1}(\beta\mathcal{B}'\mathcal{P}\mathcal{A} + \mathcal{W}'). \tag{19}$$

Substituting (17) into (16) yields a closed form of the state space representation.

$$x_{t+1} = \mathcal{A}_o x_t + \mathcal{C}w_{t+1} \tag{20}$$

where $\mathcal{A}_o = \mathcal{A} - \mathcal{B}\mathcal{F}$.

Let y_t denote a vector of observer variables: $y_t = [q_t\ c_t\ i_t\ g_t\ k_t\ h_t\ n_{1t}\ n_{2t}]'$. Let Y_t denote the corresponding vector of untransformed (upper case) observer variables. All observer variables are written as linear functions of the state variables.

$$y_t = \mathcal{G}x_t. \tag{21}$$

The matrix $\mathcal{G}$ is formed by stacking the coefficients of the linear equations of the observer variables as functions of the state variables. Total hours, h_t, is defined such that $\exp(h_t) = H_t \equiv h_1 N_{1t} + h_2 N_{2t}$. Since output, consumption, investment and hours are nonlinear functions of the state variables, Taylor approximations are used. The remaining four variables in y_t are either state or control variables.

In this model there are two corner solutions that must be avoided to ensure the accuracy of the log-linear-quadratic approximation. When the shocks to the economy are higher than anticipated, firms increase the number of persons working overtime. However firms cannot employ more overtime workers than straight time workers. Likewise, when the shocks to the economy are lower than anticipated, firms reduce the number of persons working overtime; however firms cannot reduce the number of persons working overtime below zero. Parameter values must be set such that the constraint $0 < \pi_{2t} < \pi_{1t}$ never binds.

If measurement error is added to equation (21), then the linear system (20) and (21) can be used to compute a Gaussian likelihood function. Economically sensible parameter values for the vector θ that maximize the likelihood function can then be extracted from the data. This estimation procedure is discussed in more detail in Anderson, Hansen, McGrattan and Sargent (1994).

Assume the vector y_t is observed with measurement error. Replace (21) with:

$$y_t = \mathcal{G}x_t + v_t \tag{22}$$

where v_t is measurement error such that: $v_t = \mathcal{D}v_{t-1} + \eta_t$ and

$$\begin{aligned} E\eta_t &= 0 \\ E\eta_t\eta_s' &= \begin{cases} \mathcal{R} & \text{if t=s} \\ 0 & \text{if t} \neq \text{s} \end{cases} \\ E\eta_t v_s &= 0 \ \forall t > s. \end{aligned}$$

Assume $\mathcal{R}$ is a diagonal matrix. Hence the state space system can be written as:

$$\begin{aligned} x_{t+1} &= \mathcal{A}_o x_t + \mathcal{C}w_{t+1} \\ y_t &= \mathcal{G}x_t + v_t \\ v_t &= \mathcal{D}v_{t-1} + \eta_t. \end{aligned}$$

Defining $\bar{y} = y_{t+1} - \mathcal{D}y_t$ and $\bar{\mathcal{G}} = \mathcal{G}\mathcal{A}_o - \mathcal{D}\mathcal{G}$ allows the state space system to be represented by:

$$\begin{aligned} x_{t+1} &= \mathcal{A}_o x_t + \mathcal{C}w_{t+1} \\ \bar{y}_t &= \bar{\mathcal{G}}x_t + \mathcal{G}\mathcal{C}w_{t+1} + \eta_{t+1}. \end{aligned}$$

Define the following: let $\hat{E}[Y|X]$ denote the linear least squares projection of Y onto X; let $\mathcal{K}$ and Σ denote the steady-state 'Kalman gain' and 'state-covariance matrix' of the time invariant Kalman filter. Therefore, $\mathcal{K}$ and Σ must satisfy:

$$\begin{aligned}
\mathcal{K} &= (\mathcal{C}\mathcal{C}'\mathcal{G}' + \mathcal{A}_o\Sigma\bar{\mathcal{G}}')(\bar{\mathcal{G}}\Sigma\bar{\mathcal{G}}' + \mathcal{R} + \mathcal{G}\mathcal{C}\mathcal{C}'\mathcal{G})^{-1} \\
\Sigma &= \mathcal{A}_o\Sigma\mathcal{A}_o' + \mathcal{C}\mathcal{C}' - (\mathcal{C}\mathcal{C}'\mathcal{G}' + \mathcal{A}_o\Sigma\bar{\mathcal{G}}')(\bar{\mathcal{G}}\Sigma\bar{\mathcal{G}}' + \mathcal{R} + \mathcal{G}\mathcal{C}\mathcal{C}'\mathcal{G})^{-1}(\bar{\mathcal{G}}\Sigma\mathcal{A}_o' + \mathcal{G}\mathcal{C}\mathcal{C}').
\end{aligned}$$

Applying the time invariant Kalman filter to the above state space representation attains the corresponding innovations representation:

$$\hat{x}_{t+1} = \mathcal{A}_o\hat{x}_t + \mathcal{K}u_t \tag{23}$$

$$\bar{y}_t = \bar{\mathcal{G}}\hat{x}_t + u_t \tag{24}$$

where $u_t = y_{t+1} - \hat{E}[y_{t+1}|y_t, ..., y_1, \hat{x}_1]$, $\hat{x}_t = \hat{E}[x_t|y_t, ..., y_1, \hat{x}_1]$ and $Eu_tu_t' \equiv \Omega = \bar{\mathcal{G}}\Sigma\bar{\mathcal{G}}' + \mathcal{R} + \mathcal{G}\mathcal{C}\mathcal{C}'\bar{\mathcal{G}}$. Hence the Gaussian log-likelihood function for $\{y_t\}_{t=1}^T$ conditioned on $\hat{x}_1$ is given by:

$$\log L(\theta) = -(T-1)\log 2\pi - \frac{1}{2}(T-1)\log|\Omega| - \frac{1}{2}\sum_{t=1}^{T-1} u_t'\Omega^{-1}u_t - \frac{1}{2}\sum_{t=1}^{T-1}\log|\text{diag}(Y_t)^{-1}|. \tag{25}$$

The final term is necessary since the model's implications are for transformed (logged and detrended) data.[3] The parameter estimates that maximize $\log L(\theta)$ are reported in section 5. Using these parameter values decision rules can be computed.

4 Data

In this section the data set constructed for this analysis is discussed. The data are quarterly, aggregate data of the United States for the sample 1955:Q1 to 1992:Q4.[4]

The capital stock series, K_t, is the net stock in 1987 dollars of fixed private capital plus the net stock of durable goods. These series are reported in U.S. Department of Commerce (1993) and are updated in the *Survey of Current Business.* To convert these annual series to quarterly series a linear interpolation is used. Since the data are reported as end-of-year stocks, the year t observations are used as the first quarter of year $t+1$ observations.

[3]See Anderson, Hansen, McGrattan and Sargent (1994).

[4]Unless otherwise stated, the data were obtained from the Federal Reserve Bank of Chicago's database. Data on total employment and overtime employment from 1955 to 1976 were provided by Gary Hansen.

Investment, I_t, is private fixed investment plus the personal consumption expenditure on durable goods. Private consumption, C_t, is the sum of the personal consumption expenditures on nondurable goods and services and the imputed service flow from the stock of consumer durables. G_t is government purchases. The imputed service flow from the stock of consumer durables is a quarterly seasonally adjusted series from the MPS database documented by Brayton and Mauskopf (1985). The other series are quarterly seasonally adjusted data from the National Income and Product Accounts reported in 1987 dollars. Output, Q_t, is the sum of C_t, G_t, and I_t. This measure of output differs from the official GDP measure in that it excludes net exports and the change in business inventories but includes the service flow from the stock of consumer durables.

Total employment, N_{1t}, is the number of employed persons at work in nonagricultural industries who worked 35 hours and over a week; overtime employment, N_{2t}, is the number of employed persons at work in nonagricultural industries who worked 41 hours and over a week. Total hours, H_t, is the number of persons employed at work 35 hours and over a week in nonagricultural industries multiplied by the average weekly hours worked in nonagricultural industries. All these series are from the Bureau of Labor Statistics' household survey and reported in *Employment and Earnings.* These series are adjusted for holidays in the BLS's reference week using OLS and seasonally adjusted using EZ-X11. Since the labor series are monthly, each quarterly observation is the average of the three corresponding monthly observations.

In order to make these data consistent with the lower case variables in the theoretical model, all data are converted to per capita terms using the civilian, noninstitutional population, 16 years and older. All the series of flow variables are converted to per quarter rates. The series Q_t, C_t, I_t, G_t and K_t are divided by γ^t. Logarithms of each series are taken.

5 Empirical results

This section assesses the quantitative implications of the two models. First, the parameter values are computed by maximum likelihood. Second, the ability of the models, evaluated at the computed parameter values, to fit the time-series properties of the data is measured.

Third, the propagation mechanisms embodied within each model are studied along the dimensions discussed in Burnside and Eichenbaum (1994). Fourth, the relationship between effort and overtime is studied.

5.1 Parameter values

This subsection presents the parameter estimates that maximize the likelihood function (25) for both the labor precommitment model and the labor hoarding model. The estimated models are identical to the theoretical models described above with one exception; in the estimated models, government, g_t, is allowed to grow at a different rate than the other series. Its trend is denoted γ_g. To facilitate comparisons across the two models, three parameters are fixed prior to estimation; γ is set to 1.0044, γ_g is set to 1.0007 and T is set to 1369 hours per quarter. The value for γ is chosen by separately regressing q_t, c_t, i_t and k_t on a constant and a linear time trend. The coefficients on the time trend are constrained to be equal across all four regressions. Likewise γ_g is computed by regressing g_t on a constant and a linear time trend. The value for the time endowment, T, corresponds to 15 hours per day.

The innovations, u_t, used in the likelihood function are constructed recursively using (23) and (24). The time-invariant Kalman filter is employed with $\mathcal{K}$ and Ω set to their steady state values. The initial value of the state vector, x_1, is set to the steady state level of technology and the beginning of sample values for g, k and n_1. Data over the entire sample are used in constructing the innovations; however to mitigate the influence of the choice of initial conditions, the first two elements of the innovation series are not used in computing the likelihood function.

Table 1 presents the point estimates of the preference and technology parameters for the labor hoarding and the labor precommitment models. Standard errors are also reported for all the estimates with the exception of δ; δ is free during estimation but fixed when computing standard errors. This was done to avoid a singularity in the information matrix when computing standard errors.

The parameter values for the labor hoarding model are virtually identical to those found by Burnside, Eichenbaum and Rebelo. Most of the point estimates are within one standard error of BER's point estimates. For example, in the labor hoarding model, the estimate of

Table 1
Preference and technology parameter estimates

	Labor hoarding		Labor precommit	
Parameter	Value	S.E.	Value	S.E.
α	0.656	0.093	0.689	0.027
β	0.983	0.012	0.988	0.003
δ	0.0225		0.0225	
μ_λ	-0.0145	0.0309	-0.0149	0.0112
μ_g	0.0886	0.0858	0.0887	0.0859
ρ_λ	0.979	0.006	0.973	0.005
ρ_g	0.987	0.012	0.987	0.012
h_1	511.4	10.8	451.1	48.5
h_2			158.9	47.5
v	7.004	5.706	6.670	0.609
ξ	93.56	110.20		
σ_λ	0.0057	0.0006	0.0060	0.0004
σ_g	0.0089	0.0014	0.0092	0.0013

labor's share in the production function is 0.656; BER estimate labor's share to be 0.654.[5] The estimate for ρ_λ in the labor hoarding model is 0.979 while BER compute 0.982. Likewise the labor hoarding estimate for ρ_g is 0.987; this is identical to BER's estimate. Recall that the labor hoarding model presented here is a slight variation of Burnside, Eichenbaum and Rebelo's model. BER compute their parameter estimates using GMM on a different data set. This replication of their results suggests that their selection of parameter values is robust. There is good reason to be confident that these parameter values are reasonable and are not dependent on specifics in the data set. Moreover it is reassuring that applying GMM and maximum likelihood to this model yields similar parameter values.

In the labor hoarding model three parameters are estimated that are fixed by BER. First is the discount factor, β. The estimate of β is lower than usually assumed. The point estimate, 0.983, implies an annual risk-free interest rate of 8.7%. Nevertheless the point estimate is less than one and within one standard error of the values usually assumed. Second is the shift length, h_1. BER fix the shift length so steady state effort is one. In

[5]The BER parameter estimates are for their Labor Hoarding I - Overidentified model which are presented on page 256 of their paper.

Table 2
Measurement error parameter estimates
series ordered: q_t, c_t, i_t, g_t, k_t, h_t, n_{1t}, n_{2t}

	Labor hoarding		Labor precommit	
Parameter	Value	S.E.	Value	S.E.
$\mathcal{D}(1,1)$	0		0	
$\mathcal{D}(2,2)$	0.999		0.999	
$\mathcal{D}(3,3)$	0.980	0.011	0.977	0.008
$\mathcal{D}(4,4)$	0.980	0.024	0.983	0.024
$\mathcal{D}(6,6)$	0.550	0.157	0.757	0.204
$\mathcal{D}(7,7)$	0.576	0.166	0.821	0.170
$\mathcal{D}(8,8)$			0.946	0.037
$\sqrt{\mathcal{R}(1,1)}$	0		0	
$\sqrt{\mathcal{R}(2,2)}$	0.0036	0.0006	0.0033	0.0005
$\sqrt{\mathcal{R}(3,3)}$	0.0109	0.0014	0.0104	0.0014
$\sqrt{\mathcal{R}(4,4)}$	0.0064	0.0020	0.0064	0.0017
$\sqrt{\mathcal{R}(5,5)}$	0		0	
$\sqrt{\mathcal{R}(6,6)}$	0.0212	0.0049	0.0208	0.0065
$\sqrt{\mathcal{R}(7,7)}$	0.0185	0.0047	0.0179	0.0052
$\sqrt{\mathcal{R}(8,8)}$			0.0209	0.0020

the model presented here, the point estimate is economically interpretable ($h_1 = 511.4$) and corresponds to 39.3 hours per week. The third parameter is the fixed cost to working, ξ. BER state that their results are insensitive to choices of ξ between 20 and 120. These results support that statement; the point estimate for ξ is 93.56 with a standard error of 110.20.

The parameter estimates for the labor precommitment model are similar to those in the labor hoarding model. The estimate of labor's share is 0.689, a slightly higher number than often used. The point estimate of β is 0.988 – the same number King, Plosser and Rebelo (1988) use. Both models estimate δ to be 0.0225. The point estimates for the shift parameters h_1 and h_2 are 451.1 and 158.9, respectively. These point estimates correspond to a straight time shift of 34.7 hours a week and an overtime shift of 12.2 hours a week. Since the definition of total employment excludes persons working less than 35 hours a week, the point estimate of h_1 is little troubling. Not too much should be made of the point estimate however; its standard error is large, 48.5.

In general the standard errors on the preference and technology parameters of the two models are of the same order of magnitude as those reported in McGrattan (1994). Interestingly, the standard errors reported here suggest that both models have difficulty matching the hours and employment series. On the one hand, the labor hoarding model provides a tight parameter estimate of the shift length; the standard error on h_1 is 10.8. On the other hand, there are very large standard errors associated with the preference parameters v and ξ. In contrast, applying maximum likelihood to the labor precommitment model yields estimates for the shift lengths with large standard errors but a tight estimate on v.

The parameter values that govern the measurement error process are presented in table 2. Recall $\mathcal{R}$ is the variance-covariance matrix for the innovations to the measurement error vector and is assumed to be diagonal. To ensure that estimated variances are positive, the square root of the diagonal terms of $\mathcal{R}$ are estimated and reported. Several parameters in the measurement error process are fixed prior to estimation.

At first the matrix $\mathcal{D}$ was assumed to be diagonal; however to avoid estimating a unit root in the autoregressive coefficient for the measurement error of capital, the following assumption from Christiano (1988) is made. It is assumed that the original capital stock is measured without error, but investment is measured with error; therefore the measurement error on the capital stock is a weighted sum of the past measurement errors on investment. The law of motion for the measurement error on capital is constructed as follows. Let $\hat{k}_t$ and $\hat{i}_t$ be the model's one-step-ahead forecasts of k_t and i_t, respectively. Then $k_t = \hat{k}_t + v_t^k$ and $i_t = \hat{i}_t + v_t^i$ where v_t^k and v_t^i are the time t measurement errors on capital and investment, respectively. Take the exponential of both sides of each equation, solve for K_t and I_t, substitute into equation (6) and solve for v_{t+1}^k as a function of v_t^k and v_t^i. Taking a Taylor approximation yields

$$v_{t+1}^k = \frac{1-\delta}{\gamma} v_t^k + \frac{\gamma - 1 + \delta}{\gamma} v_t^i.$$

Unfortunately this assumption causes the likelihood function to reach a constrained maximum when the autoregressive coefficient of the measurement error process for consumption, $\mathcal{D}(2,2)$, is one. Therefore $\mathcal{D}(2,2)$ is set to 0.999. Though this assumption "trades" one

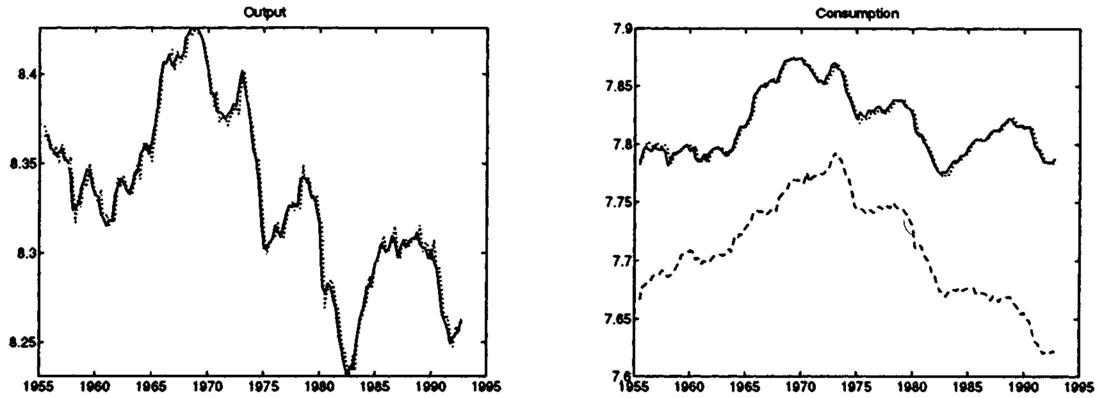

Figure 1: U.S. data (solid), fitted series (dotted), and predicted series (dashed) for output and consumption. Since output is assumed to be observed without measurement error the fitted and predicted series for output coincide.

unit root for another it improves the value of the likelihood.

Finally, output is assumed to be measured without error; so $\mathcal{D}(1,1)$ is set to 0. For numerical reasons, the variances of the innovations to the measurement error on output and capital ($\mathcal{R}(1,1)$ and $\mathcal{R}(5,5)$) are set to 1×10^{-10} rather than exactly zero.

The computed point estimates of the parameters in both models are reasonable. Moreover the standard errors are small enough for most parameters that the entire two standard error confidence interval contains economically interpretable numbers. This result is in contrast to some previous studies which had trouble computing economically interpretable parameter estimates using maximum likelihood. For example Altug (1989) and Christiano (1988) must fix β to avoid a point estimate greater than one. These results, along with studies such as McGrattan (1994), demonstrate the usefulness of applying maximum likelihood to linear-quadratic models to obtain economically interesting parameter values.

5.2 Diagnostics

This subsection assesses the performance of the two models evaluated at the parameter estimates computed above.

To visually assess the performance of the labor precommitment model, the actual data, the fitted series and the predicted series are graphed for the sample period 1955:Q3 to

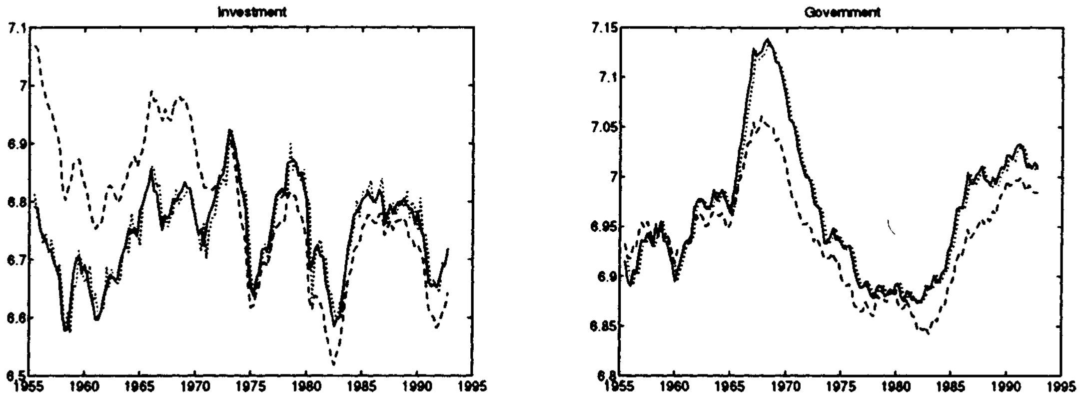

Figure 2: U.S. data (solid), fitted series (dotted) and predicted series (dashed) for investment and government

1992:Q4 in figures 1-4. The $t+1$ observation of the fitted series is $\hat{E}[y_{t+1}|y_t, y_{t-1}, ..., y_t, \hat{x}_1] = \bar{\mathcal{G}}\hat{x}_t + \mathcal{D}y_t$. Therefore the innovations, u_t, are the vertical difference between the solid line (the U.S. data) and the dotted line (the fitted series). The dashed line is the predicted series; its date t observation is $\mathcal{G}\hat{E}[x_t|y_t, y_{t-1}, ..., y_t, \hat{x}_1] = \mathcal{G}\hat{x}_t$. Recall that the actual data for output, consumption, investment, government and capital are divided by γ^t (except for government which is divided by γ_g^t) and logarithms are taken. Hours and employment are assumed stationary; only logs of these series are taken. The corresponding graphs for the labor hoarding model are not presented; they are qualitatively similar though not identical to the graphs shown.[6]

The fitted series are the model's one-step-ahead forecasts, incorporating measurement error, of the data. In contrast the predicted series ignores the measurement error and therefore displays just the contribution of the theory. Consequently, in figures 1-4 the vertical distance between the dashed line (the predicted series) and the dotted line (the fitted series) is the contribution of the measurement error. Since output is assumed to be measured without error the fitted series and the predicted series coincide (see figure 1).

Not surprisingly, the one-step-ahead forecasts (the fitted series) of the labor precommitment model matches the data very well. In figures 1-3 it is clear that the fitted series do an excellent job matching output, consumption, investment, government and capital; the

[6]The corresponding graphs for the labor hoarding model are available from the author.

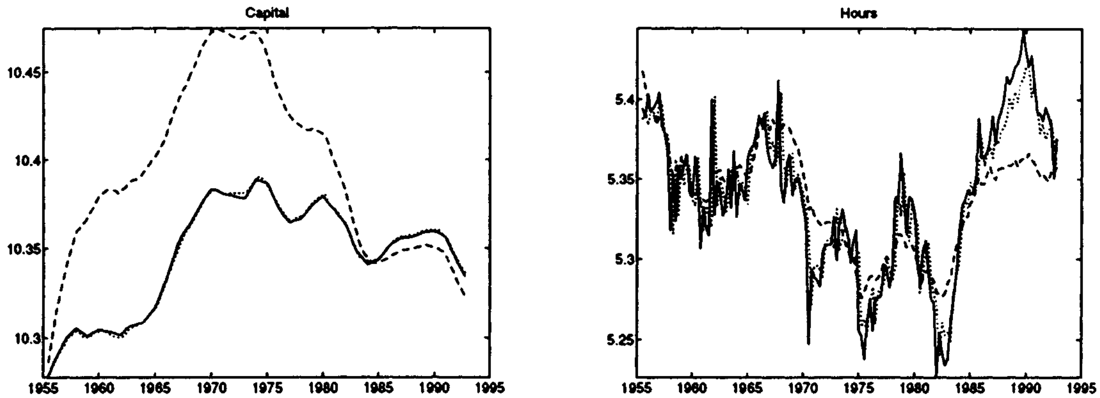

Figure 3: U.S. data (solid), fitted series (dotted) and predicted series (dashed) for capital and hours

fitted series have some difficultly matching the hours and employment data (see figure 4). In particular the model has trouble forecasting (even one step ahead) the large rise in hours and employment that occurred in the late 1980s. Unfortunately much of this excellent fit is due to the contribution of the measurement error.

The theory does an excellent job matching the output data. Unfortunately, the theory dramatically underpredicts the consumption series (see figure 1). Though the predicted series for consumption (the dashed line) and the U.S. consumption data (the solid line) appear to move in tandem, the predicted series is consistently below the actual series. This is not surprising since the autoregressive coefficient on the measurement error process for consumption was set to 0.999. Figures 2 and 3 illustrate that the theory also has some trouble reconciling the investment and capital data. For the investment series, the theory overpredicts the actual data before 1972; it underpredicts the data after 1972. Consequently, the predicted series for capital is consistently above the actual data before 1984 and consistently below after 1984. As with most business cycle models, the theory predicts hours and employment to be much less volatile than the actual data (see figures 3 and 4). Even with the total employment precommitment constraint in this model, the model still predicts a much smoother overtime employment series than is observed in the data.

One caveat is worthy of note. It can be shown that the innovations to the capital stock series are highly serially correlated. More specifically, the innovations are on average zero

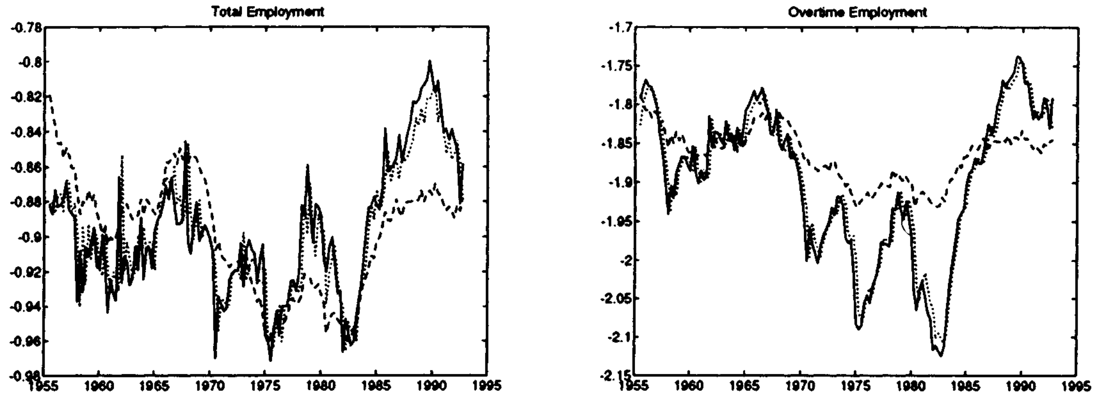

Figure 4: U.S. data (solid), fitted series (dotted) and predicted series (dashed) for total and overtime employment

but consistently above zero before 1970 and consistently below zero after 1970. Obviously either the law of motion for capital, equation (6), is misspecified or the assumption that the measurement error on capital is a weighted sum of the measurement error of investment is a poor one. Nevertheless the innovations to capital are quite small; the largest in absolute value is less than 0.004 whereas the mean of the capital stock is about 10.35. Consequently there is reason to believe this error is not serious.

At first glance it appears that in figures 1-4 the fitted series (the dotted line) leads the U.S. data (the solid line). Upon closer inspection one can see that the fitted series tends to be below the actual data when the data are increasing and above the data when the data are decreasing. Moreover the largest innovations occur at turning points in the data. To see why this happens, assume the state vector x_t is at its steady state value x_{ss}. Assume y_t does not equal its steady state value where $y_{ss} \equiv \mathcal{G}x_{ss}$. Then:

$$\begin{aligned}\hat{E}[y_{t+1}|y_t, y_{t-1}, ..., y_t, \hat{x}_1] &= \bar{\mathcal{G}}\hat{x}_t + \mathcal{D}y_t \\ &= (\mathcal{G}\mathcal{A}_o - \mathcal{D}\mathcal{G})x_{ss} + \mathcal{D}y_t \\ &= (I - \mathcal{D})y_{ss} + \mathcal{D}y_t.\end{aligned}$$

The above algebra demonstrates that the fitted value of y_{t+1} is a weighted sum of the period t realization of y and an updating vector. The larger the persistence in the measurement error, the more weight is given to the realization of y_t. Note that for investment,

Table 3
Fraction of variance explained by theory

series	q	c	i	g	k	h	n_1	n_2
Labor Hoarding	1.00	0.22	0.65	0.75	0.50	0.44	0.50	
Labor Precommitment	1.00	0.22	0.73	0.73	0.58	0.34	0.35	0.12

consumption and government the autocorrelation coefficient for the measurement process exceeds 0.97 (see table 2). It is clear that the measurement error process in this model has important implications for the model's performance.

To isolate and evaluate numerically the performance of the theory without the measurement error, the unconditional variances of the fitted time series are decomposed into the fraction due to the theory and the fraction due to the measurement error. That is,

$$\text{var}(y) = \mathcal{G}\text{var}(x)\mathcal{G}' + \text{var}(v)$$

where $\text{var}(x) = \sum_{j=0}^{\infty} \mathcal{A}^j \mathcal{C}\mathcal{C}'\mathcal{A}^j$ and $\text{var}(v) = \sum_{j=0}^{\infty} \mathcal{D}^j \mathcal{R}\mathcal{D}^j$. Table 3 reports the fraction of the total unconditional variance explained by the theory for each series for both models. This statistic is a R^2 measure of the theory for each variable in the models.

Table 3 indicates the importance of the parameter estimates for the $\mathcal{D}$ matrix. Holding all other things constant, the larger the persistence in the measurement error process, the smaller the fraction of total variance explained by the theory. Since output is assumed to be measured without error, none of its variance is explained by the measurement error. However for consumption, the theory in both models explains only 1/5 of the total variance of the fitted series. This result makes sense since $\mathcal{D}(2,2)$ is set to 0.999. The labor precommitment model explains about 3/4 of the total variance of investment and government – but only 1/3 of the total variance in hours. Disappointingly, the labor precommitment model only explains 1/8 of the variance in overtime employment. The labor hoarding model does not do much better; its theory explains about 2/3 of the variance in investment, 3/4 of the variance of government consumption and 1/2 of the variance in total employment. This measure of fit further confirms the visual assessment of figures 1-4; the labor precommitment model (as well as the labor hoarding model) does a much bet-

Table 4
Means and standard deviations of the predicted and U.S. time series

	Labor hoarding		Labor precommit		United States	
series	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
output, q	8.347	0.039	8.328	0.038	8.331	0.048
consumption, c	7.743	0.043	7.718	0.039	7.818	0.028
investment, i	6.783	0.075	6.783	0.081	6.747	0.078
government, g	6.938	0.056	6.916	0.058	6.967	0.070
capital, k	10.397	0.042	10.397	0.041	10.347	0.030
hours, h	5.330	0.023	5.329	0.023	5.341	0.048
total employment, n_1	-0.907	0.023	-0.909	0.023	-0.896	0.038
overtime employment, n_2			-1.875	0.024	-1.898	0.098

ter job explaining investment than consumption. Furthermore both models have difficulty matching the variability in the hours and the employment series.

A more conventional metric to measure the performance of the two models is to compare the models' first- and second-moment properties to the moments in the data. Table 4 reports the unconditional means and standard deviations of the models' predicted time series and the actual U.S. data. The unconditional means of the predicted series are just their steady state values, y_{ss}. Their unconditional standard deviations are the square root of the diagonal elements of the matrix $\mathcal{G}\text{var}(x)\mathcal{G}'$. None of the moments reported incorporate the effects of measurement error. The reported moments for the U.S. data are the sample moments from 1955:Q3 to 1992:Q4 for the logged and detrended (lower case) time series.

Table 4 shows that both models imply very similar first- and second-moment properties. In general the likelihood function chooses parameter values such that both models do extremely well matching the means of each series (with the exception of consumption). Both models do less well matching the standard deviations. Since output is assumed to be measured without error, it is no surprise that both models match its mean; however, both models underpredict its standard deviation. For example, the labor precommitment model explains about $(0.038^2/0.048^2) \times 100 = 63\%$ of the variance of output. As one would expect from studying the previous figures, both models do a better job matching the mean

Table 5. Correlation of apl_t with q_{t+i}

i	-4	-3	-2	-1	0	1	2	3	4
Labor hoarding	0.78	0.80	0.82	0.84	0.86	0.85	0.85	0.85	0.84
Labor precommitment	0.73	0.75	0.78	0.81	0.82	0.82	0.82	0.82	0.82
U.S. Data	0.58	0.60	0.62	0.63	0.65	0.65	0.64	0.65	0.65

and standard deviation of investment than consumption, and both models underpredict the volatility of hours and employment. It is difficult to use the implied second-moment properties of the two models to differentiate between them.

Both models capture the procylicality of the average product of labor. The average product of labor is defined as $apl_t \equiv q_t - h_t$. The correlation of the apl with q is presented in table 5. Both models overestimate the correlation or coherence of the average product of labor with output. Nevertheless both models imply that the correlations of the apl_t with the $t+1, \ldots t+4$ leads of q_t are greater than the correlations with the $t-1, \ldots t-4$ lags. This is consistent with the observation that the average product of labor is procyclical and falls at the end of an expansion.

5.3 Propagation and magnification of shocks

Using the parameters for preferences and technologies given by the maximum likelihood estimates, this subsection assesses the ability of the labor precommitment model and the labor hoarding model to propagate and magnify shocks over the business cycle. Following Burnside and Eichenbaum (1994), this analysis is executed along three dimensions. First, for each model, the parameter values computed by maximum likelihood and the impulse response functions are used to infer the strength of the model's magnification mechanisms. Second, each model's ability to account for the observed autocorrelation of output growth is evaluated. Finally, each model's implied spectrum of output growth is reported and compared to the corresponding spectrum in the data.

To analyze how each model propagates and magnifies shocks it is useful to once again consider the parameter estimates. The bottom two rows of table 1 report the parameter estimates for σ_λ and σ_g, the standard deviation of the technology innovations and govern-

Table 6. Magnification of shocks

	Labor hoarding	Labor precommit
std(λ)	0.0277	0.0260
std(q)/std(λ)	1.42	1.48

ment innovations, respectively. For both models, the standard deviation of the technology innovations is estimated to be about 0.006; likewise, for both models, the standard deviation of the government innovations is estimated to be about 0.009. Both models match the data with shocks of the same volatility.

In table 4 one can see that both models imply an unconditional standard deviation of output of about 0.038. Table 6 reports the unconditional standard deviation of the state of technology, std(λ), for each model. For the labor precommitment model it is 0.0260, while for the labor hoarding model it is 0.0277. One measure of amplification is just the unconditional standard deviation of output divided by the unconditional standard deviation of λ. This measure is reported in the second row of table 6. It implies that both models possess similarly strong magnification mechanisms. The magnification mechanism within the labor hoarding model leads to a 42% increase in the volatility of output; for the labor precommitment model the increase is 48%. This strength of the magnification mechanisms embedded in these two models is a success for both models.

These two models magnify shocks by about the same amount. This is surprising considering the different technologies the two models use to convert labor into goods. To understand the dynamic properties of the two models, it is useful to consider the impulse response functions of the linear system described by (20) and (21) for the two models. These impulse responses are plotted in figures 5 and 6. No measurement error is assumed for any of the variables in the models. These figures plot responses to a positive unit shock to λ.

Both models produce "hump-shaped" impulse response functions. To understand how the social planner responds to shocks in these two models look at figure 5. When a positive technology shock occurs in the labor hoarding model, the social planner increases effort

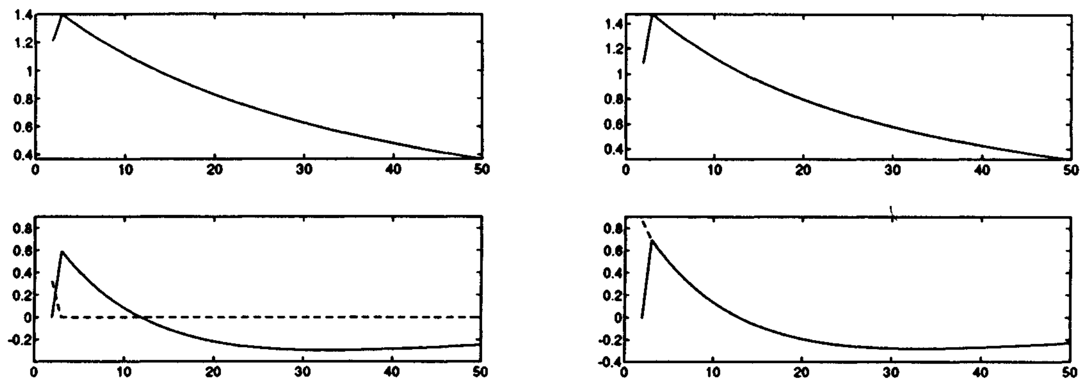

Figure 5: Left: Labor hoarding model – impulse responses from a unit shock to technology. Top - output; bottom - employment (solid) and effort (dashed).

Figure 6: Right: Labor precommitment model – impulse responses from a unit shock to technology. Top - output; bottom - total employment (solid) and overtime employment (dashed).

immediately but must keep employment fixed for one period. In the period after the shock, the social planner increases employment such that effort returns to its steady state value. This rigidity in employment causes output and investment to deviate from its non-stochastic steady state value more in the second period after the shock than in the initial period.

In a similar fashion, when a positive technology shock occurs in the labor precommitment model, the social planner increases overtime employment immediately but must keep total employment fixed for one period. This is illustrated in figure 6. In the second period the social planner adjusts total employment. Unlike effort in the labor hoarding model, however, overtime does not return to its steady state level in the second period. Instead, the ratio of total employment to overtime employment returns to its steady state level in the second period. This is an important implication of the labor precommitment model.

Consider again tables 3 and 4. The most dramatic weakness of the labor precommitment models is its inability to explain the volatility of overtime employment. From table 3, one can see that the theory explains only 12% of the unconditional variance in the fitted overtime series. Moreover, in table 4, it is clear that the overtime employment data are over twice as volatile as total employment data; however the model predicts overtime

Table 7. Autocorrelation of output growth

Autocorrelations	1	2	3	4	5	6	7	8	9	10
Labor hoarding	0.17	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.01
Labor precommitment	0.30	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.02	-0.02	-0.02
U.S. data	0.42	0.28	0.17	0.07	0.04	0.06	-0.03	-0.10	-0.04	-0.02

employment and total employment to have almost the same standard deviations.[7] Since the ratio of total employment and overtime employment returns to its steady state level in the second period following the shock, it is no surprise that the labor precommitment model has difficulty matching the relative volatility of total and overtime employment.

The differences in the labor market response to a technology shock within each model causes differences in the magnification and propagation properties of each model. For the labor hoarding model, a 1% shock to technology increases output 1.21% in the initial period; by the second period output is 1.40% higher than its non-stochastic steady state. For the labor precommitment model, a 1% shock to technology increases output only 1.09% in the initial period; but in the second period output is 1.48% higher than its non-stochastic steady state. The relative increase in output from the first to the second period is much larger in the labor precommitment model than in the labor hoarding model. This implies that it is more efficient for the social planner to increase effort in the labor hoarding model than overtime in the labor precommitment model.

A related way to measure the propagation mechanisms is to analyze the autocorrelation of output growth. Cogley and Nason (1993) demonstrates that while output growth in the data displays considerable autocorrelation, standard real business cycle models predict this autocorrelation to be white noise. Moreover Watson (1993) shows that standard real business models cannot explain the peak of the spectrum of output growth at business cycle frequencies. Table 7 presents the two models' implications for the autocorrelation of output growth. Output growth is defined to be $\log Q_t - \log Q_{t-1} = q_t - q_{t-1} + \log \gamma$.

[7]It is worth noting that adding an adjustment cost term for changes in total employment to the model can lead to the implication that overtime employment is over twice as volatile as total employment. Adjustment costs are not in the labor precommitment to simplify the contrast between the two models' implications. Results with adjustments cost are available from the author.

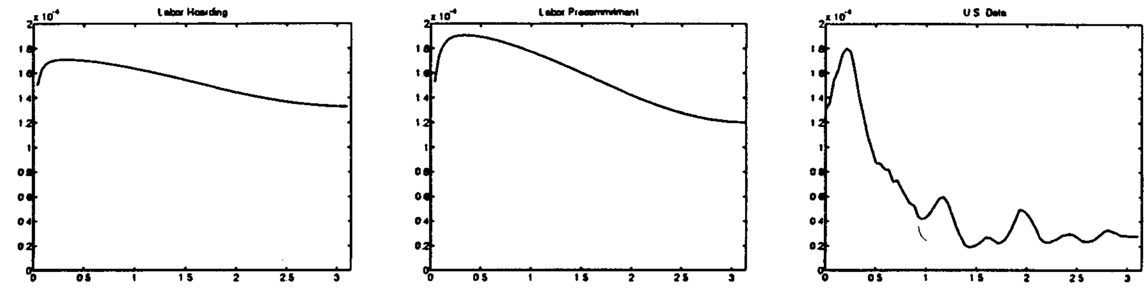

Figure 7: The spectrum of output growth in the labor hoarding model (left), the labor precommitment model (center) and the data (right).

Consider table 7. Both models imply a positive correlation at lag 1.[8] The intuition behind this result is illustrated by the impulse response functions (figure 5-6). After a positive shock in the labor precommitment model (labor hoarding model) the social planner immediately increases overtime (effort) and thus output but cannot immediately increase total employment until the next period. This first increase in output due to the increase in overtime (effort) predicts the increase in output in the following period from the increase in total employment.

It is straightforward to consider each model's implication on output growth in the frequency domain. In figure 7 the spectrum of output growth is plotted for each model. The spectrum of output growth from the data is also plotted. Both models imply spectra of the same shape. Both the labor hoarding and the labor precommitment models succeed in magnifying the shocks (which are white noise) such that a hump in the spectrum at business cycle frequencies is produced. This hump is more pronounced for the labor precommitment model than for the labor hoarding model. Although it is clear that both models dramatically overestimate the variance of output growth at high frequencies, this hump in the spectrum is a success for both models.

The analysis in this subsection demonstrates that the labor precommitment model does as well as the labor hoarding model in propagating and amplifying shocks.

[8]If adjustment costs to changing total employment are incorporated, both models can imply positive correlation coefficients for lags greater than one. Results of the labor precommitment model with quadratic adjustment costs to total employment are available from the author.

5.4 Overtime and effort

The purpose of this subsection is to use the overtime employment data to assess whether the predicted effort series from the labor hoarding model is reasonable.

The intuition motivating the labor hoarding model and the labor precommitment model is the same. It is costly to adjust total employment quickly; therefore, after observing a shock to the economy firms first adjust worker effort (in the labor hoarding model) or overtime employment (in the labor precommitment model). Clearly firms make their contemporaneous labor decisions along more than just the one dimension each of these models assumes; thus these models are not mutually exclusive. However since the labor hoarding model makes use of an unobservable time series, effort, it is more difficult to evaluate its implications.

Just as RBC theorists ask of their models, "what is the implied state of technology series measuring?" one can ask, "what is the implied effort series in the labor hoarding model measuring?" A useful way to try to answer this question is to find instruments that are expected to co-move with effort. If these instruments exhibit the same time-series properties that the labor hoarding model predicts for effort, then one can develop increased confidence that effort is moving in the direction implied by the model.

One of the advantages of using the Kalman filter to estimate the labor hoarding model is that the predicted series on the logarithm of effort, $\log e_t$, can be constructed by multiplying the matrix of time-invariant decision rule coefficients, $-\mathcal{F}$, and the constructed $\hat{x}_t$ series. Consequently one can ask: is this predicted effort series reasonable? does it behave like the overtime employment series? Recall that the overtime employment series, n_{2t}, is not used in constructing or estimating the labor hoarding model. Moreover one might think that when agents are working harder they are also working more overtime. Consequently overtime employment is a suitable instrumental variable for effort.

To study the relationship between the predicted effort series, $\log e_t$, from the labor hoarding model and the actual U.S overtime employment series, the two time series are plotted in figure 8. In this figure both series are normalized to have a mean of zero and a variance of one. At first glance the two series appear to have little relationship to each other; the correlation coefficient for the two series is just 0.26. However a regression of the

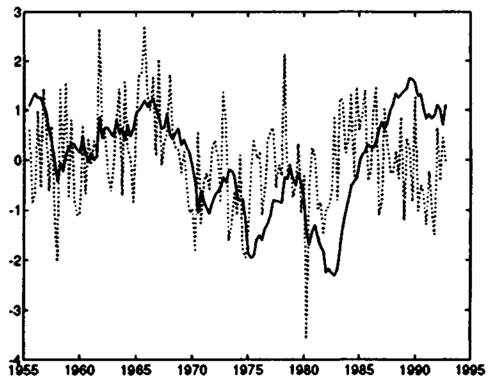

Figure 8: Scaled overtime employment (solid) and effort (dotted)

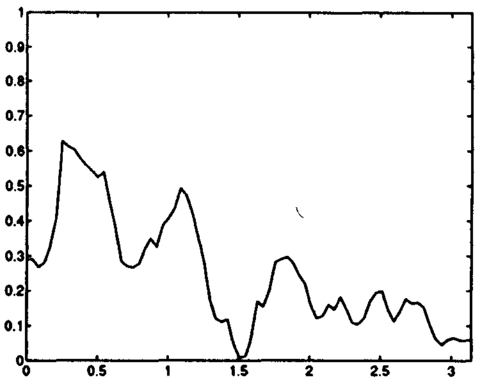

Figure 9: The coherence between overtime employment and effort

first difference (growth rate) of overtime employment on the first difference (growth rate) of the predicted effort series yields:[9]

$$\Delta \log e_t = \underset{\substack{(0.011)\\ [2.32]}}{0.026} \quad \Delta n_{2t}.$$

The significant positive regression coefficient implies that in periods in which overtime employment increases (or decreases) so does predicted effort.

Moreover the predicted effort series and the actual overtime employment series move together over the business cycle. In figure 9 the coherence between the two series is plotted. This figure indicates that at business cycle frequencies, the variance of the overtime employment series explains between 1/3 and 2/3 of the variance of the predicted effort series. Indeed the coherence between the two series peaks at the frequency corresponding to a periodicity between four and five years. At high frequencies the two series display a low coherence. Interestingly, but not surprisingly, the pairwise coherence of the two series at the seasonal frequency (period equals four quarters) is less than 0.01.

The results from this subsection suggest that the predicted effort series of the labor hoarding model makes sense. Although the effort series displays much larger volatility at high frequencies than the observed overtime employment series, the growth rates of the two

[9]The standard error is in parentheses, and the t-statistic is in braces.

series are correlated. Moreover the two series display a high pairwise coherence at business cycle frequencies. These results are consistent with the belief that overtime and effort move together over the cycle.

6 Conclusion

The labor hoarding and labor precommitment models are not mutual exclusive models. Clearly firms make labor decisions along both the effort margin and the straight time/overtime margin. In this sense these are complementary models – not competing models. But since both models ignore one of these margins, both models overstate the role of the other.

The purpose of this paper has been to assess the implications of assuming that firms contemporaneously adjust the labor input along either the effort margin or the overtime margin. To that end, this paper demonstrates three things. First, it illustrates the usefulness of applying maximum likelihood to compute reasonable parameter values for linear-quadratic models. Second, it demonstrates that the standard real business cycle model can be modified to embody quantitatively important propagation mechanisms without employing additional unobserved variables. Third, it finds that the effort series implied by the labor hoarding model moves with overtime employment over the cycle.

One interpretation of these results is as follows. Theorists need not incorporate time-varying effort into their models in order to construct models with quantitatively important propagation mechanisms. However if they chose to employ time-varying effort, there is good reason to believe that the predicted effort series from their model is reasonable. Moreover, since the predicted effort series and the U.S. overtime data display a large coherence at business cycle frequencies, there is good reason to believe that effort is procyclical.

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