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**ASSET RETURN VOLATILITY WITH EXTREMELY SMALL
COSTS OF CONSUMPTION ADJUSTMENT**

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Asset Return Volatility With Extremely Small Costs of Consumption Adjustment

Abstract

Extremely small non-convex costs of consumption adjustment are introduced into an equilibrium consumption capital asset pricing model (CCAPM), in an effort to reconcile the low variance of consumption growth with the high variance of equity returns and the large mean equity premia observed in the data. Using the approach of Hansen and Jagannathan (1991), I present evidence that the CCAPM is consistent with observed first and second moments of consumption growth and asset returns if changing the level of consumption involves an adjustment cost whose maximum value is less than $(1/1000)$ th of average within-period consumption. In practice, costs of this magnitude are undetectable. I conclude that the CCAPM is non-robust to the introduction of extremely small frictions, and that small costs of consumption adjustment can reconcile the model with the Hansen-Jagannathan mean-variance restrictions. However, this extreme non-robustness implies that the CCAPM is unlikely to yield an empirically useful model of the short-run co-movements of aggregate consumption and asset returns.

ASSET RETURN VOLATILITY WITH EXTREMELY SMALL COSTS OF CONSUMPTION ADJUSTMENT

Equilibrium consumption capital asset pricing models (CCAPMs) have attracted a great deal of attention since their introduction in the seminal papers by Lucas (1978), Breeden (1979), and Grossman and Shiller (1982). The CCAPM is attractive to financial economists because it implies an observable expression for the intertemporal marginal rate of substitution (IMRS) as a function of aggregate consumption data. According to the theory, all asset prices are determined by the conditional covariance between this IMRS and the asset's payoff. In contrast, returns-based pricing models, such as the Arbitrage Pricing Theory of Ross (1976), treat the IMRS as an unobservable process to be proxied by the returns on one or more mimicking portfolios. Unlike the CCAPM, returns-based models do not derive an explicit theoretical linkage between asset returns and aggregate quantity variables.

Unfortunately, the CCAPM has not fared well in empirical tests. When preferences are assumed to be time- and state-separable, the CCAPM cannot reconcile the low variance of consumption growth with the high variance of equity returns without implausibly high levels of risk aversion.¹ These difficulties are summarized by the failure of the IMRS implied by these models to lie within the Hansen-Jagannathan mean-variance bounds. (See Hansen and Jagannathan (1991).)

Researchers have responded to the CCAPM's empirical failures by adopting a richer

¹ See, for example, Hansen and Singleton (1982), Eichenbaum, Hansen and Singleton (1988), Grossman, Melino, and Shiller (1987), Hansen and Jagannathan (1991), Eichenbaum and Hansen (1990), Mehra and Prescott (1985), Cochrane and Hansen (1992).

preference specification or by abandoning market completeness.² In this paper, however, I argue that the difficulties with the CCAPM run deeper than a mis-specification of preferences or market structure. I argue that the implications of the CCAPM are fundamentally non-robust to extremely small perturbations in the economic environment. In particular, undetectably small costs of consumption adjustment are sufficient to reconcile the model with the Hansen-Jagannathan (1991) restrictions. While this suggests that some variant of the CCAPM may be (approximately) correct, developing an empirically useful version of the CCAPM would require accurate measurement of these extremely small adjustment costs. I conclude that the CCAPM is unlikely to yield a useful, robust model of short-run movements in asset returns.

Cochrane (1989) provides the intuitive reason why small adjustment costs can have a big effect on asset returns. He shows that the utility gains from adjusting consumption in response to most changes in investment opportunities are extremely small. If agents faced even a tiny cost of adjusting consumption, they would forego this trivial utility gain and choose an extremely smooth consumption path. In order to induce even the moderate consumption variability found in aggregate data, asset prices would have to be highly variable.

Why might it be costly for an agent to change her consumption rate? There may be time costs in implementing a new consumption/savings plan, or search costs in finding vendors for the new, better (or worse) quality goods to be purchased. More generally, costs of consumption adjustment can be interpreted as a proxy for what Cochrane (1989) calls "near-rational" behavior.

² Time-nonseparable preferences are studied by Gallant and Tauchen (1989), Constantinides (1990), Ferson and Constantinides (1991), and Heaton (1991); state non-separable preferences are studied by Epstein and Zin (1989,1991), Weil (1990). Market incompleteness due to uninsurable idiosyncratic risk is incorporated into asset pricing models of Lucas (1994), Heaton and Lucas (1994), Marcet and Singleton (1990), Telmer (1993), Constantinides and Duffie (1992).

Regardless of their interpretation, the costs I introduce into the CCAPM are too small to be detected in practice: I do not consider costs in excess of $(1/1000)^{\text{th}}$ of per capita consumption per period.

Consumption adjustment costs affect the asset price process in at least two ways. First, adjustment costs induce time-nonseparability into an individual's consumption/portfolio problem. It is known, from the work of Constantinides (1990), among others, that time-nonseparability can substantially alter the asset-pricing implications of these models. Second, in the presence of non-convex costs of consumption adjustment, assets are not priced by the IMRS of a fictitious "representative agent", whose consumption mimics the aggregate consumption. In general, non-convex adjustment costs induce some agents to keep consumption at the previous period's level, so the entire change in aggregate supply must be absorbed by the remaining agents. Assets are priced by the IMRS of these marginal agents, so asset prices display more variability than would be seen in a representative agent model.

This paper is closely related to Grossman and Laroque (1990). These authors were the first to suggest that small costs of consumption adjustment might explain the empirical failure of equilibrium asset pricing models, although they did not pursue this conjecture in a general equilibrium framework. This paper is also related to the growing literature documenting how costs of consumption adjustment at the individual level can cause aggregate consumption to deviate substantially from the predictions of frictionless models.³ This paper complements recent work in which costs of portfolio adjustment are introduced into equilibrium asset pricing models.⁴

The results of this paper support my conjecture that extremely small costs of consumption

³Caballero (1993), Heston (1992), Beaulieu (1991), Marshall and Parekh (1994).

⁴Luttmer (1993), He and Modest (1992), Aiyagari and Gertler (1990), Heaton and Lucas (1992).

adjustment may account, in part, for the failure of the CCAPM. For example, when agents' coefficient of relative risk aversion is set to five, I find that the Hansen-Jagannathan criterion is satisfied with adjustment costs whose maximum value is approximately $(6/10,000)^{\text{th}}$ of average within-period consumption. I conclude that the implications of the CCAPM are non-robust to the introduction of small frictions, and that such frictions can reconcile the first and second moments of consumption growth with the Hansen-Jagannathan restrictions. On one hand, this result is encouraging for the CCAPM, since it suggests that the empirical failure of this model need not represent a failure of equilibrium asset pricing theory. On the other hand, for a model to be useful empirically, it must be robust to small perturbations. If the CCAPM is as non-robust as this paper suggests, it is essentially useless as an empirical model of the short-run movements of asset returns.

The paper is organized as follows. Section 1 presents a simple example, which illustrates how consumption adjustment costs affect asset returns. Section 2 presents the general model studied in the remainder of the paper. Section 3 discusses the approach I use to solve the model. In section 4, I use the Hansen-Jagannathan methodology to evaluate the model when there are two types of agents. In section 5, I discuss whether the results of the two-agent case generalize to multiple types of agents. Section 6 is a brief conclusion.

1. A SIMPLE EXAMPLE OF ASSET PRICING WITH FIXED COSTS OF CONSUMPTION ADJUSTMENT⁵

In this section, I use a simple example to illustrate how small costs of consumption adjustment can substantially increase asset price variability. In the standard frictionless CCAPM with

⁵I am grateful to John Heaton for suggesting this example.

complete markets, aggregation obtains: asset returns are determined by IMRS of a fictitious representative agent who consumes the aggregate consumption. When costs of consumption adjustment are introduced into this model, aggregation breaks down. For some agents, the adjustment cost exceeds (in utility terms) any possible gain from adjusting consumption. These agents keep their consumption unchanged. The remaining agents must absorb any change in the aggregate supply of the consumption good. Since asset returns are determined by the IMRS of this second group, which has a higher variance than the representative agent IMRS, the asset returns are more variable than in the frictionless model.

In the illustrative model of this section, there are three time periods and two agents, who act as price takers. The consumption of agent i in period t is denoted c_t^i . Agents face fixed costs of adjusting consumption: if $c_t^i \neq c_{t-1}^i$, agent i must forego α units of consumption in period t . The aggregate endowment, denoted e_t , is known with perfect certainty, but is higher in period 2 than in period 1 or period 3. In particular, I set $e_1 = e_t + \alpha$, $e_2 = e_h$, and $e_3 = e_t$, where $e_h > e_t + \alpha > e_t$. The initial conditions, c_0^i , are set at the average per capita endowment: $c_0^i = (e_1 + e_2 + e_3)/6$, $i=1,2$. Preferences are given by:

$$U(c_1^i, c_2^i, c_3^i) = \log(c_1^i) + \beta \log(c_2^i) + \beta^2 \log(c_3^i), \quad 0 < \beta < 1, \quad i = 1, 2.$$

If $\alpha = 0$, the equilibrium allocation is standard. Regardless of the initial distribution of wealth, each agent consumes a fixed proportion of the aggregate endowment in each period:

$$c_t^1 = \delta e_t, \quad c_t^2 = (1-\delta)e_t \tag{1}$$

where δ is a constant between zero and one. The interest rate process R_t supporting this allocation as an equilibrium is: $R_1 = e_2/(\beta e_1)$, $R_2 = e_3/(\beta e_2)$. If $\alpha > 0$, however, allocation (1) cannot be supported as an equilibrium for all wealth distributions. If agent 1 is sufficiently poor relative to

agent 2, she will be unwilling to bear the cost of adjusting consumption each period. For some such cases, however, there is an alternative equilibrium in which agent 1's consumption is constant, and agent 2 absorbs the entire fluctuation in aggregate supply:

$$\begin{aligned}c_t^1 &= \bar{c}, \quad t = 1, 2, 3; \\c_t^2 &= c_t^2 \equiv e_t - \bar{c} - \alpha, \quad t \text{ odd}; \\c_2^2 &= c_h^2 \equiv e_h - \bar{c} - \alpha.\end{aligned}\tag{2}$$

Since agent 2 is at the margin, equilibrium asset prices are determined by her intertemporal IMRS. The interest rate process supporting allocation (2) as an equilibrium is: $R_1 = c_h^2/(\beta c_t^2)$, $R_2 = c_t^2/(\beta c_h^2)$. Since agent 2's consumption is more variable than the aggregate endowment, the asset returns in equilibrium (2) are more variable than in the frictionless equilibrium (1).

Table 1 shows that extremely small adjustment costs are sufficient to shift the economy from the standard equilibrium (1) to the alternative equilibrium (2).⁶ In this table, I display four parameterizations of this example. In all these parameterizations, I normalize $e_t + e_h = 2$ and I set $\beta = .999$. In Panel A of Table 1, I seek to replicate the variability we actually observe in the growth rate of monthly aggregate consumption. When a two-state first-order Markov chain is fit to the growth rate of U.S. monthly consumption from 1959-1986 (see equation (15) in section 5, below), the difference between consumption growth in the two states is .00886. In Panel A of Table 1, I match this feature of the data by choosing $e_t = .99779$ and $e_h = 1.00221$, so that $e_h/e_t - e_t/e_h = .00886$. In this case, a fixed cost of consumption adjustment of 2×10^{-6} (approximately one millionth of the average per-capita endowment per period) rules out the standard equilibrium (1). Intuitively, the gains from adjusting consumption in response to the time-varying interest rate are so

⁶The procedure for solving this simple example is discussed in detail in the Appendix.

small that even this trivial adjustment cost induces at least one of the agents to choose a flat consumption path. With this level of fixed cost, however, an equilibrium of form (2) exists for $\bar{c}$ as high as .29. (That is, approximately 29% of the wealth in the economy is owned by agent 1.) When $\bar{c} = .29$, the spread between R_1 and R_2 is 40% larger than the corresponding spread when $\alpha = 0$. With a slightly higher adjustment cost of 4×10^{-6} , equilibrium (2) obtains for values of $\bar{c}$ as high as .41, at which point the spread between R_1 and R_2 is 70% larger than the corresponding spread in the frictionless equilibrium. In both cases, a trivial adjustment cost induces a shift in the nature of the equilibrium and a substantial increase in asset-return variability.

With larger endowment fluctuations, larger fixed costs are needed to break the standard equilibrium. Examples are presented in the remaining panels of Table 1. Yet, even in Panel C, where the variability of aggregate endowment (as measured by $e_h - e_l$) is twenty times higher than in Panel A, the standard equilibrium (1) is broken by $\alpha = .0008$, which implies a fixed cost of consumption adjustment less than $(2/1000)$ of average per capita endowment. Only in Panel D, where real interest rates fluctuate 40 percentage points each period, does the fixed cost required to break the standard equilibrium approach detectability.

This example illustrates that aggregation can be disrupted by small adjustment costs if the gains from adjusting consumption differ across agents. This intuition carries over to the dynamic, growing economy modelled in section 2, below. Unlike the example of this section, the agents in the model of section 2 have (approximately) the same wealth. They have different propensities to adjust consumption because they have different lagged consumptions. As the aggregate endowment trends upward, those agents who have not adjusted recently find that their consumption level is substantially below their permanent income. These agents derive substantial gain from adjusting, so they are willing to bear the adjustment cost. As a result, the role of marginal consumer alternates

among the agents.

2. AN ASSET PRICING MODEL WITH COSTS OF ADJUSTING CONSUMPTION

A. Basic Structure

In this section, I describe the dynamic equilibrium model studied in the remainder of this paper. The model extends the simple exchange economy of Lucas (1978) and Hansen and Singleton (1982) by introducing a small non-convex cost of consumption adjustment. As in these earlier papers, there is a single non-storable consumption good, and there exists a complete market in state-contingent claims, payable in the consumption good. The only exogenous stochastic process is the aggregate endowment, denoted $\{e_t\}_{t=1}^{\infty}$. I assume that the endowment growth rate e_t/e_{t-1} is a first-order Markov chain. The state of the economy at date t , denoted s_t , is the history of realizations of the aggregate endowment: $s_t = \{e_1, e_2, \dots, e_t\}$; the set of possible realizations of s_t is denoted S_t . Let $\pi_t(s_t)$ denote the price at date zero of a claim to one unit of consumption at date t when the state is s_t , and let $\Pr(s_t)$ denote the probability of state s_t , conditional on date 0 information.

There are N types of agents, indexed by i . Let $c_t^i(s_t)$ denote the consumption of agent i in period t when the state is s_t . The initial wealth of agent i is denoted W_0^i . Unlike the traditional CCAPM, I assume that it is costly to change consumption. In particular, $A(c_t, c_{t-1})$ denotes the cost (in units of consumption good) of choosing c_t when consumption in the previous period was c_{t-1} . Define $x_t^i(s_t)$ as consumption inclusive of adjustment costs of agent i in period t when the state is s_t :

$$x_t^i(s_t) \equiv c_t^i(s_t) + A(c_t^i(s_t), c_{t-1}^i(s_{t-1})). \quad (3)$$

Agent i maximizes the expected value of a time separable utility function, (assumed to be the

same for all agents) subject to the budget constraint defined by the state prices $\{\pi_t(s_t)\}_{t=1}^\infty$:

$$\text{MAX}_{\{c_t^i(s_t), s_t \in S_t\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t \sum_{s_t \in S_t} \text{Pr}(s_t) \frac{c_t^i(s_t)^{1-\zeta}}{1-\zeta} \quad (4)$$

s.t

$$\sum_{t=0}^\infty \left[\sum_{s_t \in S_t} \pi_t(s_t) x_t^i(s_t) \right] \leq W_0^i \quad (5)$$

B. Fixed and Quasi-Fixed Costs of Consumption Adjustment

Suppose agents face fixed costs of adjusting consumption, with the cost a fraction α of the per-capita endowment. The cost function $A(c_t, c_{t-1})$ would then be given by:

$$A(c_t, c_{t-1}) = \begin{cases} 0 & \text{if } c_t = c_{t-1} \\ \alpha \frac{e_t}{N} & \text{if } c_t \neq c_{t-1} \end{cases} \quad (6)$$

Fixed costs are inconvenient because the cost function is discontinuous in the agents' choice variable. For this reason, I use the following smooth approximation to the fixed cost function (6), which I call a quasi-fixed cost adjustment function:

$$A(c_t, c_{t-1}) = \alpha \left(\frac{e_t}{N} \right) \left[1 - \exp \left[-\frac{\gamma}{2} \left(\log \frac{c_t}{c_{t-1}} \right)^2 \right] \right] \quad \gamma > 0, \alpha > 0 \quad (7)$$

Figure 1 displays a graph of A , as a function of $\log(c_t/c_{t-1})$, for two different values of γ . As $\gamma \rightarrow \infty$, A converges pointwise to the pure fixed cost function (6). If c_t/c_{t-1} is a stationary stochastic process,

then $A(c_t, c_{t-1})/e_t$ is stationary.

C. Equilibrium

For a given stochastic process $\{e_t\}_{t=0}^{\infty}$ and given initial conditions $\{c_{-1}^i, W_0^i\}_{i=1}^N$, an equilibrium is a vector $\{\pi_t(s_t), c_t^i(s_t)\}_{t=1}^{\infty}$, $s_t \in S_t$, satisfying:

- (i) For each i , $\{c_t^i(s_t)\}_{t=0}^{\infty}$ solves individual problem (4) - (5), given prices $\{\pi_t(s_t)\}_{t=0}^{\infty}$;
- (ii) The aggregate resource constraint holds, period by period:

$$\sum_{i=1}^N x_t^i(s_t) = e_t \quad (8)$$

Conventional approaches to proving the existence of equilibrium do not apply to this economy, due to the non-convex adjustment cost A . (In section 3.B, below, I address the problem of verifying that a proposed allocation constitutes a competitive equilibrium.)

The first-order conditions for problem (4) - (5) imply:

$$\pi_t(s_t) = \beta^t \Pr(s_t) \frac{\lambda_t^i(s_t)}{\lambda_0^i(s_0)} \quad (9)$$

where

$$\lambda_t^i(s_t) \equiv \frac{1}{1 + A_1(c_t^i(s_t), c_{t-1}^i(s_{t-1}))} \left[[c_t^i(s_t)]^{-\zeta} + E \left(\sum_{j=1}^{\infty} \beta^j [c_{t+j}^i(s_{t+j})]^{-\zeta} \prod_{k=1}^j \frac{-A_2(c_{t+k}^i(s_{t+k}), c_{t+k-1}^i(s_{t+k-1}))}{1 + A_1(c_{t+k}^i(s_{t+k}), c_{t+k-1}^i(s_{t+k-1}))} \mid s_t \right) \right] \quad (10)$$

The variable $\lambda_t^i(s_t)$ is the marginal value of wealth in period t . The intuitive content of (10) is perhaps clearer if one solves equation (10) recursively, to yield the following Euler equation:

$$[c_t^i]^{-\zeta} = \lambda_t^i [1 + A_1(c_t^i, c_{t-1}^i)] + \beta E_t[\lambda_{t+1}^i A_2(c_{t+1}^i, c_t^i)], \quad i = 1, \dots, N. \quad (11)$$

For simplicity, I have suppressed the dependence of λ and c on s_t . Equation (11) extends the familiar envelope condition. The left-hand side of (11) gives the marginal utility of increasing current consumption. The right-hand side gives the marginal disutility of the wealth reduction needed to pay for this increase in consumption. If there were no adjustment costs, this marginal disutility would equal λ_t , the marginal utility of wealth. With adjustment costs, however, a change in consumption also changes the adjustment costs the agent must pay, both now and in the future. The effect of these adjustment cost changes on current and future wealth are captured by including the marginal adjustment costs, A_1 and A_2 , in equation (11).

Equation (9) implies that any asset return r_t must satisfy

$$1 = \beta E_t \left[\frac{\lambda_{t+1}^i}{\lambda_t^i} r_{t+1} \right], \quad \forall i. \quad (12)$$

Agent i 's IMRS between wealth at t and wealth at $t+1$ is given by $\beta(\lambda_{t+1}^i/\lambda_t^i)$. Securities markets are complete, so this IMRS is equated across agents. Dropping the superscript " i ", $\beta(\lambda_{t+1}/\lambda_t)$ corresponds to the IMRS studied in Hansen and Jagannathan (1991).

D. Implications of the Model for Asset Returns

The non-convex adjustment costs introduce non-convexities and time non-separability into the individual's decision problem. To illustrate how these features affect asset returns, it is useful to regard x_t (consumption inclusive of adjustment costs, defined in equation (3)) as the choice variable. Figure 2 displays two indifference curves in (x_t, x_{t+1}) space under perfect certainty. The

upper indifference curve looks like one implied by standard concave preferences except for two perturbations near points a and b . At point a , the agent sets $c_t = c_{t-1}$, so the adjustment cost at date t is zero. Since the agent can consume the entire x_t , rather than losing a portion of the x_t in adjustment costs, she can achieve the given level of utility with a lower value of x_t . Similarly, point b is where $c_{t+1} = c_t$. The lower indifference curve passes through the point d , where $c_{t-1} = c_t = c_{t+1}$.

Figure 2 illustrates the qualitative implications of the model for asset pricing. In models with quasi-fixed costs of adjustment, agents typically make infrequent large adjustments. Between these large adjustments, agents make very small adjustments. That is, agents tend to be near points like a , b , or d , where a maximum of one large adjustment is made during the time periods $\{t, t+1\}$. As can be seen from Figure 2, the local curvature of the indifference curve is much greater near these points than at other points. Even if utility defined over c_t displays modest curvature, the local curvature of the derived utility defined over x_t is very large in the neighborhood of the optimally chosen allocations.

From the work of Hansen and Jagannathan (1991), it is known that the time- and state-separable model fails, in part, because both the mean and the variance of the IMRS implied by that model are too low. Figure 2 suggests why the model with adjustment costs might perform better in this regard. The IMRS ($\beta(\lambda_{t+1}/\lambda_t)$) is the slope of the indifference curve in $\{x_t, x_{t+1}\}$ space at the equilibrium allocation. Near points a , b , or d , this slope is extremely sensitive to small perturbations of the equilibrium allocation, so one would expect this model to induce greater variability in the IMRS than in comparable models without adjustment costs. Similarly, one would expect that this higher level of curvature would induce agents to value risk-free assets more highly. This would imply a lower risk-free rate, and therefore a higher mean value of the IMRS, than in comparable models without adjustment costs.

3. SOLVING THE MODEL

It is not feasible to solve the competitive equilibrium directly. This paper computes the competitive equilibrium in three distinct steps: (i) I use a discrete state-space dynamic programming algorithm to solve the equally-weighted optimal resource allocation (Pareto) problem corresponding to the conjectured equilibrium. (ii) I use equations (9) and (10) to compute the state-contingent claims prices corresponding to this Pareto optimum, and verify numerically that these prices in fact decentralize the Pareto optimum as a competitive equilibrium. (iii) Given this equilibrium state-contingent consumption allocation, I simulate the consumption process $\{c_t^i\}_{t=0}^T$, using as the initial condition the most common state in the discrete grid. I compute the $\{\lambda_t\}$ process corresponding to $\{c_t^i\}$ by numerically solving integral equation (11), and I study the IMRS $\beta(\lambda_{t+1}/\lambda_t)$ using the approach of Hansen and Jagannathan (1991). In the following I describe each of these steps in detail.

A. Solving the Optimal Resource Allocation Problem

The equally-weighted optimal resource allocation problem corresponding to the competitive model of section 2, above, is:

$$\text{MAX}_{\{c_t^i, t=0, \dots, \infty, i=1, \dots, N\}} E \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^N \frac{[c_t^i]^{1-\zeta}}{1-\zeta} \quad (13)$$

subject to the aggregate resource constraint (8), with initial conditions $\{c_{-1}^i\}_{i=1}^N$ given. The first order conditions for this problem are identical to equation (11), where, for all i , λ_t^i equals the Lagrange multiplier associated with the resource constraint (8).

Problem (13) cannot be solved merely by finding a vector of stochastic processes $\{c_t^i\}_{t=1}^{\infty}$ which satisfy the first order conditions (11). In a non-convex economy there generally are

suboptimal solutions to the first-order conditions. I proceed by directly solving the dynamic programming problem (13), using value function iteration over discretized state and control spaces. (Details are in the appendix.) The value function associated with problem (13) is homogeneous of degree $1-\zeta$, so the problem can be transformed into one involving only the stationary state variables $\{e_t/e_{t-1}, c_{t-1}^i/e_t\}_{i=1}^N$, and stationary control variables $\{c_t^i/e_t\}_{i=1}^N$. The exogenous driving process is the growth rate of the aggregate endowment, e_t/e_{t-1} , which is assumed to follow a stationary two-state Markov chain (as in Mehra and Prescott (1985)). The endogenous state vector $\{c_{t-1}^1/e_t, \dots, c_{t-1}^N/e_t\}$ is discretized as finely as needed to achieve an acceptable level of accuracy. While this method can approximate the true solution arbitrarily well if the state and control grids are made sufficiently fine, it suffers from the "curse of dimensionality" as N increases. For this reason, I follow the recent literature on asset pricing with heterogeneous agents (e.g., Lucas (1994), Heaton and Lucas (1994), Marcet and Singleton (1990), Telmer (1993)) by setting $N = 2$.

B. Verifying that the Pareto Optimum Can Be Decentralized

The Pareto optimal allocation need not be decentralizable as a competitive equilibrium. Consider Figure 3, which reproduces the two-period indifference map from Figure 2. If the Pareto optimal allocation for a particular agent was near point a , a risk-free interest rate equal to the inverse of the IMRS between x_t and x_{t+1} at that point would support that allocation. Line AA, tangent to the indifference curve at point a , gives the set of (x_t, x_{t+1}) pairs which are budget-feasible at this interest rate. Notice that line AA is below the indifference curve except at the point of tangency. In contrast to this well-behaved allocation, allocations near regions of nonconvexity cannot be supported as equilibria. An example is point f in Figure 4. If the interest rate were given by the inverse IMRS at point f , the agent could trade along line FF to an allocation which is preferred to f .

The task at hand, then, is to rule out the stochastic equivalent of Figure 4. In the Appendix I describe in detail how this is done. Basically, for each initial state s_0 to be considered,⁷ I compute the state-contingent claims prices, $\{\pi(s_t)\}$, according to equations (9) and (10), evaluated at the Pareto optimal state-contingent allocation, denoted $\{\hat{c}(s_t)\}$. By construction, $\{\hat{c}(s_t)\}$ satisfies the budget constraint implied by prices $\{\pi(s_t)\}$. I then perturb date 0 consumption away from $\hat{c}(s_0)$, and I search for an alternative state-contingent consumption plan $\{c(s_t), t > 1\}$ that satisfies the budget constraint implied by prices $\{\pi(s_t)\}$ and that yields a higher date t expected utility than $\{\hat{c}(s_t), t > 1\}$. If such a consumption plan exists, the Pareto optimum cannot be decentralized as a competitive equilibrium. If the search algorithm fails to find such a consumption plan for any candidate state s_0 , I interpret this Pareto optimal allocation as a competitive equilibrium.

C. Computing the equilibrium IMRS

Once it has been verified that $\{\hat{c}^i(s_t)\}$ is indeed an equilibrium consumption allocation, it remains to construct a $\{\lambda^i(s_t)\}_{t=1}^N$ process which satisfies integral equation (11). I do so using the parameterized expectations algorithm. (See Marcet and Marshall (1994).) A description of this algorithm can be found in the appendix. The IMRS process, $\{\beta\lambda_t^i/\lambda_{t-1}^i\}$ can then be simulated.

An implication of the model is that, in any given state, the IMRS is equated across the two agents. That is,

$$\beta\lambda_t^1/\lambda_{t-1}^1 = \beta\lambda_t^2/\lambda_{t-1}^2. \quad (14)$$

Nothing in the solution procedure constrains equation (14) to hold, so the correlation coefficient

⁷ In the simulations of section 4, below, I implement this algorithm for the four (discretized) states with the highest probability.

between $\beta\lambda_t^1/\lambda_{t-1}^1$ and $\beta\lambda_t^2/\lambda_{t-1}^2$ can be used as a test of the accuracy of the solution method.

4. RESULTS FROM SIMULATING THE MODEL

In this section I describe the implications of the model for asset prices, using the Hansen-Jagannathan (1991) methodology. The timing interval is one month, and the number of agent-types is set equal to 2. The subjective discount factor, β , equals .999 (implying an annual discount rate of approximately 1.2%). The coefficient of relative risk aversion (CRRA), ζ , is varied from 2 to 10. The parameter α (which determines the maximal adjustment cost, as a fraction of current per-capita endowment) is varied from .0002 to .001. The parameter γ determines the curvature of the adjustment cost function: the inflection points in Figure 1 occur when $\Delta\log(c_t) = \pm\sqrt{1/\gamma}$. With fixed costs of consumption adjustment, agents choose either a large change in consumption or no change at all. With the quasi-fixed adjustment cost (7), the choice is between a large change and a change substantially less than the distance to the inflection point. In the simulations I present, γ is varied from 200,000 to 600,000, implying that the region of small adjustment varies from $\Delta\log(c_t) = \pm.00224$ to $\Delta\log(c_t) = \pm.00126$.

The two-state Markov chain characterizing (e_t/e_{t-1}) is calibrated to the monthly growth rate of US consumption of nondurables and services from 1959:1 through 1986:12. Specifically, let g_t denote the gross growth rate of observed consumption, and let $\tilde{g}_t$ denote the two-state Markov approximation. I follow Mehra and Prescott (1985) in calibrating the two possible values of $\tilde{g}_t$ and the state transition probabilities so that $E(\tilde{g}_t) = E(g_t)$, $\text{var}(\tilde{g}_t) = \text{var}(g_t)$, $\text{cov}(\tilde{g}_t, \tilde{g}_{t-1}) = \text{cov}(g_t, g_{t-1})$, and the unconditional probabilities of the two states are equal. In the data, the sample mean of g_t is 1.002658, the sample standard deviation of g_t is .004429, and the sample first-order serial correlation

of g_t is -.2454. These statistics imply the following two-state discretization for $\tilde{g}$:

$$\begin{aligned}\tilde{g} \in \{ \tilde{g}_1, \tilde{g}_2 \} &\equiv \{ 0.99823, 1.00709 \} \\ \text{prob}(\tilde{g}_t = \tilde{g}_1 \mid \tilde{g}_{t-1} = \tilde{g}_1) &= \text{prob}(\tilde{g}_t = \tilde{g}_2 \mid \tilde{g}_{t-1} = \tilde{g}_2) = .3773.\end{aligned}\tag{15}$$

Table 2 displays the results from the simulations. The rows labelled "correlation" give the sample correlations between the IMRS of agent 1 and of agent 2. In Panels A and B, when γ equals 200,000 and 400,000 respectively, these correlations are very close to unity, indicating that the solution procedure is quite accurate for these parameterizations. It is more difficult to maintain this degree of accuracy when the adjustment cost function has a higher degree of curvature. In Panels C and D, when $\gamma = 500,000$ and $600,000$, the correlation is still above 0.9 for most parameter combinations. The correlation does drop as low as 0.565 for one set of parameters.

In Table 2, the rows labelled "decentralize" indicate whether agents could break the proposed equilibrium by moving away from the Pareto optimal allocation.⁸ For most parameter values, I found no evidence that this was possible. For three parameter combinations in Panel A, four combinations in Panel B, and one in Panel C, I found that a 10% reduction in consumption allowed each agent to trade to a consumption allocation with a slightly higher expected utility. (All other perturbations away from the Pareto optimal allocation unambiguously reduced expected utility.) This could be interpreted as evidence that the Pareto optimal allocations associated with these eight parameter combinations cannot be decentralized as competitive equilibria. Alternatively, these small irregularities may simply be artifacts of the discretization used in the solution algorithm.

Figures 5 through 8 plot the IMRS means and standard deviations from Table 2. In these

⁸ I implemented the algorithm of Appendix C for the four most probable states in each model. The results for all four were, in every case, virtually identical, so Table 2 reports results for only the most probable state.

figures, the V-shaped solid line is the Hansen-Jagannathan frontier in mean-standard deviation space, computed using monthly asset return data from 1959:1 to 1986:12 as in Hansen and Jagannathan (1991, Figure 5).⁹ The three contour lines moving from southwest to northeast plot the means and standard deviations of the IMRS as the fixed cost parameter α varies from .0002 (at the extreme southwest of each contour) to .001 (at the extreme northeast of each contour). The lowest contour corresponds to a CRRA of two. The remaining two contours correspond to a CRRA of 5 and 10 as one moves to the northwest.

The main effect of increasing risk aversion is to shift the contour lines to the left. As in Hansen and Jagannathan (1991, Figure 5), increased risk aversion reduces the mean of the IMRS far more than it increases the variance, and the impact of increased risk aversion within the range studied here is not great. However, the effect of increasing α from zero to .001 is dramatic. For example, in Figure 5, the mean and variance of the IMRS satisfy the Hansen-Jagannathan criterion with relative risk aversion of 5 when the fixed cost parameter α is set at about .0006. In Figure 6 (with $\gamma = 400,000$), the criterion is satisfied for this level of risk aversion when $\alpha = .0004$. In other words, the asset pricing anomalies associated with the Hansen-Jagannathan diagnostic can be resolved by introducing a non-convex adjustment cost whose maximum value is (4/10,000) of per-capita monthly consumption. To give an idea of the magnitude of these costs, in 1992 per-capita consumption expenditures equalled approximately \$16,000 per year, so the monthly consumption expenditure of a family of four averaged \$5,333. If the adjustment cost function set α equal to .0004, the average

⁹The bounds were computed using two return series and six scaled returns. The returns were the real one-month return to the value-weighted portfolio of New York Stock Exchange equities and the real return to one-month Treasury bills. These returns were also scaled by lagged returns and by lagged consumption. (See Hansen and Jagannathan (1991).) I would like to thank Ravi Jagannathan for providing me with the data used to construct the mean-standard deviation frontier.

family would face a maximal monthly adjustment cost of \$2.13. Costs of this magnitude are undetectable. I conclude that tiny changes in the costs of consumption transactions can have an impact on asset pricing far greater than substantial changes in risk aversion.

Figures 7 and 8 are analogous to Figures 5 and 6, except that γ is set to 500,000 and 600,000 respectively. At these extreme levels of curvature in the adjustment cost function, the effect of small adjustment costs on the variability of the IMRS is somewhat attenuated. The reason is that the curvature of the adjustment cost function affects the variability of the IMRS in two different ways. First, for a given consumption process, increasing γ increases the variability of A_1 and A_2 , the first derivatives of the adjustment cost function. This effect directly increases the variability of λ_{t+1}/λ_t through equation (11). However, increasing γ also shrinks the region between the inflection points of $A(c_t, c_{t-1})$, which affects the individual consumption choices. In particular, the optimal c_t for those who choose a small consumption change will be closer to c_{t-1} . This endogenous shift in the consumption process decreases the variability of λ_{t+1}/λ_t . These two effects work in offsetting directions. For moderately high values of γ (such as in Figures 5 and 6), the direct effect of increased γ on the variability of A_1 and A_2 dominates. For extremely high values of γ (as in Figures 6 and 7), the effect of increased γ on the optimal c_t/c_{t-1} process dominates.

5. INCREASING THE NUMBER OF AGENT TYPES

In section 4 I assumed that there were only two types of agents. Do the results of section 4 still hold when the number of agent types is increased? An equivalent way of posing this question is to ask whether the aggregate demand-for-consumption function in the two-agent economy is a good approximation for the aggregate demand function with multiple agents. Unfortunately, this question cannot be answered directly. As in Lucas (1994), Heaton and Lucas (1994), Marcet and

Singleton (1990), and Telmer (1993), it is not feasible to solve the full model with three or more types of agents. However, we can gain insight into this question by studying a simpler adjustment-cost model, and looking at how the shape of the aggregate demand function changes as the number of agent types increases.

In the simple model I wish to study, there are two time periods and N agents (indexed by i). Let c_t^i denote agent i 's consumption and e_t denote the aggregate endowment in period t , and let π denote the price in period 1 of a claim to one unit of consumption in period 2. The wealth of the i^{th} agent consists of W^i claims to period 2 consumption, where $\sum_i W^i = e_2$. Demand for consumption in period 1 is increasing in π : in order to induce agents to increase period 1 consumption, the relative price of period 2 consumption must rise. In the appendix, I describe a version of the model with a particularly simple individual demand function:¹⁰

$$c_1^i = \pi \frac{W^i}{2} \tag{16}$$

Equation (16) implies that aggregate demand is proportional to π .

Now, let us introduce a fixed cost of adjusting period 1 consumption. The fixed cost function has the form given in equation (6). Demand function (16) no longer holds. To induce an agent to change period 1 consumption, the asset price must move enough to make the utility gain from consumption adjustment greater than the utility loss from paying the adjustment cost. Formally, for

¹⁰ I have relegated the details of this simple model and the derivation of the demand functions to the appendix.

each individual there exists a region of inactivity $(\underline{\pi}^i, \bar{\pi}^i)$ with $\underline{\pi}^i < \bar{\pi}^i$, such that

$$c_1^i = \begin{cases} c_0^i & \text{if } \underline{\pi}^i < \pi < \bar{\pi}^i \\ \pi \left[\frac{W^i - \alpha e_2/N}{2} \right] & \text{otherwise} \end{cases} \quad (17)$$

I shall refer to $|\bar{\pi}^i - \underline{\pi}^i|$ as agent i 's "adjustment aversion". The aggregate demand function sums equations (17) over i .

In Figure 9, I display an example of this aggregate demand function for the cases $N = 2$, $N = 3$, and the limiting case¹¹ as $N \rightarrow \infty$, as well as the aggregate demand in the absence of adjustment costs, which is implied by equation (16). In this particular example, I impose a uniform distribution over $\{W^i\}_{i=1}^N$ by setting $W^i = 2e_2i/(N^2+N)$, and I set $c_0^i = W^i/2$. One can verify from demand functions (16) and (17) and the individual budget constraints (given in the appendix in equations (25) and (28)) that $\{e_1 = 50, e_2 = 100, \pi = 1, c_1^i = c_2^i = c_0^i\}$ constitutes an equilibrium, regardless of the value of α . That is, in this equilibrium there is never any incentive to adjust consumption. However, as shown in Figure 9, consumption adjustment costs induce a discontinuity in the aggregate demand function at $e_1 = 50$, so fluctuations in aggregate supply e_1 in the neighborhood of 50 induce more variation in π when $\alpha > 0$ than when $\alpha = 0$. The magnitude of this effect depends on N . First, the size of the discontinuity equals the minimal adjustment aversion among agents in the economy. If there are more agent types, this minimal adjustment aversion (and therefore the discontinuity in aggregate demand) will be smaller. However, simply adding a small number of agents with low adjustment aversion does not necessarily change the qualitative

¹¹ The limiting case was computed by setting $N = 2000$, although all values of N above 200 imply an aggregate demand curve indistinguishable from the limiting case displayed in the figure.

implications of the model. These new agents must also represent a significant fraction of aggregate demand. In other words, the two-agent case should approximate the limiting case if, in the limiting model, the agent types with very low adjustment aversion represent only a small fraction of aggregate demand.

Let us apply the intuition from this simple model to the model of sections 2 through 4. In that model, the only source of heterogeneity in adjustment aversion is the cross-sectional distribution of lagged consumption: those agents whose lagged consumption is very far from their optimal current consumption (in the absence of adjustment costs) display low adjustment aversion, since the utility gain from optimally adjusting consumption is relatively high. To argue that the two-agent approximation is fundamentally flawed, one would have to argue that a substantial fraction of agents each period experience a large gap between lagged consumption and (frictionless) optimal consumption. This could be the case if the aggregate supply were highly volatile. It is unlikely to be a major issue in the simulations of section 4, since the aggregate supply process in these simulations is calibrated to post-war US consumption of nondurables and services, a relatively smooth series.

6. CONCLUSIONS

I draw the following conclusions from the exercises in this paper:

(1) The implications of the CCAPM for the period-by-period joint behavior of consumption growth and asset returns are non-robust to the introduction of extremely small frictions in the consumption-transaction process.

(2) It is not difficult to reconcile the observed means and variances of consumption growth and asset returns in the context of an equilibrium model with rational, optimizing

agents. In particular, the observed low variability of consumption growth is compatible with the observed high variability of asset returns if agents face small non-convex costs of consumption adjustment.

(3) The CCAPM is unlikely to yield an empirically useful model of the short-run co-movements of aggregate consumption and asset returns. If the quantitative implications of the CCAPM are extremely sensitive to small frictions, then this model is empirically useful only if these frictions can be accurately measured. It is unlikely that accurate measures of adjustment costs this small can be obtained. Furthermore, these small frictions are sufficient to render invalid the representative-agent paradigm, so disaggregated consumption data would be required to implement the model empirically.

More generally, this paper leads one to be pessimistic that short run asset price movements can be explained by an equilibrium model of aggregate quantity variables: the nonrobustness documented in consumption-based models is likely to be found in other equilibrium models. Returns-based models of asset pricing do not suffer from this problem. These models treat the IMRS as an unobserved latent variable; fluctuations in the IMRS are proxied by the returns on portfolios that are maximally correlated with the factors determining the IMRS. The effects of adjustment costs are incorporated in these mimicking portfolios' returns, so adjustment costs need not be treated explicitly. The results of this paper argue that this returns-based approach is likely to provide more reliable models of short run movements in asset prices.

APPENDIX

A. Characterization of Equilibrium in the Simple Example of Section 1

Each agent is endowed with the consumption allocation in the conjectured equilibrium, plus the adjustment costs associated with that allocation. Let π_t denote the price, in units of period 1 consumption, of the consumption good in period t under the conjectured equilibrium. Agent i 's wealth, denoted W^i , is the present value of agent i 's endowment under prices $\{\pi_t\}$. For period t , $t = 1, 2, 3$, each agent can either set $c_t^i = c_{t-1}^i$ or let $c_t^i \neq c_{t-1}^i$, implying eight possible strategies.

To verify whether the conjectured consumption allocation can be supported by prices $\{\pi_t\}$, I compute the optimal consumption under each of the eight strategies: I derive the first order necessary conditions for optimality under each of the eight strategies, and impose the constraint that the present value of each optimal consumption plus required adjustment costs does not exceed W^i . I then compute the value of utility under each of these eight strategies to determine which strategy is dominant for each agent.

In the model of section 1, the conjectured equilibrium prices are $\pi_1 = 1$, $\pi_2 = \beta(c_1^2/c_0^2)$, and $\pi_3 = \beta^2$. The conjectured equilibrium (2) can be supported by these prices if the dominant strategy for agent 1 is $\{c_1^1 \neq c_0^1, c_1^1 = c_2^1, c_2^1 = c_3^1\}$, and the dominant strategy for agent 2 is $\{c_0^2 \neq c_1^2, c_1^2 \neq c_2^2, c_2^2 \neq c_3^2\}$. In Table 1, I display ranges of parameters for which this condition holds. To see whether the standard allocation (1) can be supported, I repeat this exercise, but setting $\{\pi_t\}$ equal to the equilibrium prices in this standard equilibrium: $\pi_2 = \beta[(e_1 - \alpha)/(e_2 - \alpha)]$, $\pi_3 = \beta^2[(e_1 - \alpha)/(e_3 - \alpha)]$. The standard equilibrium can be supported if, at these prices, the dominant strategy for both agents is $\{c_0 \neq c_1, c_1 \neq c_2, c_2 \neq c_3\}$.

B. Solving the Optimal Resource Allocation Problem using Discrete Dynamic Programming

The Bellman recursion corresponding to problem (13) is:

$$V(e_t, c_{t-1}^1, \dots, c_{t-1}^N; s_t) = \max_{\{c_t^i\}_{i=1}^N} \left[\sum_{i=1}^N \frac{[c_t^i]^{1-\zeta}}{1-\zeta} + \beta E(V(e_{t+1}, c_t^1, \dots, c_t^N) \mid s_t) \right] \quad (18)$$

subject to the aggregate resource constraint (8). The value function $V(\cdot; s_t)$ is homogeneous of degree $1-\zeta$, so (18) can be rewritten as a stationary problem. Let the stationary state process and control variables be denoted s_t^* and c_t^{*i} , as follows:

$$s_t^* \equiv (e_t/e_{t-1}, \{c_{t-1}^i/e_t\}_{i=1}^N) \in S^*,$$

$$c_t^{*i} \equiv \{c_t^i/e_t\}_{i=1}^N \in C^*,$$

where S^* and C^* denote the stationary state and control spaces. Let V^* be defined by:

$$V^*(s_t^*) \equiv V(1, c_{t-1}^1/e_t, \dots, c_t^N/e_t; s_t).$$

Problem (18) can now be written as follows:

$$V^*(s_t^*) = \max_{\{c_t^{*i}\}_{i=1}^N} \left[\sum_{i=1}^N \frac{[c_t^{*i}]^{1-\zeta}}{1-\zeta} + E \left(\beta \left[\frac{e_{t+1}}{e_t} \right]^{1-\zeta} V^*(s_t^*) \middle| \frac{e_t}{e_{t-1}} \right) \right] \quad (19)$$

subject to

$$\sum_{i=1}^N \left[c_t^i/e_t + A^*(c_t^i/e_t, c_{t-1}^i/e_t) \right] \leq 1 \quad (20)$$

where the stationary adjustment cost function A^* equals $A(c_t, c_{t-1})/e_t$, as follows:

$$A^* \left(\frac{c_t}{e_t}, \frac{c_{t-1}}{e_t} \right) \equiv \frac{\alpha}{N} \left[1 - \exp \left[\frac{-\gamma}{2} \left(\log \frac{c_t/e_t}{c_{t-1}/e_t} \right)^2 \right] \right] \quad (21)$$

Notice that the conditioning set in (18) is the entire history of realizations of e_t , while the conditioning set in (19) is simply e_t/e_{t-1} . This is a consequence of the assumption that e_t/e_{t-1} is a first-order Markov process.

I discretize S^* and C^* , and compute a discrete-state approximation to V^* by successive approximations. In section 4, $N = 2$, and the endowment growth, e_t/e_{t-1} , is allowed to take on two values. The endogenous state vector, $\{c_{t-1}^1/e_t, c_{t-1}^2/e_t\}$ is allowed to take on either 400 or 500 values. This algorithm also delivers the state transition function, T , and the optimal control function C , satisfying $s_{t+1}^* = T(s_t^*, e_{t+1}/e_t)$, $c_t^* = C(s_t^*)$.

The state prices $\pi_t(s_t)$ can be written as functions of the stationary state process $\{s_t^*\}$. From equation (10), one can write

$$\lambda_t(s_t) = e_t^{-\zeta} \lambda_t^*(s_t^*) \quad (22)$$

where

$$\lambda_t^*(s_t^*) \equiv \frac{1}{1+A_1^*(s_t^*)} \left[[c_t^*]^{-\zeta} + E \left(\sum_{j=1}^{\infty} \beta^j [c_{t+j}^*]^{-\zeta} \prod_{k=1}^j \frac{-A_2^*(s_{t+k}^*)}{1+A_1^*(s_{t+k}^*)} \frac{e_{t+k}}{e_{t+k-1}} \middle| \frac{e_t}{e_{t-1}} \right) \right] \quad (23)$$

(In equation (23), $A_j^*(s_t^*)$, $j = 1, 2$, denotes the derivative of $A^*(c_t/e_t, c_{t-1}/e_t)$, defined in (21), with respect to its j^{th} argument. Also, I have dropped the superscript "i" for notational convenience.) It follows from (9) and (22) that

$$\pi_t(s_t) = \beta^t \Pr(s_1^*, \dots, s_t^* | s_0^*) \left[\prod_{j=1}^t \frac{e_j}{e_{j-1}} \right]^{\zeta} \frac{\lambda_t^*(s_t^*)}{\lambda_0^*(s_0^*)} \quad (24)$$

so $\pi_t(s_t)$ is a function of the stationary state variables $\{s_0^*, s_1^*, \dots, s_t^*\}$.

C. Verification that the Pareto Optimum Can Be Decentralized

The Pareto optimum, computed using the discrete dynamic programming algorithm of part B of this appendix, is proposed as an equilibrium allocation. The proposed equilibrium is broken if a state-contingent allocation exists that satisfies the budget constraint, but yields a higher level of utility than Pareto optimum. I use the following algorithm to search for such an allocation:

(i) Choose a state s_0^* and an initial endowment e_0 . The dynamic programming algorithm discretizes the state space S^* into 800 or 1000 distinct states. Most of these are frequented rarely (if ever) in equilibrium. For each set of structural parameters used, I choose the four states most frequently visited. Given the homotheticity of this model, the initial endowment level e_0 can be set to any arbitrary number.

(ii) Use T and the known transition function for e_t/e_{t-1} to compute all possible state-paths $\{s_1^*, \dots, s_t^*\}_{t=1}^J$ starting from s_0^* , where $J \geq 1$. If fully specified, a state-contingent consumption plan would give consumption in all possible states for $t = 1, \dots, \infty$. It is not possible to compute an infinite number of state-contingent consumptions, so I truncate the horizon at a finite number J . This is equivalent to constraining agents in step (vi), below, to set $\{c_t^i\}_{t=J+1}^\infty = \{\hat{c}_t^i\}_{t=J+1}^\infty$. Since e_t/e_{t-1} is a two-state first-order Markov chain, there are $2^{J+1} - 2$ state-paths be computed.

(iii) Use (24) to compute the contingent claims prices $\pi_t(s_t)$ associated with each of the $(2^{J+1}-2)$ paths computed in step (ii). The infinite sum in (23) is truncated when the next term

affects the cumulative sum by less than one part in 100 million. This happens rapidly, since the cumulative product in (23) goes to zero very quickly. The expectation in (23) can be computed exactly, since the transition function T and the transition matrix of $\{e^t/e^{t-1}\}$ are known.

(iv) Using C and the arbitrary initial endowment level e_0 , compute the Pareto-optimal state-contingent consumption plan, which I denote $\{\hat{c}_t^i\}_{t=0}^J$. Consumption plan $\{\hat{c}_t^i\}_{t=0}^J$ is the proposed equilibrium state-contingent consumption allocation starting from initial state s_0^* .

(v) Compute the cost (in present value terms) of the proposed equilibrium consumption allocation, $\{\hat{c}_t^i\}_{t=0}^J$, at the claims prices computed in step (iii), above. Formally, if $S_t(s_0)$ is the set of states that are possible at date t when the initial state is s_0 , the cost of $\{\hat{c}_t^i\}_{t=0}^J$ is

$$\sum_{t=0}^J \sum_{s_t \in S_t(s_0)} \pi_t(s_t) [\hat{c}_t^i(s_t) + A(\hat{c}_t^i(s_t), \hat{c}_{t-1}^i(s_{t-1}))].$$

(vi) **Perturb c_0^i .** For each perturbation, maximize the expected utility of agent i , over budget-feasible values of $\{c_t^i\}_{t=1}^J$. The expected utility is $\sum_{t=0}^J \beta^t \sum_{s_t \in S_t} \Pr(s_t) \frac{c_t^i(s_t)^{1-\zeta}}{1-\zeta}$. The budget-

feasible state-contingent consumption plans $\{c_t^i\}_{t=0}^J$ are those plans which have the same present-value cost as $\{\hat{c}_t^i\}_{t=0}^J$, computed in step (v). (In Figure 3, if $\{\hat{c}_t^i\}_{t=0}^J$ were at point a , the set of budget-feasible consumption plans would lie on line AA .) In the simulations of section 4, I perturb c_0^i by setting the initial consumption of agent i equal to $\hat{c}_0^i[1+\eta]$ for 15 values of η ranging from $-.5$ to 1 .

(vii) **For each perturbation considered in step (vi), compare the maximized expected utility with the expected utility provided by the proposed equilibrium.** This is equivalent to comparing If this maximized expected utility exceeds that provided by the proposed equilibrium for any perturbation, the Pareto optimum cannot be decentralized.

D. Solving Equation (11) Using the Method of Parameterized Expectations

Equation (11) can be rewritten as :

$$\lambda_t = \frac{[c_t]^{-\zeta}}{1 + A_1^*(s_t^*) + \beta E \left[\frac{\lambda_{t+1}}{\lambda_t} A_2^*(s_{t+1}^*) \mid s_t^* \right]} \quad (25)$$

where, for notational convenience, I have suppressed the superscript " i ". The conditional expectation in (21) is an unknown function of s_t^* . This unknown function is approximated by a polynomial in the orthogonalized elements of s_t^* , which can be denoted $P(s_t^*; \Theta)$. Θ is a vector of polynomial

coefficients to be determined. Thus, (21) is replaced by

$$\lambda_t = \frac{[c_t]^{-\xi}}{1 + A_1^*(s_t^*) + \beta P(s_{t+1}^*; \Theta)} . \quad (26)$$

An approximate equilibrium will be given by a coefficient Θ^* such that, if the agent uses $P(s_t^*; \Theta^*)$ as her predictor of $(\lambda_{t+1}/\lambda_t)A_2^*(s_{t+1}^*)$ in choosing the c_t vector, then $P(s_t, \Theta^*)$ will in fact be the best predictor ex post in the least squares sense.

The parameter vector Θ^* is found as follows: A sample path for the equilibrium consumption vector, $\{c_t\}_{t=1}^T$, is computed by simulating the equilibrium state-contingent consumption rule. The starting values are the consumptions of agents 1 and 2 in the most common state. Equation (22) is then evaluated for $t = 1, \dots, T$ using an arbitrary value for Θ . The resulting $\{\lambda_t\}_{t=1}^T$ series is used to compute a series $\{(\lambda_{t+1}/\lambda_t)A_2^*(s_{t+1}^*)\}_{t=1}^T$ which, when regressed on the function $P(s_t^*; \Theta)$, yields the "ex post rational" value of Θ , which can be denoted $F(\Theta)$. Θ^* is the fixed point of the mapping $F(\cdot)$. In practice, this fixed point is found by successive approximations. I specify $P(\cdot, \Theta)$ as a third-order polynomial. (There was no appreciable change when the order was increased beyond three.)

A more complete description of the method of parameterized expectations can be found in den Haan and Marcet (1993), or Marcet and Marshall (1994). Marcet and Marshall (1994) gives conditions under which the approximate solution delivered by this algorithm can be made arbitrarily close to the true equilibrium if both the polynomial order and the sample size T are sufficiently large.

E. A Complete Description of the Simple Model Used in Section 5.

In this appendix I describe the simple two-period model of consumption demand used in section 5. This example is constructed to yield an aggregate demand function of the simple form given in equation (16) when there are no adjustment costs.

There are two periods, denoted 1 and 2, and there is a single non-storable consumption good. There are two classes of agents, denoted I and II, and there are N types of agents in class II. Agents of class I are endowed with the consumption good only in period 1, but derive utility from consumption only in period 2. Agents of class II are endowed with the consumption good in period 2, and derive utility from consuming in both periods 1 and 2. In particular, I assume that agents of class II maximize a utility function of the form

$$U(c_1^i, c_2^i) = \log(c_1^i) + \log(c_2^i), \quad i = 1, 2, \dots, N \quad (27)$$

where c_t^i denotes the consumption of type i in period t , $t=1,2$. Let e_t denote the aggregate endowment of the consumption good in period t , and let W^i denote the endowment of a class II agent of type i , to be received in period 2. Notice that $\sum_i W^i = e_2$. Assume that W^i is known in period 1 with certainty. The only asset in this economy is a claim to period 2 consumption. Let π denote the price of this asset, denominated in units of period 1 consumption.

For any $\pi > 0$, the optimal action of the class I agents is to exchange their entire endowment of period 1 consumption for claims to period 2 consumption at price π . Class II agents seek maximize objective function (28) subject to the budget constraint

$$c_1^i + \pi c_2^i \leq \pi W^i. \quad (28)$$

The first order conditions of this optimization problem are

$$c_1^i = \frac{\pi W^i}{2}; \quad c_2^i = \frac{W^i}{2}. \quad (29)$$

Only class II agents consume in period 1, so the aggregate demand for period 1 consumption is

$$: \quad \sum_{i=1}^N c_1^i = \frac{\pi e_2}{2}. \quad (30)$$

Now, let us assume that adjusting period 1 consumption involves a fixed cost of the form given in equation (7). For simplicity, I assume that only class II agents face the adjustment cost, and that the cost is assessed on the agent's claims to period 2 consumption. Budget constraint (28) must be replaced by:

$$c_1^i + \pi c_2^i + \pi A(c_1^i, c_0^i) \leq \pi W^i. \quad (31)$$

If the agent decides that it is optimal to adjust consumption, the optimal choice is governed by first-order conditions similar to (29):

$$c_1^i = \frac{\pi(W^i - \alpha e_2/N)}{2}; \quad c_2^i = \frac{W^i - \alpha e_2/N}{2}. \quad (32)$$

If the agent does not choose to adjust consumption, then consumption is simply given by:

$$c_1^i = c_0^i; \quad c_2^i = W^i - c_0^i/\pi. \quad (33)$$

Agent i will choose either (32) or (33), whichever yields the highest utility. For any given W^i and c_0^i , there is a range of π 's, which I denote $(\underline{\pi}^i, \bar{\pi}^i)$, for which (33) yields the higher utility. This is the region of inaction for agent i . Finally, the aggregate demand function is the sum of the individual demands.

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Table 1
Simulations of Simple Three-Period Model

Panel A: $e_t = .99779$; $e_h = 1.00221$

Equilibrium interest rates when $\alpha = 0$: $R_1^0 = .54\%$; $R_2^0 = -.34\%$

	Range of $\bar{c}$ for which standard equilibrium exists	Range of $\bar{c}$ for which adjustment cost equilibrium exists	R_1	R_2	Percent increase in interest rate spread
$\alpha = 0$	All values $\bar{c}$	Does not exist			
$\alpha = .0000006$	$\bar{c} \geq .19$	$0 < \bar{c} \leq .13$	.61%	-.41%	12.4%
$\alpha = .0000008$	$\bar{c} \geq .25$	$0 < \bar{c} \leq .17$	.63%	-.43%	15.0%
$\alpha = .000001$	$\bar{c} \geq .31$	$0 < \bar{c} \leq .19$	.65%	-.45%	23.5%
$\alpha = .000002$	Does not exist	$0 \leq \bar{c} \leq .29$	.73%	-.52%	40.9%
$\alpha = .000004$	Does not exist	$.19 \leq \bar{c} \leq .41$	.86%	-.65%	69.6%
$\alpha = .000006$	Does not exist	Does not exist			

Panel B: $e_t = .99$; $e_h = 1.01$

Equilibrium interest rates when $\alpha = 0$: $R_1^0 = 2.1\%$; $R_2^0 = -1.9\%$

	Range of $\bar{c}$ for which standard equilibrium exists	Range of $\bar{c}$ for which adjustment cost equilibrium exists	R_1	R_2	Percent increase in interest rate spread
$\alpha = 0$	All values $\bar{c}$	Does not exist			
$\alpha = .00001$	$\bar{c} \geq .15$	$0 < \bar{c} \leq .11$	2.4%	-2.1%	12.4%
$\alpha = .00002$	$\bar{c} \geq .30$	$0 < \bar{c} \leq .19$	2.6%	-2.3%	23.5%
$\alpha = .00004$	Does not exist	$0 < \bar{c} \leq .29$	3.0%	-2.7%	41.0%
$\alpha = .00006$	Does not exist	$0 \leq \bar{c} \leq .36$	3.3%	-3.0%	56.3%
$\alpha = .00008$	Does not exist	$.17 \leq \bar{c} \leq .41$	3.6%	-3.2%	69.9%
$\alpha = .0001$	Does not exist	$.34 \leq \bar{c} \leq .45$	3.8%	-3.5%	82.4%
$\alpha = .0002$	Does not exist	Does not exist			

Table 1 (continued)

Panel C: $e_a = .95$; $e_h = 1.05$ Equilibrium interest rates when $\alpha = 0$: $R_1^0 = 10.6\%$; $R_2^0 = -9.4\%$

	Range of $\bar{c}$ for which standard equilibrium exists	Range of $\bar{c}$ for which adjustment cost equilibrium exists	R_1	R_2	Percent increase in interest rate spread
$\alpha = 0$	All values $\bar{c}$	Does not exist			
$\alpha = .0002$	$\bar{c} \geq .12$	$0 < \bar{c} \leq .09$	11.7%	-10.3%	10.1%
$\alpha = .0006$	$\bar{c} \geq .36$	$0 < \bar{c} \leq .21$	13.6%	-11.8%	27.3%
$\alpha = .0008$	Does not exist	$0 < \bar{c} \leq .25$	14.4%	-12.4%	34.3%
$\alpha = .001$	Does not exist	$0 < \bar{c} \leq .29$	15.3%	-13.1%	42.2%
$\alpha = .002$	Does not exist	$.18 \leq \bar{c} \leq .41$	18.7%	-15.6%	71.5%
$\alpha = .004$	Does not exist	Does not exist			

Panel D: $e_a = .9$; $e_h = 1.1$ Equilibrium interest rates when $\alpha = 0$: $R_1^0 = 22.3\%$; $R_2^0 = -18.1\%$

	Range of $\bar{c}$ for which standard equilibrium exists	Range of $\bar{c}$ for which adjustment cost equilibrium exists	R_1	R_2	Percent increase in interest rate spread
$\alpha = 0$	All values $\bar{c}$	Does not exist			
$\alpha = .001$	$\bar{c} \geq .15$	$0 < \bar{c} \leq .11$	25.5%	-20.1%	12.9%
$\alpha = .002$	$\bar{c} \geq .30$	$0 < \bar{c} \leq .19$	28.4%	-21.9%	24.5%
$\alpha = .004$	Does not exist	$0 < \bar{c} \leq .29$	33.1%	-24.7%	43.1%
$\alpha = .006$	Does not exist	$0 < \bar{c} \leq .34$	36.2%	-26.5%	55.2%
$\alpha = .008$	Does not exist	$.19 \leq \bar{c} \leq .35$	37.7%	-26.9%	59.9%
$\alpha = .01$	Does not exist	Does not exist			

"Standard equilibrium" denotes a consumption allocation of the form (1). "Adjustment cost equilibrium" denotes an allocation of the form (2) in the text. " α " denotes the fixed cost of adjusting consumption. R_1^0 and R_2^0 denote the net interest rates in periods 1 and 2 respectively when $\alpha = 0$: $R_1^0 = e_a/(\beta(e_a - \alpha)) - 1$; $R_2^0 = e_h/(\beta e_h) - 1$. R_1 and R_2 denote the net interest rates supporting the adjustment cost equilibrium (2) at the maximal value of $\bar{c}$ for which this equilibrium exists: $R_1 = c_a^2/(\beta(c_a^2 - \alpha)) - 1$; $R_2 = c_h^2/(\beta c_h^2) - 1$. "Percent increase in interest rate spread" equals $[(R_1 - R_2) - (R_1^0 - R_2^0)] / (R_1^0 - R_2^0)$, expressed as a percentage. For all these computations, $\beta = .999$.

Table 2

Panel A: $\gamma = 200,000$ Inflection points: $\Delta \log(c_j) = \pm 0.00224$

Relative risk aversion		Values of α				
		.0002	.0004	.0006	.0008	.001
2	Mean IMRS	0.996	1.000	1.009	1.019	1.036
	Std. IMRS	0.068	0.120	0.174	0.228	0.293
	Correlation	0.993	0.991	0.997	0.995	0.990
	Decentralize	ok	ok	0.085%	0.284%	0.644%
5	Mean IMRS	0.989	0.996	1.005	1.016	1.031
	Std. IMRS	0.083	0.140	0.195	0.247	0.304
	Correlation	0.991	0.998	0.998	0.997	0.991
	Decentralize	ok	ok	ok	ok	ok
10	Mean IMRS	0.979	0.988	0.999	1.012	1.025
	Std. IMRS	0.107	0.170	0.230	0.279	0.325
	Correlation	0.998	0.994	0.993	0.993	0.996
	Decentralize	ok	ok	ok	ok	ok

Panel B: $\gamma = 400,000$ Inflection points: $\Delta \log(c_j) = \pm 0.00158$

Relative risk aversion		Values of α				
		.0002	.0004	.0006	.0008	.001
2	Mean IMRS	0.996	1.000	1.012	1.035	1.102
	Std. IMRS	0.066	0.115	0.195	0.295	0.507
	Correlation	0.985	0.989	0.996	0.997	0.998
	Decentralize	ok	ok	0.090%	0.353%	1.645%
5	Mean IMRS	0.989	0.995	1.006	1.031	1.095
	Std. IMRS	0.083	0.134	0.203	0.310	0.504
	Correlation	0.993	0.992	0.997	0.997	0.997
	Decentralize	ok	ok	ok	ok	1.047%
10	Mean IMRS	0.979	0.986	0.999	1.023	1.089
	Std. IMRS	0.112	0.161	0.229	0.325	0.520
	Correlation	0.999	0.994	0.997	0.997	0.997
	Decentralize	ok	ok	ok	ok	ok

Table 2 (continued)

Panel C: $\gamma = 500,000$ Inflection points: $\Delta \log(c_t) = \pm 0.00141$

Relative risk aversion		Values of α				
		.0002	.0004	.0006	.0008	.001
2	Mean IMRS	0.994	0.997	0.998	1.010	1.015
	Std. IMRS	0.037	0.080	0.092	0.185	0.210
	Correlation	0.946	0.921	0.960	0.876	0.905
	Decentralize	ok	ok	ok	0.110%	ok
5	Mean IMRS	0.988	0.990	0.993	1.002	1.012
	Std. IMRS	0.063	0.091	0.118	0.182	0.233
	Correlation	0.979	0.969	0.985	0.947	0.885
	Decentralize	ok	ok	ok	ok	ok
10	Mean IMRS	0.978	0.982	0.985	0.994	1.008
	Std. IMRS	0.100	0.134	0.155	0.207	0.269
	Correlation	0.988	0.984	0.983	0.985	0.977
	Decentralize	ok	ok	ok	ok	ok

Panel D: $\gamma = 600,000$ Inflection points: $\Delta \log(c_t) = \pm 0.00126$

Relative risk aversion		Values of α				
		.0002	.0004	.0006	.0008	.001
2	Mean IMRS	0.994	0.995	0.995	0.997	1.003
	Std. IMRS	0.029	0.049	0.056	0.085	0.132
	Correlation	0.926	0.939	0.828	0.892	0.565
	Decentralize	ok	ok	ok	ok	ok
5	Mean IMRS	0.987	0.989	0.990	0.993	0.999
	Std. IMRS	0.055	0.073	0.095	0.116	0.162
	Correlation	0.977	0.967	0.966	0.898	0.730
	Decentralize	ok	ok	ok	ok	ok
10	Mean IMRS	0.976	0.979	0.983	0.986	0.994
	Std. IMRS	0.083	0.112	0.140	0.159	0.207
	Correlation	0.966	0.981	0.968	0.958	0.930
	Decentralize	ok	ok	ok	ok	ok

α and γ are parameters in adjustment-cost function (8). "Correlation" denotes the correlation coefficient in the simulated data between $\beta\lambda_t^1/\lambda_{t-1}^1$ and $\beta\lambda_t^2/\lambda_{t-1}^2$. In the row labelled "Decentralize", "ok" indicates that the algorithm described in Section C of the Appendix found no budget-feasible allocation that breaks the proposed equilibrium. A numerical entry in this row indicates the percentage increase in expected utility associated with a 10% reduction in consumption in the most common state. (No other consumption perturbation resulted in an increased expected utility.)

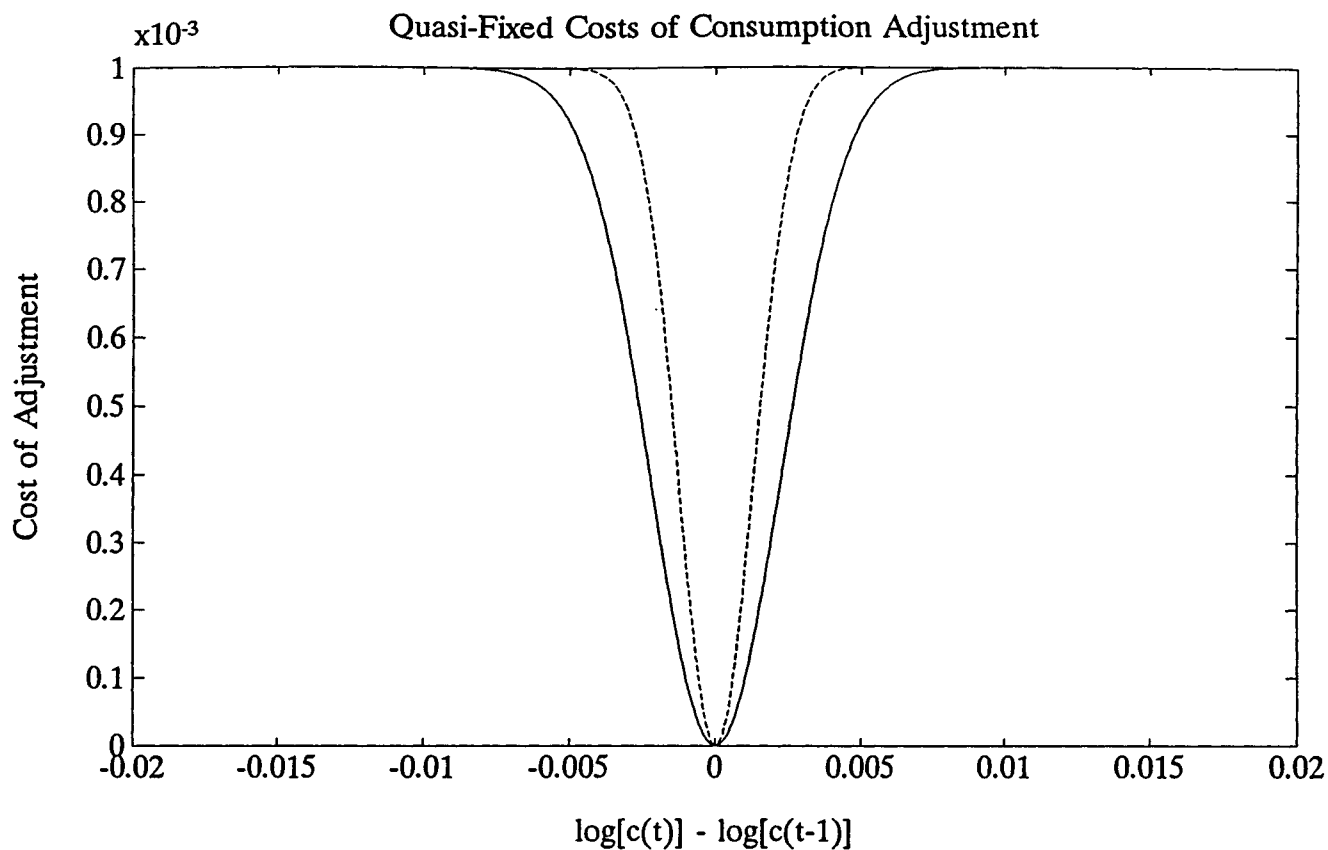


Figure 1: The quasi-fixed adjustment cost function given in equation (7) is plotted as a function of $\log(c_t) - \log(c_{t-1})$. The solid line corresponds to $\gamma = 200,000$. The dashed line corresponds to $\gamma = 600,000$. For both cases, $\alpha(e_t/N)$ is set to .001.

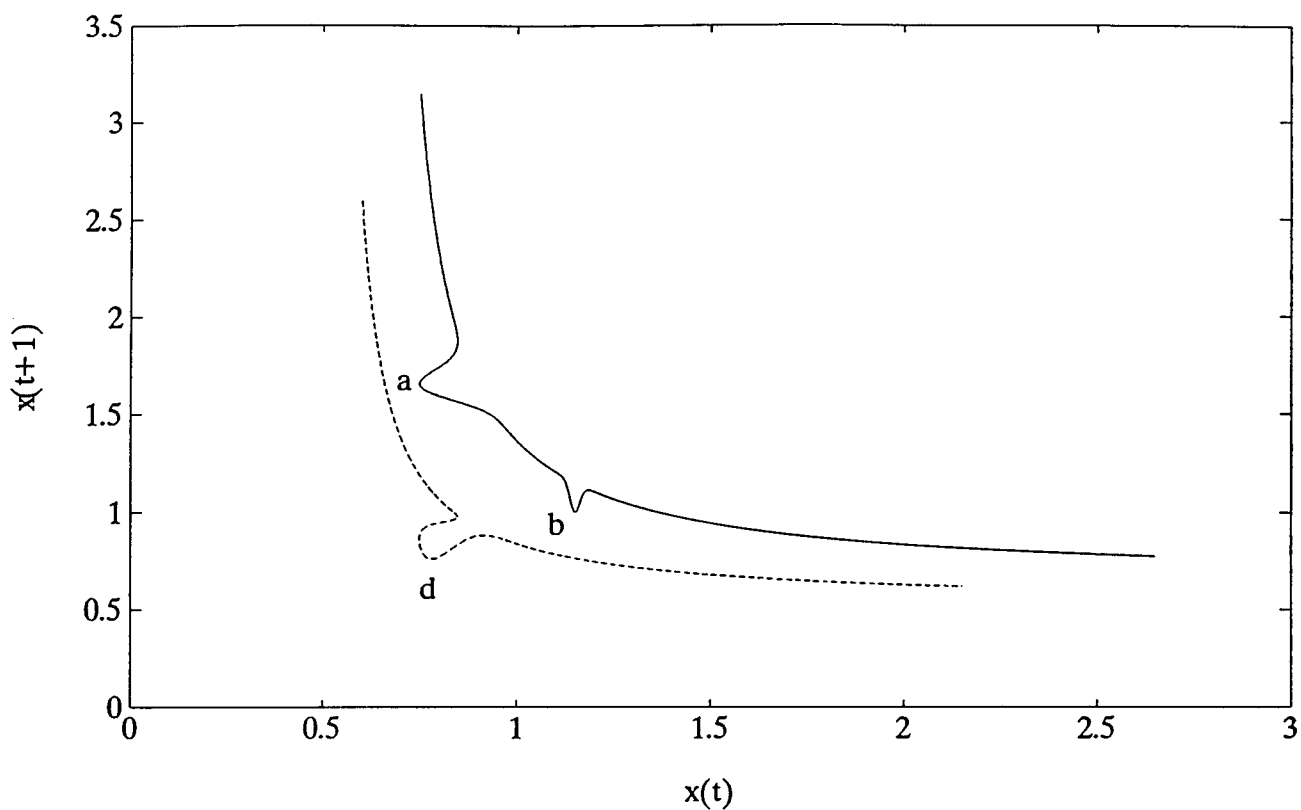


Figure 2: Two indifference curves are plotted in (x_t, x_{t+1}) space, where x_t denotes consumption inclusive of adjustment costs, as in equation (3). (The predetermined value of lagged consumption c_{t-1} is set to 0.75.) Point *a* corresponds to $c_t = c_{t-1}$. Point *b* corresponds to $c_t = c_{t+1}$. Point *d* corresponds to $c_{t-1} = c_t = c_{t+1}$.

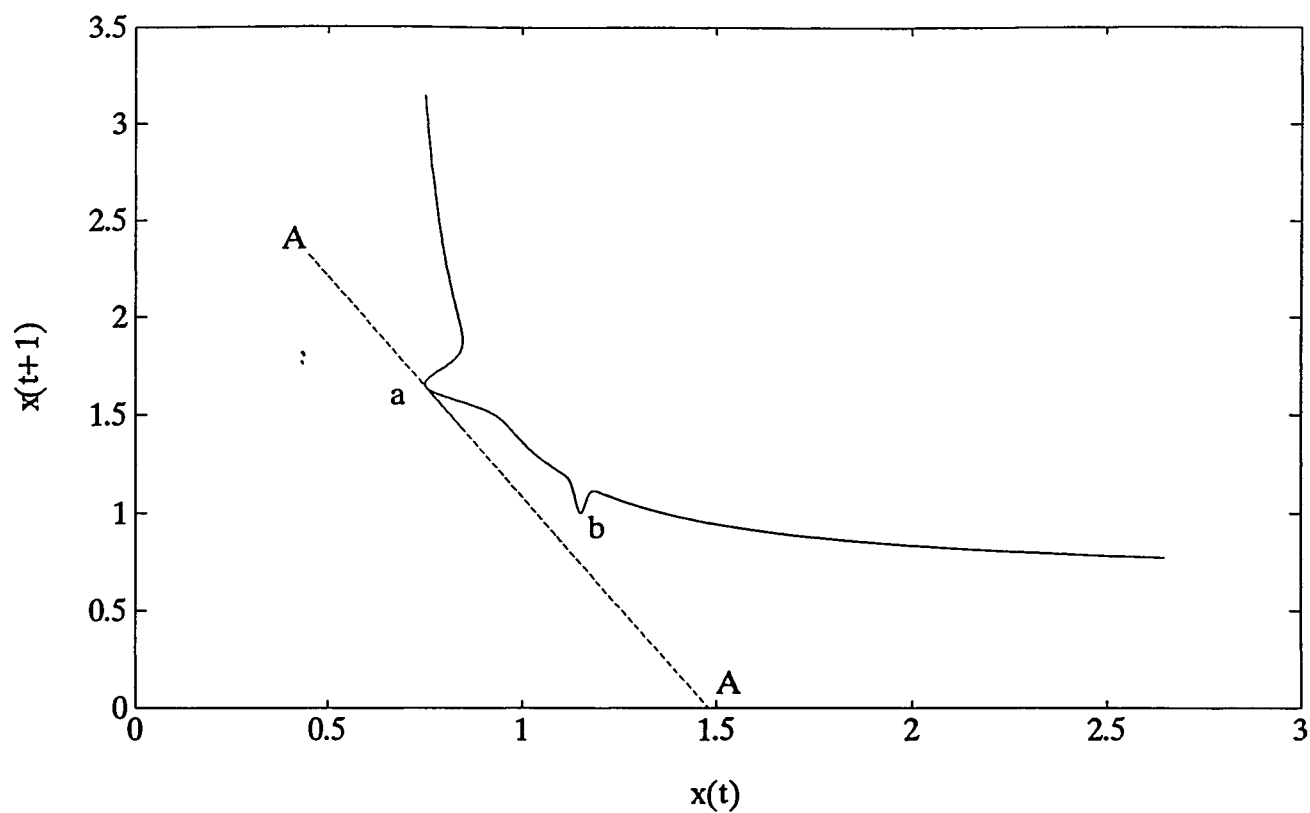


Figure 3: Allocation a can be supported as a competitive equilibrium by a risk-free interest rate given by the slope of line AA. Line AA gives the frontier of allocations which are budget-feasible at that interest rate.

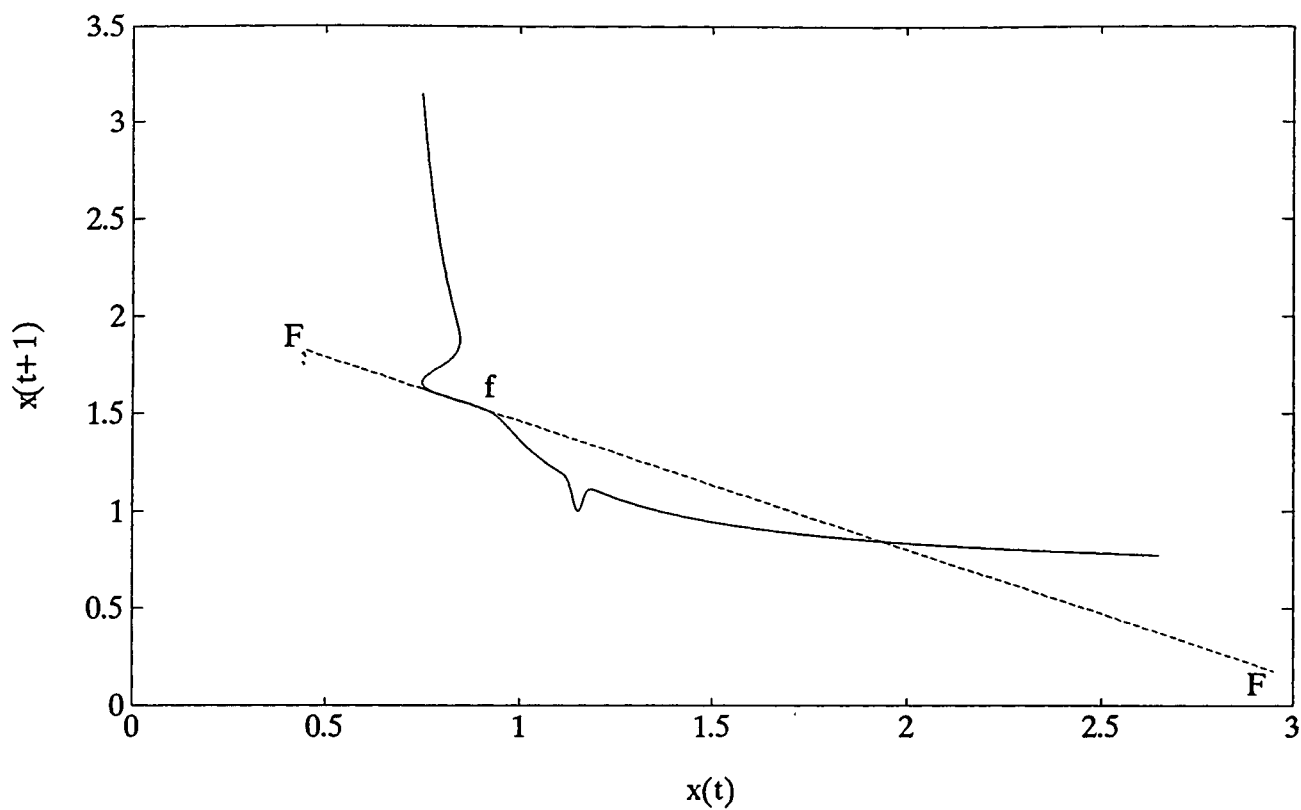


Figure 4: Allocation f cannot be supported as a competitive equilibrium. The line FF is tangent to the indifference curve through f . If the risk-free interest rate were given by the slope of line FF , allocations preferable to f would be feasible.

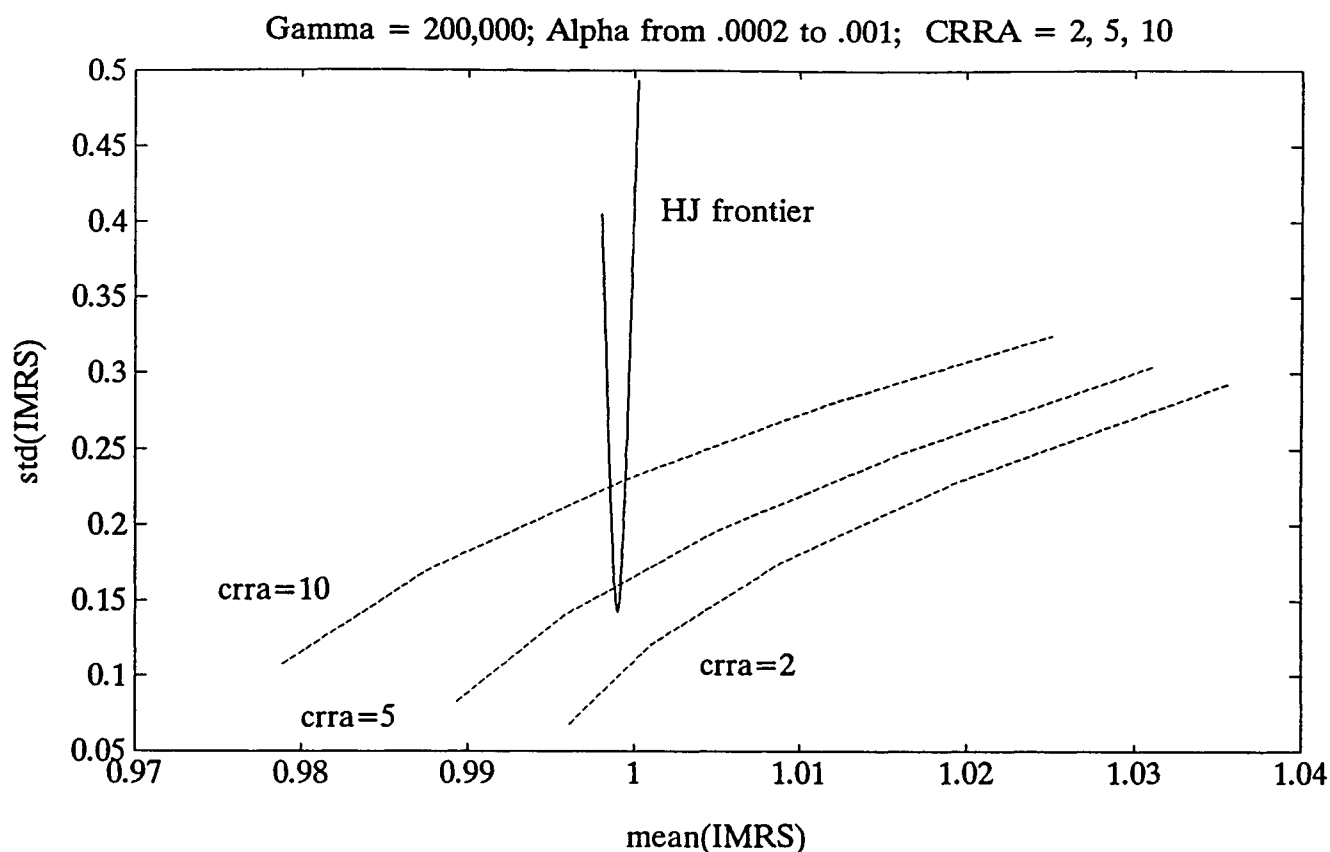


Figure 5: The means and standard deviations of the intertemporal marginal rate of substitution (IMRS) implied by the model are plotted for various parameterizations. In this figure, $\gamma = 200,000$ and $\beta = .999$. The lower-right contour line displays results when the coefficient of relative risk aversion (CRRA) equals 2; For the middle contour, CRRA = 5, and for the upper-left contour, CRRA = 10. Each contour plots mean-standard deviation pairs for the IMRS for various levels of the adjustment-cost parameter α : $\alpha = .0002$ corresponds to the lower-left end of each contour line, rising to $\alpha = .001$ at the upper-right end of each contour line. The V-shaped solid line in the middle of the figure is the Hansen-Jagannathan mean-standard deviation frontier, computed by those authors from monthly asset-return data from 1959:1 to 1986:12. (See Hansen and Jagannathan (1991, figure 5).) For an IMRS to satisfy the Hansen-Jagannathan criterion, $(\text{mean}(\text{IMRS}), \text{std}(\text{IMRS}))$ must lie above the Hansen-Jagannathan frontier.

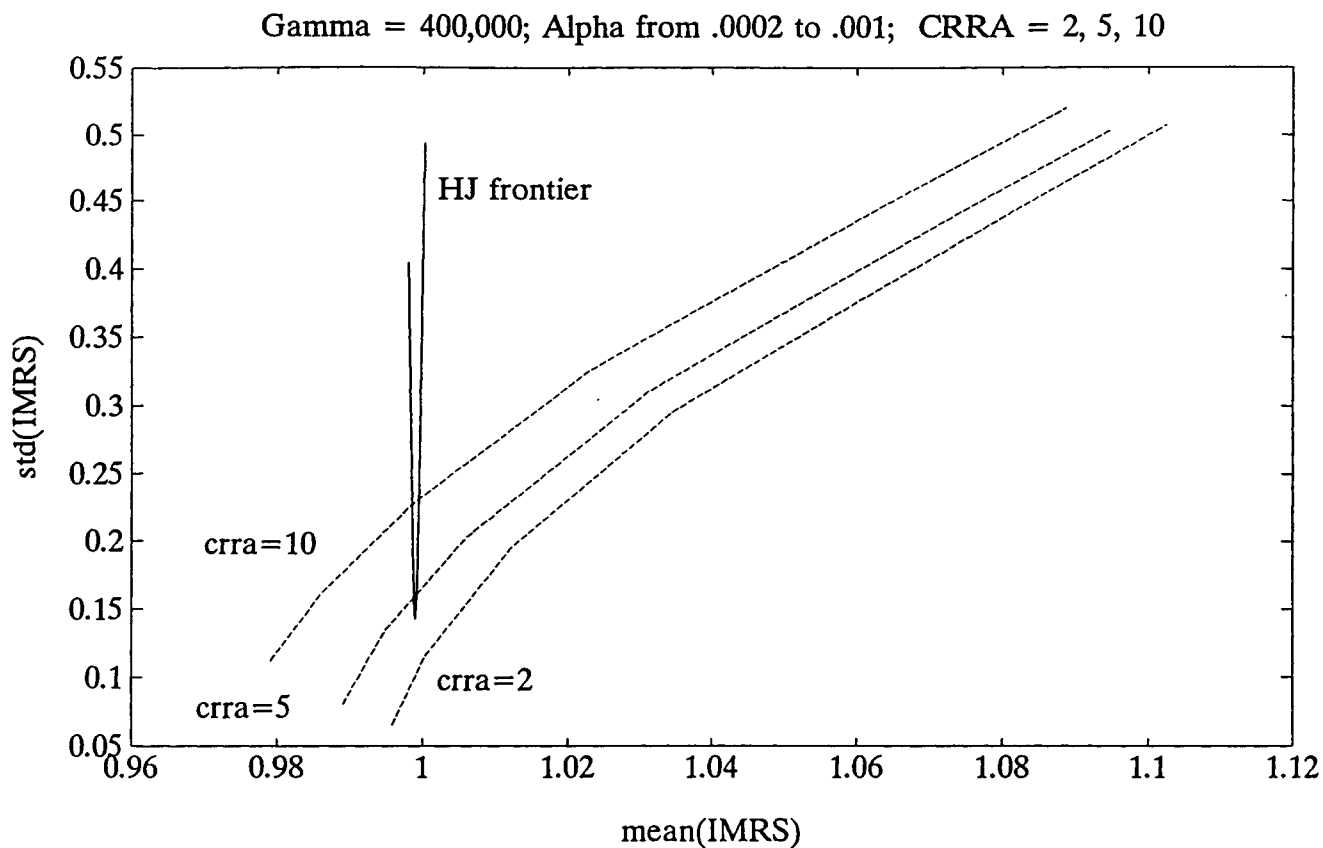


Figure 6: The means and standard deviations of the intertemporal marginal rate of substitution (IMRS) implied by the model are plotted, as in Figure 5. The parameterization is identical to Figure 5, except that $\gamma = 400,000$. The interpretation of the figure is the same as for Figure 5.

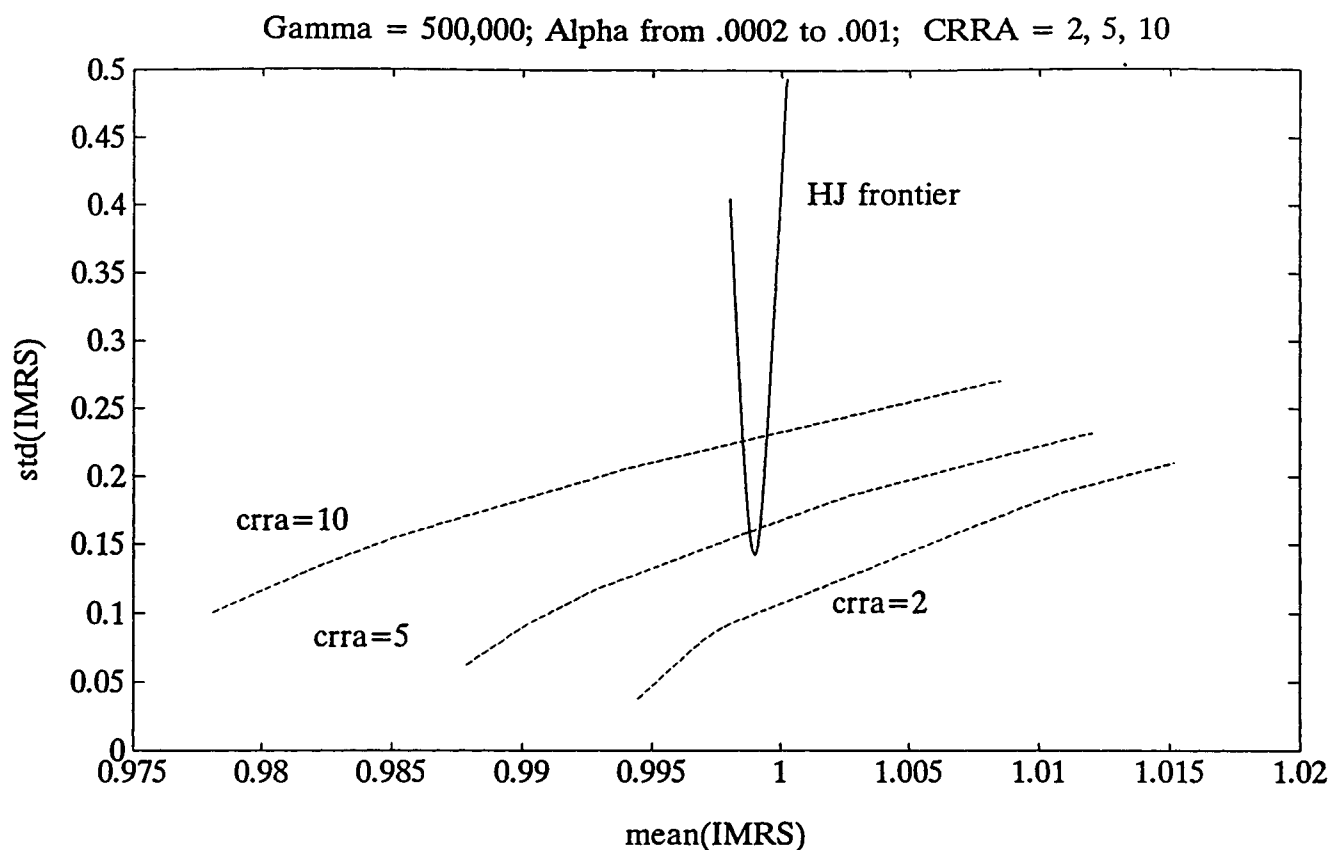


Figure 7: The means and standard deviations of the intertemporal marginal rate of substitution (IMRS) implied by the model are plotted, as in Figure 5. The parameterization is identical to Figure 5, except that $\gamma = 500,000$. The interpretation of the figure is the same as for Figure 5.

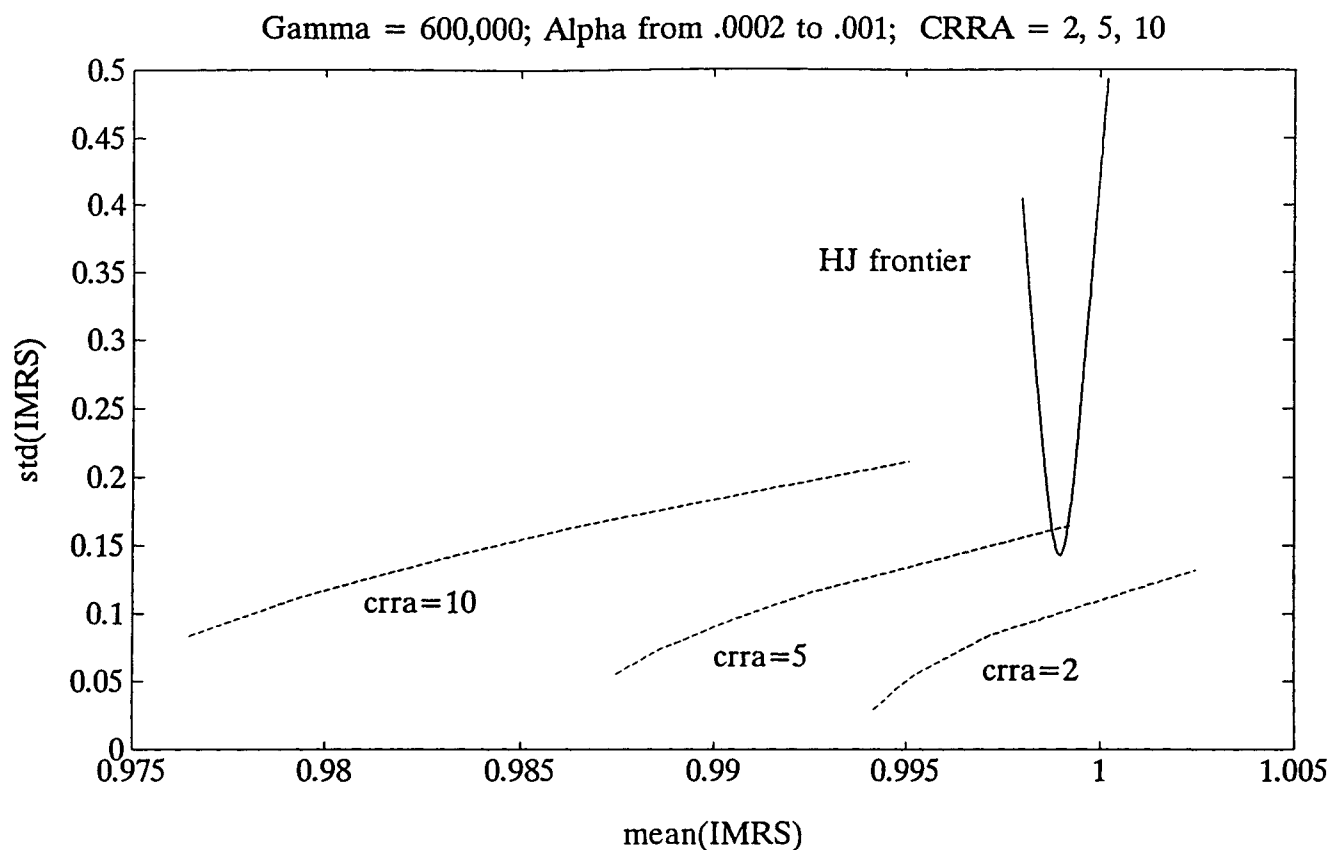


Figure 8: The means and standard deviations of the intertemporal marginal rate of substitution (IMRS) implied by the model are plotted, as in Figure 5. The parameterization is identical to Figure 5, except that $\gamma = 600,000$. The interpretation of the figure is the same as for Figure 5.

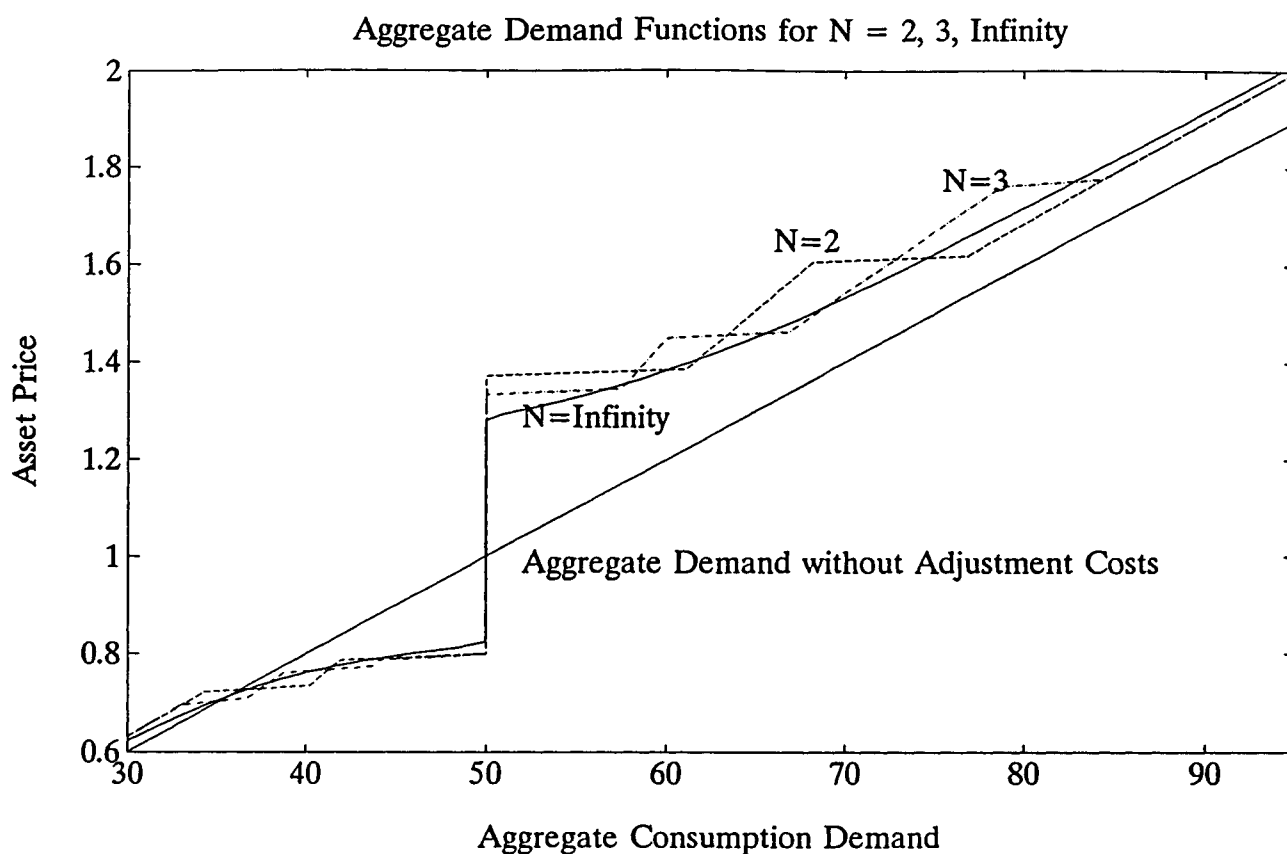


Figure 9: The aggregate demand function for the model of section 5 is plotted for four different cases. The vertical axis gives the relative price π of period 2 consumption in units of period 1 consumption; the horizontal axis gives aggregate demand for period 1

consumption, $\sum_{i=1}^N c_1^i$, as a function of π . The straight diagonal line gives the demand function

in the absence of consumption-adjustment costs ($\alpha = 0$). The remaining three lines give the demand function when $\alpha = .05$ for $N = 2, 3$, and the limiting case as $N \rightarrow \infty$.

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