## Cyclical Productivity in a Model of Labor Hoarding

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# Cyclical Productivity in a Model of Labor Hoarding 

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#### Abstract

This paper develops a dynamic model of labor hoarding to explain the cyclical behavior of total factor productivity. The model features convex costs of adjusting the labor force which induce firms to vary the intensity of labor utilization over the cycle. In particular, cyclical variations in labor 'effort' take place as a response to expected future changes in industry conditions. The restrictions imposed by the model are tested for several two-digit manufacturing industries. The estimated parameters are then used to simulate the model in order to evaluate whether it is able to generate a response of hours and productivity to aggregate innovations similar to that found in the data.


[^0]
## 1. Introduction

In many industries it is observed that output rises and falls at cyclical frequencies more than can be accounted for by changes in measured capital and labor inputs, assuming constant returns to scale. This procyclical productivity is a long-standing puzzle in the empirical study of business cycles. Macroeconomists have proposed a variety of explanations. ${ }^{1}$ Real business cycle theorists interpret the procyclical behavior of measured productivity as indicating actual shocks to technology. Others have argued that it reflects increasing returns, either internal or external to the firm.

This paper reexamines an old explanation, "labor hoarding" (Solow 1964). It is costly for firms to adjust labor hours. Hence they respond to short-run increases in demand by obtaining increased effort from their workers. Measured productivity then rises with shortrun increases in output.

To give structure to this hypothesis, I present a dynamic model of labor demand that incorporates costs of adjusting hours and a variable rate of utilization of labor. The aim is to explain the procyclical behavior of productivity, and also the observed dynamic correlations of aggregate variables with sectoral productivity:

The correlation between aggregate variables and sectoral productivity is of particular interest because it allows one to discriminate among alternative possible explanations of the corclation between sectoral productivity and sectoral output. If one assumes that true technological progress should be uncorrelated across sectors, an association between aggregate activity and productivity in an individual sector can be taken to indicate that true shifts in technology are not the cause. Furthermore, if, as found in Caballero and Lyons (1990, 1992), aggregate activity continues to predict sectoral productivity when sectoral input growth is also an explanatory variable, one can also exclude explanations, such as the hypothesis of internal increasing returns, that imply that measured productivity should vary whenever the scale of sectoral activity changes. Caballero and Lyons interpret their findings as evidence of external effects of activity in other sectors on the production possibilities in a given sector. Here I show that labor hoarding is also a possible explanation for their findings.

[^1]The idea is that aggregate variables may enter significantly in a sectoral productionfunction regression not because of any true production externality, but because they act as proxies for an omitted variable in the sectoral production function, sectoral labor utilization. In my model, sectoral output depends upon the effective labor input, defined as reported hours times an unmeasured utilization rate. I show that variations in aggregate variables may provide information about this utilization rate beyond what can be inferred from the measured sectoral inputs. The reason is simple. Firms tend to 'hoard' workers when production is temporarily low. This under-utilization of workers may take the form of variation in work effort, of the kind reported by Schor (1987), or variations in the number of workers assigned to non-production tasks, such as maintenance and training, as in Bean (1989). As a result, expectations about how future output and employment in a sector will compare to present levels are an important determinant of labor utilization. Aggregate variables are relevant to this decision problem because they help to forecast future conditions in the sector. In particular, if a higher growth rate of an aggregate variable forecasts lower growth of sectoral employment in the future, then this aggregate variable should affect measured sectoral productivity positively. For in this case, firms subject to costs of adjusting employment would prefer a higher present level of utilization of a smaller number of workers.

Not only can the model account for an association between aggregate variables and sectoral productivity, but it can account, at least roughly, for the dynamic response of sectoral productivity to an imovation in aggregate output or aggregate consumption. In Sbordone (1993) I studied the dynamic response of productivity in the two-digit manufacturing industry to fluctuations in the output of the whole manufacturing, and found that permanent increases in aggregate output result in permanent increases in sectoral output, but only a transitory increase in sectoral productivity (with the effect largely reversed after the first year). This discrepancy between the time patterns of the response of sectoral output and productivity again indicates that internal increasing returns alone cannot explain the productivity response. Similarly, the discrepancy between the time patterns of the response of aggregate output and sectoral productivity means that the hypothesis of a simple, contemporancous external effect of activity in other sectors (as in the model of Baxter and King (1991)) cannot account for it. Here I show that a labor hoarding model can account for this aspect of sectoral productivity variations as well.

The model describes the behavior of an individual industry. The empirical application
is to the two-digit sectors of U.S. manufacturing. The model is empirically evaluated along two dimensions. First, I estimate the first order condition of the firm's cost minimization problem for a number of sectors, test whether the restrictions they impose on the data hold, and whether significant costs of adjusting hours, relative to the cost of increasing labor utilization, are found. Secondly, I evaluate whether the model correctly predicts the measured response of sectoral total factor productivity to aggregate innovations. The model evaluation is based on a simulation exercise, in which some parameters are determined by direct estimation.

The structure of the paper is the following. In section 2 I describe the model and the strategies for its evaluation; in section 3 I present the Euler equation estimates and relate the results to previous literature on adjustment costs. Section 4 discusses the simulated response to an aggregate innovation, and section 5 concludes.

## 2. The model

Consider a sector i of the economy, in which a representative firm chooses inputs to use in production, while facing each period a stochastic shock to its technology. The labor input $L$ (effective hours) is composed of measured hours $I /$ and unobserved effort $e: L=e H$. The production function is

$$
\begin{equation*}
Q_{i t}=F\left(K_{i t}, \epsilon_{i t} H_{i t} \Theta_{i t}\right) \tag{2.1}
\end{equation*}
$$

where $c_{i t}$ is the rate of utilization of labor (effort) and $\Theta_{i t}$ is a labor-augmenting technological change. While $\Theta_{i t}$ need not be stationary, I assume that $\gamma_{\Theta t}^{i}=\Theta_{i t} / \Theta_{i, t-1}$ is a stationary rariable. $F$ is assumed homogeneous of degree $\eta$ in $K$ and $(\epsilon H Q)$, so I write

$$
\begin{equation*}
Q_{i t} / K_{i t}^{\eta}=f\left(e_{i t} H_{i t} \Theta_{i t} / K_{i t}^{\prime}\right) \tag{2.2}
\end{equation*}
$$

Firms face technological costs in changing the number of workers or of hours. I write total labor costs as (for ease of notation, I omit the subscript ifrom now on)

$$
C_{t}=W_{t} H_{t}\left(g\left(e_{t}\right)+\lambda\left(H_{t} / H_{t-1}\right)\right)
$$

where $W_{t}$ denotes the wage level and $g()$ indicates the proportional increase in the cost of hours that are more fully utilized. The function $\lambda()$ represents the increase in costs associated with rapid adjustment of the labor force. For simplicity, I consider the case of
adjustment costs related only to the change of the current level with respect to the previous one. One way to interpret this component of the total cost is that it represents the cost of training new workers. Therefore this cost goes up with the wages to be paid for the hours of training.

I assume that $g()$ is a positive and strictly convex function, and that $H \lambda()$ is a nonnegative, convex function of $H$. To get to the restricted cost function (i.e., the minimum real expenditure on the variable input, conditional on the quantity $Q$ to be produced, the capital $K$ and the quasi-fixed factor $H$ ) I solve eq. (2.2) for effort, getting

$$
\begin{equation*}
e_{t}=\frac{K_{t}}{H_{t} \Theta_{t}} \varphi\left(Q_{t} / K_{t}^{\eta}\right) \tag{2.3}
\end{equation*}
$$

where $\varphi()$ is the inverse of $f()$, and substitute this value into the cost function to obtain

$$
C\left(H_{t}, H_{t-1}, K_{t}^{\prime}, Q_{t}, \Theta_{t}, W_{t}\right)=W_{t} H_{t}\left[g\left(\frac{K_{t}}{H_{t} \Theta_{t}} \varphi\left(Q_{t} / K_{t}^{\eta}\right)\right)+\lambda\left(H_{t} / H_{t-1}\right)\right]
$$

The assumptions made above on the functions $g$ and $\lambda$ are sufficient for this cost function to be convex in $H_{t}$ and $H_{t-1}$.

A necessary condition for the firm's cost minimization problem is to choose the sequence $\left\{H_{t}\right\}$ to minimize the expected sum of discounted costs

$$
E_{t} \sum_{j=0}^{\infty} R^{j}\left\{C\left(H_{t+j}, H_{t+j-1}, K_{t+j}^{\prime}, Q_{t+j}, \Theta_{t+j}, W_{t+j}\right)\right\}
$$

where $R$ represents a real discount factor $(R=1 /(1+r)$, and $0<R<1)$ and $E_{t}$ denotes expectations conditional on knowledge of all the variables up to time $t$. The Euler equation for this problem is

$$
\begin{equation*}
C_{1}\left(H_{t}, H_{t-1}, K_{t}, Q_{t}, \Theta_{t}, W_{t}\right)+R E_{t}\left[C_{2}\left(H_{t+1}, H_{t}, K_{t+1}, Q_{t+1}, \Theta_{t+1}, W_{t+1}\right)\right]=0 \tag{2.4}
\end{equation*}
$$

for all $t$. To find a stationary solution, I transform the variables in order to eliminate the sources of non-stationarity (which are in the $H, K$ and $Q$ processes). I define the following variables

$$
x_{t}=Q_{t} / h_{t}^{\eta}, \gamma_{h t}=H_{t} / H_{t-1}, \gamma_{k t}=K_{t} / K_{t-1}, \gamma_{\Theta t}=\Theta_{t} / \Theta_{t-1}, \omega_{t}=W_{t} / \Theta_{t}, \kappa_{t}=K_{t} / H_{t} \Theta_{t}
$$

and rewrite the problem in terms of these variables. The firm chooses processes $\left\{\gamma_{h t}, \kappa_{t}\right\}$ to minimize the expected sum of discounted costs (per unit of initial capital)

$$
E_{t} \sum_{j=0}^{\infty}\left[R^{j} \Pi_{s=0}^{j} \gamma_{k(t+s)}\right] \tilde{C}\left(\gamma_{h t+j}, \kappa_{t+j}, x_{t+j}, \omega_{t+j}\right)
$$

where $\widetilde{C}\left(\gamma_{h}, \kappa, x, \omega\right) \equiv \frac{\omega}{\kappa}\left[g(\kappa \varphi(x))-\lambda\left(\gamma_{h}\right)\right]$, subject to the evolution equation

$$
\begin{equation*}
\kappa_{t}=\frac{1}{\gamma_{h t}}\left(\frac{\kappa_{t-1} \gamma_{k t}}{\gamma_{\Theta t}}\right) \tag{2.5}
\end{equation*}
$$

and taking as given stationary stochastic processes for the evolution of the variables $\left\{\gamma_{\Theta_{t}}, x_{t}, \gamma_{k t}, \omega_{t}\right\}$.
The optimal choice of $\left(\gamma_{h t}, \kappa_{t}\right)$ will then be a function of ( $\left.\kappa_{t-1}, \gamma_{\theta t}, x_{t}, \gamma_{k t}, \omega_{t}\right)$ and the conditional probability distribution at time $t$ for the future values of $\left(\gamma_{\theta t+j}, x_{t+j}, \gamma_{k t+j}, \omega_{t+j}\right)$ for all $j \geq 1$. Given the assumption of stationarity for the driving processes $\left\{\gamma_{\ominus t}, x_{t}, \gamma_{k t}, \omega_{t}\right\}$, it follows that $\left\{\gamma_{h t}, \kappa_{t}\right\}$ will be stationary stochastic processes as well. These will satisfy an Euler equation of the form

$$
\begin{gather*}
\omega_{t}\left[g\left(\kappa_{t} \varphi\left(x_{t}\right)\right)+\lambda\left(\gamma_{h t}\right)-\left[\kappa_{t} \varphi\left(x_{t}\right)\right] g^{\prime}\left(\kappa_{t} \varphi\left(x_{t}\right)\right)+\gamma_{h t} \lambda^{\prime}\left(\gamma_{h t}\right)\right]- \\
-R E_{t}\left[\omega_{t+1} \gamma_{\theta t+1}\left(\gamma_{h t+1}\right)^{2} \lambda^{\prime}\left(\gamma_{h t+1}\right)\right]=0 \tag{2.6}
\end{gather*}
$$

If we assume that all information at time $t$ about both current and expected future values of the variables $\left\{\gamma_{\theta t+j}, x_{t+j}, \gamma_{k t+j}, \omega_{t+j}\right\}$ can be summarized by a finite vector of state variables $z_{t}$ (which includes among its elements $\gamma_{\theta t}, x_{t}, \gamma_{k t}, \omega_{t}$ ), and furthermore that $\left\{z_{t}\right\}$ is a stationary Markor process, then we can write the optimal decision rules in the form

$$
\begin{aligned}
\gamma_{h t} & =\Gamma_{h}\left(\kappa_{i-1}, \tilde{t}_{t}\right) \\
\kappa_{t} & =\Psi\left(\kappa_{t-1}, \tilde{z}_{t}\right)
\end{aligned}
$$

These functions, together with the process $\left\{\tilde{z}_{t}\right\}$, describe the evolution of the complete set of stationary variables with which we are concerned. Note that stationary processes for $\left\{\kappa_{t}, x_{t}\right\}$ imply stationary fluctuations in effort, since

$$
\begin{equation*}
\epsilon_{t}=\kappa_{i} \varphi\left(x_{t}\right) \tag{2.7}
\end{equation*}
$$

I characterize these decision rules by taking a log-linear approximation for the functions $\Gamma_{h}$ and $\Psi$ in the neighborhood of a constant vector $\left(\kappa^{*}, z^{*}\right)$ such that the unconditional mean of $\log \tilde{z}_{t}$ is $\log {z^{*}}^{*}$, and $\kappa^{*}=\Psi\left(\kappa^{*}, z^{*}\right)$. If the fluctuations in the variables $\log z_{t}$ around their mean values are sufficiently small, the log-linear decision rules will provide an adequate approximation to the equilibrium dynamics.

To guarantee that a solution for $\kappa^{*}$ exists, I make the following assumptions (detailed proof is given in appendix Al):
(i) $g(c)$ goes to infinity as some finite upper bound for $e$ is approached, which implies that $c g^{\prime}(c) / g(c)$ is a monotonically increasing function, varying between 0 and $+\infty$ as $e$ varies between zero and its upper bound;
(ii) $\lambda\left(\gamma_{h}^{*}\right)$ and $\lambda^{\prime}\left(\gamma_{h}^{*}\right)$ are both equal to 0 - the interpretation of this assumption is that adjustment costs reach their minimum value of zero when growth of hours is at the steady state rate.

To characterize the decision rules, I first assume that the Markov process for $\log z_{t}$ is a linear autoregressive process of the form

$$
\begin{equation*}
\widehat{z}_{t+1}=V \widehat{z}_{t}+v_{t+1} \tag{2.8}
\end{equation*}
$$

where $\hat{z}_{t}$ denotes $\log \left(z_{t} / z^{*}\right)$, and $\left\{v_{t}\right\}$ is a vector white noise process. (I will from now on consistently use a hat to denote the percentage deviation of a variable from its steady state value.)

Then, I obtain a similar log-linear form for the evolution equations for $\left\{\gamma_{h t}, \kappa_{t}\right\}$ by a $\log$-linearization of the Euler equation (2.6) and the evolution equation for $\kappa$ (2.5) around the steady state solution. These are respectively

$$
\begin{equation*}
\alpha_{0} \hat{\omega}_{t}-\alpha_{1} \hat{x}_{t}+\alpha_{2} \hat{\gamma}_{h t}-\alpha_{4} \hat{\kappa}_{t}-\alpha_{3} E_{t} \hat{\gamma}_{h t+1}=0 \tag{2.9}
\end{equation*}
$$

and

$$
\begin{equation*}
\widehat{\kappa}_{t}=\widehat{\kappa}_{t-1}+\hat{\gamma}_{k t}-\hat{\gamma}_{h t}-\hat{\gamma}_{\theta t} \tag{2.10}
\end{equation*}
$$

where the coefficients a are defined by ${ }^{2}$

$$
\begin{align*}
& \alpha_{0}=g\left(\kappa^{*} \varphi\left(x^{*}\right)\right)-\left[\kappa^{*} \varphi\left(x^{*}\right)\right] g^{\prime}\left(\kappa^{*} \varphi\left(x^{*}\right)\right) \\
& \alpha_{1}=r^{*} \frac{\varphi^{\prime}\left(r^{*}\right)}{\gamma^{\left(x^{*}\right)}}\left[\kappa^{*} \varphi\left(x^{*}\right)\right]^{2} g^{\prime \prime}\left(\kappa^{*} \varphi\left(x^{*}\right)\right) \\
& \alpha_{2}=\left(\gamma_{h}^{*}\right)^{2} \lambda^{\prime \prime}\left(\hat{\gamma}_{h}^{*}\right)  \tag{2.11}\\
& \alpha_{3}=\left(\gamma_{h}^{*}\right)^{2}\left[R \gamma_{k}^{k}\right] \lambda^{\prime \prime}\left(\gamma_{h}^{*}\right) \\
& \alpha_{4}=\left[\kappa^{*} \varphi\left(x^{*}\right)\right]^{\prime \prime} g^{\prime \prime}\left(\kappa^{*} \varphi\left(x^{*}\right)\right)
\end{align*}
$$

Finally, I solve eqs.(2.9) and (2.10) for the evolution of ( $\hat{\gamma}_{h t}, \hat{n}_{t}$ ), taking as given the crolution of the vector $\hat{z}_{i}$ (eq.(2.8)) and initial values ( $\left.\hat{\kappa}_{-1}, \widehat{z}_{0}\right)^{\prime}$. Specifically, defining $\widehat{y}_{t+1}=$ $\left[\hat{\gamma}_{h t+1}, \hat{\kappa}_{t}, \hat{z}_{t+1}\right]^{\prime}$. I write the system of equations (2.9), (2.10) and (2.8) compactly as

$$
\begin{equation*}
A E_{t} \hat{y}_{t+1}=B \hat{y}_{t} \tag{2.12}
\end{equation*}
$$

Given $\left(\hat{\kappa}_{t-1}, \hat{z}_{t}\right)^{\prime}$, I solve for $\hat{\gamma}_{h t}$ and $\widehat{\kappa}_{t}$ as functions of $\left(\hat{\kappa}_{t-1}, \hat{z}_{t}\right)$ : these solutions are the lincar approximations to the functions $\Gamma_{h}$ and $\Psi$.(Details of these derivations are given in appendix A2.)

[^2]Before turning to the implications of these solutions, I want to suggest the following interpretation of the Euler equation (2.9). A log-linearization of the effort equation (2.7) gives

$$
\begin{equation*}
\hat{e}_{t}=\widehat{\kappa}_{t}+x^{*} \frac{\varphi^{\prime}\left(x^{*}\right)}{\varphi\left(x^{*}\right)} \widehat{x}_{t}=\widehat{\kappa}_{t}+\frac{\alpha_{1}}{\alpha_{4}} \widehat{x}_{t} \tag{2.13}
\end{equation*}
$$

Therefore, the Euler equation (2.9) can be written as

$$
\alpha_{0} \widehat{\omega}_{t}-\alpha_{4} \hat{e}_{t}+\alpha_{2} \hat{\gamma}_{h t}-\alpha_{3} E_{t} \hat{\gamma}_{h t+1}=0
$$

which, using the fact that $\alpha_{0}=0$ and $\alpha_{2}$ is approximately equal to $\alpha_{3}$, gives the behavior of effort in terms of the expected deviation of future hours growth from current growth

$$
\begin{equation*}
\hat{e}_{t}=-\frac{\alpha_{3}}{\alpha_{4}}\left(E_{t} \hat{\imath}_{t t+1}-\hat{\gamma}_{h t}\right) \tag{2.14}
\end{equation*}
$$

This interpretation shows that current deviations in effort are negatively related to expected future growth of hours. The intuition for that is that when hours are expected to grow, firms start to increase labor today (the marginal cost of increasing labor is lower today, taking into account the reduction of future adjustment costs), so decreasing effort today. The slow response of labor to cyclical variations. due to costs of adjustment, generates an immediate response of effort, which is the most variable factor.

I now consider the implications of the derived solution for the co-movements of the observable stationary variables

$$
v_{t}=\left[\gamma_{h t}, x_{t}, \hat{\gamma}_{k t}, \hat{\gamma}_{t}\right]
$$

Where by $\left\{\rho_{\mathrm{ht}}\right\}$ I denote a vector of stationary aggregate variables which belong to the vector $z_{t}$ and therefore provide information about the future evolution of the variables $\left\{x_{t+j \cdot} \cdot \gamma_{t+j}, \omega_{t+j}\right\}$. Note that neither $\hat{\jmath}$ nor $\kappa_{t}$, which are components of the vector $z$, are among the observables. ${ }^{3}$ This means that I cannot directly conduct tests on the functions $\Gamma_{h}$ and $\Psi$.

I can, however, test certain implications of the model about innovations in the aggregate variables, if I make further assumptions about the process $\left\{\hat{z}_{t}\right\}$. Specifically, I now assume

[^3](i) that the evolution of the aggregate variables is independent of sector-specific shocks (in particular, of sectoral technology shocks), so that I can write
$$
\gamma_{A t}=W(L) \gamma_{A t-1}+v_{A t}
$$
where $W(L)$ is a finite-order matrix lag polynomial, and the vector white noise $\boldsymbol{v}_{\boldsymbol{A} t}$ is independent of the sectoral shocks;
(ii) that sectoral technology $\Theta_{t}$ follows a random walk, so that I can write
$$
\gamma_{\Theta t}=v_{\Theta t}
$$
where $v_{\Theta t}$ is a white noise variable, independent of $v_{A t}$, and also independent of ( $\kappa_{t-1}, z_{t-1}$ ).
With these assumptions, I can try to assess whether this model is able to explain the procyclical behavior of productivity observed in the data. Specifically, I try to see if the model is able to replicate the dynamic response of sectoral variables to aggregate innovations, so that the mechanism of labor hoarding and rarying labor utilization works as an important propagation mechanism of aggregate perturbation. Before turning to this experiment, the next section explains some direct estimation of the model.

## 3. Euler Equation Estimation

Tables A1-B3 present estimates of the Euler equation (2.9), with a test of the moment conditions and the restrictions imposed on the parameters, for a number of two-digit sectors of the manufacturing industry. ${ }^{4}$ While these estimates provide a first test of the model, they are essentially aimed at recovering some parameters to be used in the simulation below. The particular parameter of interest is the coefficient on future hours $\alpha_{3}$, which I will call the adjustment cost parameter ${ }^{5}$.

In order to deal with an equation in observable variables, I take first differences of eq.(2.9) - after dividing through $\alpha_{1}$ - and obtain a relation among the rate of growth of output, capital. hours and wages

$$
\begin{equation*}
\Delta q_{t}=\pi_{0} \Delta h_{t}-\pi_{2} \Delta h_{t-1}-\pi_{3}\left(E_{t} \Delta h_{t+1}-E_{t-1} \Delta h_{t}\right)+\pi_{4} \Delta k_{t}+\pi_{5} \Delta w_{t}+\pi_{6} \varepsilon_{t} \tag{3.1}
\end{equation*}
$$

[^4]where $\pi_{0}=\left(\alpha_{2}+\alpha_{4}\right) / \alpha_{1}, \pi_{2}=\alpha_{2} / \alpha_{1}, \pi_{3}=\alpha_{3} / \alpha_{1}, \pi_{4}=\eta-\alpha_{4} / \alpha_{1}, \pi_{5}=\alpha_{0} / \alpha_{1}, \pi_{6}=$ $\left(\alpha_{4}-\alpha_{0}\right) / \alpha_{1}$, and $\varepsilon_{t}=\log \gamma_{\theta t}$. Using the fact that $\Delta h_{t}=E_{t-1} \Delta h_{t}+\nu_{h t}$, and setting $\pi_{1}=\pi_{0}+\pi_{3}$, I write this equation as
$$
\Delta q_{t}=\pi_{1} \Delta h_{t}-\pi_{2} \Delta h_{t-1}-\pi_{3} E_{t} \Delta h_{t+1}+\pi_{4} \Delta k_{t}+\pi_{5} \Delta w_{t}+u_{t}
$$
where the error term $u_{t}$ includes $\nu_{h t}$, the time $t-1$ forecasting error for $\Delta h_{t}$. I estimate this equation through an overidentified linear GMM procedure, instrumenting current and future variables on the right hand side with variables in the information set which are uncorrelated with the error term. The estimates are obtained under three restrictions imposed by the model on the parameters $\pi$. First, because the steady state level of effort is at the point of unitary elasticity of the function $g$ (see Appendix A1), the parameter $\alpha_{0}=0$, and therefore $\pi_{5}=0 .{ }^{6}$ Second, when the rate of growth of capital is sufficiently small, and the discount factor approximately equal to one, $\alpha_{2}$ is approximately equal to $\alpha_{3}$. Third, the ratio $\alpha_{4} / \alpha_{1}=$ $\eta \tilde{s}_{H}$, where $\tilde{s}_{H}$ is the share of labor in total costs $\left(\tilde{s}_{H}=\frac{w H}{w H+r K}\right)$. This last equality follows from the definition of $\alpha_{4} / \alpha_{1}$.

From (2.11) $\alpha_{4} / \alpha_{1}=\left[x^{*} \frac{\left.y^{\prime}\left(x^{*}\right)\right)^{-1}}{\varphi\left(x^{*}\right)}\right]^{\text {and }}$ from eq.(2.3) we have that $\varphi(x)=e H \Theta / K$. Therefore

$$
\begin{equation*}
x^{\star} \frac{\varphi^{\prime}\left(x^{\star}\right)}{\varphi\left(x^{\star}\right)}=\left[\frac{d \log \left(Q / K^{\prime \eta}\right)}{d \log \left(\epsilon H \Theta / K^{\prime}\right)}\right]^{-1}=\left[\frac{\left(c H \Theta / K^{\prime}\right) f^{\prime}}{Q / K^{\prime \eta}}\right]^{-1} \tag{3.2}
\end{equation*}
$$

where the sccond inequality follows from eq. (2.2). The value of the numerator can be derived from the condition that the cost-minimizing choice of (K, H) satisfy

$$
\begin{aligned}
& \partial Q / \partial K=\mu r \\
& \partial Q / \partial H=\mu w
\end{aligned}
$$

for some $\mu>0 .{ }^{7}$ From the definition of $\rho$, these two conditions are respectively

$$
\eta K^{\eta-1} f-K^{\eta} f^{\prime}\left(\frac{e H \Theta}{K^{2}}\right)=\mu r
$$

and

$$
K^{\prime \eta} f^{\prime}\left(\frac{e \Theta}{K}\right)=\mu w
$$

[^5]so that, multiplying the first by $K$ and the second by $H$, we get
$$
\eta K^{\eta} f-K^{\eta} f^{\prime}\left(\frac{e H \Theta}{K}\right)=\mu r K
$$
and
$$
K^{\eta} f^{\prime}\left(\frac{e H \Theta}{K}\right)=\mu w H
$$

These two expressions allow to derive the value of the elasticity of supply to effective labor in (3.2). The ratio $\frac{(e H \Theta / K) f^{\prime}}{\eta\left(Q / K^{\eta}\right)}=\frac{\mu w H}{\mu w H+\mu r K}=\tilde{s}_{H}$, so the ratio $\alpha_{4} / \alpha_{1}=\eta \tilde{s}_{H}$. This makes $\pi_{4}=$ $\eta-\alpha_{4} / \alpha_{1}=\eta-\eta \tilde{s}_{H}=\eta \tilde{s}_{K}$, where $\tilde{s}_{K}$ is the share of capital in total costs $\left(\tilde{s}_{K}=\frac{r K}{w H+r K}\right)$.

Moreover, the definition of the $\pi s$ implies that $\pi_{1}+\pi_{4}-\left(\pi_{2}+\pi_{3}\right)=\eta$. This constraint, together with the derived expression for $\pi_{4}$, can be written as

$$
\begin{equation*}
\left[\pi_{1}-\left(\pi_{2}+\pi_{3}\right)\right] \tilde{s}_{\kappa}=\pi_{4} \tilde{s}_{H} \tag{3.3}
\end{equation*}
$$

Alternatively, noting that

$$
\mu\left[\frac{w H+r K}{Q}\right]=\eta \frac{f}{Q / K^{\eta}}=\eta
$$

one can use the relation between shares in total revenue and shares in total costs

$$
\begin{aligned}
& \tilde{s}_{K}=\frac{r K}{w H+K K}=\frac{\mu}{\eta} \frac{r K}{Q}=\frac{\mu}{\eta} s_{K} \\
& \tilde{s}_{H}=\frac{u H}{w H+r K}=\frac{\mu}{\eta} \frac{u H}{Q}=\frac{\mu}{\eta} s_{H}
\end{aligned}
$$

to rewrite $\pi_{4}$ as $\pi_{4}=\mu s_{K}=\eta-\mu s_{H}$, and the constraint (3.3) as

$$
\left[\pi_{1}-\left(\pi_{2}+\pi_{3}\right)\right]\left(1-\frac{\mu}{\eta} s_{H}\right)=\pi_{4}\left(\frac{\mu}{\eta} s_{H}\right)
$$

In the estimation below I therefore impose and jointly test the restrictions that $\pi_{5}(=$ $\left.\alpha_{0} / \alpha_{1}\right)=0, \pi_{2}\left(=\alpha_{2} / \alpha_{1}\right)=\pi_{3}\left(=\alpha_{3} / \alpha_{1}\right)$, and that $\pi_{1}-\left(\pi_{2}+\pi_{3}\right)+\pi_{4}=\eta$. To impose the last constraint, I first consider the case of constant returns to scale, i.e. I set $\eta=1$, and allow two possible market structures. In table A1 I also set the mark up $\mu$ equal to 1 (perfect competition case), which means that the coefficient of capital is $\pi_{4}=1-s_{H}$. In table A2 and A3 I allow some degree of market power. In table A2 I report the estimation where the mark up $\mu$ takes its maximum value of $1 / s_{H}$, obtained by imposing $\pi_{4}=0$. In table A3 I impose $\pi_{4}$ to be equal to the average of the two boundary values, 0 and ( $1-s_{H}$ ). Secondly, to allow $\eta$ to be different from 1, I impose that $\left[\pi_{1}-\left(\pi_{2}+\pi_{3}\right)\right]\left(1-\frac{\mu}{\eta} s_{H}\right)=\pi_{4}\left(\frac{\mu}{\eta} s_{H}\right)$ and use the bounds on the possible levels of pure profits (the ratio $\mu / \eta$ ). For profits to be non
negative, $\mu / \eta \geq 1$. For $r$ to be non negative, $\mu / \eta$ is bounded from above by the inverse of the labor share. In the case of non constant returns to scale I therefore perform the estimation assuming in turn for $\mu / \eta$ the two boundary values, 1 and $1 / s_{H}$, and an intermediate value chosen as the simple average of the two boundary values. In table B1 I report results of the estimation under the hypothesis that there are no pure profits. Table B 2 gives the results of the estimation obtained assuming that $\mu / \eta$ have the maximum value of $1 / s_{H}$, while table B3 assumes for $\mu / \eta$ the mean value of the interval $\left[1,1 / s_{H}\right]$ for all sectors.

Each table reports the estimated parameter values, the statistic $J$ to test for the overidentifying restrictions, and the statistic D , constructed as the difference between the J statistic of the restricted model and the J statistic of the corresponding unrestricted, which tests the joint restrictions on the parameters. ${ }^{8}$

On the basis of these two statistics alone it would be hard to reject any specification of the model. However, in basically all the sectors the hypothesis of constant returns to scale cannot be rejected.

Looking first at the tables A1-A3, the adjustment cost parameter is estimated quite precisely in all the experiments, and it is significant in all the sectors in the perfect competition model, and in about half of the sample in the case of constant returns-maximum mark up model. Its size is decreasing in the amount of mark up allowed, but it remains significant in all sectors when an intermediate mark up is allowed. The mark up estimated in the model, however, at most ranges between 1.3 and 1.8 across the sectors.

In the 13 tables, where the hypothesis of $\eta=1$ is released, although the results do not suggest rejection of the restrictions, the implicit estimate of $\eta$ turns out to be either very close to 1 , or such imprecisely estimated that one cannot reject the hypothesis that it is equal to 1. By the way, in no case increasing returns are found. As in the constant returns case, the size of the adjustment cost parameter is inversely proportional to the allowed degree of departure from competition, being the highest in the case of zero pure profits. Overall, this parameter point estimate is higher than in the case in which returns to scale are constrained to be constant.

As a whole, these results suggest that, for most sectors, the theoretical framework of adjustment costs and variable rate of utilization for labor is a sensible mechanism to model

[^6]the dynamics of labor demand. Whether allowing or not for a moderate degree of market power, the model fits the data pretty well.

The estimate of a positive adjustment cost in the manufacturing industry is in line with other models in the literature (for a survey see Nickell (1989)). By using a convex cost of changing hours from one period to the next, this model is similar to those estimated, for example, by Pindyck and Rotemberg (1983), Sargent (1978), Shapiro (1986) and Sims (1974). However, here the introduction of a labor utilization parameter modifies the form of the production function. The projection of the labor utilization rate onto the space of variables known at the time of decision making introduces dynamic elements in the production function itself. ${ }^{9}$ Although I do not choose any specific functional form for the cost function, I am able to test whether there are significant costs of adjusting hours relative to the cost of increasing labor utilization.

Other authors have also studied models in which output does not depend solely upon the total number of man-hours employed. For example. Bils and Cho (1993) assume that increases in the number of employees and increases in the hours worked per employee affect output differently, while Hansen and Sargent (1988) assume that increases in straight time hours and increases in overtime hours have different effects. In these models, cyclical shifts in the composition of total man-hours are predicted, due to differential adjustment costs associated with the two margins, and the composition variable is essentially an omitted variable in the standard aggregate production function (relating output to man-hours), like the "utilization rate" in the present model. Thus the qualitative implications of models of those types are quite similar to those of the model presented here. The difference is primarily that those models suggest a particular measurable aspect of the labor input that could be used to climinate the omitted variable problem, while the model presented here (that is more general) does not commit itself to any particular source of utilization variations, and thus must use a more indirect estimation strategy to deal with the omitted variable problem.

A dynamic similar to the one considered here could be generated as well by a model where what changes in response to increases in demand is instead the utilization rate of capital. If direct observations on capital utilization are not available, this coefficient may be

[^7]solved for in terms of observables in the same way I do for labor in this paper. Although disentangling the two mechanisms may be worth investigating, if one makes the reasonable assumption of complementarity between bodies and machines (a more intensive utilization of machines requires more human effort) the results of that more general model would go in the same direction. ${ }^{10}$

Finally, note that the lag structure of eq.(2.9) depends on the specification of the adjustment cost. Were the costs associated to adjusting hours spread over more than one period, that equation would have a longer lag structure.

With this first assessment of the model, I now proceed to explore whether it may also be the case that aggregate variables act as a proxy for the unobserved rate of utilization of labor in the industry sectors. For this I conduct a simulation as described below, using in turn the sets of estimated values for the structural parameters $\alpha_{3} / \alpha_{1}$ and $\alpha_{4} / \alpha_{1}$ that I just described. Because of no evidence in favor of departure from constant returns, I will limit the simulations to the case of $\eta=1$.

## 4. Simulation of the Model

In this section I investigate whether the dynamic response of sectoral hours and productivity to aggregate innovations, generated by the model, traces the one generated by a simple vector autoregressive model fitted to the data. As aggregate variable (the one indicated by $\gamma_{A}$ in the model) I chose aggregate consumption of non durables and services, which has two characteristics. First, it does forecast sectoral activity - below I show evidence on this, for the sectors under study. Sccondly, it can be plausibly assumed to be uncorrelated with the unobserved technology of the sector, so to satisfy the assumption I made for the variable $\gamma_{A}$. Finally, because consumption is non stationary, I can interpret the response of the sectoral variables as responses to a permanent aggregate shock, and, as the discussion in the introduction showed, this model implies that there should be no long run response to persistent aggregate shocks.

The hypothesis of the model is that the variable rate of utilization of labor is the main factor driving the cyclical behavior of total factor productivity and it is able to explain the

[^8]transitory response of productivity in a sector to shocks to aggregate activity. ${ }^{11}$ Figs. 1-10 show the result of the analysis performed on the same two-digit sectors of the manufacturing industry analyzed in the previous section.

I first fit for each sector a VAR (1) to a four dimensional vector including aggregate consumption and capital, hours and output of the sector. In the estimation I impose the cointegrating restriction that the capital/output ratio in the sector is a stationary variable. This is consistent with the estimation results that show no significant evidence of departure from the constant returns to scale assumption. Aggregate consumption, capital and hours in the sectors are modeled as $\mathrm{I}(1)$ processes. ${ }^{12}$ The vector of variables $y_{i t}$ is therefore defined as $y_{i t}=\left[\Delta c_{t}, q_{i t}-k_{i t}, \Delta k_{i t}, \Delta h_{i t}\right]$, where lowercase letters denote natural logarithms. To support the choice of consumption as an aggregate variable, I perform a block exogeneity test to assess whether aggregate consumption forecasts sectoral activity. Results of this test are reported on table C. The hypothesis that aggregate consumption does not forecast sectoral variables is strongly rejected in all but two sectors (non electrical and electrical machinery). However, since the coefficient on lagged consumption in the equation for hours is significant also for these two sectors, I conduct the simulation exercise for these two sectors as well.

From the estimated VAR I compute the dynamic response of aggregate consumption, capital, hours and output/capital ratio in each sector to a unit innovation in aggregate consumption. Part a) of each figure (upper left corner) graphs the cumulative responses of these variables together with the implied response of total factor productivity (TFP) - which is the cumulate of productivity growth as measured by the Solow residuals.

The evidence from these figures is that a positive unit innovation in consumption raises its level, and has a significant and positive long lasting effect on both capital and hours. It also affects total factor productivity (although in few sectors I cannot exclude that the impact is not significantly different from zero). This effect, however, is very short-lived. Productivity tends to return to its steady state value of zero within at most two periods from the shock. The impact effect has the highest coefficient in all the sectors but primary

[^9]metals, where the response peaks instead at lag 1.
These are the 'facts' that I want the model to be able to explain. According to the model, the behavior of productivity reflects short-run variation in effort. As a consequence, variations in productivity should be larger in sectors with higher adjustment costs (like Chemicals, Paper, Primary Metals) and for which aggregate consumption has higher forecasting power. I turn therefore to discuss the model implications of a unit innovation in consumption.

To simulate the model response to a unit aggregate innovation, I do the following. For each sector $i$ I estimate the matrix $V$ in the system

$$
\hat{z}_{i t+1}=V \hat{z}_{i t}+v_{i t+1}
$$

in which the vector $\hat{z}_{i t}$ is defined as $\left(\hat{x}_{i t}, \hat{\gamma}_{k t}^{i}, \hat{\gamma}_{A t}\right)$, and compute the impact response of the variables in $\widehat{z}$ to a unit innovation in $\widehat{\gamma}_{A}$.

I then find the unstable root of the matrix $A^{-1} B$ and compute the associated left eigenvector e. To do this, I assign to the elements of $A^{-1} B$ (the structural coefficients $\alpha$ ) the values I estimated for the Euler equation (below I will be more specific about which parameters I pick for each exercise).

Finally, I solve for $\hat{\bar{i}}_{i t}^{i}$ as a function of $\left(\hat{\kappa}_{i t-1}, \hat{z}_{i t}\right)$. Specifically, this is

$$
\hat{\hat{i}}_{h t}^{i}=-c_{N_{2}} \hat{\kappa}_{i t-1}-c_{N_{3}} \hat{x}_{i t}-c_{\lambda_{4}} \hat{\gamma}_{i k t}^{i}-c_{N_{5}} \hat{\gamma}_{A t}
$$

where $x_{j}(j=2 \ldots 5)$ are the nomalized (by dividing through $c_{1}$ ) elements of the vector $\mathbf{e}^{13}$.
Substituting this value in eqs.(2.9) and (2.10) I compute $\hat{\gamma}_{h t+1}^{i}$ and all future values for 20 periods. I call ithtj $(\mathrm{j}=1 \ldots 20)$ the "simulated" response of hours in the model to a unit innoration in $\hat{\gamma} .4$. The response of capital and output is computed analogously. Finally, the simulated TFP is the cumulate of the Solow residuals constructed from the computed responses of output, capital and hours.

The craluation of the model is then based on a comparison of the impulse response derived from the VAR fitted to the data and the one computed from the model solution.

Given that I want to focus on the ability of the model to replicate the behavior of hours and productivity, I graph only the response of these variables. There are three graphs for each sector (part b), c) and d) of each figure]. In graphs b) and c) the estimated responses, respectively of hours and productivity, are plotted together with the responses simulated

[^10]by the model under the three different parametrizations discussed in the previous section. The lines labeled Mod1, Mod2 and Mod3 are the simulated responses under the assumption, respectively, of perfect competition, of maximum mark up, and of an intermediate mark up value. Using the relations between the $\alpha$ parameters of the matrix $A^{-1} B$ and the $\pi s$ of the estimated Euler equations $\left(\alpha_{3} / \alpha_{1}=\pi_{3}\right.$ and $\left.\alpha_{4} / \alpha_{1}=1-\pi_{4}\right)$, each simulation uses the estimated values of the adjustment cost and the elasticity of supply to effective labor reported in tables A1-A3. The thick lines in the graphs are two standard error bands around the estimated values. ${ }^{14}$ The performance of the model is assessed by its ability to generate impulse responses that are 'close' to the estimated response, in the sense of being within the standard error bands.

Part d) of the figures report the simulated response of 'effort' under the three model parametrizations. In these graphs I include in the legend the value of the mark-up $\mu$ and of the adjustment cost $\pi_{3}$ that characterize each parametrization.

The model performs someway better when some moderate mark up is allowed. The ability to trace the pattern of data varies moderately across the sectors. The response of total factor productivity is reproduced pretty closely for most of the sectors, while there is some overstatement of the short run response of hours in few sectors (chemicals and primary metals in particular). The response of 'effort' notably depends on the market structure. As eq.(2.3) says, $\hat{e}_{t}=\widehat{\kappa}_{t}+x^{*} \frac{\varphi^{\prime}(x \times)}{\varphi(x \times)} \widehat{x}_{t}=\hat{\kappa}_{t}+\left(1 / \mu s_{H}\right) \hat{x}_{t}$, so that the cyclical behavior of effort is decreasing in the degree of mark up. The last two graphs of each picture summarize the trade off implicit in the model's explanation of the productivity behavior. Total factor productivity is measured here as ${ }^{15}$ TFP $=\mu s_{H} \hat{e}_{t}-(\mu-1) s_{H} \hat{h}_{t}$. If $\mu=1$ its cyclical behavior is totally driven by cffort. With $\mu>1$ however, since the response of $\hat{\kappa}_{t}$ is typically "countercyclical", total factor productivity has a pronounced cyclical pattern even when the effort effect is small, and more so depending on the size of $\mu$.

Before concluding I want to point out how to translate these results - specifically the transitory response of measured sectoral TFP to aggregate innovations - into a traditional production-function regression framework.

Let $E_{t} \hat{y}_{t+1}=P\left[\widehat{\kappa}_{t-1}, \widehat{z}_{t}\right]^{\prime}$ be the solution for the whole vector $\hat{y}$, and denote by $P_{1 .}=\left\{p_{1 j}\right\}$

[^11]the first row of the matrix $P{ }^{16}$ Then the solution for $E_{t} \hat{\gamma}_{h t+1}$ can be written as
\[

$$
\begin{equation*}
E_{t} \hat{\gamma}_{h t+1}=P_{1 .}\left[\widehat{\kappa}_{t-1}, \hat{z}_{t}\right]^{\prime} \tag{4.1}
\end{equation*}
$$

\]

If we restrict the vector $z$ to include only the variables $\left(\gamma_{\theta}, x, \gamma_{k}, \gamma_{A}\right)$, we can explicitly write the solution (4.1) and recover the term in expected future hours that appears in eq.(3.1), which I rewrite here for convenience

$$
\Delta q_{t}=\pi_{0} \Delta h_{t}-\pi_{2} \Delta h_{t-1}-\pi_{3}\left(E_{t} \Delta h_{t+1}-E_{t-1} \Delta h_{t}\right)+\pi_{4} \Delta k_{t}+\pi_{5} \Delta w_{t}+\pi_{6} \varepsilon_{t}
$$

Solving for the term $\left(E_{t} \Delta h_{t+1}-E_{t-1} \Delta h_{t}\right)$ using (4.1) we obtain

$$
\begin{gather*}
E_{t} \Delta h_{t+1}-E_{t-1} \Delta h_{t}=\left(p_{14}-p_{13}\right) \Delta k_{t}+\left(p_{11}-p_{14}\right) \Delta k_{t-1}-p_{11} \Delta h_{t-1}+ \\
p_{13} \Delta q_{t}+p_{15}\left(\Delta a_{t}-\Delta a_{t-1}\right)+p_{12} \varepsilon_{t}-\left(p_{11}+p_{12}\right) \varepsilon_{t-1} \tag{4.2}
\end{gather*}
$$

Substituting this expression in eq.(3.1) we get a production function that depends on current and past values of the inputs and also on current and past values of the aggregate variable

$$
\begin{equation*}
\Delta q_{t}=\vartheta_{1} \Delta h_{t}+\vartheta_{2} \Delta h_{t-1}+\vartheta_{3} \Delta k_{t}+\vartheta_{4} \Delta k_{t-1}+\vartheta_{5}\left(\Delta a_{t}-\Delta a_{t-1}\right)+\vartheta_{6}(1+\rho L) \varepsilon_{t} \tag{4.3}
\end{equation*}
$$

where the parameters are respectively defined as

$$
\begin{align*}
& v_{1}=\left(\alpha_{2}+\alpha_{4}\right) /\left(\alpha_{1}+\alpha_{3} p_{13}\right) \\
& \vartheta_{2}=\left(\alpha_{3} p_{11}-\alpha_{2}\right) /\left(\alpha_{1}+\alpha_{3} p_{13}\right) \\
& \vartheta_{3}=\left[\alpha_{1} \eta-\alpha_{4}-\alpha_{3}\left(p_{14}-p_{13}\right)\right] /\left(\alpha_{1}+\alpha_{3} p_{13}\right) \\
& \vartheta_{4}=\left[\alpha_{3}\left(p_{14}-p_{11}\right)\right] /\left(\alpha_{1}+\alpha_{3} p_{13}\right)  \tag{4.4}\\
& v_{5}=-\alpha_{3} p_{15} /\left(\alpha_{1}+\alpha_{3} p_{13}\right) \\
& v_{6}=\left(\alpha_{4}-\alpha_{3} p_{12}\right) /\left(\alpha_{1}+\alpha_{3} p_{13}\right) \\
& \rho=\left[\alpha_{3}\left(p_{11}+p_{12}\right)\right] /\left(\alpha_{4}-\alpha_{3} p_{12}\right)
\end{align*}
$$

Eq. $(-1.2)$ shows that aggregate variables are correlated with the expected future labor growth because they are good forecasting variables. This channel brings them into the production-function regression (4.3) with a specific pattern of coefficients: coefficients on consecutive lags are the same but have opposite sign, so that the effect in each period vanishes in the next. As I show in Sbordone (1993), this is a testable implication in the regression analysis context. I argue there that my empirical results do not support the interpretation of aggregate variables as a measure of external increasing returns, as in models like Baxter-King (1991). because they have no long run effect on the level of sectoral productivity.

[^12]Moreover, expressions (4.4) show that, in the absence of adjustment costs, because the coefficients $\alpha_{2}$ and $\alpha_{3}$ are both equal to zero, the parameters $\vartheta_{2}, \vartheta_{4}$ and $\vartheta_{5}$ in (4.3) are all zero. Therefore there is no dynamics in the production function regression and no dependence on aggregate variables. The intuition for this result is that all the dynamic implications of the model come from the movement of effort (see eq.(2.14)). With no adjustment costs there is no cyclical variation of effort ( $e_{t}$ is always equal to its equilibrium value $e^{*}$ ) and, as a result, there is no movement in Solow residuals beyond pure variations in technology.

## 5. Conclusion

In this paper I construct a model of labor demand under the hypothesis that firms, because they face some costs in adjusting hours of work, respond to cyclical movements in activity by varying the rate of utilization of labor. The purpose of the model is to rationalize the observed procyclical movements in total factor productivity, giving at the same time an interpretation of the empirical results about the effect of aggregate activity on the cyclical behavior of sectoral productivity. This interpretation stresses the information content of aggregate variables for the decisions of individual sectors about labor inputs.

The model performs reasonably well. First, its implied restrictions pass the test with the data. Second, variations in labor utilization as a response to aggregate innovations generate short run dynamics in total factor productivity close to that displayed by actual data.

## A. Proofs

## A.1. Existence of a steady state vector ( $\gamma_{h}^{*}, \kappa^{*}$ ).

Consider the case of constant (steady state) values for $\gamma_{k}, \gamma_{\theta}, x$, and $\omega$, respectively indicated by $\gamma_{k}^{*}, \gamma_{\theta}^{*}, x^{*}$ and $\omega^{*}$. In this case there exists a steady state solution in which $\gamma_{h}$ and $\kappa$ are constant as well (given appropriate initial conditions). I denote these constant values respectively by $\gamma_{h}^{*}$ and $\kappa^{*}$. The existence of a constant solution for $\kappa$ may be verified from the Euler equation. In particular, the Euler equation gives

$$
\begin{gathered}
\omega^{*}\left[g\left(\kappa^{*} \varphi\left(x^{*}\right)\right)+\lambda\left(\gamma_{h}^{*}\right)-\left[\kappa^{*} \varphi\left(x^{*}\right)\right] g^{\prime}\left(\kappa^{*} \varphi\left(x^{*}\right)\right)+\gamma_{h}^{*} \lambda^{\prime}\left(\gamma_{h}^{*}\right)\right] \\
-R\left[\omega^{*} \gamma_{\theta}^{*}\left(\gamma_{h}^{*}\right)^{2} \lambda^{\prime}\left(\gamma_{h}^{*}\right)\right]=0
\end{gathered}
$$

or

$$
\left[\kappa^{*} \varphi\left(r^{*}\right)\right] g^{\prime}\left(\kappa^{*} \varphi\left(x^{*}\right)\right)=g\left(\kappa^{*} \varphi\left(x^{*}\right)\right)+\lambda\left(\gamma_{h}^{*}\right)+\gamma_{h}^{*} \lambda^{\prime}\left(\gamma_{h}^{*}\right)\left[1-R \gamma_{\theta}^{*} \gamma_{h}^{*}\right]
$$

Therefore $\stackrel{\kappa}{*}^{*}$ must satisfy

$$
\begin{equation*}
\left[\kappa^{*} \varphi\left(x^{*}\right)\right] \frac{g^{\prime}}{g}\left(\kappa^{*} \varphi\left(x^{*}\right)\right)=1+\frac{\lambda\left(\gamma_{h}^{*}\right)}{g\left(\kappa^{*} \varphi\left(x^{*}\right)\right)}+\frac{\gamma_{h}^{*} \lambda^{\prime}\left(\gamma_{h}^{*}\right)}{g\left(\kappa^{*} \varphi\left(x^{*}\right)\right)}\left[1-R \gamma_{k}^{*}\right] \tag{A.1}
\end{equation*}
$$

where I substituted $\gamma_{k}^{*}=\gamma_{h}^{*} \gamma_{\Theta}^{*}$ in the last square brackets. Assumption i) in the text guarantees that a unique positive solution for $\kappa^{*}$ exists as long as we assume values for $\gamma_{\Theta}^{*}$ and $\gamma_{k}^{*}$ that make the right hand side of this equation positive. By assumption ii) - $\lambda\left(\gamma_{h}^{*}\right)$ and $\lambda^{\prime}\left(\gamma_{h}^{*}\right)$ are both equal to 0 - the solution for $\kappa^{*}$ implies that the steady state level of effort is at the point of unitary elasticity of the function $g$, i.e., at the level $e^{*}$ that minimizes $g(e) / e$, as would be optimal in the absence of adjustment costs. The existence of a constant value for $\kappa, \kappa^{*}$, implies that $\gamma_{k}=\gamma_{\Theta} \gamma_{h}$. Hence $\gamma_{h}^{*}$ is indeed constant, and $\gamma_{h}^{*}=\gamma_{k}^{*} / \gamma_{\Theta}^{*}$.

## A.2. Log-linear approximation to the functions $\Gamma_{h}$ and $\Psi$.

Consider now the case of small, stationary fluctuations in the variables $\log z_{t}$ around their mean values $\log z^{*}$. The complete system of equations is

$$
A E_{t} \hat{y}_{t+1}=B \hat{y}_{t}
$$

where $\hat{y}_{t+1}=\left[\hat{\gamma}_{h t+1}, \hat{\kappa}_{t}, \hat{z}_{t+1}\right]^{\prime}$. Given $\left(\hat{\kappa}_{t-1}, \hat{z}_{t}\right)^{\prime}$, we want to solve for $\hat{\gamma}_{h t}$ as a function of $\left(\hat{\kappa}_{t-1}, \hat{z}_{t}\right)$ such that the vector lies in the subspace spanned by the right eigenvectors of $A^{-1} B$ with eigen values that are less than one in modulus.

A unique linear solution exists, because the matrix $A^{-1} B$ has exactly one eigenvalue with modulus greater than one. To see this, let the upper left $2 \times 2$ blocks of the matrices $A$ and $B$ be denoted $N$ and $M$ respectively, (say $A=\left[\begin{array}{cc}N & 0 \\ 0 & I\end{array}\right]$ and $B=\left[\begin{array}{cc}M & S \\ 0 & V\end{array}\right]$ ). Then the cigenvalues of $A^{-1} B$ are just the two eigenvalues of $N^{-1} M$ and the eigenvalues of $V$. The process $\hat{z}_{t}$ is by assumption stable, so the eigenvalues of $V$ are of modulus less than one. $N^{-1} M$ has one cigenvalue of modulus less than one and one with modulus greater than one, because of the following inequalities

$$
\begin{aligned}
\operatorname{Tr}\left(N^{-1} M\right)-\operatorname{Dct}\left(N^{-1} M\right) & =\frac{a_{3}+\alpha_{4}}{\alpha_{3}}>1 \\
\operatorname{Tr}\left(N^{-1} M\right)+\operatorname{Dct}\left(N^{-1} M\right) & =\frac{2 a_{2}+a_{3}+\alpha_{4}}{a_{3}}>-1
\end{aligned}
$$

(These inequalities follow from $a_{2}, a_{3}, a_{4}>0$.) Thus $A^{-1} B$ has exactly one eigenvalue with modulus greater than one. Denoting by $\mathrm{e}^{\prime}$ the associated left eigenvector, the solution can be found by setting

$$
\begin{equation*}
\mathbf{e}^{\prime}\left[\widehat{\gamma}_{h t}, \widehat{\kappa}_{t-1}, \widehat{z}_{t}\right]=0 \tag{A.2}
\end{equation*}
$$

Solving this cquation for $\hat{\gamma}_{h t}$ as a linear function of $\left(\hat{\kappa}_{t-1}, \hat{z}_{t}\right)$, I obtain a log-linear approximation to the function $\Gamma_{h}$. Substituting this solution into (2.10) I obtain $\hat{\kappa}_{t}$ as a linear function of ( $\widehat{\kappa}_{t-1}, \hat{z}_{t}$ ) as well, which provides a log-linear approximation to the function $\Psi$.

## B. Data Description and Sources

Industrial Production is from 'Industrial production', 1986 edition by the Board of Governors of the Federal Reserve System. The series were updated using the Federal Reserve Bulletin. Data on value added are from the NIPA as published in the Survey of Current Business (July issue); the capital stock is net constant dollar fixed private capital, as published in the Survey
of Current Business (August issue). Total hours of production workers are constructed as the product of employment and average weekly hours of production workers; real wage is average hourly earnings deflated by the industry gnp deflator. All labor data are from "Employment, Hours and Earnings, United States 1909-1984", vol. I, by U.S. Dept. of Labor, Bureau of Labor Statistics, March 1985; update to 1988 is from Supplement to Employment and Earnings, August 1989. The labor share is computed as the average of total labor compensation over nominal GNP, both from the NIPA, as published in the Survey of Current Business (July issue). Aggregate consumption is the sum of consumption of nondurables and services, in constant 82 dollars, from CITIBASE.

## References

[1] ABBOTT Thomas A. III, Zvi GRILICHES and Jerry A. HAUSMAN (1988) "Short Run Movements in Productivity: Market Power versus Capacity Utilization", unpublished manuscript.
[2] BARTELSMAN Eric J., Ricardo J. CABALLERO and Richard K. LYONS (1991) "Short and Long Run Externalities", NBER Working Paper No. 3810.
[3] BASU Susanto (1993) "Procyclical Productivity: Overhead Inputs or Cyclical Utilization?" manuscript, University of Michigan.
[4] BAXTER Marianne and Robert G. IING (1991) "Productive Externalities and Business Cycles", Institute for Empirical Macrocconomics Discussion Paper 53.
[5] BEAN Charles R. (1990) "Endogenous Growth and the Procyclical Behavior of Productivity", European Economic Review 34, pp. 355-363.
[6] BERNANKE Ben S. and James L. POWELL (1986) "The Cyclical Behavior of Industrial Labor Markets: A Comparison of the Prewar and Postwar Eras", in Robert J. GORDON, ed., The American Business Cycle: Continuity and Change, Univ. of (hicago Press.
[7] BERNANKE Ben S. and Martin L. PARKINSON (1991) "Procyclical Labor Productivity and Competing Theories of the Business Cycle: Some Evidence from Interwar U.S. Manufacturing Industries", Journal of Political Economy 99, pp. 439-469.
[8] BILS Mark and Jang Ok CHO (1993) "Cyclical Factor Utilization", Institute for Empirical Macroeconomics, Discussion Paper n. 79.
[9] BRAUN Anton R. and Charles L. EVANS (1991) "Seasonal Solow Residuals and Christmas: A Case for Labor Hoarding and Increasing Returns", Univ. of Virginia Discussion Paper No. 227.
[10] BURNSIDE Craig, Martin EICHENBAUM and Sergio REBELO (1990) "Labor Hoarding and the Business Cycle", Journal of Political Economy 101, pp.245-273.
[11] CABALLERO Ricardo J. and Richard K. LYONS (1990) "Internal versus External Economies in European Manufacturing", European Economic Review 34, pp. 805-830.
[12] CABALLERO Ricardo J. and Richard K. LYONS (1992) "External Effects in U.S. Procyclical Productivity", Journal of Monetary Economics 29, pp. 209-225.
[13] DOMOWITZ Ian, R. Glenn HUBBARD and Bruce C. PETERSEN (1988) "Market Structure and Cyclical Fluctuations in U.S. Manufacturing", Review of Economics and Statistics 70, pp. 55-66.
[14] EDEN Benjamin and Zvi GRILICHES (1993) "Productivity, Market Power and Capacity Utilization When Spot Markets are Complete", American Economic Review 60, Papers \& Proceedings, pp. 219-223.
[15] FAY Jon A. and James MEDOFF (1985) "Labor and Output over the Business Cycle: Some Direct Evidence", American Economic Review 75, pp. 638-655.
[16] GORDON Robert J.(1990) "Are Procyclical Productivity Fluctuations a Figment of Measurement Errors?", manuscript, Northwestern University.
[17] HALL Robert E. (1988) "The Relation between Price and Marginal Cost in U.S. Industry", Journal of Political Economy 96, pp. 921-947.
[18] HALL Robert E. (1991) "Invariance Properties of Solow's Productivity Residual", in P.A. DIAMOND, ed., Growth/ Productivity/ Unemployment, M.I.T. Press.
[19] HANSEN Gary D. and Thomas J. SARGENT (1988) "Straight Time and Overtime in Equilibrium", Journal of Monctary Economics 21, pp. 281-308.
[20] LUCAS Robert E. (1970) "Capacity, Overtime and Empirical Production Functions", Amcrican Economic Revicw 60, Papers \& Proccodings. pp. 23-27.
[21] MORRISON Catherine J. and Ernst R. BERNDT (1981) "Short Run Labor Productivity in a Dynamic Model", Journal of Econometrics 16, pp. 339-365.
[22] NELVEY Whitney K. and Kenneth D. WEST (1987) "Hypothesis Testing with Efficient Method of Moments Estimation" International Economic Review, vol.28, pp.777-787.
[23] NICKELL Stephen J. (1986) "Dynamic Models of Labor Demand", in O. Ashenfelter and R. Layard (eds.) Handbook of Labor Economics, North Holland, vol.1, ch. 9.
[24] PINDYCK Robert S. and Julio J. ROTEMBERG (1983) "Dynamic Factor Demands and the Effects of Energy Price Shocks", American Economic Review 73, pp. 1067-1079.
[25] ROTEMBERG Julio J. and Larry SUMMERS (1990) "Labor Hoarding, Inflexible Prices and Procyclical Productivity", Quarterly Journal of Economics 105, pp. 851-874.
[26] SBORDONE Argia M. (1993) "Is Procyclical Productivity Due to Externalities?" manuscript, Univ. of Chicago.
[27] SCHOR Juliet B. (1987) "Does Work Intensity Respond to Macroeconomic Variables? Evidence from British Manufacturing, 1970-1986", manuscript, Harvard University.
[28] SHAPIRO Matthew D. (1986) "The Dynamic Demand for Capital and Labor", Quarterly Journal of Economics 101, pp. 513-541.
[29] SHAPIRO Matthew D. (1987) "Measuring Market Power in U.S. Industry", NBER Working Paper No. 2212.
[30] SHAPIRO Matthew D. (1993) "Cyclical Productivity and the Workweek of Capital", American Economic Review 60; Papers \& Proceedings, pp. 229-233.
[31] SHEA John (1991) "Accident Rates, Labor Effort and the Business Cycle", manuscript, University of Wisconsin.
[32] SIMS Christopher A. (1974) "Output and Labor Input in Manufacturing", Brookings Papers on Economic Activity, pp. 695-735.
[33] SOLOW Robert M. (1964) Draft of the Presidential Address to the Econometric Society on the Short-Run Relation Between Employment and Output.

TABLE A1 ${ }^{1}$

## EULER EQUATION ESTIMATES

CONSTANT RETURNS TO SCALE - PERFECT COMPETITION MODEL
annual data 1950/1988

| $\Delta \mathrm{q}_{\mathrm{it}}=\pi_{1} \Delta \mathrm{~h}_{\mathrm{it}}-\pi_{2} \Delta \mathrm{~h}_{\mathrm{i}, \mathrm{l}-1}-\pi_{3} \mathrm{E}_{\mathrm{t}} \Delta \mathrm{h}_{\mathrm{i}, t+1}+\pi_{4} \Delta \mathrm{k}_{\mathrm{it}}+\pi_{5} \Delta \mathrm{w}_{\mathrm{it}}+\mathrm{u}_{\mathrm{it}}{ }^{2}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\pi_{1}$ | $\pi_{2}=\pi_{3}$ | $\pi_{4}$ | J [df. $=7{ }^{3}$ | D [df. $=4]^{4}$ |
| Paper | 1.300 (.105) | . 325 (.052) | . 350 (.006) | 10.20 (.177) | 2.069 (.723) |
| Print. \& Pub. | 1.471 (.225) | . 362 (.112) | . 253 (.004) | 9.198 (.239) | 11.21 (.024) |
| Chemicals | 1.701 (.258) | . 571 (.129) | . 442 (.006) | 8.018 (.331) | 3.684 (.450) |
| Rubber | 1.014 (.082) | . 143 (.041) | . 272 (.007) | 10.21 (.177) | 4.563 (.335) |
| Clay,Gl.,St. | 1.126 (.090) | . 215 (.045) | . 304 (.007) | 9.907 (.193) | 6.749 (.149) |
| Prim. Met. | 1.266 (.078) | . 276 (.039) | . 286 (.011) | 6.958 (.433) | 4.201 (.379) |
| Fabr. Met. | 1.082 (.062) | . 154 (.031) | . 226 (.005) | 7.943 (.337) | 6.855 (.144) |
| NonEl. Mach. | 1.034 (.074) | . 143 (.037) | . 253 (.007) | 2.732 (.909) | 2.782 (.595) |
| El. Mach. | 1.030 (.068) | . 127 (.034) | . 224 (.006) | 10.04 (.186) | 5.854 (.210) |
| Transp. | 1.009 (.076) | . 139 (.038) | . 270 (.018) | 11.71 (.110) | 2.215 (.696) |
| Misc. Man. | 1.054 (.137) | . 160 (.068) | . 266 (.007) | 6.709 (.460) | 4.591 (.332) |

[^13]| TABLE A $2^{1}$ <br> EULER EQUATION ESTIMATES <br> CONSTANT RETURNS TO SCALE - maximum value of mark up ( $\mu=1 / s_{h}$ ) annual data 1950/1988 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta \mathrm{g}_{\mathrm{it}}=\pi_{1} \Delta \mathrm{~h}_{\mathrm{it}}-\pi_{2} \Delta \mathrm{~h}_{\mathrm{i}, \mathrm{r}-1}-\pi_{3} \mathrm{E}_{\mathrm{i}} \Delta \mathrm{h}_{\mathrm{i},+1}+\pi_{4} \Delta \mathrm{k}_{\mathrm{it}}+\pi_{\mathrm{s}} \Delta \mathrm{w}_{\mathrm{it}}+\mathrm{u}_{\mathrm{it}}{ }^{2}$ |  |  |  |  |  |
|  | $\pi_{1}$ | $\pi_{2}=\pi_{3}$ | $\mu$ | $\mathrm{J}\left[\mathrm{df}=7{ }^{3}\right.$ | D [df=4] ${ }^{4}$ |
| Paper | 1.375 (.091) | . 187 (.045) | 1.54 | 7.115 (.417) | 0.931 (.920) |
| Print. \& Publish. | 1.528 (.216) | . 264 (.108) | 1.34 | 6.135 (.524) | 7.965 (.093) |
| Chemicals | 1.732 (.227) | . 366 (.113) | 1.79 | 7.996 (.333) | 2.235 (.692) |
| Rubber \& Plastic | 1.095 (.076) | . 047 (.038) | 1.37 | 6.766 (.454) | 3.286 (.511) |
| Clay,Glass,Stone | 1.168 (.072) | . 084 (.036) | 1.44 | 9.290 (.232) | 3.493 (.479) |
| Primary Metals | 1.325 (.101) | . 163 (.034) | 1.4 | 7.068 (.422) | 3.381 (.496) |
| Fabricated Metals | 1.162 (.060) | . 081 (.029) | 1.29 | 5.633 (.583) | 3.878 (.423) |
| Nonel. Machinery | 1.012 (.064) | . 006 (.032) | 1.34 | 7.011 (.428) | 7.263 (.123) |
| Electr. Machinery | 1.077 (.067) | . 038 (.033) | 1.29 | 12.71 (.079) | 7.021 (.135) |
| Transp. Equipm. | 1.116 (.070) | . 058 (.035) | 1.37 | 10.27 (.173) | 1.316 (.858) |
| Misc. Manufact. | 1.185 (.134) | . 092 (.067) | 1.36 | 5.339 (.619) | 3.308 (.508) |

1 GMM estimation. Instruments are two lags of hours in the sector, lagged capital, two lags of hours in the manufacturing industry (excluding the sector itself), lagged wages and two lags of aggregate consumption. The value of $\mu$ in column 4 is derived as the inverse of the labor share. This follows from imposing the restriction that $\pi_{4}=0$, where, as explained in the text, $\pi_{4}=1-\mu s_{4}$. Standard errors are reported in parentheses.
${ }^{2} \Delta x_{i n}$ indicates the log difference of variable $x$ in sector $i$ at time $t . q$ is industrial production, $h$ is hours of production workers, k is net capital stock in constant dollars, w is real wage.
${ }^{3}$ The statistic J is distributed as a chi-square with seven degrees of freedom. It tests for the overidentifying restrictions. Probability values are reported in parentheses.
4 The statistic $D$, constructed as the difference between the $J$ statistic of the restricted model and that of the unrestricted (not reported here), is distributed as a chi-square with four degrees of freedom. It tests for the parameter restrictions $\pi_{2}=\pi_{3}, \pi_{4}=0$, $\pi_{1}-\left(\pi_{2}+\pi_{3}\right)+\pi_{4}=1, \pi_{5}=0$. Probability values are reported in parentheses.

TABLE A3 ${ }^{1}$

## EULER EQUATION ESTIMATES

CONSTANT RETURNS TO SCALE - intermediate value of mark up
annual data 1950/1988

| $\Delta \mathrm{q}_{\mathrm{i}}=\pi_{1} \Delta \mathrm{~h}_{\mathrm{it}}-\pi_{2} \Delta \mathrm{~h}_{\mathrm{i},-1}-\pi_{3} \mathrm{E}_{\mathrm{t}} \Delta \mathrm{h}_{\mathrm{i}+1}+\pi_{4} \Delta \mathrm{k}_{\mathrm{it}}+\pi_{\mathrm{s}} \Delta \mathrm{w}_{\mathrm{it}}+\mathrm{u}_{\mathrm{it}}{ }^{2}$ |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\pi_{1}$ | $\pi_{2}=\pi_{3}$ | $\mu$ | $\mathrm{~J}[\mathrm{df}=7]^{3}$ | $\mathrm{D}[\mathrm{df}=4]{ }^{4}$ |  |
| Paper | $1.339(.097)$ | $.256(.048)$ | 1.27 | $8.879(.261)$ | $1.387(.846)$ |  |
| Print. \& Publish. | $1.494(.219)$ | $.310(.109)$ | 1.17 | $7.637(.366)$ | $7.787(.100)$ |  |
| Chemicals | $1.717(.241)$ | $.469(.121)$ | 1.39 | $7.842(.347)$ | $2.754(.600)$ |  |
| Rubber \& Plastic | $1.055(.079)$ | $.095(.039)$ | 1.19 | $8.023(.331)$ | $3.639(.457)$ |  |
| Clay,Glass,Stonc | $1.147(.080)$ | $.149(.040)$ | 1.22 | $9.213(.238)$ | $4.230(.376)$ |  |
| Primary Metals | $1.298(.073)$ | $.221(.036)$ | 1.2 | $6.710(.460)$ | $3.384(.496)$ |  |
| Fabricated Metals | $1.122(.060)$ | $.117(.030)$ | 1.14 | $6.241(.512)$ | $4.538(.338)$ |  |
| Nonel. Machinery | $1.019(.068)$ | $.072(.034)$ | 1.17 | $4.178(.759)$ | $4.051(.399)$ |  |
| Elect. Machinery | $1.052(.067)$ | $.082(.033)$ | 1.14 | $10.83(.146)$ | $6.145(.188)$ |  |
| Transp. Equipm. | $1.063(.072)$ | $.096(.036)$ | 1.19 | $10.33(.170)$ | $1.402(.844)$ |  |
| Misc. Manuf. | $1.120(.135)$ | $.127(.067)$ | 1.18 | $5.705(.575)$ | $3.574(.467)$ |  |

${ }^{1}$ GMM estimation. Instruments are two lags of hours in the sector, lagged capital, two lags of hours in the manufacturing industry (excluding the sector itself), lagged wages and two lags of aggregate consumption. The coefficient on wages is imposed to be zero. The coefficient on capital is constrained to be the average between the two boundary values of 0 and ( $1-s_{b}$ ), and the mark up value is computed as the ratio ( $\left.1-\pi_{4}.\right) / s_{k}$. Standard errors are in parentheses.
${ }^{2} \Delta x_{i t}$ indicates the $\log$ difference of variable x in sector $\mathrm{i} . \mathrm{q}$ is industrial production, h is total hours of production workers, k is net capital stock in constant dollars, and $w$ is real wage.
${ }^{3}$ The statistic J, distributed as a chi-square with seven degrees of freedom., tests for the overidentifying restrictions. Probability values are reported in parentheses.
4 The statistic $D$, constructed as the difference between the J statistic of the restricted model and that of the unrestricted (not reported here), is distributed as a chi-square with four degrees of freedom. It tests for the parameter restrictions $\pi_{2}=\pi_{3}, \pi_{1}$ $-\left(\pi_{2}+\pi_{3}\right)+\pi_{4}=1, \pi_{4}=\operatorname{avg}\left(0,1-s_{b}\right)$, and $\pi_{5}=0$. Probability values are reported in parentheses.

## TABLE B1 ${ }^{1}$

## EULER EQUATION ESTIMATES

NON CONSTANT RETURNS TO SCALE - no pure profits $(\mu / \eta=1)$
annual data 1950/1988

| $\Delta \mathrm{g}_{\mathrm{it}}=\pi_{1} \Delta \mathrm{~h}_{\mathrm{it}}-\pi_{2} \Delta \mathrm{~h}_{\mathrm{i}, \mathrm{t}-1}-\pi_{3} \mathrm{E}_{\mathrm{i}} \Delta \mathrm{h}_{\mathrm{i},+1}+\pi_{4} \Delta \mathrm{k}_{\mathrm{it}}+\pi_{5} \Delta \mathrm{w}_{\mathrm{it}}+\mathrm{u}_{\mathrm{it}}{ }^{2}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\pi$ | $\pi_{2}=\pi_{3}$ | $\pi_{4}$ | $\mu=\eta$ | $\mathrm{J}[\mathrm{df}=6]^{3}$ | D [df=3] ${ }^{4}$ |
| Paper | 1.564 (.136) | . 468 (.096) | . 118 (.131) | 0.341 | 7.812 (.252) | 1.493 (.683) |
| Print. \& Publ. | 1.470 (.229) | . 386 (.146) | . 236 (.073) | 0.936 | 8.879 (.180) | 11.06 (.011) |
| Chemicals | 1.493 (.257) | . 675 (.126) | . 112 (.175) | 0.255 | 5.345 (.500) | 1.037 (.792) |
| Rubber \& Pl. | 0.910 (.112) | . 235 (.074) | . 164 (.074) | 0.604 | 7.449 (.281) | 3.836 (.280) |
| Clay,Glass,St. | 1.089 (.101) | . 283 (.080) | . 229 (.073) | 0.75 | 8.209 (.223) | 5.981 (.112) |
| Primary Metals | 1.152 (.114) | . 356 (.073) | . 176 (.080) | 0.616 | 4.291 (.637) | 2.669 (.445) |
| Fabric. Metals | 1.064 (.083) | . 168 (.049) | . 212 (.039) | 0.94 | 8.173 (.225) | 6.723 (.081) |
| Nonel. Machin. | 1.021 (.079) | . 189 (.062) | . 218 (.038) | 0.861 | 1.807 (.936) | 1.857 (.602) |
| Electr. Machin. | 0.938 (.093) | . 222 (.072) | . 143 (.053) | 0.637 | 6.759 (.343) | 4.104 (.250) |
| Transp. Equip. | 1.027 (.098) | . 130 (.048) | . 284 (.045) | 1.05 | 11.71 (.069) | 2.172 (.537) |
| Misc. Manuf. | 1.009 (.175) | . 205 (.129) | . 217 (.119) | 0.816 | 6.710 (.348) | 4.392 (.222) |

1 GMM estimation. Instruments are two lags of hours in the sector, lagged capital, two lags of hours in the manufacturing industry (excluding the sector itself). lagged wages and two lags of aggregate consumption.
$=\Delta x_{i t}$ indicates the $\log$ difference of variable $x$ in sector $i$ at time $t . q$ is industrial production, $h$ is hours of production workers, k is net capital stock in constant dollars, w is real wage.
${ }^{3}$ The statistic J is distributed as a chi-square with six degrees of freedom. It tests for the overidentifying restrictions. Probability valucs are reported in parentheses.
${ }^{4}$ The statistic D, constructed as the difference between the $J$ statistic of the restricted model and that of the unrestricted (not reported here), is distributed as a chi-square with three degrees of freedom. It tests for the parameter restrictions $\pi_{2}=\pi_{3}, \pi_{5}=0$, and $\left[\pi_{1}-\left(\pi_{2}+\pi_{3}\right)\right]\left(1-s_{\mathrm{b}}\right)=\pi_{4} \mathrm{~s}_{\mathrm{b}}$. Probability values are reported in parentheses.

## TABLE B2 ${ }^{1}$

## EULER EQUATION ESTIMATES

NON CONSTANT RETURNS TO SCALE - maximum value of pure profits $\left(\mu / \eta=1 / s_{h}\right)$
annual data 1950/1988

| $\Delta \mathrm{q}_{\mathrm{it}}=\pi_{1} \Delta \mathrm{~h}_{\mathrm{it}}-\pi_{2} \Delta \mathrm{~h}_{\mathrm{i},-1}-\pi_{3} \mathrm{E}_{\mathrm{t}} \Delta \mathrm{h}_{\mathrm{i}, \text { tr1 }}+\pi_{4} \Delta \mathrm{k}_{\mathrm{it}}+\pi_{\mathrm{s}} \Delta \mathrm{w}_{\mathrm{it}}+\mathrm{u}_{\mathrm{it}}{ }^{2}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\pi_{1}$ | $\pi_{2}=\pi_{3}$ | $\eta$ | $\mu$ | $\mathrm{J}[\mathrm{df}=6]^{3}$ | D [df=3] ${ }^{4}$ |
| Paper | 1.283 (.134) | . 301 (.123) | . 681 (.324) | 1.05 | 6.521 (.367) | 0.765 (.858) |
| Print. \& Publish. | 1.533 (.223) | . 267 (.140) | . 999 (.226) | 1.34 | 6.072 (.415) | 5.474 (.140) |
| Chemicals | 1.516 (.270) | . 607 (.196) | . 302 (.480) | 0.54 | 5.760 (.450) | 1.047 (.790) |
| Rubber \& Plastic | 1.001 (.116) | . 140 (.088) | . 721 (.243) | 0.99 | 5.604 (.469) | 2.427 (.488) |
| Clay,Glass,Stone | 1.127 (.085) | . 169 (.081) | . 789 (.176) | 1.13 | 6.507 (.369) | 2.354 (.502) |
| Primary Mctals | 1.193 (.101) | . 289 (.077) | . 615 (.206) | 0.861 | 3.238 (.778) | 1.186 (.756) |
| Fabric. Metals | 1.110 (.078) | . 130 (.048) | . 850 (.130) | 1.1 | 5.115 (.529) | 2.586 (.460) |
| Noncl. Mach. | 0.985 (.070) | . 138 (.062) | . 709 (.112) | 0.95 | 1.128 (.980) | 0.908 (.823) |
| Elect. Machinery | 0.945 (.088) | . 193 (.075) | . 559 (.188) | 0.72 | 6.249 (.396) | 3.514 (.319) |
| Transp. Equipm. | 1.062 (.089) | . 086 (.046) | . 890 (.117) | 1.22 | 9.912 (.128) | 1.096 (.778) |
| Misc. Manuf. | 1.140 (.196) | . 131 (.137) | . 878 (.381) | 1.12 | 5.346 (.500) | 3.152 (.369) |

[^14]
## TABLE B3 ${ }^{1}$

## EULER EQUATION ESTIMATES

NON CONSTANT RETURNS TO SCALE - intermediate value of pure profits $\left(\mu / \eta\right.$ in $\left.\left(1,1 / s_{h}\right)\right)$ annual data 1950/1988

| $\Delta \mathrm{q}_{\mathrm{it}}=\pi_{1} \Delta \mathrm{~h}_{\mathrm{it}}-\pi_{2} \Delta \mathrm{~h}_{\mathrm{i}, \mathrm{t}-1}-\pi_{3} \mathrm{E}_{\mathrm{t}} \Delta \mathrm{h}_{\mathrm{i},+1}+\pi_{4} \Delta \mathrm{k}_{\mathrm{it}}+\pi_{5} \Delta \mathrm{w}_{\mathrm{it}}+\mathrm{u}_{\mathrm{it}}{ }^{2}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\pi_{1}$ | $\pi_{2}=\pi_{3}$ | $\pi_{4}$ | $\eta$ | $\mu / \eta$ | $\mathrm{J}[\mathrm{df}=6]^{3}$ | $\mathrm{D}[\mathrm{df}=3]^{4}$ |
| Paper | 1.229 (.137) | . 378 (.114) | . 087 (.055) | . 560 (.319) | 1.3 | 7.443(.282) | 1.086 (.780) |
| Print. \& Publ. | 1.501 (.224) | . 304 (.143) | . 111 (.028) | 1.00 (.152) | 1.19 | 7.358 (.289) | 7.456 (.058) |
| Chemicals | 1.507 (.263) | . 642 (.153) | . 064 (.099) | . 292 (.383) | 1.39 | 5.509 (.480) | 1.028 (.079) |
| Rubber \& Pl. | 0.963 (.114) | . 183 (.082) | . 098 (.037) | . 695 (.231) | 1.18 | 6.484 (.371) | 2.993 (.392) |
| Clay,Gl.,St. | 1.115 (.091) | . 213 (.081) | . 122 (.031) | . 812 (.169) | 1.22 | 7.356 (.289) | 3.573 (.311) |
| Primary Met. | 1.179 (.108) | . $318(.076)$ | . 091 (.034) | . 635 (.214) | 1.2 | 3.757 (.709) | 1.768 (.622) |
| Fabric. Met. | 1.086 (.081) | . 146 (.049) | . 106 (.018) | . 900 (.131) | 1.14 | 6.477 (.372) | 4.193 (.241) |
| Nonel. Mach. | $1.001(.073)$ | . 159 (.062) | . 098 (.016) | . 781 (.093) | 1.17 | 1.292 (.972) | 1.143 (.767) |
| Elect. Mach. | 0.945 (.090) | . 203 (.074) | . 070 (.024) | . 609 (.183) | 1.14 | 6.449 (.375) | 3.747 (.290) |
| Transp. Eqp. | 1.052 (.093) | . 103 (.047) | . 136 (.019) | . 982 (.120) | 1.18 | 10.39 (.109) | 1.428 (.699) |
| Misc. Man. | 1.093 (.188) | . $150(.136$ ) | . 127 (.059) | . 92 (.380) | 1.18 | 5.771 (.449) | 3.534 (.316) |

[^15]TABLE C BLOCK EXOGENEITY TEST FOR AGGREGATE CONSUMPTION annual data 1950/1988

|  | LR $^{\text {1 }}$ | p-value |
| :--- | :---: | :---: |
| Paper | 15.98 | 0.0011 |
| Chemicals | 13.26 | 0.0041 |
| Rubber and Plastic | 12.24 | 0.0066 |
| Clay, Glass, Stone | 12.92 | 0.0048 |
| Primary Metals | 20.4 | 0.0001 |
| Fabricated Mctals | 11.56 | 0.009 |
| Nonclectrical Mach. | 5.22 | 0.1564 |
| Electrical Machinery | 3.4 | 0.3339 |
| Transport. Equipment | 6.8 | 0.0785 |
| Miscell. Manufactures | 11.22 | 0.0106 |

[^16]$$
\text { Fig. } \quad \text { AOR }
$$


Fig. 2


Fig. 3 - RUBBER and PLASTIC


Fig. 4 - CLAY, GLASS and STONE


Fig. 5 - PRIMARY METALS


Fig. 6 - FABRICATED METALS


Fig. 7 - NONELECTRICAL MACHINERY
a) Estimated Response to o unit inn. in Consumption

c) Estimoted and Simulated TFP

b) Estimoted and Simuloted Hours

d) Simulated Effort


Fig. 8 - ELECTRICAL MACHINERY


Fig. 9 - TRANSPORTATION EQUIPMENT


Fig. 10 - MISCELLANEOUS MANUFACTURES



[^0]:    *This paper is a revised version of chapter 3 of my Ph.D. dissertation at the University of Chicago. I thank my committee members, proff. John H. Cochrane, Lars P. Hansen and Robert E. Lucas, and the participants at workshops at the Universidad Carlos III in Madrid, the IGIER Research Center in Milan, the Chicago Fed, the NBER Summer Institute and the University of Michigan for their comments. My special thanks go to Mike Woodford for his encouragment and advice.

[^1]:    ${ }^{1}$ Among the most recent contributions, see Abbott-Griliches-Hausman (1988), Bernanke-Parkinson (1991), Hall (1987,1989), Gordon (1990), Caballero-Lyons (1990, 1992), Baxter-King (1991), Basu (1993).

[^2]:    ${ }^{2}$ Note that in the derivation of eq.(2.9) the terms in $E_{t} \widehat{\omega}_{t+1}$ and $E_{t} \widehat{\gamma}_{\theta t+1}$ cancel because their coefficients depend upon $\lambda\left(\gamma_{h}^{*}\right)$ and $\lambda^{\prime}\left(\gamma_{h}^{*}\right)$ which I assumed to be 0 . Also, as explained below, $\alpha_{0}=0$.

[^3]:    ${ }^{3} 1$ camot take the sectoral Solow residuals to be a measure of $\gamma_{\theta}$ because it neglects the existence of variations in labor effort. Furthermore, as is discussed further below, I do not necessarily wish to assume that firms are competitive. so that factor shares in total revenues need not represent production function clasticities as it is assumed in the construction of Solow residuals.

[^4]:    ${ }^{4}$ Appendix $B$ contains a description of the data used. The selection of the actual sectors investigated is based on the extent to which various sectors show cyclical behavior of productivity.
    ${ }^{5}$ Below I will also denote the ratio $\alpha_{3} / \alpha_{1}$ as the adjustment cost parameter. This ratio more properly denotes the cost of adjusting hours relative to the cost of increasing labor utilization.

[^5]:    ${ }^{6}$ This restriction is driven by the form of the cost function, where the wage enters multiplicatively. llowever, it does not mean irrelevance of the wage process, since the choice of labor is conditioned on a given output.
    ${ }^{7} \mu$ indicates the degree of mark-up pricing $(\mu=P / M C)$. There is some empirical evidence of a quite large degree of market power in several sectors of the manufacturing industry (see Hall (1988) and Domowitz, IIubbard and Peterson (1988)). Hall's estimates, for example, show a particularly high mark-up for sectors like Papers, Chemicals and Primary Metals.

[^6]:    ${ }^{5}$ This is analogous to the test based on the difference between the restricted and unrestricted sum of the squared residuals. See Newey-West (1987) for the discussion of this statistic.

[^7]:    ${ }^{9}$ Because few statistics on labor utilization are available (for the U.K., there is a study by Schor (1987), while Shea (1990) tries to construct a series for the U.S. using accident rates), some authors have used variables that they argue are proxies for effort, as overtime hours (Caballero-Lyons (1992)) or the number of hours per employee (Abbott- Griliches-Hausman (1988) and Eden-Griliches (1993)).

[^8]:    ${ }^{10}$ The complementarity assumption is made, for example, by Abbott et al. (1988), who use hours per cmployee as a proxy for both capital and labor utilization. Shapiro (1993) uses a direct measure of the workweek of capital, from an unpublished panel of observations at plant level, to show that a variable workweek of capital solves the puzzle of procyclical total factor productivity.

[^9]:    ${ }^{11}$ As a first approximation, I assume that the time interval in the model is a year, so that the generated time series are annual observations: this allows a direct comparison with the dynamic pattern estimated on annual data (the only frequency available for sectoral data on capital and value added). A more interesting model may be written for a time interval of a quarter or even a month. This is under investigation, to match other empirical facts that emerge from higher frequency data.
    ${ }^{12}$ Standard unit root tests are conducted to justify the specification of the model in first differences; a
    two-step cointegration test is also conducted to test for the stationarity of the output/capital ratio.

[^10]:    ${ }^{13}$ This is just solution (A.2) of Appendix A.

[^11]:    ${ }^{14}$ The standard errors are computed with bootstrapping on 200 simulations.
    ${ }^{15}$ This definition does not include the technology term because of the assumed independence of that term from aggregate consumption, which is the only perturbation considered here.

[^12]:    ${ }^{16}$ The elements of the matrix $P$ are a combination of the parameters of the optimization and the forecasting processes and therefore, as it is well known in the rational expectations literature, the optimal decision rule for $\hat{\mathrm{T}}_{\mathrm{h}}$ is not invariant to changes in the stochastic process of the forcing variables.

[^13]:    ${ }^{1}$ GMM estimation. Instruments are two lags of hours in the sector, lagged capital, wo lags of hours in the manufacturing industry (excluding the sector itself). lagged wages and two lags of aggregate consumption. The coefficient on wages is imposed to be zero. The coefficient of capital is imposed to be equal to ( $1-s_{b}$ ). Standard errors are reported in parentheses.
    ${ }^{2} \Delta x_{u}$ indicates the $\log$ difference of variable x in sector i at time t . q is industrial production, h is total hours of production workers, $k$ is net capital stock in constant dollars, $w$ is real wage.
    ${ }^{3}$ The statistic J is distributed as a chi-square with seven degrees of freedom. It tests for the overidentifying restrictions. Probability values are reported in parentheses.
    4 The statistic D. constructed as the difference between the J statistic of the restricted model and that of the unrestricted (not reported here), is distributed as a chi-square with four degrees of freedom. It tests for the parameter restriction $\pi_{2}=\pi_{3}, \pi_{4}$ $=1-s_{4} \cdot \pi_{1}-\left(\pi_{2}+\pi_{3}\right)+\pi_{4}=1, \pi_{5}=0$. Probability values are reported in parentheses.

[^14]:    ' GMM estimation. Instruments are two lags of hours in the sector, lagged capital, two lags of hours in the manufacturing industry (excluding the sector itself), lagged wages and two lags of aggregate consumption. The parameter $p_{4}$ is constrained to be 0 , so the value of $h$ reported in column 4 is derived as the sum of the estimated $\pi_{1}, \pi_{2}$, and $\pi_{3}$. The value of $\mu$ in column 5 is computed as the ratio $\eta / s_{\mathrm{n}}$. Standard errors are reported in parentheses.
    ${ }^{2} \Delta \mathrm{x}_{\mathrm{i}}$ indicates the $\log$ difference of variable x in sector i at time t . q is industrial production, h is total hours of production workers, $k$ is net capital stock in constant dollars, and $w$ is real wage.
    3 The statistic J is distributed as a chi-square with six degrees of freedom. It tests for the overidentifying restrictions. Probability values are reported in parentheses.
    ${ }^{4}$ The statistic $D$, constructed as the difference between the J statistic of the restricted model and that of the unrestricted (not reported here), is distributed as a chi-square with three degrees of freedom. It tests for the parameter restrictions $\pi_{2}=\pi_{3}, \pi_{4}$ $=0$, and $\pi_{5}=0$. Probability values are reported in parentheses.

[^15]:    1 GMM estimation. Instruments are two lags of hours in the sector, lagged capital, two lags of hours in the manufacturing industry (excluding the sector itself), lagged wages and two lags of aggregate consumption. The coefficient on wages is imposed to be zero. The ratio $\mu / \eta$ is chosen to be the average value between 1 and the inverse of the labor share. Then $\eta$ is computed as $\eta=\pi_{1}-\left(\pi_{2}+\pi_{3}\right)+\pi_{4}$, and the implied value for $\mu$ is derived from it. Standard errors are reported in parentheses.
    ${ }^{2} \Delta x_{i n}$ indicates the $\log$ difference of variable $x$ in sector $i$ at time $t$. $q$ is industrial production, $h$ is total hours of production workers, k is net capital stock in constant dollars, and w is real wage.
    ${ }^{3}$ The statistic $\mathbf{J}$ is distributed as a chi-square with six degrees of freedom. It tests for the overidentifying restrictions. Probability values are reported in parentheses.
    4 The statistic D. constructed as the difference between the J statistic of the restricted model and that of the unrestricted (not reported here), is distributed as a chi-square with three degrees of freedom. It tests for the parameter restrictions $\pi_{2}=\pi_{3}$, $\pi_{4}=1-s_{h}$, and $\pi_{5}=0$. Probability values are reported in parentheses.

[^16]:    $1 \operatorname{LR}=(T-q) *\left(\log \left|\Sigma_{r}\right|-\log \left|\Sigma_{u}\right|\right)$ where $\Sigma_{\mathrm{t}}$ and $\Sigma_{u}$ are the residual covariance matrices respectively of the restricted (excluding lagged aggregate consumption) and unrestricted VAR estimated on each sector, $T$ is the number of observations and $q$ the number of parameters estimated in each unrestricted equation.

