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Technical Change, Diffusion, and Productivity

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Abstract

This paper presents a quantitative theoretical framework which addresses two empirical regularities. The first is the sizable volume of resource reallocation between plants. The second is the large and procyclical fluctuations in measured total factor productivity. It investigates the role of capital reallocation in generating productivity fluctuations by modeling plant entry and exit as a channel for the diffusion of process innovations. Only new plants have access to the leading edge production process, but the scrap capital from exiting plants is available for their construction. Therefore, concurrent increases in entry and exit cause measured productivity to rise. Because it explicitly accounts for technological heterogeneity across plants, the model is capable of structuring a simultaneous analysis of the plant level and macroeconomic data. In a version of the model estimated with only macroeconomic data, the measured Solow residual mimics the univariate time series properties of its empirical counterpart. Consideration of heterogeneous plants' roles in technology diffusion provides a more reasonable alternative to the interpretation of the Solow residual as a neutral, exogenous, productivity shock.

1 Introduction

This paper presents a quantitative theoretical framework which simultaneously addresses two sets of empirical regularities, one found in plant level data and the other in the macroeconomy. The first concerns the nature of resource reallocation between plants. Using data on manufacturing plants, Dunne, Roberts, and Samuelson (1988;

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1989a; 1989b) observed that the volume of resource reallocation between plants is enormous. Between 40% to 50% of plants operating during a given census year no longer exist 5 years later. Exiting plants produce nearly 20% of manufacturing output. The second empirical regularity is the macroeconomic observation that fluctuations in the Solow (1957) residual are large and procyclical. The standard error of total factor productivity growth is 0.85%,and its correlation with output growth is 0.80.[1]

The identification of total factor productivity movements with exogenous, neutral, technological change is a common feature of research from the real business cycle literature. The models of Kydland and Prescott (1982), Hansen (1985), and King, Plosser, and Rebelo (1988a; 1988b) are prominent examples of work which embodies this interpretation. As Summers (1986) noted, a problem with this understanding of total factor productivity movements is the lack of obvious candidate sources for such large shocks. The difficulty in finding technological advances which uniformly expand all producers' production possibilities makes this interpretation of total factor productivity fluctuations problematic.[2]

Many examples of technological innovation come to mind, but few of them are costless to implement. Furthermore, their diffusion throughout the economy's production sector is rarely instant or uniform. A reasonable explanation of productivity fluctuations based on technological change must account for these facts. Restricting the impact of technical innovations to new capital goods is one way of addressing these issues. In the vintage capital models of Solow (1960; 1962a; 1962b) and Greenwood, Hercowitz, and Krusell (1992), the only channel for technology diffusion is the accumulation of new capital goods. A shortcoming of this interpretation is its inability to explain the observed productivity fluctuations. Because the flow of new investment is small relative to the stock of capital, fluctuations in the productivity of new capital can have only minor immediate effects on the Solow residual.

To improve the vintage capital model's performance along this dimension, this work examines a second channel for technology diffusion: capital reallocation. Presumably, reallocating capital from exiting plants to their replacements increases its productivity. In a study of plant level data from the U.S. manufacturing sector, Bartlesman and Dhrymes (1992) argue that the reallocation of economic activity between production sites is an important source of *aggregate* productivity growth. Reallocating existing capital goods does not change their technical characteristics, so this can not be a channel for product diffusion. However, production processes do vary across plants. Focusing on process diffusion, rather than on product diffusion, allows capital reallocation and vintage capital effects to be simultaneously incorporated into a technology diffusion model.

The large observed volume of plant entry and exit suggests that this is a major medium for capital reallocation. To investigate its role in the generation of Solow

[1]Sections 5 and 6 provide details about these estimates.

[2]For an alternative identification which relies less on a literal interpretation of "technology", see Hansen and Prescott (1993).

residual fluctuations, the paper integrates a selection model of entry and exit, such as those of Jovanovic (1982) and Hopenhayn (1992a; 1992b), into a standard, general equilibrium, business cycle framework with aggregate uncertainty. In the model economy, only newly constructed plants have access to the leading edge production process. The entering plants implement these new production processes with varying degrees of success. At any time, a fraction of each plant's capital is recoverable as scrap. In a competitive equilibrium, plants with productivity below an endogenously determined threshold exit. Increasing the return to exiting, the plants' scrap value, raises the threshold. A higher threshold increases average plant productivity and the volume of exit. The construction of new plants can use the scrap capital from exiting plants. As in the model of Greenwood, Hercowitz, and Huffman(1988), the economy's only source of aggregate uncertainty is the productivity of new plants. An exogenous improvement in the leading edge production process increases the return to capital reallocation, so the pace of entry and exit accelerates. When the productivity difference between exiting and entering plants is large, the rise in reallocation causes a sizable increase in measured total factor productivity.

The union of a selection model of entry and exit with a business cycle model is capable of addressing issues which neither could alone. In contrast to standard macroeconomic models which use a representative plant to stand for the economy's production sector, the framework explicitly recognizes the observed productivity heterogeneity among plants. On the other hand, the inclusion of aggregate uncertainty in a computable model of entry and exit allows an analysis of the cross sectional productivity response to process innovations. A surprising result of this study is that a lower cost of reallocation *reduces* the model's ability to generate large productivity fluctuations. The productivity difference between entering and exiting plants is lower when the scrap value of old plants is high, so a given amount of reallocation has a smaller effect on measured aggregate productivity.

Whether the model adequately reproduces the behavior of the observed Solow residual is an empirical issue. Many of the model's parameters can be estimated using macroeconomic data. Others, those governing the uncertainty of process implementation and the cost of capital reallocation, are most naturally estimated using plant level data. For example, the parameters governing the plants' idiosyncratic uncertainty can be estimated using employment and capital asset data from an unbalanced panel of plants. Because a plant exits when it falls below a productivity threshold, the estimation easily fits into the Tobit model employed by Heckman (1976; 1979). Because this data is not readily available, this paper presents preliminary results based entirely on macroeconomic data and published work with the Longitudinal Research Database. The model's parameters are estimated using a just identified version of Hansen's (1982) generalized method of moments estimator. The empirical results are encouraging. In the estimated model, the volume of capital reallocation jumps between 30% and 35% following an unexpected 1% improvement in the productivity of the leading edge production process. This generates an observed Solow residual which resembles a random walk. Following the reallocation, output, consumption, and investment all respond much as they would to an exogenous technology shock. The increase in capital reallocation is transitory, only lasting one period. Thereafter, entry and exit temporarily decline because there are relatively few low productivity plants remaining. The decline coincides with the increase in output, so capital reallocation is countercyclical. This agrees with the observation of Davis and Haltiwanger (1990; 1992) that the reallocation of workers across sites is countercyclical.

Wald tests largely support the null hypothesis that the model adequately reproduces the Solow residual's univariate time series properties. The finding that process innovation diffusion can cause Solow residual fluctuations like those observed in the U.S. economy provides a more reasonable alternative to their interpretation as neutral, exogenous, technology shocks. Unfortunately, this paper's explanation shares a weakness with the real business cycle interpretation: neither of them provides a reasonable account of productivity decline.[3]

Computation of the model's equilibrium required an extension of standard dynamic solution techniques to handle plant level heterogeneity. A set of Euler equations and a transversality condition characterize the model's competitive equilibrium. As in the solution method of King, Plosser, and Rebelo (1987), log-linear expansions around a non-stochastic steady state growth path approximate the equations. The continuous specification for plant level heterogeneity implies that they are non-trivial functional equations. To compute the solutions of this linear dynamical system, the functional equations are approximated using quadrature methods. This produces a dynamical system with a large, but finite, dimensional state space. The analysis of this approximate system can use standard methods, such as those of Blanchard and Kahn (1980) and King, Plosser, and Rebelo (1987).

The paper's next two sections describe the model economy and discuss its competitive equilibrium. Section 4 decomposes changes in the traditionally measured Solow residual into those reflecting capital quantity and capital quality mismeasurement. Section 5 presents the identification and estimation strategy used to evaluate the model's empirical performance. In particular, it shows how plant level data could be used to estimate parameters governing the capital reallocation process. Section 6 presents the results of the empirical analysis. The paper's last section contains concluding remarks.

[3]Other authors have offered different explanations for the procyclicality of observed productivity. Summers (1986) and Burnside, Eichenbaum, and Rebelo (1990) and Sbordone (1993) entertain the hypothesis that firms' labor hoarding behavior is the source of these fluctuations. Baxter and King (1991), Caballero and Lyons (1992), focus on the presence of aggregate externalities. Hall (1988) showed that imperfect competition in product markets can cause spurious procyclical productivity movements.

2 The Model

The model of this section differs from a standard equilibrium macroeconomic framework by explicitly modeling plant level technological heterogeneity. As in Solow (1960; 1962a; 1962b), and Greenwood, Huffman, and Krusell (1992), newly constructed plants have access to the leading edge technology. Unlike these models, previously produced capital goods may be reallocated to new plants. This provides a second channel for the diffusion of technological innovations through the economy.

The remainder of the model is standard. There are many identical consumers, who provide the economy's labor and own its equity. Capital goods are traded in complete markets. A single representative firm rents all factors of production. Its manager chooses the allocation of labor among the plants and makes the capital reallocation decisions. She makes these choices to maximize the firm's profits. The constant returns to scale technology ensures that profits are zero in a competitive equilibrium.

This section describes the technology available to the model economy's production sector, the representative consumer's preferences and endowments, and the physical environment within which they operate. Section 3 describes the market structure, presents the agents' optimization problems, and defines a competitive equilibrium.

2.1 The Production Sector

A continuum of atomistic plants populates the economy's production sector. This sector uses labor and plants as inputs to produce the aggregate good. This good may then be used either for consumption or for the construction of new plants. Each plant has a capital stock, s^i, and an idiosyncratic productivity level, θ^i. The superscript i denotes variables which are specific to a single plant. The production technology available to these plants is

$$y = (s^i e^{\theta^i})^{1-\alpha} n^{\alpha}. \tag{1}$$

The plant's labor input is n and its output of the aggregate good is y. Restricting the elasticity of output with respect to labor input, α, to lie strictly between zero and one guarantees that the plant's technology is strictly concave. Because this technology obeys constant returns to scale, the plants' size distribution (as measured by capital) does not effect the economy's production possibilities. The production sets available to a single plant with one unit of capital and N otherwise identical plants each with $s^i = 1/N$ are the same.

Building a new plant requires investing one unit of the aggregate good. When the plant is constructed, it has access to the leading edge production process. New plants implement this process with varying degrees of success. Their original productivity levels reflect this. If the plant is constructed in period t, its initial value of θ^i is normally distributed with mean z_t and standard error σ.

$$\theta^i \sim N(z_t, \sigma^2) \tag{2}$$

The productivity of a plant with an average implementation of the leading edge production process is z_t. This is an index of the leading edge technology. It follows a random walk with a positive drift.

$$\begin{aligned} z_t &= \mu_z + z_{t-1} + \varepsilon_t^z \\ \varepsilon_t^z &\sim N(0, \sigma_z^2) \end{aligned} \tag{3}$$

The exogenous technological progress embodied in z_t is the model economy's only source of growth and uncertainty. Although new plants are *on average* more productive than existent plants, nothing distinguishes a new plant with idiosyncratic productivity level θ^i from an otherwise identical incumbent.

At the end of every period, a fraction, δ, of each plant's capital is lost to depreciation. The plants may *not* engage in replacement investment. What remains of the plant may then be used in one of two ways. It may be retired, in which case s units of the aggregate good are recovered as its scrap value. The scrap value is positive but less than one. Alternatively, the plant may be left intact. In that case, it receives an idiosyncratic shock to its productivity level before the next period.

$$\begin{aligned} \theta_{t+1}^i &= \theta_t^i + \varepsilon_{t+1}^i \\ \varepsilon_{t+1}^i &\sim N(0, \sigma^2) \end{aligned} \tag{4}$$

That is, the plant's idiosyncratic productivity level follows a random walk. The unit root in the plant productivity process implies that the level of the leading edge production technology during its construction, z_t, will have a permanent effect on its productivity. In this sense, the model includes vintage capital effects like those in the models of Solow (1960; 1962a; 1962b) and Greenwood, Hercowitz, and Krusell (1992). The random walk's innovation is normally distributed with zero mean and standard error σ. The standard error is the same as that governing the distribution of the plant's initial value of θ^i. This is not essential and is adopted only for the sake of parsimony.

2.2 Consumers

There are many identical infinitely lived consumers who value two goods, consumption and leisure. The population of consumers grows exogenously at the constant rate μ_p. Each consumer has a time endowment of one unit each period, which she must allocate between leisure and labor. The utility function $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t)$ gives her preferences over state contingent sequences of these two goods. Her discount factor is β, which lies strictly between zero and one. Her momentary utility function, $u(c_t, 1 - n_t)$, is

$$u(c_t, 1 - n_t) = \ln(c_t) + v(1 - n_t) \tag{5}$$

To ensure the concavity of $u(\cdot, \cdot)$, $v(\cdot)$ is continuous, twice differentiable, and weakly concave. If $v(1 - n_t) = \kappa \ln(1 - n_t)$, then her preferences take the familiar log-log form. If $v(1-n_t) = \kappa(1-n_t)$, then her preferences correspond to those derived by Hansen (1985) in an indivisible labor model. These two preference specifications will appear below in the model's computational analysis.

In the model economy, firms and consumers trade labor, the aggregate good, and capital goods in complete markets. Given the specification of preferences and technology, the next section describes the market structure governing trade, presents the agents' optimization problems, defines an equilibrium for the economy, and discusses computational issues.

3 Competitive Equilibrium

In a competitive equilibrium, the economy's agents trade the aggregate good, labor, and capital goods of all productivity types in complete markets. This section begins by outlining the model economy's market structure. Although several different market structures can support a competitive equilibrium, one is chosen to simplify the analysis. Presentations of firms' and consumers' maximization problems follow this. It finishes by defining a competitive equilibrium and outlining the computational method used.

3.1 Market Structure

Three types of agents populate the model economy, production firms, construction firms, and consumers. They trade capital goods, labor, and the aggregate consumption good in competitive markets. As in the models of Lucas (1978) and Prescott and Mehra (1980), the equilibrium prices for capital assets are of particular interest. They are the basis for capital reallocation decisions. Therefore, the market structure in the model economy is that from Prescott and Mehra (1980).

At the beginning of each period, each consumer owns an identical portfolio of the economy's productive assets. They sell these assets and their labor services to the production firms. Production firms only exist for one period. They produce the aggregate good with the technology described by equation (1). After production, the firms decide which of the surviving plants to keep intact and which to salvage for their scrap value. Then the firms sell their stock of the aggregate good, consisting of what they produced and recovered as scrap, to the construction firms and the consumers. The aggregate good is the numeraire, and its spot price always equals one. It is perishable, so the consumer must consume her purchases within the current period. The construction firms also exist for only one period. They purchase the aggregate good from the production sector and use it to create new plants. At the end of the period, firms in both the production sector and the construction sector liquidate, selling their stock of plants to the consumers. To maintain wealth homogeneity, a fraction of each consumer's portfolio is taxed and granted to young consumers. In the next period, nothing differentiates a young from an old consumer.

This market structure, as opposed to one in which consumers rented capital goods to firms every period, naturally prices the economy's capital assets.[4] The technology available to firms in both sectors obeys constant returns to scale. Therefore, firms earn zero profits in equilibrium. As in a standard macroeconomic model, each sector acts as if it has a single, representative, price taking firm. This division of the production sector into two representative firms is arbitrary but analytically useful.

3.2 The Production Problem

Given the prices of all plants and the wage rate, the representative production manager hires labor and trades plants to maximize its profits. The size distribution of the plants does not change the firm's production possibilities, so without loss of generality, we may consider the case where all of its plants have one unit of capital. Let w_t denote the wage rate in period t and let $q_t^0(\theta)$ and $q_t^1(\theta)$ respectively designate prices of a plant with one unit of capital and productivity level θ at the beginning and end of this period. If asset prices equal discounted expected dividend streams, then asset prices will be increasing in θ. A plant's scrap value is invariant to its value of θ, so the representative firm will chose to scrap only those plants below a threshold, θ_t. Those plants with productivity levels above the threshold will remain in production. This threshold scrap rule is similar to those found in Hopenhayn (1992a) and Jovanovic (1982). After incorporating the plant retirement decision, the representative production firm's profit maximization problem is

$$
\begin{aligned}
\max_{k(\theta),\acute{k}(\theta),n(\theta),n} \quad & y + s(1-\delta)\int_{-\infty}^{\theta_t} k(\theta)d\theta \\
& + \int_{\theta_t}^{\infty} q_t^1(\theta)\acute{k}(\theta)d\theta - \int_{-\infty}^{\infty} q_t^0(\theta)k(\theta) - w_t n \\
\text{subject to:} \qquad y &= \int_{-\infty}^{\infty} k(\theta)e^{\theta(1-\alpha)}n(\theta)^{\alpha}d\theta \qquad\qquad (6)\\
n &= \int_{-\infty}^{\infty} k(\theta)n(\theta)d\theta \\
\acute{k}(\acute{\theta}) &= (1-\delta)\int_{\theta_t}^{\infty} \frac{1}{\sigma}\phi\left(\frac{\acute{\theta}-\theta}{\sigma}\right)k(\theta)d\theta
\end{aligned}
$$

The objective function's first two terms are the firm's total output and the value of its scrap capital. The third term is the value of its stock of capital at the end of the period. The number of plants of type θ which the firm may sell at the end of the period is $\acute{k}(\theta)$. The final terms are the cost of its beginning of period capital purchases and its wage bill. The first constraint on the firm's problem says that

[4] Additional markets in state contingent claims on capital assets and the aggregate good could be added at the expense of considerable extra notation, but equilibrium prices for physical assets would not change. Because the consumers have identical preferences and endowments, the net trade in contingent claims must be zero.

summing the output of all plants yields the firms total output. The amount of labor the firm assigns to a single plant with productivity θ is $n(\theta)$. The second constraint restricts the total labor allocated among the plants to equal that purchased by the representative firm. The last constraint gives the stock of plants the firm may sell at the end of the period as a function of its purchases and scrap decisions. It embodies the capital depreciation rate δ, and the law of motion for θ. At the end of the period, each of its plants has $1-\delta$ units of capital. These will have the same value to the firm as $1-\delta$ plants of the same productivity level, each with one unit of capital. The function $\phi(z)$ denotes the standard normal p.d.f.

The envelope theorem allows this problem to broken into two steps. First consider the problem of maximizing the firm's output given its capital and labor inputs. This is the labor allocation problem.

$$\begin{aligned} y = \max_{n(\theta)} \int_{-\infty}^{\infty} k(\theta) e^{\theta(1-\alpha)} n(\theta)^{\alpha} d\theta \\ \text{subject to:} \quad n \quad = \quad \int_{-\infty}^{\infty} k(\theta) n(\theta) d\theta \end{aligned} \tag{7}$$

This problem has a simple and familiar solution. Define the firm's *effective capital stock* to be

$$\bar{k} = \int_{-\infty}^{\infty} e^{\theta} k(\theta) d\theta. \tag{8}$$

The effective capital stock is the sum of the number of plants of each type, weighted by their productivity level. With this notation in place, the solution to the labor allocation problem is

$$y = \bar{k}^{1-\alpha} n^{\alpha} \tag{9}$$

A Cobb-Douglas production function in labor and *effective* capital represents the firm's production possibilities. Using the same functional form, Solow (1960) derived this result in a model of vintage capital.

Substituting the solution to the labor allocation problem into the profit maximization problem yields

$$\begin{aligned} \pi_t = \max_{n,\theta_t,k(\theta),\acute{k}(\theta)} \bar{k}^{1-\alpha} n^{\alpha} + s(1-\delta) \int_{-\infty}^{\theta_t} k(\theta) d\theta \\ + \int_{\theta_t}^{\infty} q_t^1(\theta) \acute{k}(\theta) d\theta - \int_{-\infty}^{\infty} q_t^0(\theta) k(\theta) d\theta - wn \\ \text{subject to:} \qquad \bar{k} \quad = \quad \int_{-\infty}^{\infty} e^{\theta(1-\alpha)} k(\theta) d\theta \\ \acute{k}(\acute{\theta}) \quad = \quad (1-\delta) \int_{\theta_t}^{\infty} \tfrac{1}{\sigma} \phi(\tfrac{\acute{\theta}-\theta}{\sigma}) k(\theta) d\theta \end{aligned} \tag{10}$$

The first order conditions for this problem are

$$w \quad = \quad \alpha \left(\frac{\bar{k}}{n} \right)^{1-\alpha} \tag{11}$$

$$s = \int_{-\infty}^{\infty} q_t^1(\acute{\theta})\frac{1}{\sigma}\phi(\frac{\acute{\theta}-\theta_t}{\sigma})d\acute{\theta} \tag{12}$$

$$\begin{aligned} q_t^0(\theta) &= (1-\alpha)\left(\frac{\bar{k}}{n}\right)^{-\alpha} e^{\theta} + 1\{\theta < \theta_t\}(1-\delta)s \\ &\quad +1\{\theta \geq \theta_t\}(1-\delta)\int_{-\infty}^{\infty} q_t^1(\acute{\theta})\frac{1}{\sigma}\phi(\frac{\acute{\theta}-\theta}{\sigma})d\acute{\theta} \end{aligned} \tag{13}$$

The indicator function, $1\{\cdot\}$, equals one if the condition it contains is true, and zero otherwise. Equation (11) is a standard labor demand condition equalizing the marginal product of labor and the wage rate. Equation (12) defines the firm's optimal choice of θ_t, the capital reallocation threshold. It says that the scrap value of the marginal plant must equal the expected end of period asset price if it remains in production. Equation (13) is an asset pricing equation. It constrains an asset's beginning of period price to equal the dividends it returns plus its value at the end of the period. If the asset is scrapped, this value equals that of the scrap capital. Otherwise, it equals the expected end of period asset price.

3.3 The Construction Problem

A large number of construction firms use a one to one technology for converting the aggregate good into new plants. Firms are risk neutral and maximize their expected profits. In a competitive equilibrium, the firms must earn zero profits. The condition which guarantees this is

$$1 = \int_{-\infty}^{\infty} q_t^1(\theta)\frac{1}{\sigma}\phi\left(\frac{\theta - z_t}{\sigma}\right)d\theta. \tag{14}$$

The left hand side of equation (14) is the cost of constructing one plant. The right hand side is the expected price of the completed plant. In this sense, marginal q always equals one.

3.4 The Consumer's Problem

Each consumer maximizes her expected utility by choosing state contingent sequences of consumption, $\{c_t\}_{t=0}^{\infty}$, labor, $\{n_t\}_{t=0}^{\infty}$, and asset holdings $\{k_t(\theta)\}_{t=1}^{\infty}$ taking wages, asset prices, and her initial asset holdings as given.

$$\max_{\{c_t\}_{t=0}^{\infty},\{n_t\}_{t=0}^{\infty},\{k_t(\theta)\}_{t=1}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t (\ln(c_t) + v(1-n_t))$$

$$\begin{aligned} \text{subject to:} \qquad 0 &= c_t + e^{\mu_p} \textstyle\int_{-\infty}^{\infty} q_t^1(\theta) k_{t+1}(\theta) d\theta \\ &\quad - w_t n_t - \textstyle\int_{-\infty}^{\infty} q_t^0(\theta) k_t(\theta) d\theta \end{aligned} \tag{15}$$

$$k_0(\theta) \quad \text{given}$$

To maintain wealth homogeneity, a fraction of each consumer's portfolio is taxed at the end of every period and granted to the young consumers. The presence of e^{μ_p} in the consumer's budget constraint reflects this inter-generational transfer assumption. The mathematical expectations operator with respect to the information set at time t is E_t. In addition to the static budget constraint in (15), the necessary conditions for a solution to the consumer's problem are

$$\frac{1}{c_t} e^{\mu_p} q_t^1(\theta) = E_t \beta \frac{1}{c_{t+1}} q_{t+1}^0(\theta) \tag{16}$$

$$\frac{1}{c_t} w_t = v'(1-n_t) \tag{17}$$

Equation (16) is familiar from the asset pricing literature. It says that the expected product of the intertemporal marginal rate of substitution with the returns to holding an plant with productivity θ equals one. It relates an asset's price at the end of the current period to its price at the beginning of the subsequent period. Equation (17) is a standard static labor supply equation. A set of decisions for consumption, labor supply, and asset holdings will be a solution for this problem if they satisfy equation (16), equation (17), and the transversality condition,

$$\lim_{t\to\infty} E_0 \beta^t \frac{1}{c_t} \int_{-\infty}^{\infty} q_t^0(\theta) k_t(\theta) d\theta = 0. \tag{18}$$

3.5 Market Clearing, Equilibrium, and Computation

In a competitive equilibrium the firms' and consumers' problems are connected through the imposition of market clearing conditions.

Definition 1 *A competitive equilibrium for the economy is a state contingent sequence of per capita consumption, $\{C_t\}_{t=0}^{\infty}$, labor, $\{N_t\}_{t=0}^{\infty}$, gross investment, $\{I_t\}_{t=0}^{\infty}$, asset holdings, $\{K_t(\theta)\}_{t=1}^{\infty}$, wage rates, $\{w_t\}_{t=0}^{\infty}$, and asset prices, $\{q_t^0(\theta)\}_{t=0}^{\infty}$ and $\{q_t^1(\theta)\}_{t=0}^{\infty}$, such that*

1. *$\{C_t\}_{t=0}^{\infty}$, $\{N_t\}_{t=0}^{\infty}$, and $\{K_t(\theta)\}_{t=1}^{\infty}$ is a solution to the consumer's problem given $\{w_t\}_{t=0}^{\infty}$, $\{q_t^0(\theta)\}_{t=0}^{\infty}$, and $\{q_t^1(\theta)\}_{t=0}^{\infty}$.*

2. $K_t(\theta)$, $e^{\mu_p}K_{t+1}(\theta) - \frac{1}{\sigma}\phi\left(\frac{\theta - z_t}{\sigma}\right) I_t$, *and* N_t *solve the representative firm's problem at time t given* w_t, $q_t^0(\theta)$, *and* $q_t^1(\theta)$.

3. *The production and construction firms earn zero profits.*

4. *The sequences of per capital asset holdings and gross investment satisfy the capital accumulation restriction.*

$$e^{\mu_p}K_{t+1}(\acute{\theta}) = (1-\delta)\int_{\theta_t}^{\infty} \frac{1}{\sigma}\phi\left(\frac{\acute{\theta} - \theta}{\sigma}\right) K_t(\theta)d\theta + \frac{1}{\sigma}\phi\left(\frac{\acute{\theta} - z_t}{\sigma}\right) I_t$$

Because the first and second welfare theorems apply to the model economy, its competitive equilibrium quantities will correspond to those a social planner would choose to maximize the utility of a representative consumer.

$$\max_{\substack{\{C_t\}_{t=0}^{\infty}, \{N_t\}_{t=0}^{\infty}, \{I_t\}_{t=0}^{\infty}, \{Y_t\}_{t=0}^{\infty} \\ \{\theta_t\}_{t=0}^{\infty}, \{S_t\}_{t=0}^{\infty}, \{K_t(\theta)\}_{t=1}^{\infty}}} E_0 \sum_{t=0}^{\infty} \beta^t(\ln(C_t) + v(1 - N_t))$$

$$\begin{aligned}
\text{subject to:} \qquad 0 &= C_t + I_t - Y_t - S_t \\
Y_t &= N_t^{\alpha}\left(\textstyle\int_{-\infty}^{\infty} e^{\theta} K_t(\theta)d\theta\right)^{1-\alpha} \\
S_t &= (1-\delta)s \textstyle\int_{-\infty}^{\theta_t} K_t(\theta)d\theta \\
e^{\mu_p}K_{t+1}(\acute{\theta}) &= (1-\delta)\textstyle\int_{\theta_t}^{\infty} \frac{1}{\sigma}\phi\left(\frac{\acute{\theta}-\theta}{\sigma}\right) K_t(\theta)d\theta \\
&\quad + \tfrac{1}{\sigma}\phi\left(\tfrac{\acute{\theta}-z_t}{\sigma}\right) I_t
\end{aligned} \tag{19}$$

The solution to this problem is a set of decision rules expressing the social planner's choice variables as functions of the current state, $K_t(\theta)$, and the exogenous shock, z_t. The strategy for computing the model economy's competitive equilibrium is to approximate the solution of problem (19).

The solution of similar social planning problems is common in the real business cycle literature. What distinguishes this problem from those previously studied is the nature of the choice variable, $K_t(\theta)$. Because it is a function rather than a scalar, the standard solution methods are not immediately applicable. This hurdle is overcome by using quadrature approximations of the relevant functional equations. This approximation reduces a functional dynamical system to a standard vector dynamical system with a large, but finite, state space. Applying standard methods for solving linear dynamical systems then produces the desired solution.

Eliminating all sources of non-stationarity is the first step in solving a problem like (19). First note that the center of the distribution $K(\theta)$ will continually shift to the right as z_t grows. Recasting the problem in terms of $K(\theta - z_t)$ removes this source of non-stationarity. The aggregate production function can be re-written as

$$Y_t = (e^{\frac{1-\alpha}{\alpha}z_t}N_t)^\alpha(\int_{-\infty}^{\infty} e^{\theta - z_t}K_t(\theta - z_t)d\theta)^{1-\alpha} \tag{20}$$

The Cobb-Douglas functional form for the production function guarantees that technical change can be written in *labor* augmenting form, even though it is embodied in the plants. This technical progress is a source of growth for the economy. Scaling all of the social planner's choice variables but hours worked by $e^{\frac{1-\alpha}{\alpha}z_t}$, yields a social planning problem for an equivalent economy which is stationary. This transformation is familiar from the work of King, Plosser, and Rebelo (1988a; 1988b). Following Christiano (1988), this is called the star economy. The star economy is locally stable around its non-stochastic steady state.

To find an approximate solution to this social planning problem, replace its first order necessary conditions with log-linear approximations around its steady state. Because the capital stock is a function rather than a scalar, these approximate first order conditions are *functional* equations. Quadrature approximations, the evaluation of which only requires the function's values at a finite number of points, replace the functional equations.[5] This approximation produces a finite dimensional linear dynamical system. Although its dimension is much greater than that of a standard problem, its solutions can be found by applying standard linear algebraic techniques.

The approximate system of equations possesses a continuum of solutions. The unique one which also satisfies the social planning problem's transversality condition is an approximate solution to problem (19). Rescaling the solution by $e^{\frac{1-\alpha}{\alpha}z_t}$ yields the desired approximate solution to the original problem.

The log-linear nature of the approximation method yields decision rules of the form

$$\begin{aligned} \ln(K_{t+1}(\theta)) &= g(\ln(K_t(\theta), z_t) \\ \ln(N_t) &= h(\ln(K_t(\theta), z_t) \\ &\vdots \end{aligned} \tag{21}$$

The functionals $g(\cdot,\cdot)$ and $h(\cdot,\cdot)$ are linear in their arguments. Composing $g(\cdot,\cdot)$ with itself produces the moving average representation of $\ln(K_t(\theta))$ in terms of z_t. The moving average representations of all the other variables can then be computed by exploiting the log-linearity of the policy functions. With these in hand, computing the correlations and standard errors for stationarity inducing transformations of the endogenous variables is straightforward. A technical appendix (in progress) describes the computational strategy used here in greater detail.

[5]See Press, Teukolsky, Vetterling, and Flannery (1992) for an explanation of quadrature approximation of integrals.

4 Solow Residual Measurement

An econometrician measuring total factor productivity with data on output, Y_t, labor input, N_t, and *effective* capital input, $\bar{K}_t$, would find no evidence of technology shocks. However, the measure of capital input used in most traditional Solow residual accounting exercises is at best a poor approximation of effective capital input. The capital stock data found in the national income and product accounts suffers from two shortcomings. First, it does not capture the increase in average plant productivity resulting from capital reallocation and new investment. This results in mismeasurement of average plant *quality*. Second, variation in the capital reallocation rate will cause the effective capital depreciation rate to change. The capital input data are constructed by assuming a constant rate of depreciation. This will cause mismeasurement of capital *quantity*. Together, these two forms of effective capital mismeasurement will cause the measured Solow residual to vary, even absent a technology shock of the sort found in real business cycle models.

Traditional Solow residual accounting uses data on output, labor input, and a measure of capital input, $\hat{K}_t$, to measure total factor productivity as

$$x_t = \ln(Y_t) - \alpha \ln(N_t) - (1-\alpha)\ln(\hat{K}_t). \tag{22}$$

Under the assumption of competitive labor and output markets, as in the model economy, the elasticity of output with respect to labor input, α is correctly identified with labor's share of output.

The most common source of $\hat{K}_t$ is the national income and product accounts. To construct it, the Department of Commerce applies a perpetual inventory method to the flow of newly constructed equipment. In the model economy, newly constructed equipment, NI_t, equals gross investment, I_t, less the scrap capital from exiting plants, S_t.

$$\hat{K}_{t+1} = (1-\hat{\delta})\hat{K}_t + NI_t \tag{23}$$

The national income accountant assumes that a fraction of the capital stock, $\hat{\delta}$, is destroyed every period. Newly constructed equipment replenishes this and adds to next period's capital stock.

To decompose the Solow residual into its two sources of measurement error, consider using the true stock of plants, $K_t^s \equiv \int_{-\infty}^{\infty} K_t(\theta)d\theta$. Movements of the Solow residual constructed with this measure of capital will only reflect shifts of the productivity distribution of plants, not the mismeasurement of their number. We can write the measured Solow residual as

$$x_t = (1-\alpha)(\ln(\bar{K}_t) - \ln(K_t^s)) + (1-\alpha)(\ln(K_t^s) - \ln(\hat{K}_t)). \tag{24}$$

The *ideal* Solow residual is first term on the right hand side of (24). Its only source of variation is quality mismeasurement. It is proportional to the log of the average plant productivity. Exogenous growth of the leading edge technology causes this to grow. Furthermore, changes in the diffusion of the leading edge technology will cause analogous movement in x_t. The remainder is error from quantity mismeasurement.

Changes in the capital reallocation rate will cause both sources of measurement error to fluctuate. If the response of capital reallocation to shocks in z_t is large enough, their combined effect will produce large fluctuations in the measured Solow residual. The next two sections assess whether the model adequately reproduces the Solow residual's observed behavior. Section 5 presents the identification and estimation methods used to assign values to the model's parameters and measure moments in the data of particular interest. Section 6 reports the results of the empirical analysis. In the estimated model, the measured Solow residual mimics the random walk behavior of its empirical counterpart.

5 Identification and Estimation

This section and the next address two quantitative questions. First, can the measurement error associated with technology diffusion adequately explain the observed Solow residual's behavior? Second, if this view of technology shocks is correct, can they account for a substantial fraction of output variance? The econometric strategy for answering the first question is to jointly estimate the model's parameters and the Solow residual's univariate autocorrelations using a just identified version of Hansen's (1982) generalized method of moments estimator. The hypothesis that the model reproduces the Solow residual's autocorrelations can be cast as a non-linear restriction on the estimated parameters and moments. Because they are jointly estimated, Wald tests of this restriction are then easily constructed. A measure of how well the model replicates observed output fluctuations is the ratio of output's standard error in the model economy to that in the U.S. economy. As in Eichenbaum (1991), this ratio is introduced as a parameter into the estimation. This allows quantification of the uncertainty about the model's performance along this dimension.

The estimation uses six quarterly time series. The first is the non-institutional civilian population in the United States over age 16. The remainder all cover the non-farm, non-government sector of the U.S. economy. They include labor's share of income, per capita real output (in 1987$), per capita real measured capital input, and two measures of per capita hours worked, household hours and establishment hours. The sample period begins in the first quarter of 1965 and ends in the last quarter of 1987. Data extending through the first quarter of 1993 is available, but is not yet entered into the computer. The data appendix contains more details about data sources and construction.

Throughout this exercise, three of the model's parameters will be held fixed. The consumers' subjective discount rate, β, is set equal to $1.03^{-1/4}$. This implies a 3% annual risk free interest rate on the steady state growth path of an analogous economy with no aggregate uncertainty. The capital depreciation rate, δ, is set equal to 0.025. Finally, the fraction of a consumer's time endowment spent at work in non-stochastic steady state is set equal to 1/3. The last two parameters are set to maintain comparability with the real business cycle literature.

The model's remaining parameters can be divided into two groups. The first are familiar from the real business cycle literature. They include the elasticity of output with respect to labor input, α, the mean and standard error of technological progress, μ_z and σ_z, and the rate of population growth, μ_p. These can be identified using only restrictions involving macroeconomic observables. The remainder describe the productivity distribution of the production sector and the cost of capital reallocation. With plant level data, their estimation would be relatively straightforward. However, without access to a data set such as the Longitudinal Research Database (LRD)[6], estimation requires an alternative strategy. The preliminary empirical investigation uses only macroeconomic data and the results of published empirical work with the LRD.

Identification of the first set of model parameters and second moments of the U.S. economy is the topic of the subsection 5.1. That which follows details how the second set could be estimated with plant level data and explains the procedure actually used in its absence. Subsections 5.3 and 5.4 set forth the joint estimation and testing procedures respectively.

5.1 Macroeconomic Identification

In addition to the standard error of output growth σ_y, the population second moments of interest are the standard error of Solow residual growth, σ_x, the contemporaneous correlation between these two variables, ρ_{xy}, and Solow residual growth's first four autocorrelations, $\rho_x^1 \ldots \rho_x^4$. With accurate data on per-capita output, per-capita hours worked, and the measured per-capita capital input, these can be easily estimated using the orthogonality restrictions which define them. Data on output and capital input are readily available. There exist two measures of hours worked. The first is from the Bureau of Labor Statistics' survey of establishment payrolls. The second is derived from the Current Population Survey. These are referred to as establishment hours and household hours respectively. Substituting these two series into equation (22) yields the establishment and household Solow residuals. Use ex_t and hx_t to denote them.

Following Prescott (1986) and Christiano and Eichenbaum (1992), the estimation allows for the possibility that establishment and household hours are error ridden measures of true labor input. Because the primary data sources are different, it is reasonable to suppose that the two measurement errors are independent. In this case, the cross moments computed with ex_t and hx_t consistently estimate the variance and autocorrelations of the true, unobserved Solow residual. In the presence of measurement error in hours worked, the following orthogonality conditions identify

[6] For a description of this data set, covering the population of manufacturing plants, see McGuckin and Pascoe (1988).

the population second moments of interest from the U.S. economy.

$$\begin{aligned}
E[\Delta y_t - \mu_y] &= 0 \\
E[\Delta ex_t - \mu_{ex}] &= 0 \\
E[\Delta hx_t - \mu_{hx}] &= 0 \\
E[(\Delta y_t - \mu_y)^2 - \sigma_y^2] &= 0 \\
E[(\Delta ex_t - \mu_{ex})(\Delta hx_t - \mu_h x) - \sigma_x^2] &= 0 \\
E[(\Delta y_t - \mu_y)(\Delta ex_t - \mu_{ex}) - \rho_{xy}\sigma_x\sigma_y] &= 0 \\
E[(\Delta hx_t - \mu_{hx})(\Delta ex_{t-1} - \mu_{ex}) - \rho_x^1\sigma_x^2] &= 0 \\
&\vdots \\
E[(\Delta hx_t - \mu_{hx})(\Delta ex_{t-4} - \mu_{ex}) - \rho_x^4\sigma_x^2] &= 0
\end{aligned} \tag{25}$$

The mean growth rates of output, the establishment Solow residual, and the household Solow residual are μ_y, μ_{ex}, and μ_{hx} respectively. Although the model implies that the three growth rates are the same, they are left unconstrained in the estimation.

Simple first moment restrictions provide identification for three of the model's parameters, α, μ_z, and μ_p. Under the assumption of competitive labor and output markets, the elasticity of output with respect to labor input equals labor's share of income. This allows the identification of α with the restriction

$$E[\alpha - \frac{w_t N_t}{Y_t}] = 0. \tag{26}$$

The model economy's average population growth rate is μ_p. This is easily identified with its empirical analog.

$$E[\Delta p_t - \mu_p] = 0 \tag{27}$$

As noted above, the economy's only source of per capita growth is the exogenous advancement of the leading edge technology. The growth rate of output will equal that of technological change when expressed in *labor* augmenting form. This implies that μ_z can be identified with the condition

$$\mu_y - \frac{1-\alpha}{\alpha}\mu_z = 0. \tag{28}$$

The log-linear approximation of the social planning problem's Euler equations and their numerical solution are invariant to the value of σ_z. Because the endogenous variable's correlations are scale invariant, they can be calculated using their moving average representations without taking a stand on the size of the unobserved technology shocks. This is not true when considering the standard error of output. In the real business cycle literature, the technology shock's standard error is chosen to match the Solow residual's. A similar procedure is available to estimate the standard error of this model's unobserved technology shocks. This is chosen so that the standard error of the traditionally measured Solow residual in the model economy is the same as that in the U.S. data.

$$\sigma_x - V_{xz}^m \sigma_z = 0 \tag{29}$$

The relative volatility of the measured Solow residual's growth rate and that of the true technology shock in the model economy is V_{xz}^m. This is a function of the model's parameters, but this notation is suppressed for convenience. To calculate the measured Solow residual from the model economy, the construction of the NIPA capital stock data must be simulated. This was done using a log-linear approximation of (23) along the non-stochastic steady state growth path. The assumed depreciation rate, $\hat{\delta}$, was chosen so that the capital measure is correct along this path.[7]

With the estimate of σ_z in hand, the definition of λ can provide its identification.

$$\sigma_y \lambda - V_{yz}^m \sigma_z = 0 \tag{30}$$

5.2 Microeconomic Identification

The scrap value of old firms, s, and the standard error of the idiosyncratic productivity shock, σ, do not have obvious empirical counterparts in the macroeconomic data. However, with a plant level data set, they can be estimated using the model's implications for the productivity and employment distribution across plants. The following discussion presumes that the Longitudinal Research Database (LRD) is available for the estimation.

Consider σ, the standard error of the plant level productivity innovation. If two adjoining quarterly observations on plant level value added, capital, and labor input were available, then σ could be easily estimated with the sample standard error of productivity growth across those plants which survived the first period. This strategy has two shortcomings. First, such data is not available in the LRD. Second, the assumption that θ_t^i follows a random walk implies that the unconditional distribution of a plant's growth rate is the same as that conditional upon its survival. It is not hard to modify the model so that this condition is violated but its macroeconomic implications are the same. To estimate σ using a cross section of plant level data, both of these problems must be overcome.

[7]In the estimated models, this involves only a small adjustment to δ, the true rate of capital depreciation.

To deal with the first problem, considerr the representative firm's labor allocation problem. Its solution implies that employment at a plant with productivity θ_t^i and capital stock s_t^i equals

$$N_t^i = e^{\theta_t^i} s_t^i N_t / \bar{K}_t \tag{31}$$

The model implies a one to one correspondence between a plant's labor input, (adjusted for its size) and its productivity level. This allows inference about the process governing θ_t^i using only data on N_t^i and s_t^i. Now consider a plant in production in period t that survives until period $t+1$. The growth rate of its employment to capital ratio over this period will be

$$n_{t+1}^i - n_t^i = \varepsilon_{t+1}^i + \ln(N_{t+1}/\bar{K}_{t+1}) - \ln(N_t/\bar{K}_t) \tag{32}$$

The logarithm of the plant's labor to capital ratio is n_t^i. The first term reflects the contribution of the idiosyncratic shock to employment growth at the plant. The remainder is the growth rate of the aggregate labor to effective capital ratio over this period. The LRD contains annual observations on a plant's beginning and end of period capital stock as well as quarterly observations on hours worked. This allows the construction of two adjoining quarterly observations, from the last quarter of one year and the first quarter of the next, of n_t^i. Equation (32) can substitute for equation (4) to provide an estimate of σ.

This alternative estimator is implementable, but it also suffers a sample selection problem. As Heckman (1976; 1979) showed, if the distribution of a plant's growth rate conditional upon its survival is different from the unconditional distribution, then the regression equation (32) is misspecified. The usual estimates of κ and σ are inconsistent. If a plant's productivity level follows a random walk, this will not be a problem. To change this without effecting the model's macroeconomic implications, add a temporary, idiosyncratic productivity disturbance. That is, let $\tilde{\theta}_t^i$ be plant i's productivity level at time t. It is the sum of two components, a permanent part and a transitory part.

$$\tilde{\theta}_t^i = \theta_t^i + \eta_t^i \tag{33}$$

The permanent component obeys the same stochastic process as it does above. The temporary component is *i.i.d.* across plants and across time.

$$\eta_t^i \sim N(0, \sigma_\eta^2) \tag{34}$$

The rule for labor allocation across plants remains the same. Plant i's labor input is a function of $\tilde{\theta}_t^i$ and s_t^i.

$$N_t^i(\theta) = e^{\tilde{\theta}_t^i} s_t^i N_t / \bar{K}_t \tag{35}$$

This distribution of η_t^i across plants is time invariant. Because η_t^i is also independent from θ_t^i, the representative firm's aggregate production function is unchanged up to a multiplicative constant. Furthermore, the asset pricing equations, (13) and (16), the zero profit condition on entry, (14), and the exit threshold, (12), remain unchanged. That is, only the distribution of the permanent component has aggregate implications.

The existence of η_t^i breaks the simple relationship between a plant's productivity growth and the growth of its labor to capital ratio. Modifying equation (32) to reflect this yields

$$n_{t+1}^i - n_t^i = \varepsilon_{t+1}^i + \eta_{t+1}^i - \eta_t^i + \ln(N_{t+1}/\bar{K}_{t+1}) - \ln(N_t/\bar{K}_t) \tag{36}$$

The variance of the right hand side of (36) is $\sigma^2 + 2\sigma_\eta^2$, so using the cross sectional standard error of adjusted labor growth to estimate σ is inappropriate.

Adding temporary idiosyncratic productivity shocks also weakens the tight connection between a plant's observed productivity level and its exit behavior. It is still the case that a plant will exit if $\theta_t^i < \theta_t$. However, the exit condition changes when written in terms of observables.

$$n_t^i - \eta_t^i < \theta_t + \ln(N_t/\bar{K}_t) \tag{37}$$

This induces a sample selection problem. The error term in inequality (37) is correlated with a plant's observed adjusted labor growth rate. This implies that the distribution of the growth rate conditional on survival is not the same as the unconditional distribution. Therefore, the sample variance of adjusted labor growth is not even a consistent estimator of $\sigma^2 + 2\sigma_\eta^2$.

In the presence of temporary idiosyncratic productivity shocks, the use of ordinary least squares to estimate equation (32) is inappropriate. However, the application of maximum likelihood methods yields consistent and asymptotically efficient estimates of σ and σ_η. In the taxonomy of Amemiya (1985), equation (36) and inequality (37) form the basis of a type 2 Tobit model.

To use the type 2 Tobit framework, rewrite equation (36) and inequality (37) in terms of unobserved dependent variables, the plant's adjusted labor growth rate and the difference between its permanent productivity level and the exit threshold, and observed regressors, the plant's current labor to capital ratio and a constant.

$$\begin{aligned} y_{i1}^* &= \kappa_1 + \varepsilon_{i1} \\ y_{i2}^* &= \kappa_2 + x_{i2} + \varepsilon_{i2} \\ y_{i1} &= \begin{cases} y_{i1}^* & \text{if } y_{i2}^* > 0 \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{38}$$

The plant's (possibly) unobserved growth rate is y_{i1}^*. The difference between θ_t^i and θ_t is y_{i2}^*. The exit threshold in period t and growth rate of the aggregate capital to labor ratio from t to $t+1$ are subsumed in κ_1 and κ_2. The plant's observed labor to capital ratio in the first period is x_{i2}. The error terms are idiosyncratic to the plant, and so are *i.i.d.* across observations.

$$\begin{aligned}
y_{i1}^{\star} &= n_{t+1}^{i} - n_{t}^{i} \\
y_{i2}^{\star} &= n_{t}^{i} - \eta_{t}^{i} - \theta_t - \ln(N_t/\bar{K}_t) \\
x_{i2} &= n_{t}^{i} \\
\varepsilon_{i1} &= \varepsilon_{t+1}^{i} + \eta_{t+1}^{i} - \eta_{t}^{i} \\
\varepsilon_{i2} &= -\eta_{t}^{i} \\
\kappa_1 &= \ln(N_{t+1}/\bar{K}_{t+1}) - \ln(N_t/\bar{K}_t) \\
\kappa_2 &= -\theta_t - \ln(N_t/\bar{K}_t)
\end{aligned}$$

The variance-covariance matrix of the disturbance vector, $\vec{\varepsilon_i} = (\varepsilon_{i1}, \varepsilon_{i2})'$ is

$$E[\vec{\varepsilon_i}\vec{\varepsilon_i}'] = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} = \begin{bmatrix} \sigma^2 + 2\sigma_\eta^2 & \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 \end{bmatrix} \tag{39}$$

Divide the sample of plants in production during period t into two groups, those which exit before period $t+1$ and those which remain in production. The first N_0 observations are in the former group, and there are N total observations. Using equation (10.7.6) in Amemiya (1985), the model's likelihood function can be written

$$\begin{aligned}
L &= \Pi_{i=1}^{N_0}[1 - \Phi\left(\tfrac{1}{\sigma_2}(\kappa_2 + x_{i2})\right)] \\
&\times \Pi_{i=N_0+1}^{N}\Phi\left(\{\tfrac{1}{\sigma_2}(\kappa_2 + x_{i2}) + \tfrac{\sigma_{12}}{\sigma_1^2\sigma_2}(y_{i1} - \kappa_1)\}\right. \\
&\times \left.\{1 - \tfrac{\sigma_{12}^2}{\sigma_1^2\sigma_2^2}\}^{-1/2}\right)\tfrac{1}{\sigma_1}\phi\left(\tfrac{y_{i1}-\kappa_1}{\sigma_1}\right).
\end{aligned} \tag{40}$$

The function $\Phi(\cdot)$ is the c.d.f. of a standard normal distribution. After imposing the restrictions on σ_1, σ_2, and σ_{12} from (39), the function may be maximized over κ_1, κ_2, σ, and σ_η to produce consistent and asymptotically efficient estimates of these parameters.[8]

In the absence of the necessary plant level data, this estimation strategy is infeasible. The preliminary empirical work presented in section 6 used a value of σ derived from previous work with the LRD. Hopenhayn and Rogerson (1993) report the results of an ordinary regression analysis on an equation like (32). There are

[8]Note that in this case, σ_η, the standard error of the selection equation, is identified. This is because the coefficient on the plant's adjusted employment in this equation is known a priori to equal one.

three differences between the equation they estimate and (32). First, instead of using the ratio of hours to the capital stock, they regress employment on lagged employment and a constant. Second, the span of time is five years instead of one quarter. Third, it does not impose a random walk on the process for θ. Although it is clearly inappropriate for the present purposes, this is the best available estimate of σ. The implied estimate of σ from this work is 0.0364.

The cost of reallocating capital to a plant with the leading edge technology is $1-s$. Raising this cost will, all else equal, lower the threshold below which a manager retires a plant for its scrap capital. The change in the scrap decision will imply changes in both the rate of capital reallocation and the cross sectional productivity distribution. This suggests that exit rates, productivity levels, or other functions of the cross sectional productivity distribution could be used to identify s.

To build intuition for how changing s effects the productivity distribution across plants, figure 1 displays the probability distribution of $\theta - \mu_z t$ and the exit threshold, $\theta_t - \mu_z t$ along the steady state growth path for three different values of s. For these calculations, α was set to equal labor's average share of income in the non-farm private economy, μ_z was set so that the average rate of growth in the model equaled that of output in the data, μ_p was set to the average population growth rate, and $\sigma = 0.0364$. The next subsection contains details the data used to produce these estimates.

Lowering s causes plants to delay exit. This reduces the volume of capital reallocation, and increases the productivity advantage that a newly constructed plant enjoys over incumbents. Table 1 reports statistics calculated from these distributions. The capital reallocation rate, the fraction of plants exiting in any period, doubles when s is increased from 0.8 to 0.9. The difference between the productivity of the leading edge process and that of the production sector's average plant declines comparably. Two forces influence this difference: a selection effect and a vintage capital effect. The exit of less productive plants will decrease the difference, and the exogenous improvement of new plants will increase it. Clearly, the latter effect dominates. The average plant's productivity level is that of the leading edge technology one to three years earlier. Decreasing s depresses the selection effect, and so increases this difference.

In the absence of data on plant entry and exit, indirect evidence must be employed. The model incorporates two channels of technology diffusion, capital reallocation and new plant construction. The volume of new investment is small relative to that of the total stock of plants. Therefore, if the accumulation of new plant's is an important channel of technology diffusion relative to capital reallocation, then productive innovations will only gradually diffuse throughout the production sector. Because s determines the cost of using capital reallocation, it is reasonable to suppose that it would effect the relative importance of the two technology diffusion channels. Even in the absence of rapid technology diffusion, output is responsive to changes in z_t. An innovation in the leading edge production process will change the relative price of investment (in efficiency units), and consumption. The intertemporal income and substitution effects associated with this price change will cause output to fluctuate even if technology diffusion is gradual.

This intuition suggests that it is possible to identify s using a second moment from the macroeconomic data. Consider one, the contemporaneous correlation of output growth and Solow residual growth. Figure 2 plots this moment versus s over the range $[0.75, 0.95]$. When s lies below this range, the volume of exit is ridiculously small.[9] If s is close to one, the impact of selection on average plant productivity overcomes the vintage capital effect, so new entrants are less productive than incumbents. In this case, increases in investment actually decrease average productivity.[10] Clearly, over much of the parameter space, the correlation is decreasing in s. To understand the intuition for this, consider the response of technology diffusion to an innovation in the leading edge technology. As noted above, increasing s raises the productivity threshold below which plants exit. This increases the volume of capital reallocation, so a given percentage change in its value will have a greater impact on average plant productivity. On the other hand, the average productivity of exiting plants is higher, so any given amount of reallocation will increase average productivity less. Over most of the parameter space, the second effect dominates the first. The impact of a shock to z_t on average plant productivity is decreasing in s. Only when s is close to one does decreasing the cost of diffusion increase the volatility of effective capital input.

Although it is preferable to work with plant level data, this variation in the correlation of output growth and Solow residual growth allows the identification of s with the moment restriction

$$\rho_{xy} - \rho_{xy}^{m} = 0. \tag{41}$$

The first term is the correlation of output growth and Solow residual growth estimated from the data. The second term is the analogous statistic implied by the model.

5.3 Estimation

Collect the orthogonality conditions, (25), (26), (27), (28), (29), (30), and (41) into a vector equation.

$$E[g(\gamma_0, X_t)] = 0 \tag{42}$$

The vector γ is a parameter vector containing the model parameters and population moments of interest. The true value of γ is γ_0. The operator $E[\cdot]$ denotes unconditional expectation. The vector X_t contains the data required to evaluate the function $g(\cdot,\cdot)$.

[9] When $s = 0.75$, the exit rate along the non-stochastic steady state growth path is 0.007.

[10] When $s = 0.95$, there is no productivity difference between the average incumbent and entering plants.

$$\gamma = \begin{bmatrix} \alpha \\ \mu_p \\ \mu_z \\ \sigma_z \\ \lambda \\ s \\ \sigma_y \\ \sigma_x \\ \rho_{xy} \\ \rho_x^1 \\ \vdots \\ \rho_x^4 \\ \mu_y \\ \mu_{ex} \\ \mu_{hx} \end{bmatrix} \quad X_t = \begin{bmatrix} \Delta y_t \\ \Delta k_t \\ \Delta en_t \\ \vdots \\ \Delta y_{t-4} \\ \Delta k_{t-4} \\ \Delta en_{t-4} \\ \Delta hn_t \\ \Delta p_t \\ ls_t \end{bmatrix} \tag{43}$$

The growth rates of per-capita establishment and household hours worked are Δen_t and Δhn_t respectively. Labor's share of output is ls_t, and the population growth rate is Δp_t. The dimension of $g(\cdot,\cdot)$ equals that of γ, so the parameters are just identified. In this case, Hansen's (1982) GMM estimator satisfies the sample analog of (42).

$$\frac{1}{T}\sum_{t=0}^{T} g(\hat{\gamma}, X_t) = 0 \tag{44}$$

The function $g(\cdot,\cdot)$ is continuous and differentiable in all of its arguments. Under the hypothesis that the model is the true data generating process, X_t is a stationary, normally distributed process. If it can be shown that the condition (42) is only satisfied at γ_0, then the consistency and asymptotic normality results of Hansen (1982) can be applied to give the distribution of $\hat{\gamma}$. Conditional on a value of s, there exists a unique solution to the remaining orthogonality conditions in (42). Figure 2 suggests that if the parameter space is restricted to exclude large and small values of s, then a unique value of s satisfies equation (41).[11]

With this caveat, $\sqrt{T}(\hat{\gamma} - \gamma_0)$ is asymptotically normally distributed.

$$\begin{aligned} \sqrt{T}(\hat{\gamma} - \gamma_0) &\sim N(0, \Sigma_\gamma) \\ \Sigma_\gamma &= \Gamma^{-1} S \Gamma'^{-1} \end{aligned} \tag{45}$$

The matrix $\Gamma = E_0[\frac{\partial g(\gamma_0, X_t)}{\partial \gamma}]$, and S is the spectral density matrix of $g(\gamma_0, X_t)$ evaluated at frequency zero. To compute the variance-covariance matrix of $\hat{\gamma}$, Γ is estimated with its sample analog and S with the procedure suggested by Newey and West (1987).

[11]With the model specifications estimated, there was always a unique solution to the sample analog of (41).

5.4 Testing

In a standard model of vintage capital, like that of Greenwood, Hercowitz, and Krusell (1992), the only channel for technology diffusion is the accumulation of new capital. Because the flow of investment is small relative to the stock of capital, this implies that technical innovations only gradually diffuse through the economy. In this case, total factor productivity growth is positively autocorrelated. The observed Solow residual's growth rate is uncorrelated with its lagged values. Simple vintage capital models fail to replicate this observation. The model of this paper adds a second diffusion channel, capital reallocation. Unless this channel is relatively important, the model will also fail to predict that the Solow residual follows a random walk. This suggests that a reasonable strategy for evaluating the ability of capital reallocation to account for observed total factor productivity fluctuations is to test whether the model can reproduce the Solow residual's autocorrelation structure.

The null hypothesis that the model reproduces the Solow residual's autocorrelation structure can be written as

$$H_0: \begin{array}{rcl} \rho_x^1 - \rho_x^{m1} & = & 0 \\ \rho_x^2 - \rho_x^{m2} & = & 0 \\ & \vdots & \\ \rho_x^j - \rho_x^{mj} & = & 0. \end{array} \tag{46}$$

The j'th autocorrelation of the Solow residual implied by the model is ρ_x^{mj}. The analogous population moment is ρ_x^j. This null hypothesis can be expressed as j continuous and differentiable non-linear restrictions on the elements of γ.

$$H_0: \quad R(\gamma_0) = 0 \tag{47}$$

As Ogaki (1992) shows, applying the delta method and (45) yields the asymptotic distribution of $\sqrt{T}R(\hat{\gamma})$ under H_0.

$$\begin{array}{rcl} \sqrt{T}R(\hat{\gamma}) & \sim & N(0, \Sigma_R) \\ \Sigma_R & = & \Gamma_R^{-1}\Sigma_\gamma\Gamma_R^{-1'} \end{array} \tag{48}$$

The matrix Γ_R is the gradient of $R(\cdot)$ with respect to γ. Under H_0, the standard distributional result about quadratic forms of normally distributed vectors implies that the Wald test statistic is asymptotically χ^2 distributed with j degrees of freedom.

$$TR(\hat{\gamma})'\Sigma_\gamma^{-1}R(\hat{\gamma}) \sim \chi_j^2 \tag{49}$$

The next section reports the results of applying these Wald tests to the estimated model when $j = 1, \ldots, 4$.

6 Empirical Results

This section contains the results of applying the empirical methodology detailed in the previous section to the U.S. data. The estimation procedure was applied to two different model specifications. In the first, the consumers' momentary preferences are log-linear in leisure. In the second, they are linear in leisure. These are referred to as log and Hansen preference specifications respectively. In both of the estimated models, capital reallocation is an important channel for technology diffusion. The volume of entry and exit increases following a positive shock to z_t. This produces a sizable, permanent increase in the measured Solow residual. Thereafter, the innovation gradually diffuses through the production sector through the accumulation of new capital goods. The model's Solow residual adequately mimics the dynamic behavior of its empirical counterpart. The Wald tests provide very little evidence against the null hypothesis that the autocorrelations of Solow residual growth are the same in the model and U.S. economies.

6.1 Productivity in the U.S. Economy

The macroeconomic phenomenon which motivates this work is the large, procyclical movement in the Solow residual. Table 2 provides estimates of the standard errors of total factor productivity and output growth, the contemporaneous correlation between these two variables, and the first four autocorrelations of Solow residual growth. It also provides the estimates' standard errors. Quarterly fluctuations in Solow residual growth are nearly as large as those of output growth. The standard errors of both are close to 1%, and the estimated correlation between the two is 0.81. Individually, the autocorrelations of Solow residual growth are all insignificantly different from zero. This is the foundation of the observation that the Solow residual essentially follows a random walk.

6.2 The Estimated Models

Tables 3 and 4 report the estimates of the model's parameters for both preference specifications. By construction, the parameter estimates for α, μ_p, and μ_z are the same across the specifications. Table 3 reports their estimates standard errors. Labor's average share of output is 2/3. Annual population growth averages 1.6%. The average annual rate of per capita output growth over this period is 1.5%. Equation (28) implies that the productivity of the leading edge production process grows at double this rate.

The parameter estimates for s, σ_z, and λ differ across specifications. Table 4 reports their estimates and standard errors. Both specifications indicate that the cost of reallocating a unit of capital between plants is about 0.11 units of the aggregate good. This implies that the productivity distribution along the non-stochastic steady state growth path will be nearly identical in the two specifications. Figure 3 graphs this distribution when $s = 0.89$. Table 5 reports its population statistics. The distribution is slightly skewed and centered to the left of the leading edge productivity level. The average plant's productivity is that obtainable with the leading edge production process one year earlier. However, the marginal exiting plant is 24% less productive than the average entrant. About 2% of all plants exit each quarter.

In the model economy, a positive shock to the leading edge production process lowers the price of effective investment in terms of consumption goods and leisure. As Barro and King (1984) and Greenwood, Hercowitz, and Huffman (1988) have noted, this will have intertemporal income and substitution effects for a representative consumer. The substitution effect raises investment, raises labor effort, and lowers current consumption. The income effect from the price change works against this. The upper panels of figures 4 and 5 plot the responses of output, consumption, net investment, hours worked, and average labor productivity to a unit impulse in z_t. In the estimated models, the substitution effect dominates. With the log preference specification, consumption slightly declines, hours worked increase 0.25%, output increases 0.16%, and investment increases 1%. With the Hansen preference specification, labor supply is much more elastic. Accordingly, the signs of the variables' responses are the same, but their magnitudes are greater.

An improvement in the leading edge production process also influences the representative firm's plant retirement decisions. By increasing new plants' productivity, process innovations make capital reallocation more attractive. This is its primary effect. The lower left panels of figures 4 and 5 plot the responses of entry and exit to a one percent shock to z_t. In both models, a one percent innovation to the leading edge technology causes a 30% to 35% temporary increase in the volume of capital reallocation, S_t. The following period, very few low productivity firms are left, so reallocation temporarily declines. The surge in capital reallocation moves capital from the lowest productivity plants to the highest. This has noticeable effects upon the measured Solow residual. The lower right hand panel of the figures plots the response of the ideal Solow residual and that measured using the NIPA capital stock to a shock to z_t. The increase in average plant productivity causes a 0.45% jump in the ideal Solow residual. However, the increase in the depreciation rate significantly dampens this effect. The measured Solow residual increases only 0.15% following the reallocation. Thereafter, it slowly climbs to a permanently higher level.

In the period following the capital reallocation, the resulting productivity increase causes macroeconomic aggregates to react much as they would in an RBC model to a standard technology shock. Output appears to follow a random walk. There is a temporary increase in investment, and consumption slowly grows to its new permanent level. Hours worked rise temporarily and then fall back to their steady state. As would be expected, hours react more severely when the preferences are linear in leisure.

The timing of reallocation and output growth produces an interesting result. Although the productivity increase from capital reallocation causes output growth, exit is *countercyclical.* After the surge in exit, few low productivity plants remain. This causes both entry and exit to decline one period after a shock to z_t. At the same time, the increase in average plant productivity causes output to expand. In the estimated model with the log preference specification, the correlation between output growth and exit growth is -0.40. With the Hansen preference specification, it is -0.43. In studies of job reallocation across manufacturing plants, Davis and Haltiwanger (1990; 1992) observed that the reallocation of jobs across manufacturing plants is countercyclical. The selection model with stochastic production process improvement naturally reproduces the finding of countercyclical resource reallocation.

The measured Solow residuals from the model economies do not follow a random walk, but they are not dissimilar from one either. Table 6 presents the first four autocorrelations of Solow residual growth in the estimated models. In both cases, they are all very close to zero. Wald tests of the null hypothesis, (46), for $j = 1, \ldots, 4$ confirm this judgment. Table 7 presents these test statistics. With the log preference specification, the test statistics never exceed conventional significance levels. With Hansen preferences, the test statistic exceeds the 10% significance level when $j = 4$. In the other cases, the tests fail to reject the null hypothesis.

The response of the measured Solow residual to an innovation in z_t is weak relative to that of output. The estimates of λ and σ_z in table 4 reflect this. In order to match the Solow residual's standard error, the underlying technology shock must fluctuate 6% per quarter with the log preference specification. With the Hansen preferences, this estimate is only slightly lower. Although these seem large, they are not ridiculously so. In a standard RBC model with a Cobb-Douglas production function, the standard error of capital embodied technological change must equal 0.255% to match σ_x. The large point estimates of λ reflect the estimates of σ. The standard error of output growth in the model with log preferences is 1.5 times its value in the U.S. economy. The analogous statistic for the model with Hansen preferences is 2.3. It would be ridiculous to say that the model accounts for 230% of the observed fluctuations in output. The statistic $1/\lambda$ has a more sensible interpretation. This is the fraction of Solow residual fluctuations which the model reproduces when σ_z is set to match the observed volatility of output. With this identifying assumption for σ_z, the log and Hansen specifications account for 66% and 43% of Solow residual volatility respectively. Neither of these identifying assumptions for σ_z is satisfactory. An estimate of σ_z based on the volatility of capital reallocation in the U.S. economy, would provide a sounder judgement of whether capital reallocation and technology diffusion are responsible for significant fractions of output and productivity fluctuations.

6.3 Productivity Dynamics and Reallocation Costs

In light of the seemingly precise estimates of the model's parameters in tables 3 and 4, it seems odd that the estimates of λ have such large standard errors. The source of this imprecision is the sensitivity of the observed Solow residual to changes in s. Figures 6, 7, and 8 illustrate this by graphing the Solow residual's response to process innovations for three different values of s. In figure 6, $s = 0.8944$, its estimate with the log preference specification. The standard error of this estimate, 0.016 is added and subtracted from this baseline value in figures 7 and 8. In the estimated model, the Solow residual increases 0.13% after a 1% innovation in z_t. After the reallocation, output has increased 0.4%. Lowering s increases the response of the Solow residual without greatly changing the path of output. When $s = 0.878$, the Solow residual increases by 0.16% in the period after a technology shock. On the other hand, when $s = 0.91$, this is dampened to 0.09%.

Increasing plants' scrap value has two effects on the response of productivity to increases in capital reallocation. First, if s is higher, then plants with higher productivity are retired. Second, a greater volume of plants will be retired. The first effect implies that a given amount of capital reallocation will have *less* of an impact on average plant productivity. However, the greater volume associated with the second effect may overcome this. In the estimated model the first effect dominates the second. The upper right panels of figures 6, 7, and 8 confirm this. Increasing s dampens the response of effective capital input, $\bar{K}_t$, to innovations in z_t. The greater volume of capital reallocation can not counter the smaller average productivity improvement, but it does cause the volume of capital lost during reallocation to increase. This is in spite of the decline in the average capital lost at each retiring plant. These two effects combine to dampen the measured Solow residual's fluctuations and leave the ideal Solow residual unchanged.

Examining larger changes in s makes the influence of capital reallocation costs on productivity dynamics more apparent. Figure 9 is analogous to figure 4, but with $s = 0.925$. After an improvement in the leading edge production process, the measured Solow residual slowly rises. In no way could this reproduce the movements of observed total factor productivity. Figures 10 and 11 sharply contrast with this. They graph the model's responses when $s = 0.85$ and $s = 0.75$ respectively. In the first case, the model's behavior is qualitatively the same, but the influence of capital reallocation on the Solow residual is much stronger. Measured total factor productivity increases 0.2% following a 1% innovation in z_t. When s is further lowered to 0.75, the standard macroeconomic aggregates react to technology shocks in a different fashion. Capital reallocation causes the measured Solow residual to jump almost instantly to its new, long run level. The income effect associated with this productivity increase causes hours worked, investment, and output to *fall* in the period of a process innovation. In the following period, they rise as they do in the estimated model. Because the Solow residual is much more responsive to process innovations, the model requires smaller shocks to replicate σ_x. In this case, $\lambda = 1.25$.

7 Concluding Remarks

Although the empirical results are tentative, they are also promising. In principle, the endogenous diffusion of process innovations through capital reallocation is capable of reproducing the observed Solow residual's behavior. Exogenous advancement in the leading edge technology causes the volume of capital reallocation to drastically rise. The shift of resources from the least productive plants to those with access to the leading edge technology causes measured total factor productivity to suddenly and permanently rise. Tests of the hypothesis that this reproduces the Solow residual's random walk behavior can not reject it.

The interpretation this work gives the Solow residual stands in contrast to that of the Real Business Cycle literature. Authors such as Kydland and Prescott (1982), Hansen (1985), and King, Plosser, and Rebelo (1988a; 1988b) identify it with an exogenous, neutral technology shock effecting the economy's entire production sector. The explanation for the Solow residual fluctuations based on technology diffusion has two advantages over this simple model of technological improvement. First, the technology shocks only effect a subset of plants, those under construction. In this sense, the model relies less on exogenous changes in the aggregate production set to generate fluctuations. Second, the model's microeconomic implications allow the application of plant level data to test the theory.

One such implication concerns the cyclical behavior of resource reallocation. Because it eliminates low productivity plants, the increase in reallocation causes a reduction in exit the following period, when output growth increases. This produces a negative correlation between the growth rate of exit and that of output. Although there are no jobs in the model, this provides an interesting perspective of the observation by Davis and Haltiwanger (1990; 1992) that inter-plant job reallocation in the manufacturing sector is countercyclical.

The ability to address microeconomic empirical issues in the context of macroeconomic fluctuations is one of the model's strengths which the preliminary empirical analysis did not fully exploit. Using plant level data in estimation would produce more reliable estimates of the model's microeconomic parameters and test a selection model of entry and exit. This will be the focus of future empirical work. Producing an accurate assessment of the model's ability to reproduce the observed total factor productivity fluctuations is difficult without this analysis.

8 Data Appendix

The empirical work used six quarterly time series: The United States' population, real GDP, The NIPA measure of the capital stock, hours worked from the establishment survey, hours worked from the household survey, and labor's share of output.

- The population measure is the civilian non-institutional population over age 16. It is produced by the Bureau of Labor Statistics.
- Real GDP is measured as seasonally adjusted total GDP in 1987$ less the output of farms and government.
- The capital stock, as reported in the NIPA, is the fixed, non-residential private capital for all industries measured at constant cost less the analagous measure for farms.
- Establishment hours were constructed by multiplying the number of employees of private, non-farm establishments by the average weekly hours worked production or non-supervisory employees at these same establishments. This data is compiled from a survey of employer payrolls by the Bureau of Labor Statistics. The resulting time series was seasonally adjusted using the EZ-X11 computer program.
- Household hours were constructed in the same manner as establishment hours. The data were compiled from the current population survey by the Bureau of Labor Statistics.

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Table 1: Statistics from Model Cross Sectional Productivity Distribution

	$s = 0.80$	$s = 0.85$	$s = 0.90$
Mean Productivity	-0.078	-0.057	-0.032
Standard Error	0.135	0.120	0.106
Reallocation Rate	0.010	0.015	0.022
Exit Threshold	-0.380	-0.307	-0.230

The mean productivity is measured relative to z_t.
The exit threshold is measured relative to z_t.
The reallocation rate equals the fraction of plants exiting each quarter.

Table 2: Estimated Second Moments

Moment	Estimate	Standard Error
σ_x	0.0085	0.0008
σ_y	0.0113	0.0011
ρ_{xy}	0.805	0.041
ρ_x^1	0.058	0.110
ρ_x^2	0.155	0.089
ρ_x^3	0.095	0.073
ρ_x^4	0.002	0.083

σ_x = standard error of total factor productivity growth.
σ_y = standard error of output growth.
ρ_x^j = j'th autocorrelation of total factor productivity growth.

Table 3: Common Parameter Estimates

Parameter	Estimate	Standard Error
α	0.66	0.004
μ_z	0.0067	0.0026
μ_p	0.0041	0.0004

α = elasticity of output with respect to labor input.
μ_z = mean quarterly growth rate of leading edge technology.
μ_p = mean quarterly population growth rate.

Table 4: Specification Parameter Estimates

Log Preferences

Parameter	Estimate	Standard Error
s	0.8944	0.016
σ_z	0.061	0.009
λ	1.54	0.19

Hansen Preferences

Parameter	Estimate	Standard Error
s	0.8877	0.049
σ_z	0.056	0.009
λ	2.27	0.26

s = scrap value of exiting plants.
σ_z = standard error of leading edge technology improvement.
λ = ratio of output growth standard error in model and U.S. economies.

Table 5: Statistics from Cross Sectional Productivity Distribution: Estimated Model

	$s = 0.8944$
Mean Productivity	-0.034
Standard Error	0.108
Reallocation Rate	0.021
Exit Threshold	-0.239

The mean productivity is measured relative to z_t.
The exit threshold is measured relative to z_t.
The reallocation rate equals the fraction of plants exiting each quarter.

Table 6: Population Autocorrelations from Estimated Models

Log Preferences

Moment	Estimate	Standard Error
ρ_x^1	-0.050	0.032
ρ_x^2	0.033	0.012
ρ_x^3	0.054	0.012
ρ_x^4	0.061	0.012

Hansen Preferences

Moment	Estimate	Standard Error
ρ_x^1	-0.045	0.064
ρ_x^2	0.032	0.006
ρ_x^3	0.050	0.007
ρ_x^4	0.055	0.008

ρ_x^j = j'th autocorrelation of total factor productivity growth.

Table 7: Wald Tests of Solow Residual Behavior

Log Preferences

Moments	Test Statistic	Degrees of Freedom
$\rho_{\Delta x}^1$	0.813	1
$\rho_{\Delta x}^1, \rho_{\Delta x}^2$	2.64	2
$\rho_{\Delta x}^1, \ldots, \rho_{\Delta x}^3$	3.23	3
$\rho_{\Delta x}^1, \ldots, \rho_{\Delta x}^4$	8.20*	4

$\star$ Surpasses the 10% critical value.

Hansen Preferences

Moments	Test Statistic	Degrees of Freedom
$\rho_{\Delta x}^1$	0.558	1
$\rho_{\Delta x}^1, \rho_{\Delta x}^2$	2.57	2
$\rho_{\Delta x}^1, \ldots, \rho_{\Delta x}^3$	2.88	3
$\rho_{\Delta x}^1, \ldots, \rho_{\Delta x}^4$	6.93	4

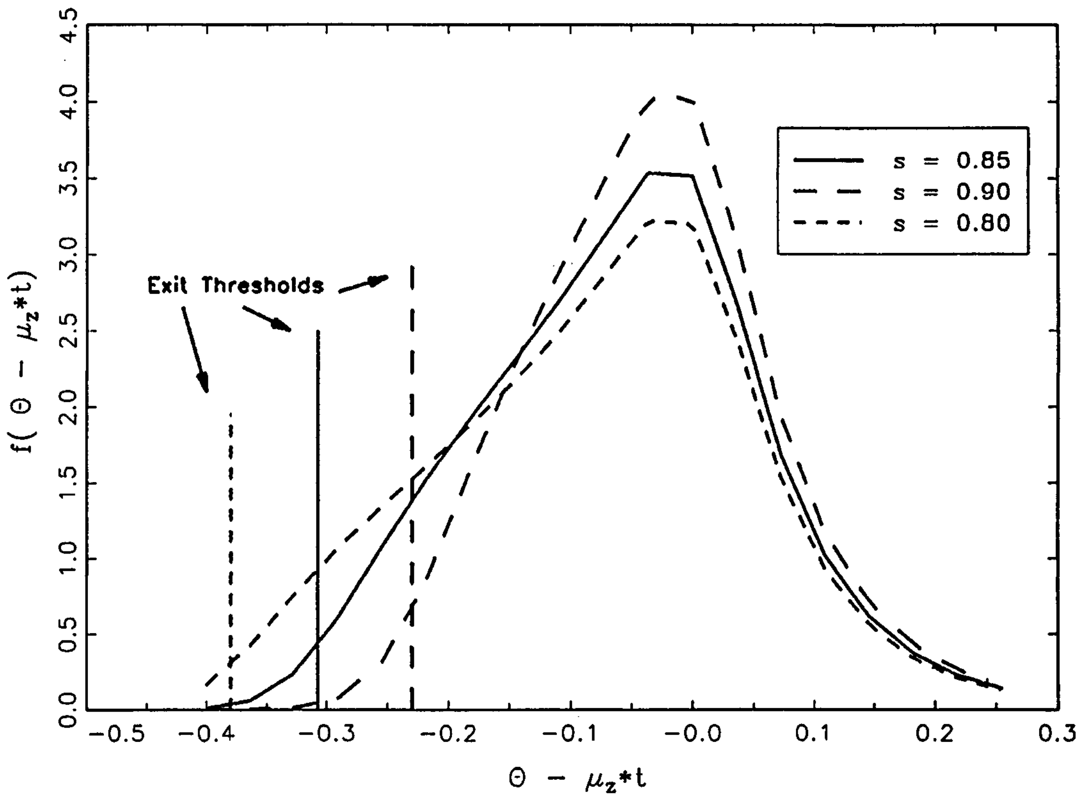

Figure 1: Cross Sectional Productivity Distributions

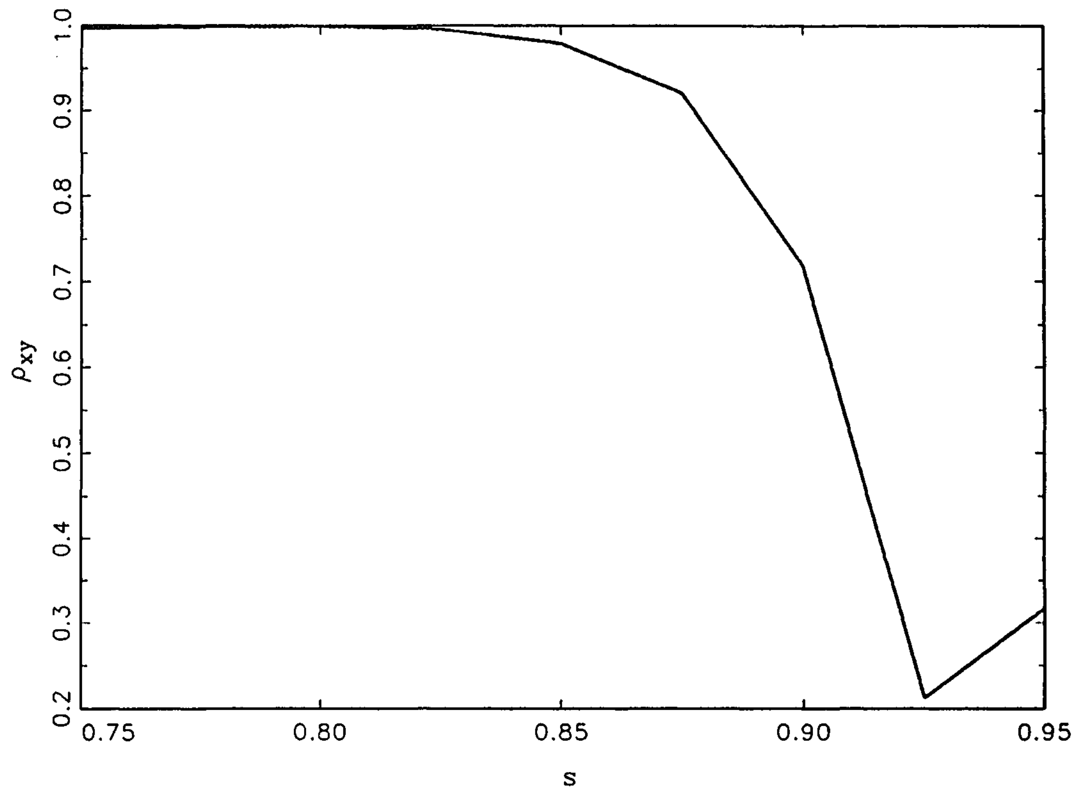

Figure 2: Identification of s

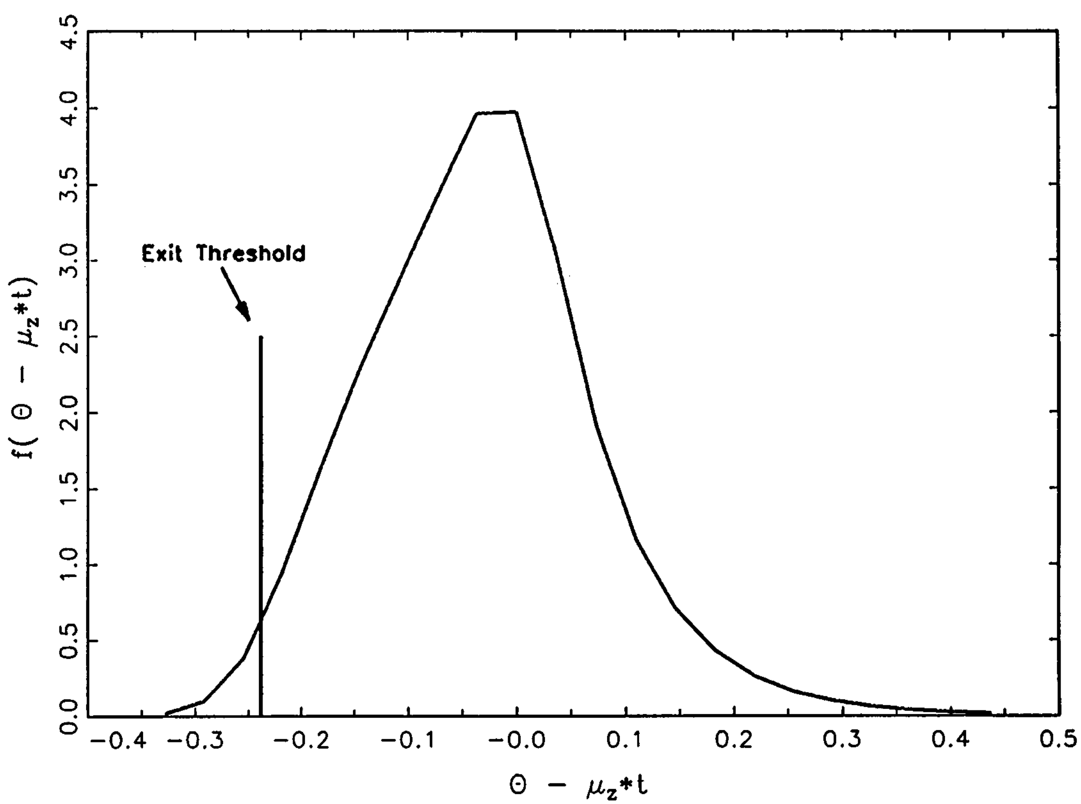

Figure 3: Cross Sectional Productivity Distribution: Estimated Model

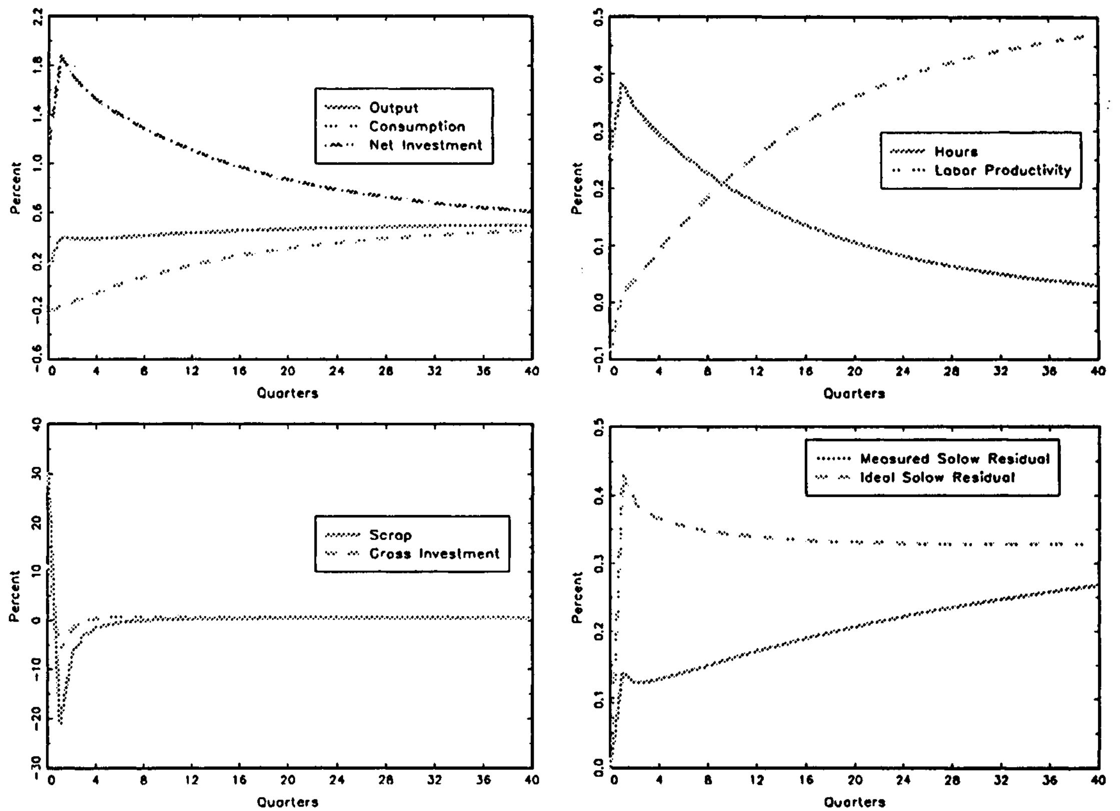

Figure 4: Responses in Estimated Model: Log Preferences

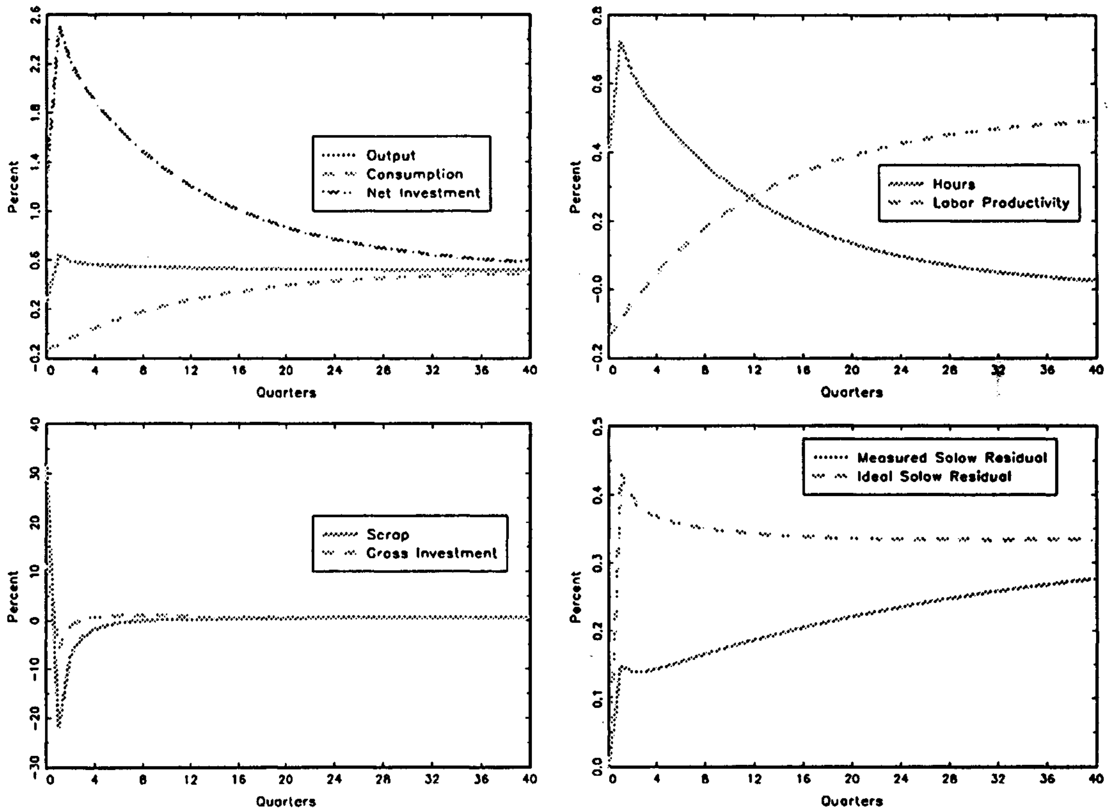

Figure 5: Responses in Estimated Model: Hansen Peferences

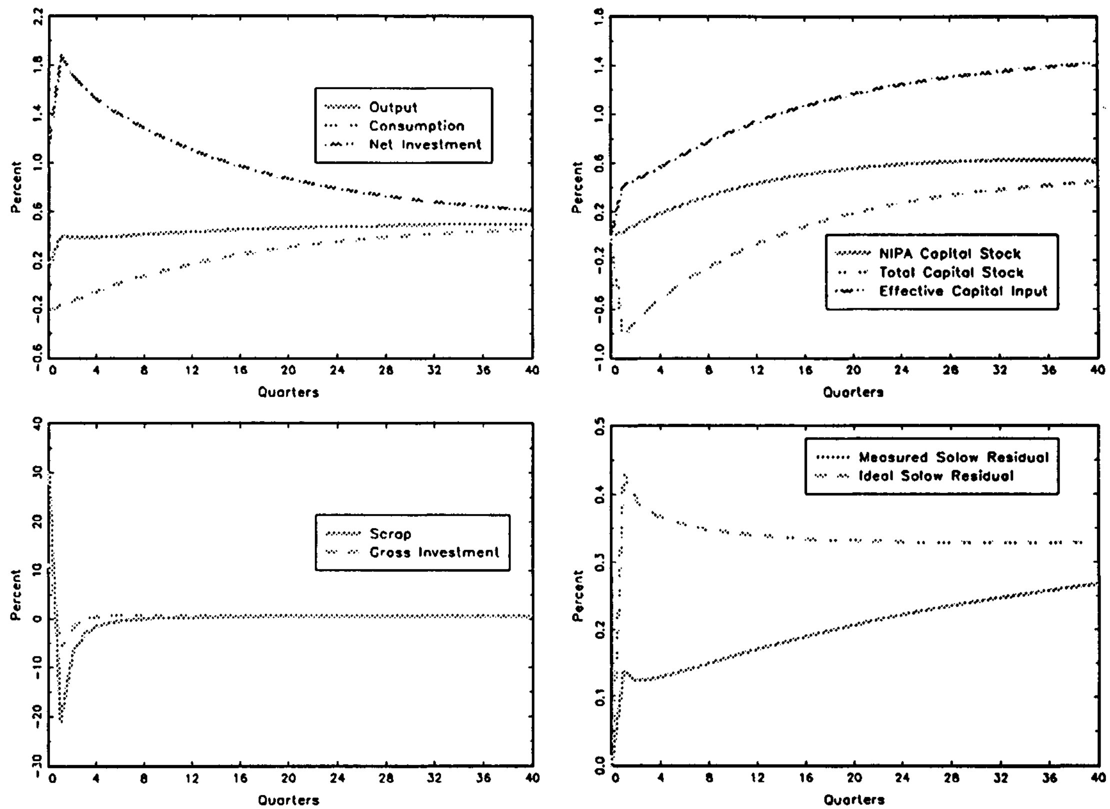

Figure 6: Productivity Response: s=0.895

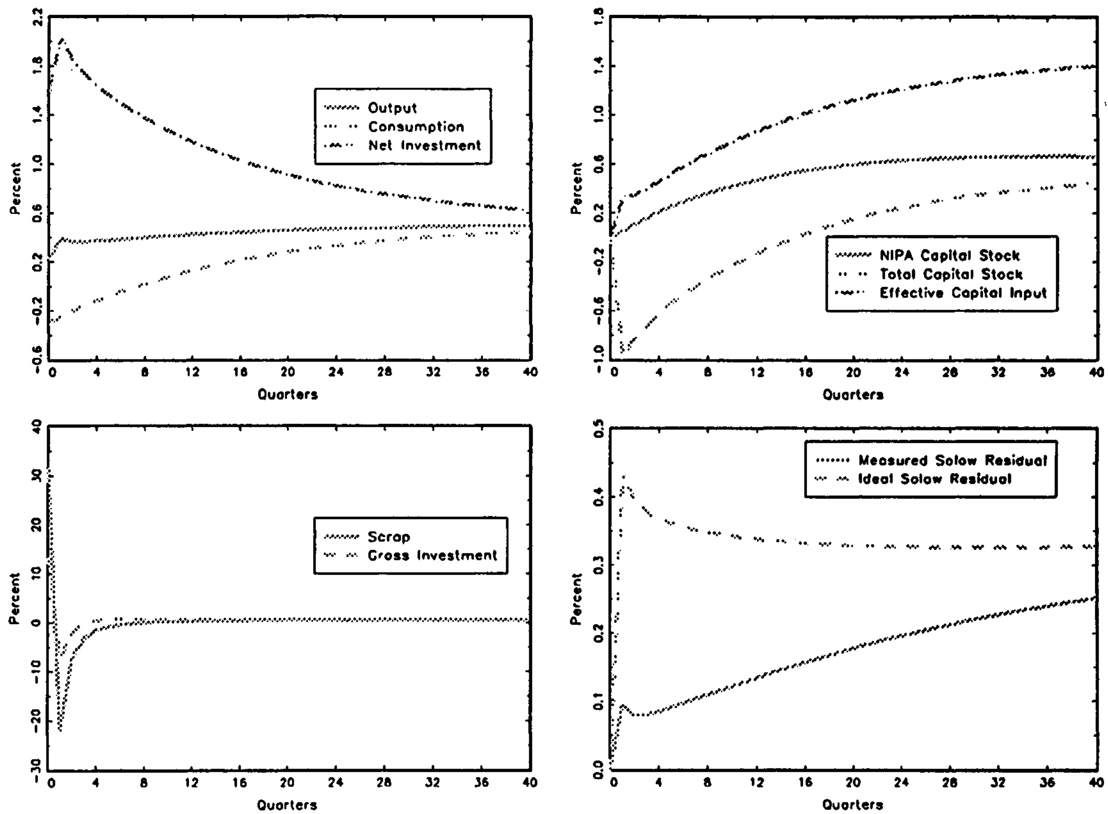

Figure 7: Productivity Response: s=0.910

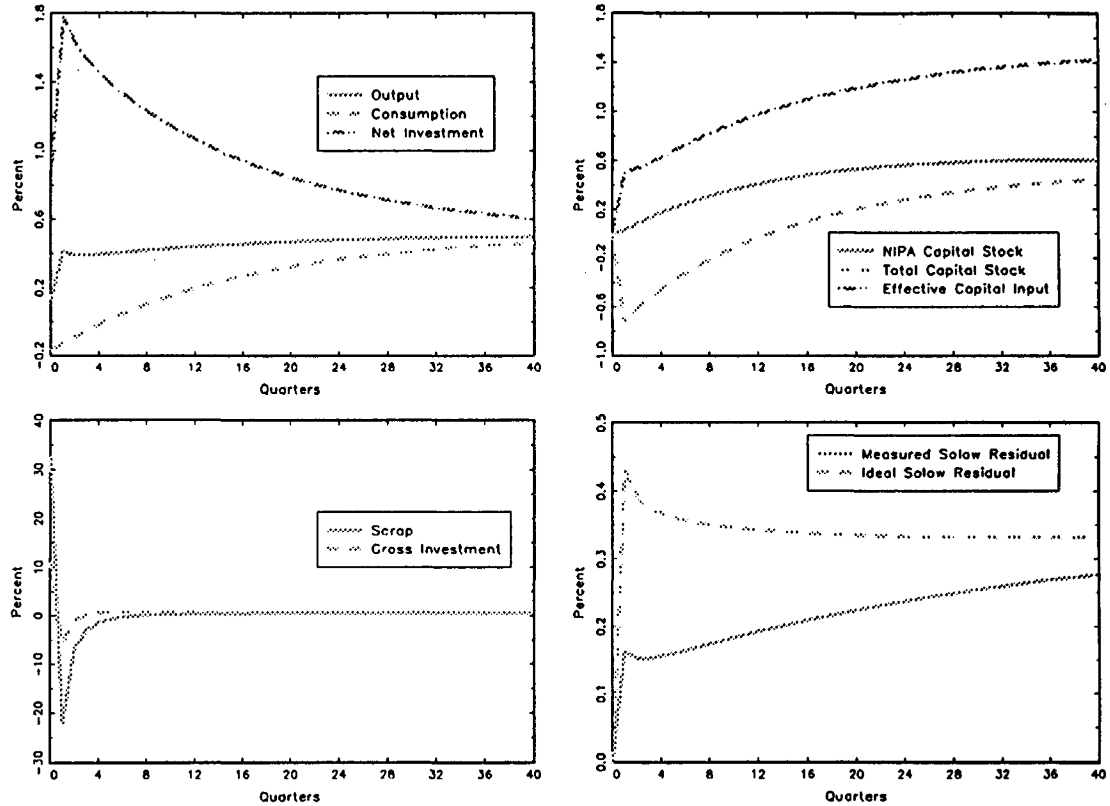

Figure 8: Productivity Response: s=0.878

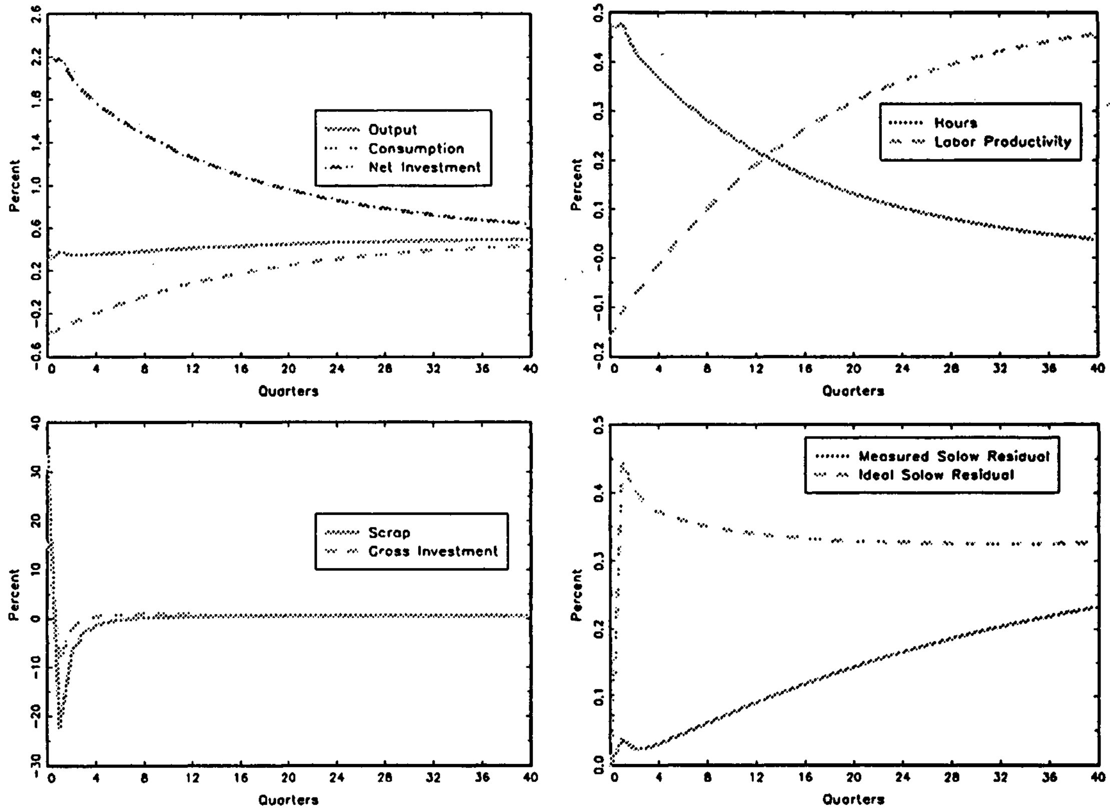

Figure 9: Productivity Response: s=0.925

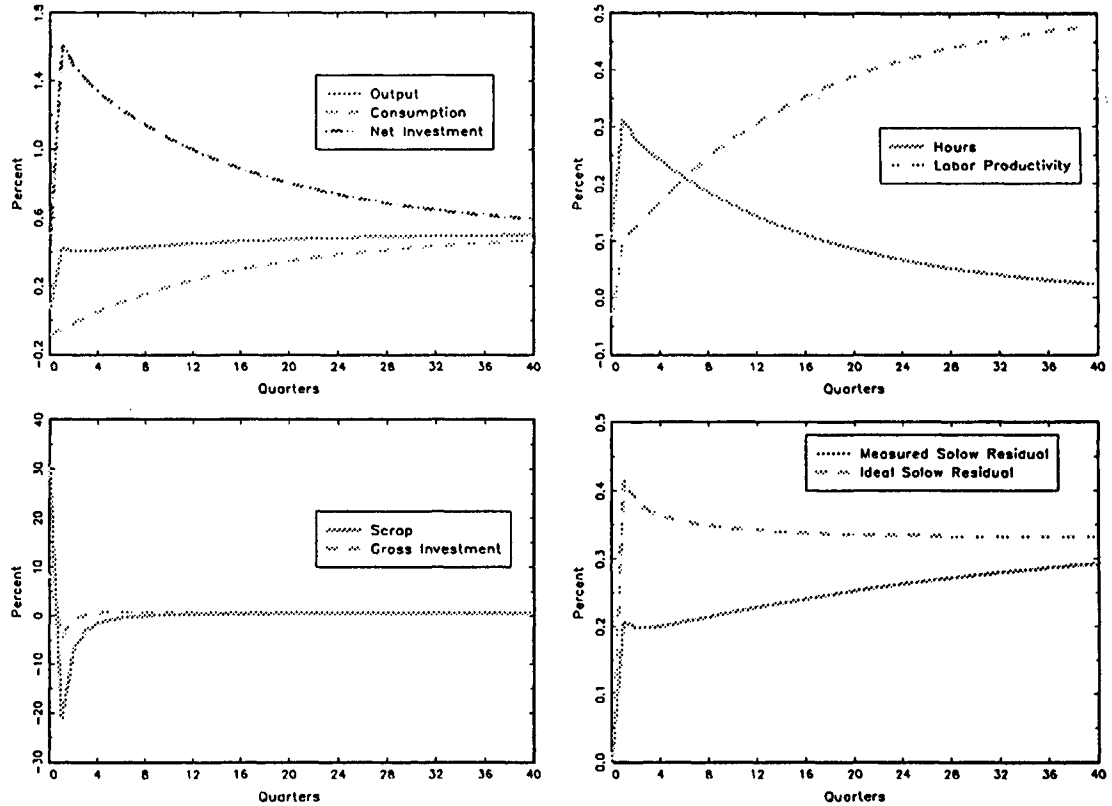

Figure 10: Productivity Response: s=0.85

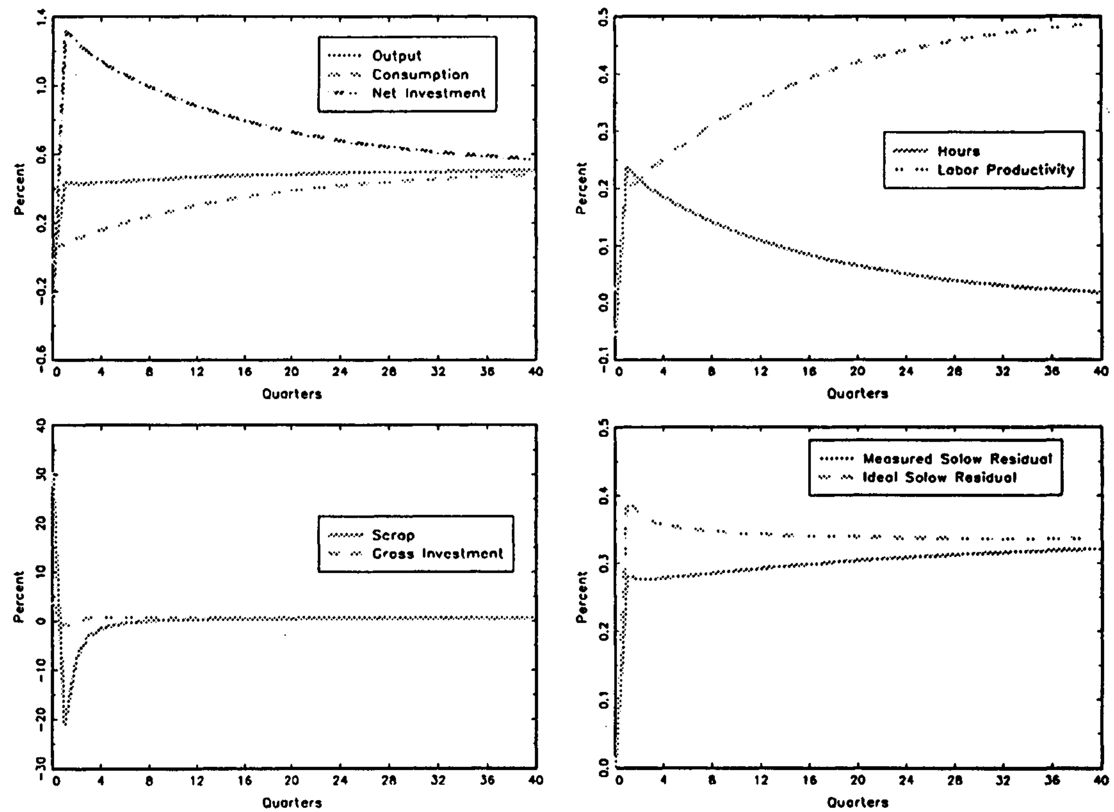

Figure 11: Productivity Response: s=0.75