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ABSTRACT

This paper proposes a new method for estimating potential output in which potential real GDP is modeled as stochastic trend, and deviations of GDP from potential affect inflation through an aggregate supply relationship. The output and inflation equations together form a bivariate unobserved-components model, which is estimated via maximum likelihood through the use of the Kalman filter algorithm. The procedure yields a measure of potential output and its standard error, and an estimate of the quantitative response of inflation to real growth and the output gap.

Keywords: Natural real GDP, stochastic detrending, Kalman filter, state-space models.

Potential output lies at the heart of many macroeconomic models, such as the familiar textbook aggregate demand and aggregate supply analysis. In these models, excess demand or the "output gap," defined as the difference between real GDP and potential output, is the primary short-run determinant of inflation. This view matches Okun's (1970, pp. 132–3) original definition as "the maximum production without inflationary pressure, ... or more precisely ... the point of balance between more output and greater stability." While potential output tends to be associated with "Keynesian" macroeconomic analysis, the underlying empirical regularity is not unique to any specific model. As Lucas (1972, p. 103) noted, the "systematic relationship between the rate of change of prices and the level of real output" is a central feature of the modern business cycle.

Conceptually, it is natural to define potential output in terms of the economy's supply side. As Hall and Taylor describe it (1991, p. 16), potential GDP is "the amount of output that would have been produced had the economy been in neither boom nor recession ... from the existing capital stock and labor force." To emphasize its interpretation as a measure of equilibrium output (as opposed to the economy's maximum level of production), Gordon (1990, p. 10) suggests the term "natural" real GDP.

Its association with stable inflation and equilibrium employment means potential output is clearly an appropriate target for macroeconomic policy (Boschen and Mills 1990). Unfortunately, its use in the policy arena is hampered by the inability to observe potential output directly. Consequently, macroeconomic policymakers have had to rely on *ad hoc* estimates that often, as in the 1970s, turned out in retrospect to be quite inaccurate. Similarly, empirical work on inflation has usually proceeded in two steps: first, constructing a proxy for potential output, and then estimating an inflation equation conditional on that estimate.

This paper proposes a new method for estimating potential output using inflation and real output data. The method treats potential real GDP as a latent stochastic trend as in Watson (1986). Deviations of GDP from this trend are linked to inflation through an aggregate supply relationship. The output and inflation equations together form a bivariate unobserved-components model, an example of the dynamic Multiple Indicator Multiple Cause (MIMIC) specification described by Watson and Engle (1983). The model is estimated by maximum likelihood through the use of the Kalman filter algorithm.

From a policy perspective, this method has three significant advantages over traditional measures of potential output. First, its explicit dependence on inflation endows it with more economic content than measures derived from purely univariate methods. Second, the stochastic trend specification allows for continuous, smooth adjustment of the estimate in real time as new data become available. The procedure's third significant advantage is its ability to estimate the uncertainty associated with the series.

The results turn up two additional findings of economic interest. First, the estimates yield a relatively precise measure of the speed of inflation's response to the output gap, which determines the "sacrifice ratio." The second implication of the bivariate model is that the estimated the unit root in real GDP is considerably larger than indicated by a univariate stochastic trend model.

The remainder of the paper proceeds as follows. To motivate the formulation of the new technique, Section 1 surveys a selection of the better-known potential output measures, and discusses some of their drawbacks. Section 2 contains a preliminary analysis of the time-series properties of output and inflation relevant to the specification of the bivariate unobserved-components potential output model developed in Section 3. Section 4 concludes.

1. A SURVEY OF POTENTIAL OUTPUT MEASURES

Numerous measures of potential output have appeared (and subsequently disappeared) in the three decades since Okun originated the idea. This section provides a brief overview of existing methods, their evolution, and a brief critical appraisal.

1.1 Univariate trend-fitting and filtering

The simplest imaginable way to construct a potential output series is to fit a linear trend through the logarithm of real GDP, and until the early 1970s, this was by far the most common method. This procedure ran into trouble, however, with the oil price shocks and productivity slowdown of the 1970s. As significant output gaps persisted for years on end, it became apparent that a linear trend was not a particularly useful description of the economy's equilibrium level of output.

As a result, the 1970s saw the abandonment of the linear trend method in favor of techniques that could accommodate a changing underlying growth rate. One alternative was to allow discrete breakpoints in the growth rate of potential output, resulting in segmented-trend series. Examples of this approach include Gordon (1975, 1977), and the "mid-expansion" series published by the Bureau of Economic Analysis, which is sometimes used as a potential output measure.

Another alternative combining draftsmanship and judgment was the "flexible ruler" method, which required fitting a smooth, time-varying trend line through a plot of real GDP. In a sense, the Hodrick-Prescott (1980) filter can be thought of as a modern, relatively judgment-free version of this procedure, and it is often used to generate a proxy for potential output. In fact, part of its attractiveness apparently lies in its resemblance to the flexible ruler technique. In offering their rationale for setting the smoothness parameter (λ) equal to 1600, Kydland and Prescott (1990, p. 9) explain that the resulting trend is "close to the one that students of business cycles and growth would draw through a plot of the series."

A third technique relies on the stochastic detrending procedure proposed by Watson (1986), and Clark (1989). As described in greater detail below in section 2.1, this procedure involves fitting a univariate unobserved-components model to real GDP, and extracting an estimate of the latent stochastic trend component.

1.2 Production function methods

The most sophisticated approach to the measurement of potential output attempts to relate the economy's productive capacity to available factor inputs and total factor productivity. Use of this technique was widespread during the 1970s, and included work by Clark (1979), Perry (1979) and Perloff and Wachter (1979). It was also used to produce the series maintained by the Council of Economic Advisers (CEA), and published in the *Economic Report of the President* until 1982.

In principle, this procedure is straightforward. Inserting full-employment labor and capital into an aggregate production function should yield a measure of the real output attainable under conditions of full employment. Its application in practice is much more complicated. Reliable estimates of the capital stock are one obstacle; another is the construction of cyclically-adjusted labor hours and labor force participation rates. Even more problematic is estimating the level of total factor productivity corresponding to full employment. As a result, while economists readily acknowledge the effects of supply-side factors on potential output (Boschen and Mills 1990), this method's formidable data requirements have led to its disuse.

1.3 Okun's law

Although Okun held the production-function approach to be the most appealing method in principle, he viewed its inherent measurement problems as largely insurmountable. These difficulties led him to develop an alternative based on the unemployment rate. Noting that the gap between the unemployment rate U and the "natural" rate U^* (thought then to be a constant 4%) bore a systematic relationship to deviations of output from trend, Okun proposed the method that became his eponymous "law,"

$$X^* = X\,[1 + 0.032\,(U - U^*)], \tag{1}$$

where X^* and X are potential and actual output, respectively. The result is a shortcut for estimating potential output that eliminates the laborious estimation of an aggregate production function.

An important drawback of Okun's Law is that it simply replaces one unobservable variable (potential output) with another (the natural rate of unemployment). Its usefulness, therefore, is limited by the inability to measure the natural rate of unemployment. When the supply shocks of the 1970s brought on increases in the level of structural unemployment, the assumption of a constant natural rate became increasingly untenable, leading economists to contemplate time-varying estimates of the natural rate based on empirical measures of structural change (Lilien 1982; Rissman 1986).

Despite this complication, this method continues to see widespread use. Examples include Clark (1983), who used a dynamic version of Okun's Law with time-varying estimates of the natural rate; and Braun (1990), who combined Okun's Law with a segmented-trend specification for the underlying path of potential output.

1.4 Multivariate filtering

Until recently, there has been little effort to base a potential output measure explicitly on inflation. The one exception to this is the recent work of Laxton and Tetlow (1992), who use a multivariate extension of the Hodrick-Prescott Filter to incorporate inflation and unemployment data while penalizing fluctuations in the growth rate of the trend component to impose smoothness.

1.5 Appraisal

One important requirement for a potential output measure is a straightforward economic interpretation. That is, it should be tied either its definition as full-employment output, or through an aggregate-supply relationship to the rate of inflation. Clearly, none of the univariate methods described in Section 1.1 satisfy this condition.

Second, the measure should be easy to construct and update in real time, and flexible enough to adjust rapidly to shocks to the underlying trend. Many of the methods surveyed above fall short on this criterion. The production function models, for example, were slow to react to the change in the trend rate of productivity growth in the 1970s. Similarly, Okun's Law techniques failed to incorporate changes in the natural rate of unemployment. Even simple segmented-trend measures suffer from this defect, since new breakpoints are typically inserted only after hindsight shows the existing trend growth rate to be untenable.

Third, many of the existing potential output measures rely on some amount of informal judgment. This applies to the selection of breakpoints for segmented-trend models, as well as the choice of natural rate for Okun's Law methods. Similarly, calibrating the Laxton-Tetlow multivariate filter requires an arbitrary choice of parameters. Finally, with the exception of the Laxton-Tetlow technique, none of the measures outlined above provides any gauge of the uncertainty associated with the estimate.

2. PROPERTIES OF OUTPUT AND INFLATION

Before specifying a fully-articulated model describing the joint behavior of output and inflation, it is useful first to examine the time-series properties of real GDP and inflation individually. This section reports the results of unit-root tests, and estimates univariate time series models for the two series using data from 1954 through 1992.

2.1 Real GDP

The first step in the empirical analysis is to determine an appropriate structure for the underlying trend component of real output. The stochastic trend specification of Watson (1986), which decomposes a difference-stationary series into an integrated trend and a stationary cycle, is a natural choice for the univariate representation of real GDP.

Letting x^* denote the unobserved trend component of log real GDP and z_t its cycle, Watson's model can be written as:

$$\begin{aligned} \Delta x_t^* &= \mu_x + e_t \\ z_t &= \phi_1 z_{t-1} + \phi_2 z_{t-2} + u_t \\ x_t &= x_t^* + z_t. \end{aligned} \tag{2}$$

Observed output, x_t, is the sum of the trend and cycle components. The trend term follows a random walk; the drift term μ_x captures the average growth rate over the sample. The cycle is assumed to be a stationary AR(2) process with coefficients ϕ_1 and ϕ_2. The white-noise e_t and u_t represent "permanent" and "transitory" shocks to real output.

An important property of this model is that it implies a unit root in real GDP. Although this is consistent with the Augmented Dickey-Fuller tests for nonstationarity reported Table 1, Christiano and Eichenbaum (1990), among others, have demonstrated the fragility of such tests. Leaving aside these inference issues, the nonstationary stochastic trend specification is a useful way to capture very low frequency movements or "variable trends" (Stock and Watson 1988) in real GDP. Moreover, unobserved-components models usually deliver smaller estimates of the size of the unit root than the ARMA specifications used by Campbell and Mankiw (1987).

Another attractive feature of the stochastic trend specification is its adaptability to linear statistical methods, unlike Hamilton's (1989) nonlinear procedure. In any case, because Hamilton's two-state model empirically captures the difference between recession and non-recession growth rates, it is inappropriate for modeling very low frequency fluctuations in potential output. An additional advantage relative to the segmented-trend specification of Braun (1990) is that it doesn't require the *ad hoc* selection of breakpoints prior to estimation.

Table 2 reports the estimate of the stochastic trend specification (2), estimated via maximum likelihood, with the Kalman filter used to evaluate the likelihood function as described in Section 3. The results are similar to those reported by Watson (1986). The estimated drift term of 0.007 implies an average annualized growth rate of 2.8%. The ϕs are indeed consistent with a stationary (but relatively persistent) cycle component. In terms of standard deviations, shocks to the cycle are on average 30% larger than the shocks to the trend component, implying a relatively larger role for the "transitory" disturbances in real output fluctuations.

2.2 Inflation

The rate of inflation used here is based on the Consumer Price Index (CPI), as its relationship to real GDP at cyclical frequencies more pronounced than it is for other series, such as the implicit deflator (Kuttner 1993). The insignificant Augmented Dickey-Fuller statistics reported in Table 3 suggest treating CPI inflation as an integrated series.

The next step is to find an appropriate autoregressive moving-average (ARMA) specification for capturing the short-run dynamics of inflation. The size of the first four autocorrelations reported in the table suggest beginning with fourth order models; the (unreported) partial autocorrelations provide no clear indication in favor of either an AR or a MA specification. Estimates of pure AR and MA models appear in the two panels of Table 4. In both cases, the fourth lag is insignificant at the 5% level, and in the AR specification, only the first two lags are significant. However, the

Q statistics show somewhat more higher-order residual autocorrelation to be present the AR specifications than in the MA models, and mixed ARMA specifications do no better than the pure AR and MA models.

Next, two lagged real growth terms are added to yield a set of transfer function models capturing the positive correlation between inflation and lagged real output growth. In both AR and MA specifications, only one lag of real growth is significant. The fourth lines of the two panels of Table 4 report the estimates of the best AR(2) and MA(3) specifications augmented with one lag of Δx. Despite its slightly less parsimonious representation, the MA(3) specification,

$$\Delta\pi_t = \mu_\pi + \gamma\Delta x_{t-1} + v_t + \delta_1 v_{t-1} + \delta_2 v_{t-2} + \delta_3 v_{t-3}, \tag{3}$$

is chosen for its smaller residual autocorrelation and standard error.

3. THE POTENTIAL OUTPUT MODEL

A model of "constant-inflation" potential output is created by including the cycle component from the stochastic trend model (2) as an additional explanatory variable in the inflation equation (3). Using the preferred MA(3) version of the inflation equation, allowing the inflation rate to depend on lagged z,

$$\Delta\pi_t = \mu_\pi + \gamma\Delta x_{t-1} + \beta z_{t-1} + v_t + \delta_1 v_{t-1} + \delta_2 v_{t-2} + \delta_3 v_{t-3}, \tag{4}$$

yields an aggregate supply relationship involving the output gap. With the change in the inflation rate expressed as a function of the gap, this specification matches the definition in Gordon (1990, p. 10) as the level of output at which the inflation rate is constant. As others have noted (Laxton and Tetlow 1992, p. 17), such "accelerationist" specifications are consistent with expectations-augmented Phillips Curve models in which the expected rate of inflation is set equal to the lagged inflation rate. Of course, there is no guarantee that this reduced-form equation, with its *ad hoc* treatment of expectations, represents a genuine output-inflation tradeoff for policymakers; see Lucas (1976).

The dependence of output and inflation on a common unobserved output gap suggests that the joint estimation of (2) and (4) should provide more information on the output gap (and the level of potential output) than (2) alone. Loosely speaking, estimating the model involves choosing the unknown parameters to yield the z_t series most closely related to inflation, subject to the smoothness restrictions implicit in the stochastic trend specification for potential GDP. In this way, the bivariate potential output model adds an element of economic content that is absent from the univariate detrending methods discussed earlier.

One convenient technique for estimating unobserved-components models is to use the Kalman filter on the model's state-space representation to evaluate a normal likelihood function; details appear in Harvey (1981, 1989). This method can be applied to the potential output model by rewriting (2) and (4) in state-space format, with current and lagged values of x^*, z, and v (potential output, the output gap, and the inflation error term) appearing in the state vector. The measurement equation expresses real GDP and the differenced CPI inflation rate as a function of these variables, lagged output growth, and a constant.

Parameter estimates are computed by maximizing the likelihood function with respect to the 13 unknown parameters. Estimates of the initial state vector and its covariance matrix are computed by using the Kalman smoother to "backcast" the required latent variables, conditional on the data and starting values for the model parameters.

3.1 Model estimates

The results from estimating the constant-inflation potential output model based on the MA(3) inflation specification appear in Table 5. In general terms, the parameter estimates resemble those of the univariate specifications in Tables 2 and 4. The diagnostic tests in Table 6 show no evidence of residual correlation; nor to they suggest including additional lags of output growth or inflation. Unreported estimates of autoregressive specifications are similar, but suffer from marginally significant high-order residual serial correlation.

From an economic perspective, the most important results concern the relationship between real output and inflation. One aspect of this is the response of inflation to the output gap, as captured by the β coefficient in (4), whose 5% statistical significance confirms its importance even controlling for output growth. The point estimate of 0.04 indicates that an output gap of 1% (*i.e.*, output in excess of potential) implies a 0.17% per quarter increase in the annualized inflation rate, an increase of 0.7% if maintained over an entire year. Negative output gaps yield symmetric effects.

The other feature of the relationship is the link between output growth and inflation embodied in γ. The statistically significant estimate of 0.11 is similar to the one obtained from the univariate model, and says that when output growth exceeds its average by 1%, the result is a 0.4% per quarter increase in the annualized inflation rate. Comparing the estimates of β and γ, it appears that the growth rate effects would dominate the output gap effects at a quarterly frequency. However, because the gap is more persistent than the growth rate, the cumulative effects of the output gap is likely to be comparable.

The estimate of the output equation from the potential output model are broadly similar to the earlier results. The one notable exception is that the "persistent" shocks (*i.e.*, the shocks to potential output) are now considerably larger than they were in the univariate specification, now surpassing the standard deviation of the "transitory" shocks by 60%.

3.2 Estimated potential GDP and its standard errors

Extracting an estimate of the latent potential output series is straightforward. Conditional on the maximum-likelihood parameter estimates, the Kalman filter generates a "one-sided" estimate of x^*. Similarly, applying the Kalman smoother yields a "two-sided" estimate that incorporates data through the end of the sample. However, as Okun (1970, p. 123) observed, potential output is "at best an uncertain estimate, and not a firm, precise measure." One of the most useful results of this potential output model is an estimate of this uncertainty.

The estimation process itself yields one ingredient, a measure of the uncertainty that comes from the fact that the underlying state variables are unobserved, and must be inferred from their laws of motion and their noisy link to the data. In estimating the latent state vector, the Kalman filter computes its variance conditional on data through the current quarter. Similarly, the Kalman smoother estimates its variance conditional on data through the end of the sample.

If the model's true parameters were known, this signal-extraction variance would be the only source of uncertainty. However, because these parameters are estimated, another source of uncertainty is the variance associated with the unknown parameters. Hamilton (1986) describes a method for computing this variance that decomposes the total variance of the state vector element of interest. Letting $x^*_{t|T}$ denote the estimate of x^*_t conditional on information through the end of the sample (*i.e.*, the estimate from the Kalman smoother), total variance can be decomposed into the signal-extraction or "filter" component,

$$E\left\{(x^*_t - x^*_{t|T,\psi_0})^2 \mid Z_T, \psi_0\right\}, \tag{5}$$

and the parameter uncertainty,

$$E_\psi\left\{(x^*_{t|T,\psi} - x^*_{t|T,\psi_0})^2 \mid Z_T\right\}, \tag{6}$$

where ψ_0 is the estimated parameter vector, and Z_T represents the data through period T. The Kalman smoother yields the filter uncertainty directly. The parameter uncertainty is computed via Monte-Carlo: an artificial sample of ψs is drawn from a multivariate normal population, generating a sample of $x^*_{t|T}$ series, which is then used to compute the sample variance of each observation in the series. A similar procedure can be used to compute the variance of the one-sided estimate.

Figure 1 displays the two-sided estimated potential output series, along with the logarithm of real GDP. While the potential real GDP series is indeed considerably smoother than actual output, it is nonetheless subject to a significant amount of low-frequency variability. This is especially apparent in its comparison to a linear trend fit through the logarithm of real GDP, depicted in Figure 2. While real GDP does appear gradually to revert to the trend, the large, persistent deviations from trend seem inconsistent with a notion of equilibrium output — some deviations exceeding 5% persist for several years.

Figure 3 shows the estimated two-sided output gap, along with the 1.00 and 1.69 standard error bounds, the latter corresponding to a 90% confidence interval. While most cyclical fluctuations exceed the one-standard-deviation mark, only four episodes surpass the 90% bounds: the 1973 and 1978 expansions, and the 1974–75 and 1981–82 recessions.

Figure 4 shows the one-sided output gap and its error bounds. Comparing it with Figure 3 is revealing in that the one-sided estimate corresponds approximately to the information that would have been available to policymakers at the time. Because of the one-sided estimate's larger variance, those output gaps are generally less significant in the one-sided estimate; the gap exceeds 90% interval only twice, during the 1974–75 and 1981–82 recessions. More recently, the gap exceeded the one-standard deviation bound during 1988–89 in the two-sided estimate, but failed to do so in the one-sided version. This is one example of how statistical "hindsight" can change one's assessment of the economy; Kuttner (1992) discusses the policy implications of this phenomenon.

More data does not always result in more significant output gaps, however. During the 1966 expansion and again in the 1990 recession, the one-sided output gap exceeds one-standard-deviation bounds, but fell short in the two-sided version. Apparently, it is also possible that subsequent data may lead to the conclusion that output fluctuations (measured as a deviation from potential) are *less* prominent than originally believed.

Table 7 reports the average size of the error attributable to filter and parameter uncertainty for both the one- and the two-sided estimates. For the one-sided results, the filter variance is almost twice the parameter variance. In the two-sided estimate, which incorporates information from the entire sample, the filter variance is only 60% of what it was in the one-sided case. The net effect of moving to the two-sided estimate is to reduce the overall standard error from 1.42% to 1.23%.

3.3 An illustrative comparison

How does the series proposed here compare with traditional estimates of potential output? Figure 5 displays the two-sided constant-inflation estimate of the output gap, and the gap implied by potential real GDP series prepared by the Federal Reserve Board (FRB). This series is based on the method outlined in Braun (1990), and has been used as an ingredient in the P^* model (Hallman, Porter and Small 1991).

The most conspicuous difference between the two measures is that the FRB series implies much larger output gaps. That is, the constant-inflation series attributes a much larger share of real GDP fluctuations to potential output shocks. According to the FRB series, for example, the output gap at the bottom of the 1981–82 recession reached -9%, while according to the constant-inflation series, it reached only -3%

— a much less severe downturn as measured by the size of the gap. Part of this discrepancy is due to the two series differing appraisals of the output gap just prior to the recession. According to the FRB estimate, output was only 1% above potential in 1978, despite the rapid increase in the inflation rate over that period. By contrast, the constant-inflation measure reflects this price pressure, registering an output gap of 3%.

4. CONCLUSIONS

This paper has described a new method for constructing a timely and economically sensible estimate of potential output by exploiting the cyclical relationship between inflation and the output gap. Estimating the bivariate model delivers a useful measure of potential output, as well as inflation's response to deviations from potential.

Generating a potential output series in this way has a number of significant advantages over the alternatives. First, it is readily updated as new output and inflation data are released, and unlike most existing methods, specification in terms of a stochastic trend allows for its continuous adjustment in light of current economic developments. Second, no independent measure of the natural rate of unemployment is needed; nor does it require any subjective judgment. A third important feature is its measure of the time-varying uncertainty associated with the potential output series — an essential ingredient for its use in policymaking.

While the basic bivariate inflation-output model has delivered promising results, two extensions may yield additional refinements. One is to incorporate multiple measures of inflation, whose common dependence on the output gap would provide an even more reliable measure of underlying price pressures. In addition, this model could provide a framework for evaluating the distinct cyclical behavior of alternative price indices. A second extension is to include suitable factor inputs as determinants of potential output, resulting in a hybrid measure embodying certain supply-side elements. Both of these extensions could easily be incorporated into the unobserved-components apparatus used here.

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Table 1: Univariate properties of real GDP

Augmented Dickey-Fuller unit root tests

	t statistics	
lags	constant	constant, trend
2	−1.45	−2.42
4	−1.32	−1.66
8	−1.19	−1.22
12	−1.33	−1.11

Autocorrelations

1–6:	0.31	0.17	0.01	−0.02	−0.07	0.02
7–12:	−0.02	−0.14	−0.06	0.07	−0.01	−0.12

Notes: Results are based on quarterly data for 1954:1 through 1992:4.

Table 2: Estimates of the univariate stochastic trend specification for real GDP

$$\Delta x_t^* = \mu_x + e_t$$
$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + u_t$$
$$x_t = x_t^* + z_t$$

μ_x	ϕ_1	ϕ_2
0.0069	1.44	−0.47
(0.0006)	(0.16)	(0.16)

$\sigma_e = 0.0052$ $\qquad$ $\sigma_u = 0.0069$

SE = 0.0088 $\qquad$ $Q(16) = 13.62$

Observations = 156 $\qquad$ Mean log-likelihood = 4.22653

Notes: Results are based on quarterly data for 1954:1 through 1992:4. Standard errors are in parentheses.

Table 3: Univariate Properties of CPI Inflation

Augmented Dickey-Fuller unit root tests

	t statistics	
lags	constant	constant, trend
2	−2.23	−2.09
4	−2.56	−2.42
8	−2.51	−2.52
12	−1.62	−1.49

Autocorrelations

1–6:	−0.20	−0.39	0.26	0.11	−0.08	−0.08
7–12:	0.07	−0.06	−0.03	−0.06	−0.06	0.05

Notes: Results are based on quarterly data for 1954:1 through 1992:4.

Table 4: Estimated univariate inflation models

A: MA specifications

$$\Delta\pi_t = \mu_\pi + \delta(L)v_t + \gamma(L)\Delta x_{t-1}$$

	μ_π	δ_1	δ_2	δ_3	δ_4	γ_1	γ_2	SE	Q(16)
(1)	†	−0.25	−0.37	−0.40	0.05			0.0044	12.60
		(0.08)	(0.08)	(0.08)	(0.08)				
(2)	†	−0.27	−0.36	−0.41				0.0043	14.11
		(0.07)	(0.07)	(0.07)					
(3)	−0.0008	−0.31	−0.46	−0.47		0.11	0.01	0.0041	14.04
	(0.0003)	(0.07)	(0.07)	(0.07)		(0.03)	(0.03)		
(4)	−0.0008	−0.30	−0.46	−0.47		0.11		0.0041	14.15
	(0.0003)	(0.07)	(0.07)	(0.07)		(0.02)			

B: AR specifications

$$\Delta\pi_t = \mu_\pi + \theta(L)^{-1}v_t + \gamma(L)\Delta x_{t-1}$$

	μ_π	θ_1	θ_2	θ_3	θ_4	γ_1	γ_2	SE	Q(16)
(1)	†	−0.26	−0.43	0.06	0.02			0.0044	20.21
		(0.08)	(0.08)	(0.08)	(0.08)				
(2)	†	−0.29	−0.45					0.0044	22.14
		(0.07)	(0.07)						
(3)	−0.0008	−0.34	−0.51			0.08	0.04	0.0042	18.79
	(0.0003)	(0.07)	(0.07)			(0.03)	(0.03)		
(4)	−0.0007	−0.34	−0.50			0.11		0.0042	20.26
	(0.0003)	(0.07)	(0.07)			(0.03)			

Notes: Results are based on quarterly data for 1954:1 through 1992:4.
Standard errors are in parentheses.
† denotes estimates less than 0.0001.

Table 5: Estimated potential output model

A: Output equations

$$\Delta x_t^* = \mu_x + e_t$$
$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + u_t$$
$$x_t = x_t^* + z_t$$

μ_x	ϕ_1	ϕ_2
0.0070	1.57	−0.68
(0.0006)	(0.12)	(0.12)

$\sigma_e = 0.0071$ $\quad$ $\sigma_u = 0.0045$

SE = 0.0088 $\quad$ $Q(16) = 14.72$

B: Inflation equation

$$\Delta \pi_t = \mu_\pi + \gamma \Delta x_{t-1} + \beta z_{t-1} + v_t + \delta_1 v_{t-1} + \delta_2 v_{t-2} + \delta_3 v_{t-3}$$

μ_π	γ	β	δ_1	δ_2	δ_3
−0.0007	0.11	0.04	−0.38	−0.52	0.43
(0.0003)	(0.02)	(0.02)	(0.09)	(0.09)	(0.09)

$\sigma_v = 0.0038$ $\quad$ $\rho_{u,v} = 0.15$ $\quad$ $Q(16) = 14.10$

Observations = 156 $\quad$ Mean log-likelihood = 9.26167

Notes: Results are based on quarterly data for 1954:1 through 1992:4. Standard errors are in parentheses.

Table 6: Diagnostic tests for the potential output specification

A: Output equation

Residual correlation		Omitted $\Delta\pi$ terms	
lags 5–8	lags 9–12	lag 1	lags 1–4
5.32	5.43	0.07	4.32
(0.26)	(0.25)	(0.79)	(0.36)

B: Inflation equation

Residual correlation		Omitted Δx and $\Delta\pi$ terms	
lags 5–8	lags 9–12	Δx_{t-2}	$\Delta\pi_{t-1}$
4.41	3.72	1.33	0.17
(0.35)	(0.45)	(0.25)	(0.68)

Notes: The LM test statistics reported are distributed as χ^2 random variables, with degrees of freedom equal to the relevant number of restrictions. Probability values appear in parentheses.
See also notes to Table 5.

Table 7: Signal extraction statistics for potential output

	one-sided	two-sided
Filter variance	1.41×10^{-4}	8.77×10^{-5}
Parameter variance	6.37×10^{-5}	6.76×10^{-5}
Overall standard error	1.42 %	1.23 %

Notes: The parameter variances are estimated via monte-carlo with 200 draws. The results exclude the first 8 quarters of the sample to minimize the effects of the initial conditions.

Figure 1

Real GDP and two-sided potential output

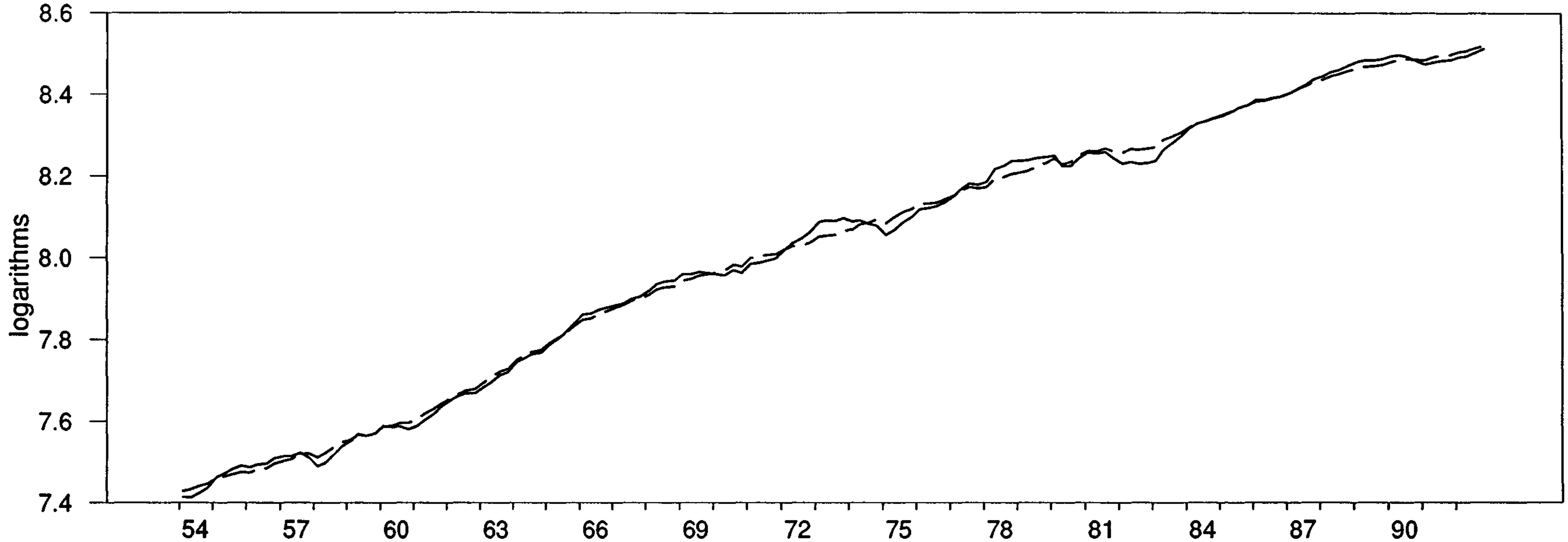

Figure 2

Real GDP and linear trend

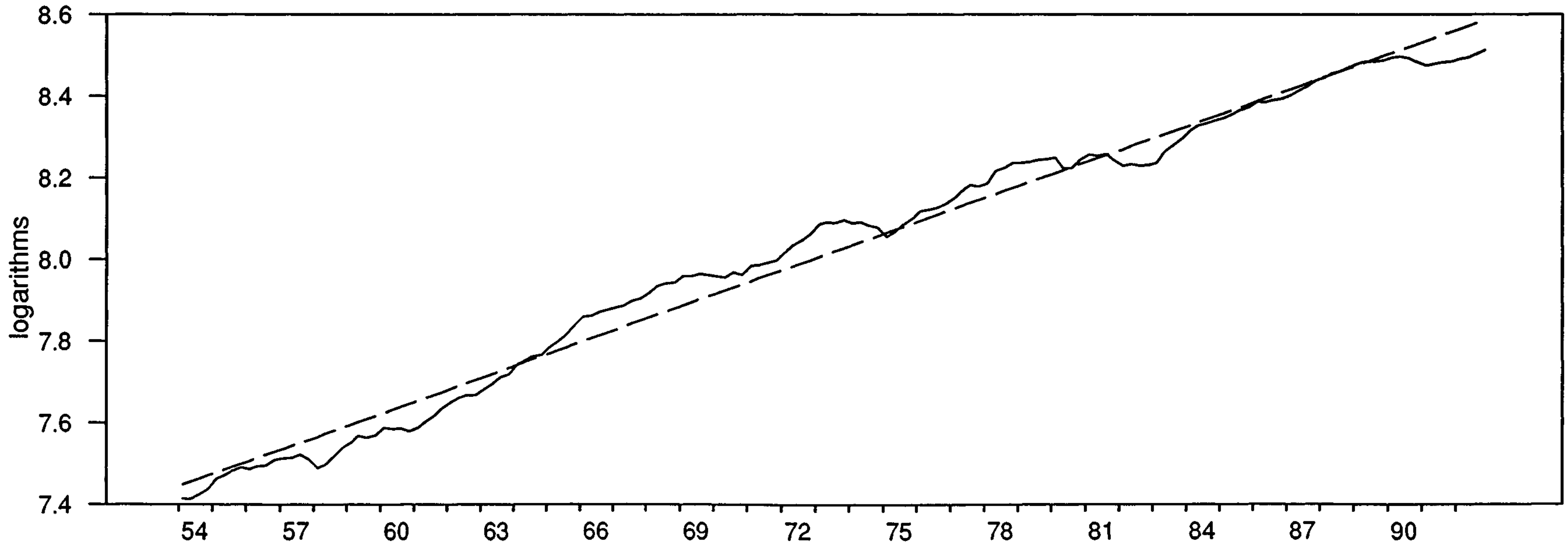

Figure 3

Two-sided output gap, 1.00 and 1.69 error bounds

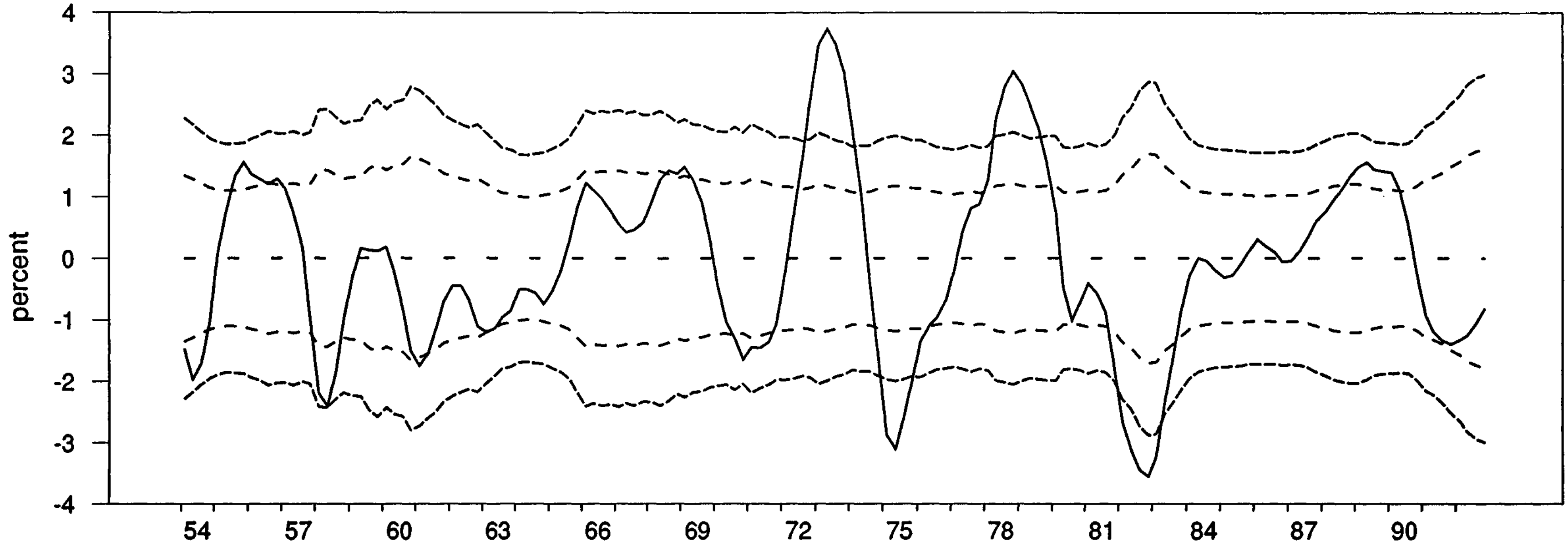

Figure 4

One-sided output gap, 1.00 and 1.69 error bounds

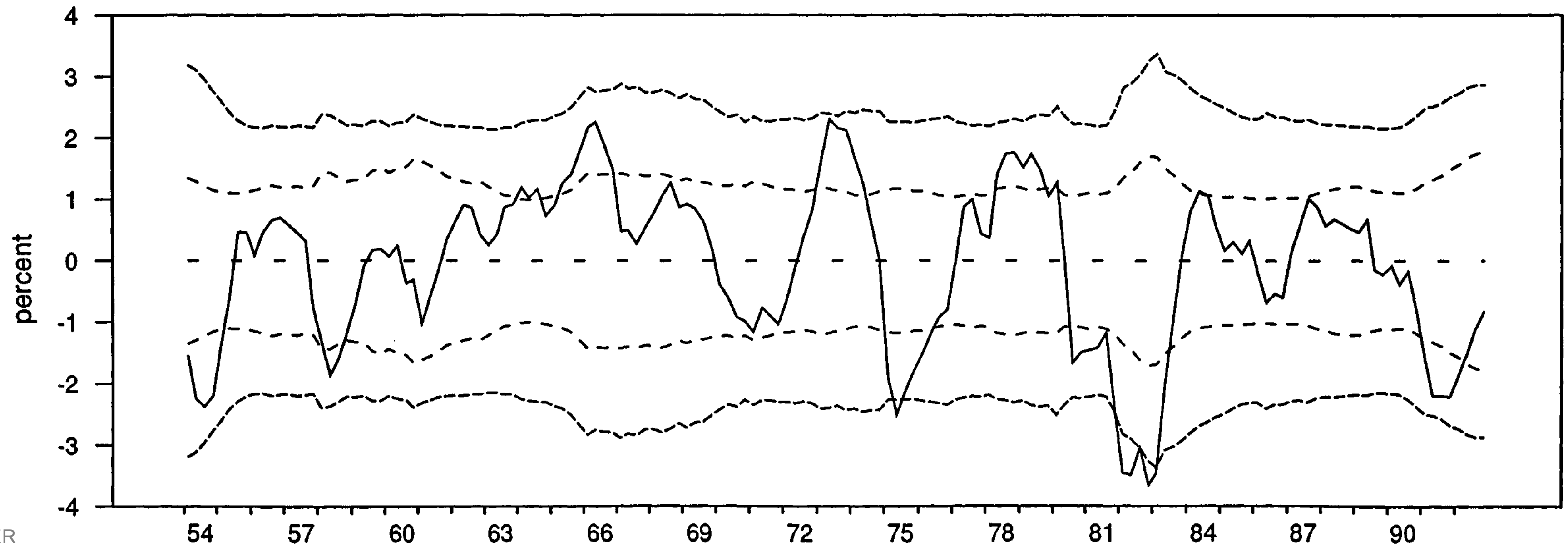

Figure 5

Comparing alternative output gap measures