Working Paper Series

A Framework for Estimating the Value and Interest Rate Risk of Retail Bank Deposits

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Working Papers Series
Issues in Financial Regulation
Research Department
Federal Reserve Bank of Chicago
December 1992 (WP-92-30)

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A Framework for Estimating the Value and Interest Rate Risk of Retail Bank Deposits*

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November 1992

Abstract

Traditional measures of interest rate risk are based on the assumption that prices of financial assets and liabilities are set in perfectly competitive markets. However, recent empirical evidence on the pricing of retail bank deposits suggests that many banks exercise market power. In this paper we provide a framework for valuing a bank's rent attributable to issuing demandable deposits. We also derive the interest rate sensitivity (duration) relevant to these retail bank deposits. Using monthly survey data on NOW accounts and MMDAs, we estimate the value of retail deposit rents and their durations for over 200 commercial banks.

*We are very grateful to Herb Baer and Nancy Andrews of the Federal Reserve Bank of Chicago for their generous help in carrying out the empirical work of this paper. We are also indebted to Herb Baer, Bjorn Flesaker, Francis Longstaff, and participants of seminars at the Federal Reserve Board and the Federal Reserve Bank of Chicago for their valuable comments.

I. Introduction

Models of interest rate risk that generate the concepts of bond duration and immunization commonly assume that fixed-income security prices are determined in perfectly competitive markets. Individuals who trade in these securities are assumed to be price-takers who treat security supply as infinitely elastic. For money managers trading in instruments such as Treasury securities and high-grade corporate bonds, these competitive market assumptions can be a useful abstraction. Hence, it is not surprising that models of interest rate risk have been most productively applied to these competitive trading environments.

However, for a number of financial institutions the competitive paradigm is less defensible. Mounting empirical evidence supports the hypothesis that many banks exercise market power in some of their deposit markets.[1] This empirical research consistently finds that a bank's behavior in setting retail deposit interest rates, such as NOW account and MMDA rates, is correlated with measures of market concentration. Retail deposit interest rates tend to be lower, and adjust more slowly and less completely to changes in market interest rates, in more highly concentrated geographic markets. An implication of this slow adjustment is that spreads between market interest rates and retail deposit rates widen as market interest rates rise and compress as market interest rates fall. These empirical characteristics of interest rates are consistent with optimal bank deposit pricing behavior predicted by some theoretical models.[2]

When changes in the spread between market and deposit interest rates are not met with equal and opposite changes in the volume of retail deposits issued, the current profitability of retail deposits will vary with movements in market interest rates. For many types of deposits, this is likely to be the

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[1]Empirical studies examining the relationship between deposit rates and competitive market interest rates include Ausubel (1990), Berger and Hannan (1989), Hannan and Berger (1991), Hannan and Liang (1990), Neumark and Sharpe (1992), and Sharpe and Diebold (1990).

[2]See Hutchison (1992) for an illustration in the context of an intertemporal asset pricing model.

case. The same macroeconomic factors that cause market interest rates to rise may cause the demand for deposits to increase as well. For banks with market power, optimal deposit pricing may then imply a widening spread between market and deposit rates, raising the current profitability of retail deposits. Hence, the present value of a bank's future deposit profits, i.e., its monopoly rent, will change as well. This implies that demandable retail deposits, despite having zero stated maturities, expose a bank to interest rate risk through changes in the value of its rent from issuing these deposits.[3] A proper measure of the duration (interest rate sensitivity) of these deposits must incorporate the change in the value of the rent associated with issuing these deposits.

The complexities in estimating the duration of retail bank deposits have been noted by other researchers.[4] Flannery and James (1984) offer an indirect solution that utilizes bank holding company data. They relate estimates of the interest rate sensitivities of bank stocks to the cross-sectional variation in the banks' holdings of different types of deposits. This enables them to infer the interest rate sensitivities of various classes of bank deposits. However, their method may be limited by its identifying assumption that durations of particular types of retail deposits are the same across different banks. If banks have differing degrees of deposit market power, this assumption may be violated. In the present paper, the direct method used to estimate durations places no restriction on an individual bank's market power.

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[3]Fixed maturity deposits, such as consumer CDs, are subject to two types of interest rate risk: one due to changes in the term structure that affect equivalent maturity competitively priced bonds, and the other due to changes in the value of the rent associated with these deposits. In related work, we have examined the value and duration of these rents. The results are comparable to those presented here.

[4]For example, Kaufman (1984, p.26) states "A number of types of bank deposit accounts, such as demand deposits, savings, NOWs, SNOWs, and MMDAs, do not have specific maturity dates....What are the durations of such deposit accounts?...If a bank's deposit rates lag increases in market rates, all deposits will not leave the bank immediately. 'Core' deposits will remain for some time and flow out only slowly. It may be possible to assign accurate probabilities to the timing of net deposit outflows, depending on the difference between the market and deposit rates....If interest rates increase, it is then possible to value these deposits at less than their par value." He later states "The correct duration awaits additional research."

Our paper extends traditional duration theory to situations in which assets trade in imperfectly competitive markets, focusing on the retail bank deposit market. The market for retail bank deposits is modeled as having a downward sloping demand curve that is a function of the deposit interest rate, the competitive market interest rate, and other local market variables. We derive a bank's optimal deposit interest rate and profit at each point in time as a function of the competitive market rate and other exogenous variables. These results are incorporated into a continuous time model of the term structure of interest rates, and contingent claims theory is used to value the bank's stochastic future stream of profits from deposit issue. This value is a function of the current competitive market interest rate, and by simple differentiation we obtain the sensitivity of the value of the retail deposit rent to changes in the market interest rate. A measure of retail deposits' duration then results. Following this, we employ our valuation technique to estimate the rents and interest rate risk of NOW accounts and MMDAs for a sample of several hundred U.S. commercial banks.

The plan of the paper is as follows. Section II presents a general model and introduces the contingent claims techniques that are used to value the rent associated with issuing demandable retail bank deposits. Section III describes additional parametric assumptions used to empirically implement the model. We derive the value of the monopoly rent as an infinite series of generalized hypergeometric functions. Section IV confronts the model with data on NOW account and MMDA interest rates and deposit balances from a sample of over 200 commercial banks. Section V provides concluding remarks.

II. The General Model

Since previous empirical studies have found a link between bank market power and retail deposit rate setting, we assume that individual banks face a downward sloping demand curve for retail deposits. The quantity of retail deposits demanded, D, is given by

(1) $$D(t) = D(r(t), r_d(t), x(t))$$

where r(t) denotes the time t rate of return on a competitive market, short (instantaneous) maturity default-free bond, $r_d(t)$ is the deposit interest rate set by the bank at time t, and x(t) is a vector of other variables affecting demand that are assumed to be independent of r(t). If the bank's marginal revenue from investing its deposits equals the market interest rate, r(t), then at each point in time the bank will set $r_d(t)$ as the value that solves

(2) $$\max_{r_d(t)} \; [r(t) - r_d(t) - c(t)]D(t)$$

where c(t) denotes the non-interest cost of issuing a retail deposit and is assumed to be independent of $r_d(t)$. The solution to problem (2) is[5]

(3) $$r_d^*(t) = r(t) - c(t) - D/(\partial D/\partial r_d)$$

Note that if a bank's retail deposit market is perfectly competitive, $\partial D/\partial r_d = \infty$ and thus $r_d(t)$ would equal the competitive rate r(t) - c(t). A less than perfectly elastic demand for deposits implies a deposit interest rate that is below r(t) - c(t).

This equilibrium deposit pricing behavior can be embedded into a model that values a bank's monopoly rent from issuing retail deposits. This monopoly rent can be considered a part of a bank's

[5]This first order condition assumes that the bank is always at an interior solution. We do not consider the possibility it may be optimal for a bank to completely cease issuing deposits.

"going concern value" or "charter value."[6] Once we obtain a valuation formula for a bank's retail deposit monopoly rent, we can examine the sensitivity of this rent to changes in the market interest rate, r(t). The result gives us a measure of the duration of retail bank deposits.

Define f(t) as the instantaneous cashflow (profit) received by the bank from issuing deposits in amount D(t) and using the proceeds to invest in short (instantaneous) maturity Treasury bills paying rate of return r(t). Substituting equations (3) into (2), we have

$$f(r(t), x(t)) = [r(t) - r_d^*(t) - c(t)]D(t) = D^2/(\partial D/\partial r_d^*) \tag{4}$$

where $r_d^*(r(t),x(t))$ is the bank's profit maximizing retail deposit interest rate.

The value of the bank's monopoly rent from issuing retail deposits is then the present value of the stochastic cashflow, f(r(t),x(t)), for all future dates. We can value this stream of cashflows by applying techniques used to price interest rate dependent claims. For analytical simplicity, we will make the assumption that the risk associated with changes in the variable x(t) bears no risk premium, i.e., it can be considered as "idiosyncratic" or "non-systematic" risk.[7] Hence, the only risk "priced" by investors is assumed to derive from changes in market interest rates, r(t).

In order to determine the present value of risky cashflows to be received at future dates T, we employ a version of Longstaff's (1990) Separation Theorem which is a stochastic interest rate extension of the Cox and Ross (1976) risk-neutral valuation approach. This theorem states that when valuing interest-sensitive risky cashflows the operation of "discounting" can be separated from the

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[6]In some instances, the rent associated with issuing retail deposits might be more accurately described as a quasi-rent. There may be fixed costs of entering a retail deposit market, such as building branch offices, that could offset the acquired rent from issuing deposits at below market interest rates.

[7]x(t) can be viewed as reflecting local market conditions.

operation of risk-adjusted "expectations-taking." Specifically, let us define P(t,T) as the price at time t of a default-free discount bond that pays \$1 at time T and, therefore, has a time until maturity defined by $\tau \equiv T - t$. Then the present (time t) value of f(r(T),x(T)), for T > t, can be expressed as

$$(5) \qquad P(t,T)E_t[f(r_\tau(T),x(T))]$$

where $r_\tau(T)$ is an appropriately transformed market interest rate process whose form depends on τ. The description of this transformation is given in Longstaff (1990, p.102).[8] Now define F as the present value of the stream of cashflows, f(r(T),x(T)), for all future dates, T. F is given by

$$(6) \qquad F \equiv \int_t^\infty P(t,s)E_t[f(r_{(s-t)}(s),x(s))]ds$$

Once F can be evaluated, we can calculate the duration of retail bank deposits as well. Let D^c be the value of competitively priced, instantaneous maturity bonds equal to the level of deposits currently issued by the bank.[9] Then $L \equiv F - D^c$ is defined as the net value of the bank's retail deposit liability. We can now use Cox, Ingersoll, and Ross's (1979) definition of duration which is a measure of an asset's risk from interest rate changes. In this sense, the duration of L is equal to the maturity of a zero coupon bond that has the same rate of return sensitivity to changes in the market interest rate,

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[8]The transformed process for r(t) is that which changes the probability distribution for r(T) into one (an equivalent martingale measure) that allows $f(r_\tau(T),x(T))$ to be valued by finding its expected value. Longstaff (1990) calculates the appropriate transformed process for r(t) when it follows the square-root process of Cox, Ingersoll, and Ross (1985b). In the Appendix, we calculate the appropriate transformed process for r(t) when it follows the Ornstein-Uhlenbeck process of Vasicek (1977).

[9]D^c is equal to the "book value" of the bank's retail deposits, $D(r,r_d,x)$.

$r(t)$, as does L. Mathematically, L is defined to have a positive duration of $T^* - t$ if $[dL(t)/dr(t)]/L(t) < 0$, and a negative duration of $-(T^*-t)$ if $[dL(t)/dr(t)]/L(t) > 0$, where T^* solves the following expression:

$$\frac{1}{P(t,T^*)}\frac{\partial P(t,T^*)}{\partial r(t)} = -\left|\frac{1}{L}\frac{dL}{dr(t)}\right| \tag{7}$$

Note that since deposits are assumed to be demandable, in the absence of any monopoly rents they would have zero duration. This implies $dD^c/dr(t) = 0$, so that $dL(t)/dr(t) = dF(t)/dr(t)$.

III. The Empirical Method

To evaluate the monopoly rent, F, given by equation (6), we need to make specific assumptions regarding the form of the bank's demand function, $D(r,r_d,x)$, and the stochastic processes for $r(t)$ and $x(t)$. This will then imply an equilibrium process for $r_d(t)$, the bank's optimal deposit interest rate. Our choice of parameterization attempts to satisfy a number of goals. First, we want to make parametric assumptions that will allow us to obtain a computationally practical way of evaluating F and $dF/dr(t)$. Second, we want to allow for sufficient generality so that our model can capture the most important regularities of the time series for bank and market interest rates, as well as bank deposit balances.[10] Third, we wish to estimate the parameters of these time series using standard econometric techniques.

With these goals in mind, we make the following assumptions. Let $x(t)$ be a 2x1 vector equal to $(\nu(t)\ \ \eta(t))'$. $\nu(t)$ is a variable that affects the elasticity of deposits with respect to the variable r_d

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[10]We specify a particular model that gives rise to equilibrium processes for deposit interest rates and deposit quantities, equations (13) and (17), below. These processes then determine the equilibrium process for bank profits that is used in our empirical work. However, there may be other models, perhaps having more realistic behavioral assumptions, that could result in the same equilibrium processes. Hence, our empirical results may be more general than implied by the paper's particular model.

and $\eta(t)$ is a variable that affects the demand for deposits without changing the elasticity with respect to r_d. This distinction will be clarified below. We can now define a 3x1 vector $s(t) = (r(t) \;\; x(t)')'$ which is assumed to follow a trivariate Ornstein-Uhlenbeck process:

$$(8) \qquad ds(t) = [\alpha + \beta s(t)]dt + \sigma_s dZ_s$$

where α is a 3x1 vector of constants, β is a 3x3 matrix of constants, and $(\sigma_s dZ_s)(\sigma_s dZ_s)' = \Sigma_s dt$ with Σ_s being a 3x3 matrix with diagonal elements σ_{si}^2, i=1, 2, and 3. For simplicity, it is assumed that $r(t)$, $\nu(t)$, and $\eta(t)$ are independent in the sense that β and Σ_s are diagonal matrices.

Under the assumption that the inverse deposit interest elasticity, $D/(\partial D/\partial r_d)$, is a linear function of $r(t)$ and $\nu(t)$:

$$(9) \qquad D/(\partial D/\partial r_d) = -d_0 - c(t) + (1-d_1)r(t) - \nu(t)$$

then the bank's optimal retail deposit rate, given by equation (3), takes the form

$$(10) \qquad r_d^* = d_0 + d_1 r(t) + \nu(t).$$

Given the parameters d_0, d_1, and the market interest rate $r(t)$, equation (10) shows that $r_d(t)$ reveals the unobserved elasticity shift variable $\nu(t)$. Further, equation (10) implies that $r(t)$ and $r_d(t)$ also follow a joint bivariate Ornstein-Uhlenbeck process. To see this, note that equation (10) implies that the retail deposit rate follows the process

$$\text{(11)} \qquad dr_d = d_1 dr(t) + dv(t) = d_1[\alpha_1 + \beta_{11} r]dt + [\alpha_2 + \beta_{22} v]dt + d_1 \sigma_{s1} dz_{s1} + \sigma_{s2} dz_{s2}$$

Substituting for v(t) from (10) into (11), we see that (11) can be written in the form

$$\text{(12)} \qquad dr_d = [a_2 + d_1(\beta_{11} - \beta_{22})r(t) + \beta_{22} r_d(t)]dt + \sigma_2 dz_2$$

where $a_2 = d_1\alpha_1 + \alpha_2 - d_0\beta_{22}$ and $\sigma_2 = (d_1^2\sigma_{s1}^2 + \sigma_{s2}^2)^{1/2}$.

Hence, if we define $R(t) = (r(t) \;\; r_d(t))'$ as the vector of the exogenous market rate, r(t), and the endogenous retail deposit rate, $r_d(t)$, the process for R(t) can be written as:

$$\text{(13)} \qquad dR(t) = [A + B\,R(t)]dt + \sigma dZ$$

where the newly defined elements are $a_1 = \alpha_1$, $b_{11} = \beta_{11}$, $b_{12} = 0$, $b_{21} = d_1(\beta_{11}-\beta_{22})$, $b_{22} = \beta_{22}$, $\sigma_1 = \sigma_{s1}$, and $\sigma_{12} = d_1\sigma_1^2$. Equation (13) implies that R(t) has a discrete-time multivariate normal distribution.

While we have assumed a form for the inverse retail interest rate elasticity of deposits, $D/(\partial D/\partial r_d)$, we have not yet fully specified the form of the demand function for deposits, $D(r,r_d,x)$. Note that equation (9) is a first order differential equation whose solution can be written as

$$\text{(14)} \qquad D(r, r_d, x) = K(r, x) e^{\frac{r_d}{-d_0 - c + (1-d_1)r - v}}$$

where K(r,x) is any arbitrary function that is independent of the bank's choice of r_d but which may be dependent on any set of exogenous variables r and/or x. Due to our freedom in choosing K, we can allow deposit demand to take on essentially any set of values. In other words, the specification of the demand elasticity given by (9) places no constraint on the equilibrium level of deposits demanded.

Therefore, we will choose a form for deposit demand that facilitates estimation. The form of the function, K, is assumed to be:

(15) $$K(r,x) = [k_1 r + k_2(d_0 + d_1 r + v) + \eta]e^{\mu t - \frac{d_0 + d_1 r + v}{-d_0 - c + (1-d_1)r - v}}$$

where μ represents the trend growth rate in deposit demand. This implies that

(16) $$D(r,r_d,x) = [k_1 r + k_2(d_0 + d_1 r + v) + \eta]e^{\mu t + \frac{r_d - (d_0 + d_1 r + v)}{-d_0 - c + (1-d_1)r - v}}$$

This functional form is convenient because it implies that when banks optimally set the retail deposit rate according to equation (10), the level of bank deposits satisfies the following relation:

(17) $$D(r,r_d,x) = [k_1 r(t) + k_2 r_d^*(t) + \eta(t)]e^{\mu t}$$

Equation (17) indicates that detrended deposit balances can be described by a linear regression model. Given the assumed process for $\eta(t)$ in equation (8), the error term in this regression model will follow a discrete-time AR(1) process, allowing for persistence in local market shocks. This model will be used to forecast deposit demand based on the expectation of future values of $r(t)$ and $r_d(t)$ that satisfy the process given in equation (13).

In order to compute the bank's expected future profits, we also need to specify a functional form for the bank's non-interest expense, $c(t)$. Allowing for required (non-interest-bearing) reserves on retail deposits, we assume $c(t) = \varsigma + (1-\rho)r(t)$, where $(1-\rho)$ is the deposits' required reserve ratio.

Given this parameterization, along with equations (13) and (17), we can now find the expectation of f(r(T),x(T)) given in equation (4):

$$
\begin{aligned}
(18)\qquad E_t[f(r_\tau(T),x(T))] &= E_t[(\rho r_\tau(T) - r_{d\tau}(T) - \varsigma)(k_1 r_\tau(T) + k_2 r_{d\tau}(T) + \eta(T))e^{\mu T}] \\
&= \{-\varsigma k_1 E_t[r_\tau(T)] - \varsigma k_2 E_t[r_{d\tau}(T)] - \varsigma E_t[\eta(T)] \\
&+ \rho k_1 E_t\left[r_\tau(T)^2\right] - k_2 E_t\left[r_{d\tau}(T)^2\right] + (\rho k_2 - k_1)E_t[r_{d\tau}(T) r_\tau(T)] \\
&+ E_t[\eta(T)]E_t[\rho r_\tau(T) - r_{d\tau}(T)]\}e^{\mu T}
\end{aligned}
$$

where $r_\tau(T)$ is the appropriately transformed process of r(t), and $r_{d\tau}(T) = d_0 + d_1 r_\tau(T) + \nu(T)$. The formulas for the moments of these variables are given in the Appendix.

In addition to finding a value for $E_t[f(r_\tau(T),x(T)]$, we need an expression for the value of a default-free discount bond maturing at date T, P(t,T), in order to utilize equation (6) which gives the value of the bank's monopoly rent, F. Given the assumed process for the instantaneous maturity interest rate r(t) in equation (13), Vasicek (1977) shows that the equilibrium value of P(t,T) is

$$
(19)\qquad P(t,T) = \exp\left[\frac{\left(1 - e^{b_{11}\tau}\right)}{b_{11}}(r(t) - r_\infty) - \tau r_\infty + \frac{\sigma_1^2}{4b_{11}^3}\left(1 - e^{b_{11}\tau}\right)^2\right]
$$

where r_∞ is the constant yield on a long-term (infinite maturity) discount bond.[11] Inserting the expressions for P(t,T) in (19) and $E_t[f(r_\tau(T),x(T)]$ in (18) into equation (6), the value of the bank's monopoly rent is determined. The Appendix shows that the integral in (6) has a solution that can be

[11]Vasicek (1977) shows that if the market price of interest rate risk is a constant, equal to q, then the equilibrium value of $r_\infty = -a_1/b_{11} - q\sigma_1/b_{11} - \frac{1}{2}\sigma_1^2/b_{11}^2$. The Vasicek model of the term-structure is consistent with a Cox, Ingersoll, and Ross (1985a) general equilibrium model of asset pricing. This is demonstrated by Campbell (1986) for the case of the univariate Vasicek model and by Pennacchi (1991) for the case of a bivariate extension of the Vasicek model.

expressed in terms of a series of generalized hypergeometric functions. dF/dr(t) can be calculated in a similar manner.

Note that equation (19) implies

(20) $$\frac{1}{P(t,T)}\frac{\partial P(t,T)}{\partial r(T)} = \frac{1-e^{b_{11}\tau}}{b_{11}}$$

so that the duration of retail bank deposits is $\pm\tau^*$ where τ^* solves

(21) $$\frac{1-e^{b_{11}\tau^*}}{b_{11}} = -\left|\frac{1}{F-D^c}\frac{dF}{dr(t)}\right|$$

and where it should be emphasized that $dF/dr = \partial F/\partial r + (\partial F/\partial r_d)\partial r_d/\partial r = \partial F/\partial r + d_1(\partial F/\partial r_d)$, i.e., we account for the bank's optimal adjustment of its deposit interest rate, $\partial r_d/\partial r = d_1$, when measuring the sensitivity of the rent to changes in market interest rates. Solving for τ^* we obtain:

(22) $$\tau^* = \frac{1}{b_{11}}\ln\left(1 + b_{11}\left|\frac{1}{F-D^c}\frac{dF}{dr(t)}\right|\right)$$

IV. Empirical Results

IV.A Data and Parameter Estimation

A series for the instantaneous maturity, competitive interest rate, r(t), was constructed using end of month Treasury bill prices collected from the Center for Research in Security Prices (CRSP) bond file. The average of bid and ask prices of 30, 90, 180, and 345 day Treasury bills, over the period 1968 through 1990, were used to find maximum likelihood estimates of the parameters of the

Vasicek (1977) term structure model. A single state variable Kalman filter was used to compute the likelihood function following the methodology described in Pennacchi (1991).[12] Table 1 reports the results of this estimation. Note that $-a_1/b_{11} = .0813$ is the estimate of the unconditional (long run) mean of r(t). $-b_{11} = .09800$ measures the speed of mean reversion of r(t) to its unconditional mean. $\sigma_1 = .02432$ is an estimate of the annualized instantaneous standard deviation of changes in r(t). These parameter estimates then provided the basis for constructing a "smoothed" time series of the instantaneous interest rate, r(t), that was used in the deposit interest rate and deposit demand regressions described below.[13]

Along with the parameter estimates in Table 1, we required an estimate of the yield on an infinite maturity zero coupon bond, r_∞, in order to evaluate the term structure of bond prices, P(t,T), used to calculate a bank's monopoly rent. Data on the average of bid and ask prices of 119 zero coupon Treasury stripped bonds, whose maturities ranged from 51 days to just under 30 years, were obtained from the Wall Street Journal for December 31 (the last trading day) of 1990. The sum of squared deviations between these prices and the theoretical Vasicek (1977) model prices, given by equation (19), were used to estimate the parameter r_∞, fixing the other parameter estimates to those given in Table 1. We obtained an estimate of $r_\infty = .08809$, with a standard error of .00055.

Retail deposit interest rate and quantity (account balance) data were obtained from the "Monthly Survey of Selected Deposits" collected by the Board of Governors of the Federal Reserve System and analyzed at the Federal Reserve Bank of Chicago. The MMDA series begins in October

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[12]The basis of the estimation was a state space model with the state transition equation being the stochastic process for r(t) and the measurement equations being the prices of 30, 90, 180, and 345 day Treasury bills which were assumed to measure their corresponding theoretical Vasicek (1977) model prices with error.

[13]A smoothed estimate of the unobserved instantaneous maturity interest rate, r(t), is an optimal estimate based on the full sample of Treasury bill prices. See Harvey (1981) for a discussion of smoothing.

1983, and extends through December 1990, providing a maximum of 87 monthly observations per bank. Prior to January 1986, both NOW account and Super NOW account data were collected, but the Super NOW account series was discontinued in January 1986, when legislative changes left no distinction between NOW and Super NOW accounts. For the sake of consistency, we use only the NOW account series beginning in January 1986 and extending through December 1990, providing a maximum of 60 monthly observations per bank. For both the MMDA and NOW account series, our analysis was limited to a subset of the 416 banks that reported continuously over the sample period.

Prior to July 1989, balances in both MMDA and NOW accounts were aggregated, whether or not banks offered multiple account categories within each account type. Since this date, data has been collected for several categories of NOW and MMDAs, allowing for different tiers of minimum balances and associated interest rates. Again to maintain consistency, we restricted our sample to those NOW or MMDA accounts at banks that reported data for only one NOW or MMDA category in the post July 1989 period. This left us with 244 NOW account series and 216 MMDA series.[14]

Using these NOW and MMDA deposit interest rate series along with the aforementioned series for the market interest rate, r(t), we estimated the continuous-time parameters of equation (13) via an ordinary least squares regression on its discrete-time analogue.[15]

Deposit demand, equation (17), was estimated in two parts. First we used ordinary least squares to estimate

[14]We did not attempt to adjust the times series of deposit quantities to take into account the effect of bank mergers or spin-offs. For this reason, and others mentioned below, our empirical results should be viewed as illustrative rather than the most precise possible.

[15]Pennacchi (1991) gives the discrete-time process that corresponds to the continuous-time process of an equation such as (13). The discrete-time process takes the form of a vector AR(1) process with normally distributed error terms. The Appendix also gives the moments of the discrete-time process in terms of the continuous time parameters.

(23) $$\ln[D(t)] = k + \mu t + \varepsilon(t)$$

Under the assumed processes for $r(t)$, $r_d(t)$, and $\eta(t)$, $\ln[k_1 r(t) + k_2 r_d(t) + \eta(t)]$ is stationary. Hence the procedure in (23) will produce unconditionally unbiased estimates of the deposit demand trend, μ. Using these estimates of μ, we then detrended the deposit demand series in order to estimate the k_i parameters in equation (17). Based on the assumed process for $\eta(t)$, equivalent to a discrete-time AR(1) process, we used the Prais-Winsten procedure (also called the two-step full transform method) to estimate the parameters and autoregressive coefficient of[16]

(24) $$D(t)\, e^{-\mu t} = k_1 r(t) + k_2 r_d(t) + \eta(t)$$

IV. B. Estimates of Rents and Durations from Retail Deposit Issue

The parameters required to evaluate a bank's retail deposit rent and duration are listed in the first column of Table 2. Except for the market interest rate parameters discussed in the previous section and the non-interest cost parameters, ς and ρ, these parameters were estimated for NOW accounts of 244 banks and MMDAs of 216 banks. The average values of these NOW account and MMDA estimates are reported in the second and third columns of Table 2. A few of the parameter estimates are worth comment. Note that the deposit interest rate's instantaneous standard deviation, σ_2, is larger for MMDAs than NOWs, .007418 versus .004124. Also as expected, a measure of the

[16]Essentially, this procedure is generalized least squares employing the residuals from an OLS estimation of the detrended demand equation in order to estimate the unknown parameter of the AR(1) process describing $\eta(t)$. The only distinction between this method and that of Cochrane and Orcutt is the use of a full transformation matrix rather than the elimination of the first observation. According to Judge et al. (1985), this procedure has been found to have good statistical properties relative to other techniques.

responsiveness of the deposit interest rate to the market rate, d_1, is greater for MMDAs than NOWs, .82920 versus .40126. The values of r(t), $r_d(t)$, and $\eta(t)$ reflect values for the last trading day of 1990.[17]

We did not attempt to estimate the non-interest cost parameter, ς, for each individual bank. This would involve measuring a bank's variable costs attributable to checking services, deposit services, and account maintenance, net of service and handling charges, for its NOW or MMDA deposits. This data was not available to us. Instead, we utilized summary data from the Federal Reserve Board's Functional Cost Analysis (FCA) which performs this cost allocation for NOW and MMDA accounts of approximately 150 banks. However, there are at least two reasons why this data is less than ideal and why our use of this data will contribute to the imprecision of our resulting estimates of deposit rents and durations. First, the FCA cost estimates are averages across banks. As we show below, calculations of deposit rents and durations are sensitive to the level of non-interest costs, and individual banks are likely to differ in their cost efficiency. Second, this data does not separate non-interest costs into its variable and fixed components, though only variable costs are relevant to our calculation of deposit rents and durations.[18] We attempted to compensate for this second deficiency by (somewhat arbitrarily) assuming that one-half of the reported costs are variable in nature. For NOW accounts, we used 50% of the average non-interest cost given in the 1986-1991 FCA reports, resulting in $\varsigma = .0066972$ (67 basis points).[19] Due to incomplete reporting on MMDA

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[17]Due to confidentiality requirements on deposit quantity data, we are unable to report the actual level of banks' deposits, and hence values for $\eta(t)$, corresponding to the last trading day of 1990. Hence, we assume that each bank's $\eta(t)$ equals its long-run mean value of $-\alpha_3/\beta_{33}$.

[18]For example, the FCA data includes expenses related to office overhead (office furniture and equipment, occupancy expense), data services, and publicity and advertizing. These costs are likely to have a significant fixed component. As mentioned earlier, fixed costs are relevant for determining whether rents associated with retail deposits are true monopoly rents or quasi-rents. However, for the purpose of calculating a retail deposit's duration, only variable costs should be considered.

[19]Each year's FCA expense estimates are broken down into three bank deposit size-based categories: banks with deposits up to $50 million; $50-$200 million; and greater than $200 million. We averaged the estimates across these three categories and across the years 1986-91.

accounts prior to 1991, we used only the 1991 FCA report to obtain an MMDA estimate of ς = .0045166 (45 basis points).[20]

The average parameter estimates in Table 2 were used to construct the graphs in Figures 1 to 4. Figures 1 and 2 graph rents per deposit, the duration of the rents (alone), and the duration of deposits (including the value of rents) as a function of the growth of deposit demand, μ. Because it is difficult to obtain a precise measure of this trend, μ, given the length of our sample period, it is worthwhile to see how NOW and MMDA rents and durations vary for a range of different growth rates. As intuition would tell us, rents per deposit are increasing functions of the growth in deposit demand. Quite simply, this reflects the higher level of future profits that a bank with a growing deposit market expects to receive. Indeed, these rents become unbounded as the growth rate approaches the infinite maturity yield, $r_\infty = .08809$. Examining the durations of rents in Figures 1 and 2, we see that the duration of NOW rents is smaller than the duration of MMDA rents for the same rate of deposit growth. This is what one would expect given the prior empirical evidence that NOW rates are less responsive to market interest rates. In addition, the duration of NOW rents is much more sensitive to changes in deposit growth than is the duration of MMDA rents. Indeed, as can be seen from Figure 1, for small deposit growth rates the NOW rent's interest rate sensitivity can be so large that duration becomes undefined.[21]

What is perhaps more interesting is that both NOW and MMDA durations are increasing functions of deposit demand growth, being negative for low growth rates, then positive for higher

[20]Prior to 1991, MMDA service and handling charges were not reported.

[21]This is due to the fact that a bond in the Vasicek (1977) model (as well as other general equilibrium bond pricing models) has a sensitivity to interest rates, $(1/P)\partial P/\partial r = (1\text{-}\exp(b_{11}\tau)/b_{11}$, that is a bounded function of bond maturity. In other words, the most this sensitivity can be is $1/b_{11}$ as $\tau \rightarrow \infty$. Hence, duration is undefined for interest sensitivities of greater magnitude.

growth rates.[22] The economic intuition for this is as follows. Because MMDA rates, and especially NOW account rates, adjust less that one-for-one with market interest rates, a rise in the market interest rate increases a bank's interest margin and, hence, its profitability.[23] This first effect would tend to increase the value of the bank's rent from deposit issue, thus giving the bank a negative duration. This is, in fact, the case for banks with low deposit demand growth rates. However, there is a second effect working in the opposite direction. An increase in the market interest rate reduces the present discounted value of the bank's future profits (cashflows) from retail deposit issue. For banks with high deposit demand growth rates, i.e., banks whose future profits contribute significantly to the value of its monopoly rent, this second effect dominates, causing the value of its rent to fall when market interest rates increase, i.e., giving the bank a positive duration.

Figures 1 and 2 also show the duration of NOW and MMDA deposits, i.e., the duration of $L = F - D^c$ which is given by equation (22) above. Since the competitive component of these demandable deposits, D^c, has a zero duration, we see that the signs of deposit durations are opposite to the signs of rent durations and have a magnitude that depends on the size of the rents per deposit. For the parameter estimates used in Figures 1 and 2, these deposit durations are small in magnitude, except for large growth rates of deposits when the value of rents per deposit become large.

Figures 3 and 4 consider the value of NOW and MMDA rents as a function of a bank's non-interest cost per dollar of deposit, ς.[24] As expected, rents decline as ς increases from 0 to .02 (0 to 200 basis points). Durations of deposits increase, which imply that durations of rents are decreasing,

[22]Note that by referring to a deposit rent having a negative duration of x years, we mean that its interest rate sensitivity is of equal magnitude, but of opposite sign, to that of an x year maturity zero coupon bond.

[23]Cashflows from deposit issue will be expected to increase at all future dates, but more so in the near-term than farther in the future. This is because of the mean-reverting behavior of market interest rates.

[24]A zero growth in deposit demand, μ=0, is assumed in Figures 3 and 4.

as this cost increases. The intuition for this is similar to that of a declining growth rate. An increase in market interest rates increases the bank's lending-borrowing spread due to the slow adjustment of NOW and MMDA rates (effect 1), which tends in increase current bank profitability. But, the greater market rate also reduces the present discounted value of future profits (effect 2). When ς is relatively small, making future profits significant, the second effect dominates, giving the bank's rent a greater (more positive) duration and, hence, the bank's deposit a smaller (more negative) duration. When ς is large and the value of future profits small, an increase in the market interest rate increases the value of the bank's rent, giving rents a smaller (more negative) duration and deposits a larger (more positive) duration.

We now turn to results using individual bank parameter estimates rather than averages of individual bank estimates. Figures 5 to 12 report distributions (histograms) for individual bank NOW and MMDA rents and deposit durations. The first four histograms, Figures 5 to 8, were constructed using the actual estimated growth in deposit demand, μ, for each individual bank. However, we had to exclude those NOW and MMDA observations for which the estimated growth rate, μ, exceeded r_∞ = .08809 since the estimated value of these rents would be unbounded. Based on this truncated subsample of 98 NOW accounts and 188 MMDAs, we observe that the median NOW account rent per deposit (median = 3.49%) is slightly smaller than the median rent for MMDAs (median = 4.97%). However, there is significant variation in these NOW and MMDA rents across individual banks. Comparing durations, we are not surprised to find NOW account deposit durations are generally much higher (median = 11.12 years) relative to those of MMDAs (median = .29 years). Similar to the rent estimates, durations also vary significantly across individual banks.

In Figures 9 to 12, we calculate the distribution of rents and durations for the individual bank NOW accounts and MMDAs, but under the assumption that the growth in deposit demand is zero for each bank's accounts. Doing this allows us to use the full sample of 244 NOW accounts and 216

MMDAs. Assigning all banks the same growth rate has the effect of reducing the variation in the estimates of the individual bank's deposit rents and durations. NOW accounts have a median rent of 6.55% and a significantly smaller median duration of 6.69 years. MMDAs have a slightly higher median rent of 7.88% and a median duration of .37 years.

While it is difficult to calibrate the precision of our estimates, they certainly appear to be in a range that most would consider reasonable. It is interesting to note that our median NOW and MMDA rent estimates are comparable to estimates obtained by Berkovec and Liang (1991) from data on failed bank merger premiums. Based on prices paid to the FDIC for banks that failed over the period January 1987 to October 1990, Berkovec and Liang (1991) present a hedonic approach to estimating deposit premiums that takes into consideration the FDIC's failure resolution method. For bank failures resolved using a purchase and assumption transaction, they estimate a NOW account premium of 4.9% (versus our actual growth and zero growth medians of 3.49% and 6.55%, respectively). For MMDAs, they find a premium of 4.0% (versus our actual growth and zero growth medians of 4.97% and 7.88%, respectively).

V. Conclusion

This paper analyzes the interest rate risk of retail bank deposits by recognizing the existence of market power associated with these bank liabilities. Accounting for retail deposit rents is essential to correctly evaluating a deposit's duration. By valuing this rent using contingent claims techniques and deriving its interest rate sensitivity, we have provided a framework for evaluating interest rate risk. Given additional parametric assumptions, we presented a practical method of estimating retail deposit rents and their deposit durations. Our results have a number of potential applications. Examples include: calculating the "going concern" value of a distressed bank's retail deposits so that regulators can be make a better decision regarding the cost of keeping a bank open or closing it via a purchase and assumption or liquidation; calculating a merger premium associated with one bank purchasing another bank's retail deposit market; measuring the value and duration of a bank's "core" deposits, as part of an exercise to estimate the duration of a bank's overall net worth. This last application should be of interest to bank managers seeking to limit interest rate risk as well as regulators attempting to set interest rate risk-based capital standards.

The valuation technique presented here could potentially be extended to other fixed-income assets or liabilities traded in imperfectly competitive markets. One class of callable assets might be revolving lines of credit to consumers or small or medium-sized firms. In principle, incorporating default-risk in the valuation of the rents from these assets is not an insurmountable task. However, there are likely to be additional empirical complications in estimating rents from these assets since the (promised) interest rates charged on these loans will not be equivalent to their actual rates of return. Tackling this problem is left to future research.

Appendix

This appendix shows how the value of a bank's monopoly rent from retail deposit issue, F, can be computing using equation (6) and the expression $E_t[f(r_\tau(T),x(T))]$ and P(t,T) given in equations (18) and (19), respectively.

Following the line of argument made by Longstaff (1990 p.102), the appropriate transformation of the market interest rate process, r(t), is

$$\text{(A.1)} \qquad \begin{aligned} dr_\tau(t) &= [a_1 + \sigma_1 q + \frac{\sigma_1^2}{b_{11}}(1-e^{b_{11}\tau}) + b_{11}r_\tau]dt + \sigma_1 dz_1 \\ &\equiv [a_{1\tau} + b_{11}r_\tau]dt + \sigma_1 dz_1 \end{aligned}$$

Since, by equation (10), the equilibrium deposit rate is a linear function of r(t), repeating the steps taken in equations (11) and (12) in the text, we find that the transformed deposit rate, $r_{d\tau}$, follows the process

$$\text{(A.2)} \qquad \begin{aligned} dr_{d\tau}(t) &= [d_1 a_{1\tau} + \alpha_2 - d_0\beta_{22} + d_1(\beta_{11} - \beta_{22})r_\tau + \beta_{22}r_{d\tau}]dt + \sigma_2 dz_2 \\ &\equiv [a_{2\tau} + b_{21}r_\tau + b_{22}r_{d\tau}]dt + \sigma_2 dz_2 \end{aligned}$$

Hence, the transformed processes, $r_\tau(t)$ and $r_{d\tau}(t)$, are equivalent to their untransformed processes but with constants added to their unconditional means. One can now calculate $E_t[f(r_\tau(T),x(T))]$ given in equation (18) which depends on some of the first and second moments of $r_\tau(T)$, $r_{d\tau}(T)$, and $\eta(T)$. These moments are given below. See Pennacchi (1991, p. 63-64) for discussion of the formulas.

$$E_t[r_\tau(T)] = r(t)e^{b_{11}\tau} + \frac{a_{1\tau}}{b_{11}}(e^{b_{11}\tau}-1)$$

(A.3)

$$= r(t)e^{b_{11}\tau} + \frac{a_1 + \sigma_1 q}{b_{11}}(e^{b_{11}\tau}-1) - \frac{\sigma_1^2}{b_{11}^2}(e^{b_{11}\tau}-1)^2$$

$$E_t[r_{d\tau}(T)] = r_d(t)e^{b_{22}\tau} + \frac{a_{2\tau}}{b_{22}}\left(e^{b_{22}\tau} - 1\right) + r(t)\frac{b_{21}}{b_{11}-b_{22}}\left(e^{b_{11}\tau} - e^{b_{22}\tau}\right)$$

(A.4)

$$+ \frac{a_{1\tau}b_{21}}{b_{11}-b_{22}}\left[\frac{\left(e^{b_{11}\tau} - 1\right)}{b_{11}} - \frac{\left(e^{b_{22}\tau} - 1\right)}{b_{22}}\right]$$

(A.5)

$$E_t\left[r_\tau(T)^2\right] - E_t\left[r_\tau(T)\right]^2 = \frac{\sigma_1^2}{2b_{11}}\left(e^{2b_{11}\tau} - 1\right)$$

$$E_t\left[r_{d\tau}(T)^2\right] - E_t\left[r_{d\tau}(T)\right]^2 = \frac{\sigma_2^2}{2b_{22}}\left(e^{2b_{22}\tau} - 1\right)$$

(A.6)

$$+ \frac{2\sigma_{12}b_{21}}{b_{11}-b_{22}}\left[\frac{\left(e^{(b_{11}+b_{22})\tau} - 1\right)}{b_{11}+b_{22}} - \frac{\left(e^{2b_{22}\tau} - 1\right)}{2b_{22}}\right]$$

$$+ \frac{b_{21}^2\sigma_1^2}{\left(b_{11}-b_{22}\right)^2}\left[\frac{\left(e^{2b_{11}\tau} - 1\right)}{2b_{11}} - \frac{2\left(e^{(b_{11}+b_{22})\tau} - 1\right)}{b_{11}+b_{22}} + \frac{\left(e^{2b_{22}\tau} - 1\right)}{2b_{22}}\right]$$

$$E_t[r_\tau(T)r_{d\tau}(T)] - E_t[r_\tau(T)]E_t[r_{d\tau}(T)] = \sigma_{12}\frac{\left(e^{(b_{11}+b_{22})\tau} - 1\right)}{b_{11}+b_{22}}$$

(A.7)

$$+ \frac{b_{21}\sigma_1^2}{b_{11}-b_{22}}\left[\frac{\left(e^{2b_{11}\tau} - 1\right)}{2b_{11}} - \frac{\left(e^{(b_{11}+b_{22})\tau} - 1\right)}{b_{11}+b_{22}}\right]$$

$$E_t[\eta(T)] = \eta(t)e^{\beta_{33}\tau} + \frac{\alpha_3}{\beta_{33}}(e^{\beta_{33}\tau}-1)$$

(A.8)

$$= [D(t)e^{-\mu t} - k_1 r(t) - k_2 r_d(t)]e^{\beta_{33}\tau} + \frac{\alpha_3}{\beta_{33}}(e^{\beta_{33}\tau}-1)$$

Inserting these expressions into equation (18), we find that the expected profit at date T takes the form:

(A.9)
$$E_t[f(r_\tau(T), x(T)] = [f_0 + f_1 e^{\gamma_1\tau} + \dots + f_{12}e^{\gamma_{12}\tau}]e^{\mu\tau}$$

where the f_i's are independent of τ and functions solely of $r(t)$, $r_d(t)$, $D(t)$, and the (constant) parameters of the model. The γ_i's are linear functions of the constants b_{11}, b_{22}, and β_{33}. Given the functional forms of $E_t[f(r_\tau(T),x(T))]$ and $P(t,T)$, the bank's monopoly rent requires computing integrals of the form:

(A.10)
$$I \equiv c_0\int_0^\infty e^{(\gamma_1+\mu-r_\infty)\tau} e^{c_1(1-e^{b_{11}\tau})(r(t)-r_\infty)} e^{c_2(1-e^{b_{11}\tau})^2} d\tau$$

where c_0, c_1, and c_2 are constants, with $c_1 = 1/b_{11}$ and $c_2 = \sigma_1^2/(4b_{11}^3)$. Assuming that $b_{11} < 0$ (which is a requirement for $r(t)$ to follow a stationary process), and making the change in variables $y \equiv 1-e^{b_{11}\tau}$, (A.10) can be written as:

(A.11)
$$I \equiv \frac{c_0}{-b_{11}} \int_0^1 (1-y)^{e-1} e^{c_1 y(r(t)-r_\infty)} e^{c_2 y^2} dy$$

where $\varepsilon \equiv (\gamma_i+\mu-r_\infty)/b_{11}$. Note that if we expand the term $e^{c_1 y(r(t)-r_\infty)}$ around the point $r(t) = r_\infty$, we obtain the following series:

(A.12)
$$e^{c_1 y(r(t)-r_\infty)} = 1 + c_1 y(r(t)-r_\infty) + \frac{1}{2!}c_1^2 y^2 (r(t)-r_\infty)^2 + \frac{1}{3!}c_1^3 y^3 (r(t)-r_\infty)^3 + \dots$$

Hence, the above integral can be written as:

(A.13)
$$I = \frac{c_0}{-b_{11}} \sum_{j=0}^{\infty} \left[\frac{c_1^j}{j!}(r(t)-r_\infty)^j \int_0^1 (1-y)^{\varepsilon-1} y^j e^{c_2 y^2} dy \right]$$

The integral in the right hand side of (A.13) is now in the form of a Riemann-Liouville (fractional) integral of order ε. See Erdelyi (1954, Chapter XIII). It can be evaluated in terms of a generalized hypergeometric function as:

(A.14)
$$\int_0^1 (1-y)^{\varepsilon-1} y^j e^{c_2 y^2} dy = \frac{\Gamma(\varepsilon)\Gamma(j+1)}{\Gamma(\varepsilon+j+1)} \, {}_2F_2\left(\frac{j+1}{2}, \frac{j+2}{2}; \frac{\varepsilon+j+1}{2}, \frac{\varepsilon+j+2}{2}; c_2\right)$$

where ${}_2F_2(\omega_1, \omega_2; \omega_3, \omega_4; z) = \sum_{k=0}^{\infty} \frac{(\omega_1)_k \, (\omega_2)_k}{(\omega_3)_k \, (\omega_4)_k} \frac{z^k}{k!}$ and where the notation

$$(\omega)_k = \frac{\Gamma(\omega+k)}{\Gamma(\omega)} = \omega(\omega+1) \dots (\omega+k-1), \quad (\omega)_0 = 1.$$

Inserting (A.14) into (A.13), the value of I can be written as:

(A.15) $$I = \frac{c_0}{-b_{11}} \sum_{j=0}^{\infty} \left[\frac{c_1^j}{j!} (r(t)-r_\infty)^j \, \frac{\Gamma(\varepsilon)\Gamma(j+1)}{\Gamma(\varepsilon+j+1)} \; {}_2F_2\left(\frac{j+1}{2}, \frac{j+2}{2}; \frac{\varepsilon+j+1}{2}, \frac{\varepsilon+j+2}{2}; c_2 \right) \right]$$

This expression simplifies to

(A.16) $$I = \frac{c_0}{-b_{11}} \sum_{j=0}^{\infty} \left[\left(\frac{r(t)-r_\infty}{b_{11}}\right)^j \frac{1}{\varepsilon(\varepsilon+1)\cdots(\varepsilon+j)} \; {}_2F_2\left(\frac{j+1}{2}, \frac{j+2}{2}; \frac{\varepsilon+j+1}{2}, \frac{\varepsilon+j+2}{2}; c_2 \right) \right]$$

The duration of retail bank deposits requires computation of the derivative $\partial F/\partial r$, which, in turn, requires the derivative $\partial I/\partial r$. Using (A.16) we have:

(A.17) $$\frac{\partial I}{\partial r} = \frac{c_0}{-b_{11}^2} \sum_{j=0}^{\infty} \left[\left(\frac{r(t)-r_\infty}{b_{11}}\right)^j \frac{j+1}{\varepsilon(\varepsilon+1)\cdots(\varepsilon+j+1)} \; {}_2F_2\left(\frac{j+2}{2}, \frac{j+3}{2}; \frac{\varepsilon+j+2}{2}, \frac{\varepsilon+j+3}{2}; c_2 \right) \right]$$

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Table 1

Maximum Likelihood Estimates of the Vasicek Model Parameters

(Standard Errors in Parentheses)

$-a_1/b_{11}$	$-b_{11}$	σ_1
.08131	.09800	.02432
(.05329)	(.01902)	(.00110)

Data: End of month 30, 90, 180, and 345 day Treasury bill prices over period 1968 through 1990, 267 observations.

Table 2

Averages of Parameter Estimates
(Used in Figures 1 to 4)

Parameter	NOWs	MMDAs
$r(t)$	.06240	.06240
r_∞	.08809	.08809
a_1	.007968	.007968
b_{11}	-.09800	-.09800
σ_1	.02432	.02432
$r_d(t)$	.04926	.05648
b_{22}	-3.3207	-2.0022
σ_2	.004124	.007418
σ_{12}	-2.776E-6	-1.4308E-5
$\alpha_2 - d_0\beta_{22}$	.15695	.00511
$\eta(t)$	140112.39	414101.81
α_3	683845.52	848320.81
β_{33}	-7.2414	-1.9952
d_1	.40126	.82920
k_1	735439.04	-9682.92
k_2	-1566220.62	-1833831.38
μ*	0	0
ς	.0066972	.0045166
ρ	.88	1.0

*The estimated average annual growth rate of deposits was .1266 for NOWs and -.0257 for MMDAs. However, a value of $\mu = 0$ was used in Figures 3 and 4.

Figure 1

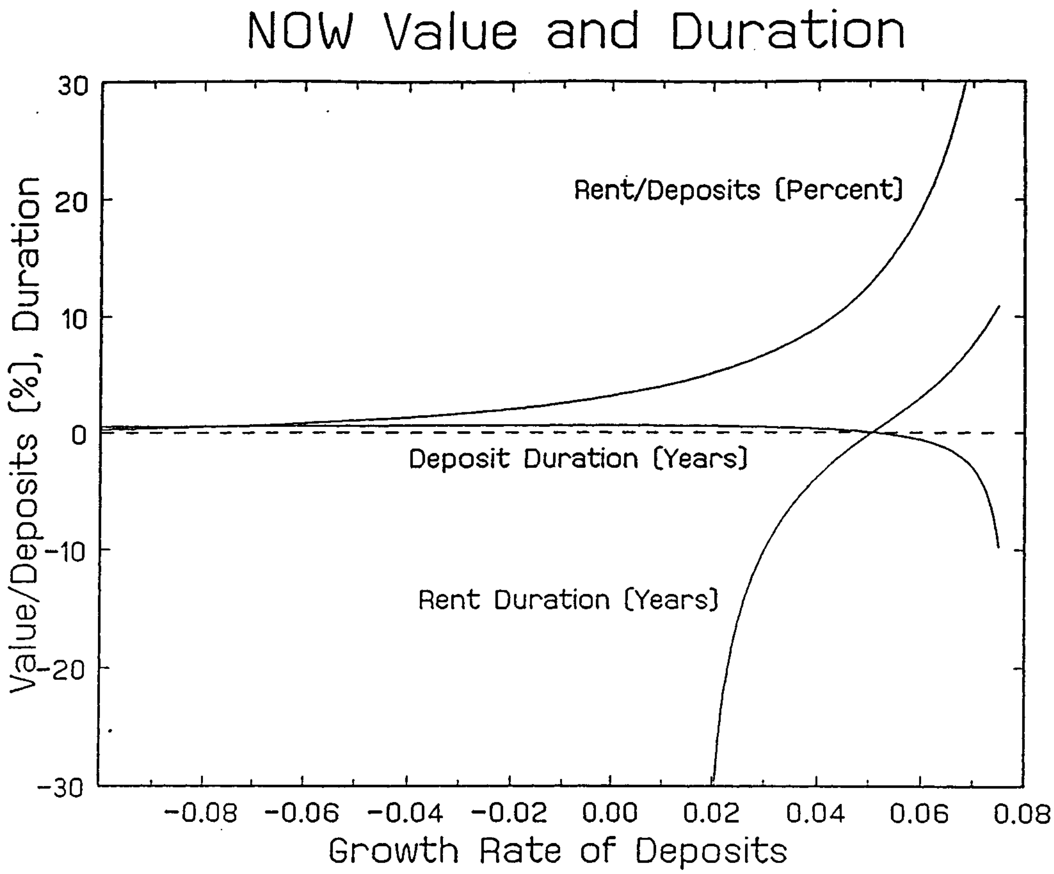

Figure 2

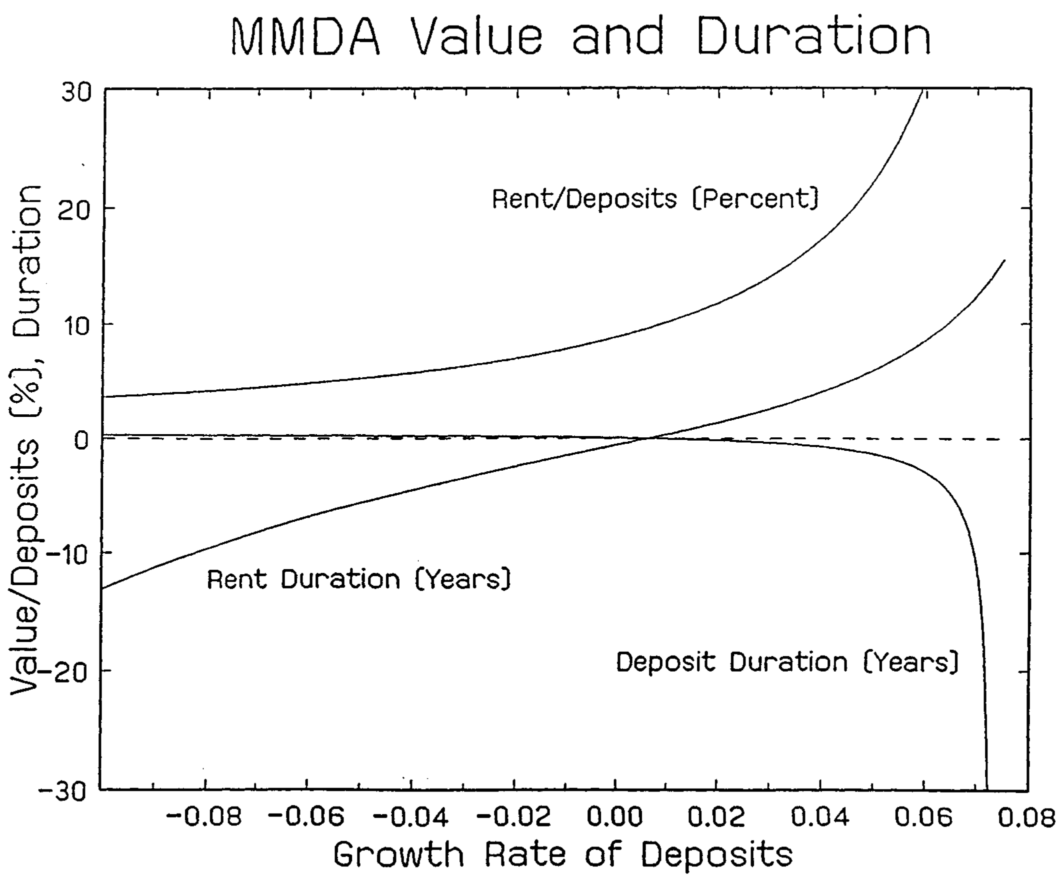

Figure 3

NOW Value and Duration

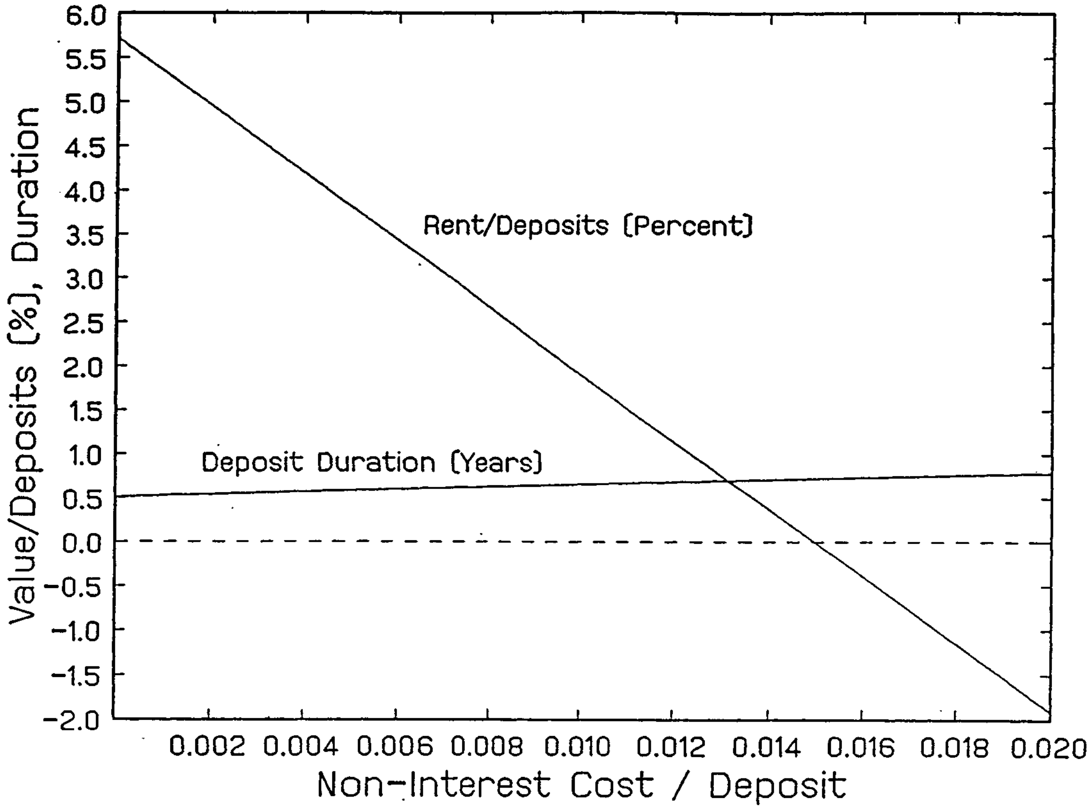

Figure 4

MMDA Value and Duration

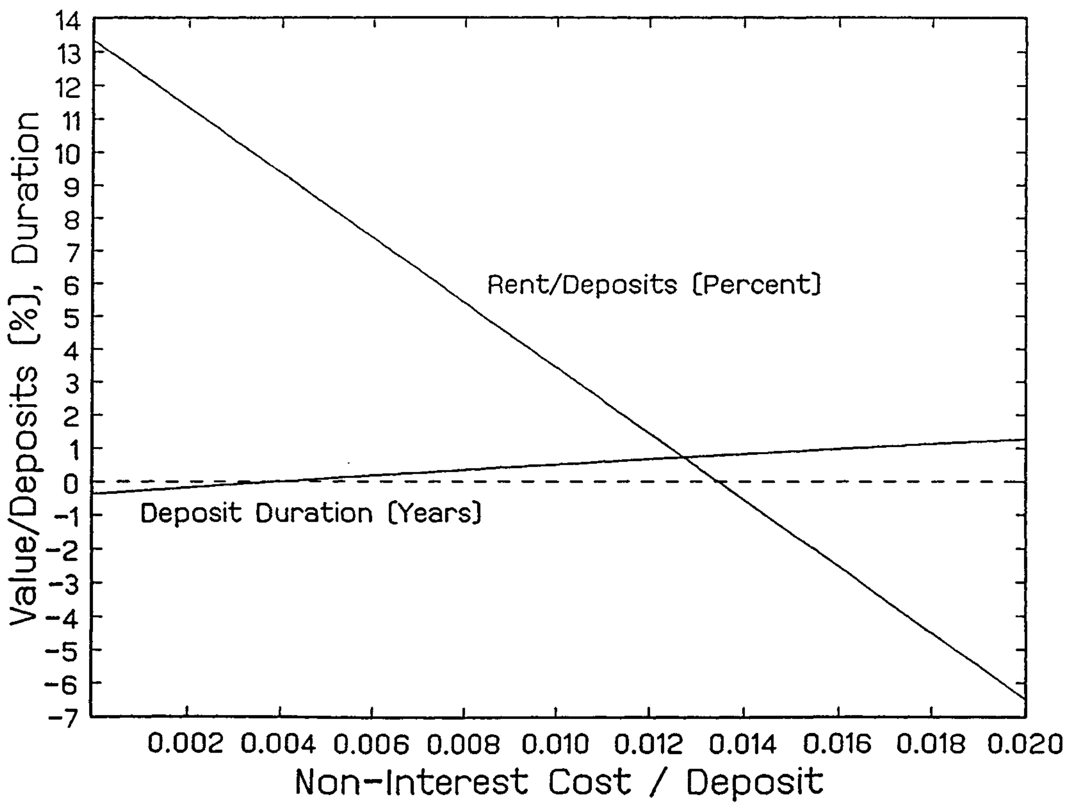

Figure 5

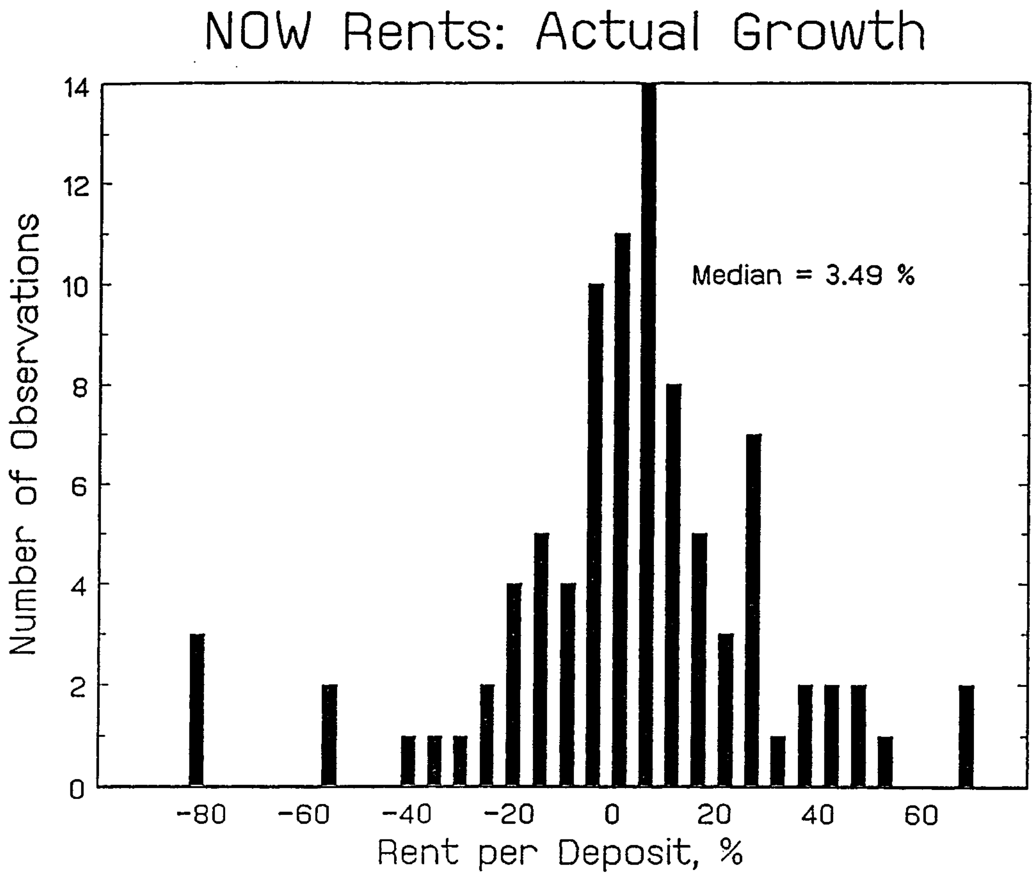

Figure 6

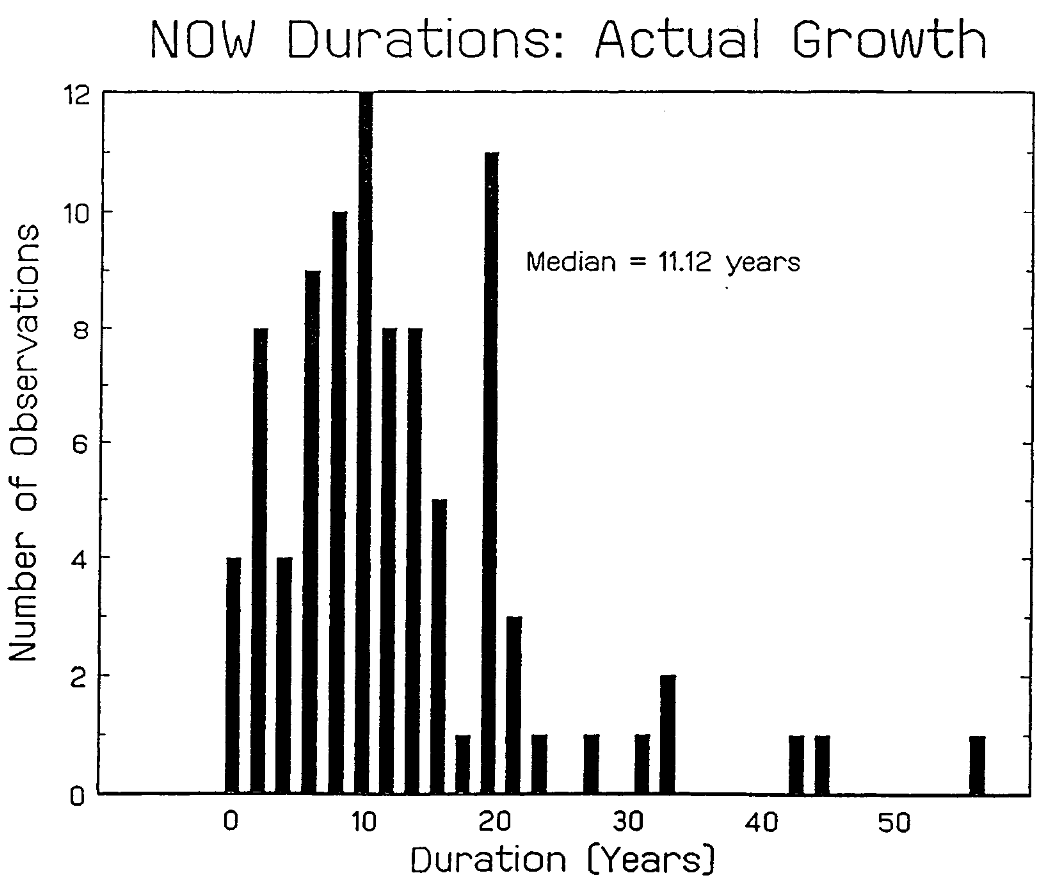

Figure 7

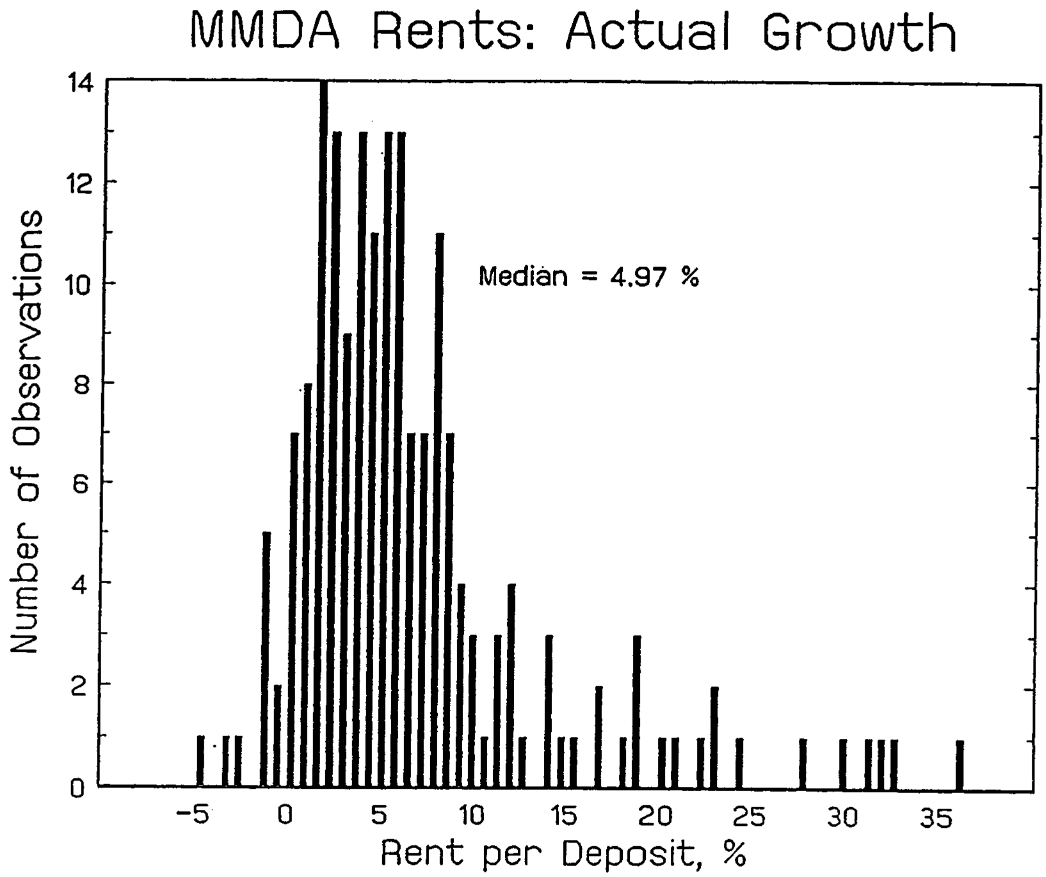

Figure 8

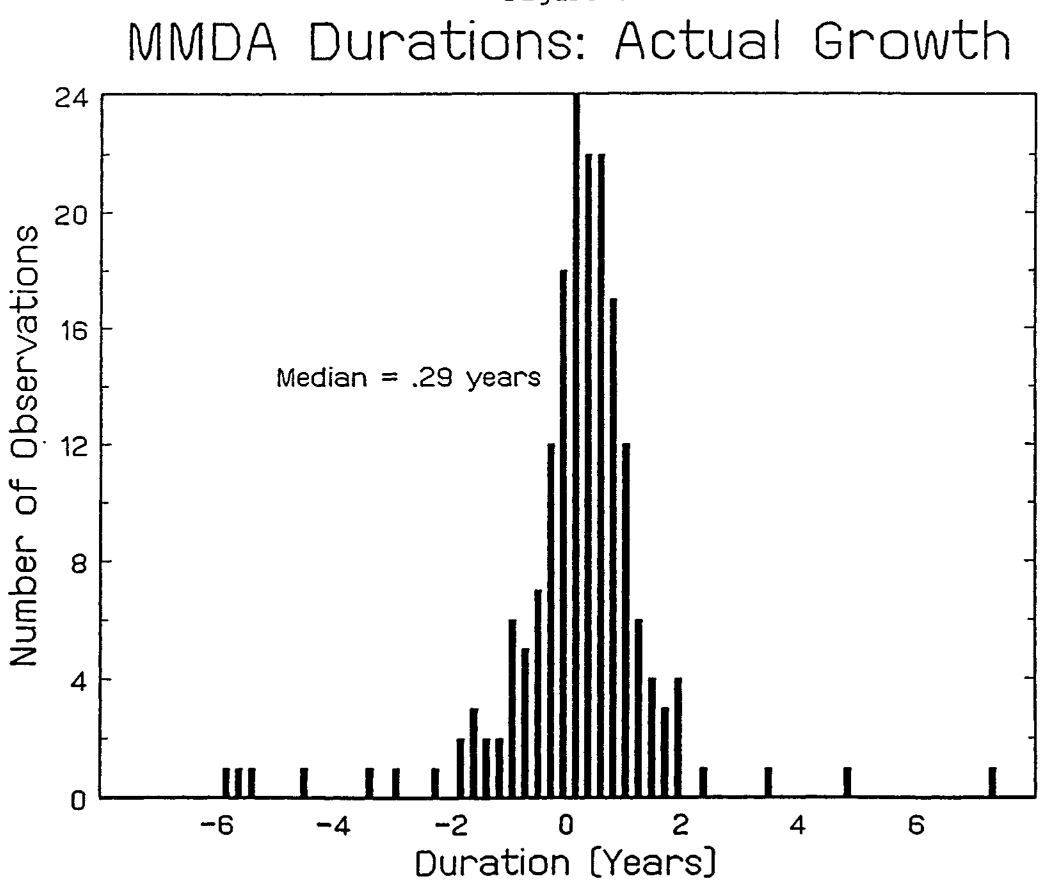

Figure 9

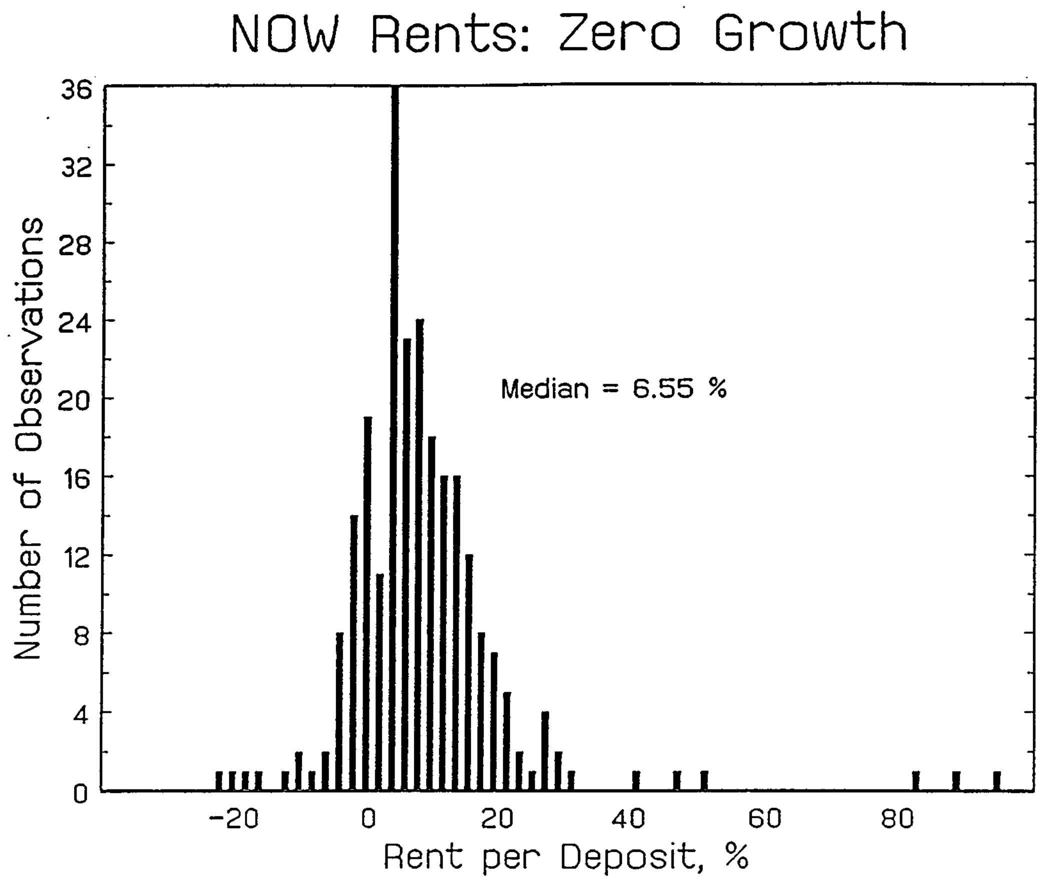

Figure 10

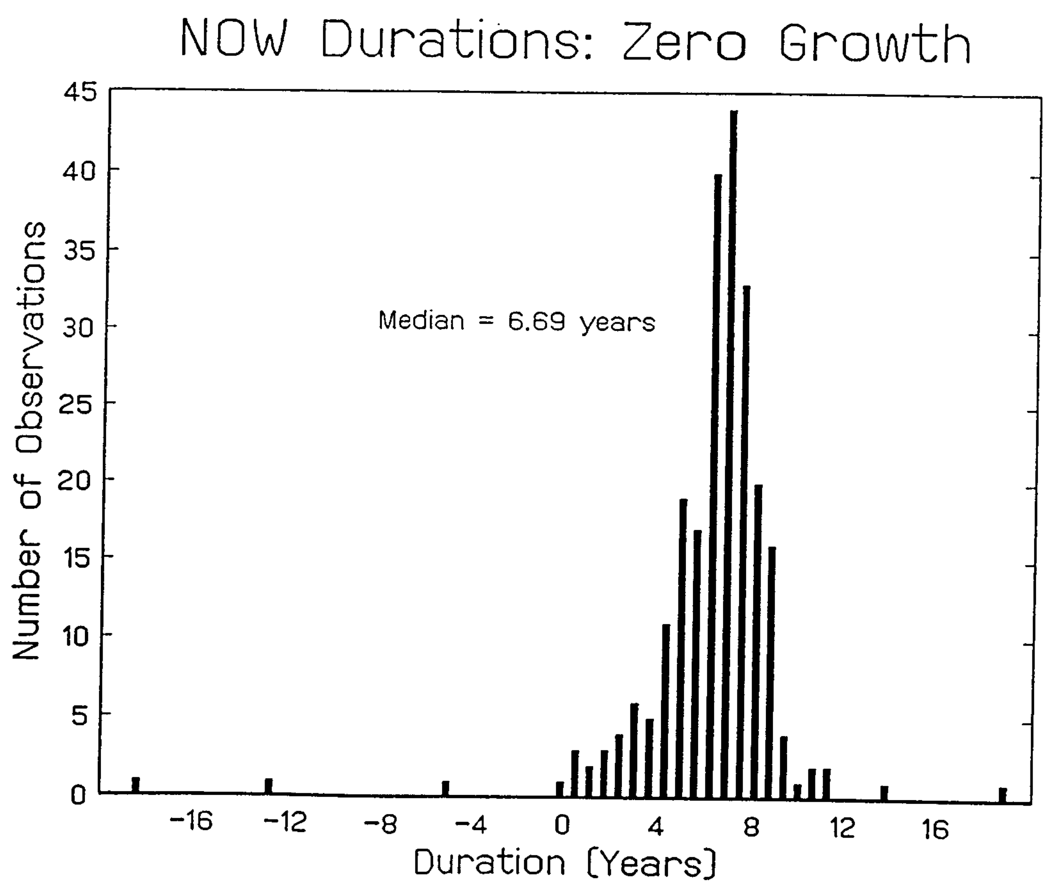

Figure 11

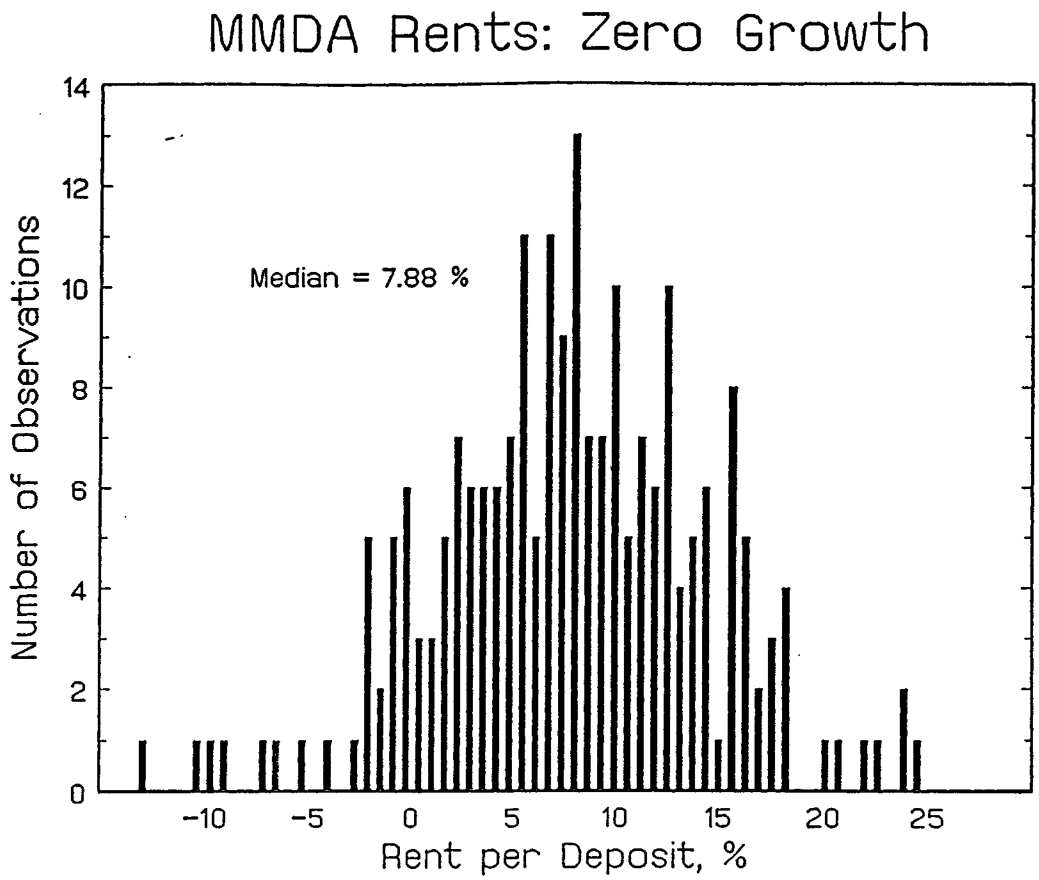

Figure 12