Working Paper Series

Seasonality and Equilibrium Business Cycle Theories

R. Anton Braun and Charles L. Evans

Working Papers Series
Issues in Macroeconomics
Research Department
Federal Reserve Bank of Chicago
December 1991 (WP-91-23)

FEDERAL RESERVE BANK
OF CHICAGO

Seasonality and Equilibrium Business Cycle Theories

R. Anton Braun*

Charles L. Evans**

March 1990
Revised November 1991

Abstract

Barsky-Miron [1989] find that the postwar U.S. economy exhibits a regular seasonal cycle, as well as the business cycle phenomenon. Are these findings consistent with current equilibrium business cycle theories as surveyed by Prescott [1986]? We consider a dynamic, stochastic equilibrium business cycle model which includes deterministic seasonals and nontime-separable preferences. We show how to compute a perfect foresight seasonal equilibrium path for this economy. An approximation to the stochastic equilibrium is calculated. Using postwar U.S. data, GMM estimates of the structural parameters are employed in the perfect foresight and simulation analyses.

The nontime-separable model predicts most of the seasonal patterns found in aggregate quantity time series; a notable exception is the seasonal pattern in labor hours. An evaluation of the model's predictions for deseasonalized second moments finds support for the parameterization. This model broadly displays a seasonal cycle as well as the business cycle phenomenon.

JEL Classification(s): E32, E20

*Department of Economics
University of Virginia
Charlottesville, VA 22901
(804) 924-7845

**Research Department
Federal Reserve Bank of Chicago
P.O. Box 834
Chicago, IL 60690-0834
(312) 322-5812

For helpful comments, we thank Fabio Canova, Marty Eichenbaum, Eric Ghysels, Jim Hamilton, Valerie Ramey and seminar participants at the 1990 NBER Summer Institute, Rutgers, Queens', the Federal Reserve Bank of Chicago, and the Universities of Montreal, South Carolina, and Virginia. An earlier version of this paper was presented at the 1990 Winter Econometric Society meetings. Any opinions, findings, conclusions, or recommendations expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Chicago, or the Federal Reserve System.

1. Introduction

The postwar U.S. economy exhibits a regular seasonal cycle, as well as the business cycle phenomenon: this is the principal finding by Barsky and Miron [1989]. These researchers analyze aggregate data which has not been adjusted for seasonality and find that deterministic seasonals account for between 50 - 95% of the variation in the growth rates of aggregate quantity variables such as GNP, consumption, and investment. Are these findings consistent with current equilibrium business cycle theories as surveyed by Prescott [1986]? Prescott concludes that variations in the rate of technological change are an important source of economic fluctuations, accounting for about 70% of cyclical fluctuations (Kydland and Prescott [1989]). Theory predicts cyclical fluctuations, but does it also predict seasonal cycles? Answering this question is a potentially important step in assessing the validity of equilibrium theories. The similarities between the seasonal cycle and the business cycle suggest that the economic mechanism generating business cycle fluctuations also generates seasonal cycles. Consequently, a proper theory should predict seasonal cycles as well as business cycles.

We consider a dynamic, stochastic equilibrium business cycle model which includes deterministic seasonals. Our seasonal specification is parsimonius: we include only a technology seasonal, a preference seasonal, and a government spending seasonal. As in Kydland and Prescott [1982], Eichenbaum, Hansen, and Singleton [1989], and Braun [1990], preferences exhibit nontime-separability in consumption goods and leisure. This model is tractable. In particular, we show how to compute a perfect foresight seasonal equilibrium path for this economy. An approximation to the stochastic equilibrium is also calculated around this equilibrium path.

Using a Generalized Method of Moments (GMM) estimator, the model's structural parameters and the seasonals are estimated using postwar U.S. data. The overidentifying restrictions of the model are not rejected at conventional levels; and the technology seasonal estimates indicate a strong seasonal pattern, sufficient to drive an equilibrium seasonal cycle.

Given the parameter estimates, the seasonal patterns and business cycle properties of the equilibrium model accord well with the data. First, the model replicates many of the seasonal patterns in the aggregate data, particularly for output, consumption, capital, average labor productivity, and the real interest rate. The principal shortcomings are with respect to second and fourth quarter investment, and labor hours in quarters two, three, and four. The model also captures the large contribution of deterministic seasonals for the total variation in most aggregate variables. Second, we find that seasonal variation in technology is essential for explaining the seasonal patterns in output. Seasonal variation in preferences and government purchases alone generate output seasonals no larger than 0.3% per quarter. Third, without nontime-separabilities in preferences, seasonal variation in output, hours, and investment is much too large. Habit-persistent preferences for leisure are an important element in the model's ability to match the magnitude of seasonal variation in aggregate hours and local durability of consumption services also proves to be important for matching the seasonal properties of consumption. Fourth, the model's predictions for deseasonalized second moments match the data's second moments with about the same accuracy as existing nonseasonal real business cycle models.

Does theory predict seasonal fluctuations as well as cyclical fluctuations? The equilibrium model with nontime-separable preferences displays the seasonal patterns emphasized by Barsky and Miron [1989] as well as the business cycle phenomenon. The fact that the model successfully offers predictions across both business and seasonal cycle frequencies is perhaps its greatest strength. The fact that the model requires large seasonal variations in technology to achieve this match, however, suggests that this model is simply a benchmark. Other theories which deliver seasonal variations in technology endogenously may encompass the findings here.

The paper is organized as follows. Section 2 presents the model economy which includes seasonality and exogenous growth, and the perfect foresight seasonal equilibrium path is defined. Section 3 presents the GMM estimation strategy, describes the data, and discusses the structural parameter estimates. Section 4 analyzes the perfect foresight seasonal equilibrium paths implied by the parameter estimates and assesses the relative contributions of technology, preferences, and government seasonals. Section 5 presents and analyzes the simulation results for the stochastic economy with seasonality. Section 6 offers conclusions.

2. An Equilibrium Business Cycle Economy with Seasonality

This section presents a one-sector, real business cycle economy which is subjected to seasonal variation in the technology, preferences, and government purchases. The model is similar to the models considered by Christiano-Eichenbaum [1990] and Braun [1990].

2.1 The economy with growth and seasonality

Consider an economy composed of a large number of identical, infinitely-lived households each of which seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \tau_t \log c_t^* + \gamma_2 \log l_t^* \right\}, \qquad \gamma_2 > 0 \qquad [2.1]$$

where c^* and l^* represent consumption and leisure services, respectively. Consumption services are related to private consumption (cp) and public consumption (g) as follows:

$$c_t^* = cp_t + \gamma_1 g_t + a\,(cp_{t-1} + \gamma_1 g_{t-1}), \quad 0 \le \gamma_1 < 1, \ |a|<1 \qquad [2.2]$$

where γ_1 governs the substitutability of public goods for private consumption goods. The parameter a governs the character and degree of nonseparability: if a is negative (positive), consumption goods are complements (substitutes) across adjacent time periods. The complementarity case can also be interpreted as habit-persistence in preferences. The variable τ_t is a deterministic preference seasonal which follows:

$$\tau_t = \tau_1 Q_{1t} + \tau_2 Q_{2t} + \tau_3 Q_{3t} + \tau_4 Q_{4t}, \quad \tau_j > 0 \text{ for all } j \qquad [2.3]$$

and the variable Q_{jt} is a dummy variable taking on the value of 1 when period t corresponds to season j, and zero otherwise; consequently, τ_j is the preference seasonal in season j. Leisure (l) is time not devoted to labor (n), leading to the time allocation constraint that $n_t + l_t = T$, where T is the maximum number of hours available per period. Preferences are defined over leisure services l_t^*:

$$l_t^* = l_t + b\, l_{t-1} \qquad |b|<1. \qquad [2.4]$$

The parameter b governs the character and degree of nonseparability: if b is negative (positive), then leisure choices are complements (substitutes) across adjacent time periods. Finally, the operator E_t is the mathematical expectations operator conditional on all information known at time t.

Each household has access to a production function of the form:

$$y_t = (k_t^d)^{\theta} (z_t n_t^d)^{1-\theta} \qquad [2.5]$$

where y is output and k^d and n^d are the quantities of capital and labor services demanded by the entrepreneur-household. The household's output can be consumed (privately or publicly) or stored in the form of additional capital next period. Each period, the existing capital stock depreciates at the geometric rate δ. The variable z_t is a labor-augmenting technology shock which includes deterministic seasonal components:

$$z_t = z_{t-1} \exp(\lambda_t) \qquad [2.6]$$

$$\lambda_t = \lambda_1 Q_{1t} + \lambda_2 Q_{2t} + \lambda_3 Q_{3t} + \lambda_4 Q_{4t} + \epsilon_t$$

where ϵ_t is a purely indeterministic, white noise random variable. Notice that $\log z_t$ is a random walk with seasonal drift: when the seasonal growth rates λ_j do not sum to zero, this economy experiences growth.

The economy possesses competitive markets in labor and capital services: suppliers of labor services receive a wage w_t, suppliers of capital services receive the rental rate r_t. Finally, the government taxes each household in a lump-sum fashion, TL_t. This leads to the household's period budget constraint:

$$cp_t + k_{t+1} = y_t + (1-\delta)k_t - w_t(n_t^d - n_t) - r_t(k_t^d - k_t) - TL_t \qquad [2.7]$$

where k and n represent the supply of capital and labor services by the household.

The government chooses a stochastic process for g_t which is uncontrollable from the household's perspective. Government purchases are assumed to contain a permanent and a transitory component. The permanent component is related to the technology shock z_t; the transitory component is an autoregressive process of order 1 with a seasonal mean. The stochastic process for g_t is:

$$\log \frac{g_t}{z_t} - \log \tilde{g}_{jt} = \rho \left(\log \frac{g_{t-1}}{z_{t-1}} - \log \tilde{g}_{j-1,t-1}\right) + u_t \ , \ 0<\rho<1 \qquad [2.8]$$

where $\tilde{g}_{jt}$ is the seasonal mean of transitory government purchases when period t corresponds to season j; and u_t is an indeterministic, white noise random variable. A specification such as this one, but without seasonality, was adopted in Christiano-Eichenbaum [1990] and Braun [1990].

In this Ricardian environment, we assume without loss of generality that the government's budget constraint is $g_t = TL_t$. This leads to the economy-wide resource constraint (in per capita terms):

$$cp_t + k_{t+1} + g_t = y_t + (1-\delta)k_t - w_t(n_t^d - n_t) - r_t(k_t^d - k_t) \qquad [2.9]$$

When the supply of labor and capital equals the demand for labor and capital (respectively), equation [2.9] is the familiar per capita national income accounting identity for this closed economy. Since all individuals are identical in this economy, each individual's net supply of either factor will be zero, in equilibrium.

As in King-Plosser-Rebelo [1988], an empirical analysis of this economy is facilitated by rescaling the economy in a way which induces a stationary environment. To this end, define the following scaled variables:

$$\tilde{k}_{t+1} = \frac{k_{t+1}}{z_t}, \quad \tilde{y}_t = \frac{y_t}{z_t}, \quad \tilde{g}_t = \frac{g_t}{z_t}, \quad \tilde{i}_t = \frac{i_t}{z_t}, \quad \tilde{c}_t^* = \frac{c_t^*}{z_t},$$

$$\tilde{cp}_t = \frac{cp_t}{z_t}, \quad \tilde{w}_t = \frac{w_t}{z_t}, \quad \tilde{TL}_t = \frac{TL_t}{z_t}$$

where i_t is gross investment. Under the assumption that the unscaled economy exhibits balanced growth, the scaled variables are stationary; the remaining unscaled variables are leisure services (l^*), labor (n) and the rental rate (r), which are stationary without any rescaling. The household's problem in the scaled economy becomes:

$$\max \; E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \tau_t \log \tilde{c}_t^* + \gamma_2 \log l_t^* + \tau_t \log z_t \right\} \qquad [2.1']$$

subject to the constraints

$$\tilde{c}_t^* = \tilde{c}p_t + \gamma_1 \tilde{g}_t + a\,(\tilde{c}p_{t-1} + \gamma_1 \tilde{g}_{t-1})\, e^{-\lambda_t} \qquad [2.2']$$

$$l_t^* = T - n_t + b\,(T - n_{t-1}) \qquad [2.4']$$

$$\tilde{y}_t = (\tilde{k}_t^d)^{\theta} (n_t^d)^{1-\theta} e^{-\theta\lambda_t} \qquad [2.5']$$

$$\tilde{c}p_t + \tilde{k}_{t+1} = \tilde{y}_t + (1-\delta)\tilde{k}_t e^{-\lambda_t} - \tilde{w}_t(n_t^d - n_t) - r_t(\tilde{k}_t^d - \tilde{k}_t)e^{-\lambda_t} - \tilde{g}_t \qquad [2.7']$$

where the uncontrollable $\tilde{g}_t$ has replaced $\tilde{T}L_t$, and the presence of log z_t poses no analytical difficulties since it is uncontrollable and stochastically dominated.

Given initial values for the capital stock, leisure, government purchases and private consumption, as well as a law of motion for $\tilde{g}_t$, the equilibrium in this economy is essentially a sequence of contingency plans $\{ \tilde{c}p_t, \tilde{k}_{t+1}, n_t;\ t>0 \}$ which satisfies: (1) the household's first-order necessary conditions for a constrained maximum of [2.1'], (2) a transversality condition on capital, (3) the economy-wide per capita resource constraint, and (4) market-clearing in the labor and capital markets. The system of equations which characterize the equilibrium of this scaled economy are:

$$\frac{\gamma_2}{l_t^*} + b\,\beta\, E_t \frac{\gamma_2}{l_{t+1}^*} = \tilde{\mu}_t\,(1-\theta)\,\tilde{k}_t^{\theta}\, n_t^{-\theta}\, e^{-\theta\lambda_t} \qquad [2.10]$$

$$\beta E_t \left\{ \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t} \left(\theta \tilde{k}_{t+1}^{\theta-1} n_{t+1}^{1-\theta} e^{-\theta\lambda_{t+1}} + (1-\delta) e^{-\lambda_{t+1}} \right) \right\} = 1 \qquad [2.11]$$

$$\tilde{\mu}_t = \frac{\tau_t}{\tilde{c}_t^*} + a \beta E_t \left\{ e^{-\lambda_{t+1}} \frac{\tau_{t+1}}{\tilde{c}_{t+1}^*} \right\} \qquad [2.12]$$

$$\tilde{c}p_t + \tilde{k}_{t+1} + \tilde{g}_t = \tilde{k}_t^{\theta} n_t^{1-\theta} e^{-\theta\lambda_t} + (1-\delta) \tilde{k}_t e^{-\lambda_t} \qquad [2.13]$$

$$\tilde{y}_t = \tilde{k}_t^{\theta} \tilde{n}_t^{1-\theta} e^{-\theta\lambda_t} \qquad [2.14]$$

$$\tilde{c}_t^* = \tilde{c}p_t + \gamma_1 \tilde{g}_t + a (\tilde{c}p_{t-1} + \gamma_1 \tilde{g}_{t-1}) e^{-\lambda_t} \qquad [2.15]$$

$$l_t^* = T - n_t + b (T - n_{t-1}) \qquad [2.16]$$

$$\tilde{w}_t = (1-\theta) \tilde{k}_t^{\theta} n_t^{-\theta} e^{-\theta\lambda_t} = (1-\theta) \tilde{y}_t/n_t \qquad [2.17]$$

$$r_t = \theta \tilde{k}_t^{\theta-1} n_t^{1-\theta} e^{-\theta\lambda_t} e^{\lambda_t} = (\theta \tilde{y}_t/k_t) e^{\lambda_t} \qquad [2.18]$$

$$\lim_{t\to\infty} \beta^t \tilde{\mu}_t \tilde{k}_{t+1} = 0 \qquad [2.19]$$

The variable $\tilde{\mu}_t$ is the Lagrange multiplier associated with the resource constraint. These conditions determine the equilibrium of the economy.

2.2 Characterizing the Seasonal Equilibrium

In general, a perfect foresight path would solve the system of equations [2.10] - [2.19], with the uncertainty removed. We restrict attention to a particular perfect foresight path. This path has the characteristic that the value of a variable x in quarter j is always equal to its realization four quarters ago. For our economy with (quarterly) seasonals, equations [2.10] - [2.16] can be reduced to twelve restrictions which { $\tilde{cp}_j$, $\tilde{k}_j$, $\tilde{n}_j$; j = 1,2,3,4 } must satisfy. These seasonal restrictions (without uncertainty) can be written as:

$$\frac{\gamma_2}{l_j^*} + b\,\beta\,\frac{\gamma_2}{l_{j+1}^*} = \tilde{\mu}_j\,(1-\theta)\,\tilde{k}_j^{\theta}\,n_j^{-\theta}\,e^{-\theta\lambda_j} \qquad [2.20]$$

$$\tilde{\mu}_j = \beta\left\{\tilde{\mu}_{j+1}\left(\theta\,\tilde{k}_{j+1}^{\theta-1}\,n_{j+1}^{1-\theta}\,e^{-\theta\lambda_{j+1}} + (1-\delta)\,e^{-\lambda_{j+1}}\right)\right\} \qquad [2.21]$$

$$\tilde{cp}_j + \tilde{k}_{j+1} + \tilde{g}_j = \tilde{k}_j^{\theta}\,n_j^{1-\theta}\,e^{-\theta\lambda_j} + (1-\delta)\,\tilde{k}_j\,e^{-\lambda_j} \qquad [2.22]$$

where the seasonal index j runs from 1 to 4. We adopt a wrap-around dating convention that when j=5, this represents the first quarter (j=1). The variables $\tilde{\mu}_j$, $\tilde{c}_j^*$, and l_j^* are functions of the essential variables $\tilde{cp}$ and n, as well as the exogenous variables $\tilde{g}$, τ, and λ:

$$\tilde{c}_j^* = \tilde{cp}_j + \gamma_1\,\tilde{g}_j + a\,(\tilde{cp}_{j-1} + \gamma_1\,\tilde{g}_{j-1})\,e^{-\lambda_j} \qquad [2.23]$$

$$l_j^* = T - n_j + b\,(T - n_{j-1}) \qquad [2.24]$$

$$\tilde{\mu}_j = \frac{\tau_j}{\tilde{c}_j^*} + a\,\beta\left\{e^{-\lambda_{j+1}}\,\frac{\tau_{j+1}}{\tilde{c}_{j+1}^*}\right\} \qquad [2.25]$$

This leads to the following definition:

<u>Definition</u> A sequence { $\tilde{cp}_i$, $\tilde{k}_i$, $\tilde{n}_i$; i = 1,2,3,4 } which satisfies equations [2.20] - [2.22] is a <u>perfect foresight seasonal equilibrium</u>.

For all parameterizations of the model which we consider below, we were able to numerically calculate a unique seasonal equilibrium: no evidence of multiple equilibria was found. Furthermore, in the course of solving for the stochastic equilibrium (in Section 5), we calculate roots for the log-linear system which exhibit the proper characteristics to ensure local stability of the perfect foresight seasonal equilibrium. That is, in the state space representation, the fundamental matrix had equal numbers of roots inside and outside the unit circle. Also, Chatterjee and Ravikumar [1989] characterize existence and uniqueness in a model similar to ours.

Given the perfect foresight path for { $\tilde{cp}_i$, $\tilde{k}_i$, $\tilde{n}_i$; i = 1,2,3,4 } and the seasonals { λ_j, τ_j, g_j ; j = 1,2,3,4 }, seasonal paths for { $\tilde{y}_j$, $\tilde{i}_j$, $\tilde{w}_j$, r_j } can be computed using seasonal counterparts to equations [2.14], [2.13], [2.17], and [2.18]:

$$\tilde{y}_j = \tilde{k}_j^{\theta} n_j^{1-\theta} e^{-\theta\lambda_j} \qquad [2.26]$$

$$\tilde{i}_j = \tilde{k}_{j+1} - (1-\delta) \tilde{k}_j e^{-\lambda_j} \qquad [2.27]$$

$$\tilde{w}_j = (1-\theta) \tilde{y}_j / n_j \qquad [2.28]$$

$$r_j = \theta \tilde{y}_j / (\tilde{k}_j e^{-\lambda_j}) \qquad [2.29]$$

To compare these seasonal means with the Barsky-Miron seasonal results, the data set must be precisely defined, the seasonals in { λ_j, τ_j, g_j ; j = 1,2,3,4 } must be estimated, and the model's other parameters set.

3. Estimation of the Structural Parameters

Given seasonally unadjusted time series data for the U.S economy, the Euler equation methods of Hansen-Singleton [1982] can be used to estimate the model's structural parameters and test the overidentifying restrictions implied by the model and choice of instruments. The parameter vector to be estimated is:

$$\Psi = (\theta, a, b, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \tau_1, \tau_4, d_1, d_2, d_3, d_4, \rho, \sigma_u, \sigma_\epsilon),$$

where the d_i seasonals are related to the log $\tilde{g}_i$ seasonals by the relationship $d_i = \log \tilde{g}_i - \rho \log \tilde{g}_{i-1}$. Notice further that there are only two τ's. In estimating the model we restrict attention to preferences that vary only in the fourth quarter in response to Christmas. Thus in quarters one through three τ takes on the value τ_1 and in quarter four τ takes on the value τ_4. The parameters β, γ_1, γ_2, δ, and T are set a priori, in accordance with previous studies. The discount factor β was chosen to be $1.03^{-.25}$, as in Christiano-Eichenbaum [1990] and Braun [1990]. The depreciation rate δ was chosen to be 2.5% per quarter, as in Kydland-Prescott [1982] and King-Plosser-Rebelo [1988]. The utility parameter γ_1 governing the substitutability of government purchases for private consumption was chosen to be 0.4, as found by Aschauer [1985]. The utility weight γ_2 on leisure was normalized to be 1. The total time endowment for the household is 1369 hours per quarter.

The moment equations chosen for the estimation consist of two stochastic Euler equations, the production function, the transitory government spending autoregression, and two variance estimates. Specifically, the household's time t decision for n_t and k_{t+1} yield the conditions:

$$\beta E_t \left\{ \left(\frac{\tau_{t+1}}{c^*_{t+1}} + \beta a \frac{\tau_{t+2}}{c^*_{t+2}} \right) \left(\theta \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) - \left(\frac{\tau_t}{c^*_t} + \beta a \frac{\tau_{t+1}}{c^*_{t+1}} \right) \right\} = 0 \qquad [3.1]$$

$$E_t \left\{ \left(\frac{\tau_t}{c^*_t} + \beta a \frac{\tau_{t+1}}{c^*_{t+1}} \right) (1-\theta) \frac{y_t}{n_t} - \frac{\gamma_2}{l^*_t} - \beta b \frac{\gamma_2}{l^*_{t+1}} \right\} = 0 \qquad [3.2]$$

where c^* and 1^* can be constructed from equations [2.2] and [2.4] given values of γ_1, a, and b. The technology specification [2.5] - [2.6] yields the stochastic equation:

$$\frac{1}{1-\theta}\left(\Delta \log y_t - \theta \Delta \log k_t - (1-\theta) \Delta \log n_t \right)$$

$$-\lambda_1 Q_{1t} - \lambda_2 Q_{2t} - \lambda_3 Q_{3t} - \lambda_4 Q_{4t} = \epsilon_t \qquad [3.3]$$

where ϵ_t is an unconditionally mean zero random variable which is not observed by the econometrician. The law of motion for transitory government spending [2.8] yields the stochastic equation:

$$\log \frac{g_t}{z_t} - \rho \log \frac{g_{t-1}}{z_{t-1}} - d_1 Q_{1t} - d_2 Q_{2t} - d_3 Q_{3t} - d_4 Q_{4t} = u_t \qquad [3.4]$$

where u_t is an unconditionally mean zero random variable which is not observed by the econometrician. The d_i seasonals are related to the log $\tilde{g}_i$ seasonals by the relationship $d_i = \log \tilde{g}_i - \rho \log \tilde{g}_{i-1}$. Finally, the residual errors ϵ_t and u_t are used to estimate the standard deviations σ_ϵ and σ_u:

$$E\left(\epsilon_t^2 - \sigma_\epsilon^2 \right) = 0 \qquad [3.5]$$

$$E\left(u_t^2 - \sigma_u^2 \right) = 0 \qquad [3.6]$$

Equations [3.1] - [3.6] are estimated simultaneously.

The instruments for equations [3.1] - [3.6] were selected as follows: in equation [3.1], four seasonal dummies, and the time t growth rates of private consumption, leisure, output-capital ratio and output-labor ratio; in equation [3.2], four seasonal dummies and the time t and t-1 growth rates of private consumption, leisure, output-capital ratio and output-labor ratio; in equation [3.3], four seasonal dummies; in equation [3.4], four seasonal dummies and the logarithm of g_{t-1}/z_{t-1}; and in equations [3.5] and [3.6], only unity. A total of 31 instruments are used to estimate the 18 parameters of the nontime-separable specification, yielding 13 overidentifying restrictions. For a time-separable specification in which the parameters a and b are set to zero a priori, only 16 parameters are estimated, yielding 15 overidentifying restrictions.

The original data set employed in this study is the Barsky-Miron [1989] data: U.S. quarterly data which has not been adjusted for seasonality. For the empirical analysis to conform to the theoretical constructs of our model, however, we redefine some of the variables as follows (and convert to per capita values). Output (y) is Gross National Product per capita. Private consumption (cp) is nondurables plus services consumption expenditures per capita. Investment (i) is the sum of Business Fixed Investment plus Durable consumption expenditures, per capita. Government (g) is Federal, State, and Local government purchases, per capita. The capital stock is computed using the flow investment expenditures, a quarterly depreciation rate of 2.5%, and an initial capital stock value for 1950. Labor hours are computed as the product of total nonagricultural employment times average hours per week of nonagricultural production workers times 13 weeks per quarter (per capita). Average labor productivity and the capital rental rate are constructed from the output, labor, and capital data. The data is converted to per capita values by using the civilian population, 16 years and older.

Table 3.1 presents two sets of parameter estimates of Ψ. The time-separable (TS) estimates set the parameters a and b equal to zero a priori; the nontime-separable estimates (NTS) allow a and b to be nonzero. The NTS estimates display local durability (or adjacent substitutability) in preferences for consumption goods and habit-persistence in preferences for leisure hours; that is, a is estimated to be positive and b is estimated to be negative.

Habit-persistence in leisure is consistent with previous empirical analyses using these preference specifications. In seasonally adjusted quarterly data, Braun [1990] has found evidence of habit-persistence in leisure. In seasonally adjusted monthly data, Eichenbaum-Hansen-Singleton [1989] have also found evidence of habit-persistence in leisure hours. A number of researchers have estimated consumption preferences which are consistent with $a>0$. For example, using monthly data on consumption and returns, Eichenbaum-Hansen-Singleton [1989], Gallant-Tauchen [1990] and Heaton[1990] find evidence of local durability in consumption. On the other hand, using quarterly consumption data, Braun finds evidence of habit-persistence and Constantinides [1990] shows that negative values of a can help explain the equity premium puzzle.

In Table 3.1, the remaining parameter estimates are similar across both sets of estimates. The capital and labor shares are tightly estimated to be around .28 and .72, respectively. The seasonal patterns in transitory government spending are similar: fourth quarter spending is low, while first quarter spending is high. For each parameterization the government spending autoregressive coefficient ρ is estimated to be approximately .88. Finally, the estimated standard deviations of u_t and ϵ_t are roughly similar across both parameterizations.

Both estimated parameterizations display a large degree of seasonal variation in technology. The fourth quarter growth in total factor productivity is estimated to be 6% (or 24% on an annualized basis). The first quarter experiences technical regress (on average), growing at a rate of -7% in one quarter. If the technological specification [2.4] is correct, and the factor inputs and output are properly measured, then these seasonals represent true seasonal variation in aggregate technology. Barro [1990] suggests that, weather conditions and seasonality in construction probably account for some of the seasonal patterns in aggregate technological growth. In this paper we adopt the interpretation that the measured seasonal variation in technology represents true variation in technology, and consider the ability of the equilibrium model to capture the seasonal patterns uncovered by Barsky-Miron.[1]

Finally, Hansen's [1982] J-statistic indicates that neither model's overidentifying restrictions can be rejected at conventional significance levels. Since the nontime-separable specification nests time separable preferences we can directly test the additional restrictions imposed by time-separability by estimating the TS specification using the converged NTS weighting matrix. This estimation produced a J-statistic of 888. Following Eichenbaum, Hansen and Singleton (1988) the difference between this statistic and the J-statistic from the nontime-separable estimates is distributed asymptotically χ^2 with two degrees of freedom. Thus, the additional restrictions imposed by time-separable preferences are sharply rejected at conventional significance levels.

4. The Perfect Foresight Seasonal Equilibrium Analysis

This section examines the roles of seasonal technology, preferences, and government purchases for generating equilibrium seasonality in our

---

[1]Our research strategy is to answer the following question: in the context of a standard neoclassical model, what factors account for the seasonal fluctuations in aggregate data? If seasonal variation in preferences τ and government g are important, while technology λ is not, then explaining the estimated seasonality in λ is unlikely to be a high priority. On the other hand, if all of the seasonality in aggregate data is explained by seasonal variations in λ, then an explanation for seasonal λ is of first-order importance (although beyond the scope of the current paper).

parameterized model. Figure 4.1 plots the seasonal growth rates of aggregate variables along the PFSE growth path for three cases: (1) technology seasonals only (TECH), (2) preference seasonals only (PREF), and (3) transitory government seasonals only (GOVT). As the discussion below indicates, this decomposition analysis highlights the important role of technology seasonals in generating seasonals in output and hours.

Case 1: Technology Seasonals

In each panel of Figure 4.1, the TECH case presents the results from solving the perfect foresight NTS model for the scenario in which the only source of seasonality is in technology. The technology seasonals have been set at their estimated values. The preference seasonals τ have been set to their weighted average value of .1879. Although the seasonals in the transitory government purchases process ($\tilde{g}_t$) have been set to their nonseasonal mean value, the government purchases panel still shows seasonal variation in the growth rate of g_t. This variation is inherited from the seasonal variation in the permanent component of government purchases.[2]

Along the seasonal equilibrium path, seasonal variation in technology has important effects on hours, output, investment and labor productivity. All of these variables are proseasonal, exhibiting positive (negative) growth rates in seasons where the growth rate of technology (λ) is positive (negative). The seasonals in output are approximately the same as the seasonals in technology; this is similar to the observation made in Christiano [1988] that the movements in output and the Solow residual (in

---

[2]Our specification for government purchases embodies the assumption that g_t and z_t cointegrate. This assumption is necessary for the economy's perfect-foresight equilibrium path to exhibit balanced growth.

adjusted data) are very close. Seasonal investment is about four times as volatile as output. Seasonal hours are only about one-half as volatile as output; as in the RBC literature, since seasonal output variability exceeds that of labor hours, labor productivity is strongly seasonal.

Private consumption purchases are smooth but slightly counterseasonal. There are two reasons for this. First, since the technology seasonal is an anticipated event, there is no wealth effect. So one source of consumption variability, unanticipated wealth shocks, is absent. Second, fourth quarter government purchases grow due to the growth in technology. Since $\gamma_1=.4$ and government purchases substitute imperfectly for private consumption, consumption demand declines in the fourth quarter. Even though the real interest rate falls, equilibrium consumption falls. If $\gamma_1=0$ and government purchases do not substitute for private consumption, the model predicts that equilibrium consumption would rise slightly. Therefore, technology seasonals do not contribute significantly to the seasonality in consumption expenditures.

Habit-persistence in leisure preferences have the effect of smoothing seasonal labor hours movements. For the time-separable economy with technology seasonals only, seasonal labor hours are approximately three times as volatile as for the NTS economy; output is about twice as volatile. The pattern of seasonality, however, is the same for both economies: output, investment, labor hours, and labor productivity are proseasonal, while consumption and the interest rate are counterseasonal. Given our estimates of the structural parameters, habit persistence in leisure plays an important role in matching the magnitude of a seasonal cycle, but not the pattern.

In summary, seasonal variation in technology leads to substantial seasonal fluctuations in output. For the NTS economy, the output fluctuations are essentially the same size as the estimated technology seasonals.

Case 2: Preference Seasonals

In each panel of Figure 4.1, the PREF case presents the results from solving the perfect foresight NTS model for the scenario in which the only source of seasonality is in preferences. Technology and government purchases do not contain any seasonal effects: the quarterly growth rates of technology and government purchases are set at the baseline average of 0.17%.

In our economy with only preference seasonals, the equilibrium effects essentially involve only consumption and investment. The first and fourth quarter movements in consumption growth closely parallel the two seasonal movements in preferences (τ_1 and τ_4). Since consumption preferences exhibit adjacent substitutability (a>0), second quarter consumption growth is positive due to the first quarter reduction in consumption expenditures. A similar explanation holds for the third quarter drop in expenditures. The transitory nature of these shifters produces seasonal changes in investment that almost exactly offset the changes in consumption, leaving the level of output unchanged. Labor hours, labor productivity, and the interest rate are also largely unchanged over this seasonal equilibrium path. These findings also hold for the TS economy: output, labor hours, and productivity display no perceptible seasonal variation due to preferences. Attempts to match the seasonal variation in GNP with consumption preference shifters would dramatically magnify the large responses of consumption and investment documented here. Thus, seasonal fluctuations in technology are crucial for generating seasonal variation in output.

Case 3: Transitory government spending seasonals

In each panel of Figure 4.1, the GOVT case presents the results from solving the perfect foresight NTS model for the scenario in which the only source of seasonality is in transitory government purchases. Technology and preferences have no seasonal variation in this case. Along the seasonal growth path, consumption and investment move together; these movements run counter to the seasonal pattern in government purchases. The TS results are about the same as for the NTS economy. The seasonal equilibrium responses to government seasonals are similar to the Case 2 analysis: changes in output, labor hours, labor productivity, and the interest rate are virtually imperceptible. Again, this reinforces the important role of seasonal variations in technology.

Case Analysis Summary

Collecting the results from Cases 1 - 3 suggests that seasonal output, labor hours, labor productivity, and interest rate movements will be determined largely by the technology seasonal. Seasonal consumption is driven primarily by the preference shifter, and investment patterns will depend on the joint configuration of technology, preference, and government spending seasonals. This decomposition is due primarily to the transient and anticipated nature of the seasonal shifters in preferences and technology.

5. Evaluation of the Stochastic Model

This section presents results from solving a stochastic version of the seasonal business cycle model. In the course of describing the empirical characteristics of this model two issues will be addressed. First, the models' predicted seasonal patterns will be compared with the data. Second, (deseasonalized) cyclical properties of the seasonal business cycle models will be examined and compared with the data.

5.1 Solving the Stochastic Model

Before discussing the results we will briefly outline the methodology used in solving for the stochastic equilibrium. In the last section the perfect foresight seasonal equilibrium was calculated and analyzed. In this section we approximate the true stochastic equilibrium. In solving the model we modify the standard method (see Kydland and Prescott [1982], King, Plosser and Rebelo [1988] and Christiano and Eichenbaum [1990]) of approximating the true stochastic equilibrium with a Taylor expansion about the steady-state. The modifications arise because the perfect foresight equilibrium we consider exhibits seasonal cycles. The nonlinear model has the characteristic that technology and preferences change with the season. Varying the location of the Taylor approximation with the season allows the linearized system to inherit this characteristic of the nonlinear model. We consider this approach to be the appropriate generalization of the strategy adopted by Hansen and Sargent [1990], Ghysels [1989], and Todd [1990] who analyze seasonality in explicitly linear-quadratic frameworks.

The first step in solving the model is to linearize the equations [2.10]-[2.16] about the perfect foresight seasonal equilibrium path calculated in section 2. The linearized system can be reduced to twelve stochastic difference equations governing the evolution of the capital stock, hours, and private consumption in each season. These twelve equations are linearized, stochastic counterparts to equations [2.20]-[2.22]. In a technical appendix we display the linearized system and describe how these twelve difference equations are mapped into a state space representation which can be solved using methods described in King, Plosser, and Rebelo [1990]. The state space representation essentially has the same structure as Todd's time-invariant linear-quadratic representation (TILQ) or Hansen and Sargent's time-varying strictly periodic equilibrium. The model's solution is a series of twelve equations that describe the optimal decision rule for capital, hours, and private consumption, one equation for each season:

$$K_{t+1} = A\ S_t \qquad [4.1]$$

where

$$K_{t+1} = [\ k^1_{t+1}\ k^2_{t+1}\ k^3_{t+1}\ k^4_{t+1}\ cp^1_t\ cp^2_t\ cp^3_t\ cp^4_t\ n^1_t\ n^2_t\ n^3_t\ n^4_t\ ]'$$

and

$$S_t = [\ K'_t\ \ g^1_t\ \ g^2_t\ \ g^3_t\ \ g^4_t\ \ \lambda^1_t\ \ \lambda^2_t\ \ \lambda^3_t\ \ \lambda^4_t\ ]'.$$

where the superscripts denote the seasons.

Given these log-linear decision rules for capital, private consumption and hours, it is straightforward to generate time series for the model economy. First, a sequence of normal variables is drawn to mimic the empirical covariance structure of the forcing processes u_t and ϵ_t. Once u_t and ϵ_t have been constructed it is straightforward to calculate λ_t and g_t. Then given an initial K_o we can construct a sequence of realizations for the capital stock, hours, and private consumption using the following method. If this is the jth quarter then use the jth, j+4th and j+8th row of matrix A along with the current states: k^j_t, cp^{j-1}_{t-1}, n^{j-1}_{t-1}, λ^j_t, and g^j_t to determine the current decisions for next period's capital, and today's consumption and hours. Given the values of next period's stock of capital, today's consumption and today's work effort, it is straightforward to determine the current choices of output, investment, real wages, and the real interest rate using equations [2.14], [2.13], [2.17], and [2.18]. In practice we choose an effective sample length of 88 and provide results based on 500 draws.

The equilibrium model developed in this paper imposes restrictions across the entire spectrum. To focus attention on a specific set of moments, researchers often decompose time series. Examples of such decompositions include first differencing to remove low frequency moments and seasonal adjustment to remove particular high frequency moments. Our objective is to investigate the model's ability to match both the seasonal patterns uncovered by Barsky and Miron, as well as the data's cyclical moments which Prescott defines to be the business cycle phenomena. To facilitate these comparisons, we adopt Barsky and Miron's decomposition of the stationary, stochastic processes into "deterministic seasonal" and "indeterministic" components. Specifically, after log-first differencing the data to induce stationarity, we regress each series on four seasonal dummies: the estimates on the dummy variables define the seasonal patterns emphasized by Barsky and Miron. We also adopt the convention of referring to moments calculated using the indeterministic residuals from these regressions as relating to cyclical or business cycle phenomena. Obviously, the properties of the "seasonal cycle" will vary depending on the particular decomposition used.

An alternative approach is to simply avoid decompositions. Hansen and Sargent [1990] observe that seasonally unadjusted data can be stationary, conditional on a starting season. In light of this result, we also report moments for seasonally unadjusted growth rates in section 5.3. This approach ignores the entire distinction between business cycles and seasonal cycles.

5.2 Seasonal Predictions of the Stochastic Model

First we report results on the seasonal properties of the model. In summarizing the results particular attention is paid to the following question: can a parsimoniously parameterized real business cycle model capture the central features of the data at seasonal frequencies? In order to facilitate comparision with Barsky and Miron's work we address this question by considering the same set of moments reported in their paper. The first set of columns in Table 5.1 present the seasonal patterns for the data set using the log first difference filter. The seasonal means are reported in terms of percentage deviations from average growth rates for the sample period 1964:I to 1985:IV. The real interest rate, which is measured by the rental rate on capital, is reported in terms of annualized rates of return. The table also includes R-square statistics for each variable which describe the percentage of the total variation in the particular time series that is attributable to the deterministic seasonal. Finally, we report standard errors for each estimate that are based on the Newey-West [1987] weighting matrix with 12 autocorrelations.

The second and third sets of columns in Table 5.1 contain simulation results for respectively the time-separable and nontime-separable preference specifications. For each specification, columns 1 through 4 label the average seasonal means for 500 draws. The fifth column contains the average R-square of the regressions.

Comparisons of the results in table 5.1 reveal several significant shortcomings of the time-separable model. The model sharply overstates the seasonal means in output, hours and investment. The second and fourth quarter consumption means are also a poor match. The predicted R-squares for these variables also exceeds the respective number in the data in each instance. Overall, the time-separable specification does not capture the seasonal properties of the data.

One way to interpret the time-separable model's shortcomings is to focus on hours. In the fourth quarter, for instance, the model predicts a large positive seasonal in hours under the first difference filter while the data indicates that hours are only slightly above their fourth quarter mean value. The model's surge in fourth quarter hours induces a large increase in output which is high already due to a positive technology seasonal. Since these seasonals are anticipated and temporary, their wealth effects on consumption are small. Instead, optimizing households choose to increase desired savings which shows up as increased equilibrium investment. Based on this analysis, it is conceivable that the excess variability in hours across the seasons is the cause for the excess variability in investment and output. Alternatively, generalizations of the model that act to smooth hours across the seasons (such as habit-persistence in leisure preferences) will also smooth output and investment.

This proposition is explored in the context of the nontime-separable preference results in Table 5.1. In order to facilitate comparision of the NTS results with the data, seasonal growth rates are also presented graphically in Figure 5.1. Recall from Section 3 that estimates of the nontime-separabilities produced evidence of local durability in consumption and habit persistence in leisure. Habit persistence in leisure acts to smooth desired labor supply between adjacent periods. Examination of Figure 5.1 reveals that the NTS specification captures many of the seasonal movements in the data. The model successfully mimics the overall seasonal patterns in output, consumption, government purchases, average productivity and capital. For these variables the model reproduces the sequential pattern of seasonal movements in the data and in most cases the magnitudes[3]. The model is somewhat less successful with respect to investment, the rental rate on capital, and hours. For the rental rate the model consistently overstates the magnitudes, although the sequential pattern is correct. In the case of investment, the model captures the sequential pattern of seasons found in the data, but understates the second quarter rise and overstates the fourth quarter rise in investment growth.

Hours represent the model's single largest failure. Although the non-time separabilities in leisure have improved the model's seasonal predictions, the magnitudes are still off in three out of four quarters and the model predicts a counter-factual rise in fourth quarter employment. One strategy that would improve the model's predictions for hours would be to introduce a more complicated pattern of preference shifters. However, we see little economic rationale for increasing the dimensionality of exogenous seasonal shifters that cannot be lined up with calendar events like Christmas. An alternative strategy that we explore in Braun and Evans(1991) involves modeling time-varying work effort.

---

[3] A cursory inspection of the graphs might suggest that the fit for capital not particularly good. Notice however, that the vertical axis is in tenths of a percent.

5.3 Cyclical Predictions of the Stochastic Model

This subsection examines the cyclical properties of the stochastic model. Two distinct parameterizations are considered: the time-separable GMM optimum (TS) and the nontime-separable global optimum (NTS).

Table 5.2 contains results relating to relative variability and cross-correlations with output for both parameterizations and the data under three different filters. The heading "one-quarter growth rate" corresponds to moments calculated using data that has been first differenced and regressed on four dummies. The heading "HP filter" corresponds to data that has been Hodrick-Prescott filtered and then regressed on four seasonal dummies. The heading "Unadjusted one-quarter rates" corresponds to moments calculated using data that has been first differenced only.

For each filter we report moments for U.S data running from 1964:1-1985:4 in the first column. The second column contains standard errors for the data's moments reported in column one. The standard errors were calculated using a Newey-West weighting matrix with 12 lags. The third and fourth columns contain results from simulating the model using the TS and NTS parameterizations. In each case the reported statistics are sample averages based on 500 draws of length 88.

Looking first at the properties of the data, observe that the two filters which incorporate deseasonalization (one-quarter growth and HP filter in table 5.5) produce the same general patterns. With respect to relative variability, investment is about twice as variable as output, government purchases are about as variable as output, and hours and consumption are less variable than output. We do observe some differences in moving from the deseasonalized one-quarter growth rates to the deseasonalized HP filtered data. In particular, there are significant rises in the relative variabilities of hours and investment and declines in the relative variabilities of consumption and government expenditures. With respect to correlations we observe similar general patterns accross the two filters. Here the most significant differences occur in the instances of investment and hours.

More important differences are observed when comparing the first two filters with the third filter, first differenced seasonally unadjusted data. Output is considerably more variable prior to removal of seasonal means and some of the correlations are quite different. For instance, government purchases have a correlation with output of about .3 under the first two filters, but a correlation of .8 under the third filter. In general, the strongest cross-correlations with output occur in data which have not been seasonally adjusted, but have been first differenced in order to induce stationarity. This is one way of interpreting Barsky and Miron's finding that aggregate variables exhibit strong comovement across seasonal frequencies as well as cyclical frequencies.

Turning to the theory, consider next a comparison of the cyclical properties of the NTS parameterization with the TS parameterization. In performing these comparisons we use the standard errors for the data as a metric. The NTS parameterization is successful in matching many of the properties of unadjusted 1-quarter growth rates. It captures most of the relative variability statistics and cross-correlation patterns found in the data. The major shortcomings lie in the failure of the model to capture precisely the relative variabilities of consumption and investment and the correlations of hours, government expenditures and rental rates with output. The TS parameterization has considerably more difficulty matching the unadjusted moments, missing the relative variability of consumption, investment, hours and average productivity by wider margins and overstating the correlation of capital with output. On the basis of these seasonally unadjusted moments, the NTS parameterization captures more features of the data.

If we compare the predictions of the NTS and TS parameterizations under the two seasonally adjusted filters, we get a different picture. In many cases the two models' predictions lie outside a two standard deviation band around the data. The largest differences appear to occur under the 1-quarter growth filter. With respect to relative variabilities the NTS specification fails to capture the relative variabilities of average productivity, employment and government expenditures. The TS specificaton fails to capture the relative variabilities of consumption, investment, capital and average productivity. In addition, the TS specification signficantly overstates the variability of output. Similar patterns emerge under the HP-filter. On net, we would argue that the NTS specification captures more aspects of the relative variabilities in the data with the failure in hours more than offset by sucesses in consumption, investment and overall variability in output.

Comparisons of seasonally adjusted contemporaneous correlations with output reveal significant failures of both specifications. Under the 1-quarter growth filter, both specifications fail to capture the contemporaneous correlations of investment, government purchases, employment and the rental rate with output. Under the HP-filter both specifications fail to capture the correlation of output with government purchases, employment and average productivity.

How do these results compare with the performance of standard business cycle models that ignore seasonality? Many of the models' predictions are similar to those of standard real business cycle models. The tendency for RBC models using estimated parameterizations to overstate the observed variability of output in the data appears in both Christiano and Eichenbaum (1990) and Braun (1990). Comparison with these other studies finds that benchmark RBC models overstate the contemporaneous correlation of hours with output and average productivity with output. This is attributed to the absence of important labor supply shifters. Christiano and Eichenbaum find that when measurement error in hours is modeled as being i.i.d., these predicted correlations drop. Braun finds that movements in income taxes shift labor supply thereby reducing both of these correlations.

Overall, we conclude that the NTS specification captures the general features of the seasonal cycle while continuing to capture the same features of the business cycle that have generated so much attention for this model. The NTS specification successfully captures important aspects of seasonal fluctuations in output, consumption, average productivity and investment but fails to capture the seasonal pattern in hours. The TS specification on the other hand fails to capture many of these moments. At business cycle frequencies the NTS specification also captures more of the variability in seasonally adjusted data than the TS specification. Smaller differences are observed when comparing contemporaneous correlations. Thus, an additional finding of this analysis is that nontime-separabilities play an important role in explaining seasonal fluctuations and contribute to the overall performance performance of business cycle models more generally.

6. Conclusions

In this paper we have introduced seasonals shifters into an equilibrium model of the business cycle. The model is tractable: the perfect foresight seasonal equilibrium is computable without any approximations, and an approximate linear solution of the stochastic model can be calculated using methods analogous to those of Hansen-Sargent [1990], and Todd [1990]. The structural parameters and seasonals were estimated using GMM with seasonally unadjusted, postwar U.S. data. The overidentifying restrictions implied by the model cannot be rejected at conventional significance levels.

Are Barsky and Miron's findings consistent with current equilibrium business cycle theories as surveyed by Prescott [1986]? Conditional on our parameterization, the nontime-separable model predicts most of the seasonal patterns found in aggregate quantity time series; a notable exception is the seasonal pattern in labor hours. The model also predicts many of the deseasonalized second moment properties of the data. Our answer to this question is yes: this equilibrium model generally displays the seasonal patterns discovered by Barsky and Miron [1989] as well as the business cycle phenomenon.

We view this model as a benchmark in the tradition of Kydland and Prescott [1982] and Long and Plosser [1983]. While the predictions of the theory match the data's properties fairly well, the assumptions of the theory require further investigation. In particular, for this class of equilibrium models, seasonal variation in technology is crucial for delivering seasonal fluctuations in output. Does the aggregate technology vary exogenously as much as our estimates suggest, or is this seasonal variation due to some misspecification of the technology? Future research should assess the plausibility of seasonality in aggregate Solow residuals by examining alternative general equilibrium economies. We conjecture that this seasonal investigation will provide new macroeconomic insights into the importance of labor hoarding (as in Summers [1986]), increasing returns due to endogenous growth (Romer [1986]), increasing returns due to market externalities (Diamond [1982], Murphy-Shleifer-Vishny [1989]), countercyclical markups of price over cost (as suggested by Hall's [1989] evidence), and propagation mechanisms in general. In principle, each of the above phenomenon could produce endogenous seasonal movements in measured Solow residuals, even if true technological advances are nonseasonal. The ability of these theories to encompass the results of this benchmark model and the seasonality in measured Solow residuals is the topic of our current research.

References

Aschauer, D. , 1985, Fiscal Policy and Aggregate Demand, American Economic Review, 117-127.

Barro, R., 1990, Macroeconomics (J. Wiley Company, New York).

Barsky, R. and J. Miron, 1989, The Seasonal Cycle and the Business Cycle, Journal of Political Economy.

Braun, R., 1990, The Dynamic Interaction of Distortionary Taxes and Aggregate Variables in Postwar U.S. Data, unpublished Ph.D. thesis, Carnegie Mellon University.

Braun, R., and C. Evans 1991, Seasonal Solow Residuals and Christmas: A Case for Labor Hoarding and Increasing Returns, manuscript, Federal Reserve Bank of Chicago.

Chatterjee, S. and B. Ravikumar, 1989, A Neoclassical Growth Model with Seasonal Perturbations, unpublished manuscript, University of Iowa.

Christiano, L. and M. Eichenbaum, 1990, Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations, unpublished manuscript, Institute for Empirical Macroeconomics.

Constantinides, G., 1990, Habit Formation: A Resolution of the Equity Premium Puzzle, Journal of Political Economy 98, pp. 519-543.

Diamond, P., 1982, Aggregate Demand in Search Equilibrium, Journal of Political Economy 90, pp. 881-894.

Eichenbaum, M. and L. Hansen, 1990, Estimating Models with Intertemporal Substitution Using Aggregate Time Series Data, Journal of Business and Economic Statistics.

Eichenbaum, M., L. Hansen, and K. Singleton, 1989, A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice Under Uncertainty, Quarterly Journal of Economics.

Gallant, R., L. Hansen, and G. Tauchen, 1990, Using Conditional Moments of Asset Payoffs to infer the Volatility of Intertemporal Marginal Rates of Substitution, Journal of Econometrics.

Gallant, R. and G. Tauchen, 1989, Seminonparametric Estimation of Conditionally Constrained Heterogeneous Processes: Asset Pricing Applications, Econometrica 57, 1091-1120.

Ghysels, E. 1991, On the Economics and Econometrics of Seasonality, Discussion Paper C.R.D.E. Universite de Montreal.

Hall, R., 1988, The Relation between Price and Marginal Cost in U.S. Industry, Journal of Political Economy 96, pp. 921-947.

Hansen, L., 1982, Large Sample Properties of Generalized Method of Moments Estimators, Econometrica 50, 1029-1054.

Hansen, L. and T. Sargent, 1990, Disguised Periodicity as a Source of Seasonality, unpublished manuscript, Hoover Institution.

Hansen, L. and K. Singleton, 1982, Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models, Econometrica 50, 1269-1286.

Heaton, J., 1990, The Interaction between Time-Nonseparable Preferences and Time Aggregation, unpublished manuscript, Sloan School, MIT.

King, R., C. Plosser, and S. Rebelo, 1988, Production, Growth, and Business Cycles, Journal of Monetary Economics 21, 309-342.

King, R., C. Plosser, and S. Rebelo, 1990, Technical Appendix to Production, Growth, and Business Cycles, unpublished manuscript, University of Rochester.

Kydland, F. and E. Prescott, 1982, Time to Build and Aggregate Fluctuations, Econometrica 50, 1345-1370.

Kydland, F. and E. Prescott, 1989, Hours and Employment Variation in Business Cycle Theory, manuscript, Federal Reserve Bank of Minneapolis.

Long, J. and C. Plosser, 1983, Real Business Cycles, Journal of Political Economy 91, 39-69.

Murphy, K., A. Shleifer, and R. Vishny, Building Blocks of Market Clearing Business Cycle Models, NBER Macroeconomics Annual 1989, pp. 247-287.

Newey, W. and K. West, 1987, A Simple, Positive Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, Econometrica 55, pp. 703-708.

Prescott, E., 1986, Theory Ahead of Business Cycle Measurement, Carnegie-Rochester Conference Series on Public Policy 27 (Autumn), 11-44.

Summers, L., 1986, Some Skeptical Observations on Real Business Cycle Theory, Quarterly Review 10 (Federal Reserve Bank of Minneapolis, Minneapolis), 23-27.

Todd, R., 1990, Periodic Linear-Quadratic Methods for Modeling Seasonality, forthcoming in Journal of Economic Dynamics and Control.

Table 3.1

GMM Estimates of the Structural Parameters

	Time – Separable		Nontime – Separable	
	Estimate	Std Error	Estimate	Std Error
θ	.2751	.0055	.2803	.0195
a	———	———	.3402	.0184
b	———	———	–.4956	.0154
λ_1	–.0710	.0027	–.0724	.0031
λ_2	.0375	.0022	.0385	.0026
λ_3	–.0190	.0015	–.0193	.0019
λ_4	.0596	.0019	.0600	.0024
τ_1	.1819	.0013	.1863	.0011
τ_4	.1929	.0012	.1929	.0011
d_1	.6115	.1747	.6048	.1521
d_2	.5807	.1743	.5712	.1518
d_3	.6162	.1747	.6072	.1523
d_4	.5469	.1748	.5382	.1520
ρ	.8756	.0368	.8779	.0319
σ_u	.0198	.0013	.0195	.0385
σ_ϵ	.0194	.0014	.0191	.0193
J–Statistics	20.87		20.72	
P–value	(232)		(.146)	
	$\chi^2(17)$		$\chi^2(15)$	

Table 5.1 Seasonal Growth Rates

	Data[1]					Time Separable Estimates[2]					Non-Time Separable Estimates[3]				
X-variables	Winter	Spring	Summer	Fall	R^2	Winter	Spring	Summer	Fall	R^2	Winter	Spring	Summer	Fall	R^2
Output	-7.72	4.26	-1.04	4.16	.905	-12.31	5.64	-3.20	9.87	.932	-7.35	3.04	-1.48	5.79	.868
	(.59)	(.42)	(.22)	(.35)											
Consumption	-7.30	2.59	0.04	4.34	.936	-6.34	.55	-.20	7.08	.958	-7.29	2.29	-1.25	6.25	.914
	(.48)	(.40)	(.09)	(.26)											
Investment	-14.64	11.79	-2.15	4.33	.903	-35.39	24.79	-14.60	25.20	.946	-10.55	5.41	-4.21	9.35	.830
	(.68)	(1.08)	(.34)	(.58)											
Government	-4.89	2.86	0.49	1.31	.696	-4.13	2.48	.87	.78	.353	-4.10	2.48	.80	.82	.357
	(.37)	(.79)	(.46)	(.32)											
Capital	.16	-.26	.09	.02	.214	.19	-.22	.54	.51	.761	.023	-.082	.19	-.13	.198
	(.10)	(.08)	(.10)	(.10)											
Labor Hours	-3.26	2.31	.73	.07	.842	-10.00	4.77	-2.56	7.79	.973	-2.96	.96	-.12	2.12	.898
	(.16)	(.21)	(.20)	(.17)											
Avg. Prod.	-4.47	1.95	-1.78	4.09	.846	-2.31	8.72	-.64	2.08	.627	-4.39	2.08	-1.37	3.67	.844
	(.52)	(.40)	(.30)	(.40)											
Rental Rate on Capital	4.02	3.91	4.49	3.41	.084	4.46	3.95	5.50	3.59	.618	4.28	4.05	4.94	3.83	.336
	(.48)	(.44)	(.49)	(.42)											

[1] The seasonal growth rates for the data were estimated using GMM and a Newey-West weighting matrix.

[2] The time separable results are sample averages based on simulations with 500 replications of draws of length 88.

[3] The non-time separable results are sample averages based on simulations with 500 replications of draws of length 88.

Table 5.2 Selected Second Moment Properties, Various Filters[1]

1. Relative Volatility: (Standard deviation of x)/(Standard deviation of Output)

	1-Quarter Growth				HP Filter				Unadjusted 1-Quarter Rates			
X-variables	Data	Std. Err	NTS	TS	Data	Std. Err	NTS	TS	Data	Std. Err	NTS	TS
Output[2]	.016	.002	.019	.023	.026	.002	.032	.031	.051	.003	.054	.089
Consumption	.73	.054	.79	.43	.60	.036	.51	.55	.88	.033	.97	.55
Investment	1.94	.26	1.82	2.69	2.42	.12	2.24	2.17	1.96	.069	1.61	3.03
Government	1.21	.19	1.70	1.50	.86	.24	1.31	1.36	.66	.063	.77	.47
Capital[3]	.18	.026	.13	.097	.32	.060	.28	.24	.07	.009	.052	.052
Labor Hours	.52	.065	.33	.50	.73	.033	.39	.35	.42	.025	.37	.79
Avg. Prod.	.89	.050	.69	.55	.75	.11	.63	.69	.70	.024	.64	.24
Real rate[3]	.36	.073	.30	.25	-	-	-	-	.28	.086	.13	.10

2. Contemporaneous correlation: x with Output

	1-Quarter Growth				HP Filter				Unadjusted 1-Quarter Rates			
X-variables	Data	Std. Err	NTS	TS	Data	Std. Err	NTS	TS	Data	Std. Err	NTS	TS
Consumption	.72	.075	.85	.79	.87	.030	.90	.92	.96	.010	.98	.89
Investment	.72	.059	.84	.96	.93	.024	.97	.96	.94	.010	.97	.98
Government	.38	.081	.76	.78	.30	.11	.85	.86	.83	.030	.69	.64
Capital[2]	.26	.11	.24	.48	.28	.070	.36	.41	.50	.060	.46	.89
Labor Hours	.46	.085	.96	.94	.67	.10	.97	.93	.81	.020	.99	.99
Avg. Prod.	.82	.040	.99	.95	.59	.080	.99	.98	.93	.010	.995	.91
Real rate	.25	.098	-.25	-.25	-	-	-	-	-.09	.095	-.35	-.50

[1]The sample period of the data is 1964:II -1985:IV. The standard errors are for the data's moments; and 12 lags are employed in the Newey-West estimator. The Stochastic models were simulated 500 times using draws of length 88.

[2]The "output" rows reports the standard deviation of output.

[3]The capital stock refers to K_{t+1}, whereas the other variables are X_t.

[4]The real rate is not filtered, for comparability with Barsky-Miron [1989].