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**Seasonal Solow Residuals and Christmas:
A Case for Labor Hoarding and Increasing Returns**

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Abstract

In aggregate unadjusted data, measured Solow residuals exhibit large seasonal variations. Total Factor Productivity grows rapidly in the fourth quarter at an annual rate of 24% and regresses sharply in the first quarter at an annual rate of -30%. This paper considers two potential explanations for the measured seasonal variation in the Solow residual: labor hoarding and increasing returns to scale. Using a specification that allows for no exogenous seasonal variation in technology and a single seasonal demand shift in the fourth quarter, we ask the following question: How much of the total seasonal variation in the measured Solow residual can be explained by Christmas? The answer to this question is surprising. With increasing returns and time varying labor effort, Christmas is sufficient to explain the seasonal variation in the Solow residual, consumption, average productivity and output in all four quarters. Our analysis of seasonally unadjusted data uncovers important roles for labor hoarding and increasing returns which are difficult to identify in adjusted data.

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1. Introduction

Prescott [1986] has argued that the variability of Solow's residual is a reasonable estimate of the variability of exogenous technology shocks. When Solow's residual is measured using seasonally unadjusted data for the postwar U.S. economy, it exhibits large seasonal variations, growing rapidly in the fourth quarter at an annual rate of 24% and falling sharply in the first quarter at an annual rate of -30%. This paper starts from the premise that it is implausible to attribute seasonal variation of this magnitude to changes in the state of technology.¹ We present a model in which all seasonal fluctuations arise from a single demand shift, Christmas. This demand shift together with misspecification of the traditional production function leads to large seasonal variation in the Solow residual. We consider two candidates for misspecification, labor hoarding and external increasing returns to scale. Even when technological growth is aseasonal, either candidate in isolation can induce spurious seasonality in the Solow residual. Our general equilibrium analysis indicates that: (1) the economy's seasonal patterns in all four quarters may be a response simply to a fourth quarter consumption demand shift, and (2) a combination of labor hoarding and external increasing returns are important for replicating these patterns in aggregate quantity variables for the postwar U.S. economy. Since our analysis identifies important roles for labor hoarding and increasing returns, these results have implications for nonseasonal macroeconomic models.

By focusing on seasonal fluctuations we are able to exploit information

¹Beaulieu and Miron [1991] cast doubt on weather explanations by showing that the first quarter is a period of negative output growth in Australia and Argentina as well as the United States. In the Southern Hemisphere, there is no reason to expect a negative technology seasonal in the first quarter.

that is typically ignored in empirical macroeconomic research.² This is important because the research strategy of selecting a macroeconomic theory based upon its ability to match certain statistical moments in seasonally adjusted data often fails to discriminate among competing theories. For example, Murphy-Shleifer-Vishny [1989] survey the many similarities between technology shock-driven and increasing returns equilibrium theories. Ignoring seasonality, they conclude that both theories can be modified in plausible ways to generate the same implications for aggregate quantity variables. In a similar vein, Cooper and Haltiwanger [1989] conclude that production bunching may arise due to either nonconvexities in production or the arrival of technology shocks in bunches.³ The literature on labor hoarding and procyclical productivity also produces mixed results. Rotemberg and Summers [1990] show that the combination of labor hoarding and price inflexibility can generate procyclical total factor productivity without appealing to technology shocks. On the other hand, Burnside, Eichenbaum, and Rebelo [1990] find that labor hoarding, price flexibility, and relatively small exogenous technology shocks can generate procyclical total factor productivity.

The principal difficulty in selecting one theory, of course, is that each school of thought interprets the postwar period as being dominated by either supply shocks (RBC) or demand shocks (Keynesian labor hoarding, increasing

²Other researchers have noted this also. Barsky and Miron [1989] argue that the seasonal cycle facts for the U.S. contain information for evaluating alternative macroeconomic theories. Ghysels [1991] disputes the claim that the seasonal cycle is like the business cycle, but agrees that seasonally unadjusted data contain useful information for identifying propagation mechanisms.

³Evidence favoring increasing returns is uncovered by Hall [1989], Ramey [1991], and Caballero and Lyons [1990]. Their conclusions rely on the assumed exogeneity of their instruments, which is controversial. Further evidence of increasing returns which does not rely on these instruments would strengthen their case. Chirinko [1991] concludes that increasing returns is more important than either labor hoarding or technology shocks.

returns)-- but not both. If economists could identify demand shocks, however, the predicted countercyclical response of labor productivity in RBC models could be compared with the predicted procyclical response in increasing returns and labor hoarding theories. Needless to say, such an identification is difficult to achieve unambiguously in postwar seasonally adjusted time series data.

Seasonal fluctuations offer valuable identifying restrictions. For instance, few economists would dispute that Christmas is an important seasonal event which produces an increased demand for consumption services in the fourth quarter. On the other hand, it is difficult to identify bonafide seasonal shifters of technology. The contention that weather is an important seasonal impulse is weakened considerably by Beaulieu and Miron's [1991] finding that seasonal patterns in Southern Hemisphere countries resemble patterns in the U.S. Together our assumptions that technology is aseasonal and that Christmas is an important shift in demand provide identifying restrictions that have strong discriminatory power.⁴ For instance, Braun and Evans [1990] find that seasonal Real Business Cycle models predict totally aseasonal patterns in output under these identifying assumptions. Due to the transient and anticipated nature of Christmas, households simply draw down their savings and increase consumption by equal amounts in the fourth quarter leaving the level of output unchanged. RBC models require implausibly large seasonal variations in technology to explain the seasonal pattern in output. Thus, our identifying assumptions offer strong evidence against one explanation for the seasonal pattern in output. In this paper we demonstrate

⁴Our use of seasonal identifying restrictions is similar to Bernanke and Parkinson's analysis. Using interwar data, Bernanke and Parkinson [1991] investigate procyclical productivity in industrial markets. Under the plausible identifying assumption that the Great Depression was not caused by a series of large technology shocks, they find evidence in favor of increasing returns and labor hoarding.

that labor hoarding in conjunction with increasing returns can generate many of the observed patterns in output, consumption, and the measured Solow residual.

Modeling seasonal fluctuations with a single Christmas demand shift requires us to model economic agents' responses to anticipated and transitory impulses. First, the anticipated nature of the Christmas seasonal shift leads us to model variations in labor effort as driven by convex costs of adjusting employment. Burnside, Eichenbaum and Rebelo [1990] model labor hoarding by assuming that employment is fixed at the beginning of the period and only labor effort can respond within a period to shocks. In their framework labor effort responds only to unanticipated shocks, exhibiting no noticeable persistence.⁵ To induce seasonal labor hoarding, the costs of adjusting quasi-fixed factors must be modeled explicitly. Second, the transitory nature of seasonal shifts leads us to consider convex costs of adjusting capital. In an economy with external increasing returns, Baxter and King [1990] found a negligible response of output to a purely transitory increase in consumption demand: consumption rose but investment fell, leaving output unchanged. In the absence of adjustment costs, a similar result is to be expected for the case of a fourth quarter Christmas seasonal. If increasing returns is to have a chance, it must be costly to adjust investment. Third, the nontime-separable preferences emphasized by Kydland and Prescott [1982], Eichenbaum, Hansen, and Singleton [1988], and others for business cycle variability also play an important role in propagating the Christmas demand shock beyond simply the fourth quarter. Thus, modeling seasonal fluctuations leads to a specification that incorporates the same propagation mechanisms that receive wide attention in models of the business cycle.

⁵In Burnside-Eichenbaum-Rebelo [1990], the impulse response functions of labor effort to innovations in technology and government purchases appear to be zero after the initial period's response.

Many of the model's parameters governing returns to scale, the magnitude of adjustment costs and elasticities for work effort are difficult to pin down on a priori grounds. A Generalized Method of Moments (GMM) estimation strategy is used to produce estimates of these parameters. These estimates are then used to evaluate the seasonal growth rates implied by the model. We find that both labor hoarding and increasing returns mechanisms are important for capturing the U.S. economy's seasonal fluctuations. For each of the real variables in the model, the hypothesis that the predicted seasonal fluctuations match the data's seasonals cannot be rejected. The estimated parameterization proves to be remarkably successful at capturing the seasonal pattern in the measured Solow residual as well as the seasonal pattern in output, consumption, and average productivity. Results reported in section 4 suggest further that labor hoarding and nontime-separabilities play the biggest role in propagating the Christmas demand shock. Increasing returns prove to be important for amplifying the seasonal patterns generated by the other features of the model.

Finally, this model also offers an explanation for the similar seasonal patterns across countries which Beaulieu and Miron [1991] have documented. Christmas-like celebrations induce fourth quarter preference shifts in consumption demand in many Northern and Southern Hemisphere countries. To the extent that increasing returns to scale and labor hoarding are important features of these other economies as well, then our model predicts a seasonal pattern similar to that found in the U.S.

An outline of the remainder of the paper follows. In section 2 the model is described and the seasonal equilibrium growth path is defined. Section 3 contains a description of the data, the estimation strategy and a summary of the estimation results. Section 4 evaluates the seasonal implications of the estimated parameterization and explores the role of the various components of

the model. In section 5 we conclude by summarizing our results.

2. The Economic Model

In this section we describe the model economy. The presentation of the economy leads naturally to an optimization problem whose solution is the competitive equilibrium allocation. This solution is not Pareto optimal due to a productive externality. As we pose the problem, the planner does not take account of the externality. A benevolent social planner could do better by allowing agents to coordinate. This strategy for calculating competitive allocations in distorted economies is discussed at length in Romer[1988].

Preferences

The household's period preferences depend upon consumption services c_t^* , leisure services l_t^* , and (negatively upon) the intensity of labor effort v_t . The period utility function is:

$$r_t \log c_t^* + \alpha \log l_t^* - \frac{\xi}{2} (v_t - \bar{v})^2, \quad \xi > 0, \alpha > 0 \quad [1]$$

where r_t is a seasonal preference shifter. The preference shifter captures the household's increased desire to consume during the Christmas season:

$$r_t = \bar{r} Q_{1t} + \bar{r} Q_{2t} + \bar{r} Q_{3t} + r_4 Q_{4t}, \quad r_4 > \bar{r} > 0 \quad [2]$$

where Q_{it} is a quarterly seasonal dummy variable taking on the value of 1 when period t corresponds to season i and zero otherwise. Consumption and leisure services are defined as follows:

$$c_t^* = cp_t + a cp_{t-1}, \quad |a| < 1 \quad [3]$$

$$l_t^* = T - n_t + b (T - n_{t-1}), \quad |b| < 1 \quad [4]$$

where cp_t are consumption expenditures and T represents the total time allocation. If $a > 0$ consumption expenditures have a durable quality and are substitutable across adjacent periods. If $a < 0$ consumption expenditures are complements across adjacent periods, and consumption preferences exhibit habit

persistence. The same interpretations hold for b and leisure preferences.

The interpretation of labor effort in the period utility function depends upon the value of \bar{v} . If $\bar{v} > 0$, \bar{v} is a bliss point for labor effort. In this case, deviations from \bar{v} provide disutility for the household. Here \bar{v} can be interpreted as a normal level of labor effort: workers view periods of inactivity on the job with the same dissatisfaction as comparable periods of overactivity. If $\bar{v} < 0$, labor effort provides increasing disutility. In this case, less work effort is strictly preferred to more.

Production

The representative household has access to a technology which produces goods (y) using capital (k), labor hours (n), and labor intensity (v):

$$y_t = \phi_t k_t^\theta (z_t n_t v_t)^{1-\theta} J_t, \quad 0 < \theta < 1 \quad [5]$$

$$z_t = z_{t-1} \exp(\lambda + \epsilon_t) \quad [6]$$

$$\phi_t = \phi_1 (\bar{y}_t / z_t)^{\phi_2}, \quad \phi_1, \phi_2 > 0 \quad [7]$$

Aside from the choice of factor inputs k , n , and v , the level of production is influenced by three additional factors: exogenous variation in the state of technology z_t , a productive externality ϕ_t , and convex adjustment costs J_t on capital and labor (with the specification described below). Each of these factors will now be discussed separately.

The technology variable z_t is a random walk process in logarithms with constant drift λ . The impulse ϵ_t is an independent, serially uncorrelated random variable. Three observations on the role of z_t in our analysis are noteworthy. First, the constant drift term λ is nonseasonal-- this is our identifying restriction that the true technology is aseasonal. Second, all growth in this economy originates with z_t since our specification of the productive externality exhibits local increasing returns (discussed below). In the balanced growth equilibrium that we analyze, therefore, all trending

variables share the same trend as z_t . Third, the variability of ϵ_t plays no role in our analysis of perfect foresight seasonal growth paths, but its presence satisfies a necessary condition for our econometric relationships to be well-posed in Section 3.

Increasing returns in production are captured by the Marshallian externality variable ϕ_t , where \bar{y}_t represents the economy-wide level of per capita output. Marshallian productive externalities have been considered by Murphy, Shleifer, and Vishny [1989], Caballero and Lyons [1990], and Baxter and King [1990]. In our framework, the representative household is too small to influence the economy-wide output, so ϕ_t is taken to be beyond the household's control. Since the externality is expressed relative to the level of technology z_t , this specification embodies local increasing returns-- as aggregate economic activity rises relative to trend, the economy becomes more productive.⁶

The variable J_t is a factor which relates to the cost of adjusting capital and labor hours in terms of lost output:

$$J_t = \exp \left\{ - \frac{\psi_1}{2} \left(\frac{k_{t+1} - k_t \exp(\lambda + \epsilon_t)}{k_t} \right)^2 - \frac{\psi_2}{2} \left(\frac{n_t - n_{t-1}}{n_{t-1}} \right)^2 \right\} \quad [8]$$

where ψ_1 and ψ_2 are positive, and λ is the average growth rate of capital as well as the technology z_t . The first term states that it is costly to increase the capital stock at a rate other than its growth rate. The firm has in place

⁶Alternatively, if z_t were deleted from the specification of ϕ_t in [7], the externality would grow with economy-wide output, and this would be global increasing returns. Given our econometric methodology in Section 3, these two specifications are observationally equivalent. Specifically, for local IR all growth is exogenous; whereas for global IR the exogenous growth is magnified by the ϕ_t process so that some growth is endogenous. In the global case, there is a lower value of λ which interacts with the same value of ϕ_2 as in the local case to produce the same equilibrium as we report in Section 4. Applying our estimation procedure to the global case would produce this lower value of λ .

a technology for assimilating new capital into the production process. This technology costlessly accepts the normal level of new investment, but other levels create congestion in the production process. Likewise, the second term states that it is costly to increase labor hours at a rate other than its unconditional growth rate, which is zero. For this specification, the adjustment cost factor J_t is in the interval $(0,1]$ and in a nonseasonal steady state $J_t=1$.⁷

Period Budget Constraint

The household's period budget constraint is given by:

$$\phi_t k_t^\theta (z_t n_t v_t)^{1-\theta} J_t = c p_t + k_{t+1} - (1-\delta) k_t \quad [9]$$

where δ is the rate of capital depreciation per quarter. Fiscal policy could be introduced into the model and constraint [9] (as in Braun and Evans [1991]), but our focus in this paper is the single demand seasonal Christmas since that is a relative constant across countries.

Planner's Problem

The competitive equilibrium allocations in a decentralized version of this economy are identical to the solution of the following optimization problem.⁸ At time 0, choose a sequence of contingencies $(c p_t, n_t, v_t, k_{t+1}; t \geq 0)$ to solve the following:

⁷Due to the inclusion of the growth term in J_t , the nonseasonal steady state of this economy will be the same as an economy which omits adjustment costs. Besides being plausible, the growth term allows greater comparability with previous studies.

⁸This is a solution strategy previously employed by Romer [1986].

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ r_t \log(cp_t + a cp_{t-1}) + \alpha \log(T - n_t + b(T - n_{t-1})) - \frac{\xi}{2} (v_t - \bar{v})^2 \right. \\ \left. + \mu_t \left[\phi_t k_t^\theta (z_t n_t v_t)^{1-\theta} J_t - cp_t - k_{t+1} + (1-\delta)k_t \right] \right\} \quad [10]$$

where μ_t is a Lagrange multiplier, and the initial values k_0 , cp_{-1} , and n_{-1} are given. Notice that the planner ignores the productive externality, treating ϕ_t as given: while this is suboptimal, it is the analogous problem to the one faced by small households and firms. It is well-known that the optimal allocations which solve this problem are characterized by the first-order conditions for cp_t , n_t , v_t , k_{t+1} , and a transversality condition related to capital (for an example, see Braun and Evans [1991]). Furthermore, assets can be priced using intertemporal marginal rates of substitution in the usual way.

A Perfect Foresight Seasonal Equilibrium Growth Path

This economy grows over time at the rate of exogenous technological progress which is given by λ per period. Since preferences shift over the calendar year, however, these growth rates may vary seasonally. A perfect foresight seasonal equilibrium growth path for this economy is a generalization of the standard definition of a balanced growth path.⁹ The relevant new feature is that the seasonal growth path is indexed by season. Thus, consumption in year t and quarter i is linked to consumption in year $t+1$ quarter i by the following relationship: $c_{t+1,i} = e^{4\lambda} c_{t,i}$. Along the seasonal growth path, consumption will always grow $x_1\%$ in the winter, $x_2\%$ in

⁹For example, see King, Plosser, and Rebelo [1988] for a standard definition of a balanced growth path.

the spring, x_3 in the summer, and x_4 in the fall.¹⁰

Seasonality in Measured Solow Residuals

Suppose that a researcher attempts to measure Solow residuals for this economy as Prescott [1986] does. Armed with the precise knowledge of θ , the measured Solow residual will be:¹¹

$$\begin{aligned} S_t &= [\Delta \log y_t - \theta \Delta \log k_t - (1-\theta) \Delta \log n_t] / (1-\theta) \\ &= [(1-\theta-\phi_2) \Delta \log z_t + \phi_2 \Delta \log y_t + (1-\theta) \Delta \log v_t + \Delta \log J_t] / (1-\theta) \end{aligned} \quad [11]$$

Assuming that the deterministic component of technological growth (z_t) is aseasonal, then seasonality in S_t can arise from: (1) increasing returns if output is seasonal, (2) labor hoarding if variations in labor effort are seasonal, and (3) seasonal adjustments in capital and labor hours. If the fourth quarter increased desire to consume is strong enough to generate a seasonal increase in fourth quarter output, then measured Solow residuals will be proseasonal due to the productive externality. If the higher output is achieved by a seasonal increase in work effort (without a correspondingly large increase in adjustment costs), then the demand effect is reinforced.¹² Whether or not a single Christmas seasonal in preferences can explain seasonality in Solow residuals, in all four quarters, depends upon the model's ability to generate seasonality in output and labor effort across the entire calendar year.

¹⁰For an explicit characterization of this type of seasonal equilibrium path, see Braun and Evans or Chatterjee and Ravikumar [1989].

¹¹We assume that S_t is an attempt to measure $\Delta \log z_t$ rather than $(1-\theta)\Delta \log z_t$.

¹²Evans [1991] documents that Prescott's measure of the Solow residual is not exogenous when seasonally adjusted data is used. The finding that money, interest rates, and government spending Granger-cause Prescott's residual could be due to increasing returns or unobserved variations in labor effort of the form modeled here.

3. Econometric Estimation of the Model's Structural Parameters

The vector of structural parameters Ψ contains 16 elements:

$$\Psi = (\delta, \beta, T, \bar{r}, r_4, \alpha, a, b, \xi, \bar{v}, \theta, \psi_1, \psi_2, \lambda, \phi_1, \phi_2).$$

In assigning parameter values, there are three categories of parameters: (1) parameters which can be normalized a priori because their values have no influence upon the analysis; (2) parameters which are customarily set a priori because their values are not well-identified in the data; and (3) parameters which are econometrically estimated by Generalized Method of Moments. First, the parameters $(\alpha, T, \phi_1, \bar{v})$ are inherently unidentified. Since utility is ordinal, we normalize $\alpha=1$ and estimate the consumption preference parameters \bar{r} and r_4 . The time allocation is set to $T=1369$ hours per quarter (as in Christiano and Eichenbaum [1991]). Labor effort v_t is an index variable whose level depends upon \bar{v} : we set \bar{v} at a level which guarantees that average labor effort will be 1.¹³ The parameter ϕ_1 simply defines the units of measure for commodities (thousands of dollars, billions of yen, etc.): its value can be selected arbitrarily without affecting the analysis. Second, the discount factor β is not well-identified in aggregate time series data. We set β equal to $1.03^{-.25}$ as in Christiano and Eichenbaum. Third, the lack of seasonally unadjusted quarterly data on the capital stock leads us to construct capital from investment flows assuming that the depreciation rate δ is 2.5% per quarter. The remaining parameters are econometrically estimated by Generalized Method of Moments.

3.1 GMM estimation

The parameters $(\theta, \bar{r}, r_4, \psi_1, \psi_2, a, b, \lambda, \phi_2, \xi)$ are estimated by

¹³This normalization ensures that the average labor input in the model corresponds to the average level of labor hours in the data.

imposing jointly two sets of moment conditions: (1) orthogonality conditions based upon the household's intratemporal Euler equation for choosing consumption and leisure; and (2) explicitly equating a set of first moments in the data with the model's predictions for these moments. For the first set of moments, the Euler equation can be written (in terms of observables) as:

$$\mu_t \left[(1-\theta) \frac{y_t}{n_t} - \psi_2 \frac{y_t}{n_{t-1}} \left(\frac{n_t}{n_{t-1}} - 1 \right) \right] + \beta E_t \left[\mu_{t+1} \psi_2 \frac{y_{t+1} n_{t+1}}{n_{t-1}^2} \left(\frac{n_{t+1}}{n_t} - 1 \right) \right] - \left[(T-n_t) + b(T-n_{t-1}) \right]^{-1} - b \beta \left[(T-n_{t+1}) + b(T-n_t) \right]^{-1} = 0 \quad [12]$$

where $\mu_t = r_t / (c p_t + a c p_{t-1})$. Any variable in the time t information set is a valid instrument for estimating the parameters in this equation. The instrument set includes the time t and $t-1$ growth rates (x_t/x_{t-1} and x_{t-1}/x_{t-2}) of labor hours, capital, consumption, and output, as well as four seasonal dummy variables.

To describe the second set of moment restrictions, let $H(x_t)$ refer to the following transformations of the data:

$$H(x_t) = \begin{bmatrix} (\Delta \log y_t) q_t' & (\Delta \log c p_t) q_t' & (\Delta \log y_t/n_t) q_t' & (\Delta \log k_t) q_t' \\ (\Delta \log (1+r_t)) q_t' & k_t/y_t & n_t & \end{bmatrix}'$$

where q_t is a 4×1 vector of seasonal dummies, r_t is a real interest rate, and the symbol $'$ denotes transposition, so $H(x_t)$ is a 22×1 vector. Accordingly, the first 20 elements of the expected value of $H(x_t)$ are the seasonal growth rates of output, consumption, labor productivity, and capital, and the seasonal change in the real interest rate. The last two elements correspond to the average capital-output ratio and average labor hours. Given this definition, the model predicts that

$$H(x_t) = h(\Psi) + u_t$$

where $h(\Psi)$ corresponds to the model's predicted first moments of $H(x_t)$ and u_t is a vector mean zero, serially correlated random variable. Based upon these moment restrictions, our estimator of Ψ attempts to set the sample mean of u_t to zero, as well as the sample moments based upon equation [12].

Our choice of moment restrictions is motivated by two concerns. First, since labor effort is unobserved, the parameter ξ cannot be estimated by Euler equation methods. Second, estimating ϕ_2 from production function residuals seems hopeless due to the presence of unobserved variations in labor effort: no exogenous instruments are available.¹⁴ However, these parameters can be estimated by forcing the model to confront the seasonal growth rates in the data by choosing ϕ_2 and ξ , as well as the other parameters.

Finally, Sims [1990] and Hansen and Sargent [1991] have argued that econometricians who use seasonally unadjusted data and misspecify the seasonal mechanisms may do much worse than econometricians who discard the potential information content at seasonal frequencies and simply use seasonally adjusted data. On the other hand, Ghysels [1991] has pointed out that great efficiency gains may be possible if seasonally unadjusted data is used. Thus, there is a potential trade-off involved in using seasonally unadjusted data, efficiency gains versus misspecification bias. We try to address the bias issue by comparing our parameter estimates with other econometric studies which used seasonally adjusted or annual data.¹⁵

¹⁴ Hall [1988] has noted that his set of instruments would fail to be exogenous in this setting.

¹⁵ Another interesting statistical issue is whether the unadjusted time series data are better characterized by purely indeterministic seasonality, purely deterministic seasonality, or a mixture of both. We identify Christmas effects with a fourth quarter mean of r_t which is larger than the other three quarters. This single nonzero seasonal mean induces nonzero seasonal means in other economic aggregates. Even if r_t is stochastic, our economic theory predicts that economic time series will possess some deterministically seasonal components. This argues against purely indeterministic models of

3.2 Data

The original data set employed in this study is the Barsky-Miron [1989] data for the sample period 1964-1985: U.S. quarterly data which has not been adjusted for seasonality. For the empirical analysis to conform to the theoretical constructs of our model, however, we redefine some of the variables as follows (and convert to per capita values). Output (y) is Gross National Product per capita. Private consumption (cp) is nondurables plus services consumption expenditures per capita. Investment (i) is the sum of Fixed Investment plus Durable consumption expenditures, per capita. The capital stock is computed using the flow investment expenditures, a quarterly depreciation rate of 2.5%, and an initial capital stock value for 1950. Labor hours are computed as the product of total nonagricultural employment times average hours per week of nonagricultural production workers times 13 weeks per quarter (per capita). The real interest rate is the ex post return on three-month Treasury Bills, not seasonally adjusted as reported in Citibase. The data is converted to per capita values by using the civilian population, 16 years and older.

3.3 Estimation Results

Table 1 presents our estimation results. The estimation imposes 34 moment restrictions in estimating 10 structural parameters; in principle, there are 24 overidentifying restrictions which are tested by Hansen's [1982] J-statistic. The statistic is 20.86 with a probability value of 0.65, uncovering no evidence against these restrictions. Recall that 20 of the 34 restrictions involve matching the model's seasonal predictions against the

seasonality.

data's seasonal growth rates. Informally, this test suggests that the model successfully captures the data's seasonal properties. This claim is examined in more detail in Section 4 where we consider a variety of other tests that focus explicitly on the model's seasonal predictions.

Turning to the individual parameters, our estimates using seasonally unadjusted data are similar to other estimates in the literature which have employed seasonally adjusted data. Our estimate of θ is .279 which is close to Prescott's [1986] value of .25 (when output is identified with GNP and does not include the services of durable consumption goods). The weighted average value of \bar{r} and r_4 is .2363. The inverse $1/\bar{r}$ corresponds to the leisure preference parameters estimated by Christiano and Eichenbaum; our value of 4.23 falls within the range 3.92 and 5.15 they report. The Christmas consumption effect is estimated to be $r_4/\bar{r} = 1.022$; this is the percentage increase in the marginal utility of consumption services, holding consumption services fixed. This value does not seem to be implausibly large.

The unusual precision of $\hat{\theta}$, $\hat{\bar{r}}$, and \hat{r}_4 is due to the two moment restrictions in $H(x_t)$ which are related to the capital-output ratio and the level of labor hours. If these two moment conditions are dropped and Ψ is re-estimated, the parameter estimates are essentially unchanged, but the standard errors for $\hat{\theta}$, $\hat{\bar{r}}$, and \hat{r}_4 rise to .0352, .0105, and .0107, respectively. Therefore, the unusual precision of these parameter estimates is due to the inclusion of strong identifying restrictions from the model's equilibrium predictions.

The nontime-separability parameters a and b are similar to other researchers' estimates. The value of $a=.445$ indicates that consumption goods have a durable quality: in seasonally adjusted data, this has been found by

Eichenbaum, Hansen, and Singleton [1988] and Gallant and Tauchen [1990].¹⁶ The value of $b = -.517$ indicates that leisure preferences exhibit habit-persistence: in seasonally adjusted data, this has been found by Eichenbaum, Hansen, and Singleton as well as Braun [1990]. This feature makes leisure and labor hours relatively smooth; in addition to adjustment costs for labor hours, habit-persistence in leisure will smooth labor hours and lead to greater variations in labor effort in response to exogenous shocks.

The adjustment cost parameters are significantly different from zero.¹⁷ The capital and labor estimates are 28.45 and 0.258, but these numbers are a poor indication of their relative effects. On a quarterly basis, the standard deviations for the growth rates of capital and labor are 0.35% and 2.21%. The percentage reduction in output due to adjusting capital (only) and labor (only) by one standard deviation above average is -0.018% and -0.006%. So the capital adjustment penalty is only about 3 times larger than the labor penalty. Also, these numbers indicate that the direct effect of adjustment costs on measured Solow residuals is negligible. That is, the effects of $\Delta \log J_t$ in equation [11] are small. As was noted in Section 2, however, the indirect effect may be large: in response to a consumption demand shock, reducing investment may now be costly enough to induce a large response in output.

Our estimate of the output elasticity with respect to external increasing returns is $\phi_2 = .2389$. The elasticity is significantly different from zero. This value is within the range of estimates reported by Caballero and Lyons

¹⁶On the other hand, Constantinides and Ferson [1990] find evidence of habit-persistence in consumption goods preferences ($a < 0$). In simulations of an equilibrium business cycle model with seasonality, Braun and Evans [1990] found that durability in consumption ($a > 0$) helped the model match key business cycle moments better than habit-persistence.

¹⁷Ghysels [1988] observes that there is a lot of spectral power at seasonal frequencies for identifying adjustment cost parameters.

[1989] and Baxter and King [1990], although Baxter and King use a larger value of .33 in their model evaluation. The size and statistical significance of our estimate provides some evidence that external increasing returns are important for explaining seasonal fluctuations; a quantitative assessment is offered in Section 4. Nevertheless, since our identifying restrictions differ from those of Caballero-Lyons [1990] and Baxter-King [1990], our estimate of ϕ_2 provides evidence which is both independent of theirs and complementary.

Our estimate of ξ is 0.0241, so the disutility of labor effort deviations may in fact be small enough to induce sizable variations. The standard error is .0348, so the estimate is reasonably imprecise. It is important to note that the hypothesis that variations in labor effort are small would imply that ξ is large: the point estimate and standard error do not support this. As we will see in Section 4, estimates of ξ in the range reported are capable of yielding substantial variations in work effort. Thus, the estimated habit-persistence in leisure preferences, costly adjustment of labor, and relatively small disutility associated with varying labor effort jointly provide evidence for the labor hoarding hypothesis. Finally, the value of \tilde{v} implied by the estimates and normalization is negative. Thus, utility is strictly decreasing in work effort.

Overall, the estimated parameterization seems reasonable. The similarity of many estimates with previous studies suggests that if we had chosen to "calibrate" our model using these other studies, the resulting parameterization would not have been very different. Finally, the overidentifying restrictions cannot be rejected.

4. Evaluating the Model's Seasonal Implications

In this section we evaluate the seasonal properties of the estimated

parameterization and offer evidence on the relative importance of labor hoarding and increasing returns in explaining the seasonal patterns in the data. Two criteria are used to evaluate the model's seasonal predictions. First, a series of hypothesis tests are reported. These tests have the benefit of taking into consideration sampling error in the summary statistics for the data and sampling error in the estimated parameterization. Second, the seasonal growth rates of the data and the model are simply plotted together. This latter approach provides a visual summary of the seasonal properties of the model relative to the data.

Table 2 contains results from a series of hypothesis tests. The results in table 2 are aimed at providing information on the following three questions: Is there evidence of seasonality in the data? Does the model predict significant seasonality? Does the model predict the same seasonal patterns found in the data? Column one reports Wald statistics that offer evidence on the first question for each variable individually. The maintained null underlying the column one results is that the four seasonal dummies for a particular time series are equal (equation (1), Table 2). These statistics are constructed from GMM estimates of the average seasonal growth rates in the data and use a Newey-West covariance estimator. The p-values for each statistic indicate that the null hypothesis of no seasonality is overwhelmingly rejected for each time series. These results are representative of findings reported by Barsky and Miron [1989]¹⁸. Column two reports Wald statistics that offer evidence on the second question. The maintained null hypothesis is that the model's predicted seasonal growth rates are equal.¹⁹

¹⁸ Barsky and Miron also find that there is statistically significant seasonality in the real interest rate although the magnitude of the estimated seasonals (in levels) is small.

¹⁹ The model's predicted seasonal growth rates are a highly nonlinear function

The null hypothesis of no seasonality is also sharply rejected for each of the time-series that the model offers predictions for. On the basis of the results from these two tests, we conclude that both the model and the data offer strong refutable predictions at seasonal frequencies.

Certainly the most important question is the third one: Does the model predict the seasonal patterns in the data? Column three of Table 2 provides one metric for evaluating the model's "fit" at seasonal frequencies. The maintained null hypothesis in column three is that the model's predicted seasonal growth rates for the j th time-series equal the corresponding values in the data in each of the four seasons. This LaGrange multiplier (or LM) test is formally a test of particular moment restrictions that were imposed in the course of estimation. For hours, the Solow residual, and investment, the statistics were calculated using the fact that these time series can be expressed as (log) linear combinations of other time series that were included in the estimation. Eichenbaum, Hansen and Singleton [1984] and Newey and West [1987b] describe the details of implementing LM tests in the context of GMM estimation. Column three contains surprisingly little evidence against the null of a common seasonal pattern in all instances. As a check we also calculated a GMM analog to the likelihood ratio statistic and found that the two statistics were virtually identical.

This collection of statistics provides two important conclusions. First, the tests reported in columns one and two demonstrate that the statistics have sufficient power to reject the null hypothesis of no seasonality for the model and the data. Second, the column three results find no evidence against the hypothesis that the model correctly predicts the pattern of seasonality found

of the estimated structural parameter vector $\hat{\Psi}$. The asymptotic covariance of the predicted seasonals is computed using the covariance estimator of Ψ and the gradient of the nonlinear function. The Wald statistics are constructed from these objects in the usual way.

in the data.

Turning to the specific predictions of the model, we report plots of the seasonal growth rates for the data and model in figures 1-3 (the actual numbers are presented in Table 3). These diagrams complement the previous hypothesis tests in that they offer summary information on the ability of the estimated parameterization to capture particular aspects of the seasonal pattern in the data. We will focus on two aspects of the seasonal pattern: the magnitude of the model's predicted seasonal in a particular quarter relative to the data and the ability of the model to mimic the sequential relationship of seasons found in the data. The Solow residuals labeled "data" are calculated using $\theta = .28$. As was noted in section 3 this number is qualitatively close to the value of .25 used by Prescott [1986].

One of the principal aims of this paper is to investigate the possibility that increasing returns and/or time-varying labor effort can explain the large seasonal variation in the Solow Residual. Figure 1 confirms the results reported in table 2: the model is quite successful in this respect. The predicted Solow residual has the same sequential pattern and captures the magnitudes found in the data. These results offer strong support for our contention that the observed seasonal pattern in the Solow residual is driven largely by demand shocks.

In addition to capturing the seasonal pattern in the Solow residual, the model also mimics important features of seasonality in output, consumption and average productivity. In all of these instances the model correctly predicts the sequential seasonal pattern of the data. For consumption we do see a tendency for the model to understate the third quarter deceleration found in the data and for output the model understates the second quarter rise. However, the hypothesis tests indicate that both of these disparities can be attributed to sampling error. These successes across the entire calendar year

are particularly striking given that the only seasonal shifter is a fourth quarter shift in preferences.

Figure 3 displays the seasonal patterns in labor effort. Since this is an unobservable, the data's seasonals cannot be reported. Fourth quarter output rises on the strength of higher than normal labor effort. In combination with increasing returns, fourth quarter effort is only 3% above normal in generating an annualized output growth of 18%.²⁰ Opposing forces are at work in the first quarter. These variations in effort do not seem implausible.

If we ignore sampling error, the figures suggest that the model fails to account for some aspects of the seasonal pattern in other variables. For hours and investment the magnitudes are off in all four quarters, for the capital stock they are off in three quarters and for interest rates they are off in two quarters. However, even for these time series the model captures many features of the sequential pattern in the data. The fact that the hypothesis tests in table 2 fail to reject a common seasonal pattern in individual time series suggests that there may be considerable sampling error. The most likely sources for this sampling error are in $\hat{\xi}$ and $\hat{\phi}_2$, parameters which govern respectively the roles of time-varying labor effort and increasing returns. Both parameters are estimated with sizable standard errors. The case analyses below demonstrate that variations in these two parameters lead to a deterioration in the model's seasonal predictions relative to the data.

It is also interesting to examine the role of the various features of the model in explaining these seasonal patterns. Notice first, that the increasing returns can be shut down by setting $\phi_2 = 0$. Second, time variation in work effort can be ruled out by setting $\xi = \infty$ and $\psi_2 = 0$. For these values of

²⁰The fourth quarter growth rate of labor effort is only 3.8%.

ξ and ψ_2 it is never desirable to vary effort, while it is costless to adjust labor in production. By examining these two special cases of the estimated parameterization we can get some feel for the contributions of labor hoarding and increasing returns to scale.

The results from these exercises are displayed in Figures 4, 5, and 6 along with the data for purposes of comparison. Consider first output and the Solow residual. With only increasing returns the Solow residual and output are essentially flat across all four seasons. Labor hoarding, on the other hand, does in isolation produce the correct sequential pattern for these two series while dramatically understating the magnitudes. On the basis of these diagrams it would appear that labor hoarding plays a crucial role in generating the correct seasonal pattern in output and the Solow residual. The primary role of increasing returns then is to magnify these patterns. In our estimated parameterization increasing returns and labor hoarding are clearly interacting to deliver the seasonal patterns in output and the Solow Residual.

Consumption and investment, on the other hand, continue to display some seasonality with only increasing returns. This can be attributed to two properties of the model. The fourth quarter seasonal demand shift helps explain the fluctuations in the fourth and first quarters. In the second and third quarters investment and consumption are adjusting endogenously via the nontime-separabilities in consumption which imply that consumption in adjacent periods are substitutes.

5. Conclusion

This paper demonstrates that the seasonal cycle contains valuable information for uncovering the roles of labor hoarding and increasing returns. In contrast to business cycles which are arguably induced by both demand and technology shocks of varying persistence, seasonal fluctuations are

anticipated, transient, and easily identified with calendar events like Christmas. Our findings indicate that increasing returns to scale alone does not directly explain the seasonality in measured Solow residuals. However, it plays an important role in magnifying small variations in work effort. Hall [1988] has argued that labor hoarding requires implausibly large variations in work effort to explain cyclical fluctuations in Solow residuals. For seasonal fluctuations this is not the case. Our estimated parameterization implies labor effort variation of no more than 5.5% on a quarterly basis. With increasing returns these variations are magnified, thereby producing fluctuations in total factor productivity that are of the same magnitude observed in the data. Finally, since our explanation rests on phenomena which are not country-specific-- Christmas celebrations, productive externalities, and labor hoarding-- this model offers an explanation for the similar cross-country seasonal patterns documented by Beaulieu and Miron [1991].

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Table 1: GMM Parameter Estimates

<u>Parameter</u>	<u>Estimate</u>	<u>Standard Error</u> ²¹
θ	.2790	0.00091
\bar{r}	.2350	0.00045
r_4	.2402	0.00047
λ	.0021	0.00005
ψ_1	28.448	3.64360
ψ_2	.2576	0.05645
a	.4453	0.01123
b	-.5173	0.00854
ϕ_2	.2389	0.09240
ξ	.0241	0.03481
J-statistic	20.86	
Degrees of Freedom	24	
P-value	.6469	

²¹A Newey-West procedure [1987a] with four lags was used to compute the optimal GMM weighting matrix.

Table 2: Hypothesis Test Results

- (1) $\Delta x_t = d'q_t + w_t$, H_0^d : The data do NOT exhibit deterministic seasonality. The elements of d are equal.
- (2) $\Delta x_t = f(\Psi)'q_t + w_t$, H_0^f : The model does NOT exhibit deterministic seasonality. The elements of $f(\Psi)$ are equal. ²²
- (3) $f(\Psi) = d$, H_0^j : The model's predicted seasonality equals the data's seasonality.

Variable X	H_0^d ²³	H_0^f	H_0^j ²⁴
Solow	303.64 (.000)	437.74 (.000)	0.125 (.998)
Output	413.90 (.000)	348.08 (.000)	0.201 (.995)
Consumption	715.28 (.000)	197.32 (.000)	0.058 (.999)
Investment	1097.45 (.000)	27.35 (.000)	0.137 (.997)
Capital	280.26 (.000)	31.21 (.000)	0.301 (.989)
Labor Hours	831.49 (.000)	112.31 (.000)	0.216 (.994)
Labor Effort	-----	615.61 (.000)	-----
Labor Prod	227.14 (.000)	391.25 (.000)	0.167 (.996)
Real Rate	31.06 (.000)	54.03 (.000)	0.034 (.999)

²²Equation (2) is predicted by our theoretical model, but our test is not regression-based. See the text for a description.

²³For both columns 1 and 2, the Wald test statistics are asymptotically distributed χ^2 with four degrees of freedom. The numbers in parentheses are probability values of the test statistic.

²⁴The Lagrange Multiplier test statistic is asymptotically distributed χ^2 with four degrees of freedom.

Table 3: Seasonal Growth Rates²⁵

<u>Variable</u>	<u>Season</u>	<u>Model</u>	<u>Data</u>
Solow Residual	Winter	-7.451	-7.323
	Spring	3.512	3.889
	Summer	- .703	-2.019
	Fall	5.466	5.889
Output	Winter	-6.501	-7.356
	Spring	2.924	4.623
	Summer	- .345	- .674
	Fall	4.746	4.529
Consumption	Winter	-6.744	-6.876
	Spring	3.500	3.015
	Summer	-1.322	.461
	Fall	5.390	4.767
Investment	Winter	-5.637	-14.065
	Spring	.854	12.365
	Summer	3.141	-1.578
	Fall	2.465	4.896
Capital	Winter	.297	.640
	Spring	.136	.219
	Summer	.155	.572
	Fall	.235	.506
Labor Hours	Winter	-1.681	-3.126
	Spring	0.491	2.438
	Summer	.165	.863
	Fall	1.025	.197
Labor Prod.	Winter	-4.821	-4.229
	Spring	2.433	2.186
	Summer	- .510	-1.537
	Fall	3.721	4.332
Real Rate	Winter	2.901	.701
	Spring	0.812	1.247
	Summer	0.409	- .397
	Fall	-4.122	-1.162

²⁵Quarterly rates of growth in percentages, except for the real interest rate which is the quarterly change in annualized yields (i.e., $\Delta \log (1+r_t)$, with r_t at annual rates).

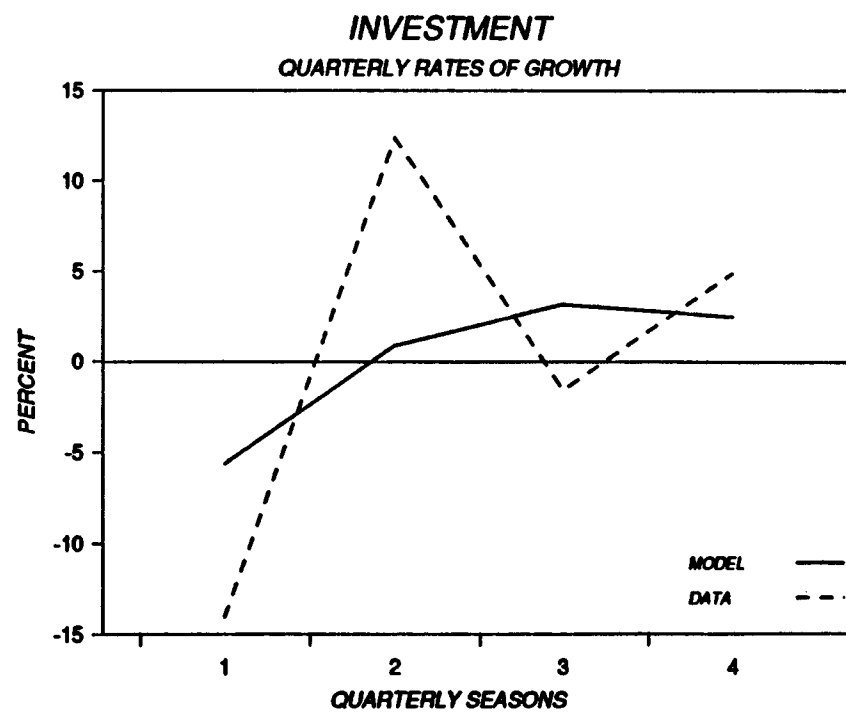
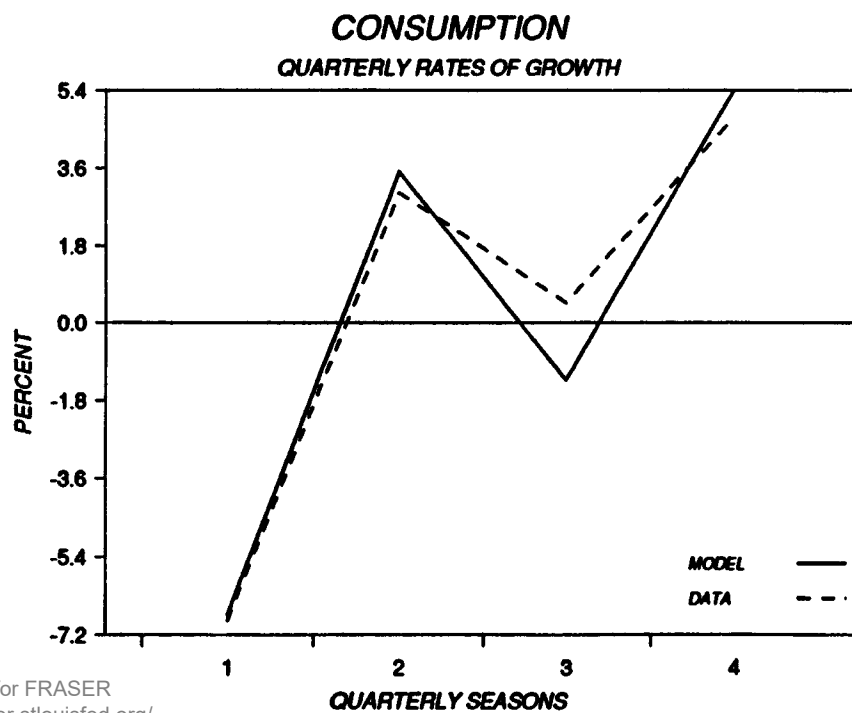
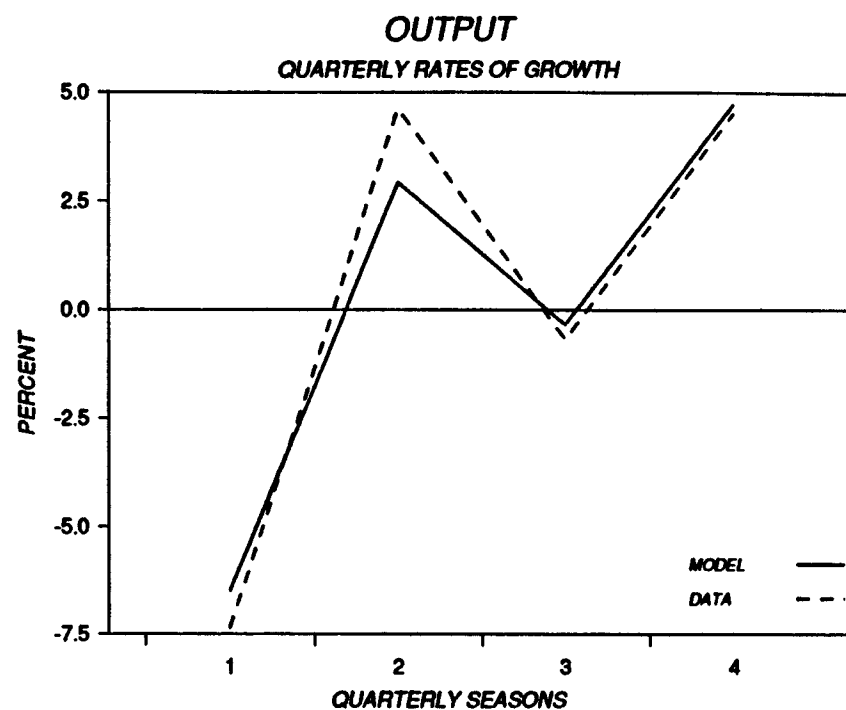
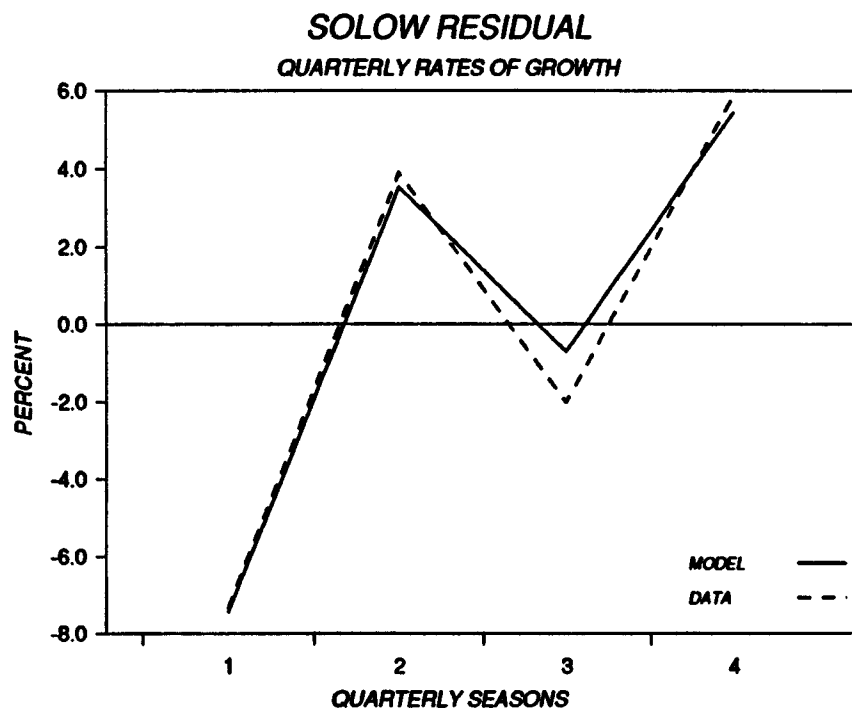


Figure 1

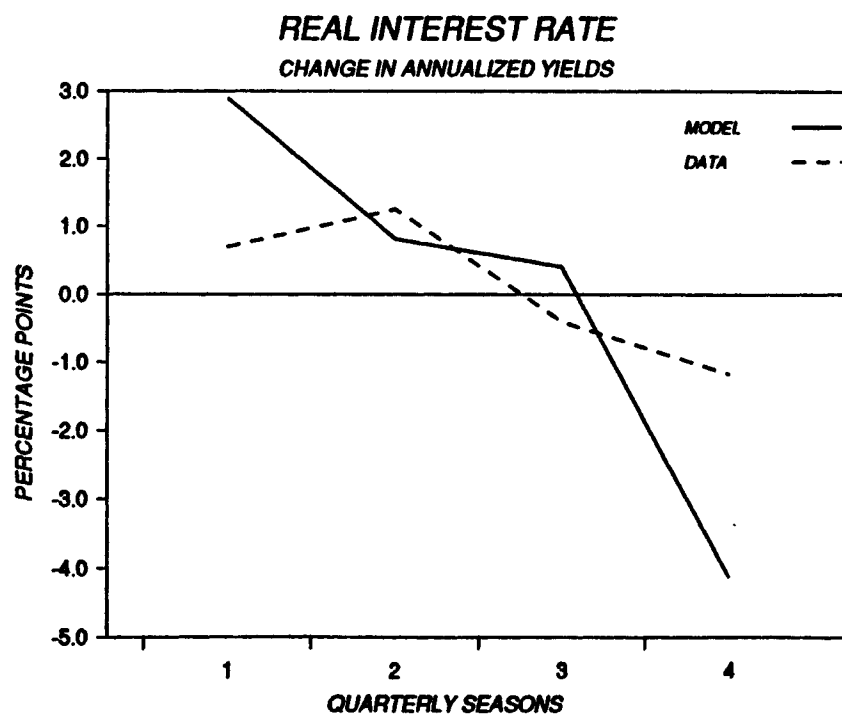
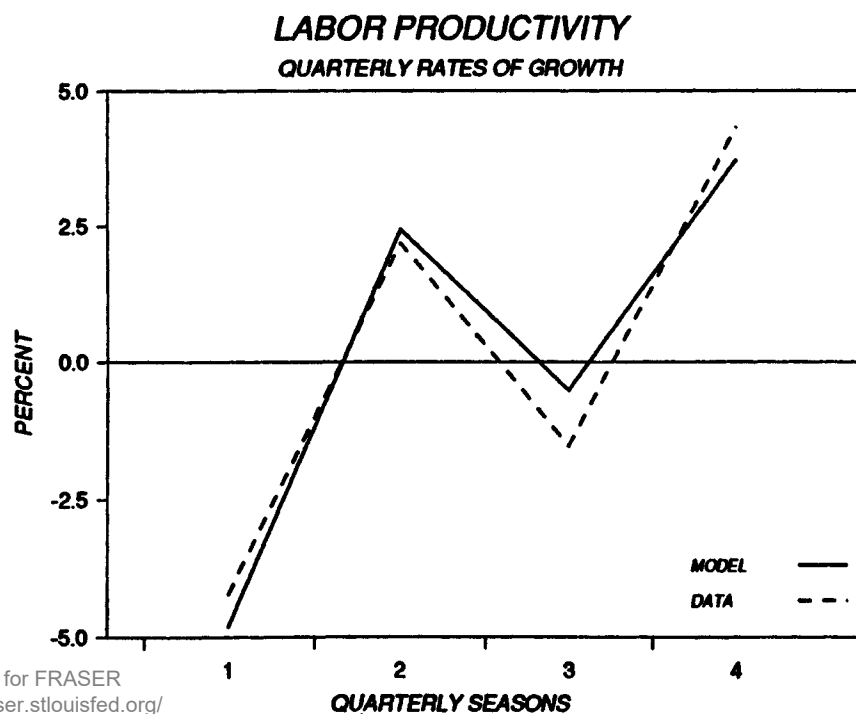
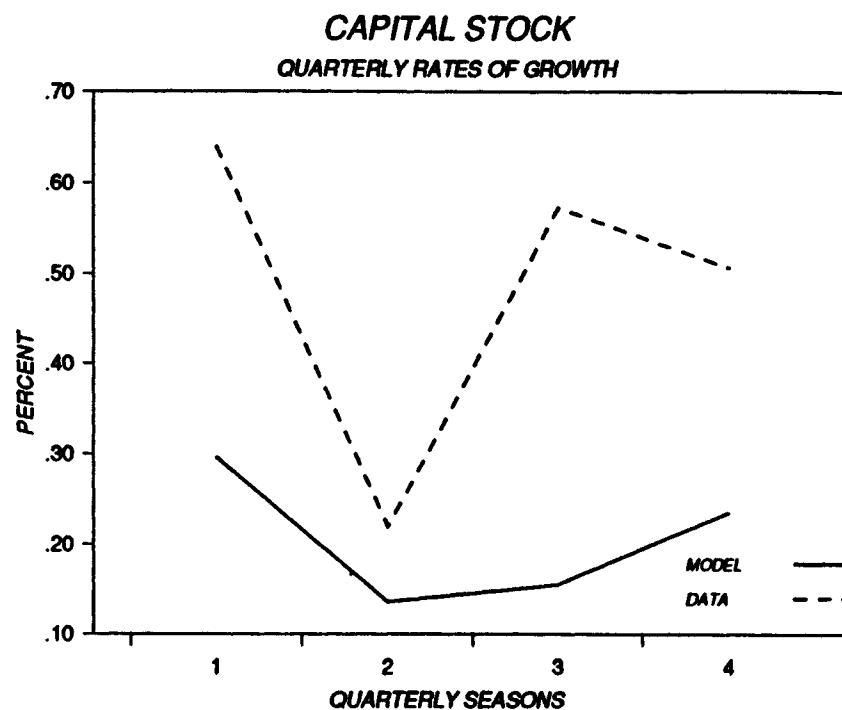
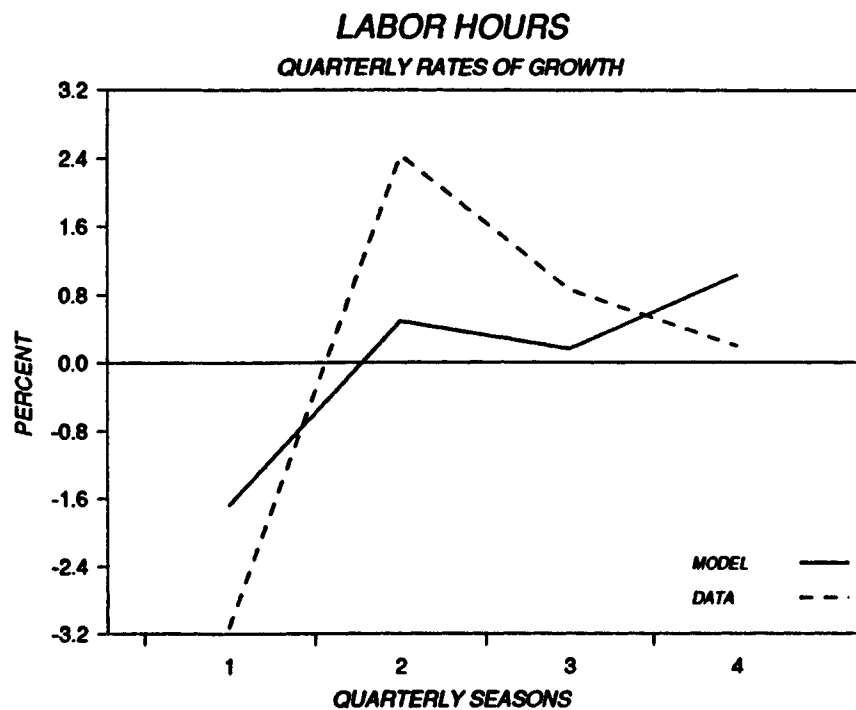
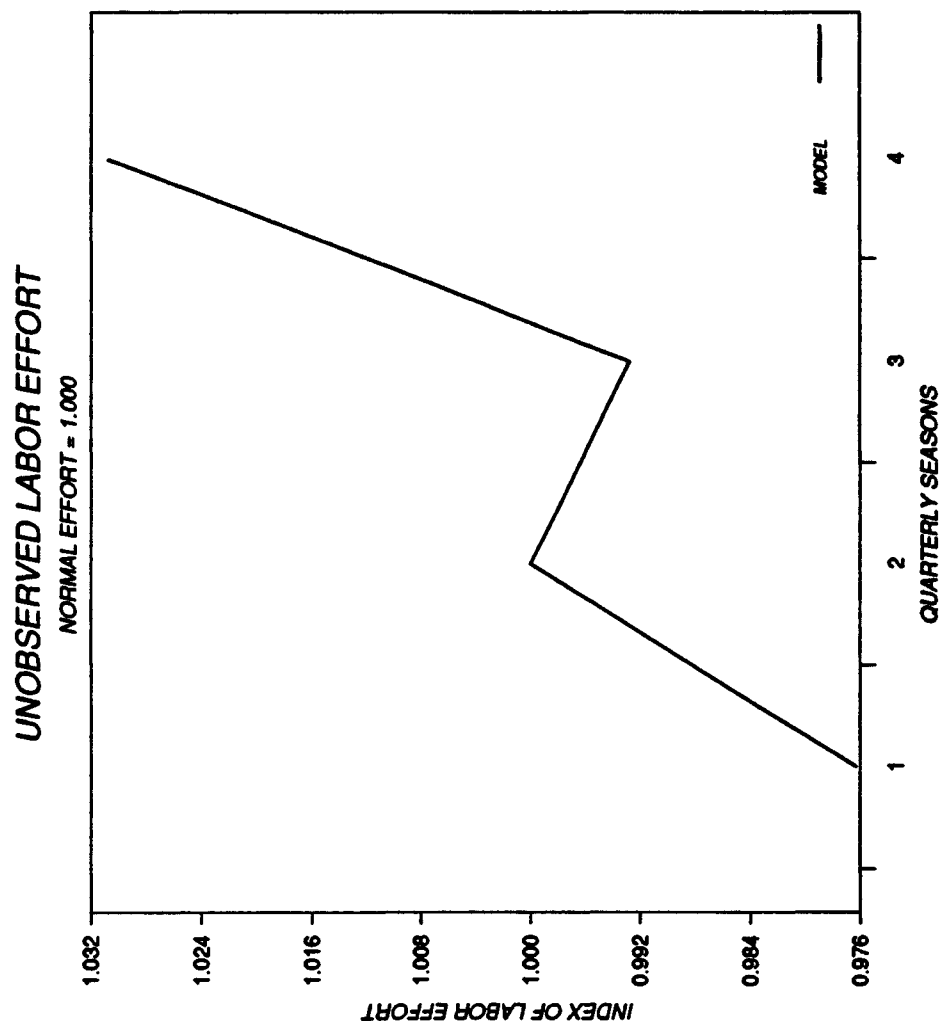


Figure 2

Figure 3



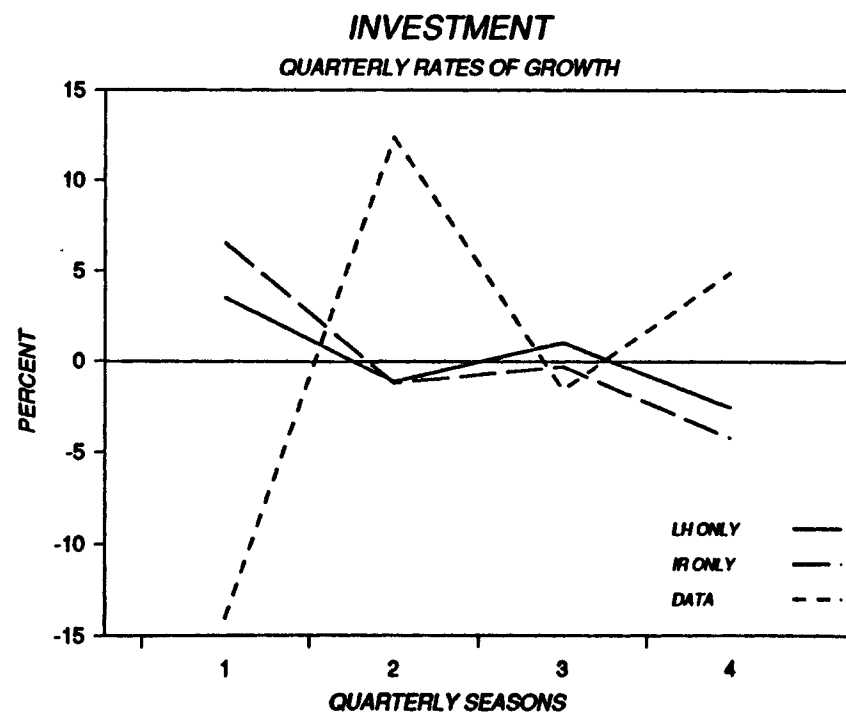
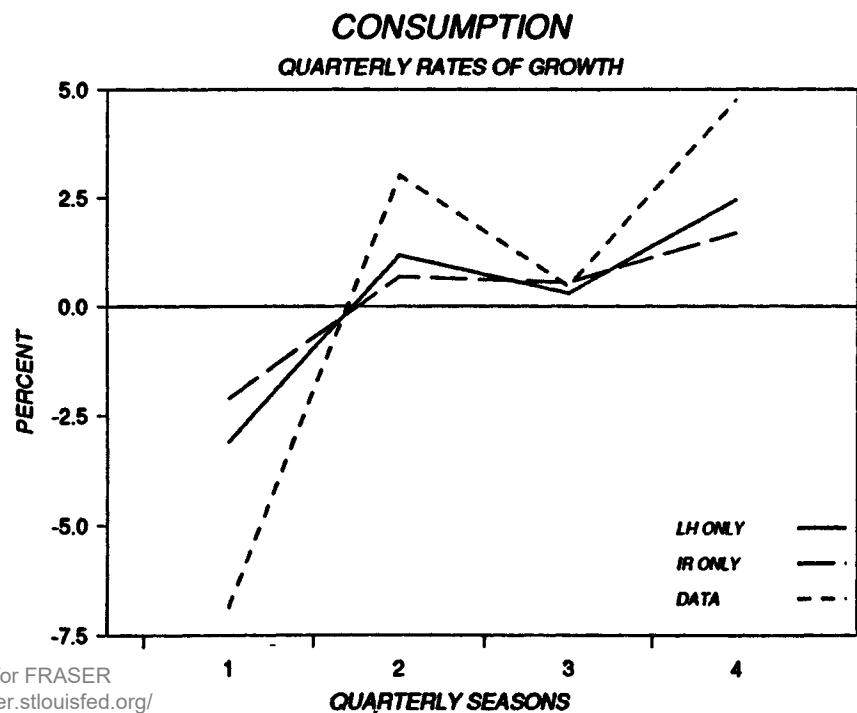
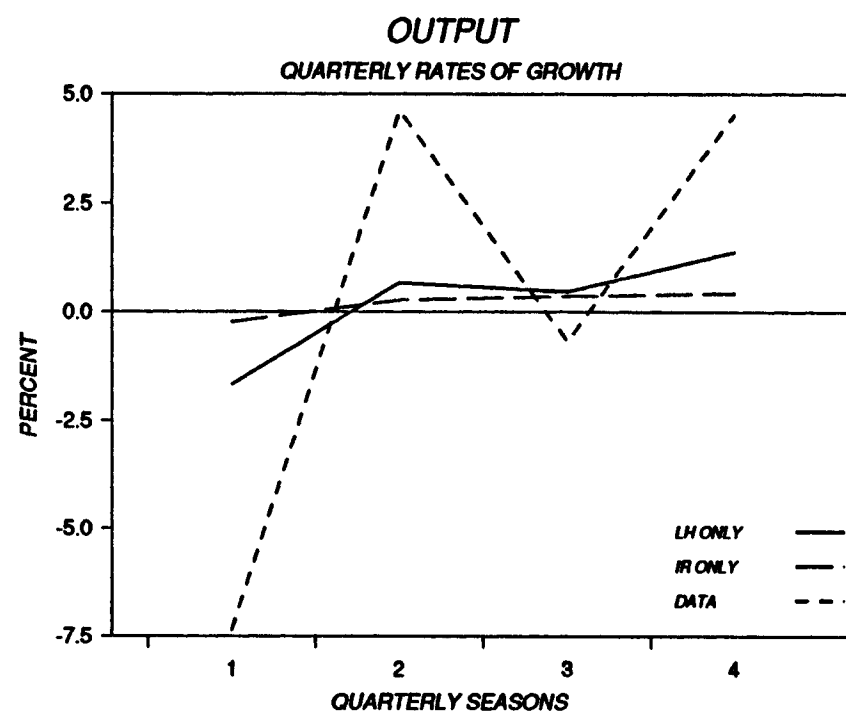
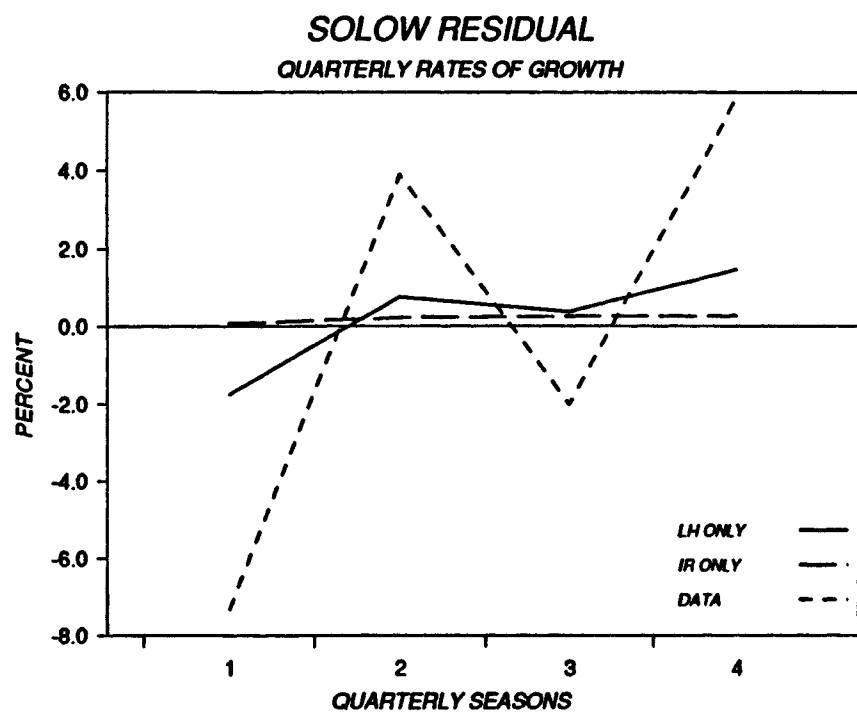


Figure 4

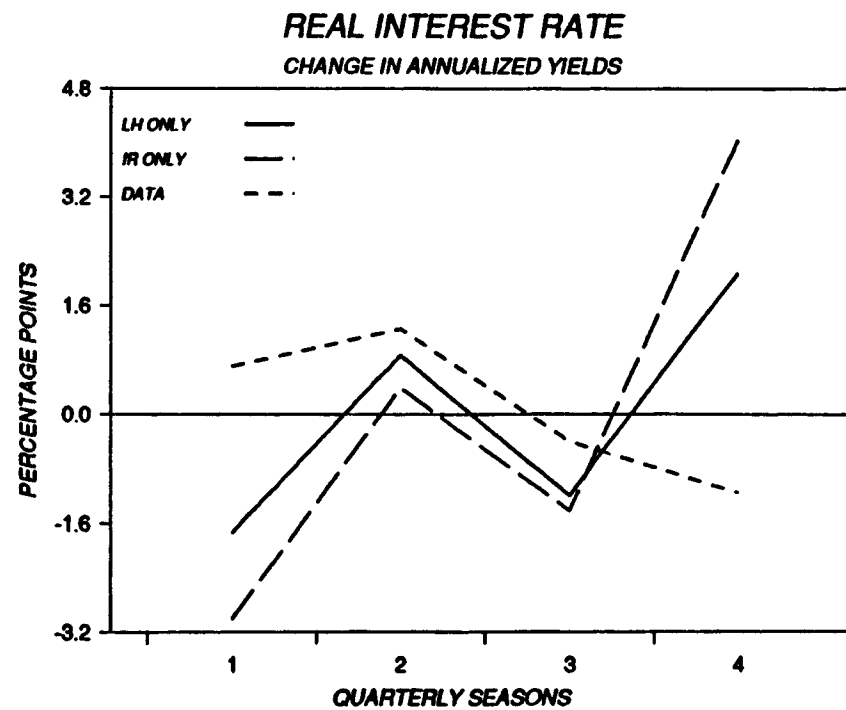
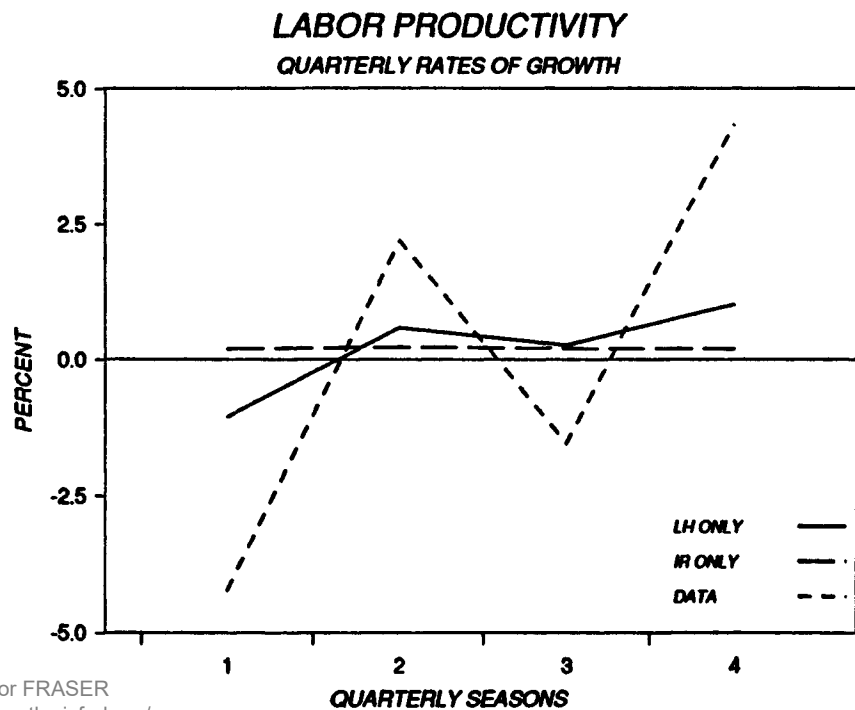
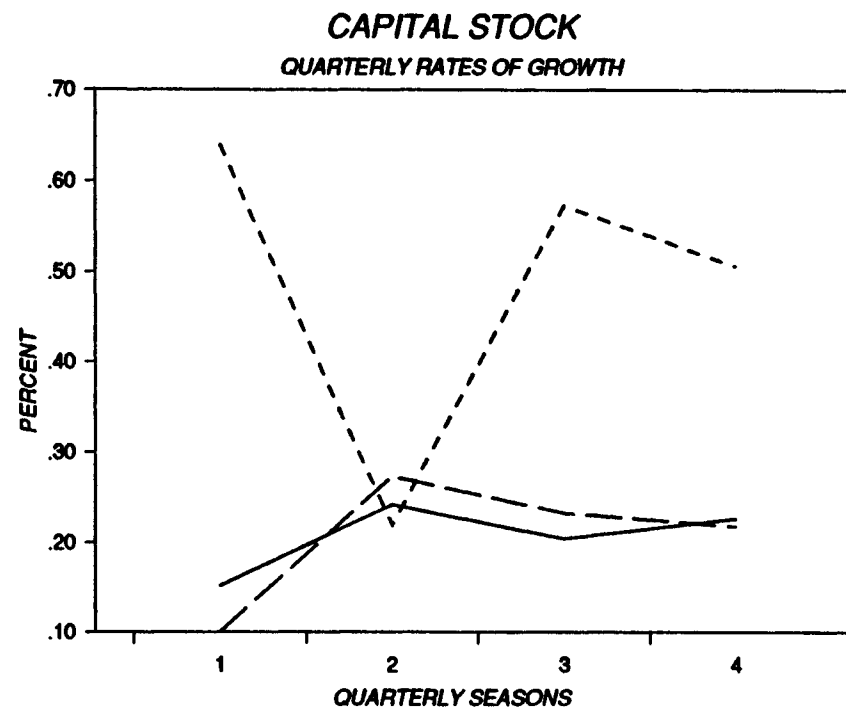
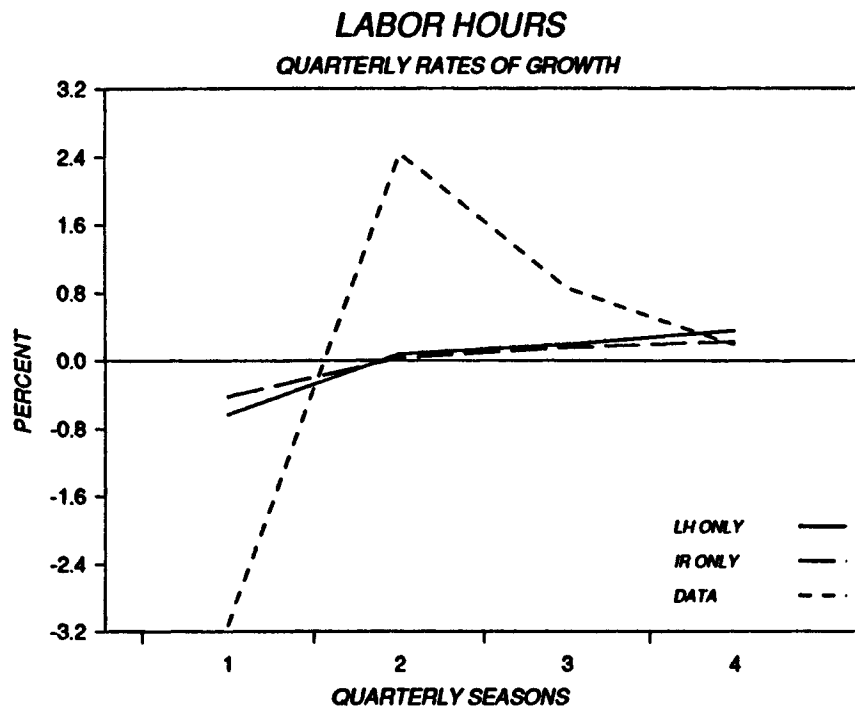


Figure 5

Figure 6

