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Scale Elasticity and Efficiency For U.S. Banks

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Scale Elasticity and Efficiency For U.S. Banks

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In the early bank cost literature many of the studies found significant scale economies. As a result, the authors suggested that changes in industry structure could produce cost savings through increased efficiency. Recent bank cost studies improved upon previous methodologies by utilizing flexible functional forms, accounting for multiproduct production processes, estimating scale measures at both the branch and firm level, distinguishing between branch and unit bank technologies resulting from regulatory restrictions, etc. The typical finding from the recent studies is that relatively minor scale economies exist in banking. This reported finding is usually followed by a statement claiming that the potential cost gains from exploiting scale advantages via merger or growth activities appear to be relatively minor. The implication from the conclusions drawn by the authors of these studies is that scale elasticity and scale efficiency are synonymous. It would appear that the derivation of one automatically provides the other.

The purpose of this article is to clarify an apparent confusion in the lit-

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erature between scale elasticity and scale efficiency. The bank production process is one of the most extensively researched aspects of bank behavior. With few exceptions, however, studies have not evaluated scale inefficiency. Instead, scale elasticity estimates have been used as a proxy for efficiency and elasticity measures close to 1.0 are taken to imply that scale inefficiency is trivial. It appears that scale inefficiency is typically assumed to equal one minus the elasticity measure, e.g., see Berger and Humphrey (1991).[1] However, scale elasticity and scale efficiency are two distinct concepts. Very slight scale economies *do not* imply slight scale inefficiency; nor do significant economies imply substantial scale inefficiency. We formalize the scale inefficiency measure and find that the relationship between elasticity and efficiency is not the relationship typically implied in the bank cost literature. We empirically apply the new scale efficiency measure to a group of large U.S. banks, and also apply it to the results of previous studies to highlight the distinction between scale elasticity and efficiency. The findings suggest that viewing elasticity alone to determine scale efficiency is inappropriate and can produce misleading conclusions.

1 Elasticity and efficiency measures

The scale elasticity measure, $\mathcal{E} = \partial \ln C / \partial \ln Q$, where C is cost and Q output, is a point elasticity associated with a particular output level and indicates the relative change in cost associated with an incremental change from this output level. Scale inefficiency, I, can be measured as the aggregate cost of F inefficient firms ($\mathcal{E} \neq 1.0$) relative to the cost of a single efficient

[1] In fact, when first discussing this distinction with colleagues, the most common response was that although the two concepts may not be equivalent, they are closely related and scale inefficiency could not exceed one minus the scale elasticity measure. As shown below, this is incorrect.

firm ($\mathcal{E}$ = 1.0), where F = the size of the efficient relative to the inefficient one. That is, $I = [F \cdot C_I \ / \ C_E]$ - 1.0, where C_I and C_E are the cost of production at the inefficient and efficient firms, respectively.

Intuitively, the two concepts differ because they measure different things: elasticity is related to incremental changes in output, and inefficiency to significantly larger changes. For inefficiency we are interested in the difference in total or average cost at two output levels. The scale elasticity at the inefficient level of output suggests the *initial* path to the efficient output level. However, the initial path itself is inadequate to determine the efficient output. In Figure 1, the average cost relationships for three production technologies are shown. Although each produces the same degree of scale inefficiency, the path from the inefficient level of production to the efficient one, and the scale elasticity measure at the inefficient output level, are significantly different. The scale elasticity measure at output Q_I says nothing about the level of scale inefficiency found in these three technologies. Alternatively, Figure 2 presents average cost relationships for three technologies which have the same point elasticity at output level Q_I. The three technologies, however, exhibit significantly different levels of scale inefficiency for production at this output level. The cost savings realized by an incremental increase in output by a scale inefficient firm is irrelevant for measuring inefficiency since *this is not the savings realized by producing at the efficient scale.* The elasticity measure is important in determining scale inefficiency *only* to the extent that it can be used to derive the cost differential over a broader range of outputs, i.e., between the output of the scale efficient and inefficient firms. The elasticity value is not even needed to calculate scale inefficiency if direct information is available on the value of average cost at the efficient and inefficient levels of output.

More formally, we next derive a general measure of scale inefficiency employing a standard translog cost function. Let

$$\begin{aligned}
&\ln C = \\
&\alpha_0 + \sum_i \beta_i \ln(P_i) + 0.5 \sum_i \sum_j \gamma_{ij} \ln(P_i)\ln(P_j) + \sum_i \beta_i \ln(Z_i) + 0.5 \sum_i \sum_j \phi_{ij} \ln(Z_i)\ln(Z_j) \\
&+\beta_Q \ln Q + \sum_i \gamma_{iQ} \ln Q \ln(P_i) + \sum_i \theta_{iQ} \ln Q \ln(Z_i) \\
&+0.5\beta_{QQ}(\ln Q)^2 \\
&\forall\ i \text{ and } j. \qquad (1)
\end{aligned}$$

where P denotes factor prices, Z exogenous variables relevant to the bank cost production process, and the other variables are as previously defined. For simplification, we rearrange equation (1):

$$\begin{aligned}
&\ln C = \\
&[\alpha_0 + \sum_i \beta_i \ln(P_i) + 0.5 \sum_i \sum_j \gamma_{ij} \ln(P_i)\ln(P_j) + \sum_i \beta_i \ln(Z_i) + 0.5 \sum_i \sum_j \phi_{ij} \ln(Z_i)\ln(Z_j)] \\
&+[\beta_Q + \sum_i \gamma_{iQ} \ln(P_i) + \sum_i \theta_{iQ} \ln(Z_i)] \cdot \ln Q \\
&+[0.5\beta_{QQ}] \cdot (\ln Q)^2 \\
&\forall\ i \text{ and } j. \qquad (2)
\end{aligned}$$

and allow the term in brackets in each of the three lines of equation (2) to be replaced by the coefficients a, b, and c, respectively. Therefore:

$$\ln C = a + b_Q(\ln Q) + .5c(\ln Q)^2 \qquad (3)$$

represents the bank cost relationship where a is independent of output levels by construction, c is independent by the nature of the functional form, and b_Q is a function of output levels (thus the subscript). We normalize output levels around the level produced by the inefficient firm so that $Q_I = 1.0$ and Q_E is a multiple, F, of Q_I. For the inefficient firm,[2]

$$\ln C_I = a + b(\ln Q_I) + .5c(\ln Q_I)^2 = a, \tag{4}$$

and the scale elasticity,

$$\mathcal{E}_I = \partial \ln C_I / \partial \ln Q_I = b. \tag{5}$$

For the scale efficient firm,

$$\ln C_E = a + b \ln(F \cdot Q_I) + .5c[\ln(F \cdot Q_I)]^2, \tag{6}$$

and

$$\mathcal{E}_E = \partial \ln C_E / \partial \ln(F \cdot Q_I) = 1.0. \tag{7}$$

Realizing that $Q_I = 1.0$, by taking the difference between equation (4) and equation (6), and with substitution, it can be shown that (see appendix)

$$I = [F \cdot C_I / C_E] - 1.0 = F^{.5(1-b)} - 1.0. \tag{8}$$

Since b is the elasticity coefficient resulting from the normalization of output around that of the inefficient firm, the inefficiency measure in equation (8) can be generalized:

$$I = [F \cdot C_I / C_E] - 1.0 = F^{.5(1-\mathcal{E}_I)} - 1.0. \tag{9}$$

[2]Neither the simplification in equation (2) nor the output normalization will alter the analysis. The chosen functional form, however, is important. The translog function is discussed because it is the most common form used in bank cost studies. For the quadratic form, $C = a+b \cdot Q+.5c \cdot Q^2$, the inefficiency measure will equal $[(b+c \cdot Q_I)/(b+c \cdot Q_E)][1/\mathcal{E}_I]$.

The scale inefficiency measure, in general, is obviously not equal to $1 - \mathcal{E}_I$. In fact, information about scale elasticity alone is inadequate to derive the inefficiency measure because of the integral role played by the output differential between the efficient and inefficient firms.

Alternatively, since F is determined by the characteristics of the cost function, we can solve for the level of scale inefficiency in terms of the cost parameters only. Solving for F in terms of c (the second derivative of $\ln C$) from equations (6) and (7), and substituting into equation (9) gives

$$I = e^{(.5/c)(1-\mathcal{E}_I)^2} - 1.0. \tag{10}$$

That is, scale inefficiency is a function of the first and second derivatives of the cost function with respect to output. Setting equation (10) equal to $1 - \mathcal{E}_I$ we can solve for c to see when the two measures are equal:

$$c^* = .5(\mathcal{E}_I^* - 1)^2 / \ln(2 - \mathcal{E}_I^*). \tag{11}$$

There is only one point on the cost function corresponding to $I = 1 - \mathcal{E}_I$. For $\mathcal{E}_I^* < 1.0$, elasticities greater than $\mathcal{E}_I^*$ produce inefficiencies less than $1 - \mathcal{E}_I^*$; and elasticities less than $\mathcal{E}_I^*$ produce inefficiencies greater than $1 - \mathcal{E}_I^*$. Similar conjectures can be made for $\mathcal{E}_I^* > 1.0$. Figure 3 presents the relationship between scale inefficiency and elasticity for three different values of the second derivative.[3]

[3]The middle relationship corresponds to the empirical results presented in tables 1 through 3 and discussed later in the text.

2 Supporting evidence

Having shown that scale efficiency and elasticity are distinct concepts, we next present evidence of differences in the two measures. A translog cost function, equation (1), with traditional parameter restrictions was estimated using 1987 data for 164 banks which were holding company members and were ranked in the top 500 U.S. banks over the 1972-1987 period. Exogenous variables include holding company affiliation and the number of branches. The variable definitions, estimation results, and properties of the estimates are presented in Table 1.[4] Scale elasticities for output quartiles are presented in Table 2. The estimated relationship is a well behaved cost function having all the desirable properties, and similar scale estimates to that found in the bank cost literature, suggesting that our findings are not driven by unique cost characteristics.[5] However, more is required to evaluate scale inefficiency. Table 3 presents point scale elasticities for various observations. Viewing specific data points obviously reveals detail not available in the calculations based on output quartiles. Significant economies of scale are found

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[4]Use of an aggregate output measure can be criticized for incorrectly specifying bank output. This can result in biases toward finding greater scale advantages. However, our purpose is to illustrate the difference between the two concepts; not to accurately capture the intricacies of the bank production process or to advance the bank cost literature. The aggregate output measure simplifies the model and in no way distorts the distinction between scale elasticity and scale inefficiency. However, multiproduct production does make the analysis of scale economies more complex since the product mix can vary with bank size. The ray scale measure, $\sum_i \partial \ln C / \partial \ln Q_i$, assumes banks expand along an output ray with product-mix held constant. While this enables us to use the formulas developed here to analyze scale inefficiency by using the ray scale measure and summing across outputs to obtain the second derivative, this is obviously a simplification and brings into question the usefulness of the ray scale measure in evaluating output expansion to levels very far removed from the point at which it is evaluated. For an analysis of scale inefficiency in a multiproduct framework, see Berger (1991).

[5]All regularity conditions are satisfied: positivity and homogeneity by model construction, monotonicity by having all predicted factor shares positive, and concavity by having factor shares range between four and 96 percent (well within our predicted range). See Evanoff, Israilevich, and Merris (1990), particularly footnote 10.

for the smaller banks, and diseconomies set in at approximately $3.3 billion in output. From equation (10), calculated values for scale inefficiency for these same observations are presented in column 4. As implied by equation (11), scale inefficiency is greater than $1 - \mathcal{E}_I$ for some observations, and less for others.

As further evidence of the distinction between the two concepts, Table 4 presents the findings of a number of bank cost studies, and presents estimates of scale inefficiency based on the assumptions listed in the note to the table. The distinction between the two measures is more pronounced in some of these studies. This occurs because of the sample range and particular cost function characteristics. The findings again suggest that inefficiency for specific firms can be substantially greater than that indicated by the elasticity measure, and substantially greater than that typically referred to in the bank cost literature. Even minor deviations from a value of one for the scale elasticity measure can be associated with significant inefficiency. Again, the elasticity measure is calculated based on incremental changes in output. To generate scale efficiency measures, the required changes in output shown in tables 3 and 4 are *not* incremental.

3 Concluding comments

Given the relatively large levels of scale inefficiency found in these banking studies, one is compelled to ask: Why have the inefficient firms survived? The most standard reason given for continued inefficient production is the lack of competitive forces; i.e., regulatory induced market protection. This has been given as the cause for both input and output induced inefficiency; see Evanoff and Israilevich (1991). However, given the changing regulatory environment, this protection appears to be eroding. Certain banks may not be able to survive in the future unless changes are made to produce significant efficiency gains.

Finally, what do the results imply about the propensity for merger activity in banking? The evidence suggests that, for certain banks, there are significant scale efficiency gains to be achieved by growing via internal means or by merger. The merger data also suggests that gains are being realized in the vast majority of mergers. Rhoades (1985) found that between 1960-83, over 93 percent of acquired banks had assets less than $100 million. Nearly all bank cost studies find scale advantages up to this size. Additionally, equation (9) indicates that efficiency gains increase as the difference between the output levels of the efficient and inefficient firms increase, i.e., F. Viewing bank merger data for the second quarter of 1991, the largest size differential between the acquiring and acquired firm, was 651; i.e., the acquirer was 651 times the size of the acquiree. The mean differential was 69; see Matthews (1991). Thus, the size differentials in bank acquisitions appear to be quite large suggesting significant potential gains.[6]

The efficiency gains, however, may not be realized by the larger banks which recently have been so aggressive in pursuing acquisitions. In fact, the larger banks in most cost studies exhibit inefficiency resulting from diseconomies of scale. Instead, it is the elimination of small, inefficient banks, which will improve industry inefficiency. For example, assume a small, scale

[6]It has been argued that the scale elasticity measure may fairly accurately indicate the marginal gains from changes in bank size that *actually* occur [Berger and Humphrey (1991)]. This may or may not be true. However, if the merger data discussed here is representative of the population of mergers, it casts doubt on the statement. If one is interested in evaluating scale efficiency, the direct measure (of which the scale elasticity measure is one component) would appear to be a more appropriate measure. It should also be emphasized that the efficiency gains we are discussing are those resulting from altering the size of the inefficient firm to achieve an appropriate scale of production, e.g., by merger. One could also view the gains of the acquiring firm in a merger or the net gains resulting from gains (losses) by both parties.

inefficient bank is acquired by a larger, efficient bank. Assume also that the acquisition leaves the scale elasticity for the larger firm unchanged (i.e., a flat average cost relationship exists over the output range considered). The output of the acquired firm will now be produced more efficiently. The acquiring firm, however, achieves no efficiency gains. Nevertheless, ceteris paribus, regulators and society should view these acquisitions favorably since they increase industry efficiency by eliminating inefficient firms.[7]

It should be emphasized that much of the gains of the magnitude discussed in the tables are based on the nation's smallest banks combining to achieve efficient size. While the structure of the industry is such that this is possible, it would require much greater consolidation than that typically projected for the industry. Less consolidation, or mergers between banks which are each producing near minimum efficient scale would produce fewer gains. However, as we have shown here, none of these potential gains can be detected by simply evaluating the scale elasticity measure.

[7]The public policy implications carry a number of caveats. It is assumed, for example, that the cost relationship and outputs are properly modeled in these studies. However, these are the same caveats which have existed in the past when cost studies were used to make public policy recommendations based on the belief that no scale inefficiency existed. It may be that analysis of balance sheet information does not allow the researcher to capture the true characteristics of bank production. It is also possible that the gains from scale may be partially offset by other factors. The average cost across various size groups of banks have been shown to be remarkably similar; see Humphrey (1990). In evaluating perspective acquisitions, the parties involved and regulators obviously need to consider more than scale advantages alone.

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Appendix- Derivation of the scale inefficiency measure

Reproducing some of the earlier equations:

$$\ln C = a + b(\ln Q) + .5c(\ln Q)^2 \tag{12}$$

$$\ln C_I = a + b(\ln Q_I) + .5c(\ln Q_I)^2 = a, \tag{13}$$

$$\mathcal{E}_I = \partial \ln C_I / \partial \ln Q_I = b. \tag{14}$$

$$\ln C_E = a + b \ln(FQ_I) + .5c(\ln FQ_I)^2, \tag{15}$$

$$\mathcal{E}_E = \partial \ln C_E / \partial \ln(FQ_I) = b + c \ln(FQ_I) = 1.0. \tag{16}$$

Rearranging equation (16), and realizing $Q_I = 1.0$, gives,

$$c = (1 - b)/\ln(F). \tag{17}$$

Substituting (17) into (15) gives,

$$\ln C_E = a + .5(1 + b)\ln(F). \tag{18}$$

To solve for scale inefficiency, i.e.,

$$I = [F(C_I)/C_E] - 1.0 \tag{19}$$

$$\ln(C_I/C_E) = \ln C_I - \ln C_E = -.5(1 + b)\ln(F) \tag{20}$$

and

$$I = F(e^{-.5(1+b)\ln(F)}) - 1.0 \tag{21}$$

and

$$I = F(F^{-.5(1+b)}) - 1.0 = F^{-.5(1-b)} - 1.0, \tag{22}$$

or, more general,

$$I = F^{-.5(1-\mathcal{E}_I)} - 1.0 \tag{23}$$

which is equation (9).

To generate equation (10), from equation (17)

$$\ln F = (1-b)/c. \tag{24}$$

Substituting (24) into (23) gives

$$I = e^{[(1-b)/c]^{.5(1-b)}} - 1.0 = e^{(.5/c)(1-\mathcal{E}_I)^2} - 1.0 \tag{25}$$

which is equation (10).

Table 1: Bank cost estimates

Coefficients	Model estimates
α_0	10.045 (1.7)
β_L	0.561 (8.8)
β_Q	-1.105 (1.4)
β_{QQ}	0.147 (2.6)
β_B	0.626 (1.7)
β_{BB}	0.078 (2.2)
β_H	-0.950 (0.6)
γ_{LL}	0.037 (5.2)
γ_{LQ}	0.003 (0.8)
γ_{LB}	0.000 (0.1)
γ_{LH}	0.020 (1.5)
θ_{QB}	-0.051 (1.8)
θ_{QH}	0.078 (0.6)
θ_{BH}	-0.081 (1.0)

Costs are defined as the sum of expenditures on labor and capital; output as the dollar value of total loans and securities; a BLS wage index was used for the price of labor and, as a proxy for the cost of physical capital, the ratio of additions to plant, equipment, furniture, and physical premises to the book value of bank premises, furniture and physical equipment was calculated; wage data were from BLS reports and the remaining data information was taken from Call Reports. All data are for 1987. For additional details see Evanoff, Israilevich, and Merris (1990).
Absolute t-values are in parenthesis.

Table 2: Scale Elasticity Estimates

Output quartile	Output (billions)	Mean $\mathcal{E}$
1	< 1.2	.85
2	1.2 - 2.5	.92
3	2.5 - 5.3	.99
4	> 5.3	1.13
entire sample		.98

$\mathcal{E}$ denotes scale elasticity.

Table 3: Scale Elasticity and Efficiency Estimates

Output($ billions)	$\mathcal{E}_I$	$(1 - \mathcal{E}_I) \times 100$	$I_I(\%)$
.47	.72	28	32.0
.52	.73	27	28.0
.80	.80	20	14.4
1.72	.90	10	3.4
2.27	.95	5	.9
3.02	.99	1	.04
3.26	1.00	0	0
3.49	1.01	1	.03
4.61	1.05	5	.9
6.55	1.10	10	4.0
13.00	1.20	20	15.0
30.00	1.32	32	42.0
mean			8.7

$\mathcal{E}_I$ denotes scale elasticity and I_I denotes scale inefficiency.

Table 4

Calculated scale inefficiencies from banking studies

Author	Scale elasticity*	Scale inefficiency
Berger and Humphrey (1991)[a]	.98	4.2%[b]
	.92	11.4%[u]
	.91	19.2%[b]
	.84	27.7%[u]
Clark (1984)[c]	.96	18.3%
Gilligan, Smirlock and Marshall (1984)	.93	6.9%[u]
	.94	5.9%[b]
Hunter, Timme, and Yang (1990)	.86	26.6%
Lawrence and Shay (1986)	.92	5.5%
Noulas, et al. (1990)	.97	2.1%
Shaffer (1984)[d]	.94	12.2%
Shaffer (1988)[d]	.95	10.0%

Note: The reported inefficiencies were derived using equation (9) assuming prices, exogenous variables, and product mix were constant across banks and that the cost representation could be approximated as discussed in equations (1)-(7). The inefficiency presented is that resulting from production in the range of increasing returns to scale. The same methodology could be used to calculate inefficiency resulting from production in the range of decreasing returns. In the studies considered, findings for alternative specifications are frequently presented. When this occurred the results reported here are based on findings for the most recent year, on earning assets as the output measure, and on the specification using the intermediation approach. Scale measures are also typically reported for various size ranges. Unless noted, the scale elasticity is that for the smallest size group, and the values for F were derive directly from the studies as the most inefficient bank, or the mean bank in the most inefficient size group, relative to the bank or group where scale economies were exhausted. The F values used were (in order presented) 60, 15, 50, 20, 31.25, 60.7, 6.7, 6.7, 10, 4, 14.2, and 10. More details are available from the authors. By holding product mix constant we restrict the cost savings to scale effects only; precluding any savings resulting from altering the mix. This implicitly assumes either that the mix is actually invariant over the banks considered or that the scale efficient bank analyzed is equal in size to the scale efficient bank observed in the data. Given the assumptions employed and the relatively broad size categories reported in the studies considered, the reported inefficiencies should be considered rough approximations. See Evanoff and Israilevich (1991) for additional information.

*For the inefficient firm.

[a]The first two calculated values are for the "thick" frontier estimates and the latter two for the conventional cost estimates.

[b]Denotes branch banks.

[c]The scale elasticity for the "efficient" bank was .9637 since scale advantages were not exhausted in the data sample. The calculated inefficiency would be larger if we extrapolated outside of the sample.

[d]The inefficiency measure is biased downward because data limitations necessitated using an "inefficient" size bank which was not the most inefficient in the sample.

[u]Denotes unit banks.

Figure 1
Cost representations with different scale elasticities and equivalent scale inefficiencies

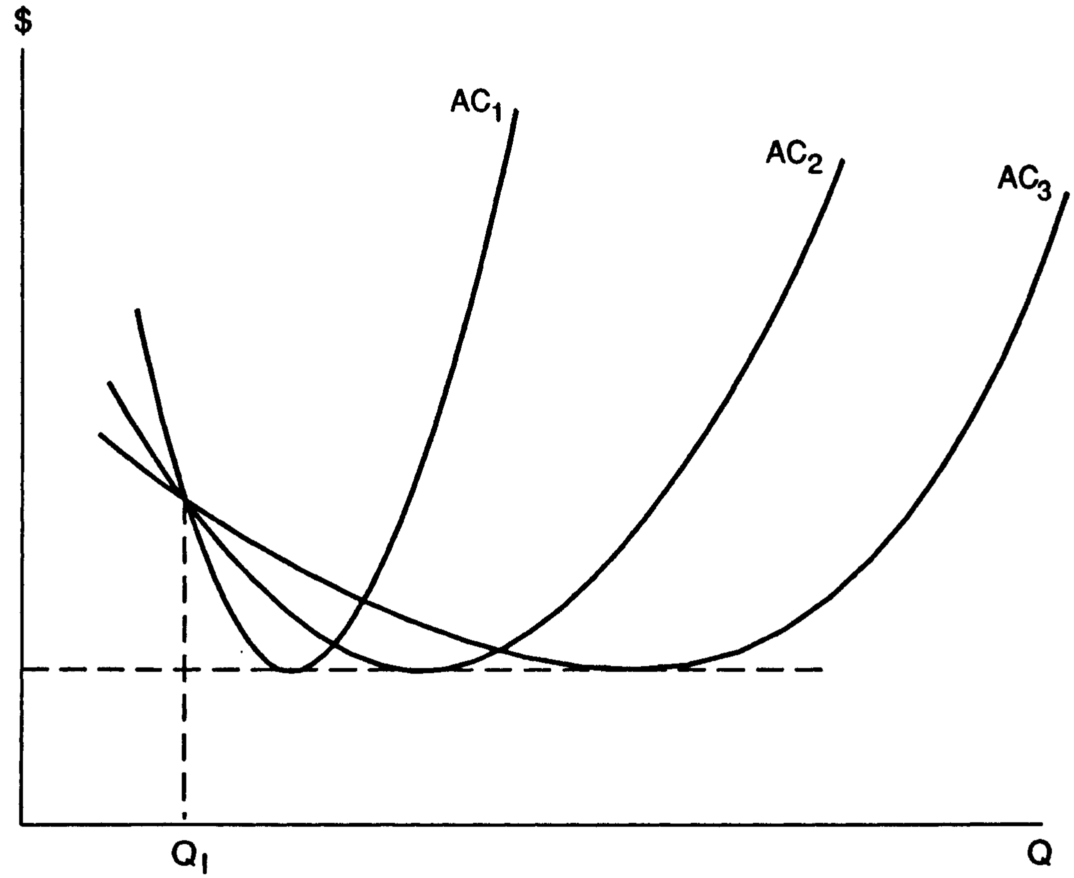

Figure 2
Cost representation with equal scale elasticities and different scale inefficiencies

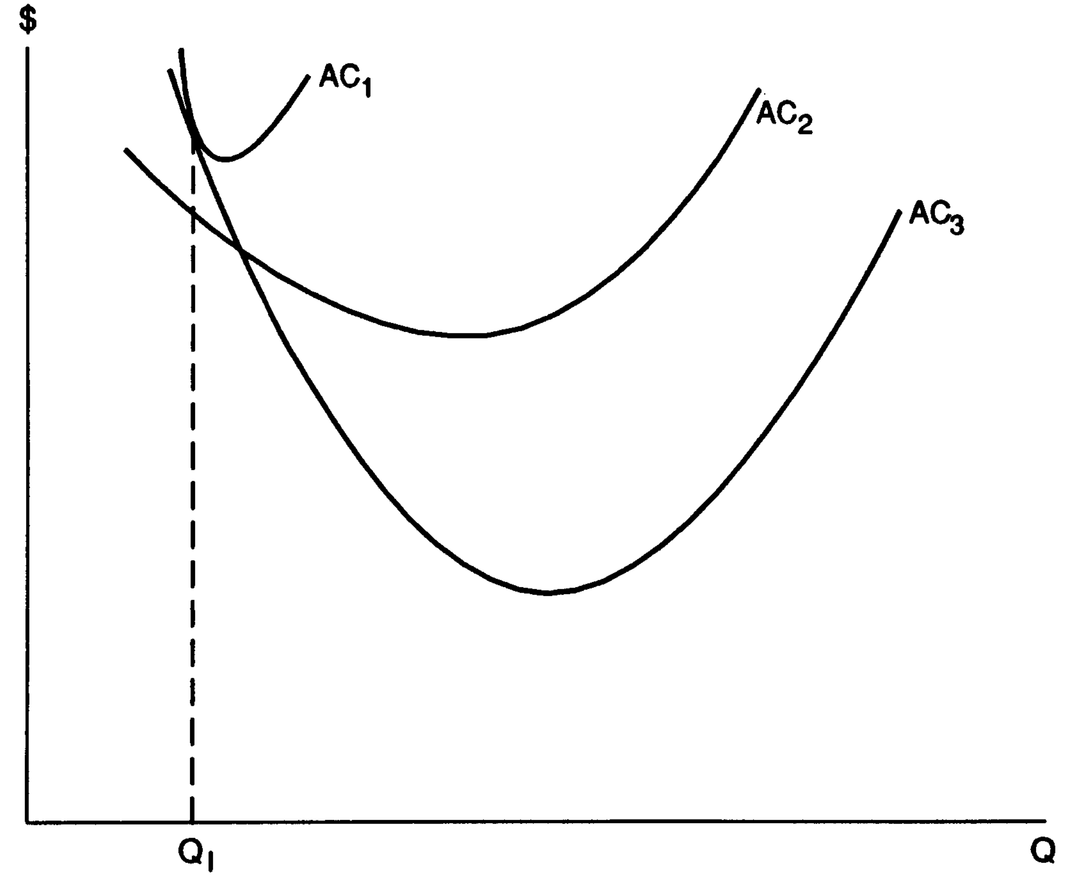

Figure 3: The Relationship Between Scale Inefficiency and Scale Elasticity