# USING NOISY INDICATORS TO MEASURE POTENTIAL OUTPUT

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# Using Noisy Indicators to Measure Potential Output

Kenneth N. Kuttner*

## Introduction

Potential output, like the real interest rate and the natural rate of unemployment, is one of the key unobservable economic variables of macroeconomics and monetary policy. While there is little agreement on which observable quantity best substitutes for potential output, most economists would endow it with a variety of distinct, but not mutually exclusive, attributes. Arthur Okun's original definition emphasizes inflation. According to this interpretation, potential output is the level of production that does not add to inflationary pressures. A second characterization is as the sustainable level of output — that is, a level consistent with stable future rates of output growth. A third corresponds to the full-employment level of production, where unemployment is equal to the natural rate.[1]

This paper synthesizes these definitions by modeling real GNP growth, inflation, and the unemployment rate as three distinct indicators of a single underlying unobservable variable, which we call potential output or potential GNP. The resulting model is an example of a dynamic multiple indicator specification, which optimally combines data from these imperfect sources in an estimate of the underlying level of potential output. [2]

This multiple-indicator approach to measuring potential output has two chief advantages relative to more traditional measures. The first is that by incorporating a number of alternative definitions of potential, it constructs a best-fit measure based on a variety of indicators. Such a composite promises to be more precise and reliable than measures based on only a single proxy variable.

A second advantage is that this technique delivers natural measures of the standard error associated with the estimates of potential output. Although Okun himself acknowledged that it was "at best an uncertain estimate, and not a firm, precise measure," the estimates in this paper are the first to include a set of confidence

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bounds. Some gauge of the precision of the estimates are essential to policymakers acting on the basis of a measure of the economy's potential.

**A brief history of potential output**

The origins of the potential output idea can be traced to Arthur Okun's work from the 1960s. He viewed it as essentially an aggregate supply measure, or an index of productive capacity, describing it as "the maximum production without inflationary pressure, ...or more precisely... the point of balance between more output and greater stability."[3] Deviations of output from potential were attributed to an inappropriate level of aggregate demand. While his definition of potential output rests on its relationship to inflation, as an empirical matter, Okun used the unemployment rate as the sole indicator of the gap between output $X$ and potential $X^*$, according to the equation that became known as "Okun's Law":

$$X^* = X\,[1 + 0.032\,(U - U^*)],$$

where the natural rate of unemployment, $U^*$, was assumed to be a constant 4 percent.

Until the 1970s, a simple log-linear trend served as an excellent approximation to the potential output series derived from Okun's Law with a time-invariant natural rate. (In fact, Okun himself recommended the trend method, preferring to ignore the "wiggles and jiggles" in the measure introduced by random fluctuations in the unemployment rate.) After the supply shocks of the 1970s, however, economists interested in the relationship between aggregate demand and inflation recognized the importance of changes in the growth rate of potential output, and replaced the linear trend with some time-varying measure of potential, as in Gordon (1975, 1977).

Most traditional measures of potential output have employed one of three alternative techniques in their construction: segmented trends, adjustments to Okun's Law, or an aggregate production function. The segmented trend group includes some of the more familiar measures: the high-employment and mid-expansion series of the Bureau of Economic Analysis, for example, which use business cycle dates as the basis for the segments' endpoints. Other examples are the Federal Reserve Board's new potential GNP series,[4] and the "trend output" series in Gordon (1990a, 1990b).

The best example of the modern approach to Okun's Law is Clark (1983), who suggests using the smoothed difference between the observed unemployment and a time-varying natural rate as the basis for potential output. Those who have proposed aggregate production function measures include Perry (1977), Clark (1979),



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Perloff and Wachter (1979), and (indirectly) Boschen and Mills (1990). Among these studies, the Perloff and Wachter paper deserves note; while based on an aggregate production function, their measure is unique for its use of inflation data as an indicator of an unobserved time-varying production function parameter.

The technique described in this paper bears a resemblance to that of Perloff and Wachter, in that it uses the inflation rate to extract information on an unobservable. However, here potential output is itself the unobservable of interest, and the latent-variable econometric framework explicitly addresses the statistical noise associated with its indicators. As decomposition of aggregate output fluctuations into persistent and transitory components, it is also related to the unobserved component model of Watson (1986), and the structural VARs of Blanchard and Quah (1989) and Galí (1989).

# Potential output as an unobserved stochastic trend

Constructing a meaningful potential output series first requires specifying the form of its time variation. An attractive alternative to the traditional segmented-trend method is to model potential as a stochastic trend, along the lines suggested by Stock and Watson (1988). Specifically, (log) potential output, $x_t^*$, is assumed to follow an integrated stochastic process — a random walk with drift:

$$\Delta x_t^* = \mu^* + \varepsilon_t. \qquad (1)$$

This specification allows its growth rate to depart from its long-run growth rate $\mu^*$ over the short run, generating persistent deviations from a deterministic linear trend.

Measured real GNP can now be modeled as an error-correction equation in which (log) real output, $x$, reverts to potential over time:

$$\delta^x(L)\Delta x_t = \mu^x + \theta(x_t^* - x_t) + \lambda^x(L)\eta_t^x. \qquad (2)$$

The $\delta^x(L)$ and $\lambda^x(L)$ lag polynomials and the coefficient $\theta$ are to be estimated; the constant term $\mu^x$ is set equal to $\delta^x(1)\mu^*$ to ensure that potential and measured output grow at the same rate on average. The $\eta_t^x$ innovation is uncorrelated with $\varepsilon_t$ by assumption.

In the context of the unit roots literature, (1) and (2) imply that potential and measured real output are I(1) processes, while their difference — the "output gap" — is stationary. Despite the fact that $x^*$ is unobservable, the interpretation of error-correction mechanism is straightforward: when output exceeds potential, real GNP



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growth is expected to fall below its mean. Conversely, output below potential implies higher-than-average future growth rates. The disturbance term $\eta_t^x$ induces purely transitory deviations in output from potential.[5]

Viewed as a latent variable model, (2) says that $\Delta x_t$ (real GNP growth) may be used as an indicator of the unobserved $x_t^*$. Thus, with the appropriate restrictions on the dynamics of the error-correction equation, the two equations may be estimated, and an estimate of potential output may be extracted. This is particularly easy to verify in the one-indicator case, as the two equations may be rearranged to express $\Delta x_t$ in canonical form[6] as an ARMA process with a composite error term:

$$[\delta^x(L)\Delta + \theta]\Delta x_t = \theta\varepsilon_t + \Delta\lambda^x(L)\eta_t^x.$$

Therefore, while the model's spectral density is that of an ARMA, estimating (1) and (2) parameterizes the spectrum of $\Delta x$ in terms of two distinct shocks; innovations to the extracted potential output "signal" correspond to the low-frequency portion of the spectrum of real GNP growth. Further discussion of the models' spectral properties appears in a subsequent section of the paper.

The real payoff to the latent variable methodology comes from the use of additional variables as indicators of the level of potential output. This improves the precision of the estimates, and it adds to the economic interpretation of the model, associating the unobserved $x^*$ with alternative descriptions of potential output.

We begin by explicitly linking potential output to the rate of inflation by way of a simple dynamic aggregate supply relation:

$$\delta^\pi(L)\Delta\pi_t = \alpha(L)(x_t - x_t^*) + \lambda^\pi(L)\eta_t^\pi + \gamma v_t, \tag{3}$$

where $\Delta\pi_t$ is the change in the inflation rate, the vector $v_t$ includes exogenous determinants of inflation not related to the current state of aggregate demand, and $\eta^\pi$ is a disturbance term. Specification in terms of the *change* in inflation is appropriate given the apparent non-stationarity of the inflation rate over the 1960–90 sample period. It also means that an output gap equal to zero is consistent with any constant rate of inflation, rather than zero inflation; in other words, potential GNP corresponds to the "non-accelerating-inflation" (or, more precisely, the non-*increasing*-inflation) level of output. Excluding $\Delta x_t$ from direct inclusion in the equation for $\Delta\pi_t$ imposes a factor structure on the co-movements of output and inflation growth; all covariation between the two is assumed to come from the common factor, $(x_t^* - x_t)$.

With the addition of inflation as a second indicator of the unobservable level of potential output, equations 2–4 form what is referred to as a dynamic multiple indicator model.[7] The specifications discussed in the following section of the paper



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are one-indicator output-only and two-indicator output-inflation versions. A subsequent section of the paper introduces the unemployment rate as a third indicator of potential.

## Estimating the model

For the purposes of estimation, the indicator equations (2) and (3) need to be restricted; $\delta^x(L)$ is taken to be a second-order lag polynomial, while the $\lambda^x(L)$ and $\lambda^\pi(L)$ moving-average polynomials are both assumed to be first-order. The $\delta^\pi(L)$ polynomial is set to unity, so that inflation changes are entirely a function of the output gap, plus a moving-average error term. These restrictions are consistent with (unreported) Wald tests of more profligate specifications.

Next, setting $\alpha(L)$ equal to $\alpha_1 L + \alpha_2 L^2$ specifies inflation rate changes as a function of two lags of the output gap. Finally, one exogenous variable is chosen for inclusion in $v_t$: the lagged difference between the growth rates of M2 and nominal income, or the opposite of the differenced velocity of M2. The idea, which is consistent with an error-correction mechanism between the growth rates of M2 and nominal income, is that this difference represents the inflationary pressure from any monetary expansion not yet incorporated into nominal income.

One convenient way to estimate multiple indicator models like the one presented here is through the application of the Kalman Filter to its state-space representation,

$$z_t = Az_{t-1} + Be_t + Gw_t$$
$$y_t = Cz_t + Du_t + Hw_t,$$

where $z$, $y$ and $w$ are a vectors of state, observable and exogenous variables, respectively. The model in equations 1–3 can be expressed by including current and lagged values of $x^*$, $\eta^x$ and $\eta^\pi$ in the state vector. The observable $y_t$ obviously includes $\Delta\pi_t$ and $\Delta x_t$, while $w_t$ includes a vector of ones, lagged observables, and the monetary term discussed above.

As described in Harvey (1981, 1989), the Kalman filter computes the minimum mean square error forecast of the period $t$ state vector and its error variance, conditional on period $t-1$ information, according to the prediction equations ,

$$z_{t|t-1} = Az_{t-1|t-1} + Gw_{t-1}$$
$$\Omega_{t|t-1} = A\Omega_{t-1|t-1}A' + BB'.$$



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Conditional on period $t$ data, the filter extracts the optimal estimate of the state vector, and computes the period $t$ signal-extraction error, $\Omega_{t|t}$, by way of the updating equations ,

$$z_{t|t} = z_{t|t-1} + K_t\,[y_t - (Cz_{t|t-1} + Hw_{t-1})]$$
$$\Omega_{t|t} = \Omega_{t|t-1} - K_t C \Omega_{t|t-1}, \text{ where}$$
$$K_t = \Omega_{t|t-1} C'[C\Omega_{t|t-1} C' + DD'].$$

If the elements of the matrices $A$, $B$, $C$, $D$, $G$, and $H$ were known, extracting estimates of the unobserved $z_t$ and its variance would be trivial. As they are not known, the series of uncorrelated prediction errors generated by the Kalman filter can be used to evaluate a likelihood function under the assumption of normally distributed disturbances. Standard optimization routines[8] can compute maximum-likelihood estimates of the parameters.

The one additional piece of information required for estimation is the initial value of the state vector, $z_0$, and the variance associated with that value. The Bayesian interpretation is that $z_0$ embodies prior information on the value of the state vector at the beginning of the sample. The estimates are not generally sensitive to the choice of $z_0$: as of 1959:4, log potential GNP is assumed to equal the Federal Reserve Board's estimate of 7.428, with a standard error of 5%.[9] The initial values of the $\eta$ disturbances are set to zero, with standard errors of 3%.

The lack of consistent pre-1959 money stock data requires beginning the estimation in 1960; the sample ends in 1991:1, for a total of 125 observations. While the obvious measure of real output is standard NIPA real GNP series, the choice between alternative price series is less clear. Because it is less volatile and less subject to fluctuations in oil prices, the fixed-weight GNP deflator is used as the basis for the inflation rate.[10] Relative to the implicit deflator, the fixed-weight version is preferable as a better measure of pure price changes.[11]

### Parameter estimates

Maximum-likelihood estimates of the one- and two- indicator models appear in Table 1. The estimated $\theta$ and $\sigma_\varepsilon$ (respectively, the coefficient on the error-correction term and the standard deviation of the potential output innovation) generally support the idea of time-varying potential GNP. Although these two parameters are not individually significant in the single-indicator model (specification I), they are much more precisely estimated when the inflation rate is included as a second indicator in specification II; their asymptotic standard errors fall by one-half. As $\sigma_\varepsilon$



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is expressed in percentage terms, its specification II estimate of 0.73 corresponds to a quarterly potential GNP shock with a variance of roughly half a percent.

Both models yield estimates of $\mu^*$, the mean growth rate of potential output, close to 0.74% on a quarterly basis (2.96% annualized). The estimates of the standard deviation of the transitory $\eta^x$ shocks are both roughly 0.95, again in quarterly percentage terms. In contrast, the estimated lag polynomials $\delta^x$ and $\lambda^x$ change somewhat from specification I to II. However, these changes are small relative to the coefficients standard errors, and have little impact on the models' spectral density. Because specification I (except for the effect of the initial conditions) is essentially a reparameterized ARMA(3,2), its maximized likelihood is virtually equal to that of the estimated ARMA.

The estimate of aggregate supply relation in specification II produces no surprises. The $\alpha$ coefficients indicate a strong effect of aggregate demand on inflation. The estimates of $\alpha_1$ and $\alpha_2$ suggest a specification in terms of the first difference of the output gap. However, including the level of the output gap allows aggregate demand to contribute to some of the non-stationarity in the inflation rate, which the first-difference specification would not allow. Finally, $\gamma$, the coefficient on lagged adjusted M2 growth, appears as a significant additional determinant of inflation.

### Extensions to the basic model

The models discussed above used output growth and the inflation rate as indicators of the latent potential GNP. This section presents two extensions to the basic model, both of which include the unemployment rate as a third indicator variable.

#### *Including unemployment data*

The foundation for the unemployment indicator is a modified version of Okun's Law:

$$\delta^u(L)U_t = U_t^* + \phi(L)(x_t^* - x_t) + \xi_t \tag{4}$$

where $\phi(L)$ and $\delta^u(L)$ are lag polynomials specific to the unemployment equation, and $\xi_t$ is the disturbance term. The $U_t^*$ term represents the natural rate of unemployment.

There are two problems with using this equation as an indicator of the output gap. The first is the problem of time variation in the natural rate. While it was thought to be roughly constant at 4% in the 1960s, its apparent upward trend in the 1970s led to considerable research on time-variation in its structural component.[12] A related problem is the non-stationarity of the unemployment rate. Perhaps due to highly persistent changes in its structural component, the usual statistical tests are



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**Table 1**
Basic Estimation Results

| | Specification I | | Specification II | |
|---|---|---|---|---|
| | Estimate | Std Error | Estimate | Std Error |
| $\mu^*$ | 0.730 | 0.090 | 0.740 | 0.075 |
| $\sigma_\varepsilon$ | 0.553 | 0.618 | 0.728 | 0.278 |
| $\theta$ | 0.064 | 0.070 | 0.106 | 0.054 |
| $\delta_1^x$ | 0.276 | 0.442 | 0.435 | 0.314 |
| $\delta_2^x$ | 0.215 | 0.158 | 0.200 | 0.148 |
| $\lambda_1^x$ | –0.001 | 0.456 | –0.165 | 0.318 |
| $\sigma_x$ | 0.940 | 0.069 | 0.948 | 0.074 |
| $\alpha_1$ | ... | | 0.117 | 0.030 |
| $\alpha_2$ | ... | | –0.092 | 0.028 |
| $\lambda_1^\pi$ | ... | | –0.680 | 0.224 |
| $\gamma$ | ... | | 0.058 | 0.028 |
| $\sigma_\pi$ | ... | | 0.256 | 0.038 |
| $\overline{LLF}$ | 4.2103 | | 9.562 | |
| $SE_{\Delta x}$ | 0.900 | | 0.875 | |
| $SE_{\Delta \pi}$ | ... | | 0.296 | |

Specification I: single indicator

$$\Delta x_t^* = \mu^* + \varepsilon_t$$

$$\Delta x_t = (1 - \delta_1^x - \delta_2^x)\mu^* + \theta(x_t^* - x_t) + \delta_1^x \Delta x_{t-1} + \delta_2^x \Delta x_{t-2} + \eta_t^x + \lambda_1^x \eta_{t-1}^x$$

Specification II: adds inflation indicator

$$\Delta \pi_t = \alpha_1 (x_{t-1} - x_{t-1}^*) + \alpha_2 (x_{t-2} - x_{t-2}^*) + \gamma \Delta(\ln(M2) - \ln(GNP))_{t-1} + \eta_t^\pi + \lambda_1^\pi \eta_{t-1}^\pi$$



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unable to reject the null hypothesis of a unit root in the unemployment rate over the postwar sample.

One possible solution is to model the natural rate as an additional non-stationary latent variable, and specify the measured unemployment rate as a function of the output gap and the time-varying natural rate. However, because the focus of this paper is potential GNP rather than the natural rate of unemployment, an appealing alternative approach is simply to remove the unit root by first differencing.

This would be a perfect solution to the time-varying natural rate problem only if $U^*$ were a pure random walk, $U_t^* = U_{t-1}^* + \nu_t$, whose errors were uncorrelated with the stochastic components of the output gap. In this case, first differencing (4) yields:

$$\delta^u(L)\Delta U_t = \phi(L)\Delta(x_t^* - x_t) + \nu_t + \Delta\xi_t;$$

or, combining the $\nu_t$ and $\xi_t$ into a single MA(1) composite error term $\lambda^u(L)\eta_t^u$,

$$\delta^u(L)\Delta U_t = \phi(L)\Delta(x_t^* - x_t) + \lambda^u(L)\eta_t^u, \qquad (5)$$

where $\eta^u$ is uncorrelated with $\eta^x$ and $\varepsilon$ at all lags. In practice, these assumptions are rather dubious. While serial correlation in $\nu_t$ could be taken care of by expanding the $\lambda^u(L)$ polynomial, the literature on sectoral shifts suggests sources of nonzero correlation between the structural unemployment innovations, $\nu$, and the other disturbances, $\eta^x$ and $\varepsilon$, violating the dynamic factor assumptions. At best, one would hope that first-differencing would remove most low-frequency structural unemployment fluctuations. Adding the unemployment rate as a third indicator of the output gap, therefore, may introduce a tradeoff between bias and precision. While the additional indicator reduces the signal extraction error associated with estimated potential GNP, it also introduces the possibility that estimated fluctuations in the output gap may be contaminated by changes in structural unemployment.

***Including population growth***

A second extension involves including population as a determinant of potential output. The simplest way to do this is to subtract the logarithm of population from the log of potential output, $x_t^* - n_t \equiv \tilde{x}_t^*$, and express *per capita* potential output, $\tilde{x}_t^*$ as a stochastic trend:

$$\Delta\tilde{x}_t^* = \mu^* + \varepsilon_t, \qquad (6)$$

as before in equation 1. The three indicator equations 2, 3, and 5, remain the same, except the sum of the latent variable $\tilde{x}_t^*$ and the (exogenous) log of population now appear in place of $x_t^*$.



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**Table 2**
Extended Estimation Results

| | Specification III | | Specification IV | |
|---|---|---|---|---|
| | Estimate | Std Error | Estimate | Std Error |
| $\mu^*$ | 0.733 | 0.052 | 0.349 | 0.060 |
| $\sigma_\varepsilon$ | 0.505 | 0.134 | 0.545 | 0.164 |
| $\theta$ | 0.084 | 0.048 | 0.076 | 0.038 |
| $\delta_1^x$ | 0.411 | 0.356 | 0.494 | 0.273 |
| $\delta_2^x$ | 0.209 | 0.160 | 0.222 | 0.162 |
| $\lambda_1^x$ | −0.132 | 0.364 | −0.188 | 0.285 |
| $\sigma_x$ | 0.948 | 0.078 | 0.954 | 0.076 |
| $\alpha_1$ | 0.096 | 0.021 | 0.089 | 0.022 |
| $\alpha_2$ | −0.075 | 0.019 | −0.072 | 0.021 |
| $\lambda_1^\pi$ | −0.549 | 0.095 | −0.505 | 0.082 |
| $\gamma$ | 0.051 | 0.026 | 0.044 | 0.025 |
| $\sigma_\pi$ | 0.273 | 0.021 | 0.279 | 0.021 |
| $\phi_0$ | 21.11 | 2.17 | 21.27 | 2.13 |
| $\phi_1$ | 6.62 | 3.66 | 7.71 | 3.49 |
| $\delta_1^u$ | 0.338 | 0.106 | 0.300 | 0.103 |
| $\lambda_1^u$ | −0.283 | 0.198 | −0.351 | 0.293 |
| $\sigma_u$ | 0.170 | 0.026 | 0.157 | 0.039 |
| $\overline{LLF}$ | 10.648 | | 10.637 | |
| $SE_{\Delta x}$ | 0.846 | | 0.894 | |
| $SE_{\Delta \pi}$ | 0.291 | | 0.296 | |
| $SE_{\Delta U}$ | 0.207 | | 0.204 | |

Specification III: adds unemployment indicator

$$\Delta U_t = \phi_0 \Delta(x_t^* - x_t) + \phi_1 \Delta(x_{t-1}^* - x_{t-1}) + \delta_1^u \Delta U_{t-1} + \eta_t^u + \lambda_1^u \eta_{t-1}^u$$

Specification IV: potential output in per capita terms

$$x_t^* = n_t + \tilde{x}_t^* \text{ where } \Delta \tilde{x}_t^* = \mu^* + \varepsilon_t$$



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The results for these two models, designated specifications III and IV, appear in table 2. Adding the unemployment rate indicator has a relatively small impact on most of the parameter estimates. The most noticeable change is in the size of the shocks to potential output; specification III delivers an estimated $\sigma_\varepsilon$ of only 0.505, compared with 0.728 in specification II.

The specification IV results are similar to those from III. The main difference is that $\mu^*$ now represents the growth rate of per captia potential output, whose estimated annual growth rate is roughly 1.4%. Surprisingly, as measured by the standard error of the one-step-ahead forecasts, the output and inflation equations' fit declines slightly relative to specification III.

**Estimates of potential GNP**

Perhaps the most useful results of these models are the estimated paths of potential GNP and their standard errors. Extracting an estimate of the latent $x^*$ is straightforward; conditional on the maximum-likelihood parameter estimates, the Kalman filter can compute one-sided estimates of potential output and its signal extraction error, denoted $x_{t|t}$ and $\Omega_{t|t}$. Applying the Kalman smoothing algorithm yields analogous two-sided estimates, $x_{t|T}$ and $\Omega_{t|T}$.

However, these estimates of the signal extraction error neglect the uncertainty from the parameter estimates' variance. To compute this variance, we adopt the procedure proposed by Hamilton (1986). This involves decomposing the total variance of state vector element of interest, say $x^*_{t|T}$, into the signal extraction or "filter" uncertainty,

$$E\left\{(x_t^* - x^*_{t|T,\beta_0})^2 \mid Z_T, \beta_0\right\},$$

and the parameter uncertainty,

$$E_\beta\left\{(x^*_{t|T,\beta} - x^*_{t|T,\beta_0})^2 \mid Z_T\right\},$$

where $\beta_0$ is the estimated parameter vector, and $Z_T$ denotes the data through period $T$. The $\Omega_{t|T}$ from the Kalman smoother approximates the filter uncertainty. The parameter uncertainty is estimated via Monte-Carlo: an artificial sample of $\beta$s is drawn from a multivariate normal population, generating a sample of $x^*_{t|T}$ series, which is then used to compute the sample variance of each observation in the series. The same procedure applies to the one-sided estimates, $x^*_{t|t}$.

Figure 1 shows the two-sided estimate of the potential GNP series from specification II (the two-indicator model with inflation), along with the 5th and 95th percentiles of its empirical distribution (analogous to the 90% confidence interval).



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**Figure 1**

Estimated potential GNP from specification II, 2-sided

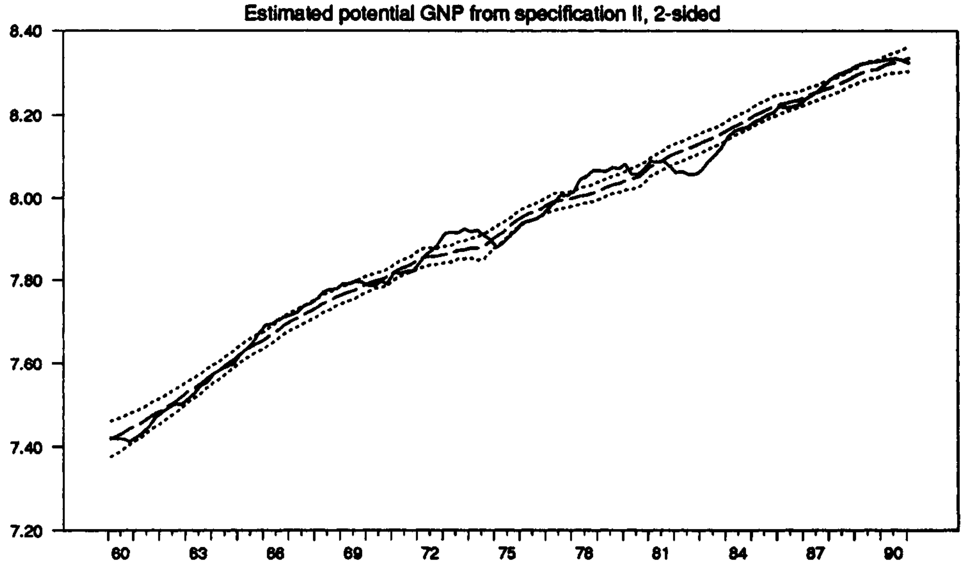
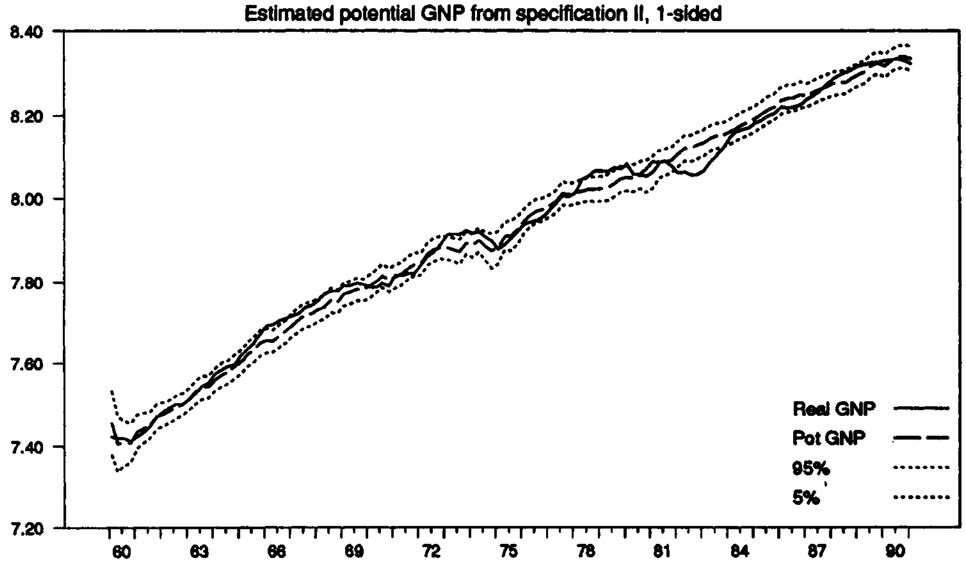



**Figure 2**

Estimated potential GNP from specification II, 1-sided



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## Figure 3

Estimated potential GNP from specification III, 1-sided

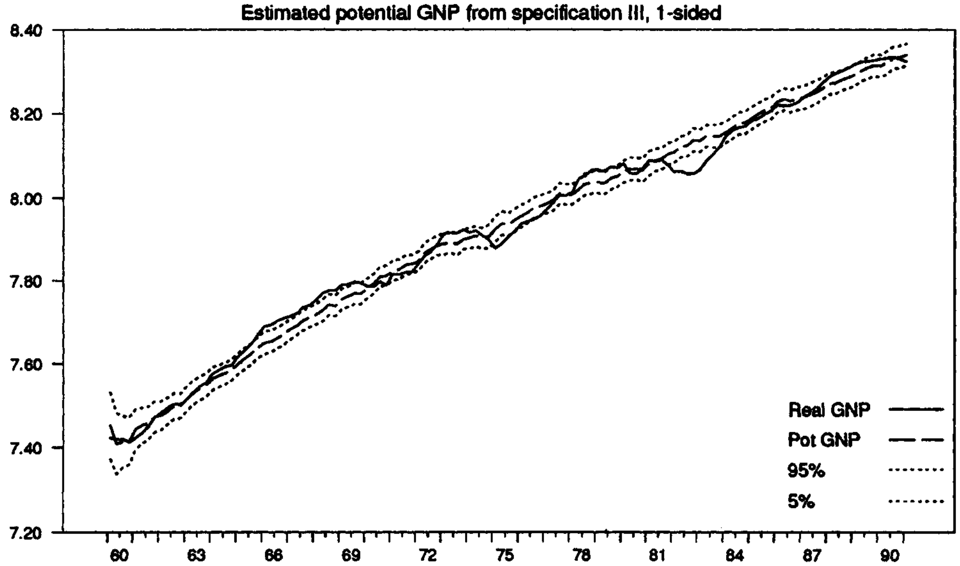




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**Table 3**

Sources of uncertainty in potential GNP

| | Specification | | | |
|---|---|---|---|---|
| | I | II | III | IV |
| One-sided | | | | |
| Filter variance | 7.68 | 2.07 | 1.74 | 2.08 |
| Parameter variance | 9.54 | 1.02 | 0.64 | 1.17 |
| Overall standard error | 4.07 | 1.75 | 1.54 | 1.80 |
| Two-sided | | | | |
| Filter variance | 4.44 | 1.03 | 0.95 | 1.12 |
| Parameter variance | 6.80 | 0.65 | 0.40 | 0.87 |
| Overall standard error | 3.30 | 1.29 | 1.16 | 1.41 |

The estimated parameter variances are based on 200 draws.

The reported averages exclude the first 8 quarters of the sample.

The solid line plots log real GNP. The local trend in potential output varies considerably over the sample; from a relatively rapid growth rate in the early 1960s, it begins to slow late in the decade, a tendency which continues throughout the 1970s. In the early 1980s, however, its growth rate picks up once again.

As measured by the two-indicator specification II, the output gap is statistically significant during only four episodes: the 1965–9, 1973–4 and 1978–9 expansions and the 1981–2 recession. The 1974–5 recession and the late-80s expansion are marginal episodes. Here, the deviations barely exceed the two-sided 90% confidence bounds, but not the one-sided bounds shown in figure 2. Figure 3 shows the one-sided estimate from specification III. Including the unemployment rate as a third indicator makes it possible to distinguish the 74–75 recession, although the late 80s remain within the 90% bounds.

Except for the fact that the parameters are estimated over the entire sample, the one-sided estimates correspond to the estimates that would have been available to policymakers in "real time." The fact that some of the deviations of output from potential are discernible in the two-sided but not in the one-sided estimates illustrates the point that discerning the appropriate course of monetary policy is frequently easier with the benefit of hindsight.



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Table 3 reports the average uncertainty associated with the estimates of potential GNP, including the separate contributions of filter and parameter uncertainty. The table demonstrates the significant reduction in the standard error obtained in going from a one- to a two-sided estimate; in the case of specification II, it shrinks by 30%. The table also shows the beneficial effect of adding additional indicators of the output gap. The reduction in uncertainty is particularly spectacular in going from specification I to II, in which the overall standard errors fall by over 60%. Including the unemployment rate as an additional indicator yields an additional improvement in the standard errors. Moving to the per capita specification, however, increases the standard errors, perhaps due to the deterioration in the model's fit in the output equation. Except in the case of the (somewhat overparameterized) specification I, signal extraction contributes more uncertainty to the estimates than the sampling variance of the parameter estimates.

**Spectral properties of the estimated models**

Another interesting way to view the results is from the frequency domain, examining the spectral density function of $\Delta x$ implicit in the specification. The contributions of the two shocks to the spectrum of the change in log real GNP can be computed from their transfer functions:

$$f_{\Delta x}(\omega) = \sigma_\varepsilon^2 \left| \frac{\theta}{\delta^x(e^{-i\omega})(1-e^{-i\omega})+\theta} \right|^2 + \sigma_x^2 \left| \frac{(1-e^{-i\omega})\lambda^x(e^{-i\omega})}{\delta^x(e^{-i\omega})(1-e^{-i\omega})+\theta} \right|^2 ;$$

because the $\varepsilon$ and $\eta^x$ shocks are uncorrelated, the spectral density of $\Delta x$ is just the sum of the two shocks' individual contributions. The spectral density from the two-indicator specification II appears in figure 4. Except for small differences in the size of the frequency-zero variance, the other specifications' spectra are virtually identical.

The shapes of the shocks' individual contributions reflect the error-correction structure of equation 2. The transitory $\eta^x$ shocks contribute nothing to the spectrum at frequency zero, while the persistent $\varepsilon$ shocks contribute spectral mass *only* near the zero frequency; by $\omega = 0.5$ (12.5 quarters), its contribution is virtually zero. The decomposition of the $k$-step-ahead forecast error variance, shown in table 4, is an alternative way to view the information in the spectral density function. While almost none of the one-quarter-ahead variance comes from the potential output shocks, that proportion rises sharply at horizons longer than 12 quarters. At 16 quarters, the $\varepsilon$ shock accounts for more than three-quarters of the variance.

As mentioned earlier, the univariate representation of $\Delta x_t$ implied by these potential output specifications is equivalent to an ARMA(3,2). An interesting restriction on these specifications involves imposing $\theta = 0$ (which also renders $\sigma_\varepsilon$ irrelevant).



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**Figure 4**
Spectrum of output growth, specification II

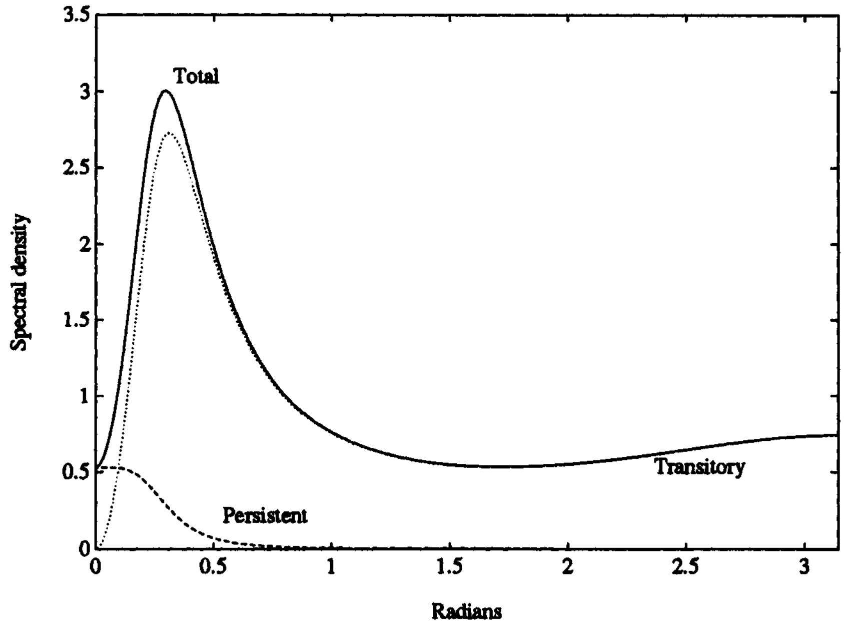


**Table 4**
Decomposition of forecast variance

| $k$ | $k$-step-ahead variance, % $\eta^x$ shocks | $\varepsilon$ shocks |
|---|---|---|
| 1 | 94 | 6 |
| 4 | 88 | 12 |
| 8 | 82 | 18 |
| 12 | 71 | 29 |
| 16 | 23 | 77 |



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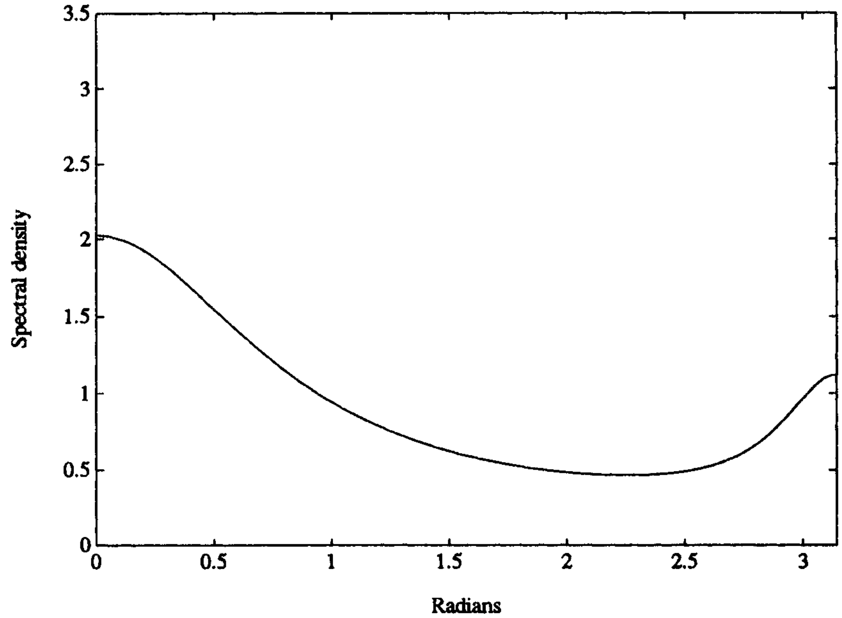
**Figure 5**

Spectrum of output growth, ARMA(2,1)




Doing so produces redundant unit roots in both the autoregressive and moving-average polynomials; the $(1 - L)$ factors cancel, and the model collapses to an ARMA(2,1). Imposing this restriction and estimating the restricted ARMA yields the following model: $(1 + 0.33L - 0.32L^2)\Delta x_t = (1 + 0.59L)e_t$.

The comparison between the restricted and unrestricted models reveals a lot about their low-frequency behavior. The restricted ARMA(2,1) spectrum, which appears in figure 5, differs significantly from the unrestricted model's — particularly near frequency zero, where the ARMA(2,1) spectral density function attains a peak of roughly 2.0. By contrast, the unrestricted model with its ARMA(3,2) representation allows for a much richer characterization of the low frequencies of $\Delta x$, an observation consistent with the Christiano and Eichenbaum (1990) analysis of the low-frequency properties of alternative ARMA specifications.

Examining the roots of the models' characteristic polynomials is another way to understand this behavior. The effect of moving from zero to positive values of $\theta$



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and $\sigma_\varepsilon$ is move the redundant unit MA root away from the unit circle. Likewise, a positive $\theta$ moves the AR root away from unity. The result is to create two *nearly* redundant roots. In specification II, for example, the largest MA root is –0.90, and the modulus of the largest AR root is 0.81.

## Some comparisons

### An Unobserved Components model

In his discussion of univariate detrending methods, Watson (1986) proposes decomposing difference-stationary time series as the sum of an I(1) component and a stationary autoregressive component. For real GNP, Watson uses the following Unobserved Components (UC) specification:

$$x_t = \tau_t + c_t \text{ where}$$
$$\tau_t = \mu + \tau_{t-1} + e_t^\tau \text{ and}$$
$$c_t = a_1 c_{t-1} + a_2 c_{t-2} + e_t^c ,$$

a random walk stochastic trend $\tau_t$, and a stationary AR(2) cyclical component, $c_t$. The estimated parameters of this model for the 1960:1–1991:1 sample period are: $\hat{\mu} = 0.721$, $\hat{a}_1 = 1.604$, $\hat{a}_2 = -0.647$, $\hat{\sigma}_\tau = 0.702$, and $\hat{\sigma}_c = 0.476$. The standard error is 0.8996.

As a decomposition in which the stochastic trend is modeled as a latent variable, the UC model has much the same flavor of the latent-variable specification proposed in this paper. Indeed, the spectrum of $\Delta x$ it implies, shown in figure 6, is almost identical to the that of the potential GNP model. The difference between them lies in the nature of the two shocks' contributions to the spectrum: in the case of the UC model, the shock to the trend $\tau$ contributes to the spectrum across *all* frequencies. In contrast, by virtue of the error-correction specification of the potential GNP model, the shock to the potential output trend contributes only at very low frequencies. The result is a somewhat smoother estimated trend, free of the higher-frequency movements that characterize the UC trend.

### The Hodrick-Prescott filter

Another interesting comparison is between the estimated potential output series and the trend component from the Hodrick-Prescott (HP) filter, which is essentially a sophisticated two-sided moving-average.[13] Figure 7 plots the two-sided specification II estimate of potential along with the HP trend, with $\lambda$ set to 1600. For reference, figure 7 also includes a deterministic linear trend. In general appearance, the two series are comparable; the smoothness imposed by the stochastic trend specification for $x^*$ is similar to the smoothness delivered by $\lambda = 1600$.



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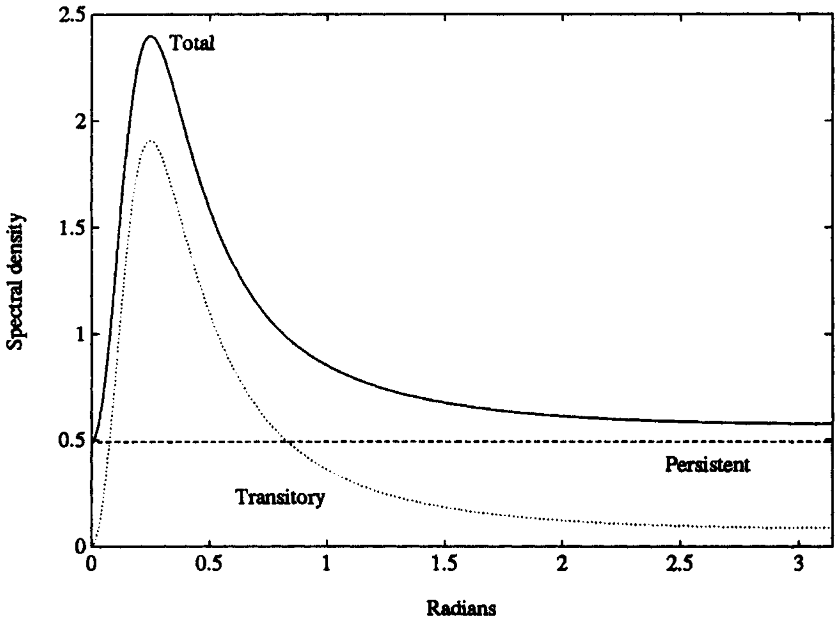
**Figure 6**

Spectrum of output growth, unobserved components model




One obvious distinction between the two procedures is that without an underlying statistical model, the HP filter cannot deliver the standard errors computed by the Kalman filter.

The series themselves also reflect a number of important discrepancies. Two of these occur in 1973 and 1979, when potential output growth takes a dip not reflected in the HP trend. The estimate of potential apparently associates the increasing rate of inflation over those periods with a depressed level of potential output. Or, to attach a more structural interpretation, the potential GNP estimate successfully picks out the 1973 and 1979 OPEC-induced supply shocks.

Another important discrepancy occurs in 1982–4, where the HP trend is depressed relative to estimated potential. Because the HP technique simply involves smoothing the observed output series, some portion of the 1981–2 recession appears in the filtered trend component. By contrast, the potential GNP procedure attributes



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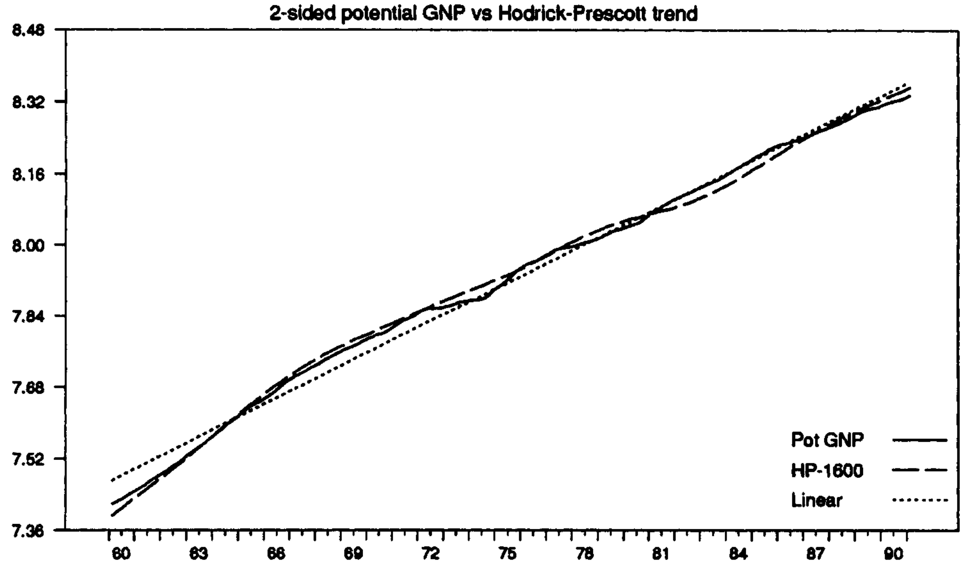
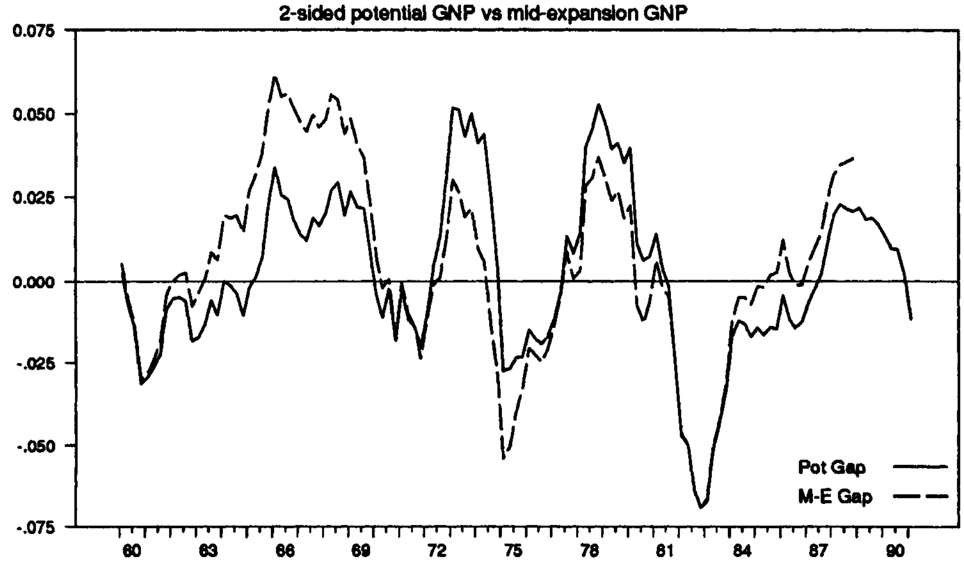
## Figure 7

2-sided potential GNP vs Hodrick-Prescott trend




## Figure 8

2-sided potential GNP vs mid-expansion GNP



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virtually all of the 1981–2 decline in output to a widening output gap — in other words, a pure demand shock.

### Mid-expansion trend GNP

A final instructive comparison is between this estimate of potential GNP and the more traditional mid-expansion measure constructed by the Bureau of Economic Analysis, which, because it uses *ex post* business cycle dating information, can also be thought of as a two-sided estimate. Figure 8 compares the "output gaps" implied by the two measures, where the gap is defined as actual GNP less potential.

Again, the two series differ significantly during a number of important episodes. Like the HP trend, the mid-expansion measure misses the 1973 and 1979 dips in potential, implying smaller output gaps than those based on the specification II measure. The discrepancy goes the other way in the late 60s. During this period, the mid-expansion gap is about twice the size of the one based on the specification II estimate. Only during the 1981–2 recession do the two measures coincide.

## Conclusions

The goal of this paper has been to apply new econometric techniques to the estimation of a key unobservable macroeconomic quantity, potential output. The paper proposed using multiple indicators — output growth, inflation, and unemployment — to extract information on the level of potential real GNP, which was modeled as a latent stochastic trend.

While consistent with Okun's original conception of potential output, these new estimates embody two important advantages relative to traditional measures. First, they distill disparate sources of information into a single estimate of potential output, promising to improve its reliability and precision. Second, they explicitly recognize the uncertainty involved in the estimation and provide a way of gauging that uncertainty.

Another way to interpret these results is as a decomposition of real GNP fluctuations into potential output shocks, and deviations from output around potential (traditionally, " supply" and "demand" shocks). The results presented here are consistent with the idea that shocks to potential are a very small source of output variance at horizons shorter than three years. However, over four-year and longer horizons, shocks to potential output are responsible for most of the variance in real GNP.



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One possible extension to the method outlined above is to expand the supply side of the potential output specification, which was modeled as a stochatic trend or as a simple function of population. Augmenting that specification to include measures of factor inputs is a promising area for future work.



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# Footnotes

1. Some economists would dissent from this characterization. On one hand, classical economists criticize the potential output concept for its lack of a general equilibrium foundation. Plosser and Schwert (1979) make this case. On the other side, DeLong and Summers (1988) argue that potential output should correspond to the highest feasible level of production.

2. Since the 1970s, latent-variable models have found extensive application. A partial list includes Aigner *et al.* (1986), Engle and Watson (1985), Hamilton (1985), and Norrbin and Schlagenhauf (1988). A comprehensive guide to their application to econometrics is Harvey (1989).

3. Okun (1970), pp. 132–3.

4. Although this series remains unpublished, it is widely used in Federal Reserve Board research, such as Hallman Porter and Small (1989).

5. As the $\varepsilon$ shocks produce persistent changes in the sustainable level of output, while the $\eta^x$ shocks produce only transitory fluctuations around that level, one might interpret them as "supply" and "demand" shocks respectively, as in Blanchard and Quah (1989). While appealing, this interpretation is not essential to the model.

6. See Nerlove, Grether and Carvalho (1979).

7. A useful taxonomy of latent variable econometric models appears in Watson and Engle (1983).

8. These results use the BFGS and BHHH algorithms from the Gauss MAXLIK procedure.

9. In the single-indicator model, real GNP changes alone provide very little information on the level of potential output. Therefore, when a very loose prior on $x_0^*$ is specified, the procedure manages to fit a trend-stationary model by raising the initial value of log potential GNP upwards, to over 7.5. Tying down a more plausible initial condition with only a 2% standard error on $x_0^*$ forces the procedure to fit a difference stationary model with nonzero $\varepsilon$ variance.

10. In practice, results based on the CPI are very similar to those obtained with the fixed-weight deflator when oil price changes are appropriately controlled for.



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11. Statistical releases from the Department of Commerce caution against computing an inflation rate from the implicit deflator: "[Because] the prices are weighted by the composition of GNP in each period, ...the Implicit Price Deflator reflects not only changes in prices but also changes in the composition of GNP, and its use as a measure of price change should be avoided."

12. See, for example, Lilien (1982) and Rissman (1986).

13. See King and Rebelo (1989).



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Federal Reserve Bank of St. Louis

## References


Aigner, D.J., F. Schneider, and D. Ghosh (1986), "Me and My Shadow: Estimating the Size of the U.S. Hidden Economy from Time Series Data" in Barnett, Berndt, and White, (ed.), *Dynamic Econometric Modeling.* Cambridge: Cambridge University Press.

Blanchard, O.J. and D. Quah (1989), "The Dynamic Effects of Aggregate Demand and Supply Disturbances," *American Economic Review* **79**, September 655–73.

Boschen, J. and L. Mills (1990), "Monetary Policy with a New View of Potential GNP," Federal Reserve Bank of Philadelphia *Business Review,* July–August, 3–10.

Bureau of Economic Analysis, Commerce Department (1991), "GNP Report and Tables," various issues.

Christiano, L.J. and M. Eichenbaum (1990), "Unit Roots in Real GNP: Do We Know, and Do We Care?" Federal Reserve Bank of Chicago Working Paper #WP-90-2.

Clark, P.K. (1979), "Potential GNP in the United States, 1948–80," *The Review of Income and Wealth* **25**, June 141–166.

Clark, P.K. (1983), "Okun's Law and Potential GNP," Federal Reserve Board of Governors, Washington, D.C..

DeLong, J.B. and L.H. Summers (1988), "How Does Macroeconomic Policy Affect Output?" *Brookings Papers on Economic Activity* **1**, 433–80.

Galí, J. (1989), "How Well does the IS-LM Model Fit Postwar U.S. Data?" Manuscript: Department of Economics, Massachusetts Institute of Technology.

Goldberger, A.S. (1974), "Unobservable Variables in Econometrics" in Zarembka, (ed.), *Frontiers in Econometrics.* New York: Academic Press.

Gordon, R.J. (1975), "The Impact of Aggregate Demand on Prices" *Brookings Papers on Economic Activity* **3**, 613–662.

Gordon, R.J. (1977), "Can the Inflation of the 1970s Be Explained?" *Brookings Papers on Economic Activity* **1**, 253–76.





Digitized for FRASER
http://fraser.stlouisfed.org/
Federal Reserve Bank of St. Louis

Gordon, R.J. (1990a), "What Is New-Keynesian Econonomics?" *Journal of Economic Literature* **28**, September 1115–1171.

Gordon, R.J. (1990b), *Macroeconomics* 5th edition. Glenview, IL: Scott Foresman.

Hallman, J.J., R.D. Porter, and D.H. Small (1989), "M2 per Unit of GNP as an Anchor for the Price Level," Board of Governors of the Federal Reserve System: Staff Study #157.

Hamilton, J.D. (1985), "Uncovering Financial Market Expectations of Inflation," *Journal of Political Economy* **93**, December 1224–1241.

Hamilton, J.D. (1986), "A Standard Error for the Estimated State Vector of a State-Space Model," *Journal of Econometrics* **33**, December 387–398.

Harvey, A.C. (1981), *Time Series Methods.* New York: Wiley.

Harvey, A.C. (1989), *Forecasting, Structural Time Series Models, and the Kalman Filter.* Cambridge: Cambridge University Press.

King, R.G. and S.T. Rebelo (1989), "Low Frequency Filtering and Real Business Cycles," Manuscript: Economics Department, Northwestern University.

Lilien, D. (1982), "Sectoral Shifts and Sectoral Unemployment," *Journal of Political Economy* **90**, August 777–93.

Nerlove, M., D. Grether, and J. Carvalho (1979), *Analysis of Economic Time Series: A Synthesis.* New York: Academic Press.

Norrbin, S.C. and D.E. Schlagenhauf (1988), "An Inquiry into the Sources of Macroeconomic Fluctuations," *Journal of Monetary Economics* **22**, 43–70.

Okun, A.M. (1970), *The Political Economy of Prosperity.* Washington, D.C.: The Brookings Institution.

Perloff, J.M. and M.L. Wachter (1979), "A Production Function – Nonaccelerating Inflation Approach to Potential Output: Is Measured Potential Output Too High?" *Carnegie Rochester Conference Series on Public Policy* **10**, 113–164.

Perry, G.L. (1977), "Potential Output: Recent Issues and Present Trends," in *U.S. Productive Capacity: Estimating the Utilization Gap*, Center for the Study of American Business Working Paper #23, Washington University.



Digitized for FRASER
http://fraser.stlouisfed.org/
Federal Reserve Bank of St. Louis

Rissman, E.R. (1986), "What Is the Natural Rate of Unemployment?" Federal Reserve Bank of Chicago *Economic Perspectives* **X**, September–October 3–17.

Plosser, C.I. and G.W. Schwert (1979), "Comment on Perloff and Wachter," *Carnegie Rochester Conference Series on Public Policy* **10**, 179–186.

Stock, J.H. and M.W. Watson (1988), "Variable Trends in Economic Time Series," *Journal of Economic Perspectives* **2**, Summer 147–74.

Watson, M.W. (1986), "Univariate Detrending Methods with Stochastic Trends," *Journal of Monetary Economics* **18**, 49–75.

Watson, M.W. and R.F. Engle (1983), "Alternative Algorithms for the Estimation of Dynamic Factor, MIMIC, and Varying Coefficient Regression Models," *Journal of Econometrics* **23**, 385–400.



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