9113

ISSUES IN FINANCIAL REGULATION

Working Paper Series

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FEDERAL RESERVE BANK
OF CHICAGO

WP - 1991 / 13

Optimal Contingent Bank Liquidation Under Moral Hazard

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April, 1991

Abstract

This paper uses models of asymmetric information and incentive compatibility to explain two distinctive features of the arrangements used by banks to obtain financing—demandable debt and the sequential service constraint. The option of early withdrawal and contingent liquidation of the bank serves as a disciplinary device to keep the banker's portfolio choice in line with depositors' preferences. The possibility of default and the first-come, first-served rule give depositors the incentive to monitor the banker.

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Support from the Garn Institute of Finance and the Herbert V. Prochnow Educational Foundation of the Graduate School of Banking, Madison, Wisconsin is gratefully acknowledged. We thank David Bizer, V.V. Chari, George Pennacchi and Jay Ritter for useful comments.

1 Introduction

A fundamental challenge for corporate finance is to understand the institutional structure of financial intermediaries. This paper uses models of asymmetric information and incentive compatibility to investigate two distinctive features of the arrangements used by banks to obtain financing. These two special features are *demandable debt* and the *"sequential service constraint"* by which repayment is allocated to those first in line when demand for repayment exceeds available reserves. For centuries the vast majority of externally financed investments have been funded by banks, for which demandable debt instruments (bank notes and demand deposits) have been the principal source of funds. Demandable debt warrants explanation because it appears more costly than available alternative contracting structures. When combined with a sequential service constraint, demandable debt carries the risk of bank runs, as depositors rush to be first in line to withdraw funds. Since bank runs are apparently costly, it becomes important to ask if there are circumstances in which the bank's institutional structure and the resultant runs play a constructive role.

In our view demandable debt serves a useful role as a disciplinary device to keep a bank manager's interests in line with those of depositors. When a depositor withdraws his deposits he is in effect voting for the replacement of the bank's management. If enough depositors take this action the bank is liquidated. In general, monitoring the activities of the bank management is costly. If there are many depositors, each would normally have an incentive to free ride on the monitoring efforts of the other depositors. The sequential service constraint helps eliminate free riding and makes it incentive compatible for depositors to monitor. This works as follows: Since non-monitors only base withdrawal decisions on observations of runs already in progress, monitors have a better chance than non-monitors of being at the front of the line. By rewarding those who are first in line when a liquidation actually occurs, the structure gives depositors an incentive to be diligent in monitoring. By providing an incentive compatible arrangement that effectively monitors the management's actions, the institution allows banks to acquire funds at lower costs than they would face under alternate arrangements.

Although our work is the first to focus on a justification for these two particular features,[1] the general literature on the structure of banks is vast. Some authors have investigated the relative merits of debt and equity financing (for example Diamond [1984] and Pennacchi [1985]). Others have investigated the "maturity-mismatch" problem, considering the advantages and disadvantages of financing long-term loans with short-term instruments. Cone [1983] shows that mismatch can lead to excessive risk of liquidation in a world of full information. McCulloch [1981] and Kareken [1985] argue that maturity mismatch is an artifact of regulation, but such a view is hard to reconcile with the historical importance of demandable debt in a variety of regulatory regimes. On the other hand, Gorton [1985] and Diamond [1989] describe incentive advantages from maturity mismatch.

Pennacchi [1983] and Flannery [1990] also develop models which demonstrate the incentive advantages that arise from demandability. These papers are actually part of the maturity mismatch literature, since for these authors short term debt and demandable debt are equivalent. This equivalence ultimately is due to the assumption that it is costless for monitors to observe the investments made by the intermediary; so that the market for the bank's short term debt is perfectly informed. By issuing short-term debt, the firm commits itself to behave properly, since improper behavior would raise the cost of debt service in the future. In our account, the costliness of monitoring means that the bank's short term debt would trade in an imperfect market. In our account, demandable debt is *not* equivalent to relying on a secondary market for short term (non-demandable) debt, since such a market might

[1]The implications of (but not the rationales for) the sequential service constraint are investigated by Diamond and Dybvig [1983].

not allow monitors to reap the rewards of their monitoring.

Finally, the issue of the use of financial structure to ensure socially desirable liquidation is also a theme which has engaged the interest of both finance and economics researchers in recent years. In papers in this literature, liquidation, while costly, has advantages over allowing management to remain in control when times become sufficiently bad. Harris and Raviv [1990] use this insight to devise models which predict debt-equity mixes: the inability to pay off debt holders becomes a signal that the firm should be liquidated. Kahn and Huberman [1988] and Hart and Moore [1989] use the insight to understand the role of collateral. In the banking context, Calomiris and Kahn [1989] show the use of demandable debt to forestall ex post fraud by bank management. When such fraud is likely, it becomes desirable to remove the managers from control. While studies of bank failures historically assign a prominent place to fraud in the list of causes, historical experience also suggests that banks have much latitude in choosing the type of projects they will finance, and that under some circumstances, banks may opt for "excessively risky" loan portfolios. Banks may choose to do so because under debt contracting with limited liability, they retain marginal upside gains and share downside losses (cf., Black, Miller and Posner (1979)). In this paper, unlike the others described above, the emphasis is on ex ante moral hazard by bank management in choice of portfolio.

We show that this contracting structure induces some depositors to invest in monitoring the bank's portfolio, and to choose whether to force bank liquidation contingent on their observation of the signal. Monitoring is incentive-compatible because of potential gains from being first in line when excess risk taking is detected. The potential for monitoring causes banks to reduce their propensity to cheat. The social costs of implementing the equilibrium (costs of monitoring) are justified by the benefit of reducing the costs associated with default. In essence, demandable debt provides a beneficial tradeoff of ex ante monitoring for the costs of asset transfer in default.

For the threat of liquidation to be an effective disciplinary device, it must be credible that depositors desire reorganization of banks which have managed their portfolios badly. In our model—as in many other models of intermediation—a bank derives its value from diversification. A badly diversified bank is less valuable to its depositors and they therefore have less incentive to hold it together. This means that contingent liquidation of badly diversified firms is in the interest of depositors, which in turn means that contracts allowing contingent liquidation will dominate other contracts in our setting.

The next section describes the physical assumptions of the model. Section three examines as a special case the situation of a one-depositor bank and derives circumstances under which contingent liquidation, demandable debt contracts will be equilibria. Section four considers a multi-depositor case and shows that the first-come, first-serve rule mitigates against free riding. The final section shows that demandable debt as described dominates other simple contracts and briefly considers extensions to the model and links to historical reserve banking.

2 The Model

We begin with two types of agents: entrepreneurs and investors. Investors each own one unit of capital; entrepreneurs each own a project requiring S units of capital. All investors are risk neutral and derive utility only from final consumption.[2] If an investor does not lend his unit of capital to an entrepreneur, he obtains an alternative return $\bar{u}$.

The value of the matured project i is a random variable x_i. The value is observable by the entrepreneur, but not verifiable by the public. We assume

[2]This essentially means that there are no liquidity traders. In section 5.2 we consider what happens if there is a certain percentage of liquidity traders, i.e. some consumers who derive utility from early consumption.

that in such a world *direct investment* takes the following form: In return for the necessary capital the entrepreneur promises to pay the investors a total amount $\bar{C}$.[3] If the borrower defaults on that promise he can be forced to turn over an equal fraction of the matured project to each investor. Collecting on a defaulted investment is a costly procedure; we assume that a cost of v is incurred by each investor when the matured project is handed over.[4] Suppose the distribution of payoffs from the project is denoted by the function $D(.)$ and let f be the probability that the payoff is at least $\bar{C}$. If there are z direct investors in project i and we represent the net expected payoff to a direct investor by $\psi(z)$, then

$$\psi(z) = \frac{1}{z} \int \min\{x_i, \bar{C}\}\, dD(x_i) - (1 - f)v$$

In this situation we would expect some agents to become intermediaries so as to economize on the costs of asset transfers in default. Rather than making this decision endogenous we simply assume that there is a third group of agents who operate "banks," which act as specialized intermediaries and have lower costs (in fact zero costs) of collecting when a defaulted project is handed over.[5] Banks make direct investments in entrepreneurs' projects. Contracts between banks and investors take a similar form. In a *simple debt contract* each depositor is promised a constant payment $\bar{R}$, which we call the "face value" of the contract. If the bank reneges on its promise, each depositor receives a share of the bank's assets. (These assets are payments the

[3] $\bar{C}$ is determined by the supply of projects. We assume projects are in perfectly elastic supply at a fixed required return for the entrepreneur. This has the effect of fixing $\bar{C}$ exogenously. We can therefore treat $\bar{C}$ as a constant for the rest of this paper.

[4] These assumptions are slightly simpler than Townsend (1979) but have the same effect.

[5] In Diamond (1984) the lower costs of banks stem from eliminating duplicate liquidation costs. For our analysis the source of the lower costs is immaterial. All that is important is that banks be able to achieve low risk portfolios at less cost than individuals could on their own. It will also be clear to the reader that we can easily make the choice to become a bank endogenous. This extension is outside the main focus of the paper and is therefore omitted.

bank has received from its debtors plus the matured projects of entrepreneurs who have defaulted.) Again we assume that each depositor incurs a cost v if the bank defaults. Thus the net-expected value to a depositor of the simple debt contract with face value $\bar{R}$ is given by

$$R(y) = \begin{cases} \bar{R}, & \text{if } y \geq \bar{R}; \\ y - v, & \text{otherwise,} \end{cases}$$

where y is the total value of the bank's holdings divided by the number of depositors.

More general forms of contracts are also possible and will be discussed below. But throughout this paper, the terms of any contract will only depend on observables. In particular monetary payments cannot depend on the value of defaulted projects. Also, for simplicity, we will only deal with contracts which are *anonymous*: depositors' treatment in the contract depends on their actions in the contract, but not on their names.[6]

Thus far, our framework captures the situation of Diamond (1984) while avoiding many of its complications. In other words, intermediaries in the model we have described thus far play a role identical to Diamond's delegated monitors. By processing the defaulted investments themselves, banks economize on the costs of default. In such a framework simple debt contracts are optimal. The following two additional assumptions are what make *demandable* debt useful in this environment: First, depositors can, at a cost, monitor the banker's portfolio choice. Second, liability is limited; there is no non-pecuniary cost depositors can impose on the banker. Limited liability creates the need for penalizing bad portfolio choices by stripping the banker of his assets. Monitoring makes that possible.

Specifically, we assume that the banker has the option of picking one of two portfolios of projects. He can either pick a *well-diversified* portfolio X or an *undiversified* portfolio Y. In a well-diversified portfolio, all projects

[6]This restriction simplifies the analysis without changing our main points.

are independent; in an undiversified portfolio, they are correlated. While the two portfolios give the bank the same expected returns, they have different variances. In the main model in section 4, we will focus on banks which invest in a large number of entrepreneurs; thus a well-diversified portfolio will converge to a riskless portfolio. In section 3 we will examine a simplified version of the story, and there we will treat the investment X as literally riskless. Since the banker has limited liability, he will prefer the undiversified portfolio. Unfortunately, because depositors bear the costs of default, this portfolio is less valuable to them and socially inefficient.

Banks maximize their expected profits. A bank takes deposits. Some of the deposits are placed in one of the two types of portfolios; the remainder we will call reserves. The level of reserves is publicly observable. However, depositors cannot costlessly observe which type of portfolio the banker has chosen. Depositors can learn a bank's portfolio choice by paying a cost m (which can be thought of as effort). Only the depositor himself knows whether he has paid the costs.[7]

Finally it is possible to have a bank liquidated before its projects mature. Liquidation is defined as the removal of the banker from control of the bank's assets, and the distribution of those assets among depositors. In other words liquidation means reversion to a direct-investment equilibrium. We let l be the transactions cost (per depositor) associated with a decision to liquidate the bank.[8]

[7]The effect of this assumption is to rule out contracts in which compensation is explicitly tied to monitoring. This assumption seems natural to us; even if the monitor is observed to engage in monitoring, the quality of the monitoring and the information received in the process will typically be private. If monitors are to be rewarded they must receive their rewards in an incentive-compatible fashion. The sequential-service constraint will turn out to be an incentive-compatible way to reward monitors.

[8]An explicit transaction cost is only necessary in the single depositor case examined in Section 3. In the multiple depositor case, consumers can also have the option of directly investing in projects initially. However, in equilibrium banks will drive out direct investors, as in Diamond (1984), Williamson (1986) or Krasa-Villamil (1990).

The timing of these actions is as follows:

1st Period	2nd Period	3rd Period
Bank takes deposits; Portfolio decision.	Depositors can monitor; bank can be liquidated	Projects mature; entrepreneurs pay up or default.

A bank's charter specifies the payments to be made to each depositor if the bank is liquidated, the payments to be made to each depositor if the bank defaults, and the payments to be made to each depositor if neither liquidation nor default occurs. The charter also specifies conditions for early liquidation. Contracts cannot be written on the portfolio structure of the bank or on monitoring effort, neither of which is directly observable.[9] The decision to liquidate a bank must be a function of observable variables. The only possible variable is an announcement by a depositor as to which signal he has observed. In our structure the "announcement" will come in the form of an early withdrawal from the bank. Contracts will also specify the payment to a depositor who chooses early withdrawal. Since reserves are assumed observable, the contract can simply specify the liquidation decision as a function of the level of reserves remaining after early withdrawals.

3 The Case of a Single Depositor

In this section we examine the special case in which the banker deals with only one depositor. Clearly, the economic interpretation of the single depositor case is limited. However, it allows us to simplify the exposition by focusing first on the incentive problem of the banker. We will show that demandable

[9]This assumption is natural since the portfolio is observed by all depositors individually, and also if we generalize our model where their observations are subject to random errors.

debt solves this incentive problem. In the general model of Section 4, demandable debt works essentially the same way. In addition, however we have to introduce the sequential service constraint in order to solve the incentive problem for the depositors, i.e., to prevent depositors from "free riding" on other depositors' monitoring.

In this section, we specialize the analysis to deal with a situation in which the risky portfolio has a two-point distribution. If the project is successful, the bank receives $\bar{C}$; the probability of success is f. For further reduction in notational requirements we assume that if the project is unsuccessful, its value is exactly equal to v so that if the banker defaults when the project is not successful, the residual to the investor is entirely eaten up by the default proceedings. In the simplified model of this section, the well-diversified portfolio is literally riskless: it gives a payoff of

$$\bar{X} = f\bar{C} + (1 - f)v$$

with certainty. We assume that the required expected return $\bar{u}$ is greater than v; otherwise there is no incentive problem. We assume it is less than the payoff from the riskless project, otherwise there is no profitable investment.[10]

When a simple debt contract has a face value lying between v and $\bar{X}$, the banker's payoff is $\bar{X} - \bar{R}$ if the banker chooses the riskless investment. His expected payoff is $(\bar{C} - \bar{R})f$ if he chooses the risky investment. Thus, as long as

$$\bar{X} - \bar{u} < (\bar{C} - \bar{u})f,$$

the simple debt contract always has a default risk, since the banker will find it tempting to invest in the risky asset. In such a circumstance, of course, the

[10] If the total payoff of the bank's portfolio were observable then it would be possible to distinguish the bank's portfolio choice in this example since the two portfolios have different supports. More general examples could be developed in which the supports are identical so that this possibility does not arise. The reader may prefer to consider this example as the limiting situation in which the well diversified portfolio converges in distribution to the riskless portfolio.

payment to the depositor in the contract will have to incorporate a default premium; the depositor will demand a face value of $\bar{u}/f$.

Finally, suppose the cost of liquidation l is less than $(1-f)v$, the expected cost of default if the risky investment is chosen. Then simple contracts are dominated by a contract that always mandates liquidation, with the banker receiving $\bar{X} - \bar{u} - l$. Since the asset passes to the depositor, the banker has no incentive to pick the risky portfolio if the liquidation technology is used.[11]

Nonetheless the liquidation technology has a cost associated with it as well, namely the amount l. Are there more complicated arrangements which would reduce the transactions cost? After all, if the banker could be convinced to take the riskless choice, then the simple debt contract could be used without suffering any default costs (as long as $\bar{R} \leq \bar{X}$). In fact, the simple liquidation contract is dominated by a *stochastic liquidation* contract in which the liquidation occurs with some probability less than one. (In such a contract, liquidation is determined by an exogenous random event – e.g., a coin toss – not by either the depositor or the banker.) If liquidation does not occur the depositor is paid the value v, the maximum promised payment the banker can credibly make. If liquidation does occur, the depositor is given the entire value of the liquidated project. By maximizing the payment to the depositor in the default states, the banker is able to minimize the probability of liquidation, and hence expected liquidation costs. This contract is optimal among the stochastic liquidation schemes. The probability of liquidation λ is adjusted so that the weighted average of these two payments is equal to the required return. Thus,

$$\lambda = \frac{\bar{u} - v}{\bar{X} - l - v}.$$

Since the banker never defaults under this scheme, the loss in profits stems en-

[11]Liquidation essentially means that the contracts are reversed, i.e., the bank now holds the simple debt contract. The moral hazard problem disappears since the bank does not have access to the upside gains of the distribution any more.

tirely from the costs associated with liquidation. Since liquidation no longer occurs with certainty, the associated costs are reduced from l to $l\lambda$.

While stochastic liquidation is the best that can be achieved without monitoring, monitoring will in general improve the contract. If monitoring were costless, the optimal arrangement would be the following: The depositor always monitors. He is given the choice of calling for liquidation of the bank. Whether or not the bank is liquidated, the depositor is promised a constant amount. The banker receives the remnant.

The behavior of each party under such a contract is as follows: Since monitoring is costless, the depositor always monitors. If he observes a risky investment he calls for liquidation, and receives his payment. If he observes the riskless investment he does not call for liquidation. With a riskless investment, the banker will never choose to default, and the depositor will receive his payment with certainty. Since the residual is greater when the bank is not liquidated, the banker prefers to choose the riskless strategy.[12] Note therefore that in the case where monitoring is costless, the outcome is first best: The non risky investment is always chosen and all liquidation and default costs are avoided.

When monitoring is costly, the situation is much more complicated. In general, the first best will no longer be feasible. Nonetheless as long as monitoring is not too expensive, the optimal contract always involves a positive probability of monitoring, as the following theorem shows. The theorem is proved by finding a demandable debt contract which induces the depositor to monitor and thereby induces the banker to choose the less risky investment with positive probability. The contract we find gives the depositor his required return and gives the banker a greater return than he can achieve in any contract without monitoring.

Theorem 1. *For positive but sufficiently small costs of monitoring, the*

[12]This is the essence of the models of Pennacchi or Flannery.

optimal contract involves a positive probability of monitoring. In the optimal contract, the bank picks the risky outcome with positive probability.

Proof. We have already described the optimal contract with zero monitoring. It is therefore sufficient to demonstrate that the optimal zero-monitoring contract is dominated by some contract with positive levels of monitoring.

The contract we will examine works as follows: The depositor is given the option of demanding "early withdrawal." If he does not call for early withdrawal, then the contract becomes a simple debt contract with face value $\bar{R}$. If he does call for early withdrawal, then the bank is liquidated, the depositor receives the payoff $\bar{R} - l$, and the residual goes to the banker.[13]

In this contract the banker chooses the risky investment with some probability p strictly between 0 and 1 and the depositor always monitors the banker's actions. The depositor calls for early withdrawal when and only when the banker has chosen the risky investment. We begin by deriving the necessary and sufficient conditions for this to be equilibrium behavior under the contract. First, note that conditional on the depositor's behavior, the banker's payoff is $\bar{X} - \bar{R}$, regardless of his choice of investment. Thus randomizing between them is indeed a best response.

The depositor's behavior yields him a payoff of $(1-p)\bar{R} + p(\bar{R} - l) - m$. Suppose instead that the depositor always called for early withdrawal. Then his payoff would be $\bar{R} - l$. On the other hand, if he chose never to call for early withdrawal, his expected payoff would be $(1-p)\bar{R} + pf\bar{R}$.

Thus the prescribed behavior dominates the alternatives if the following inequalities hold:

$$(1-p)l \geq m \tag{3.1}$$

[13]The banker's residual under liquidation could be implemented by having him hold a portfolio which includes bank debt. This residual is necessary to implement the contract when there is a single depositor. As long as there are multiple depositors, demandable debt can be implemented in an incentive compatible fashion with the banker retaining none of the profits of the bank in the case of liquidation. See the next section for an example.

and

$$p(\bar{R} - l - f\bar{R}) \geq m \tag{3.2}$$

It can be readily verified that if the prescribed behavior dominates these two alternatives, it dominates all other possible behavior for the depositor (for example, randomly monitoring or randomly calling for liquidation). Thus for the contract to induce the prescribed behavior, conditions (3.1) and (3.2) are necessary and sufficient. If in addition

$$\bar{R} - pl - m = \bar{u}. \tag{3.3}$$

then the contract yields the depositor the competitive payoff $\bar{u}$.

In any contract the expected payoff to the banker is the expected return on the investment less the expected payoff of the depositor less any costs of liquidation or default. Thus the banker's expected profits in the contract we describe are

$$\bar{X} - \bar{u} - pl,$$

since the liquidation costs are borne whenever the banker chooses to engage in the risky investment. The banker's expected profits in the optimal contract with no monitoring is

$$\bar{X} - \bar{u} - \lambda l,$$

where λ was defined above. Thus there exists a contract with monitoring which dominates the optimal contract without monitoring as long as there exists a pair $(\bar{R}, p)$ satisfying equations (3.1)–(3.3) with p less than λ. It can be verified that as long as m is sufficiently small, such a pair always exists. Thus the contract with zero monitoring is not optimal.

Once we know that the optimal contract involves positive monitoring, it is easy to see that the banker must randomize. For if in equilibrium he chose one portfolio with certainty, it would not be in the depositor's interest actually to pay the cost of monitoring. This proves the theorem.

In the contract described above, monitoring occurs with certainty. It may be that the globally optimal contract for this problem entails monitoring less than one hundred per cent of the time. However, if this is the case, it can be shown that the optimal contract specifies that when the monitoring does not occur, liquidation will not be called for; in other words, the simple debt contract will be the default option.

In summary, the optimal arrangement in the one-depositor case has the following characteristics: The depositor monitors the portfolio choice of the banker. If the monitor observes a risky choice, he calls for liquidation of the bank. If he does not observe a risky choice, then he leaves the banker in control of the bank's assets. The monitor's incentives to monitor are maintained by the following considerations: If liquidation occurs, the monitor receives less than he would had he allowed the banker to continue in control, provided default does not occur. If the banker engages in a risky portfolio choice, default becomes more likely, and the monitor prefers to force the bank to liquidate rather than bear the losses default would entail. The banker himself is indifferent between the two choices given the monitor's response, but he must choose the risky portfolio occasionally, just to give the depositor an incentive to monitor. For if the banker knew the depositor was not monitoring, the banker would actually prefer to choose the risky portfolio all the time, wrecking the contract.

Thus we have shown that monitoring with contingent liquidation is optimal given that investment is taking place through a bank. As long as the costs of monitoring are sufficiently low and the depositor cannot achieve riskless portfolios on his own, the optimal bank contract dominates direct investment.

4 The Equilibrium With Multiple Depositors

In this section we extend the analysis to a multi-depositor problem. In the multi-depositor problem the sequential service constraint will prevent the depositors form free-riding on other depositors' monitoring. There are two key differences between the model of this section and the model of the previous section. First, in this section the well-diversified portfolio is only asymptotically riskless. Second, we no longer need to assume an exogenous cost to liquidation. The breakup of the diversified portfolio into parcels for each depositor in liquidation entails a cost which can take the place of the explicitly described parameter l.

In the multi-depositor problem, the contract induces a game among the depositors and the banker. An optimal contract and associated equilibrium of the game must maximize the profit of the bank subject to the expected utility of depositors being greater than the reservation level $\bar{u}$. We propose a *demandable debt* contract in which bank liquidation occurs when at least one monitor observes that the bank has cheated. We will demonstrate that this contract fulfills desirable properties in aligning the incentives of the banker and all the depositors. In section 5 we show that this contract dominates other simple contracts and dominates direct investment. It should be noted that making the number of projects as large as possible is always desirable in the contracts since it allows reduction in default costs through diversification. Our strategy in this section is to treat the number of projects the banker can manage as a technologically-determined constraint, and derive results as the number of projects s becomes large.

In this section we proceed as follows: In 4.1 the game is formally specified; 4.2 describes the demandable debt contract and the sequential service constraint; in 4.3 we compute the payoffs to the bank and to the investors for their choice of strategies; in 4.4 we finally show that demandable-debt is incentive compatible (i.e, is a Nash equilibrium) for depositors and the bank. All proofs for the sections 4 and 5 can be found in the appendix.

4.1 Description of the Game

In the induced games we consider symmetric equilibria in mixed strategies. The game works as follows. The bank offers deposit contracts to depositors in period 1 and chooses a probability distribution over all feasible portfolio choices. Since the level of reserves is publicly observable, this probability distribution can be simply described by the probability p_q of choosing a portfolio of type $q = X, Y$. In what follows we will often denote the probability of cheating (investing in the undiversified portfolio Y) by p. In equilibrium the bank will choose p to maximize expected profits.

Every depositor can choose a probability π of monitoring the bank. As before, a monitoring depositor observes the banker's portfolio choice without error. When the depositor monitors, his decision to withdraw will be a function of which portfolio he observes. A depositor who does not monitor can also randomize over withdrawing or not withdrawing. Every depositor will choose withdrawing and monitoring strategies to maximize his expected payoff, given the deposit contracts the bank offers, the bank's portfolio choice strategy p, and the monitoring-withdrawing strategy for the remaining depositors. For a given monitoring-withdrawing strategy for all the depositors other than depositor θ, let $\eta(W_\theta; q)$ be the probability that W_θ of the others attempt to withdraw their deposits.[14]

4.2 Description of the Demandable Debt Contract

The demandable debt contract works as follows: depositors are given the option of early withdrawal, with withdrawals backed by reserves. An agent who

[14] If every depositor monitors with the same probability and no depositor attempts to withdraw without monitoring, then this is clearly a Binomial distribution (cf. Lemma 4).

withdraws early receives the fixed payment T. If there are more withdrawal requests than reserves, then only the first to request withdrawal receive early payment. This is the "sequential service constraint," or "first-come, first-served rule." If there are more reserves than early withdrawal requests, then other depositors, having observed early withdrawal can also be paid $\bar{T}$ as long as there are reserves left. If reserves are depleted, then the bank is liquidated and remaining depositors seize the remaining assets. Otherwise, the bank is not liquidated, and remaining depositors have a simple debt contract with payment $\bar{R}$. Thus the terms of a demandable debt contract are described by indicating the pair $(T, \bar{R})$ and by specifying the number of initial depositors and the number of entrepreneurs the bank finances with its deposits. (Reserves are the difference between deposits and loans.)

We describe an equilibrium in which monitors start a run on the bank when they observe the banker has cheated. Once non-monitors observe reserve depletion, they too will attempt to withdraw funds (we refer to late queuing by non-monitors as "follow-up" withdrawals).[15] Thus observing the risky investment will always lead to full reserve depletion and bank liquidation, even if the number of monitors is small relative to the level of reserves.[16]

Suppose the bank receives deposits from zs depositors (each of whom

[15]These "follow-up" withdrawals are a non-essential feature of the model. One could, equivalently, liquidate the bank immediately after the initial round of withdrawals. Under risk neutrality, these two rules have the same expected payoffs to each depositor, and therefore, under risk neutrality they are equally desirable. Under risk aversion, it would be better to liquidate the bank before allowing follow-up withdrawals by non-monitors, because follow-up withdrawals increase the riskiness of each depositors returns for the same level of expected payoff. We allow follow-up withdrawals for algebraic simplicity, and without loss of generality (that is, the desirability of demandable debt in no way depends on this assumption).

The essential feature is that monitors have a better chance than non-monitors of being first in line contingent on observation of Y. A non-monitor could of course choose always choose to run—and thereby be first in line regardless of the banker's portfolio choice—but this strategy is dominated by other behavior.

[16]Again, we could equivalently and without loss of generality make bank liquidation depend on the bank's receiving one or more withdrawal requests.

provides one dollar of deposits) and invests Ss dollars in s projects; the remnant of the bank's deposits is held as reserves r. Thus, $r = (z - S)s$. Let F^q denote the distribution of the value of the bank's portfolio of projects where $q = X, Y$ determines whether the bank has invested in type X or in type Y projects.[17]

If the bank is liquidated, we revert to a direct investment equilibrium. Each remaining depositor will receive a share of the liquidated portfolio and will have to bear any transactions costs for defaults that result. Under the circumstances transactions costs will be minimized if duplication is avoided—that is, if liquidation is arranged so that each holder concentrates his holdings in a single project rather than dispersing them. If zs depositors remain at liquidation, then the net payoff to each is $\psi(z)$. Note therefore that the net payoff in liquidation does not depend on which loan portfolio the banker chose.[18]

If the bank is not liquidated the contract becomes a simple debt contract with face value $\bar{R}$. Since the per capita value of the bank is $(x + r)/zs$ where x is the value of the loan portfolio, we conclude that if the banker chooses portfolio q, a simple demandable debt contract with face value $\bar{R}$ gives a depositor a net payoff of

$$\int R(\frac{x + r}{zs})\, dF^q(x).$$

Let $\xi(q)$ denote the bank's profit from a simple debt contract when the bank has chosen portfolio q. Then

$$\xi(q) = \int \max\{x + r - zs\bar{R}, 0\}\, dF^q(x).$$

The first thing to note is that a under a simple debt contract the banker always has an incentive to cheat.

[17] F^X is the distribution of the random variable $\sum_{i=1}^{s} \min\{\bar{C}, x_i\})$, and F^Y is the distribution of $s \min\{\bar{C}, x_i\})$.

[18] Recall that individual projects all have the same distribution; once the bank is broken up, benefits of diversification are lost.

Lemma 1. *As s increases,*

$$\frac{\xi(X)}{\xi(Y)} \to \xi,$$

where $0 < \xi < 1$ *as long as the face value* $\bar{R}$ *lies within the support of the per-capita value of the bank.*

Since a simple debt contract in which $\bar{R}$ does not lie between the maximum and the minimum per-capita values of the bank is nor feasible (either it would give negative profits to the bank, or would not give adequate returns to the depositors, we get the following Corollary.

Corollary 1. *In any feasible simple debt contract the banker prefers to invest in the undiversified portfolio.*

4.3 Computation of the Payoffs of the Game

We now turn to computing the payoffs of the agents in the game given the demandable debt contract we have described. First we derive the expected payoff for a depositor θ. We begin by calculating the payoff conditional on (a) the depositor's choice of attempting to withdraw or not; (b) the bank's portfolio; and (c) the other depositors' strategies. The depositor's payoff depends on other depositors' strategies only through W_θ, the number of other depositors who attempt to withdraw in the first round. Call $\zeta_w(q, W_\theta)$ the payoff to a depositor who attempts to withdraw when W_θ other depositors are also attempting to withdraw. Call $\zeta_n(q, W_\theta)$ the payoff to a depositor who does not attempt to withdraw when W_θ other depositors attempt to withdraw.

First we calculate ζ_w. If the depositor attempts to withdraw the bank is liquidated. If the bank has r dollars in reserves, then it can only pay $r/\bar{T}$ of its depositors with early withdrawals.[19] All remaining depositors receive an

[19]We ignore integer problems. They can be accounted for without substantive change but at the cost of a large increase in notational requirements.

equal share of the liquidated bank's assets.[20]

Assume that the bank randomly pays the first $r/\bar{T}$ of the depositors who request withdrawal, and that monitoring depositors who request withdrawal in the first round are equally likely to be included in the first $r/\bar{T}$ requests. If the depositor attempts to withdraw and $W_\theta + 1 \leq r/\bar{T}$ then he receives $\bar{T}$ with probability one. Otherwise he receives $\bar{T}$ with probability $\frac{r/\bar{T}}{W_\theta+1}$. After the reserves are exhausted, $zs - (r/\bar{T})$ depositors remain. Each receives an equal share of the liquidated firm, for a net payoff of $\psi(z - (r/\bar{T}s))$. Thus $\zeta_w(q, W_\theta)$ is a weighted average of $\bar{T}$ and ψ.

Next we calculate ζ_n, the payoff to a depositor who initially leaves his deposit in the bank when W_θ other depositors are attempting to withdraw. Suppose W_θ is positive. Then the depositor attempts a "follow-up" withdrawal. His payment ζ_n is ψ with certainty if $W_\theta \geq r/\bar{T}$. If $W_\theta < r/\bar{T}$ then his payment ζ_n is $\bar{T}$ with probability $(r/\bar{T} - W_\theta)/(z - W_\theta)$ and ψ otherwise.

On the other hand suppose W_θ equals zero—that is, no one else attempts to withdraw. Then the depositor receives the payoff of a simple debt contract with face value $\bar{R}$ as calculated in the previous section.[21]

Now we use ζ_n and ζ_w to analyze the expected returns to depositors, given the bank's portfolio choice and the equilibrium distribution of strategies of all other depositors. The equilibrium strategies for all other depositors imply that $W_\theta = 0$ with probability one if $q = X$; it has a binomial distribution in the symmetric equilibrium if $q = Y$.

There are four possible actions over which depositors randomize, i.e. any combination of monitoring/not monitoring, and withdrawing/not withdraw-

[20]We could generalize the set of contracts considered by allowing the possibility that only a portion of the reserves are returned to the depositors in liquidation. But it should be clear that it is always optimal to pay off some of the depositors with reserves if possible since this cuts down the total costs spent for the transfer of assets. Aside from this there is no loss of generality in returning to depositors the entirety of any residual reserves remaining as a result of integer problems.

[21]Note therefore that only in the case of $W_\theta = 0$ does q affect ζ_n.

ing. We now describe the payoffs for each action. First, we consider the case where depositor θ monitors. Let β_{wq} be the expected utility of withdrawing given that the monitor observes the investment $q = X, Y$, and let β_{nq} be the expected utility of not withdrawing given that the monitor observes investment $q = X, Y$. These are given by the following formulas:

$$\beta_{wq} = \int \zeta_w(q, W_\theta)\, d\eta(W_\theta; q). \tag{4.1}$$

$$\beta_{nq} = \int \zeta_n(q, W_\theta)\, d\eta(W_\theta; q). \tag{4.2}$$

Finally, consider the cases in which the depositor does *not* monitor. Let α_w be the expected utility of the depositor if he withdraws, and let α_n the expected utility if he does not withdraw. α_w and α_n are given by

$$\alpha_w = \sum_{q=X,Y} p_q \beta_{wq} \qquad \alpha_n = \sum_{q=X,Y} p_q \beta_{nq}.$$

Let $w_\theta(q)$ be the function that describes the decision of depositor θ given that he observes q. Thus, let $w_\theta(q) = 1$ if he withdraws, and $w_\theta(q) = 0$, otherwise.

Depositor θ then chooses an optimal probability π of monitoring, a function $w_\theta(q)$, and a probability ρ of withdrawal without monitoring. Consequently

$$\begin{aligned} \pi, \rho, w_\theta(q) \in \arg\max_{\pi,\rho,w_\theta(q)} & (1-\pi)\{\rho\alpha_w + (1-\rho)\alpha_n\} - \pi m \\ & + \sum_{q=X,Y} \pi p_q \{w_\theta(q)\beta_{wq} + (1 - w_\theta(q))\beta_{nq}\}. \end{aligned} \tag{4.3}$$

The right side of (4.3) is the utility of a depositor for given demandable debt contract $(\bar{R}, \bar{T})$ and for given strategies of the banker and the other depositors.

Next, we derive the decision problem of the bank—that is the choice of p. Let W be the number of consumers who choose early withdrawal. The distribution of W depends on all depositors' decisions and on q. If $W > 0$ then the bank is liquidated, and the banker's payoff is zero. If $W = 0$ then the banker's payoff is $\xi(q)$. Then for given $\bar{R}$ and π the bank solves

$$p \in \arg\max_{p} p\xi(Y)P\{(W = 0)\} + (1 - p)\xi(X). \tag{4.4}$$

The right side of (4.4) is the profit of the bank given a contract and strategies for the depositors.

4.4 The Equilibrium of the Game

Our next task is to show that in the class of demandable-debt contracts described above, there are contracts and associated strategies which solve the incentive constraints (4.3) and (4.4) and in which the depositor's utility (right side of (4.3)) is at least his reservation value $\bar{u}$.

Lemma 2. *If the number of projects s is sufficiently large and the cost of monitoring m is sufficiently low, then there exists a demandable debt contract $(\bar{R}, \bar{T})$ with zs depositors in which the following behavior occurs in equilibrium:*

(1) Each depositor monitors with probability π where $0 < \pi < 1$.

(2) A depositor attempts early withdrawal if and only if he monitors and observes that the bank has chosen the risky portfolio. (All non-monitoring depositors attempt follow up withdrawals if they observe any early withdrawal.)

(3) The banker chooses the risky portfolio with a probability p where $0 < p < 1$.

Next we show that the demandable debt contract exists whenever direct investment is feasible.

Lemma 3. *Let m be sufficiently small and s be sufficiently large. Whenever direct investment is feasible then the demandable debt contract is also feasible – that is, the set of incentive compatible demandable debt contracts yielding depositors their reservation values and yielding a positive profit for the bank is non-empty. Further, there exists an optimal contract in this set.*

5 Optimality of Demandable Debt

Having shown that the demandable debt contract can be implemented, it remains to show that it dominates other natural mechanisms for financial intermediation. The set of contracts we will examine can be summarized as follows: Contracts either utilize monitoring or they do not. Contracts without monitoring either always specify bank liquidation in the second period, or they never do. Monitoring contracts are also of two types: (a) the demandable debt contract, in which liquidation occurs when the bad signal is observed—where the incentive-compatibility of demandable debt is ensured by the availability of reserves—or (b) non-liquidation contingent contracts that allow varying payoffs conditional on the signal observed in period 2. We will show that demandable debt is superior to the three alternative contracts.

The simple liquidation contract is equivalent to direct investment: The assets are split up and returned to the depositors. Financial intermediation dominates direct investment provided that the costs m of monitoring are not too high and the number s of projects is sufficiently large that the benefits of diversification apply.

As we have seen, in a simple debt contract bankers will choose project Y. Since the bank is no better diversified than in the direct investment this outcome is again dominated by schemes which induce diversification as long as monitoring costs are not too high.

Finally, consider the non-liquidation contract with varying payoffs in period 2. This contract will never be optimal because liquidation will always be preferred to risking default, given that depositors are aware that the bank has chosen the Y projects. Again, once banks are known to have cheated, direct investment will be superior because it avoids duplication of verification costs.

The result of all this is the following Theorem.

Theorem 2. *There exists $\bar{m} > 0$ and an integer $\bar{s}$ such that for a bank with at least $\bar{s}$ projects and a monitoring cost $m < \bar{m}$ there exists an incentive compatible demandable debt contract with zs depositors and corresponding probabilities of monitoring π, and probability p of moral hazard of the bank which gives higher profits than any other simple contract. As $s \to \infty$, in the demandable debt contract, the costs of bank default per depositor converge to $p(1 - \bar{\xi}) > 0$, and the total costs of monitoring the bank's portfolio converge to $-m \log \bar{\xi} > 0$.*

It is important to note that the ability to maintain demandable debt contracting presumes an efficient legal system, which enforces some (socially beneficial) contracts, and not others. In particular, our equilibrium requires that banks and monitors not be able to collude privately to the detriment of other depositors. To do so without depleting observable reserves (and thereby initiating bank liquidation) bankers would have to offer special contracts to monitors in period 2 that promised additional payments in period 3. If such payments were enforced by the legal system, our equilibrium clearly would be unsustainable. Thus it will be efficient (and necessary for our equilibrium) for such side-payment contracts to be disallowed. Clearly, the legal system is designed to prevent such abuses—for example, through enforcement of debt covenants that preclude subordination or the accumulation of further debt.[22]

[22]For further examples of the effects of side trades in banking see Bizer and DeMarzo (1990) and Kahn and Mookherjee (1989).

5.1 The Role of Reserves in Implementing Demandable Debt

In our model, reserves are useful because they provide a means for rewarding monitors for their efforts through the first-come first-served rule. Given that the bad signal has been received, monitors are better off accepting a lower payment from reserves in period 2, than holding on to their original higher promise of payment in period 3. The actions of monitors have positive externalities because of the observability of reserve withdrawals, which allows the social decision of bank liquidation to be made easily.

Next, we show that the bank only has to hold a constant amount of reserves independent of the number of depositors.

Lemma 4. *For every $\varepsilon > 0$ there exists a C such that the probability that more than C depositors monitor in equilibrium is smaller than ε independent of the size of the bank. Consequently, the probability that the demand of monitors for reserves can be covered by the bank can be made arbitrarily small with a fixed amount of reserves.*

Lemma 4 and (8) now imply that the bank will hold a strictly positive level of reserves as $s \to \infty$, since the costs per depositor approaches zero, whereas the gain per depositor due to a decrease of p remains strictly positive. This economizing on reserves means that the demandable debt contract dominates a similar contract in which depositors withdraw when they do not monitor. This has two disadvantages. It means that solvent entrepreneurs will be liquidated unless large stocks of reserves are held. With sufficiently large stocks of reserves this new contract could mimic a demandable debt, however, holding such a large level of reserves is very costly.

One could imagine alternative means for implementing the mechanism of contingent bank liquidation that would reward monitors for truthfully reporting observed bad signals. For example, suppose that depositors were offered the opportunity to exchange their contracts at the beginning of period 2 for a new contracts (limited in number, as in the case of reserves) that offered lower returns in period 3, but subordinated other depositors' claims on the bank to these new promised payments. The legal system could make a special exception for such recontracting (to avoid encouraging collusion between banks and monitors) by making the enforcement of such contracts depend on their public observability, or equivalently, on the condition that the bank is placed into liquidation in period 2. This arrangement, like reserve banking, rewards monitors without encouraging non-monitors to announce bad outcomes. As in the case of reserve pre-payment, the monitor will only choose to convert to lower-paying "preferred" debt when he observes the bad state. In this alternative arrangement, after liquidation, the depositors in the new direct-investment equilibrium would each owe the monitors a fixed share of the amount of the secured debt (new promised payment) received by monitors in period 2.

Given the availability of this alternative, why would banks use reserves to implement contingent liquidation, given that reserve holding entails opportunity costs of foregone productive investments? One reason might be that banks have other reasons for holding reserves (i.e., because of liquidity traders, which we discuss below), and thus can use reserves to "kill two birds with one stone." Another reason might be the prohibitively high transaction cost of writing and enforcing new contracts between the monitors and the (now liquidated) bank.

5.2 The Transactability of Bank Claims

An important property of demandable debt claims historically has been their transactability. Bank deposits and notes often out-competed other financial claims as media of exchange. Calomiris and Kahn (1989) argue that the transactability of these claims does not depend in a physical sense on the withdrawal of reserves on demand. Rather, transactability depends on the shared information of agents engaging in exchange.

While we do not model an exchange process among agents in this paper, our results do suggest reasons why demandable-debt bank claims might be more transactable. In the contingent-liquidation equilibrium, the bank's portfolio decision is revealed to monitors, and their decision whether to run the bank is observable and fully revealing to non-monitors. This implies that information regarding the value of the bank claims is effectively symmetric among depositors, both before and after the critical moment in which monitors receive and act on the signal.

Harris and Raviv [1990] make a similar argument for firms. They argue that the fact that a firm is able to pay interest on its debt provides valuable information to the market about the firm's quality. In the case of a bank, the criterion is more stringent and the resultant information is an assurance of even higher quality. Since a bank would be closed if even a small percentage of its depositors lacked confidence, information about low quality banks becomes public much more easily than in the case of firms with a non-demandable financial structure. Hence, the fact that a bank's doors remain open provides the public with a strong indication of the bank's strength. We would argue that this strong indication itself becomes the basis of the liquidity of demandable debt holdings.

The extreme result of full information sharing follows from the fact that the bank can only choose two types of portfolios. Even in more complicated contexts, however, one could argue that monitors are unable to make fine distinctions regarding the state of the bank's portfolio, and thus the simple run/no-run rule may be still optimal, and the extent of shared information between monitors and non-monitors may still be high (even though neither is as informed as the banker).

5.3 Early Consumption Demand, Liquidity Traders

It is straightforward to add liquidity traders to our model. Assume that a proportion q of the zs depositors will require reserves to finance consumption in period 2. The incidence of this demand for reserves is determined randomly, and there is no aggregate uncertainty. Now the sequence of events is as follows: The bank invests. After the signal is received, either monitors demand reserves or not, and simultaneously liquidity traders demand early withdrawal. Again the bank randomizes over which depositors are served. Liquidity traders who are not served by the bank directly can trade their claim on third period income for reserves with monitors who have withdrawn from the bank.

In equilibrium the bank will have to hold enough reserves to cover the demand of liquidity traders in period 2. Lemma 4 now immediately implies that the probability that the demand of a monitor is not fulfilled approaches zero as $s \to \infty$. As in the model without liquidity traders the change in the level of reserves reveals the signal monitors have observed. Now, however, the costs of holding reserves to implement demandable debt is zero for all sufficiently large s, since this source of demand is not the binding constraint on reserves. However, β_Y will have to be lower than in the model without liquidity traders, because now the second period payment $\bar{T}$ is not only a "reward" for monitors.

The assumption of no aggregate uncertainty is of course unrealistic. But, we would argue that historical experience shows that banks dealt with aggregate uncertainty through arrangement on the level of banking systems rather than at the level of individual banks. As much recent work on banking history has emphasized (for example, Gorton (1985), Calomiris and Gorton (1991), and Calomiris and Schweikart (1989)) historically banks used suspension of convertibility to forestall economy-wide liquidation. Historical studies often view panics as resulting from readily observable aggregate shocks to the banking system, and suspension of convertibility or other mutual insurance relations as the banking system's response.

Extending the analysis to a multi-bank environment would allow investigation of many aspects of banking panics propounded in historical studies. For example, prior to the Panic of 1857, the high-risk railroad bond market collapsed, resulting in the insolvency of many bond brokers with large outstanding bank debt. In such circumstances, depositors may have incentives to withdraw funds from banks that are actually unaffected by the disturbance, and such disintermediation imposes costs on the banking system as a whole.

To generate suspensions through this sort of signal-extraction problem in our framework, one would have to begin with a multi-bank environment. Add to our model a random disturbance that uninformed depositors can observe in aggregate at no cost, but because they have not invested in information about individual bank portfolios, they cannot tell which of their banks is most at risk (cf. Gorton (1989)). This seems a fruitful means for integrating our model's emphasis on the disciplinary rule of contingent liquidation of individual banks with the use of bank suspension in response to economy-wide shocks.

6 Appendix

Proof of Lemma 1. From the definition of ξ we get

$$\begin{aligned}\frac{\xi(X)}{\xi(Y)} &= \frac{\int \max\{x + r - zs\bar{R}, 0\}\, dF^X(x)}{\int \max\{x + r - zs\bar{R}, 0\}\, dF^Y(x)} \\ &= \frac{\int \max\{\frac{x}{s} + r - z\bar{R}, 0\}\, dF^X(x)}{\int \max\{\frac{x}{s} + r - z\bar{R}, 0\}\, dF^Y(x)}. \end{aligned} \tag{6.1}$$

Since the projects in a type Y portfolio are completely correlated it follows that the denominator is equal to $\int \max\{x + r - z\bar{R}, 0\}\, dD(x)$, where D is the distribution of one project. Further, by the law of large numbers it follows that the numerator converges to $\max\{E(x) + r - z\bar{R}, 0\}$, where the expected value is taken with respect to the distribution of D. Since $x \mapsto \max\{x + r - z\bar{R}\}$ is a strictly convex function somewhere in the support of D it follows that

$$\max\{E(x + r - z\bar{R}, 0\} < E(\max\{x + r - z\bar{R}, 0\}. \tag{6.2}$$

(6.1) and (6.2) imply Lemma 1.

Proof of Lemma 2. If all zs depositors follow this strategy then the bank solves

$$p \in \arg\max_{p\in[0,1]} (1 - p)\xi(X) + p(1 - \pi)^{zs}\xi(Y). \tag{6.3}$$

As in the single-depositor case, the existence of the demandable-debt equilibrium requires randomization by the banker in choosing the project. Certainty of both monitoring and low-risk project choice cannot be an equilibrium. For randomization and cheating to occur in equilibrium, the banker must be indifferent over the choice of p. From expression (6.3) this implies that

$$(1 - \pi)^{zs} = \frac{\xi(X)}{\xi(Y)}. \tag{6.4}$$

For given z let π be the solution of (6.4). From the previous lemma recall that the right side of this equation is less than one so the solution lies strictly between zero and one.

The behavior specified for the depositors satisfies the depositors' incentive conditions if

$$\alpha_n = (1 - p)\beta_{nX} + p\beta_{wY} - m \geq \alpha_w.$$

By the definition of α_n we therefore get

$$p(\beta_{wY} - \beta_{nY}) = m, \tag{6.5}$$

and

$$(1 - p)(\beta_{nX} - \beta_{wX}) \geq m; \tag{6.6}$$

where β_{wY} and β_{nY} depend on the distribution of W, which is binomial with parameters π and zs. If m is small, it is possible to choose p so that (6.5) and (6.6) hold provided

$$\beta_{wY} > \beta_{nY}$$

and

$$\beta_{nX} > \beta_{wX}.$$

As s becomes large, $\beta_{nX} - \beta_{wX} \rightarrow \bar{R} - \bar{T}$. As s becomes large, the probability of $W = 0$ approaches $\bar{\xi}$, so $\beta_{wY} > \beta_{nY}$ provided that $\bar{T}$ is greater than ψ.[23] It is always possible to choose $\bar{R}$ and $\bar{T}$ so that these constraints are satisfied. This proves Lemma 2.

Proof of Lemma 3. By assumption, if liquidation does not occur, the contract becomes a simple debt contract. Lemma 1 of Krasa and Villamil (1990) now immediately implies that ζ_n, ζ_w, and ξ are continuous functions of $\bar{R}$. Therefore β_{wq}, β_{nq}, and α_w, α_n are all continuous functions of $(p, \pi, \bar{R}, \bar{T})$, because they are linear combinations of ξ, ζ_n and ζ_w.

Let $\mathcal{C}$ be the set of all combinations $(p, \pi, \bar{R}, \bar{T})$ for which intermediation is feasible—that means (6.4), (6.5), and (6.6) are fulfilled, and the right side of (4.3), which is the utility of a depositor, is at least $\bar{u}$. We have to show that $\mathcal{C}$ is compact. Clearly $\mathcal{C}$ is bounded. Both sides of (6.4) and (6.5) are continuous. Furthermore (6.6) and the right side of (4.3) are continuous. Therefore $\mathcal{C}$ is closed. It now remains to prove that $\mathcal{C}$ is non-empty for all sufficiently large s and for all sufficiently small m: Clearly, it is possible to find $(p, \pi, \bar{R}, \bar{T})$ that fulfill (6.3), (6.5) and (6.6). It remains to prove that the utility of depositors is at least $\bar{u}$ for these contracts and strategies. If m is small then the probability of cheating by the bank has to be small because of (6.5). This implies that the delegated monitoring costs will be small as $s \rightarrow \infty$. Since the costs of lack of diversification remain large, intermediation

[23] $\bar{T}$ greater than ψ also assures that all non-monitoring depositors attempt follow-up withdrawals.

will dominate direct investing (cf. Diamond (1984)). Since $\bar{u}$ is the utility of a direct investor this proves that $\mathcal{C}$ is non-empty.

We now choose a contract from the set $\mathcal{C}$ which maximizes the profit of the bank. This is possible since, as a consequence of the continuity of ξ, the profit of the bank (right side of (4.4)) is also continuous in $(p, \pi, \bar{R})$. We are therefore maximizing a continuous function on a compact set. This proves the Lemma.

Proof of Lemma 4. The probability that k of z depositors monitor is $\binom{z}{k}\pi^k(1-\pi)^{z-k}$. Hence,

$$P_C = \sum_{k=C}^{z} \binom{z}{k} \pi^k (1-\pi)^{z-k} \tag{6.7}$$

is the probability that at least C depositors monitor. For every $\varepsilon > 0$ we get $|\pi - \frac{a}{z}| \leq \frac{\varepsilon}{z}$, for all sufficiently large z because of (6.4). Hence, (6.7) implies

$$P_C \leq \sum_{k=C}^{z} \binom{z}{k} \left(\frac{a+\varepsilon}{z}\right)^k \left(1 - \frac{a-\varepsilon}{z}\right)^{z-k}. \tag{6.8}$$

Since $\left(1 - \frac{a-\varepsilon}{z}\right)^{z-k} \leq 1$, and since $\binom{z}{k} \leq \frac{z^k}{k!}$, (6.8) implies that

$$\lim_{C\to\infty} P_C \leq \lim_{C\to\infty} \sum_{k=C}^{\infty} \frac{(a+\varepsilon)^k}{k!}. \tag{6.9}$$

Since the series at the right-hand side of the inequality (6.9) converges, it follows the $\lim_{C\to\infty} P_C = 0$. This proves the Lemma.

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