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Abstract

Are business cycles mainly the result of permanent shocks to productivity? This paper uses a long-run restriction implied by a large class of real business cycle models -- identifying permanent productivity shocks as shocks to the common stochastic trend in output, consumption and investment -- to provide new evidence on this question. Econometric tests indicate that this common stochastic trend/cointegration implication is consistent with postwar U.S. data. However, in systems with nominal variables, the estimates of this common stochastic trend indicate that permanent productivity shocks typically explain less than half of the business cycle variability in output, consumption and investment. (JEL 131,211)

A central, surprising and controversial result of some current research on real business cycles is the claim that a common stochastic trend -- the cumulative effect of permanent shocks to productivity -- underlies the bulk of economic fluctuations. If confirmed, this finding would imply that many other forces have been relatively unimportant over historical business cycles, including the monetary and fiscal policy shocks stressed in traditional macroeconomic analysis. This paper shows that the hypothesis of a common stochastic productivity trend has a set of econometric implications that allows us to test for its presence, measure its importance and extract estimates of its realized value. Applying these procedures to consumption, investment and output for the postwar U.S., we find results that both support and contradict this claim in the real business cycle literature. The U.S. data *are* consistent with the presence of a common stochastic productivity trend. Such a trend *is* capable of explaining important components of fluctuations in consumption, investment and output in a three variable reduced form system. But the common trend's explanatory power drops off sharply when measures of money, the price level and the nominal interest rate are added to the system. The key implication of the standard real business cycle model -- that permanent productivity shocks are the dominant source of economic fluctuations -- is not supported by these data. Moreover, our empirical results cast doubt on other explanations of the business cycle: estimates of permanent nominal shocks, which are constrained to be neutral in the long-run, explain little real activity.

Our econometric methodology can determine the importance of productivity shocks within a wide class of real business cycle (RBC) models with permanent productivity disturbances. To explain why this is so, we begin by discussing three features of the research on which our analysis builds. First, there is a long tradition of empirical support for balanced growth in which output, investment and consumption all display positive trend growth but the consumption-output and investment-output "great ratios" do not (see, for example, Robert Kosobud and Lawrence Klein (1961)). Second, in large part because of this ratio stability, most

RBC models are one sector models which restrict preferences and production possibilities so that "balanced growth" occurs asymptotically when there is a constant rate of technological progress. Third, these RBC models imply that permanent shifts in productivity will induce (i) long-run equiproportionate shifts in the paths of output, consumption and investment; and (ii) dynamic adjustments with differential movements in consumption, investment and output.

The econometric procedures developed here use the models' long-run balanced growth implication to isolate the permanent shocks in productivity, and then to trace out the short-run effects of these shocks. These econometric procedures rely on the fact that balanced growth under uncertainty implies that consumption, investment and output are cointegrated in the sense of Robert Engle and Clive Granger (1987). In turn, this means that a cointegrated vector autoregression (VAR) nests log-linear approximations of all RBC models that generate long-run balanced growth. Our empirical analysis is based on such a cointegrated VAR (or vector error correction model), which is otherwise unrestricted by preferences or technology. Thus, our conclusions can be interpreted as casting doubt on the strong claims emerging from an entire class of real business cycle models.

The empirical analysis is structured around three questions. First, what are the cointegration properties of postwar U.S. data, and are these properties consistent with the predictions of balanced growth? Second, how much of the cyclical variation in the data can be attributed to innovations in the common stochastic trends? Third, a natural alternative to RBC models is one in which nominal variables play an important role. Do innovations associated with nominal variables explain important cyclical movements in the real variables?

The empirical results provide robust answers to these questions. First, cointegration tests and estimated cointegrating vectors indicate that the data are consistent with the balanced growth hypothesis. Second, in a three variable model incorporating output, consumption and investment, the balanced growth shock explains 60-75% of the variation of output at business cycle horizons (4 to 20 quarters). Moreover, the estimated response of the real variables to the balanced growth shock is similar to the dynamic multipliers implied by simple RBC models driven by random walk productivity. Third, these results change significantly when nominal variables are added to the empirical model. When money, prices, and interest rates are added, the balanced growth shock explains less of the fluctuations in output, 35% to 44%, depending on the particular specification used. Permanent nominal shocks, identified by imposing long-run neutrality for output, explain little of the variability in the real variables. Much of the short-run variability in output and investment is associated with a shock that has a persistent effect on real interest rates. These results suggest that models that rely solely on permanent productivity and/or long-run neutral nominal shocks are not capable of capturing important features of the postwar U.S. experience.

The paper is organized as follows. Section II provides theoretical background and reviews recent work on real business cycles. Section III outlines the empirical model and discusses identification. Sections IV and V present the empirical results. Section VI concludes.

II. Growth and Fluctuations: Theoretical Background

To fix some ideas and notation, this section outlines a simple real business cycle model with permanent productivity shocks. The model is of the general class put forward by Fynn Kydland and Edward Prescott (1982) and is detailed in Robert King, Charles Plosser, and Sergio Rebelo (1988). Output, Y_t, is produced via a constant returns to scale Cobb-Douglas production function:

$$(2.1) \qquad Y_t = \lambda_t K_t^{1-\theta} N_t^{\theta}$$

where K_t is the capital stock and N_t represents labor input. Total factor productivity, λ_t, follows a logarithmic random walk:

$$(2.2) \qquad \log(\lambda_t) = \mu_\lambda + \log(\lambda_{t-1}) + \xi_t$$

where the innovations, $\{\xi_t\}$, are independent and identically distributed with a mean of 0 and a variance of σ^2. The parameter μ_λ represents the average rate of growth in productivity; ξ_t represents deviations of actual growth from this average.

Within the basic neoclassical model with deterministic trends, it is familiar -- from Robert Solow (1970) -- that per capita consumption, investment and output all grow at the rate μ_λ/θ in steady state.[1] The common deterministic trend implies that the "great ratios" of investment and consumption to output are constant along the steady state growth path. When uncertainty is added, realizations of ξ_t change the forecast of trend productivity equally at all future dates: $E_t\log(\lambda_{t+s})=E_{t-1}(\lambda_{t+s})+\xi_t$. A positive productivity shock raises the expected long-run growth path: there is a common stochastic trend in the logarithms of consumption, investment and output. The stochastic trend is $\log(\lambda_t)/\theta$ and its growth rate is $(\mu_\lambda+\xi_t)/\theta$, the analogue of the deterministic model's common growth rate restriction, μ_λ/θ. With common stochastic trends the great ratios C_t/Y_t and I_t/Y_t become stationary stochastic processes.

These theoretical results have a natural interpretation in terms of cointegration. Let X_t be a vector of the logarithms of output, consumption and investment at date t, denoted by y_t, c_t and i_t. Each component of X_t is integrated of order one (I(1) or, loosely speaking, "nonstationary") because of the random walk nature of productivity. Yet, the balanced growth implication of the theory implies that the difference between any two elements of X_t is integrated of order zero (I(0) or "stationary"). In Engle and Granger's (1987) terminology, the two linearly independent cointegrating vectors, $\alpha_1=(-1, 1, 0)'$ and $\alpha_2=(-1, 0, 1)'$ isolate stationary linear combinations of X_t corresponding to the logarithms of the balanced growth great ratios.

In this basic one-sector model and variants of it, the precise dynamic adjustment process to a permanent productivity shock depends on the details of preferences and technology. For example, recent RBC research has studied alterations in the investment technology (time-to-build, adjustment costs, inventories), the production technology (variable capacity utilization, labor indivisibilities, employment adjustment costs), preferences (nonseparabilities in leisure, durable consumption goods) and serial correlation in the productivity growth process. Two general properties emerge from these investigations. First, the productivity shock sets off transitional dynamics, as capital is accumulated and the economy moves towards a new steady state. During this transition, work effort and the great ratios change temporarily. Second, there is a common stochastic trend in consumption, investment and output arising from productivity growth.[2] These two properties motivate the econometric theory and empirical research described in the next sections.

In systems that incorporate both real and nominal variables, additional cointegrating relations may plausibly arise. Two are relevant for our empirical analysis. The first is the money demand relation:

$$m_t - p_t = \beta_y y_t - \beta_R R_t + v_t \tag{2.3}$$

where m_t-p_t is logarithm of real balances, R_t is the nominal interest rate and v_t is the money demand disturbance. The second is the conventional Fisher relation:

$$R_t = r_t + E_t \Delta p_{t+1} \tag{2.4}$$

where r_t is the *ex-ante* real rate of interest, p_t is the logarithm of the price level and $E_t \Delta p_{t+1}$ denotes the expected rate of inflation between t and t+1. If real balances, output and interest rates are I(1), while the money demand disturbance in (2.3) is I(0), then real balances, output and nominal interest rates are cointegrated. If the real rate is I(0) and the inflation rate is I(1), then (2.4) implies that nominal interest rates and inflation are cointegrated. The empirical analysis investigates these cointegrating relations and isolates the common stochastic trends that they imply.

III. Econometric Methodology

This section provides an overview of the econometric techniques used to answer the questions posed in the introduction. The first question, concerning the integration and cointegration properties of the data, can be addressed using techniques that are now familiar. This section therefore focuses on the specification of an econometric model in which the trends and their impulse response functions can be identified and estimated.

Let X_t denote an n×1 vector of time series. The individual series are assumed to be I(1) (so that they must be differenced before they are stationary) and to have the Wold representation:

$$(3.1) \qquad \Delta X_t = \mu + C(L)\epsilon_t$$

where ϵ_t is the vector of one-step-ahead forecast errors in X_t given information on lagged values of X_t. The ϵ_t's are serially uncorrelated with a mean of zero and covariance matrix Σ_ϵ. Equation (3.1) is a reduced form relation, and except for purposes of forecasting, is of little inherent interest. What is of interest is the set of structural relations that leads to (3.1), and the primary purposes of this section are to discuss (i) how the balanced growth and other cointegration restrictions outlined in the last section restrict this set of structural relations, and (ii) how these restrictions can be exploited to draw inferences about structural relations from consistent estimators of C(L) and Σ_ϵ.

To be specific, consider a structural model of the form:

$$(3.2) \qquad \Delta X_t = \mu + \Gamma(L)\eta_t$$

where η_t is a n×1 vector of serially uncorrelated structural disturbances with a mean of zero and a covariance matrix Σ_η. The reduced form of (3.2) will be of the form (3.1) with $\epsilon_t=\Gamma_0\eta_t$ and $C(L)=\Gamma(L)\Gamma_0^{-1}$.[3]

The identification problem can now be stated as follows: under what conditions is it possible to deduce the structural disturbances η_t and matrix of lag polynomials $\Gamma(L)$ from the reduced form errors ϵ_t and matrix of lag polynomials $C(L)$? In the classic literature on simultaneous equation models, the identification problem is solved by postulating that certain blocks of $\Gamma(L)$ are zero, so that some of the X's are exogenous or predetermined. In linear rational expectations models, the identification problem is solved by imposing cross-equation restrictions on the various elements of $\Gamma(L)$, as described, for example in Kenneth Wallis (1980) and Lars Hansen and Thomas Sargent (1980). The literature on vector autoregressive models addressed the identification problem by imposing restrictions on the covariance matrix Σ_η and the matrix of structural impact multipliers, Γ_0. For example, in his classic paper on vector autoregressions, Christopher Sims (1980) assumes that Σ_η is diagonal and Γ_0 is lower triangular, assumptions analogous to a Wold causal chain; Olivier Blanchard and Mark Watson (1986) modify Sims' original procedure by imposing restrictions on Γ_0 analogous to those appearing in static simultaneous equation models.

In this paper, identification is achieved through two sets of restrictions. First, the cointegration restrictions impose constraints on the matrix of long-run multipliers $\Gamma(1)$ $(=\sum_{i=0}^{\infty}\Gamma_i)$ in (3.2). This identifies the permanent components. Second, the innovations in the permanent components are assumed to be uncorrelated with the innovations to the remaining transitory components. This identifies the dynamic response of the economic variables to the permanent innovations. A concise algebraic summary of the identification scheme is given in the appendix, with a more extensive discussion in King, Plosser, James Stock and Watson (1987); here we outline the major ideas and relate our procedure to other recent work.

Consider the three variable model with $X_t=(y_t,c_t,i_t)$. Because there are two cointegrating vectors, there is only one permanent innovation, the balanced growth innovation η_t^1. This shock corresponds to ξ_t in the neoclassical model of Section II. The other two shocks, η_t^2 and η_t^3 have only transitory effects on X_t. Thus, the first identification restriction (the balanced growth cointegrating vectors) implies that the matrix of long-run multipliers is

$$\Gamma(1) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{3.3}$$

where the values of the coefficients in the first column of $\Gamma(1)$ are normalized to 1 to fix the scale of η_t^1. Equation (3.3) serves to identify the balanced growth shock as the common long-run component in X_t, since the innovation in the long-run forecast of X_t is $(1\ 1\ 1)'\eta_t^1=C(1)\epsilon_t$, which can be calculated directly from the reduced form (3.1). The second restriction -- that η_t^1 is uncorrelated with η_t^2 and η_t^3 -- is used to determine the dynamic effect of η_t^1 on X_t, that is, to identify the first column of $\Gamma(L)$. The reason this assumption is needed is clear: the impulse responses given by the first column of $\Gamma(L)$ are the partial derivatives of ΔX_{t+k} with respect to η_t^1. The second restriction specifies what is being held constant in computing these second derivatives.

Another way to motivate these identifying restrictions is to rewrite the model in terms of the stationary variables $Z_t=(\Delta y_t,c_t-y_t,i_t-y_t)'$. The productivity shock, η_t^1, has a long-run effect on y_t, but no long-run effect on the stationary ratios, c_t-y_t and i_t-y_t. Thus, η_t^1 can be identified as the innovation in the long-run component of the first element of Z_t. The other two disturbances, η_t^2 and η_t^3, have temporary effects on y_t and the ratios.

Olivier Blanchard and Danny Quah (1989) used a special case of this identification scheme to analyze $\tilde{Z}_t=(\Delta y_t,u_t)$, where u_t was the unemployment rate, which they assumed to be I(0). Their disturbances were $\tilde{\eta}_t^1$, a "supply shock," and $\tilde{\eta}_t^2$, a "demand shock." These shocks were restricted to be uncorrelated, and only the supply shock, $\tilde{\eta}_t^1$, was allowed to have a long-run effect on y_t. Thus, except for the fact that their system is bivariate and ours is trivariate, the identifying restrictions are identical. Indeed, if we eliminate one of the ratios, so that our model is bivariate, our identifying restrictions are formally equivalent to those used by Blanchard and Quah.

This equivalence highlights what we consider to be two practical advantages of the empirical specification employed here. First, work in the tradition of Milton Friedman (1957) links consumption to permanent income. This suggests that the emphasis on real flow variables, rather than on unemployment and output as in Blanchard and Quah, arguably will result in better estimates of the trend components of output and the parameters of the structural model.[4] Second, our application is to multivariate systems rather than bivariate systems. This has two advantages: first, more macroeconomic variables are used to estimate the common trends, and second, by allowing for a wider range of shocks, a richer set of alternative models is considered.

To introduce nominal shocks, the three-variable real model is augmented by real balances, nominal interest rates and inflation. The resulting six variable model has three common stochastic trends, and this makes identification more complicated since the individual permanent innovations must be sorted out. We use a version of Sims' (1980) procedure for this purpose.

The general indentification problem can be described as follows. Suppose that there are k common stochastic trends driving the n×1 vector X_t. Partition the vector of structural disturbances η_t into two components, $\eta_t=(\eta_t^{1\prime}\ \eta_t^{2\prime})'$, where η_t^1 contains the disturbances that have permanent effects on the components of X_t and η_t^2 are disturbances that have only temporary effects. (In our six variable model k=3, and η_t^1 is a 3×1 vector containing the balanced growth shock, a long-run neutral inflation shock, and a real interest rate shock.) Partition $\Gamma(1)$ conformably with η_t as $\Gamma(1)=[A\ \underline{0}]$, where A is the n×k matrix of long-run multipliers for η_t^1 and $\underline{0}$ is an n×(n-k) matrix of zeros corresponding to the long-run multipliers for η_t^2. The matrix of long-run multipliers is determined by the condition that its columns are orthogonal to the cointegrating vectors, and $A\eta_t^1$ represents the innovations in the long-run components of X_t.

Identification of the individual elements of η_t^1 becomes more complicated when there is more than one permanent innovation because the unique influence of each permanent component needs to be isolated. Formally, while the cointegration restrictions identify the permanent innovations $A\eta_t^1$, they fail to identify η_t^1 because $A\eta_t^1=(AP)(P^{-1}\eta_t^1)$ for any nonsingular matrix P. The following restrictions are used to identify the model. First, as in the model with k=1, we assume that η_t^1 and η_t^2 are uncorrelated. Second, the permanent shocks, η_t^1, are assumed to be mutually uncorrelated. Third, A is assumed to be lower triangular, which permits writing $A=\tilde{A}\Pi$, where $\tilde{A}$ is a matrix with no unknown parameters (analogous to the vector of 1's in the k=1 model) and Π is a k×k lower triangular matrix. As will become clear in the next section, $\tilde{A}$ can be chosen in a way that associates each shock with a familiar economic mechanism: the first disturbance is interpreted as a balanced growth shock, the second is a long-run neutral inflation shock and the third is a permanent real interest rate shock. Finally, the constrained reduced form is estimated as a VAR with error correction terms, i.e., a vector error correction model (VECM).

IV. Empirical Results

IV.a The Data

The data are quarterly U.S. observations on real aggregate national income account flow variables, the money supply, inflation and a short term interest rate. The three aggregate real flow variables are the logarithms of per capita real consumption expenditures (c), per capita gross private domestic fixed investment (i) and per capita "private" gross national product (y), defined as total gross national product less real total government purchases of goods and services. The measure of the money supply used is M2 (per capita in logarithms, m). The price level is measured by the implicit price deflator for our measure of private GNP (in logarithms, p) and the interest rate (R) is the three-month U.S. Treasury bill rate. Regressions were run over 1949:1 through 1988:4 for statistical procedures that involve only real flows. To avoid observations that occurred during periods of price controls, the Korean War, and the Treasury-Fed accord, the regressions were run over 1954:1 to 1988:4 when money, interest rates, or prices are involved. Data prior to 1949:1 (respectively 1954:1) were used as initial observations in regressions that contain lags.[5]

Because the national product measure is not the standard one we have graphed the logarithm of the real variables (y, c, i and m-p) in Figure 1a. These plots show the familiar growth and cyclical characteristics of the data. Output, consumption and investment display strong upward trends. Investment is the most volatile component, followed by output and then consumption. Real balances (m-p) also display an upward trend. Figure 1b plots the logarithm of the consumption output ratio (c-y) and the logarithm of the investment ratio (i-y). Over the postwar period, these ratios display the stability reported by prior researchers; it is easy to view them as fluctuating around a constant mean. This suggests that the growth evident in Figure 1 occurs in a manner that is "balanced" between investment and consumption.

IV.b Integration and Cointegration Properties of the Data

Univariate analysis of these six variables indicates that the real flow variables, y, c and i can be characterized as I(1) processes with positive drift, and that R, price inflation (Δp) and nominal money growth (Δm) can be characterized as I(1) processes without drift. These results are consistent with the large literature on the "unit root" properties of U.S. macroeconomic time series.[6]

The balanced growth conditions also appear consistent with the data: we can reject the presence of unit root components in the great ratios. Augmented Dickey-Fuller t-statistics ($\hat{\tau}_{\tau}$, with 5 lags) testing for a unit autoregressive root in c-y and i-y have values of -4.21 and

-3.99 respectively; both are significant at the 1% level, suggesting that (c,y) and (i,y) are cointegrated. The log of real balances, m-p, appears to be an I(1) process with drift, even though both m and p can be characterized as I(2) processes. (The $\hat{\tau}_{\tau}$ statistic for Δ(m-p) is significant at the 1% level). The best characterization of the real interest rate is unclear. The *ex-post* real rate, R-Δp, has sample AR(1) coefficient equal to .86, suggesting stationary behavior, but the augmented Dickey-Fuller t-statistic ($\hat{\tau}_{\mu}$, with 5 lags) is only -1.8, which is consistent with one unit root in the real rate.

Tables 1-3 present a variety of statistics calculated from multivariate systems, starting with the results for the three variable (y,c,i) model. Panel A of Table 1 shows the largest eigenvalues from the companion matrix of an estimated VAR(6). The one common stochastic trend (balanced growth) model implies that the companion matrix should have one unit eigenvalue, corresponding to the common trend, and all the other eigenvalues should be less than one in modulus. The point estimates accord with this prediction. Panel B presents formal tests for cointegration using procedures developed by Soren Johansen (1988) and Stock and Watson (1988). Both procedures take as their null hypothesis that the data are integrated but not cointegrated, so that there are three unit roots in the companion matrix. The first two rows of Panel B test this against an alternative of at most 2 unit roots, while the third row tests the null against the sharper alternative of at most 1 unit root. The results are consistent with the one unit root (one common trend) specification.[7]

The final panel in Table 1 presents maximum likelihood estimates of the cointegrating vectors, conditional on the presence of one unit root in the VAR, computed using the dynamic OLS procedure of Stock and Watson (1989). The point estimates are close to (1,0,-1) and (0,1,-1), the values that imply balanced growth in output, consumption and investment. These balanced growth restrictions impose two constraints on the cointegrating vectors. In Table 1, these restrictions are tested using a Wald statistic based on the dynamic OLS point estimates and standard errors; under the null hypothesis, this statistic has an asymptotic chi-squared distribution with two degrees of freedom. Although the restriction is rejected at the 10% (but not the 5%) level, the estimated cointegrating vector is broadly consistent with the balanced growth prediction.[8]

Table 2 explores two sets of cointegrating relations suggested by the nonstationarity of the nominal and real interest rates. The top panel of the table reports an estimated cointegrating relation between real balances, output and nominal interest rates.[9] The estimate of the long-run income elasticity is close to unity (although statistically significantly larger than 1); the estimated interest rate semi-elasticity is small, but statistically significantly less than zero.

The second issue examined in Table 2 is the possibility that the consumption/output and investment/output ratios might exhibit permanent shifts resulting from permanent shifts in real rates. Estimated bivariate cointegrating relations $(c-y)=\phi_1(R-\Delta p)$ and $(i-y)=\phi_2(R-\Delta p)$ are shown in panel B of Table 2. As predicted by the long-run theory of the growth model, for example, a higher real interest rate lowers the share of product going into investment and, symmetrically, raises the share of consumption. However, the long-run effects are imprecisely estimated and small: a permanent increase in the annual real rate of one percentage point is associated with an increase in the consumption-output ratio of 0.3 percentage points.

The cointegration properties of the six variable system $(y,c,i,m-p,R,\Delta p)$ are investigated in Table 3. The theoretical analysis suggests 3 stochastic trends in the system -- a balanced growth trend, an inflation/money growth trend and, possibly, a real interest rate stochastic trend. Equivalently, three cointegrating relations should be present in the system -- two (interest rate adjusted) balanced growth relations and a long-run money demand relation. The $q_\tau^f(6,3)$ statistic reported at the bottom of Table 3, panel A provides some evidence for the three-trend specification, rejecting six unit roots against three with a p-value of 11%.

The Wald tests in panel B of Table 3 investigate various hypotheses about the cointegrating vectors, under the maintained hypothesis that the number of cointegrating vectors is correctly specified. The first hypothesis (panel B, row 1) is that the cointegrating vectors are the balanced growth and money demand cointegrating vectors. This hypothesis is rejected at the 10%, but not the 5% level. Despite this formal rejection at the 10% level, we interpret the balanced growth/money demand cointegrating restrictions as providing a good qualitative description of the cointegrating vectors for the system. The remaining lines of Table 3, panel B investigate alternative cointegration restrictions. There is strong evidence against a fourth cointegrating vector implying stationary real rates (line 2) and against the stationary velocity model, even permitting cointegration between the great ratios and real rates (line 4). The evidence is weakest against the hypothesis that the great ratios and real rates are cointegrated, combined with the money demand cointegrating vector (line 3).[10]

Taken together, these results suggest that a money demand cointegrating relation is consistent with the observed behavior of the time series. There is some evidence that the shares of consumption and investment move with permanent shifts in the real rate. Yet, this effect is negligibly small in the long-run, and the hypothesis of "balanced growth" also appears generally consistent with the data.

IV.c A Three Variable System of Real Flow Variables

The results for the three-variable system are based on an estimated VECM using eight lags of the first differences of y, c and i with an intercept and the two theoretical error correction terms, y-c and y-i. The only identifying assumption needed to analyze the dynamics of the system is that the permanent shock is uncorrelated with the transitory shocks. The impulse response of y, c and i to a one standard deviation innovation in the common trend is plotted in Figure 2, together with one standard deviation confidence bands.[11]. (The standard deviation of the balanced growth shock is 0.7%, and, as discussed in Section III, the system is normalized so that a one unit innovation eventually leads to a one unit increase in y, c, and i.) In response to a shock that leads to a one percent permanent increase in y, c and i, output and investment increase by more than one percent in the near term (one to two years), while consumption moves less. Most of the adjustment is completed within four years. The results for c and i are consistent with the simple theoretical model discussed in Section II where the capital stock rapidly increases at the short-run cost of consumption.

Are these responses large enough to explain a substantial fraction of the short-run variation in the data? This key question is addressed in Table 4, which shows the fraction of the variance of the forecast error variance attributed to innovations in the common stochastic trend, at horizons of one to twenty-four quarters. These variance decompositions suggest that innovations in the permanent component appear to play a dominant role in the variation in GNP and consumption. At the one to four quarter horizon, the point estimates suggest that 45% to 58% of the fluctuations in private GNP can be attributed to the permanent component. This increases to 68% at the two year horizon and to 81% at the six year horizon. The results for consumption are broadly similar. Notably, the permanent component explains a much smaller fraction of the movements in investment: only 31% at the one year horizon, increasing to 47% at the six year horizon.

This evidence suggests the existence of a persistent, potentially permanent, component which shifts the composition of real output between consumption and investment. (If there were temporary components with negligible effect on forecast errors after three or more years, then the population counterparts of the variance ratios in Table 4 would increase more sharply at the longer horizons.) Thus, the results motivate us to investigate the possibility of additional permanent components.

IV.d Six-Variable Systems with Nominal Variables

Augmenting output, consumption and investment by real balances, nominal interest rates and inflation yields a six variable system. The results of Section IV.b suggest that a reasonable specification incorporates three cointegrating relations (and thus three common trends) among the six variables. We have estimated a variety of six variable models, with different numbers of trends, different cointegrating relations and different ordering of the shocks. The detailed results for a benchmark model are reported in this subsection, and the results for the other models are summarized in the next subsection. The benchmark model incorporates the cointegrating relations $(c-y)=\phi_1(R-\Delta p)$, $(i-y)=\phi_2(R-\Delta p)$ and $m-p=\beta_y y-\beta_R R$. The first two relations link variation in the real ratios to permanent shifts in the real interest rate, although the estimates reported above suggest that this effect is small. The third implies that "money demand" disturbances are I(0). The estimates of ϕ_1, ϕ_2, β_y and β_R are the restricted dynamic OLS estimates given in Table 2.

The permanent components and their impulse responses are identified by specifying a structure for the matrix of long-run multipliers. In the notation of Section III, with $X=(y,c,i,m-p,R,\Delta p)$, the particular structure adopted is:

$$(4.1) \qquad A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & \phi_1 \\ 1 & 0 & \phi_2 \\ \beta_y & -\beta_R & -\beta_R \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \pi_{21} & 1 & 0 \\ \pi_{31} & \pi_{32} & 1 \end{bmatrix}$$

The 6×3 matrix A is the matrix of long-run multipliers from the three permanent shocks. In the notation of Section III, the two matrices on the right hand side of (4.1) are $\tilde{A}$ and Π respectively.

Our interpretation of the shocks follows from the structure placed on the long-run multipliers in (4.1). The first shock is a real balanced growth shock, since it leads to a unit long-run increase in y, c and i. Through the money demand relation, it also leads to a β_y increase in real balances. The second shock is a neutral inflation shock. It has no long-run effect on y, c, or i, and has a unit long-run effect on inflation and nominal interest rates. Further, the unit increase in nominal interest rates arising from this shock leads to reduction of real balances of β_R. The final permanent shock is a real interest rate shock. A one unit increase in this shock leads to a change of ϕ_1 and ϕ_2 in c-y and i-y. There is also a one unit increase in nominal interest rates and a decrease of β_R in real balances. The coefficients in Π are determined by the requirement that the permanent innovations are mutually uncorrelated. In the standard VAR terminology, following Sims (1980), the balanced growth disturbance is ordered first, the inflation disturbance is second and the real rate shock is third.

The model is estimated using a VECM with eight lags and the three error correction terms implied by the cointegrating relations. Table 5 presents the variance decompositions of the forecast errors from the benchmark model. Four aspects of this table are of particular interest. First, relative to the three variable model, the real or "balanced growth" shock is *less* important for output and consumption, especially at the one to four quarter horizon. At the three to five year horizon, however, this shock has important explanatory power: roughly one half the variation in these forecast errors is attributable to the first permanent component. Second, including the additional shocks in this expanded model does not enable the first permanent component to explain the short-run variations in investment. Third, the component associated with permanent shifts in the rate of inflation explains a considerable amount of the variation in inflation, but little else. Fourth, the component associated with permanent movements in the real interest rate explains most of the forecast errors in the nominal rate. It also is important for real activity: it explains substantial amounts of the output and investment forecast errors, particularly at the short horizons.

Figure 3 illustrates the roles played by the different shocks by plotting the forecast error at the three year horizon and the portion attributable to each stochastic trend for y, c and i. These plots highlight the negligible role of the inflation shock and the substantial role played by the balanced growth shock and the real interest rate shock. Looking at specific episodes in this figure, one finds that the balanced growth shock has particular explanatory power for the sustained growth of the 1960s. On the other hand, the real rate shock seems particularly important in the contraction of 1974 and the 1981-82 recession.

Figure 4 shows the responses of the variables to one standard deviation impulses in the balanced growth shock, the inflation shock and the real interest rate shock. The estimated standard deviations of these underlying shocks are 0.7%, 0.08% and 0.12% per quarter. The response of output to the balanced growth shock is negligible over the first few quarters, while consumption increases slightly and investment declines. After one year, major increases in output, consumption and investment are present. While these responses are smaller than those in the three variable model, they conform to how one might think a system would respond to news about technological developments. The inflation shock has very little impact on output and consumption. Investment, on the other hand, shows a large positive response to this shock for the first three quarters.

We have already noted that the real interest rate shock plays an important role in explaining the short-run behavior of output and investment. The impulse response functions make interpreting this shock as a permanent change in the real rate of interest somewhat difficult. All three of the real flow variables have an initial response to this shock that is strongly positive, before turning negative after two to three quarters. While there may be economic models that predict such responses to a permanent change in the real rate, standard ones do not.

We draw four conclusions from this analysis. First, permanent innovations account for a substantial fraction of transitory economic fluctuations. Second, the balanced growth factor retains a significant role in explaining movements at horizons greater than 2 years, although it has considerably less explanatory power in the six variable system than in the three variable system. Third, a large fraction of the short-run (0-2 year) variability in output and investment is explained by a factor that is related to persistent movements in the real rate of interest. Fourth, the impulse response functions appear consistent with the interpretation of the first shock as a real or balanced growth shock, but lead us to be uncertain about the interpretation of the third, real rate shock, at least within the context of standard macroeconomic models.

IV.e Sensitivity Analysis

It is important to explore the sensitivity of these main conclusions to changes in cointegrating vectors and changes in the ordering of the permanent components: we do this by estimating a variety of five- and six-variable models. To save space, we focus on a key measure, the fraction of the variance of the three-year-ahead forecast error in each variable explained by the balanced growth permanent innovation. The results, summarized in Table 6, suggest four conclusions. First, looking across specifications, a substantial fraction of the forecast errors in output and consumption is explained by the balanced growth innovation; the point estimates range from one-third to two-thirds. Second, in systems including nominal variables, the fraction of the forecast error variance of investment explained by the balanced growth real permanent component is never large (at most 27% in model M.1.) Third, little changes when balanced growth is imposed by setting ϕ_1 and ϕ_2 equal to zero. Fourth, changing the ordering of the shocks (for example, putting the balanced growth shock last in the Wold causal ordering, as in model M.3) does not change the main qualitative features of the results. In short, the sensitivity analysis indicates that the principle results for the base six-variable model are robust to a wide variety of changes in the identifying restrictions.[12]

V. Analysis of Trend Components of Private GNP

In the neoclassical growth framework of Section II, the common long-run movements in aggregate variables arise from changes in productivity. Is there any evidence that productivity movements are related to innovations in the balanced growth component of GNP? We investigate this by comparing these estimated innovations to a popular estimate of the change in total factor productivity in the economy, Solow's (1957) residual. If the economy can be described by a Cobb-Douglas production function -- as in the theoretical model of Section 2 -- the Solow residual has the convenient interpretation of being exactly ξ_t in (2.2). We use two measures of this productivity residual: Robert Hall's (1988, Table 1) for total manufacturing and that produced by Prescott (1986).[13]

The time path of the Solow residual and the change in the balanced growth trend component of private GNP from the 6 variable model are plotted in Figure 5a for Hall's measure and in Figure 5b for Prescott's measure. The graphs suggest a very modest relation between Hall's Solow residual and our estimated balanced growth shock (the correlation is .19) and a stronger relationship between Prescott's Solow residual and the estimated shock (the correlation is .48).

The Solow residual is an imperfect measure of technical change. For example, Prescott (1986) points to errors in measuring the variables used in its construction and Hall (1988) suggests that this measure of productivity misrepresents true technological progress in noncompetitive environments where price exceeds marginal cost. Nonetheless, these results suggest some link between the real permanent shocks from the model and the two measures of the Solow residual. These comparisons thus lend some credence to the interpretation in Section IV of the permanent real shocks as measuring economy-wide shifts in productivity.

The focus so far has been to use the empirical model to evaluate a class of real business cycle models. However, the empirical model also provides a solution to a classic problem in descriptive economic statistics: how to decompose an economic time series into a "trend" and "cyclical" component. A natural definition of the trend is the long-run forecast of the variables (see Harvey (1989), chapter 6), and some simple algebra (see the appendix) shows that changes in this trend are just linear combinations of the permanent innovations. Thus, the empirical model can be used to form a multivariate generalization of the trend-cycle decomposition proposed by Stephen Beveridge and Charles Nelson (1981).

The implied trend component of output computed using the six-variable model is shown in Figure 6 along with Edward Denison's (1985) estimate of real potential GNP per capita.[14] Despite the very different approaches used to construct the two trend estimates they are broadly similar. The three major differences between the two series are the prolonged growth of the 1960s, the 1974 contraction and the slowdown of the late 1970s.

VI. Conclusion

In this paper we analyzed the stochastic trend properties of postwar U.S. macroeconomic data to evaluate the empirical relevance of standard RBC models with permanent productivity shocks. Several aspects of these results are consistent with the central proposition of most real business cycle models. Real per capita output, consumption and investment (as well as real balances and interest rates) appear to share common stochastic trends. The cointegrating relations among the real flow variables are consistent with balanced growth; in addition, money, prices, output and interest rates are consistent with a long-run money demand cointegrating relation. In a three variable real model, innovations in the balanced growth component account for more than two-thirds of the unpredictable variation in output over the two- to five-year horizon.

Yet much evidence is at odds with one-sector RBC models in which permanent productivity changes play a major role. Even in the three-variable model, the balanced growth innovation accounts for less than two-fifths of the movements of investment at horizons up to six years. The explanatory power of the balanced growth innovation for output is reduced to under 45% by introducing nominal variables. Moreover, the explanatory power arises mainly from some specific episodes, notably the sustained growth of the 1960's. The balanced growth innovation sheds little light on other important episodes, such as 1974-75 and 1981-82. Thus, we are led to conclude that the U.S. data are not consistent with the view that a single real permanent shock is the dominant source of business cycle fluctuations.

What are the omitted sources of the business cycle. From a monetarist perspective, it is surprising that such a small role is played by the inflation shock. Accelerations and decelerations in money growth and inflation, which are assumed to have no long-run effect on real flow variables and real interest rates, explain a trivial fraction of the variability in output and consumption, and a small fraction of the variability in investment. The results point toward an additional permanent (or at least highly persistent) component associated with real interest rates, which has large effects on investment.

Appendix

This appendix presents a discussion of identification and estimation. The definitions in the text are:

(A.1) Reduced Form: $\Delta X_t = \mu + C(L)\epsilon_t$.

(A.2) Structural Model: $\Delta X_t = \mu + \Gamma(L)\eta_t$.

The identifying restrictions are:

(A.3) $\Gamma_0\eta_t = \epsilon_t$, where Γ_0^{-1} exists.

(A.4) $\Gamma(1) = [\tilde{A}\Pi \; \underline{0}]$,

where $\tilde{A}$ is a known n×k matrix with full column rank, Π is a k×k lower triangular matrix with full rank and 1's on the diagonal, and $\underline{0}$ is a n×(n-k) matrix of 0's. The covariance matrix of the structural disturbances is assumed to be:

$$(A.5) \quad \Sigma_\eta = E(\eta_t\eta_t') = \begin{bmatrix} \Sigma_{\eta 1} & 0 \\ 0 & \Sigma_{\eta 2} \end{bmatrix},$$

where Σ_η is partitioned conformably with $\eta_t=(\eta_t^{1\prime}\ \eta_t^{2\prime})'$, where η_t^1 is k×1 and η_t^2 is (n-k)×1, and where $\Sigma_{\eta 1}$ is diagonal.

Equations (A.1) and (A.2) are the definitions of the reduced form and structural models given in (3.1) and (3.2) in the text. Assumption (A.3) says that the structural disturbances are in the space spanned by current and lagged values of X_t, and that there are no singularities in the structural model. Assumption (A.4) is discussed in the text explicitly for the six-variable model. It also applies to the three-variable model also by defining $\tilde{A}$ to be a vector of 1's. In assumption (A.4) the diagonal elements of Π are normalized to unity without loss of generality, since the variances of η_t^1, in (A.5), are unrestricted.

The permanent innovations, η^1_t, can be determined from the reduced form, (A.1), as follows. From (A.1)-(A.3), $C(L)=\Gamma(L)\Gamma_0^{-1}$, so that $C(1)=\Gamma(1)\Gamma_0^{-1}$. Let D be any solution of $C(1)=\tilde{A}D$ (for example, $D=(\tilde{A}'\tilde{A})^{-1}\tilde{A}'C(1)$). Thus $\tilde{A}D\epsilon_t=\tilde{A}\Pi\eta^1_t$ so, since $E\eta^1_t\eta^{1\prime}_t=\Sigma_{\eta 1}$, $D\Sigma_\epsilon D'=\Pi\Sigma_{\eta 1}\Pi'$. Let Π^* be the unique lower triangular square root of $D\Sigma_\epsilon D'$, and let Π and $\Sigma^{½}_{\eta 1}$ be the unique solutions to $\Pi\Sigma^{½}_{\eta 1}=\Pi^*$, where Π and $\Sigma_{\eta 1}$ satisfy (A.4) and (A.5). Then $A=\tilde{A}\Pi$, and the first k rows of Γ_0^{-1} are given by $G=\Pi^{-1}D$. Since D is unique up to premultiplication by a nonsingular matrix, G is unique. Finally, $\eta_t=\Gamma_0^{-1}\epsilon_t$ implies that $\eta^1_t=G\epsilon_t$.

The dynamic multipliers associated with η^1_t are given by the first k columns of $\Gamma(L)$. These can be calculated as follows. First, write $\Gamma_0=(H\ J)$, where H is n×k and J is n×(n-k). Since $\Gamma(L)=C(L)\Gamma_0$, the first k columns of $\Gamma(L)$ are given by C(L)H. Finally, $\epsilon_t=\Gamma_0\eta_t$ implies $\Sigma_\eta\Gamma_0'=\Gamma_0^{-1}\Sigma_\epsilon$, so that from (A.5) $H'=\Sigma_{\eta 1}^{-1}G\Sigma_\epsilon$; thus, the dynamic multipliers for η^1_t are $C(L)\Sigma_\epsilon G'\Sigma_{\eta 1}^{-1}$.

Both the structural and reduced form lead naturally to the multivariate version of the Beveridge-Nelson (1981) decomposition used to estimated trend output and plotted in Figure 6. The structural form can be expressed, $X_t=X_0+\mu t+\sum_{s=1}^{t}\Gamma(L)\eta_s$ or, setting $X_0=0$, $X_t=\mu t+\Gamma(1)\sum_{s=1}^{t}\eta^1_s+\Gamma^*(L)\eta_t$, where $\Gamma^*_j = -\sum_{i=j+1}^{\infty}\Gamma_i$. Let $\tau_t=\sum_{s=1}^{t}\eta^1_s$; then this becomes, $X_t=X^p_t+X^s_t$, where $X^s_t=\Gamma^*(L)\eta_t$ is the stationary component of X_t and $X^p_t=\mu t+\Gamma(1)\sum_{s=1}^{t}\eta^1_s=\mu t+A\tau_t$ is the permanent component of X_t. By construction, X^p_t satisifies the natural notion of a trend as the infinitely long-run forecast of X, based on information through time t.

The only restrictions that the structural model places on the reduced form are the cointegration restrictions. This implies that efficient estimates of the structural model can be calculated in two steps: first, the reduced form is estimated imposing only the cointegration restrictions, and second, this estimated reduced form is transformed into the structural model using the relations given in the last two paragraphs. In all models reported in this paper, the reduced form was parameterized as a VECM (a cointegrated VAR). The estimated VECM was inverted to yield an estimate of the moving average representation of the reduced form in (A.1).

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Endnotes

1. This follows directly from the economy's commodity resource constraint $Y_t=C_t+I_t$, its investment technology, $K_{t+1}=(1-\delta)K_t+I_t$, with δ being the rate of depreciation, and the fact that the economy's allocation of time between work and leisure must be constant in steady state.

2. As one example of how an extension of the basic model preserves the stochastic trend implication, consider the time-to-build investment technology of Kydland and Prescott (1982). All of the stages of investment in their model inherit the common stochastic trend. Similar conclusions hold for the other examples in the text. There are two important categories of RBC models that need not display a single common stochastic trend when there are permanent productivity shocks. Multi-sector models can have separate productivity trends in each sector, as in John Long and Plosser (1983). Models of stochastic endogenous growth such as those constructed by King and Rebelo (1988) generate a stochastic trend in the level of productivity when shocks are stationary; with endogenous growth, permanent changes in taxes or in the level of exogenous productivity lead to permanent changes in the growth rates.

3. This assumes that the structural disturbances lie in the space spanned by current and lagged values of X_t.

4. This notion, that consumption might provide a good measure of permanent income, has been recently exploited by John Cochrane and Argia Sbordone (1988), Andrew Harvey and James Stock (1988), Eugene Fama (1990), and Cochrane (1990).

5. All data were obtained from Citibase. Using the Citibase mnemonics for the series, the precise definitions of the variables are GC82 (consumption), GIF82 (investment) and (GNP82-GGE82) (real private output). The Citibase M2 series (FM2) was used for 1959:1-85:4; the earlier M2 data was formed by splicing the M2 series reported in Banking and Monetary Statistics, 1941-1970, Board of Governors of the Federal Reserve System to the Citibase data in January 1959. The monthly data were averaged to obtain the quarterly observations. The price deflator was obtained as the ratio of nominal private GNP (the difference between Citibase series GNP and GGE) to real private GNP (the difference between Citibase series GNP82 and

GGE82). The interest rate is FYGM3. It is measured as an annual percentage (a typical value is 10.0%). Price inflation was also measured as an annual percentage ($400*\ln(P_t/P_{t-1})$). Output, consumption, investment and money are on per capita basis using total civilian noninstitutional population (P16).

6. Because the techniques and results are now familiar, they are omitted here; interested readers are referred to an earlier version of this paper (King, Plosser, Stock and Watson (1987)) for details.

7. The multiviarate unit root tests in Tables 1-3 are based on the $J_\mu(r)$, $J_\tau(r)$, $q_\mu^f(k,m)$, and $q_\tau^f(k,m)$ statistics. The J(r) statistics are Johansen's (1988) test of the null of r cointegrating vectors against >r cointegrating vectors, and the $q^f(k,m)$ statistics are Stock and Watson's (1988) test of the null of k unit roots in the multivariate system against the alternative of m ($m<k$) unit roots; "μ" and "τ" subscripts respectively denote the tests computed using demeaned data and data that have been both demeaned and linearly detrended. (Detrending is appropriate if the series have a nonzero drift under the null.) Asymptotic critical values for q_μ^f and q_τ^f are taken from Stock and Watson (1988). Asymptotic p-values for J_μ and J_τ differ from those tabulated in Johansen (1988) because of the demeaning/detrending. It is straightforward to derive and to compute the asymptotic null distribution of J_μ and J_τ using the results in Sims, Stock and Watson (1990). We have done this, and the p-values shown in the table are based on these asymptotic distributions. All multivariate tests were computed using six lagged levels, which these procedures parameterize as five lags in first differences and a single lagged level.

8. The cointegrating vectors reported in Tables 1-3, and the estimated cointegrating vectors used as the basis of the VAR analysis in Tables 4-6, were estimated using the Stock-Watson (1989) dynamic OLS procedure, which is asymptotically equivalent to the Gaussian maximum likelihood procedure for a triangular error correction system. If there are r cointegrating vectors, then there are r regression equations; each equation has n-r regressors in levels (where n is the number of variables), a constant, and m leads, m lags, and the contemporaneous values of the differences of right-hand side levels variables. Standard errors, calculated using a VAR(4) estimator of the spectral density matrix of the errors in these r equations, are given in

parentheses in Tables 1-3. All results are based on m=5; to allow for the leads, the dynamic OLS regressions end at 87:3 (all other regressions go through 88:4). Wald statistics computed using the estimated covariance matrix have the usual large-sample χ^2 distributions.

The log likelihoods provided in this and subsequent tables are provided as a guide for readers interested in exploring the shape of the likelihood surface. It should be stressed that, because of the hypothesized unit roots, the usual chi-squared inference does not always apply to likelihood ratio statistics computed from the reported values.

9. See Robert Lucas (1988), Dennis Hoffman and Robert Rasche (1989), and Benjamin Friedman and Kenneth Kuttner (1990) for recent empirical investigations of the stability of long run money demand.

10. To check robustness, the cointegrating vectors in Tables 1, 2, 3, and 6 were also estimated using Johansen's (1988) MLE for a VECM with 5 lagged differences, one lagged level, and a constant. The Johansen MLE point estimates (available from the authors upon request) are similar to the dynamic OLS point estimates. For example, the Johansen MLE of the money demand equation is m-p = 1.134y - .0093R (cf. Table 2, model (1)).

11. The standard errors for the impulse response functions and variance decompositions were approximated using 500 simulations as discussed by Thomas Doan and Robert Litterman (1986), page 19-4.

12. Additional sensitivity analyses were performed: substituting short-term private and long-term public interest rates for the short term public rate, dropping interest rates entirely and changing the number of lags. The results, available from the authors on request, are consistent with the summary conclusions in this and subsequent sections.

13. Because Hall's series is annual, Prescott's quarterly series was aggregated to an annual level.

14. Denison computed his measure of potential output by adjusting actual output using an Okun's law relationship, by adjusting for capacity utilization, and by making other adjustments such as for labor disputes, the weather, and the size of the armed forces. Source: Denison (1985), Tables 2-4.

Table 1
Cointegration Statistics:
3 Variable Model, (y, c, i), 1949:1 - 88:4

A. Results from Unrestricted Levels Vector Autoregressions
Largest Eigenvalues of Estimated Companion Matrix

VAR(6) with Constant			VAR(6) with Constant and Trend		
Real	Imaginary	Modulus	Real	Imaginary	Modulus
1.00	0.00	1.00	0.97	0.00	0.97
0.83	0.19	0.85	0.83	0.18	0.85
0.83	-0.19	0.85	0.83	-0.18	0.85
-0.62	0.46	0.77	-0.62	0.46	0.77
-0.62	-0.46	0.77	-0.62	-0.46	0.77
0.50	0.50	0.71	0.49	0.49	0.69
Log Likelihood: 2199.66			2200.95		

B. Multivariate Unit Root Statistics

Statistic	Value	(P-Value)	Null/Alternative
$J_\tau(0)$	35.4	(0.13)	3 unit roots/at most 2 unit roots
$q_\tau^f(3,2)$	-29.4	(0.21)	3 unit roots/at most 2 unit roots
$q_\tau^f(3,1)$	-29.4	(<.01)	3 unit roots/at most 1 unit root

Log Likelihood (3 Unit Roots): 2183.27
Log Likelihood (2 Unit Roots): 2193.11
Log Likelihood (1 Unit Roots): 2199.43
Log Likelihood (0 Unit Roots): 2200.95

C. Estimated Cointegrating Vectors

Variable	Null Hypothesis α_1	α_2	Estimates $\hat{\alpha}_1$	$\hat{\alpha}_2$
c	1	0	1.00^a	0.00^a
i	0	1	0.00^a	1.00^a
y	-1	-1	-1.058 (0.026)	-1.004 (0.038)

Wald test of balanced growth restrictions: $\chi_2^2 = 4.96$ (0.08)

Notes to Table 1: Values in parentheses are p-values (for the test statistics) or standard errors (for the estimators). The roots and likelihoods in panel A correspond to an unrestricted VAR(6) in levels. The multivariate unit root tests in panel B, which are described in detail in footnote 7, were computed from VAR(6)'s in levels. $J_\tau(r)$ is Johansen's (1988) test of the null of r cointegrating vectors against >r cointegrating vectors, and $q_\tau^f(k,m)$ is Stock and Watson's (1988) test of the null of k unit roots in the multivariate system against the alternative of m (m<k) unit roots, where "τ" denotes linear detrending. The log likelihoods in Panel B are for models that include a constant and linear time trend. Panel C reports the cointegrating vectors for (c,i,y), estimated by Stock and Watson's (1989) dynamic OLS (with a constant, 5 leads and 5 lags) procedure. t-statistics formed using the standard errors have asymptotic normal distributions. The Wald statistic, which tests that the cointegrating vectors lie in the hypothesized subspace, is computed using the dynamic OLS estimates and standard errors described in footnote 8.

Table 2

Estimated Cointegrating Vectors

A. Money Demand, 1954:1 - 88:4

(1)	$m - p = \underset{(0.062)}{1.197}\, y \underset{(0.004)}{-0.013}\, R$	$q_{\tau}^{f}(3,2) = -20.6$ (0.54)	$J_{\tau}(0) = 42.6$ (0.03)

Wald test of velocity restriction (β_y=1 and β_R=0): $\chi_2^2 = 12.7$ (<0.01)

B. Real Ratios and Real Interest, 1954:1-1988:4

(2)	$c - y = \underset{(0.0022)}{0.0033}\,(R - \Delta p)$	$q_{\tau}^{f}(2,1) = -70.0$ (<0.01)	$J_{\tau}(0) = 19.3$ (0.17)
		$q_{\mu}^{f}(2,1) = -62.2$ (<0.01)	$J_{\mu}(0) = 15.6$ (0.11)
(3)	$i - y = \underset{(0.0050)}{-0.0028}\,(R - \Delta p)$	$q_{\tau}^{f}(2,1) = -73.8$ (<0.01)	$J_{\tau}(0) = 26.9$ (0.02)
		$q_{\mu}^{f}(2,1) = -65.1$ (<0.01)	$J_{\mu}(0) = 24.8$ (0.01)

Notes to Table 2: Values in parentheses are p-values (for the test statistics) or standard errors (for the estimators). The cointegrating vectors (1)-(3) were estimated by dynamic OLS (with 5 leads and 5 lags), including a constant in the regression, equation-by-equation. The q_{μ}^{f} and J_{μ} tests are computed using demeaned data (see footnote 7). The Wald statistic is described in the notes to Table 1 and in footnote 8.

Table 3
Cointegration Statistics
6 Variable Model, $(y,c,i,m-p,R,\Delta p)$, 1954:1 - 88:4

A. Estimated Cointegrating Vectors

Variable	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$
c	1.00[a]	0.00[a]	0.00[a]
i	0.00[a]	1.00[a]	0.00[a]
m-p	0.00[a]	0.00[a]	1.00[a]
y	-1.118	-1.120	-1.152
	(0.050)	(0.083)	(0.063)
R	0.004	0.002	0.009
	(0.003)	(0.005)	(0.004)
Δp	0.004	0.006	0.002
	(0.003)	(0.004)	(0.003)

$q_\tau^f(6,3) = -27.5$ (p-value =0.11)

Log Likelihood = 2826.54

B. Tests of Restrictions on Cointegrating Vectors

Null Hypothesis	d.f.	Wald test
$(c-y)$, $(i-y)$, $m-p-\beta_y y+\beta_R R$	7	12.6 (0.08)
$(c-y)$, $(i-y)$, $m-p-\beta_y y+\beta_R R$, $R-\Delta p$	6	40.1 (<0.01)
$(c-y)-\phi_1(R-\Delta p)$, $(i-y)-\phi_2(R-\Delta p)$, $m-p-\beta_y y+\beta_R R$	5	7.6 (0.18)
$(c-y)-\phi_1(R-\Delta p)$, $(i-y)-\phi_2(R-\Delta p)$, $m-p-y$	7	42.0 (<0.01)

Notes to Table 3: Values in parentheses are p-values (for the test statistics) or standard errors (for the estimators). Panel A reports dynamic OLS estimates for the 3-equation system. Panel B reports tests of whether the cointegrating vectors fall in the hypothesized subspace, conditional on the number of cointegrating vectors. The Wald statistics are described in the notes to Table 1 and in footnote 8.

[a]normalized.

Table 4
Forecast Error Variance Decompositions:
3 variable real model (y, c, i), 1949:2 - 88:4

Fraction of the forecast error variance
atributed to the real permanent shock

Horizon	y	c	i
1.	0.45	0.88	0.12
	(0.28)	(0.21)	(0.18)
4.	0.58	0.89	0.31
	(0.27)	(0.19)	(0.23)
8.	0.68	0.83	0.40
	(0.22)	(0.18)	(0.18)
12.	0.73	0.83	0.43
	(0.19)	(0.18)	(0.17)
16.	0.77	0.85	0.44
	(0.17)	(0.16)	(0.16)
20.	0.79	0.87	0.46
	(0.16)	(0.15)	(0.16)
24.	0.81	0.89	0.47
	(0.16)	(0.13)	(0.16)
∞	1.00	1.00	1.00

Note: Based on an estimated vector error correction model of $X_t = (y_t, c_t, i_t)$ with 8 lags of ΔX_t, one lag each of the error correction terms c-y and i-y, and a constant. Approximate standard errors, shown in parentheses, were computed by Monte Carlo simulation using 500 replications.

Table 5
Forecast Error Variance Decompositions:
6 Variable Model (4.1), 54:1 - 88:4

A. Fraction of the forecast error variance attributed to balanced growth shock

Horizon	y	c	i	m-p	R	Δp
1.	0.00	0.02	0.11	0.79	0.14	0.30
	(0.13)	(0.09)	(0.16)	(0.23)	(0.19)	(0.13)
4.	0.05	0.15	0.06	0.76	0.11	0.22
	(0.14)	(0.13)	(0.11)	(0.23)	(0.19)	(0.08)
8.	0.22	0.31	0.14	0.70	0.11	0.20
	(0.13)	(0.18)	(0.11)	(0.24)	(0.20)	(0.07)
12.	0.44	0.48	0.27	0.72	0.11	0.17
	(0.14)	(0.21)	(0.16)	(0.25)	(0.20)	(0.06)
16.	0.54	0.59	0.32	0.74	0.11	0.16
	(0.15)	(0.21)	(0.17)	(0.24)	(0.20)	(0.07)
20.	0.59	0.63	0.33	0.75	0.12	0.15
	(0.15)	(0.19)	(0.17)	(0.22)	(0.21)	(0.07)
24.	0.62	0.65	0.33	0.77	0.14	0.14
	(0.14)	(0.17)	(0.16)	(0.22)	(0.22)	(0.08)
∞	1.00	0.92	0.97	0.78	0.23	0.04

B. Fraction of the forecast error variance attributable to inflation shock

Horizon	y	c	i	m-p	R	Δp
1.	0.00	0.02	0.08	0.01	0.03	0.43
	(0.12)	(0.11)	(0.16)	(0.13)	(0.14)	(0.19)
4.	0.04	0.01	0.23	0.04	0.04	0.37
	(0.14)	(0.10)	(0.19)	(0.14)	(0.15)	(0.13)
8.	0.04	0.01	0.20	0.01	0.02	0.36
	(0.12)	(0.11)	(0.15)	(0.14)	(0.16)	(0.12)
12.	0.03	0.01	0.12	0.01	0.02	0.45
	(0.10)	(0.12)	(0.12)	(0.14)	(0.15)	(0.11)
16.	0.02	0.01	0.10	0.01	0.03	0.49
	(0.10)	(0.12)	(0.11)	(0.13)	(0.15)	(0.11)
20.	0.02	0.02	0.10	0.01	0.03	0.53
	(0.10)	(0.12)	(0.11)	(0.13)	(0.15)	(0.12)
24.	0.02	0.02	0.09	0.01	0.02	0.55
	(0.10)	(0.11)	(0.11)	(0.13)	(0.14)	(0.13)
∞	0.00	0.02	0.00	0.00	0.01	0.96

Table 5
(Continued)

C. Fraction of the forecast error variance attributable to real interest rate shock

Horizon	y	c	i	m-p	R	Δp
1.	0.67	0.35	0.44	0.03	0.63	0.00
	(0.19)	(0.22)	(0.20)	(0.13)	(0.21)	(0.11)
4.	0.74	0.24	0.50	0.07	0.72	0.10
	(0.20)	(0.18)	(0.20)	(0.14)	(0.21)	(0.08)
8.	0.55	0.12	0.37	0.20	0.77	0.16
	(0.16)	(0.10)	(0.14)	(0.17)	(0.22)	(0.09)
12.	0.39	0.11	0.36	0.21	0.78	0.14
	(0.11)	(0.09)	(0.12)	(0.18)	(0.22)	(0.08)
16.	0.32	0.09	0.37	0.20	0.78	0.13
	(0.10)	(0.09)	(0.12)	(0.17)	(0.22)	(0.07)
20.	0.28	0.09	0.34	0.18	0.78	0.12
	(0.09)	(0.08)	(0.12)	(0.16)	(0.22)	(0.07)
24.	0.25	0.10	0.34	0.16	0.77	0.11
	(0.08)	(0.08)	(0.11)	(0.15)	(0.22)	(0.07)
∞	0.00	0.06	0.03	0.21	0.77	0.00

Note: Based on an estimated vector error correction model of $X_t = (y,c,i,m-p,R,\Delta p)$ with 8 lags of ΔX_t, one lag each of the error correction terms $c-y-\phi_1(R-\Delta p)$, $i-y-\phi_2(R-\Delta p)$, and $m-p-\beta_y y-\beta_R R$, and a constant. Approximate standard errors, shown in parentheses, were computed by Monte Carlo simulation using 500 replications.

Table 6
3 Year Ahead Forecast Error Variance Decompositions:
Summary of Results of Various Models

		Tests of Restrictions on Cointegrating Vectors			Fraction of forecast error variance attributed to the permanent real shock					
Model	Est. Per.	d.f.	Wald test	Log Lik.	y	c	i	m-p	R	Δp
R.1	49:2-88:4	2	4.96 (0.08)	2196.67	.73	.83	.43	--	--	--
M.1	54:1-88:4	5	7.60 (0.18)	2816.06	.44	.48	.27	.72	.11	.17
M.2	54:1-88:4	7	12.60 (0.08)	2814.64	.42	.52	.25	.68	.07	.16
M.3	54:1-88:4	------ same as M.1 -------			.35	.30	.12	.26	.02	.16
M.4	54:1-88:4	6	40.10 (<0.01)	2820.48	.37	.40	.15	.56	.01	.18
M.5	54:1-88:4	4	3.04 (0.55)	2812.52	.42	.47	.23	.64	.06	--
M.6	54:1-88:4	------ same as M.5 -------			.42	.36	.19	.46	.01	--

Model Description

Model R.1: Three variable (y,c,i) model with cointegrating relations c-y and i-y.

Model M.1: Six variable (y,c,i,m-p,R,Δp) baseline model of Table 5.

Model M.2: Identical to M.1, except that the coefficients ϕ_1 and ϕ_2 are set to zero in the cointegrating vectors and the A matrix, i.e., cointegration of shares and the real interest rate is dropped.

Model M.3: Identical to M.1, except that the stochastic trend innovations are reordered to place the inflation shock first, the real interest rate shock second and the balanced growth trend third.

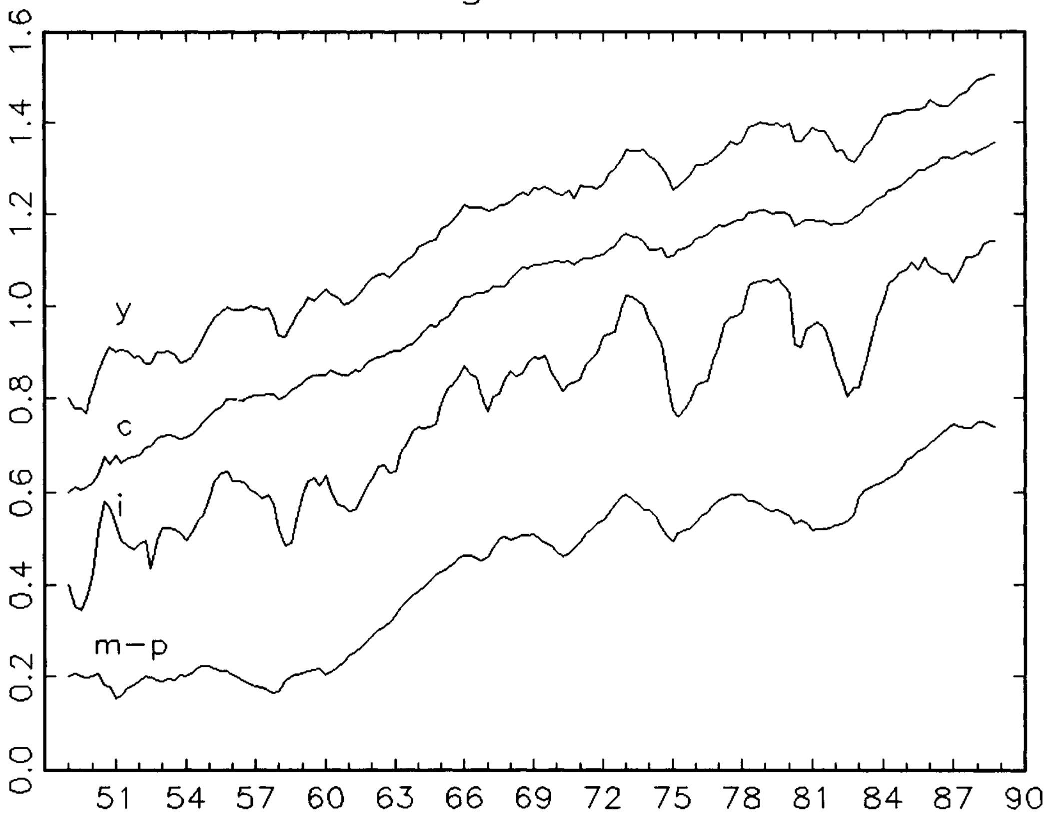
Model M.4: A two stochastic trend model for (y,c,i,m-p,R,Δp), obtained by assuming that the real interest rate is stationary. The cointegrating relations are c-y, i-y, $(m-p)-\beta_y y+\beta_R R$ and R-Δp, and $A = [A_1\ A_2]$, where $A_1 = (1\ 1\ 1\ \beta_y\ 0\ 0)'$ (balanced growth shock) and $A_2 = (0\ 0\ 0\ -\beta_R\ 1\ 1)'$ (neutral inflation shock).

Model M.5: A five variable system (y,c,i, m-p,R) with cointegrating relations c-y, i-y and $(m-p)-\beta_y y+\beta_R R$, and $A = [A_3\ A_4]$, where $A_3 = (1\ 1\ 1\ \beta_y\ 0)'$ (balanced growth shock) and $A_4 = (0\ 0\ 0\ -\beta_R\ 1)'$ (neutral interest rate shock).

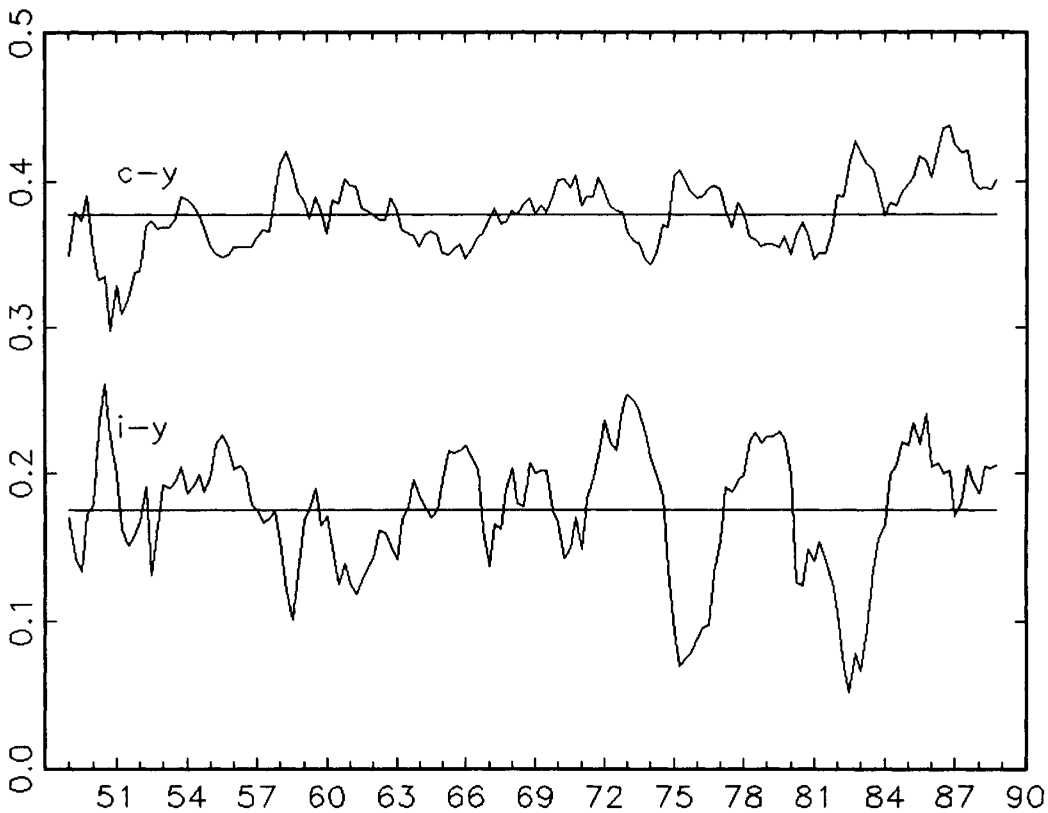
Model M.6: Identical to M.5, except that the ordering of stochastic trend innovations is reversed, so $A = [A_4\ A_3]$.

Notes to Table 6: The estimation period denotes the sample used to estimate the VECM, with earlier data used for initial conditions for the lags. The Wald statistics test the hypothesis that the true cointegrating subspace is spanned by the hypothesized cointegrating vectors, or equivalently is orthogonal to the A matrix, and are described in the notes to Table 1 and footnote 8.

Figure 1



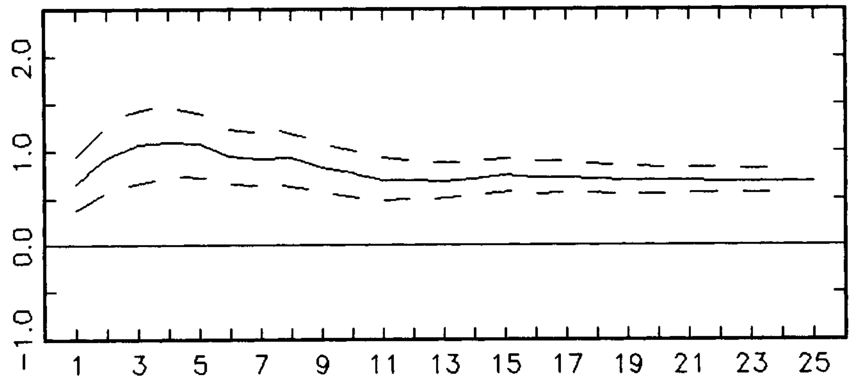
A. Logarithms of Private Output (y), Consumption (c), Investment (i) and Real Money Balances (m−p)



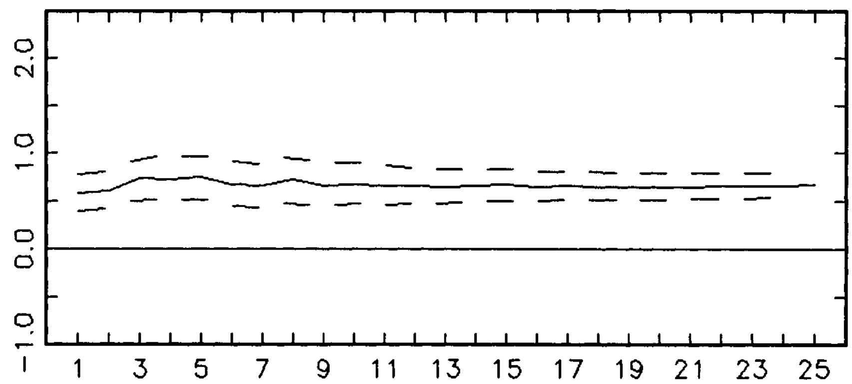
B. Logarithms of the Consumption–Output (c−y) and Investment–Output (i−y) Ratios

Figure 2

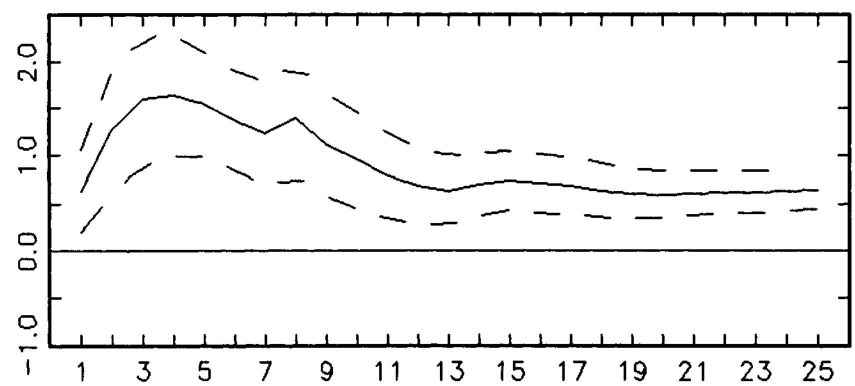
Responses in 3-variable Model to a One Std. Dev. Shock
in the Real Permanent Component



(a) y



(b) c



(c) i

Figure 3

Historical Forecast Decomposition
Six Variable Model

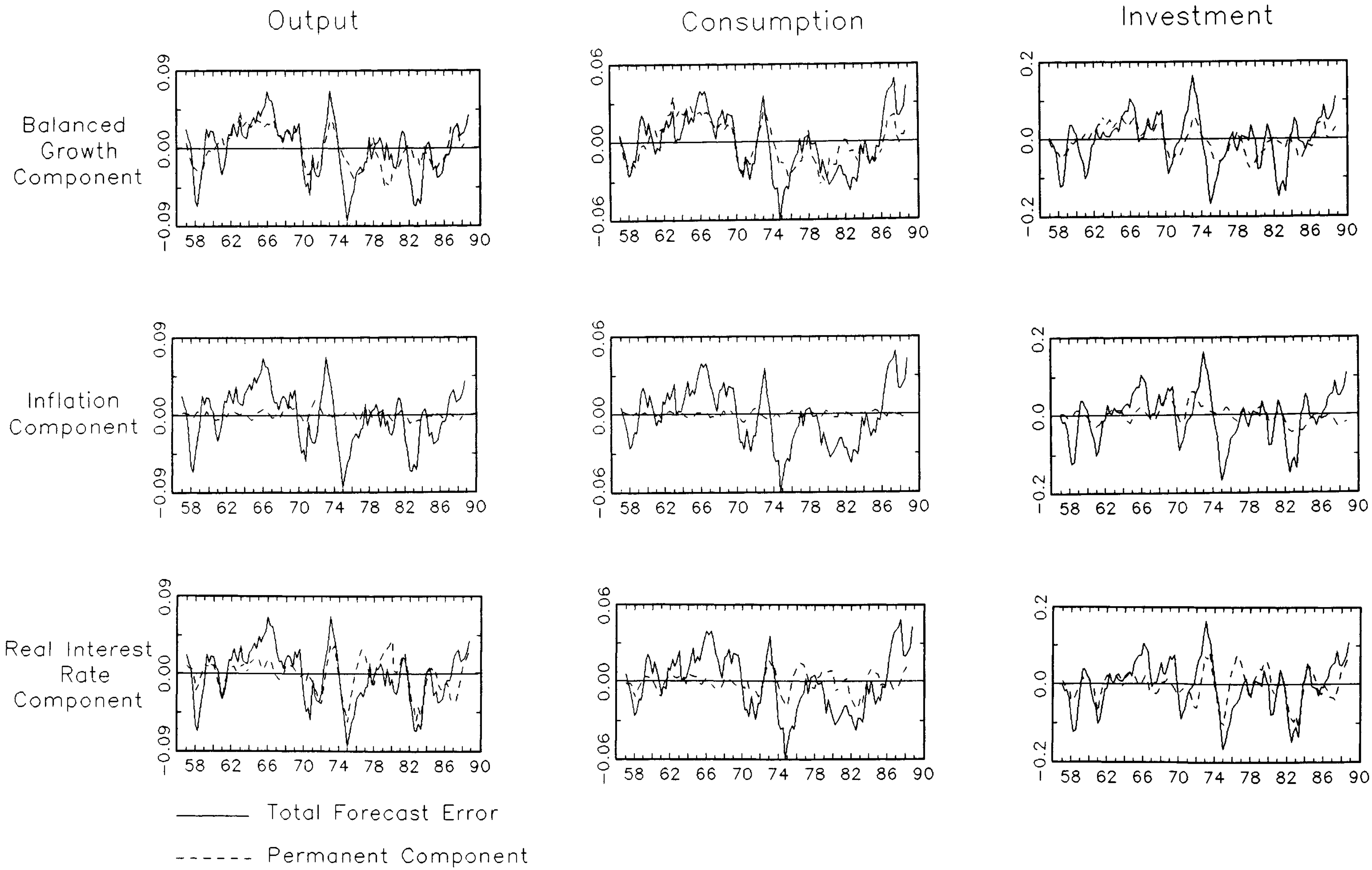


Figure 4

Selected Impulse Responses for the Six Variable Model

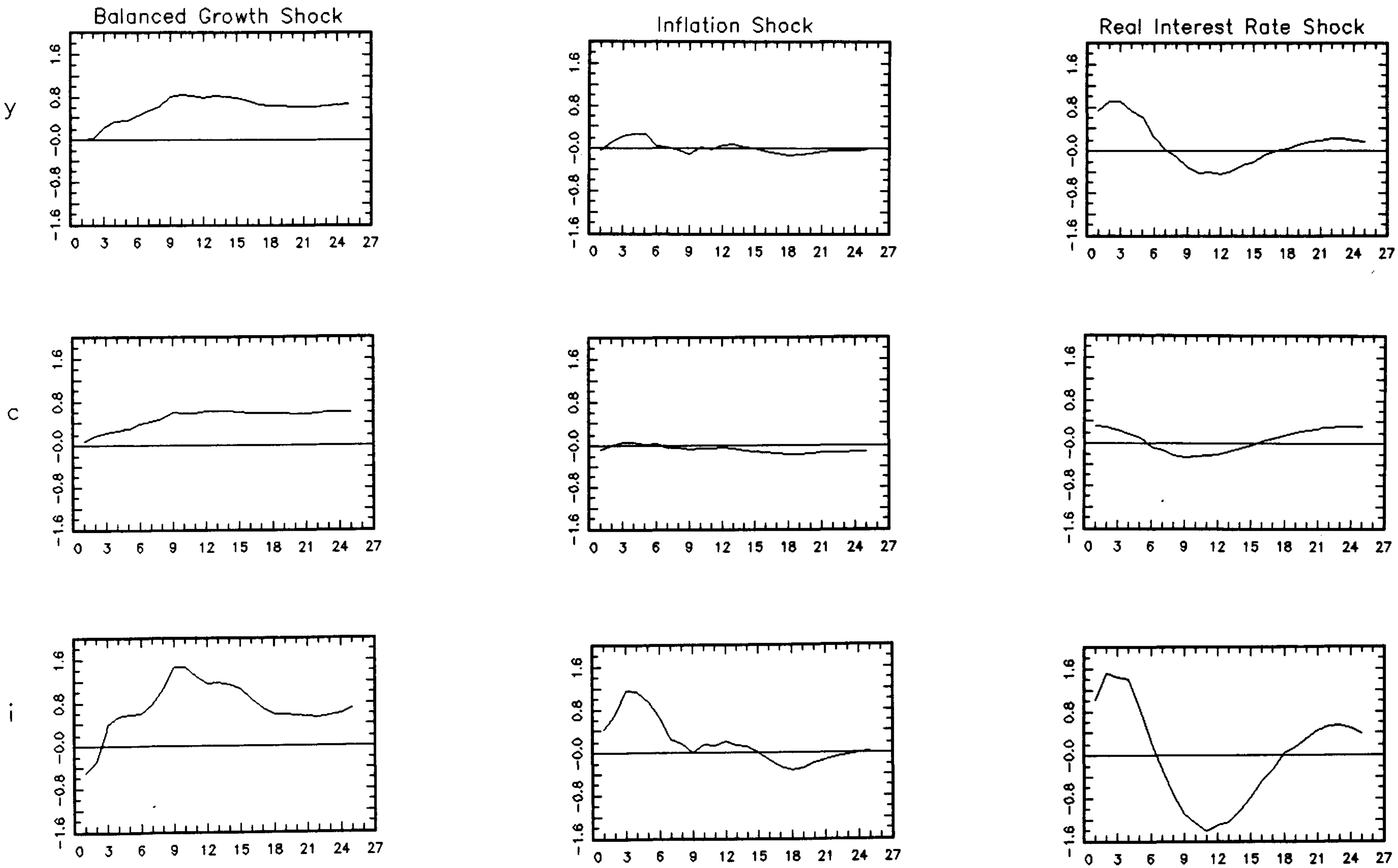
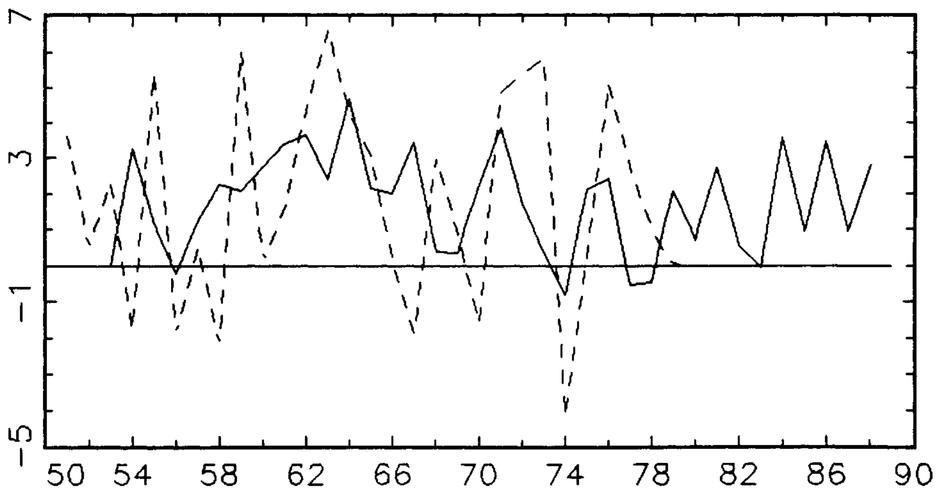


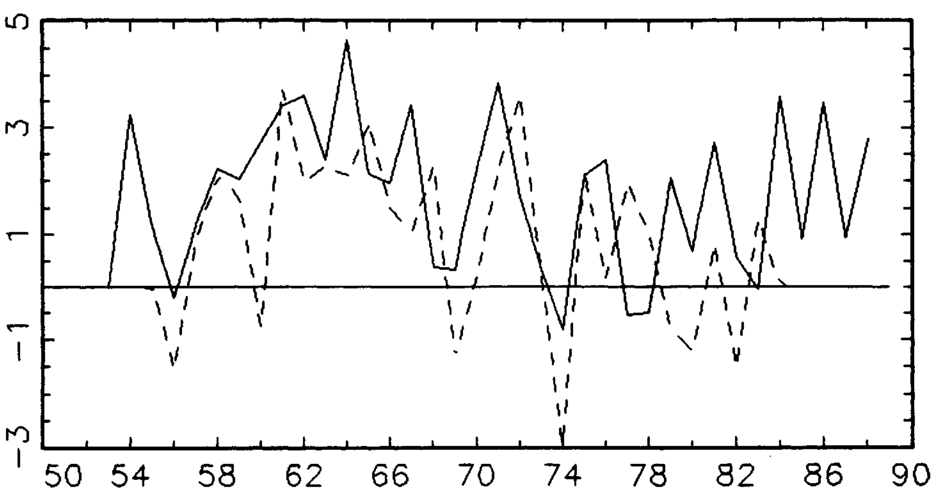
Figure 5a



- - - - Hall's Solow Residual

——— Balanced Growth Shock from the 6-variable Benchmark Model

Figure 5b



- - - - Precott's Solow Residual

——— Balanced Growth Shock from the 6-variable Benchmark Model

Figure 6

Estimates of Annual Trend Output