CONSTRUCTION OF INPUT-OUTPUT COEFFICIENTS WITH FLEXIBLE FUNCTIONAL FORMS

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1 Introduction

Input-output (I-O) models with fixed coefficients are widely recognized as the most restrictive set of production functions. An attempt to estimate I-O coefficients using neoclassical production functions with flexible functional form has been made by Hudson and Jorgenson [1974] (H-J), and later Nakamura [1984].[1] Input shares derived within the neoclassical framework were identified as I-O coefficients. However, we observe that shares of primary resources (value-added components) in the neoclassical framework are defined differently from the I-O framework, an incompatability that was neglected in the previous work.

It is shown that this incompatability can be eliminated via the use of the double deflated value-added measure. More importantly, a methodology of imposing traditional I-O constraints (on the sums of rows and columns of the I-O table) on the neoclassical model is

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[1]Nakamura's work represents clarification of the H-J assumptions and an extension of the original model. The scope of this paper does not cover numerous innovative topics addresssed in Nakamura. Therefore most of the references are drawn from the H-J work.

developed that makes it possible to perform the task of RAS (biproportional adjustments of I-O tables) with an econometric model.[2] These two aspects are presented in Section 2. In Section 3, the RAS and H-J methods of projecting I-O coefficients are combined. For this purpose, a modified RAS procedure is developed and termed Extended RAS or ERAS. To make a consistent presentation the paper begins with the brief description of I-O model, Subsection 2.1, and description of the translog cost function, Subsections 2.2 and 2.3.

2 Time-series data as an aid to the construction of input-output coefficients

2.1 I-O table as a set of input shares

An I-O table is computed from transaction flows, which are dollar values of purchases of goods and services by industries. If $\boldsymbol{F}$ is a matrix of transaction flows, then the I-O table is represented as $\boldsymbol{A} = \boldsymbol{F}\widehat{\boldsymbol{X}}^{-1}$, where $\widehat{\boldsymbol{X}}$ is a diagonal matrix with output values for each industry on its principal diagonal [$(\widehat{.})$ denotes, throughout the text, a diagonal matrix with vector (.) on its principal diagonal].

The I-O table $\boldsymbol{A}$ represents the set of shares of intermediate input values. The sum of intermediate shares for each industry (single column's sum of I-O coefficients), combined with the corresponding value added per unit of output, are equal to unity for each industry.

The $\boldsymbol{A}$ matrix is assumed to be fixed in a traditional I-O analysis. I-O table $\boldsymbol{A}$ is observed in time and can be deflated with respect to input and output prices (see Section 2.3). Therefore, via deflation, I-O tables can be made consistent over time, allowing changes in the $\boldsymbol{A}$ matrix to be due to input substitution. However, I-O analysis does not offer an explanation for such substitution. Instead, this phenomenon (under certain assumptions)

[2]The RAS method of projection of I-O tables is well known among I-O experts. For definitions see Section 3.

must be explained with a neoclassical model, as discussed in the following Subsection.

2.2 Neoclassical representation of input shares

H-J presented a methodology that allows for the use of time-series data for the estimation of all input shares, which are interpreted as I-O coefficients. Earlier, we indicated that such shares are equivalent to the I-O coefficients in value terms. In this Subsection, the H-J methodology is described. This methodology is based on a neoclassical micro framework, which is not necessarily consistent with the I-O framework. Differences between the two frameworks are discussed and, based on this discussion, a modification to the H-J framework is proposed.

Following Jorgenson [1986], price of unit of output (denote, q) can be expressed as a function of input prices and time:

$$q = Q(p, t) \tag{1}$$

where p and t are the price and time vectors, respectively. According with the neoclassical considerations the unit price function has to be positive, homogenous, monotone, and concave with respect to input prices.

The necessary condition for equilibrium is to have equality between shares of inputs in the value of output and the elasticities of respective input prices:

$$s_i = \frac{p_i x_i}{q} = \frac{\partial \ln Q}{\partial \ln p_i} \tag{2}$$

where $s = [s_i]$ is a vector of shares. Assuming constant returns to scales the sum of shares adds up to unity:

$$1' s_i = 1. \tag{3}$$

Since the unit price function is monotone in positive input prices and the sum of shares equals one, each share must be nonnegative and at least one share must be positive.

Next, the rate of technical change can be expressed as a negative of the change in price of output over time, holding input prices fixed:

$$v_t = -\frac{\partial \ln Q}{\partial t}. \tag{4}$$

Shares and technical change are derived by the first order derivative of the unit price function with respect to input prices and time. Second-order derivatives provide measures of input substitutions, biases of shares with respect to technical changes, and changes in the rate of technical change. The above notions can be determined by the first derivatives of s_i and v_t, since they, in turn, are the first derivatives of Q. The matrix of share elasticities (denote U_{pp}) is:

$$\frac{\partial s}{\partial \ln p} = U_{pp}. \tag{5}$$

Bias of shares with respect to technical change (denote u_{pt} as a technical bias) is:

$$\frac{\partial s}{\partial t} = -\frac{\partial v_t}{\partial \ln p} = u_{pt}. \tag{6}$$

Finally, the speed of the rate of change (denote v_{tt}), i.e. deceleration or acceleration of the technical changes is:

$$-\frac{\partial v_t}{\partial t} = v_{tt}. \tag{7}$$

The matrix of the second-order logarithmic derivatives of the price function Q must be symmetric. The matrix of the second-order derivatives of Q must be nonpositive definite to satisfy concavity of the price function. This implies that the matrix $U_{pp} - \widehat{s} + ss'$ must be nonpositive definite.

In most econometric studies of production processes, inputs (and input prices) are presented in a very aggregated form. To consider the disaggregated input prices, economists assume independence between disaggregated input prices of one product and aggregated prices of the rest of inputs. The problem of input price disaggregation can be resolved with the separability assumptions. Price function Q can be presented as:

$$q = Q(P(p_1, ..., p_m), p_{m+1}, ..., p_n) \tag{8}$$

where P is an aggregated price as a function of individual prices $p_1, ..., p_m$, and p_i the price of good i.

Price function Q is homothetically separable if P is linearly homogeneous in its disaggregated components $p_1, ..., p_m$, and independent of the rest of the input prices $p_{m+1}, ..., p_n$. Under the homothetic separability, P has properties identical to those of Q. This allows one to formulate two- (or greater) stage optimization of the production process. The first stage represents the determination of shares as the logarithmic differentiation of Q with respect to aggregated P, such as equation (2). In the second stage the set of "subshares" of the share derived in equation (2) can be computed as $(\partial \ln P / \partial \ln p_i)$.

H-J, for example, have utilized a two-stage procedure. In the first stage, each industry was considered to be a production process with four inputs — capital, labor, energy, and materials. In the second stage, some of these inputs were disaggregated. For example, energy prices were represented by coal, gas, oil, and other energy product prices. Nakamura presented a three-stage process, where the first two were similar to the H-J model. In the third stage, input prices of the second stage were disagregated, based on a function that consisted of domestic and import prices.

2.3 Econometric model of input shares

To determine the functional forms for the unit price of output, shares, and technical change as functions of input prices and time, an econometric approach can be utilized. In this approach, the matrix of elasticities of substitution, the vector of technical biases, and the rate of technical change are considered to be unknown.

The model of cost and production can be generated by integration of the following set of second-order partial differential equations:

$$\boldsymbol{B}_{pp} = \boldsymbol{U}_{pp}, \quad \boldsymbol{\beta}_{pt} = \boldsymbol{u}_{pt}, \quad \boldsymbol{\beta}_{tt} = \boldsymbol{u}_{tt} \tag{9}$$

where $\boldsymbol{B}_{pp}$ is a matrix of elasticities of shares with respect to input prices; $\boldsymbol{\beta}_{pt}$ is a vector of technical biases, which is expressed as the elasticity of a share with respect to time or as the elasticity of technical change with respect to input prices; and $\boldsymbol{\beta}_{tt}$ is a constant equal to the elasticity of technical change with respect to time.

By integrating system (9), one obtains a system of first-order partial differential equations:

$$\boldsymbol{s} = \boldsymbol{\alpha}_p + \boldsymbol{B}_{pp} \ln \boldsymbol{p} + \boldsymbol{\beta}_{pt} \boldsymbol{t} \tag{10}$$

$$\boldsymbol{v}_t = \boldsymbol{\alpha}_q + \boldsymbol{\beta}'_{pt} \ln \boldsymbol{p} + \boldsymbol{\beta}_{tt} \boldsymbol{t} \tag{11}$$

where vector $\boldsymbol{s}$ and $\boldsymbol{v}_t$ are vectors of shares and technical change, respectively. The intercept of equation (10) is a constant of integration. If prices are normalized to unity and the level of technology is normalized to zero, then the intercept, $\boldsymbol{\alpha}_p$, is equal to $\boldsymbol{s}$. In this case, $\boldsymbol{\alpha}_p$ represents the share of an input expenditure per unit of output for the base period, i.e., the period when all prices are set to unity, and technology is represented by time $\boldsymbol{t} = \mathbf{0}$. Similarly, a constant of integration $\boldsymbol{\alpha}_q$ for the base year establishes the level of technology, which is equal to $\boldsymbol{v}_t$ for $\boldsymbol{t} = \mathbf{0}$.

Integrating the system of partial differential equations (10) and (11), one can derive an expression for the unit price of output in translog form:

$$\ln \tilde{\boldsymbol{p}} = \boldsymbol{\alpha}_o + \boldsymbol{\alpha}_p \ln \boldsymbol{p} + \boldsymbol{\alpha}_t \boldsymbol{t} + 0.5 \ln \boldsymbol{p}' \boldsymbol{B}_{pp} \ln \boldsymbol{p} + \ln \boldsymbol{p}' \boldsymbol{\beta}_{pt} \boldsymbol{t} + 0.5 \boldsymbol{\beta}_{tt} (\boldsymbol{t})^2, \tag{12}$$

where α_o is a constant of integration and $\tilde{p}$ is unitary price of output.

Traditionally, linear homogeneity in prices is imposed on the set of equations (10) – (12). This implies the following adding-up restrictions on parameters:

$$\begin{array}{c} \mathbf{1}'\alpha_p = 1, \quad \mathbf{1}'\beta_{pt} = 0, \quad \text{and} \\ \\ B_{pp} = B'_{pp}, \quad \mathbf{1}'B_{pp} = \mathbf{0} \end{array} \tag{13}$$

where **1** and **0** are vectors of ones and zeroes.

The described system of equations can be used to model an individual industry. Nakamura used the unit price function to model a given industry. This approach is adopted here. Collecting equations (10) – (12) into one system, the H-J specification can be delineated as:

$$\begin{array}{rcl} s & = & \alpha_p + B_{pp}\ln p + \beta_{pt}t \\ v_t & = & \alpha_q + \beta'_{pt}\ln p + \beta_{tt}t \\ \ln\tilde{p} & = & \alpha_o + \alpha_p\ln p + 0.5\ln p'B_{pp}\ln p \\ & & +\ln p'\beta_{pt}t + 0.5\beta_{tt}(t)^2 \end{array} \tag{14}$$

subject to a set of constraints (13). Constant returns to scale are imposed on the system (14), due to the adding up restrictions. For the system of equations (14), input prices p are exogenous.[3]

The estimation of shares s is the primary objective of this paper. The price equation in (14) adds efficiency to the estimation process. Estimates of the output prices are beyond the scope of this paper. To simplify notations, input and output prices are assumed to be the same (denoted by p).[4]

[3]In the H-J system these prices are projected with the growth model. Therefore, by separating equations (14) from the rest of the H-J system the assumptions related to (14) are not violated.

[4]Under the assumption of linear relations between input and output prices, the simultaneity for price

2.4 Input-output and the Neoclassical approach

The H-J procedure models optimal behavior over the historical period, and shares are derived as a result of the assumed optimization. To compare the derived shares in time, the shares must be expressed in real terms, i.e. prices must have a common deflator for all periods. However, the observed shares are reported by the I-O tables in current values. Therefore, I-O tables must be transformed in time with the deflator common to the H-J model.

There are two ways to model real shares. First, estimate equation (10) with current values and then deflate shares. This is the method used by H-J and Nakamura. Our alternative is to use real shares and prices on both sides of equation (10). These two methods are illustrated as follows.

Assume $\boldsymbol{s}_i$ is input i real share of its respective output, and $\boldsymbol{p}_j$ is the real price of input j. Then $\boldsymbol{p}_j = \boldsymbol{p}_j^c/\boldsymbol{p}_j^b$, where superscripts c and b stand for the current and base periods. Then according to equation (10):

$$\boldsymbol{s}_i = (\boldsymbol{\alpha}_{p_i} - \sum_j \boldsymbol{b}_{p_i p_j} \ln \boldsymbol{p}_j^b) + \boldsymbol{b}_{p_i p_j} \ln \boldsymbol{p}_j^c + \boldsymbol{\beta}_{p_i t} \boldsymbol{t}. \tag{15}$$

Since $\boldsymbol{p}_j^b$ is constant, the components in parentheses in equation (15) represent an intercept. To derive current shares, expression $\sum_j \boldsymbol{b}_{p_i p_j} \ln \boldsymbol{p}_j^b$ is set equal to zero. Therefore, the transformation from real to current prices is derived via the change of the intercept in the translog formulation.

In contrast H-J compute $\boldsymbol{s}_i$ in current values (expression $\sum_j \boldsymbol{b}_{p_i p_j} \ln \boldsymbol{p}_j^b$ is set to zero), then $\boldsymbol{s}_i$ is multiplied by $\boldsymbol{p}_k^b/\boldsymbol{p}_i^b$ where subscript k denotes output. Obviously, projection of real shares by equation (15) or by H-J approach would yield different results. As indicated

equations in (14) can be considered. For the case of n industries, n endogenous prices and $(n^2 - n)$ share equations can be considered as endogenous. Thus, only primary resource prices would be exogenous.

in the next Subsection, equations estimated for real shares provide the necessary framework for projection of I-O tables.

As indicated, to express shares in real values H-J pre- and post-multiply shares by the base year reciprocal of input prices and the base year prices of output. This methodology was applied to both intermediate inputs and the value-added components. Although this transformation of the shares from current values to real values is consistent with the I-O methodology [this formula is derived in equation (18)] for the intermediate inputs, it is *not* consistent for the components of value added. This inconsistency can be illustrated with the dual I-O approach.

I-O analysis assumes value added for each sector as an exogenous variable. For the given vector of value added, $\boldsymbol{V}$, the I-O matrix $\boldsymbol{A} = [\boldsymbol{a}_{ij}]$ determines the vector of prices, $\boldsymbol{P}$:[5]

$$\boldsymbol{P} \;\; = \;\; [\boldsymbol{I} - \boldsymbol{A}^T]^{-1}\boldsymbol{V}. \tag{16}$$

For example, if the vector of value added, $\boldsymbol{V}$, is such that $\mathbf{1} - \sum_{i=1}^{n} \boldsymbol{a}_{ij} = \boldsymbol{v}_j$, for any j, then according to equation (16) $\boldsymbol{P} = \mathbf{1}$.[6] This example corresponds to the base year for which the price deflator is set, i.e., all real prices are equal to unity for this year. Matrix $\boldsymbol{A}$ with $\boldsymbol{P} = \mathbf{1}$ is an I-O table for a given year [textbooks report these tables as I-O tables, see footnote (5)]. Given matrix $\boldsymbol{A}$ of the current year, correspondence between real prices for any current year to real value added, is defined below. If equation (16) considers the current year to be the base year, then:

$$\mathbf{1} = [\boldsymbol{I} - \boldsymbol{A}^T]^{-1}\boldsymbol{V} \tag{17}$$

this equation can be rewritten as,

$$\widehat{\boldsymbol{P}}^{-1}\boldsymbol{P} = [\boldsymbol{I} - \boldsymbol{A}^T]^{-1}\widehat{\boldsymbol{P}}^{-1}\widehat{\boldsymbol{P}}\boldsymbol{V}$$

[5]For details see Miller & Blair [1985].

[6]This follows from $[I - A^T]\mathbf{1} = V$.

or,

$$\boldsymbol{P} = [\boldsymbol{I} - \widehat{\boldsymbol{P}}\boldsymbol{A}^T\widehat{\boldsymbol{P}}^{-1}]^{-1}\widehat{\boldsymbol{P}}\boldsymbol{V}. \tag{18}$$

Elements of the matrix $\widehat{\boldsymbol{P}}\boldsymbol{A}^T\widehat{\boldsymbol{P}}^{-1}$ are shares of current expenditures expressed in constant dollars $\boldsymbol{P}$. Vector $\boldsymbol{P}$ represents real values of one unit of output and vector $\widehat{\boldsymbol{P}}\boldsymbol{V}$ represents value added necessary to produce the output value $\boldsymbol{P}$. Equation (18) represents the current year. If this year is selected as a base year, then equation (18) transforms into equation (17).

Equation (18) presents a convenient way to deflate the $\boldsymbol{A}$ matrix. However, this equation does not define share of value added in real terms. Since equation (18) relates output in real terms to I-O coefficients in real terms, the share of value added can be computed as a residual. This is the double deflation method, which can be illustrated as follows:

$$[\boldsymbol{I} - \widehat{\boldsymbol{P}}\boldsymbol{A}^T\widehat{\boldsymbol{P}}^{-1}]\boldsymbol{1} = \boldsymbol{V}_d \tag{19}$$

where $\boldsymbol{V}_d$ is double deflated value added. It is important to note that only in the base year does $\boldsymbol{V}_d = \boldsymbol{V}$.

In transforming shares from current to real values, H-J do not use double deflation. As a result, shares may not sum up to unity. This problem is eliminated with the shares expressed in real terms using the translog formulation. In this formulation the double deflated value-added figure is necessary. Value added in real terms, $\boldsymbol{V}_d$, can be projected with the share equations (10), as the neoclassical model would suggest. Yet, for projection purporses value added in I-O analysis is treated as exogenous variable. In that case, sum of shares for the intermediate consumption are constrained(i.e. sum of shares is less or equal to unity) as opposed to the neoclassical constraints on total sum of shares (with the strict equality to unity). This case is addressed in the next Subsection.

2.5 Econometrically estimated I-O as a substitute for the RAS procedure

In this subsection the econometric I-O link is considered for the period when prices, GNP, shipments, and value added are reported. This information is sufficient to estimate margins for the I-O model.[7]

There is an extensive literature on the use of margins to update I-O tables. The most recent I-O table available is called the base table (not to be confused with the base year for the deflator). The predicted I-O is called the target table. The target table is, obviously, predicted for the year when margins are available.

The prediction is based on the choice of the objective function that is minimized subject to marginal constraints in order to find the shortest distance between the base and the target tables.

The well-known objective functions in the literature are linear, quadratic, and minimum information objective functions. Only the last of the three has a theoretic justification. However, the theoretic justification for the minimum information principle comes from information theory and has no reflection on the economic nature of the problem.[8] In the economic literature the minimum information principle for the I-O update is called the RAS procedure, which is discussed in detail in Section 3.

In this Subsection an alternative to the above mentioned techniques is provided. The proposed method is based on the H-J approach, applied to the historic period with known margins but the unknown I-O table. For example, in 1989 the latest available data for margins are for 1987. The most recent update of the U.S. I-O table is for 1983.[9] In this

[7]One set of margins for the I-O analysis are the column totals, determined as a residual between shipments and value added. Another set of margins are row totals as a residual between shipments and final demand. GNP can be considered as a proxy for the final demand.

[8]For the most fundamental reference on this subject see M. Bacharach [1970], or footnote (5).

[9]The annual estimates of I-O tables are produced, among others, by Bureau of Economic Analysis. These tables are provided in the electronic forms.

case the base I-O table is for 1983 and the target table is for 1987.

Earlier the importance of expressing I-O in real prices was emphasized. Thus, equations (14) should represent a time-series of I-O coefficients and prices in real terms [computed according to equation (19)]. The difficulty with projecting shares for the target year is in generating estimates for the double deflated value added, because the I-O table is not available for that year.

This problem can be resolved by using *the target period as a base period for the price deflator* because, for the base year, $V_d = V$. Furthermore, all real prices in the target period are equal to unity, under this consideration. In addition prices for all other periods will have to be deflated with the prices of the target period.

By designating the target period as the base year, share equations [in the system of equations (14)] are greatly simplified for the target period. Consider industry j, since prices for the target year are equal to unity, the shares for this year are computed as:

$$s^j = \alpha_p^j + \beta_{pt}^j t + b_{pp}^j \mathbf{o} \tag{20}$$

where s^j is a vector of shares for industry j, which is equivalent to the j column of the I-O table for the target year. Superscript j denotes industry-specific vector of coefficients. As a result, an estimated share for the target year represents the constant of integration plus the technical bias.

To simplify equation (20) further, the time variable can be presented in such a way that $t = \mathbf{o}$ for the target year and t is negative (in ascending order) for all the previous years. Then, new equations for the shares in the target year are:

$$s^j = \alpha_p^j + \beta_{pt}^j \mathbf{o} + b_{pp}^j \mathbf{o}. \tag{21}$$

To estimate the target I-O matrix, constraints on rows and columns should be introduced for the H-J system. Since the sum of shares of the intermediate consumption is determined by each column's margins, constraints on the shares of expenditures for the intermediate consumption have to be imposed. For example, for industry j this constraint is $\sum_{i \notin D} s_i^j = a_{.j}$, where D – denotes the set of subscripts of value added components. According to equation (21), these constraints on shares can be expressed as:

$$\sum_{i \notin D} \alpha_{p_i}^j = a_{.j}. \tag{22}$$

Another set of constraints, which is not required for the neoclassical models, is for the rows of the I-O table. It is required that $\sum_{j=1}^n s_i^j X_j = f_{i.}$, for $i \notin D$, where X_j is output in real terms for the target year and $f_{i.}$ is total sales of product i to the intermediate consumers. Again, utilizing equation (21), constraints on the intermediate sales can be expressed as

$$\sum_{j=1}^{n} \alpha_{p_i}^j X_j = f_{i.}, \quad \text{where} \quad i \notin D. \tag{23}$$

Since X_j is known, equation (23) is linear in the unknown parameters $\alpha_{p_i}^j$.

In the presented model for the I-O table, only estimation constraints on intercepts [equations (22) and (23)] are required. Therefore constraints on B_{pp} and β_{pt} in (13) can be relaxed, making the estimated model more general than the H-J model.

The method demonstrated here allows one to estimate the target I-O table for the year that has the observed variables necessary to calculate margins. The same method can be utilized to estimate the I-O table for any year with predicted margins.

To summarize, the advantages of the developed method relative to the H-J model and I-O model are listed below. Advantages relative to the H-J model include:

- To achieve consistency between the H-J and I-O models, prices in both models should be in real terms. This means that the value added should be double deflated.[10] However double deflation is not possible for the target year, unless the target year is chosen as a base year for the price deflators. The proposed method equates double deflated value added with the real value added for the target year. This insures consistency between the H-J estimation of the input shares and the traditional I-O constraints for the target period.

- The proposed method allows one to introduce constraints on the intermediate consumption [equations (22)] for the target period. The H-J model allows constraints on the sum of all shares only.

- The H-J model considers the technological structure for an individual industry. However there are no constraints on the intermediate sales. The proposed method imposes such constraints (23).

- Constraints (23) and (22) are less restrictive than the H-J constraints (13) making the presented model more general.

The I-O model benefits from the proposed method as follows:

- In the traditional approach, I-O coefficients represent a fixed technology. In the proposed approach I-O coefficients are based on the neoclassical behavioral assumptions. This allows one to determine the effect of input prices on I-O coefficients according to the estimated elasticities of substitutions.

- The traditional methods of projection of I-O tables use only one observation on I-O coefficients. The proposed method utilizes a time-series of I-O tables. This allows one to derive the statistical properties of the estimated I-O coefficients.

[10]Components of the double deflated value added should represent the dependent variables in the H-J model.

3 Extended RAS as a method of disaggregation of shares estimated by the H-J method

3.1 H-J method of disaggregation

Due to the low degrees of freedom, H-J's analysis has to be applied to a very aggregated I-O table. For example, H-J considers a 9 sector table with 4 input components. Nakamura considers 12 sectors with 5 input components.

Both works estimate aggregated I-O tables as a first stage of estimation. In the second stage, under the assumption of homothetic separability between aggregate input prices, they disaggregated individual I-O coefficients (shares). The sum of these disaggregated shares is constrained to be equal to the corresponding aggregated share. Aggregated shares correspond to the aggregated prices $\boldsymbol{P}$ and disaggregated shares correspond to prices $\boldsymbol{p}_1, ..., \boldsymbol{p}_m$ in equations (14).

Due to the limited degrees of freedom, only a small number of input prices can be considered for the estimation of the disaggregated shares (H-J and Nakamura used on average four prices for the shares' regressions). The sum of these disaggregated shares are not constrained across rows. As a result, intermediate consumption can exceed the supply of input, i.e. violate equation (23). Finally, prices are not available on the time-series bases for some inputs. This is common for different types of services (medical, business etc.). At the same time, detailed service outlays are reported in the I-O table and, therefore, can be projected with the RAS or other methods of I-O projection.

According to both works the shares estimated at the first stage of the H-J procedure present good proxy for a very aggregated I-O tables. However, due to the outlined shortcomings of the disaggregation (second and further stages of the H-J procedure), the number of estimated shares may not be adequate, as was found by Nakamura.

The procedure that combines H-J disaggregated shares with the RAS method is developed in the following Subsection.

3.2 RAS method and the notion of associations

Methods of projection of an I-O table from the base to the target period are known as nonsurvey procedures. These procedures are based on the principal that the estimate of an I-O table should be as "close" as possible to the latest observed I-O table, which means:

$$\begin{aligned} min \quad & \boldsymbol{d}(\boldsymbol{A}^{\circ}, \boldsymbol{A}^{p}) \\ s.t. \quad \sum_j \boldsymbol{a}_{ij}^{p} &= \boldsymbol{a}_{i.} \\ \sum_i \boldsymbol{a}_{ij}^{p} &= \boldsymbol{a}_{.j} \end{aligned} \tag{24}$$

where the objective function represents the distance between base I-O table $\boldsymbol{A}^{\circ}$ and target (unknown) table $\boldsymbol{A}^{p}$, and $\boldsymbol{a}_{i.}$ and $\boldsymbol{a}_{.j}$ are margins for rows and columns. This system consists of $(2n-1)$ equations, since one of the equations is linearly dependent on the rest. To derive the RAS procedure, the objective function in (24) should be expressed as an information measure of distance (for details see Bacharach). Then, system (24) is rewritten as:

$$\begin{aligned} min \quad & \sum_{i,j} \widetilde{\boldsymbol{z}_{ij}} \ln(\widetilde{\boldsymbol{z}_{ij}}/\boldsymbol{z}_{ij}) \\ s.t. \quad & \sum_j \widetilde{\boldsymbol{z}_{ij}} = \boldsymbol{f}_{i.} \\ & \sum_i \widetilde{\boldsymbol{z}_{ij}} = \boldsymbol{f}_{.j} \end{aligned} \tag{25}$$

where transaction flows $\boldsymbol{z}_{ij}$ are substituted for the direct coefficients $\boldsymbol{a}_{ij}$. It will be shown in the following that this substitution does not alter the results of the RAS procedure. Applying the Lagrange multipliers method, one derives:

$$\widetilde{\boldsymbol{z}_{ij}}(\boldsymbol{z}_{ij}/\widetilde{\boldsymbol{z}_{ij}})/\boldsymbol{z}_{ij} + \ln(\widetilde{\boldsymbol{z}_{ij}}/\boldsymbol{z}_{ij}) - \boldsymbol{\lambda}_i - \boldsymbol{\mu}_j = \mathbf{0},$$

or

$$\ln \widetilde{\boldsymbol{z}_{ij}} = \boldsymbol{\lambda}_i + \ln \boldsymbol{z}_{ij} + (\boldsymbol{\mu}_j - \mathbf{1}),$$

then

$$\widetilde{z_{ij}} = e^{\lambda_i} z_{ij} e^{\mu_j - 1}.$$

This expression can be written in matrix form as:

$$Z^p = \widehat{R} Z^\circ \widehat{S} \tag{26}$$

where $r_i = e^{\lambda_i}$ and $s_j = e^{\mu_j - 1}$. Matrix Z^p is computed via a series of iterations: the base matrix is multiplied by the ratio of row sums to row margins, then the resultant matrix is multiplied by the ratio of column sums to column margins, and so on until sums of rows and columns of the resultant matrix are identical to the corresponding margins. The RAS procedure is convergent and has a unique solution (for the proof see Bacharach).

Next, the notion of associations between the elements of a matrix is introduced. For any matrix Z, the *association* among any four elements, which are chosen at the intersection of two given rows (i and k) and two given columns (j and m) is a number:

$$D_{ij}^{km} = \frac{z_{ij} z_{km}}{z_{kj} z_{im}}$$

where $j \neq m$ and $k \neq i$.

It can be seen that the assocations for matrix Z^p projected by RAS are the same as for the base matrix Z° since:

$$D_{ij}^{km} = \frac{\widetilde{z_{ij}}\,\widetilde{z_{km}}}{\widetilde{z_{kj}}\,\widetilde{z_{im}}} = \frac{(r_i z_{ij} s_j)(r_k z_{km} s_m)}{(r_k z_{kj} s_i)(r_i z_{im} s_m)} = \frac{z_{ij} z_{km}}{z_{kj} z_{im}}. \tag{27}$$

Also, D_{ij}^{km} is the same whether we consider the transaction flows matrix Z or the direct coefficients matrix A. Since $z_{ij} = a_{ij} X_j$, the association D_{ij}^{km} can be written as:

$$D_{ij}^{km} = \frac{z_{ij} z_{km}}{z_{kj} z_{im}} = \frac{(a_{ij} X_j)(a_{km} X_m)}{(a_{kj} X_j)(a_{im} X_m)} = \frac{a_{ij} a_{km}}{a_{kj} a_{im}}.$$

RAS procedure preserves associations [equation (27)] and provides unique projection. Since matrices A and Z have the same associations the RAS will provide identical projection wether the base matrix is $\boldsymbol{A}$ or $\boldsymbol{Z}$. [11] Similarly, it can be shown that, whether the base matrix is in current or real values, the RAS projection will be the same:

$$D_{ij}^{km} = \frac{z_{ij}z_{km}}{z_{kj}z_{im}} = \frac{(p_i q_{ij})(p_k q_{km})}{(p_k q_{kj})(p_i q_{im})} = \frac{q_{ij}q_{km}}{q_{kj}q_{im}}$$

where q is a physical measure of input. The price units of the projected matrix are determined by the margins and not by the base matrix.

3.3 ERAS method as a disaggregation tool for the H-J estimates

The system (24) was formulated for $(2n-1)$ equations and, correspondingly, the RAS procedure would require $(2n-1)$ margins. In traditional I-O analysis, data on margins are considered as the only information available for the target period. The H-J procedure estimates aggregated coefficients that can be considered as totals of subcolumns of the I-O table. This can be expressed as $((2n-1)+k)$, where k is a number of subcolumns for which total sums are estimated in the form of the share. Consider one such subcolumn l with two elements, the constraint for the subcolumn is: $\widetilde{z_{kl}} + \widetilde{z_{ml}} = z_l$. This extra constraint for system (25) can be expressed as:

$$\begin{aligned} min \quad & \sum_{i,j} \widetilde{z_{ij}} \ln(\widetilde{z_{ij}}/z_{ij}) \\ s.t. \quad & \sum_j \widetilde{z_{ij}} = f_{i.} \\ & \sum_i \widetilde{z_{ij}} = f_{.j} \quad for \quad j \neq l \\ & \widetilde{z_{kl}} + \widetilde{z_{ml}} = z_l \end{aligned} \tag{28}$$

[11] It is common to suggest to begin the RAS procedure with the A matrix to derive Z matrix, since margins are given as flow values (f) see footnote (5). This is unnecessary, since the whole process can begin from Z.

Table 1: Base matrix for the RAS procedure.

	1	2	3	4
1	X	X	X	X
2	X	X	X	X
3	X	X	X	X
4	X	X	X	X

Table 2: Modification of the base matrix when a single subcolumn constraint is given. This constraint is $\widetilde{z_{15}} + \widetilde{z_{25}} = z_5$. $(\cdot)$ denotes elements replaced by zero; (X) denotes elements which are not replaced.

	1	2	3	4	5
1	X	X	0	X	·
2	X	X	0	X	·
3	X	X	X	X	0
4	X	X	X	X	0

$$\sum_{j \neq k,m} \widetilde{z_{ij}} = f_{.l} - z_l$$

Applying the Lagrangian method, in a manner similar to system (25), we derive $\widetilde{z_{ij}} = r_i z_{ij} s_j$. With the traditional set of margins, such as system (25), multipliers $\boldsymbol{R}$ and $\boldsymbol{S}$ are of dimension $\boldsymbol{n}$. However, in the above example of system (28), $\boldsymbol{S}$ is of dimension $(\boldsymbol{n} + \mathbf{1})$ because of the one additional constraint in the form of the new column. The number of rows remains the same, making $\boldsymbol{R}$ of dimension $\boldsymbol{n}$.

The difference in methodology between RAS and ERAS is in the specification of constraints, as was illustrated in system (28). Representation of the base matrix constitutes another difference between the two methods. This difference is illustrated in Tables 1 and 2.

The ERAS method has exactly the same basis in the minimum information principle as does RAS. The iterative procedure applied to ERAS is the same as for RAS, so the theorems of uniqueness and existence of RAS are equivalent for ERAS. In the above example, one subcolumn constraint was applied to two elements of one column. Obviously, system (28) can be designed for many rows and columns. The H-J system predicts shares (in our example it is z_l). ERAS can disaggregate this share following outlined modifications for margins and the base matrix.

The main difference between RAS and ERAS is the change in associations. The RAS projection postulates that associations in the base and target matrices are the same. The ERAS relaxes this assumption. This can be explained with the example based on elements from Table 2:

$$\widetilde{D_{11}^{33}} = \frac{(r_1 z_{11} s_1)(r_3 z_{33} s_3}{(r_3 z_{31} s_1)(r_1 z_{13} \sigma)} = \frac{s_3}{\sigma} D_{11}^{33} \qquad (29)$$

whereas $\widetilde{D_{31}^{43}} = D_{31}^{43}$ etc, where $\widetilde{D}$ denotes associations derived by ERAS and D represents associations derived by the RAS procedure. Therefore associations of the target matrix derived by ERAS are not necessarily identical to those of the base matrix.

4 Conclusion

H-J estimated the value of input shares in current dollars using translog relations between input prices and shares. Afterwards, the estimated shares were deflated using I-O relations. This paper has pointed out that estimation of deflated shares with the deflated input prices via translog relations would provide results different from those derived by the H-J method. It was further stated that the H-J method does not provide an appropriate deflator for value added (from the I-O point of view).

Double deflated value added should be used for the econometric estimation of deflated shares and prices. If the price of the target year is chosen as base price for deflation, then the set of constraints that is typical for the RAS method can easily be incorporated into the H-J model.

The H-J method can only provide estimates for the highly aggregated I-O table. The ERAS method presented here allows to disaggregate the estimated I-O table according with the associations of the base table.

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