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**FINITE HORIZONS, INTERTEMPORAL  
SUBSTITUTION AND FISCAL POLICY**

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and Fiscal Policy

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## Abstract

This paper synthesizes two strands of current macroeconomic research in fiscal policy by allowing for finite lifetimes in an intertemporal substitution model. Among other things, the paper establishes that in such a model public debt issuance is contractionary with respect to current output, a conclusion in strong contrast with conventional macroeconomic analysis.



## I. Introduction

This paper contains an investigation of the effects of fiscal policies on investment and output in an intertemporal substitution model where agents have finite horizons. The paper synthesizes two strands of the current literature in macroeconomic theory. In the intertemporal substitution model as popularized by Barro (1976, 1981, 1987) agents typically are taken to have infinite planning horizons by virtue of the assumption of an operative intergenerational bequest mechanism. In this model, government bonds are not "net wealth" and lump sum public financial policy is irrelevant for the determination of interest rates, consumption, investment, and output. However, public expenditure on goods--temporary or permanent--may have a contemporaneous impact on macroeconomic variables of interest.

On the other hand, in the recent work of Blanchard (1985) and Frenkel and Razin (1986) agents are given finite horizons so that government bonds constitute net wealth and public financial policy takes on relevance. However, these models allow optimization only over consumption, neglecting leisure, which eliminates the ability of fiscal policy actions to have contemporaneous output effects.

In the model of this paper, agents optimize over consumption and leisure and current production is a function of the accumulated capital stock and labor supply. The major conclusions are as follows. Under lump sum taxation, a tax cut crowds out private investment, as in conventional models, but also induces a contraction of current output. Temporary expansions of public purchases also crowd out investment but raise production. Permanent increases in government spending leave the capital stock unchanged and raise output by less than in the temporary expenditure case. Under distortional labor

taxation, a reduction in tax rates has an ambiguous effect on investment and output, but it appears plausible (in the context of this model) that such a tax cut would crowd in investment as well as raise current production levels.

## II. An Intertemporal Substitution Model with Finite Horizons

The following model, though highly stylized, is adequate for conveying the essential points of the paper. Agents in the economy have preferences given by

$$V_t^S = \sum_{\tau=t}^{\infty} \theta^{\tau-t} [\alpha \log c_{\tau}^S + (1-\alpha) \log l_{\tau}^S]. \quad (1)$$

Here, the notation  $x_i^j$  denotes the value of  $x$  at time  $i$  for a representative agent born at date  $j$ . The level of utility during any period is taken to be an increasing function of consumption,  $c$ , and leisure,  $l$ . The subjective discount factor,  $\theta$ , is assumed to lie between zero and one and measures the extent to which the agent is "impatient to consume" apart from any uncertainty about the length of life.

As in Blanchard (1985) and Frenkel and Razin (1986), individuals are assumed to have a constant, known probability of survival equal to  $p$ . Hence, the probability of an individual at time  $t$  surviving to time  $\tau$ ,  $\tau \geq t$ , is  $p^{\tau-t}$  and expected utility is

$$EV_t^S = \sum_{\tau=t}^{\infty} (p\theta)^{\tau-t} [\alpha \log c_{\tau}^S + (1-\alpha) \log l_{\tau}^S]. \quad (2)$$

The introduction of a certain probability of death effectively transforms the subjective discount factor from  $\theta$  to  $p\theta$ , making the agent more impatient than in a completely deterministic environment.

The population is normalized to unity in each period in the following way. Letting  $x$  denote the number of individuals who are born each period ( $s \leq t$ ) and recognizing that by application of the law of large numbers at time

at time  $t$  there will remain  $p^{t-s}$  individuals of cohort  $s$ , we have the total population given by

$$\sum_{s=-\infty}^t p^{t-s} x = \frac{x}{1-p}.$$

Thus, a constant population equal to unity requires  $x=1-p$  individuals to be born each period.

Now letting  $R_t$  denote the period  $t$  discount factor defined by

$R_t^{-1} = \prod_{s=-\infty}^{t-1} (1+r_s)$ , with  $r_s$  the rate of interest on regular or riskless loans in period  $s-1$ , we have  $1+r_t = R_t/R_{t+1}$ . However, the individual's borrowing rate, due to probability of death, requires an insurance premium so that the effective borrowing rate,  $\delta_t$ , is defined by

$$1+\delta_t = R_t/pR_{t+1}.$$

Using this last result, the agent's intertemporal budget constraint may be expressed as below:

$$\sum_{\tau=t}^{\infty} \frac{p^{\tau-t} R_{\tau}}{R_t} [c_{\tau}^S - (1-\lambda_{\tau})\gamma_{\tau}(1-l_{\tau}^S) + z_{\tau}] + n_t^S = 0 \quad (3)$$

where  $\gamma_{\tau}$  is the real wage in period  $\tau$ ,  $\lambda_{\tau}$  is a proportional tax on labor income in period  $\tau$ ,  $z_{\tau}$  are lump sum taxes in period  $\tau$  and  $n_t^S$  is the agent's nonhuman wealth as of period  $t$ .

Maximization of utility (2) subject to the budget constraint (3) yields the first order necessary conditions

$$\frac{\alpha\theta}{c_t^S} \tau^{-t} + \phi \frac{R_{\tau}}{R_t} = 0 \quad \tau \geq t \quad (4)$$

$$\frac{(1-\alpha)\theta}{l_t^S} \tau^{-t} + \phi \gamma_{\tau}(1-\lambda_{\tau}) \frac{R_{\tau}}{R_t} = 0 \quad \tau \geq t \quad (5)$$

where  $\phi$  is a Lagrangian multiplier. These may be used in conjunction with the budget constraint to obtain the consumption and leisure demand functions

$$c_t^S = \alpha(1-p\theta)(n_t^S + h_t^S) \quad (6)$$

$$l_t^S = \frac{(1-\alpha)(1-p\theta)}{\gamma_t(1-\lambda_t)} (n_t^S + h_t^S) \quad (7)$$

where human wealth,  $h_t^S$ , is defined by

$$h_t^S = \sum_{\tau=t}^{\infty} \frac{p^{\tau-t} R_{\tau}}{R_t} [(1-\lambda_{\tau})\gamma_{\tau} - z_{\tau}]. \quad (8)$$

Thus, consumption and leisure demand are proportional to the agent's total--nonhuman plus human--wealth. Note, however, that the factor of proportionality depends inversely on the probability of survival.

Aggregating across individuals, we have total consumption at time  $t$  given by

$$\begin{aligned} c_t &= (1-p)c_t^t + p(1-p)c_t^{t-1} + p^2(1-p)c_t^{t-2} + \dots \\ &= (1-p) \sum_{s=-\infty}^t p^{t-s} c_t^s \end{aligned} \quad (9)$$

where it will be recalled that  $(1-p)$  individuals are born each period. Similarly, aggregate leisure is described by

$$l_t = (1-p) \sum_{s=-\infty}^t p^{t-s} l_t^s \quad (10)$$

while aggregate nonhuman wealth is composed of the aggregate physical capital stock,  $k_t$ , plus government debt,  $d_t$ , and total aggregate wealth is

$$w_t = k_t + d_t + \sum_{\tau=t}^{\infty} \frac{p^{\tau-t} R_{\tau}}{R_t} [(1-\lambda_{\tau})\gamma_{\tau} - z_{\tau}]. \quad (11)$$

Turning next to the government, we recognize that as the population remains constant at unity, the public sector's planning horizon is not affected by the uncertainty of death faced by individuals. Consequently, the government may borrow or lend at the riskless rate, resulting in an intertemporal budget constraint of the form

$$d_t + \sum_{\tau=t}^{\infty} \frac{R}{R_t} g_{\tau} = \sum_{\tau=t}^{\infty} \frac{R}{R_t} [z_{\tau} + \lambda_{\tau} \gamma_{\tau} (1 - \delta_{\tau})] . \quad (12)$$

which equates the initial stock of debt plus the present value of goods expenditures,  $g$ , to the present value of lump sum and labor income tax collections.

Finally, the model is closed by asserting the market clearing conditions

$$c_t + i_t + g_t = y_t \quad (13)$$

where  $i_t$  is gross investment in physical capital and gross output,  $y_t$ , is given by the linear production technology

$$\begin{aligned} y_t &= f(\delta_t, k_t) \\ &= \gamma_t(1 - \delta_t) + (\rho_t - 1)k_t \end{aligned} \quad (14)$$

$\rho_t$  being the gross return to capital at time  $t$ .

### III. Fiscal Policies Under Lump Sum Taxation

In this section we allow the public sector to have recourse to lump sum taxation so as to study the effect of changes in taxes and public spending on wealth and thereby on consumption, output, and investment. Accordingly, we set  $\lambda_t = 0$  in all periods.

#### Government Bonds and Wealth

We begin by considering the question of whether bonds issued by the government are treated, on net, as additions to private sector wealth.

Utilizing equations (11) and (12) allows us to obtain the economy-wide wealth expression

$$w_t = \sum_{\tau=t}^{\infty} \frac{R_{\tau}}{R_t} [p^{\tau-t} \gamma_{\tau} - g_{\tau} + (1-p^{\tau-t}) z_{\tau}]. \quad (15)$$

This particular aggregate wealth expression points out the fact that government bonds are net wealth since lump sum taxes do not cancel out in the process of internalizing the government budget constraint. Clearly, as the probability of survival becomes smaller, aggregate wealth rises, reflecting the fact that the future tax obligations necessary to service or retire the outstanding public debt are being discounted at a higher rate than the risk free rate  $r$ . Only if the probability of survival equals unity--the conventional intertemporal substitution model--will the wealth expression be invariant with respect to changes in the time profile of taxes and government bonds cease to be perceived as net wealth.

In order to focus on the effects of various fiscal changes on the economy's intertemporal equilibrium, we make periods  $t+1, t+2, \dots$  a composite period by assuming  $\gamma_{t+j} = \bar{\gamma}$ ,  $g_{t+j} = \bar{g}$ ,  $k_{t+j} = \bar{k}$  and  $z_{t+j} = \bar{z}$  for all  $j \geq 1$ . The capital stock is assumed not to depreciate. The first order necessary conditions for individual optimization plus the goods market clearing conditions then allow us to obtain the solution for current investment as

$$i = \bar{k} - k = \frac{1}{(\rho-1+p)} [(1-p)(1-\rho)k + p(\gamma-g) + (1-p)(\bar{\gamma}-\bar{z}) - (\gamma-\bar{g})]. \quad (16)$$

Consider, as a benchmark, the pure intertemporal substitution model where  $p=1$ . The capital accumulation equation in this nearly steady state environment then becomes

$$i = \frac{1}{\rho} [(\gamma - g) - (\bar{\gamma} - \bar{q})] \quad (17)$$

so that investment will be positive (negative) in periods with high (low) labor productivity or low (high) government spending. In this fashion, the agent smooths goods consumption by transferring resources from periods of relative abundance to periods of scarcity. Furthermore, as was to be expected from the previous discussion, in this benchmark case the timing of taxation is irrelevant to investment expenditure. Public sector deficits, driven by tax cuts, do not "crowd out" private investment.

Returning to the general case, however, we see that the impact of a reduction in current taxes and a rise in future taxes to balance the government budget constraint over time is to reduce private investment by the amount

$$di = \frac{(\rho - 1)(1 - p)}{(\rho - 1 + p)} dz.$$

Pure public sector deficits now crowd out private investment since the rearrangement of taxes increases wealth, and by equations (6) and (7) raises current consumption as well as lowers current production in equilibrium by the amount

$$dy = \frac{(1 - \alpha)(\rho - 1)(1 - p)}{(\rho - 1 + p)} dz .$$

Perhaps surprisingly, tax cuts are contractionary, as opposed to the usual analysis where as a consequence of a tax cut an expansion of current output follows the expansion of demand. Here, agents respond to the increase in wealth not only by increasing consumption but also by reducing labor supply and so production falls.

This result that tax cuts may be contractionary is in line with similar conclusions reached on the basis of considerably different models. Blanchard (1984) argues that in an environment of slowly increasing deficits, real interest rates on long bonds rise, depressing current investment and output. Feldstein (1984) presents a two sector model in which a negative fiscal multiplier is possible through induced changes in the sectoral balance of demand. For example, a tax cut which raises consumption demand and the price level will reduce real money balances so that, given a low enough interest sensitivity of money demand, real output must fall to equilibrate the money market. Mankiw and Summers (1986) argue that an appropriate scale variable in money demand is aggregate consumption and not real output; consequently, in the context of an otherwise standard IS-LM model a tax cut may be contractionary if the effect on output via the excess demand for goods is dominated by the effect via the excess demand for money.

Next, consider various public expenditure policies. The impact of a permanent rise in government spending financed by an equal increase in taxes is to leave investment unchanged while raising output:

$$di = \frac{1}{(\rho-1+p)} [-p-(1-p)+1]dg = 0$$

$$dy = (1-\alpha)dg.$$

Here, a permanent fiscal expansion has no effect on intertemporal resource allocation because in any future period in which the representative agent expects to be alive net output will be reduced in an amount equal to the current reduction. Output rises, permanently, due to the fact that the increase in government spending--here modelled as wasteful from the standpoint of the private sector--increases work effort and production through a negative wealth effect.

A temporary, balanced budget rise in public spending reduces investment while also raising output:

$$di = \frac{-p}{(\rho-1+p)} dg$$

$$dy = \frac{(\rho-1)(1-\alpha)}{(\rho-1+p)} dg.$$

Here, the crowding out of investment arises as the agent attempts to shift resources from the future to the present to compensate for the extraordinary public sector demand for goods. Furthermore, the extent of the crowding out is more severe than in the pure intertemporal substitution case since the representative agent values current consumption at a premium over future consumption (over and above usual time preference) due to the probability of dying. As in the permanent expenditure case, output rises but by less than in the former case because the wealth effect operating on labor supply is mitigated as a result of the temporary nature of the resource drain through the government sector.

Finally, consider the joint impact of a temporary rise in public spending which is financed by future taxation. We have

$$di = \frac{[-p - (1-p)(\rho-1)]}{(\rho-1+p)} dg$$

$$dy = \frac{p(1-\alpha)(\rho-1)}{(\rho-1+p)} dg$$

so that the extent of the crowding out of investment is greater while the impact on output is smaller than in the previous cases. Thus, the present model suggests the possibility that the most stimulative of fiscal policy

actions, under lump sum taxation, would involve the government raising spending permanently and financing it by a relatively larger increase in current taxes. The most expansionary of policies would be those which lower wealth, not raise it, a result in clear contrast to those obtained on the basis of conventional, "demand side" models.

#### IV. Distortional Taxation

We now investigate the effects of changes in distortional tax rates on the economy's equilibrium. The particular exercise considered is a reduction in current tax rates on labor income matched by an increase in future tax rates to balance the government's intertemporal budget. To simplify, we set government spending to zero in each period and normalize the marginal product of labor to unity. Following a method directly analogous to the lump sum case, we obtain

$$\dot{i} = \frac{1}{(\rho-1)\beta(\lambda)+p\beta(\lambda)} \{ (1-\rho)[\beta(\bar{\lambda})-p\beta(\lambda)]k+p\beta(\lambda)-(1-\rho)(1-\bar{\lambda})-\beta(\bar{\lambda}) \} \quad (18)$$

where

$$\beta(x) = \frac{1-x}{1+\alpha x}$$

Differentiating equation (18), paying explicit attention to the government budget constraint and evaluating around an initial equilibrium with zero tax rates, we have

$$d\dot{i} = \left[ \frac{\rho p}{(\rho-1+p)} - \alpha \right] d\lambda.$$

In general, the effect on investment of a reduced tax rate is ambiguous. On the one hand, a reduction in current tax rates twists the intertemporal profile of the net of tax real wage which tends to increase current production, saving and investment. On the other hand, the tax reduction raises wealth which tends to decrease current production, raise consumption, and lower investment. The critical value of  $p$  for which these substitution and wealth effects cancel is

$$p^* = \frac{\alpha(\rho-1)}{(\rho-\alpha)} .$$

Higher (lower) values of  $p$  than  $p^*$  imply a crowding in (out) of investment subsequent to a reduction in the current tax rate. In the context of the present, highly abstract model if we were to assume a marginal physical product of capital of  $\rho-1=.05$  and a value of physical consumption's share in total consumption of  $\alpha=.5$  we have  $p^*=.05$  which appears to be sufficiently low to render likely a crowding in of private investment.

Further, note that the contemporaneous output effect of the tax policy is given by

$$dy = (1-\alpha) \left[ \frac{\rho p}{(\rho-1+p)} - \alpha \right] d\lambda$$

so that if the tax cut crowds in investment it also raises current production levels. This positive association of investment and production should be contrasted with the usual analysis of the impact of fiscal policy, where a tax cut works to stimulate output but is balanced by a crowding out of investment expenditure.

## V. Conclusion

The effect of alterations in the fiscal stance of the public sector on the economy is the subject of major dispute among various macroeconomic schools of thought. Traditional Keynesian models of the behavioral function variety attribute a strong stimulative effect of lump sum tax cuts on the economy as agents respond to the increase in disposable income and wealth by expanding consumption expenditure. The simple model presented here suggests the possibility that tax cuts may depress current, as well as future, output levels as agents expand leisure activities in response to an increase in their perceived wealth status. Indeed, the fiscal policies most capable of expanding production are those which lower wealth, not raise it. Of course, whether tax cuts actually increase wealth is a matter to be resolved by empirical research. Recently, Feldstein (1982) rejected Ricardian equivalence while Aschauer (1985) and Kormendi (1983) found no evidence that changes in the intertemporal pattern of taxation altered consumption expenditure.

The model also brings into focus the interplay between substitution and wealth effects corresponding to a reduction in contemporaneous labor income tax rates. The net effect of such a tax cut on output and investment is ambiguous, and resolution of the issue depends on accumulated empirical evidence. Dewald (1983), Evans (1985), Makin (1983), Mascaro and Meltzer (1983), and Plosser (1981) find little or no effect of tax changes on interest rates. Indeed, if anything, Evans (1985) discovers a negative relationship between tax induced budget deficits and interest rates. On the other hand, Hoelscher (1986) isolates a positive relationship between deficits and long term rates.

The model of this paper clearly is not detailed enough to provide a basis for policy evaluation. What the model does hint is that confidence about the effects of fiscal policy on the economy must be muted. Such "model pessimism" lies behind much of the distaste of newclassical economists for attempts to "fine tune" the economy along traditional Keynesian lines.

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