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Pairwise Trade, Asset Prices and Monetary Policy

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Pairwise Trade, Asset Prices, and Monetary Policy*

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Abstract

We construct a search-theoretic model in which fiat money coexists with real assets, and no restrictions are placed on the use of assets as means of payment. The terms of trade in bilateral matches are determined by a pairwise Pareto-efficient pricing mechanism. From the view point of agents with a financing need, this mechanism replicates the explicit liquidity constraints found in Kiyotaki and Moore (2005) or Lagos (2006) that are needed to generate asset pricing facts found in the data. A critical difference, however, is that we do not impose any such constraints in our environment. We show that fiat money can be valued despite being dominated in its rate of return. Moreover, real assets can generate different rates of returns even if agents are risk-neutral. An increase in inflation raises assets' prices, lowers their returns, and widens the rate-of-return differences between real assets. Finally, there is a range of inflation rates that implement the first-best allocation.

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1 Introduction

What are the determinants of an asset's liquidity? Twenty years ago, Kiyotaki and Wright (1989)—KW hereafter—provided an answer to this question in the context of a monetary model with bilateral trades. They found that the moneyness of an asset depends on its physical properties (e.g., storage costs), the fundamentals (e.g., the pattern of specialization), and on conventions (e.g., the coordination on one among multiple equilibria). These insights were derived under extreme portfolio restrictions—agents cannot hold more than one unit of an asset—and stark assumptions, such as indivisible assets and goods. More recent developments in the search-theoretic approach to monetary economics that allow for unrestricted portfolios and divisible assets—e.g., Shi (1997) and Lagos and Wright (2005)—have led to a renewed interest for the initial question that prompted this literature: What makes money money?

According to the original insights of KW, the liquidity differences among assets can stem from differences in terms of assets' physical properties. Lester, Postlewaite, and Wright (2007) focus on the recognizability property of assets, and show that fiat money is a superior means of payment because it is harder to counterfeit and easier to authenticate than other assets.[1] Fundamentals can also help explain the moneyness of assets. Rocheteau (2008) shows that in the presence of informational asymmetries the liquidity of assets depends on the properties of their dividend processes.

In this paper, we take yet a different approach. We pursue the idea that when agents interact in small groups, the possible set of equilibrium outcomes can be a non-degenerate core, and that agents use bargaining conventions to determine prices. The non-degeneracy gives the modeler some freedom, in terms of choosing the conventions that agents use, for determining the actual terms of trade that will prevail. Our approach is inspired by the work of Zhu and Wallace (2007). We are not alone in believing that such an approach can yield useful insights. Kocherlakota (2005, pp. 726-727) points out that "we are likely to continue to learn a lot about monetary economics, and economics more generally, by studying the implications of ... trade tak[ing] place in small groups." So, whereas in KW, network-like externalities can generate multiple equilibria with different payment arrangements, and a convention will dictate which equilibrium is played,[2]

[1] The recognizability of money is an old theme in monetary theory. Recent formalizations include Williamson and Wright (1994) and Banarjee and Maskin (1996).

[2] Kiyotaki and Wright (1989) consider two versions of their model, models A and B, that differ in terms of the pattern of specialization. While the equilibrium of model A is unique, model B delivers multiple equilibrium for a nonempty set of parameter values. For other examples of search-theoretic models of payments with multiple equilibria, see, e.g., Matsuyama, Kiyotaki and Matsui (1993) and Ayagari, Wallace and Wright (1996).

in our environment, the possibility of multiple equilibria presents itself in a very natural way, thanks to a large set of Pareto-efficient allocations in core, and the convention determines which allocation is chosen.

To develop our argument, we adopt the search-theoretic model with divisible money of Lagos and Wright (2005), in which a lack of double coincidence of wants and the absence of a record-keeping technology in decentralized markets generate an explicit need for a tangible medium of exchange. Besides money, we introduce real assets, "Lucas' trees," that yield a flow of real dividends.[3] We adhere to the Wallace (1996) dictum and place no restrictions on the use of assets as means of payment. Agents are free to use any quantity of their real assets and money holdings for transactions purposes. A crucial aspect of the environment is that some trades take place in bilateral meetings. Because the Pareto-frontier of the bargaining set in a bilateral match is non-degenerate, a large set of pairwise Pareto-efficient allocations are potentially implementable. We will exploit this feature of the model to select a mechanism that generates outcomes that are qualitatively consistent with some observations in the data.[4]

While most of the recent search literature assumes an axiomatic bargaining solution to determine terms of trades in bilateral meetings, we follow Zhu and Wallace (2007) and only retain the axiom of Pareto efficiency.[5] The axiom of Pareto-efficiency, however, is not enough to get a unique outcome. Instead of imposing other axioms, such as independence of irrelevant alternatives or monotonicity, we take a different route and let some qualitative aspects of the data guide our selection of a trading mechanism. We demand that the trading mechanism accounts for the rate-of-return-dominance puzzle, according to which fiat money can coexist with interest-bearing assets, and various other asset pricing anomalies. We also want the model to generate the observed negative relationship between inflation and assets' returns.

Kiyotaki and Moore (2005) are able to generate the asset pricing anomalies and the effects of monetary

[3] The first attempt to introduce capital goods into the Lagos-Wright model was by Aruoba and Wright (2003) but capital goods were not allowed to be used as means of payment in bilateral matches. Lagos and Rocheteau (2008) relax the restriction on the use of capital as a competing means of payment and show that fiat money and capital can coexist provided that there is a shortage of capital to be used as means of payment. Geromichalos, Licari and Suarez-Lledo (2007) follow a similar approach but assume that capital is in fixed supply. Lagos (2006) calibrates the model where money is replaced by risk-free bonds and capital is a risky asset, and shows that it can account for the risk-free rate and equity premium puzzles under a mild restriction on the use of capital as means of payments.

[4] Interestingly, the same feature of the labor search model is used by Hall (2005) to explain wage rigidity. See the discussion in Kocherlakota (2005). In an earlier version of their paper Zhu and Wallace (2007) called their pricing mechanism "cash-in-advance with a twist" because from the view point of the buyer, it is as if he faces a cash-in-advance constraint. To mirror this terminology, our pricing protocol could be called "Kiyotaki-Moore with a twist."

[5] Shi (1997) and Lagos and Wright (2005) assume that terms of trade in bilateral matches are determined by the generalized Nash solution. Aruoba, Waller, and Rocheteau (2007) investigate the robustness of the results to alternative bargaining solutions. Other mechanisms have been studied such as auctions and competitive posting. For a mechanism design approach where the mechanism is chosen by normative considerations, see Hu, Kennan and Wallace (2008).

policy we want to explain.[6] They do so, however, by imposing liquidity constraints on agents, where the constraint takes the form of a restriction on the fraction of real asset holdings that agents can use to finance investment opportunities. We design a family of trading mechanisms that replicate the same asset pricing patterns as the ones in the economy with liquidity constraints of Kiyotaki and Moore (2005) but we do not impose any exogenous liquidity constraints in our environment. Unless a Pareto-efficient trade is achieved, our mechanism does not leave any asset unused as a means of payment. The family of trading mechanisms we consider is parametrized by a single parameter—just like the generalized Nash solution—and it admits as particular cases the pricing mechanisms considered in Geromichalos, Licari and Suarez-Lledo (2007), Lagos (2006), Lagos and Rocheteau (2008), Zhu and Wallace (2007).

The main insights of our theory are as follows. First, fiat money can be held and valued despite being dominated in its rate of return by competing assets. In contrast to earlier works, we do not need to impose trading restrictions to generate a rate of return dominance pattern: a plain-vanilla Lagos-Wright model can account for this puzzle. Second, the model is capable of generating a liquidity-based structure of assets' yields, where real assets can exhibit different rates of returns, even if they share the same risk characteristics, or if agents are risk-neutral.[7] Those rate of return differences, which are all anomalies from the standpoint of consumption-based asset pricing theories, reflect differences in the liquidity of the assets; some assets can be liquidated at better terms of trade than others. Third, our model also has implications for the transmission mechanism of monetary policy to asset prices. Under our pricing protocol, real assets have a liquidity value—i.e., a buyer is able to capture some surplus in a bilateral meeting when paying with real assets—and, hence, an increase in inflation will raise their prices and lower their returns. Moreover, inflation widens the rate of return differences between real assets. Finally, from a normative standpoint, there is a range of inflation rates that implement the first-best allocation, including the Friedman rule. As a consequence, a small inflation above the Friedman rule does not impose a welfare cost on society.

[6] More generally, our paper is related to the recent literature in macroeconomics that takes into account the transaction role of assets in order to explain asset pricing anomalies and the effects of monetary policy on assets' returns. For instance, Bansal and Coleman (1996) explain the risk-free rate and the equity premium puzzles in a pure exchange economy in which there are different transactions costs associated with the use of different means of payment, e.g., fiat money, government bonds and credit.

[7] Wallace (2000) provides a theory of the liquidity structure of asset yields based on the indivisibility of assets. Weill (2008) explains differences in liquidity across seemingly identical assets by the presence of thick market externalities in decentralized asset markets.

2 The environment

Time is discrete and continues forever. The economy is populated with a $[0, 1]$ continuum of infinitely-lived agents. As in Lagos and Wright (2005), each period is divided into two subperiods, called AM and PM. In the AM, trade takes place in decentralized markets, where agents are bilaterally matched in a random fashion. In the PM, trade takes place in competitive markets.

In the AM decentralized market, agents produce and consume perishable goods that come in different varieties. The probability that an agent is matched with someone who produces a good he wishes to consume is $\alpha \leq 1/2$. Symmetrically, the probability that an agent meets someone who consumes the good he produces is $\alpha \leq 1/2$. For convenience, and without loss of generality, we rule out double-coincidence-of-wants meetings. In the PM subperiod, all agents are able to consume and produce a perishable (general) good.[8]

An agent's utility function is

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left[u\left(y_t^b\right)-c\left(y_t^s\right)+x_t-h_t\right]\right]$$

where y^b is consumption and y^s is production of the AM good, x is consumption of the general good, h is hours of work to produce the general good, and $\beta = (1+r)^{-1} \in (0, 1)$ is the discount factor across periods. We assume that $u(y) - c(y)$ is continuously differentiable, strictly increasing and concave. In addition, $c(0) = u(0) = 0$, $u'(0) = +\infty$, $u'(+\infty) = 0$, and there exists a $y^* < +\infty$ such that $u'(y^*) = c'(y^*)$. The technology to produce general good is linear and one-to-one in hours, i.e., h hours of work produce h units of the general good in the PM.[9]

Agents are unable to commit, and their trading histories are private information. This implies that credit arrangements are infeasible. The infeasibility of credit, in conjunction with the specialization of agents' consumption and production in the AM decentralized markets, generates a role for a medium of exchange. There are two storable and perfectly divisible assets in the economy, and both can serve as media of exchange without restrictions. There is a real asset that is in fixed supply, $A > 0$. In each PM subperiod, one unit of the real asset generates a dividend equal to $\kappa > 0$ units of the general good. There also exists an

[8] We could assume that the same goods which are traded in the AM decentralized market are also traded in the PM competitive market. However, the specialization in terms of preferences and technologies is irrelevant in a complete information, competitive environment.

[9] Following Lagos and Wright (2005), we could adopt a more general utility function in the PM, $U(x) - h$ with $U'' < 0$. Our results would not be affected provided that the nonnegativity constraint for the number of hours, $h \geq 0$, is not binding in equilibrium.

intrinsically useless asset called fiat money. The quantity of money at the beginning of period t is denoted M_t. The money supply grows at the gross rate $\gamma > \beta$, where $\gamma \equiv \frac{M_{t+1}}{M_t}$, via lump-sum transfers or taxes in the PM subperiod.[10] In the AM subperiod, producers and consumers in a bilateral match can exchange the assets for one another or for the consumption good. In the PM subperiod, assets are traded with the general good in competitive markets.

The asset prices at date t are measured in terms of the general good in the date t PM subperiod. The price of money is denoted by ϕ_t and the price of the real asset is denoted by q_t. In what follows, we will focus our attention on stationary equilibria, where $\phi_t M_t$ and q_t are constant.

3 Pricing

In this section, we describe the determination of the terms of trade in bilateral meetings in the AM decentralized market. Before we do this, however, it will be useful to show some properties of an agent's value function in the PM subperiod, W, since it tells us how agents will value the assets they give up or receive in the AM decentralized market. The value function of an agent entering the PM competitive markets holding a portfolio of a units of real asset and z units of real balances is,

$$W(a,z) = \max_{x,h,a',z'} \{x - h + \beta V(a', z')\} \tag{1}$$

$$\text{s.t. } \gamma z' + qa' + x = z + h + a(q + \kappa) + T, \tag{2}$$

where $T \equiv \phi_t(M_{t+1} - M_t)$, which is measured in terms of the general good, is the lump-sum transfer associated with money injection. At the start of the PM subperiod, each unit of the real asset can be bought or sold in a competitive market at the price q, and it generates κ units of the general good. The agent chooses his net consumption, $x - h$, and the portfolio, (a', z'), that he will bring into the subsequent decentralized market. Each unit of real balances acquired in the PM subperiod of date t will turn into $\frac{\phi_{t+1}}{\phi_t} = \gamma^{-1}$ units of real balances in date $t + 1$. Hence, to have z' units of real balances next period, an agent must acquire $\gamma z'$ units in the current period.

[10] If $\gamma < 1$ the government can force agents to pay taxes in the PM. In a related model, Andolfatto (2007) considers the case where the government has limited coercion power—it cannot confiscate output and cannot force agents to work—and the payment of lump-sum taxes is voluntary: agents can avoid paying taxes by not accumulating money balances. He shows that if agents are sufficiently impatient, then the Friedman rule is not incentive-feasible, i.e., there is an induced lower bound on deflation.

Substituting $x-h$ from (2) into (1) gives

$$W(a,z) = z + a(q+\kappa) + T + \max_{a',z'} \left\{-\gamma z' - qa' + \beta V(a',z')\right\}. \tag{3}$$

From (3), the PM value function is linear in the agent's wealth: this property will prove especially convenient in terms of simplifying the pricing problem in the AM decentralized market. Note also that the choice of the agent's new portfolio, (a',z'), is independent of the portfolio that he brought into the PM subperiod, (a,z), as a consequence of quasi-linear preferences.

Consider now a match in the AM decentralized market between a buyer holding portfolio (a,z) and a seller holding portfolio (a^s,z^s). The terms of trade are given by the output $y \geq 0$ produced by the seller and the transfer of assets $(\tau_m,\tau_a) \in [-z^s,z] \times [-a^s,a]$ from the buyer to the seller, where τ_m is the transfer of real balances and τ_a is the transfer of real assets. (If the transfer is negative, then the seller is delivering assets to the buyer.) The procedure that determines the terms of trade in the AM decentralized market generalizes the one suggested by Zhu and Wallace (2007). The procedure has two steps. The first step generates a payoff or surplus for the buyer, denoted as $\hat{U}^b$, which is equal to what he would obtain in a bargaining game if he had all the bargaining power, but was facing *liquidity constraints*. Specifically, in this "virtual game" it is assumed that the buyer can at most transfer a fraction θ of his real asset holdings, i.e., $\tau_a \leq \theta a$. Zhu and Wallace (2007) assume that $\theta = 0$; Lagos and Rocheteau (2008) and Geromichalos, Licari and Suarez-Lledo (2007) assume $\theta = 1$. In terms of real balance transfers, the buyer cannot transfer more than he holds, i.e., $\tau_m \leq z$.[11] The liquidity constraint on real asset holdings in the virtual game is chosen purposely to be reminiscent of the one used in Kiyotaki and Moore (2005), where individuals can only use a fraction of their capital goods to finance investment opportunities.[12] From the buyer's standpoint, it is as if he was trading in the Kiyotaki-Moore economy. The liquidity constraint is also reminiscent of the constraint in Lagos (2006), where $\theta = 0$ in a fraction of the matches and $\theta = 1$ in the remaining matches. Note that the output and wealth transfers that the first step generates are virtual in the sense that they are simply used to determine the buyer's surplus, $\hat{U}^b$. The *actual* output and wealth transfer are determined in the second step so as to generate a pairwise Pareto-efficient trade. The actual trade maximizes the seller's surplus subject to the constraint that the buyer receives a surplus at least equal to $\hat{U}^b$, and the only restrictions placed

[11] We do not constraint the transfer of asset holdings of the seller, but this is with no loss in generality.

[12] Kiyotaki and Moore (2005) consider an economy with two assets, capital and land. Land is in fixed supply while capital is accumulated. Both assets are inputs in the production of the final good. Individuals receive random opportunities to invest. In order to finance investment, they can use all their land—land is "completely liquid"—but only a fraction θ of their capital holdings. So land is analogous to money in our formulation, while capital is similar to our real asset.

on the transfer of either asset is that an agent cannot transfer more than he has, i.e., $-a^s \leq \tau_a \leq a$ and $-z^s \leq \tau_m \leq z$.

The first step of our pricing protocol, which determines the buyer's surplus $\hat{U}^b$, solves the following problem,

$$\begin{aligned} \hat{U}^b(a,z) = \max_{y,\tau_m,\tau_a} & [u(y) + W(a-\tau_a, z-\tau_z) - W(a,z)] \\ \text{s.t.} \quad & -c(y) + W(a^s+\tau_a, z^s+\tau_m) \geq W(a^s, z^s) \\ & \tau_m \in [-z^s, z], \quad \tau_a \in [-a^s, \theta a] \end{aligned}$$

The buyer in this virtual game maximizes his surplus, subject to the participation constraint of the seller and the constraints on the transfer of his asset holdings: while the buyer can transfer all his money balances, he can only hand over a fraction θ of his real asset.[13] Using the linearity of $W(a,z)$, the above problem can be rewritten as

$$\hat{U}^b(a,z) = \max_{y,\tau_m,\tau_a} [u(y) - \tau_m - \tau_a(q+\kappa)] \tag{4}$$

$$\text{s.t.} \quad -c(y) + \tau_m + \tau_a(q+\kappa) \geq 0 \tag{5}$$

$$-z^s - a^s(q+\kappa) \leq \tau_m + \tau_a(q+\kappa) \leq z + \theta a(q+\kappa) \tag{6}$$

From this formulation, note that what matters is the total value of the transfer of assets, $\tau_m + \tau_a(q+\kappa)$, and not its composition in terms of money and real asset. Moreover, from the seller's participation constraint, $y \geq 0$ requires $\tau_m + \tau_a(q+\kappa) \geq 0$. Thus, the constraint that the seller cannot transfer more than his wealth is irrelevant, and the buyer's payoff is independent of (a^s, z^s). From this, it is easy to see that the buyer's payoff is a function of only his *liquid* wealth, $z + \theta a\,(q+\kappa)$, the wealth he can use in the virtual game to maximize his payoff. We now describe some of the properties of buyer's surplus function, $\hat{U}^b(a,z)$.

Lemma 1 *The buyer's payoff is uniquely determined and satisfies,*

$$\hat{U}^b(a,z) = \begin{cases} u(y^*) - c(y^*) & \text{if } z + \theta a(q+\kappa) \geq c(y^*) \\ u \circ c^{-1}[z + \theta a(q+\kappa)] - z - \theta a(q+\kappa) & \text{otherwise} \end{cases} \tag{7}$$

If $z + \theta a(q+\kappa) < c(y^*)$, then $\hat{U}^b(a,z)$ is strictly increasing and strictly concave with respect to each of its arguments. Moreover, $\hat{U}^b(a,z)$ is jointly concave (but not strictly) with respect to (a,z).

[13] In principle, $\hat{U}^b$ should also have z^s and a^s as arguments. Here we anticipate the result that the terms of trade in this virtual game are independent of the seller's portfolio.

Proof. The solution to (4)-(6) is $y = y^*$ and $\hat{U}^b = u(y^*) - c(y^*)$ iff $z + \theta a(q+\kappa) \geq c(y^*)$; otherwise, $y = c^{-1}[z + \theta a(q+\kappa)]$ and $(\tau_m, \tau_a) = (z, \theta a)$.

If $z + \theta a(q+\kappa) < c(y^*)$, then

$$\hat{U}_a^b = \frac{\partial \hat{U}^b}{\partial a} = \theta(q+\kappa)\left[\frac{u'(\omega)}{c'(\omega)} - 1\right] > 0 \tag{8}$$

$$\hat{U}_z^b = \frac{\partial \hat{U}^b}{\partial z} = \frac{u'(\omega)}{c'(\omega)} - 1 > 0, \tag{9}$$

where $\omega = c^{-1}[z + \theta a(q+\kappa)]$; $\hat{U}^b(a,z)$ is increasing with respect to each of its arguments. As well,

$$\begin{aligned}
\hat{U}_{zz}^b &= \frac{u''(\omega)c'(\omega) - u'(\omega)c''(\omega)}{[c'(\omega)]^3} < 0 \\
\hat{U}_{za}^b &= \theta(q+\kappa)\left[\frac{u''(\omega)c'(\omega) - u'(\omega)c''(\omega)}{[c'(\omega)]^3}\right] < 0 \\
\hat{U}_{aa}^b &= [\theta(q+\kappa)]^2\left[\frac{u''(\omega)c'(\omega) - u'(\omega)c''(\omega)}{[c'(\omega)]^3}\right] < 0;
\end{aligned}$$

$\hat{U}^b(a,z)$ is strictly concave with respect to each of its arguments, and $\hat{U}_{aa}^b \hat{U}_{zz}^b - \left(\hat{U}_{za}^b\right)^2 = 0$. Hence, $\hat{U}^b(a,z)$ is jointly concave, but not strictly jointly concave. ■

The second step of the pricing protocol determines the seller's surplus, $\hat{U}^s(a,z)$, and the actual terms of trade, (y, τ_m, τ_a), as functions of the *buyer's* portfolio in the match, (a,z). By construction, the terms of trade are chosen so that the allocation is pairwise Pareto-efficient. The allocation solves the following problem,

$$\hat{U}^s(a,z) = \max_{y,\tau_m,\tau_a} [-c(y) + \tau_m + \tau_a(q+\kappa)] \tag{10}$$

$$\text{s.t.} \quad u(y) - \tau_m - \tau_a(q+\kappa) \geq \hat{U}^b(a,z) \tag{11}$$

$$-z^s \leq \tau_m \leq z, \quad -a^s \leq \tau_a \leq a \tag{12}$$

Notice that in this problem, the use of the real asset as means of payment is not restricted. Moreover, $\hat{U}^s(a,z) \geq 0$ since the allocation determined in the first step of the pricing protocol is still feasible in the second step. It is straightforward to characterize the solution to the seller's problem.

Lemma 2 *If $z + a(q+\kappa) \geq u(y^*) - \hat{U}^b(a,z)$, then the terms of trade in bilateral meetings satisfy*

$$y = y^* \tag{13}$$

$$\tau_m + \tau_a(q+\kappa) = u(y^*) - \hat{U}^b(a,z); \tag{14}$$

otherwise,

$$y = u^{-1}\left[z + a(q+\kappa) + \hat{U}^b(a,z)\right] \tag{15}$$

$$(\tau_a, \tau_m) = (a, z). \tag{16}$$

The seller's payoff and output are uniquely determined. The composition of the payment between money and the real asset is unique if the output is strictly less than the efficient level, y^*. If $z + a(q+\kappa) > u(y^*) - \hat{U}^b(a,z)$, then there are a continuum of transfers (τ_a, τ_m) that achieve (14).

Consider the case where $z + a(q+\kappa) < u(y^*) - \hat{U}^b(a,z)$. Here, output, y, depends on the buyer's portfolio. From (15)-(16) one can compute the quantity of output a buyer can acquire with an additional unit of wealth. If an agent accumulates an additional unit of real balances, his consumption in the AM increases by

$$\frac{\partial y}{\partial z} = \frac{u'(\omega)}{c'(\omega)u'(y)},$$

where $\omega = c^{-1}[z + \theta a(q+\kappa)]$. If the agent accumulates an additional unit of the real asset, which promises $q+\kappa$ units of output in the next PM, then

$$(q+\kappa)^{-1}\frac{\partial y}{\partial a} = \frac{1 + \theta\left[\frac{u'(\omega)}{c'(\omega)} - 1\right]}{u'(y)}.$$

If $\theta < 1$, then a claim on one unit of PM output buys more output in the AM if it takes the form of fiat money instead of the real asset. It can also be checked from Lemma 1 that

$$\frac{\partial \hat{U}^b(a,z)}{\partial a} = \theta(q+\kappa)\frac{\partial \hat{U}^b(a,z)}{\partial z}.$$

One unit of the real asset generates an increase of the buyer's surplus that is θ times the one associated with $q+\kappa$ units of real balances. In this sense, θ is a measure of the liquidity of the real asset, i.e., its ability to buy AM output at favorable terms of trade.

In Figure 1 the determination of the terms of trade is illustrated. The surpluses of the buyer and the seller are denoted $\hat{U}^b$ and $\hat{U}^s$, respectively. There are two Pareto frontiers: the lower (dashed) frontier corresponds to the pair of utility levels in the first step of the pricing protocol, where the buyer cannot spend more than a fraction θ of his real asset, and the upper frontier corresponds to the pair of utility levels in the second step of the procedure, where payments are unconstrained.[14] In Figure 1, the upper frontier is constructed

[14] If $z + \theta a\,(q+\kappa) > c(y^*)$ then both Pareto frontiers have a part in common with the efficient line $U^b + U^s = u(y^*) - c(y^*)$. In that case, $\hat{U}^b = u(y^*) - c(y^*)$ and $\hat{U}^s = 0$.

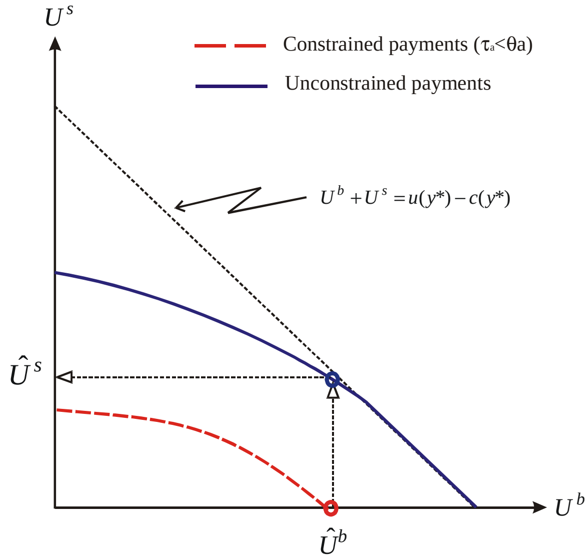

Figure 1: Pricing mechanism

in the case where the first-best level of output is incentive-feasible, i.e., $z + a(q + \kappa) > c(y^*)$. Along the linear portion of the upper frontier—moving in a north-west direction—output remains at y^* but wealth transferred to the seller increases. The linear portion ends when the wealth transference to the seller equals $z + a(q + \kappa)$; beyond that point on the upper frontier, the seller's surplus increases by having him receive all of the buyer's wealth in exchange for producing successively smaller amounts of the consumption good. The lower frontier is constructed under the assumption that $z + \theta a(q + \kappa) < c(y^*)$, i.e., $\hat{U}^b < u(y^*) - c(y^*)$. In Figure 1, in the first step of the pricing protocol, the buyer's surplus is the one he obtains in a virtual game where he offers his entire "liquid" wealth in exchange for the maximum level of output the seller is willing to produce. In the second step, the seller's surplus is chosen so that the agreement $(\hat{U}^b, \hat{U}^s)$ lies on the upper frontier in Figure 1 so that the trade is Pareto efficient. Given the configuration of Figure 1, the seller will receive all of the buyer's wealth and will produce a level of output $y < y^*$ that provides the buyer with a surplus equal to $\hat{U}^b$.

4 Equilibrium

We incorporate the pricing mechanism, described in Section 3, in our general equilibrium model. Let $y\,(a,z)$, $\tau_a\,(a,z)$ and $\tau_m\,(a,z)$, represent the output and transfer outcomes from the pricing mechanism, when the buyer in the match has portfolio (a,z). The value to the agent of holding portfolio (a,z) at the beginning of the AM subperiod, $V\,(a,z)$, is given by

$$\begin{aligned} V(a,z) &= \alpha\left\{u[y(a,z)] + W\left[a-\tau_a(a,z), z-\tau_m(a,z)\right]\right\} \\ &\quad +\alpha\mathbb{E}\left\{-c[y(\tilde{a},\tilde{z})] + W\left[a+\tau_a(\tilde{a},\tilde{z}), z+\tau_m(\tilde{a},\tilde{z})\right]\right\} \\ &\quad +(1-2\alpha)W(a,z). \end{aligned} \tag{17}$$

With probability α, the agent is the buyer in a match. He consumes $y(a,z)$ and delivers the assets $[\tau_a(a,z), \tau_m(a,z)]$ to the seller.[15] As established in Lemmas 1 and 2, the terms of trade (y, τ_a, τ_m) only depend on the portfolio of the buyer in the match. With probability α the agent is the seller in the match. He produces y and receives (τ_a, τ_m) from the buyer where (y, τ_a, τ_m) is a function of the buyer's portfolio $(\tilde{a},\tilde{z})$. The expectation is taken with respect to $(\tilde{a},\tilde{z})$, since the distribution of asset holdings might be non-degenerate, assuming that the buyer partner is chosen at random from the whole population of potential buyers. Finally, with probability $1-2\alpha$ the agent is neither a buyer nor a seller. Using the linearity of $W(a,z)$ and the expressions for the buyer's and the seller's surpluses, (17) can be rewritten more compactly as

$$V(a,z) = \alpha\hat{U}^b(a,z) + \alpha\mathbb{E}\hat{U}^s(\tilde{a},\tilde{z}) + W(a,z). \tag{18}$$

If the agent was living in an economy with exogenous liquidity constraints where he can only transfer a fraction θ of his real asset holdings, as in Kiyotaki and Moore (2005) or Lagos (2006), then the main difference would be that $\mathbb{E}\hat{U}^s(\tilde{a},\tilde{z}) = 0$ (assuming that the buyer makes take-it-or-leave-it-offers). Consequently, the buyer's choice of portfolios in both economies would be similar.

Substituting $V(a,z)$, as given by (18), into (3), and simplifying, an agent's portfolio solves

$$(a,z) \in \arg\max_{a,z}\left\{-\gamma z - qa + \beta\left[\alpha\hat{U}^b\,(a,z) + z + a(q+\kappa)\right]\right\}. \tag{19}$$

[15] Recall from Lemma 2 that even though the terms of trade (τ_a, τ_m) may not be uniquely determined, the transfer of wealth, $\tau_m + \tau_a(\kappa+q)$, is unique.

The portfolio is chosen so as to maximize the expected discounted utility of the agent if he happens to be a buyer in the next AM market minus the effective cost of the portfolio. The cost of holding an asset is its purchase price minus its discounted resale price and its dividend. Rearranging (19), it simplifies further to

$$(a,z) \in \arg\max_{a,z} \left\{ -iz - ar\left(q-q^*\right) + \alpha \hat{U}^b\left(a,z\right) \right\}, \tag{20}$$

where $i \equiv \frac{\gamma-\beta}{\beta}$ represents the cost of holding real balances, $r\left(q-q^*\right)$ is the cost of holding the real asset where $q^* \equiv \frac{\kappa}{r}$ is the discounted sum of the real asset's dividends. The first-order (necessary and sufficient) conditions for this (concave) problem are:

$$-i + \alpha\left[\frac{u'(\omega)}{c'(\omega)} - 1\right]^+ \leq 0, \quad \text{“}=\text{” if } z > 0 \tag{21}$$

$$-r\left(q-q^*\right) + \theta(q+\kappa)\alpha\left[\frac{u'(\omega)}{c'(\omega)} - 1\right]^+ \leq 0, \quad \text{“}=\text{” if } a > 0 \tag{22}$$

where $[x]^+ = \max(x,0)$ and $\omega = c^{-1}[z + \theta a(q+\kappa)]$. The term $\left[\frac{u'(\omega)}{c'(\omega)} - 1\right]^+$ in (21) represents the liquidity return of real balances, i.e., the increase in the buyer's surplus from holding an additional unit of money. From (22), the liquidity return of $\frac{1}{q+\kappa}$ units of the real asset is θ times the liquidity return of real balances.

Finally, the asset price is determined by the market clearing condition

$$\int_{[0,1]} a(j)dj = A, \tag{23}$$

where $a(j)$ is the asset choice of agent $j \in [0,1]$.

Definition 1 *An equilibrium is a list* $\{[a(j), z(j)]_{j\in[0,1]}, [y(a,z), \tau_a(a,z), \tau_m(a,z)], q\}$ *that satisfies (13)-(16), (20) and (23). The equilibrium is monetary if* $\int_{[0,1]} z(j)dj > 0$.

Consider first nonmonetary equilibria, where the real asset is the only means of payment in the AM market. In this case, $z(j) = 0$ for all j.

Proposition 1 *There is a nonmonetary equilibrium, and it is such that* $q \in [q^*, +\infty)$.

(i) If $\theta = 0$, *then* $q = q^*$.

(ii) If $\theta > 0$, *then* q *is the unique solution to*

$$\frac{q^*+\kappa}{q+\kappa} + \frac{\theta\alpha}{r}\left[\frac{u' \circ c^{-1}[\theta A(q+\kappa)]}{c' \circ c^{-1}[\theta A(q+\kappa)]} - 1\right]^+ = 1. \tag{24}$$

If $\theta A(q^*+\kappa) \geq c(y^*)$, *then* $q = q^*$; *otherwise* $q > q^*$.

Proof. Define the demand correspondence for the real asset as

$$A^d(q) = \left\{ \int_{[0,1]} a(j)dj : a(j) \in \arg\max \left\{ -ar\left(q - q^*\right) + \alpha \hat{U}^b\left(a, 0\right) \right\} \right\}.$$

The market clearing condition, (23), can then be re-expressed as $A \in A^d(q)$. First, suppose $\theta = 0$. From (7), $\hat{U}^b\left(a, 0\right) = 0$ for all $a \geq 0$. Then, $A^d(q) = \{0\}$ for all $q > q^*$ and $A^d(q^*) = [0, +\infty)$. Consequently, the unique solution to $A \in A^d(q)$ is $q = q^*$.

Next, suppose $\theta > 0$. In order to characterize $A^d(q)$ we distinguish three cases:

1. If $q > q^*$ then $A^d(q) = \{a\}$ where $a < \frac{c(y^*)}{\theta(q+\kappa)}$ is the unique solution to

$$r\left(q - q^*\right) = \alpha \hat{U}_a^b\left(a, 0\right). \tag{25}$$

   To see this, recall from Lemma 1 that $\hat{U}^b\left(a, 0\right)$ is strictly concave with respect to its first argument over the domain $\left[0, \frac{c(y^*)}{\theta(q+\kappa)}\right]$, $\hat{U}_a^b\left(0, 0\right) = \infty$ and $\hat{U}_a^b\left(a, 0\right) = 0$ for all $a \geq \frac{c(y^*)}{\theta(q+\kappa)}$. Substituting $\hat{U}_a^b$ by its expression given by (8) into (25) and rearranging, one obtains

$$\frac{q^* + \kappa}{q + \kappa} + \frac{\theta\alpha}{r}\left[\frac{u' \circ c^{-1}[\theta a(q+\kappa)]}{c' \circ c^{-1}[\theta a(q+\kappa)]} - 1\right] = 1. \tag{26}$$

   Note that the left-hand side is strictly decreasing in both q and a; hence, a is strictly decreasing in q. So, for all $q > q^*$, $A^d(q)$ is single-valued and strictly decreasing. Moreover, as $q \to q^*$, $a \to \frac{c(y^*)}{\theta(q^*+\kappa)}$ and as $q \to \infty$, $a \to 0$.

2. If $q = q^*$, then $A^d(q^*) = \text{argmax}_{a \geq 0} \left\{ \hat{U}_a^b\left(a, 0\right) \right\} = [\frac{c(y^*)}{\theta(q^*+\kappa)}, +\infty)$.

3. If $q < q^*$, then the agent's problem has no solution.

In summary, $A^d\left(q\right)$ is upper hemi-continuous over $[q^*, \infty)$ and its range is $[0, \infty)$. Hence, a solution $A \in A^d(q)$ exists. Furthermore, any selection from $A^d\left(q\right)$ is strictly decreasing in $q \in [q^*, \infty)$, so there is a unique q such that $A \in A^d(q)$. If $A \geq \frac{c(y^*)}{\theta(q^*+\kappa)}$, then $A \in A^d(q)$ implies $q = q^*$. If $A < \frac{c(y^*)}{\theta(q^*+\kappa)}$, then $A \in A^d(q)$ implies that q solves (26) with $a = A$, i.e., (24). ■

If $\theta = 0$, as in Zhu and Wallace (2007), then the real asset is fully illiquid in the sense that holding the asset does not allow the buyer to extract a surplus from his trade in the decentralized AM. The allocation coincides with the one where the seller makes a take-it-or-leave-it offer, and the asset is priced at its fundamental value, $q = q^*$.

If $\theta > 0$, then the buyer can obtain a positive surplus from holding the asset in the decentralized market. If the intrinsic value of the stock of the real asset, $A\left(q^* + \kappa\right)$, is sufficiently high, and if it is not too illiquid, i.e., θ is not too low, then the buyer can extract the entire surplus of the match. An additional unit of asset does not affect the buyer's trade surplus in the AM so that the asset has no liquidity value and its price corresponds to its fundamental price, $q = q^* = \kappa/r$. The distribution of asset holdings is not uniquely determined, but this indeterminacy is payoff irrelevant since the output traded in all matches in the AM is y^*. In contrast, if the intrinsic value of the asset is low, or if the asset is very illiquid (θ is low but positive), then the price of the asset raises above its fundamental value because it is useful to the buyer to increase his surplus in the AM. In this case, the equilibrium and the distribution of asset holdings—which is degenerate—are unique.

It is rather interesting to note from (24) that the allocation can be socially efficient, $y = y^*$, even when $q > q^*$. It can be the case that the value of the buyer's *liquid* portfolio, $\theta a\left(q + \kappa\right)$, is insufficient to purchase y^* in the decentralized AM subperiod—i.e., $\theta a\left(q + \kappa\right) < c\left(y^*\right)$—but the total value of the buyer's portfolio can be sufficient to support an output of y^*, i.e., $a(q+\kappa) \geq u(y^*) - \hat{U}^b(a,z)$.

Let's now turn to monetary equilibria.

Proposition 2 *There exists a monetary equilibrium iff*

$$A < \frac{(r - i\theta)}{\theta\kappa\left(1+r\right)}\ell(i), \tag{27}$$

where $\ell(i)$ is unique and implicitly defined by

$$1 + \frac{i}{\alpha} = \frac{u' \circ c^{-1}(\ell)}{c' \circ c^{-1}(\ell)}. \tag{28}$$

In a monetary equilibrium, the asset price is uniquely determined by

$$q = \frac{\kappa\left(1 + i\theta\right)}{r - i\theta} \geq q^*. \tag{29}$$

Proof. With a slight abuse of notation, define $\hat{U}^b\left(\ell\right) = \hat{U}^b\left(a,z\right)$ where $\ell = z + \theta a(q+\kappa)$. (Notice from Lemma 1 that (a,z) matters for the buyer's payoff only through $z + \theta a(q+\kappa)$.) Then, the agent's portfolio problem (20) can be re-expressed as

$$(a,\ell) \in \arg\max_{a,\ell} \left\{ -i\ell - a\left[\left(r - i\theta\right)q - \kappa\left(1 + i\theta\right)\right] + \alpha\hat{U}^b\left(\ell\right)\right\} \text{ s.t. } \theta a(q+\kappa) \leq \ell. \tag{30}$$

Define the demand correspondence for the real asset as

$$A^d(q) = \left\{ \int_{[0,1]} a(j)dj : \exists \ell \text{ s.t. } [a(j), \ell] \text{ is solution to (30)} \right\}.$$

The market-clearing condition (23) requires $A \in A^d(q)$ for some q. In order to characterize $A^d(q)$ we distinguish three cases:

1. If $(r - i\theta) q > \kappa (1 + i\theta)$ then, from (30), $A^d(q) = \{0\}$.

2. If $(r - i\theta) q < \kappa (1 + i\theta)$, then constraint $\theta a(q+\kappa) \leq \ell$ must bind and $z = 0$, i.e., the equilibrium is non-monetary. Hence, from the proof of Proposition 1, $A^d(q^*) = [\frac{c(y^*)}{\theta(q^*+\kappa)}, \infty)$ and, for all $q \in \left(q^*, \frac{\kappa(1+i\theta)}{(r-i\theta)}\right)$, $A^d(q) = \{a\}$ where a solves

$$r(q - q^*) = \alpha\theta(q+\kappa) \left[\frac{u' \circ c^{-1} [\theta a(q+\kappa)]}{c' \circ c^{-1} [\theta a(q+\kappa)]} - 1 \right]. \tag{31}$$

3. If $(r - i\theta) q = \kappa (1 + i\theta)$, then $A^d(q) = [0, \frac{\ell}{\theta(q+\kappa)}]$ where, from (30), ℓ solves (28).

In conjunction with (28), it can be checked that the solution to (31) for a approaches $\frac{\ell}{\theta(q+\kappa)}$ as $q \nearrow \frac{\kappa(1+i\theta)}{(r-i\theta)}$. Hence, $A^d(q)$ is upper hemi-continuous on $[q^*, +\infty)$ with range $(0, +\infty)$. Moreover, any selection from $A^d(q)$ is strictly decreasing. Therefore, there is a unique $q \in [q^*, +\infty)$, such that $A \in A^d(q)$. Moreover, if $r > i\theta$ then $q \in [q^*, \frac{\kappa(1+i\theta)}{r-i\theta}]$. See Figure 2.

A monetary equilibrium exists if $z(j) > 0$ for a positive measure of agents. From the discussion above, a monetary exists if and only if $r > i\theta$ and $A < \frac{\ell(i)}{\theta(q+\kappa)}$, where $\ell(i)$ is defined by (28) and $q = \frac{\kappa(1+i\theta)}{r-i\theta}$, i.e., equation (29). This gives (27). ■

Since the right-hand side of (27) is decreasing in i, by taking the limit as i approaches 0 we obtain the following necessary condition for the existence of a monetary equilibrium: $A < \frac{c(y^*)}{\theta(q^*+\kappa)}$. If this condition holds, (27) can be restated as $i < i_0$, where i_0 is the unique solution to[16]

$$\frac{(r - i_0\theta)}{\theta\kappa (1+r)} \ell(i_0) = A. \tag{33}$$

[16] In a monetary equilibrium, $\ell(i) > A\frac{\theta\kappa(1+r)}{r-i\theta}$ or, from (28),

$$1 + \frac{i}{\alpha} < \frac{u' \circ c^{-1}\left(A\frac{\theta\kappa(1+r)}{r-i\theta}\right)}{c' \circ c^{-1}\left(A\frac{\theta\kappa(1+r)}{r-i\theta}\right)}. \tag{32}$$

Since the left-hand side is increasing and continuous in i and the right-hand side is decreasing and continuous in i, there exists a unique i_0 such that (32) holds at equality, which implies that for any $i < i_0$, condition (32) holds and, hence, a monetary equilibrium exists.

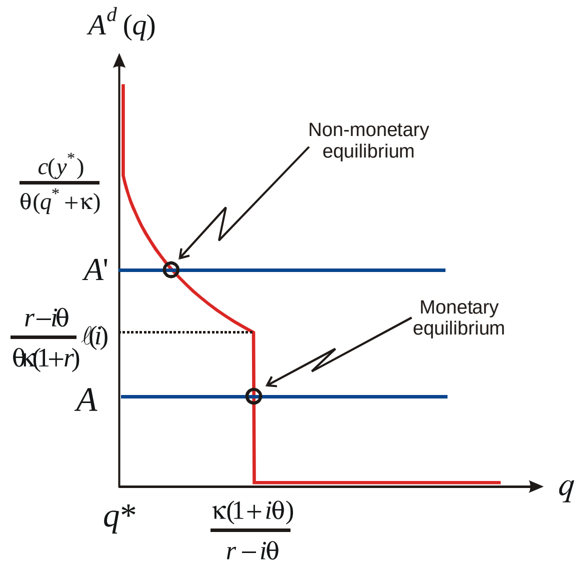

Figure 2: Asset demand correspondence

Thus, a monetary equilibrium exists whenever the asset price in the nonmonetary equilibrium is greater than its fundamental value, provided that the inflation rate is not too large. If money is valued, then the asset price is still greater than its fundamental value if $\theta i > 0$, the real asset is not fully illiquid and the cost of holding real balances is positive.

The determination of the equilibrium is characterized in Figure 2. The price q is at the intersection of the constant supply, A, and the downward-sloping demand, $A^d\,(q)$. It is uniquely determined. The equilibrium is monetary whenever the supply intersects the demand in its vertical portion.

A rather important result from above is that, if $A < \frac{c(y^*)}{\theta(q^*+\kappa)}$, then there always exists a $i > 0$, such that a monetary equilibrium exists. Diagrammatically speaking, it is easy to see this result. Suppose that $A < \frac{c(y^*)}{\theta(q^*+\kappa)}$, but $A > \frac{r-\theta i}{\theta\kappa(1+r)}\ell\,(i)$ for a given value of i; in figure 2, let $A = A'$. This implies that at the current inflation rate, the equilibrium is non-monetary, as A' intersects the strictly downward portion of $A^d\,(q)$. However, since $\frac{\kappa(1+\theta i)}{r-\theta i}$ is increasing in i, by decreasing i from its current level, the vertical portion of $A^d\,(q)$ will "move" to the left. If i is decreased sufficiently, A' will intersect the vertical portion of $A^d\,(q)$.

Since in a monetary equilibrium the "liquid wealth" of the buyer, $\ell(i)$, is uniquely determined, the buyer's payoff in the decentralized market is unique. However, there are infinitely many ways to to combine a and

z to obtain a given $\ell(i)$. As a consequence, the terms of trade (y, τ_a, τ_z) and a seller's payoff need not be unique. So, even though the asset price is unique, the real allocation may be indeterminate. In the following we restrict our attention to symmetric steady-state equilibria. All agents hold A of the real asset and $z(i) = \ell(i) - \theta A(q + \kappa)$ real balances.

5 Asset prices and monetary policy

Let us turn to asset pricing considerations and the implications for monetary policy. We have established in Proposition 2 that in any monetary equilibrium the asset price satisfies $q = \frac{\kappa(1+\theta i)}{r-\theta i}$, which can be rewritten as

$$q = q^* + \theta i \frac{q^* + \kappa}{r - \theta i}. \tag{34}$$

If $\theta = 0$—the pricing mechanism corresponds to the one in Zhu and Wallace (2007)—then the asset price is equal to its fundamental value, i.e., the discounted sum of its dividends, and it is unaffected by the money growth rate. In contrast, if the real asset is at least partially liquid, i.e., $0 < \theta \leq 1$, then the asset price is above its fundamental value.

We define the liquidity premium of the asset as the difference between q and q^*. From (34), it is equal to

$$\mathcal{L} = \theta i \frac{q^* + \kappa}{r - \theta i}. \tag{35}$$

This liquidity premium arises because the real asset allows the buyer in a bilateral match in the AM to capture some of the gains from trade. Monetary policy affects the asset price through this liquidity premium. The asset price increases with inflation, i.e., $\partial q / \partial i > 0$. As the cost of holding money gets higher, agents will attempt to reduce their real balance holdings in favor of the real asset. The price of the real asset will, therefore, increase. As the cost of holding real balances is driven to zero, $i \to 0$, then the liquidity premium vanishes and the asset price approaches its fundamental value. In this limiting case, agents can use fiat money to extract all the gain from trade in the decentralized AM market, and hence the real asset has no extra value beyond the one generated by its dividend stream.

In a monetary equilibrium, the gross rate of return of the real asset is $R = \frac{q+\kappa}{q}$ or, from (34),

$$R = \frac{1 + r}{1 + \theta i}. \tag{36}$$

From (36), the rate of return of the asset depends on preferences, r, monetary policy, i, and the characteristics of the pricing mechanism, θ.

Proposition 3 *In any monetary equilibrium, $R \leq 1 + r$. If $\theta i > 0$, then $\partial R/\partial i < 0$ and $\partial R/\partial \theta < 0$.*

Proof. Immediate from equation (36). ■

If $\theta i = 0$, then the liquidity premium of the asset is 0 and hence its rate of return is equal to the rate of time preference. In contrast, if $\theta i > 0$, then the asset price exhibits a liquidity premium and its rate of return is smaller than the rate of time preference. The model predicts a negative correlation between the rate of return of the real asset and inflation. As inflation increases, agents substitute the real asset for real money balances and, as a consequence, the asset price increases and its return decreases.[17] The rate of return of the asset also decreases with its liquidity as captured by θ for much the same reason: as the liquidity of the asset increases, the value of the asset for transactions purposes increases. Hence, the asset price increases and its return decreases.

The absolute value of the elasticity of the asset rate of return with respect to i is given by

$$\eta_{R/i} = \frac{\partial R/R}{\partial i/i} = \frac{\theta i}{1 + \theta i}.$$

This elasticity is less than one and is increasing with θ. Hence, if the asset becomes more liquid, as measured by an increase in θ, its return becomes more sensitive to inflation. Also, in high-inflation environments the rate of return of the asset is more sensitive to changes in monetary policy.

Let's now turn to the rate of return differential between the real asset and money. Since the gross rate of return of fiat money is γ^{-1}, the rate of return differential is

$$R - \gamma^{-1} = \frac{1}{\gamma}\left(\frac{1+i}{1+\theta i} - 1\right). \tag{37}$$

Proposition 4 *In any monetary equilibrium, the real asset dominates money in its rate of return iff $i > 0$ and $\theta < 1$.*

Proof. Immediate from equation (37). ■

If $\theta = 1$, as in Lagos and Rocheteau (2008) or Geromichalos, Licari and Suarez-Lledo (2007), the model is unable to explain the rate of return differential between money and the real asset. Since both capital and money are "equally liquid," in order for the two media of exchange to coexist, they must have the same rate of return. As well, since $R \geq 1$, a monetary equilibrium cannot exist if inflation is positive, i.e., if $\gamma > 1$. Both of these results are counterfactual.

[17] This finding is in accordance with the empirical evidence. See, e.g., Marshall (1992).

In our model, if $i = 0$, then fiat money and the real asset will have the same rate of return. By running the Friedman rule, the monetary authority can satiate agents' need for liquidity, in which case the rate of return of the real asset is equal to the rate of time preference—since the asset has no value as a medium of exchange—which is also the rate of return of fiat money.

However, if $\theta < 1$ and $i > 0$, then our model delivers a rate of return differential between the real asset and money. For given i, this differential decreases with θ: as θ increases, the value of the real asset increases owing to its increased benefit as a medium of exchange. As a result, its rate of return declines. One can relate the rate of return differential and the elasticity of the asset return with respect to inflation, i.e.,

$$R - \gamma^{-1} = \frac{(1+i)\left(1 - \eta_{R/i}\right) - 1}{\gamma}.$$

There is a negative relationship between the rate of return differential and the elasticity of the asset rate of return with respect to inflation.

We conclude this section by investigating the optimal monetary policy.

Proposition 5 *Assume $\theta A(q^* + \kappa) < c(y^*)$ and $\theta < 1$. Then there is $\hat{\imath} > 0$ such that for all $i \in [0, \hat{\imath}]$, $y = y^*$ at the symmetric monetary equilibrium.*

Proof. From Proposition 2, since $\theta A(q^* + \kappa) < c(y^*)$, there is a symmetric monetary equilibrium, provided that $i < i_0$ (where i_0 is defined in (33)). Moreover, $q(i) = \frac{\kappa(1+\theta i)}{(r-\theta i)}$, $\ell(i)$ solution to $1 + \frac{i}{\alpha} = \frac{u' \circ c^{-1}(\ell)}{c' \circ c^{-1}(\ell)}$ and $z(i)$ solution to

$$z(i) = \ell(i) - \theta A \frac{\kappa(1+r)}{(r - \theta i)} \tag{38}$$

are all continuous in i. Define

$$\Gamma(i) \equiv A(q(i) + \kappa) + z(i) + \hat{U}^b\left[\ell(i)\right] - u(y^*).$$

The function $\Gamma(i)$ is continuous over $[0, i_0)$ and, from Lemma 2, $y = y^*$ whenever $\Gamma(i) > 0$. Substitute $z(i)$ by its expression given by (38) to get

$$\Gamma(i) \equiv (1-\theta) A \left(\frac{\kappa(1+r)}{(r-\theta i)}\right) + \ell(i) + \hat{U}^b\left[\ell(i)\right] - u(y^*).$$

As $i \to 0$, $\ell(i) \to c(y^*)$ and $\hat{U}^b\left[\ell(i)\right] \to u(y^*) - c(y^*)$. Since $\theta < 1$,

$$\lim_{i \to 0} \Gamma(i) = (1-\theta) A \frac{\kappa(1+r)}{r} > 0$$

By continuity, there exists a nonempty interval $[0, \hat{\imath}]$ such that $\Gamma(i) > 0$. ■

In most monetary models with a single asset, the Friedman rule is optimal and it achieves the first best (provided that there are no externalities, no distortionary taxes, and the pricing is well-behaved); we have that result too.[18] However, in contrast to standard monetary models, a small deviation from the Friedman rule is neutral in terms of welfare in our model. Hence, a small inflation is (weakly) optimal. The only effect of increasing the inflation above the Friedman rule is to increase asset prices.

This finding has the following implications. First, a small inflation will have no welfare cost. Hence, we conjecture that moderate inflation (let say, 10 percent) will have a lower welfare cost than in standard models. Second, even though asset prices respond to monetary policy, these movements do not correspond to changes in society's welfare. Hence, asset prices may not be a very good indicator of society's welfare or monetary policy effectiveness. Third, since there is a range of inflation rates that generates the first-best allocation, the optimal monetary policy is consistent with a rate of return differential between fiat money and the real asset.

6 Liquidity structure of asset yields

In this section, we extend the model to allow for multiple real assets. We will show that the same model that can explain the rate of return dominance puzzle—that fiat money has a lower rate of return than risk-free bonds—can also deliver a non-degenerate distribution of assets' yields despite agents being risk neutral. We will investigate how this structure of yields is affected by monetary policy.

Suppose that there are a finite number $K \geq 1$ of infinitely-lived real assets indexed by $k \in \{1, ..., K\}$. Denote $A_k > 0$ as the fixed stock of the asset $k \in \{1, ..., K\}$, κ_k its expected dividend, and q_k its price. Agents learn the realization of the dividend of an asset at the beginning of the PM centralized market. Consequently, the terms at which the asset is traded in the AM decentralized market only depend on the expected dividend κ_k. Moreover, since agents are risk-neutral with respect to their consumption in the PM centralized market, the risk of an asset has no consequence for its price.[19]

Consider a buyer in the AM with a portfolio $(\{a_k\}_{k=1}^{K}, z)$, where a_k is the quantity of the k^{th} real asset.

[18] In search monetary models, the Friedman rule can be suboptimal because of search externalities (Rocheteau and Wright, 2005) or distortionary taxes (Aruoba and Chugh, 2008). Also, if the coercive power of the government is limited, then the Friedman rule might not be incentive-feasible (Andolfatto, 2007).

[19] The result that the price of an asset does not depend on its risk would no longer be true if the realization of the dividend was known in the AM when agents trade in bilateral matches. See Lagos (2006) and Rocheteau (2008).

The pricing mechanism is a straightforward generalization of the one studied in the previous sections. The buyer's payoff is given by

$$\hat{U}^b = \max_{y,\tau_m,\{\tau_k\}} \left[u(y) - \tau_m - \sum_{k=1}^{K} \tau_k (q_k + \kappa_k) \right] \tag{39}$$

$$\text{s.t.} \quad -c(y) + \tau_m + \sum_{k=1}^{K} \tau_k (q_k + \kappa_k) \geq 0 \tag{40}$$

$$\tau_m + \sum_{k=1}^{K} \tau_k (q_k + \kappa_k) \leq z + \sum_{k=1}^{K} \theta_k a_k (q_k + \kappa_k), \tag{41}$$

where $\theta_k \in [0,1]$ for all k. According to (39)-(41), the buyer's payoff is the same as the one he would get in an economy where he can make a take-it-or-leave-if-offer to the seller, but where he is constrained not to spend more than a fraction θ_k of the real asset k.

One can generalize Lemma 1 in the obvious way; in particular,

$$\hat{U}^b(\ell) = \begin{cases} u(y^*) - c(y^*) & \text{if } \ell \geq c(y^*) \\ u \circ c^{-1}(\ell) - \ell & \text{otherwise} \end{cases}, \tag{42}$$

where $\ell = z + \sum_{k=1}^{K} \theta_k a_k (q_k + \kappa_k)$ is the buyer's liquid portfolio. Assume that $\theta_1 \geq \theta_2 \geq ... \geq \theta_K$. Then,

$$(q_k + \kappa_k)^{-1} \frac{\partial \hat{U}^b}{\partial a_k} = \theta_k \frac{\partial \hat{U}^b}{\partial z}.$$

So $1/(q_k + \kappa_k)$ units of the k^{th} asset, which yields one unit of PM output, allows the buyer to raise his surplus in the AM decentralized market by a fraction θ_k of what he would obtain by accumulating one additional unit of real balances instead. The parameter θ_k can then be interpreted as a measure of the liquidity of the asset k, that is, the extent to which it can be used to financed consumption opportunities in the AM at favorable terms of trade. Given our ranking, the asset 1 is the most liquid one and the asset K is the least.

The second step of the pricing procedure is a generalization of (10)-(12). The seller's payoff and the actual terms of trade are determined by

$$\hat{U}^s = \max_{y,\tau_m,\{\tau_k\}} \left[-c(y) + \tau_m + \sum_{k=1}^{K} \tau_k (q_k + \kappa_k) \right]$$

$$\text{s.t.} \quad u(y) - \tau_m - \sum_{k=1}^{K} \tau_k (q_k + \kappa_k) \geq \hat{U}^b$$

$$-z^s \leq \tau_m \leq z, \quad -a_k^s \leq \tau_k \leq a_k, \qquad k = 1, ..., K$$

In the PM agents choose the portfolio, $(\{a_k\}, z)$, that they will bring into the decentralized market. The portfolio problem becomes

$$(\{a_k\}, z) \in \arg\max_{\{a_k\},z} \left\{ -iz - r\sum_{k=1}^{K} a_k\left(q_k - q_k^*\right) + \alpha \hat{U}^b\left(z + \sum_{k=1}^{K}\theta_k a_k(q_k+\kappa_k)\right)\right\}, \tag{43}$$

and $q_k^* = \kappa_k/r$. According to (43), the agent maximizes his expected utility of being a buyer in the AM decentralized market, net of the cost of the portfolio. The cost of holding asset k is the difference between the price of the asset and its fundamental value (expressed in flow terms), while the cost of holding real balances is $i = \frac{\gamma-\beta}{\beta}$, approximately the sum of the inflation rate and the rate of time preference. An agent's portfolio choice problem, (43), can be rewritten as

$$\max_{\{a_k\},\ell} \left\{ -i\ell + \sum_{k=1}^{K} a_k\left[i\theta_k(q_k+\kappa_k) - r\left(q_k - q_k^*\right)\right] + \alpha\hat{U}^b\left(\ell\right)\right\} \tag{44}$$

$$\text{s.t. } \sum_{k=1}^{K}\theta_k a_k(q_k+\kappa_k) \leq \ell. \tag{45}$$

As in Section 4, in a monetary equilibrium, constraint (45) does not bind, ℓ solves (28), and the asset prices must satisfy $i\theta_k(q_k+\kappa_k) - r\left(q_k - q_k^*\right) = 0$ or

$$q_k = \frac{1+i\theta_k}{r-i\theta_k}\kappa_k, \qquad \forall k \in \{1,...,K\} \tag{46}$$

for all $k \in \{1,...,K\}$.[20] Note that the price of the real asset k increases with inflation, provided that $\theta_k > 0$. From (45) and (46), a monetary equilibrium exists if $r > i\theta_k$ for all k and

$$\sum_{k=1}^{K}\theta_k A_k\left(\frac{1+r}{r-i\theta_k}\right)\kappa_k < \ell(i). \tag{47}$$

For money to be valued, the total stock of real assets, adjusted by their liquidity factors, must not be too large. The rate of return of asset k is given by

$$R_k = \frac{\kappa_k + q_k}{q_k} = \frac{1+r}{1+i\theta_k}, \qquad \forall k \in \{1,...,K\} \tag{48}$$

Provided that the nominal interest rate is strictly positive, the model is able to generate differences in the rates of return of the real assets, where the ordering depends on the liquidity coefficients $\{\theta_k\}$.

[20] The proof to these claims are almost identical to the proof of Proposition 2. Specifically, if $r\left(q_k - q_k^*\right) > i\theta_k(q_k+\kappa_k)$, then $A^d\left(q_k\right) = \{0\}$, which cannot be an equilibrium; if $r\left(q_k - q_k^*\right) < i\theta_k(q_k+\kappa_k)$, then constraint (45) binds and $z = 0$, i.e., the equilibrium is non-monetary; and if $r\left(q_k - q_k^*\right) = i\theta_k(q_k+\kappa_k)$, then $\sum_k A_k^d\left(q_k\right)\theta_k\left(q_k+\kappa_k\right) \in [0,\ell]$, where ℓ solves (28).

Proposition 6 *In any monetary equilibrium, $R_K \geq R_{K-1} \geq ... \geq R_1 \geq \gamma^{-1}$. Moreover,*

$$R_{k'} - R_k = (1+r)\frac{i\left(\theta_k - \theta_{k'}\right)}{(1+i\theta_k)(1+i\theta_{k'})} > 0. \tag{49}$$

Proof. Direct from (48). ■

The differences between the rates of return across assets emerge even if the assets are risk-free, or agents are risk-neutral. They result from the pricing mechanism according to which different assets are traded at different terms of trade in the decentralized market. These rate-of-return differences would be viewed as anomalies by standard consumption-based asset pricing theory.

Let us turn to monetary policy. A change in inflation affects the entire structure of asset returns. Denote $\{R_k^i\}_{k=1}^K$ the structure of asset yields when the cost of holding fiat money is equal to i.

Proposition 7 *In any monetary equilibrium, $\{R_k^i\}_{k=1}^K$ dominates $\{R_k^{i'}\}_{k=1}^K$ in a first-order stochastic sense whenever $i' > i$. Moreover, $\frac{\partial(R_{k'}-R_k)}{\partial i} > 0$ if and only if $(\theta_k - \theta_{k'})\left(1 - i^2\theta_k\theta_{k'}\right) > 0$.*

Proof. The first part of the proposition is direct from (48). From (49), if $\theta_k = \theta_{k'}$ then $R_{k'} - R_k = 0$ which is independent of i. With no loss, assume $\theta_k > \theta_{k'}$ and $i > 0$ so that $R_{k'} - R_k > 0$. From (49), differentiate $\ln(R_{k'} - R_k)$ to get

$$\frac{\partial \ln\left(R_{k'} - R_k\right)}{\partial i} = \frac{1 - i^2\theta_k\theta_{k'}}{i\left(1+i\theta_k\right)\left(1+i\theta_{k'}\right)}.$$

■

An increase in inflation raises the rates of return of all real assets because agents substitute the real assets for real balances which are more costly to hold. Moreover, the premia paid to the less liquid assets, $R_{k'} - R_k$ with $\theta_k > \theta_{k'}$, increase provided that i is not too large. So if one interpret the less liquid asset as risky equity and the most liquid one as risk-free bonds, then the equity premium increases with inflation.

7 Conclusion

The main contribution of this paper is to show that the rate-of-return-dominance and other asset pricing puzzles need not be puzzles when viewed through the lens of monetary models with bilateral trades. These seemingly-anomalous asset pricing patterns can be generated by trading mechanisms that are pairwise Pareto-efficient and that do not impose any restriction on the use of assets as means of payment, i.e., the Wallace dictum is satisfied. Liquidity differences across assets can arise because agents coordinate on a mechanism that makes it cheaper for buyers to trade some assets relative to others. For instance, sellers can agree to offer better terms of trade to buyers trading with money instead of bonds or equity. Lagos (2006) shows that a calibrated version of a search-theoretic monetary model can account for both the risk-free rate and the equity premium puzzles once a small restriction is introduced on the use of equity as means of payment. Our analysis indicates that such a restriction is, in fact, not needed once one allows for a broader class of trading mechanisms. The trading mechanisms we have considered have no axiomatic or strategic foundations, where the only property that we impose on the mechanism is that it be pairwise Pareto-efficient. Ideally, one would like to reconcile the existing approaches by providing deeper foundations for the mechanisms we have considered. We leave this for future research.

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