FEDERAL RESERVE BANK *of* CHICAGO
FINANCIAL MARKETS GROUP

PDP 2008-8

Search, Welfare and the 'Hot Potato' Effect of Inflation

Ed Nosal

December 31, 2008

POLICY DISCUSSION PAPER SERIES

Search, Welfare and the 'Hot Potato' Effect of Inflation

Ed Nosal*

December 2008

Abstract

An increase in inflation will cause people to hold less real balances and may cause them to speed up their spending. Virtually all monetary models capture the first effect. Few capture the second—'hot potato'—effect; and those that do associate negative welfare consequences with it. Since, via the inflation tax and the hot potato effect, inflation has negative effects on welfare, an optimal monetary policy will be characterized by Friedman rule. In the model presented in this paper there is a hot potato effect, but—holding all else constant—the hot potato effect has positive consequences for welfare. As a result, a departure from the Friedman rule will be socially desirable.

1 Introduction

There is a conventional wisdom that in high inflation economies, inflation speeds up trading and spending, (for recent treatments see Tommasi (1994, 1999) and Ennis (forthcoming)). The intuition behind this idea is simple: Since high inflation rapidly erodes the value of money, it is better to spend money now than to hold on to it and spend later. This effect of inflation on spending is sometimes referred to as the 'hot potato' effect. There is another conventional wisdom that it is optimal for the rate of return on money to equal to the rate of return on other assets. The intuition behind this policy is equally as simple: If the rate of return on money is less than that of other assets, people will take socially wasteful actions to reduce their money holdings, even though, in the end someone has to hold all of the money. These social costs can be avoided if money pays the same return as other assets. The policy that implements the optimal outcome—the Friedman rule—has prices

*Financial Markets Group, Federal Reserve Bank of Chicago; ed.nosal@chi.frb.org. I would like to thank Guillaume Rocheteau for numerous conversations and helpful suggestions, and to Ricardo Cavalcanti and Randy Wright for comments on earlier drafts. The views expressed here do not necessarily reflect those of the Federal Reserve Bank of Chicago or of the Federal Reserve System.

decreasing at the rate of time preference. The optimality of the Friedman rule is a robust outcome in monetary economics as it is the best policy for a wide variety of economic environments, i.e., environments where money enters the utility function or as a cash-in-advance constraint, in over-lapping generations environments, and search models of money environments. There is some distance between any sort of meaningful conversation between the hot potato effect and the Friedman rule. This should not be surprising: The former operates in high inflation economies and the latter is associated with deflation. In this paper, I construct a model where the hot potato effect exists at a low (negative) inflation rates and its existence provides an avenue for a welfare *improving* departure from the Friedman rule.

In Tommasi (1994, 1999) buyers must spend their money either early or late, and in Ennis (forthcoming) buyers have only periodic access to a general market that allows them to rebalance their portfolio. In these papers, if the monetary authority could choose the optimal policy, then it would choose a policy where the hot potato effect is not operative. For example, in Ennis (forthcoming), the optimal policy is given by the Friedman rule and, at the Friedman rule, there is no notion of buyers "speeding up" their spending. Lagos and Rocheteau (2005) model the potential for a hot potato effect by letting search intensity be a choice variable for the buyer; a hot potato effect exists if search intensity increases with inflation. Lagos and Rocheteau (2005) are able to generate a hot potato effect, but this result depends critically on how prices are determined in decentralized trade. If prices are determined by bargaining, then a hot potato effect does not exist; if they are determined by competitive price posting, then a hot potato effect exists at low inflation rates. It should be noted that, as in Tommasi (1994, 1999) and Ennis (forthcoming), if the monetary authority is free to choose an optimal policy, it would choose the a policy consistent with the Friedman rule. Finally, in Ennis (forthcoming) and Lagos and Rocheteau (2005), one should equate an operative hot potato effect with "bad" welfare outcomes since the hot potato effect has negative implications for welfare, and, in order to the generate a hot potato effect, a deviation from the Friedman rule is needed.

In the model presented here, a hot potato effect arises because there is an opportunity cost associated with the buyer accepting a trade. Although such an opportunity cost may be novel to monetary economics, these sorts of opportunity costs are central to the labor search literature, e.g., in a basic labor search model, accepting a job today, rules out the possibility of obtaining job offers—at possibly higher wages—tomorrow. One can think of a number of examples where an opportunity cost (and a search externality) associated with a buyer's decision to purchase might arise in a monetary environment. Here are two: Suppose that a buyer searches for a particular good; after is successfully matched and trades, he then searches for another good. The longer he searches for a particular good, the better he becomes at finding trading partners. Suppose further that the aggregate matching function does *not* depend upon experience, so the more experienced searcher gets a higher match probability by displacing less experienced searchers from potential matches. Or, after a buyer engages in a successful transaction he may have to temporarily exist the market, while unsuccessful buyers remain. What these two examples have in common is that a buyer who has just successfully matched and purchased a good has a lower probability of entering into a successful match next period, compared to an unsuccessful buyer. There is now an opportunity cost associated with accepting a trade. Since the probability of getting into a successful match *next period* is lower if buyers are in a successful match *this period*, buyers may become more "choosy" in their purchases *this period*. As a result, buyers may choose not to consume in a match if the match surplus is "too low." If consumers reject low surplus trades, then there may be a benefit from departing from the Friedman rule: an increase in inflation may cause buyers to be less choosy, i.e., they will accept low surplus trades, which will generate more trading activity and higher welfare. It is interesting to note that in Tommasi (1994, 1999) inflation also causes buyers to be less choosy; but in his environment, being less choosy is associated with lower welfare. In the model presented below, a social planner would want buyers to purchase whenever the surplus is positive. However, owing to the opportunity cost of accepting a trade, buyers will not want to accept low surplus trades, i.e., from the planner's perspective, the buyer is "too choosy." Hence, a policy that makes buyers less choosy, e.g., a departure from the Friedman rule, may increase social welfare.

The paper is organized as follows. The environment is described in the next section. Some basic results are presented in sections 3 and 4. Section 5 demonstrates that if the search externality—and, hence, the opportunity cost of accepting a low surplus trade by the buyer—is neutralized, then the Friedman rule characterizes optimal policy. The search externality is active in section 6 and it is shown that, because of the existence of the hot potato effect, the Friedman rule is not optimal. Section 7 concludes the paper.

2 Environment

The environment is similar to that used in Lagos and Rocheteau (2005) and Rocheteau and Wright (2005). Time is discrete and the horizon infinite. There are two types of non-storable consumption goods, called special and general goods. The economy is populated by a unit measure of agents called sellers and a measure $b \geq 1$ of agents called buyers. All agents are infinitely-lived. Both buyers and sellers have the ability to produce and consume general goods. Buyers may want to consume special goods but cannot produce them; sellers can produce special goods but do not want to consume them. Each time period is divided into two subperiods, where different types of goods are traded in different market structures. In the first subperiod, special goods are traded in a decentralized market, where agents are matched bilaterally. In the second subperiod, agents trade general goods in a centralized (Walrasian) market.

There is an intrinsically useless, perfectly divisible and storable asset called money. Let M_t denote the quantity of money at the beginning of period t. The gross growth rate of the money supply is constant over time and equal to γ; that is, $M_{t+1} = \gamma M_t$. New money is injected or withdrawn by lump-sum transfers or taxes, respectively. These transfers or taxes take place in the second subperiod. Transfers are made before the centralized market opens; taxes are paid after it closes. The price of money in terms of general goods in period t is denoted ϕ_t and is taken as given by agents in the centralized market. Assume that agents are anonymous and there are no forms of commitment or public memory that would render money inessential.

Exactly a unit measure of buyers are allowed to participate in the decentralized market. This means that in any given date $b-1$ buyers do not participate in the decentralized market. A buyer who is in the decentralized market at date t and who does not trade can participate in the date $t+1$ decentralized market. A buyer who participates in the date t decentralized market and who trades with a seller enters a "queue" with the $1-b$ buyers who did not participate in date t decentralized market. Each buyer in the queue—independent of his participation in the most recent decentralized market—has an equally likely chance of being chosen to participate in the next decentralized market. So, for example, if x buyers trade in the decentralized market, where $0 \leq x \leq 1$, then the probability that a buyer in the queue participates in the next decentralized market is $x/(b-1+x)$. I will denote the probability of exiting the queue as λ.[1] Just after the date t decentralized market closes but before the date t centralized market opens, buyers in the queue learn whether or not they will participate in date $t+1$ decentralized market.

Buyers have idiosyncratic preferences over the special goods produced by the seller. This is captured by assuming that if a buyer is matched, he receives an idiosyncratic shock ε to his marginal utility of consumption.[2] Shocks ε_t are *iid* with cumulative distribution $F(\varepsilon)$ on $[0,1]$. The instantaneous utility function of a buyer is

$$U^b(x, y, q, \varepsilon) = \varepsilon u(q) + x - y \tag{1}$$

where q is consumption of the special good in the first subperiod, x and y are the quantities of general goods consumed and produced, respectively, in the second subperiod. I assume that $u(0) = 0$, $u'(0) = \infty$, $u'(q) > 0$ and $u''(q) < 0$ for $q > 0$. Buyers discount next period utility by the factor β. The buyer's rate of time preference, r, is defined as $(1-\beta)/\beta$. The instantaneous utility function of a seller is

$$U^s(x, y, q) = -c(q) + x - y. \tag{2}$$

I assume that $c(0) = c'(0) = 0$, $c'(q) > 0$ and $c''(q) > 0$ for $q > 0$, and for some $0 < \hat{q} < \infty$, $c(\hat{q}) = u(\hat{q})$. Sellers always participate in the decentralized market;

[1] This is the simplest way to model the search externality, i.e., the probability of being in a successful match tomorrow is lower if the buyer consumes today, compared to not consuming today.

[2] This assumption is similar in spirit to Kiyotaki and Wright (1991). None of the results or insights would be affected if, instead, I assumed that the seller received an idiosyncratic shock on the cost of production.

hence, lifetime utility for a seller is given by $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^s(x_t, y_t, q_t)$, where $\mathbb{E}_0$ is the expectation operator conditional on all information available at date $t = 0$.

A match is a meeting between a seller producing a special good and a buyer who will enjoy consuming the good, i.e., $\varepsilon > 0$ for the buyer. A match need not result in trade; for example if ε is low, the buyer may choose not to trade.

There are matching frictions in the decentralized market. Let α, where $0 < \alpha < 1$, denote the probability that a buyer is matched with a seller and the probability that a seller is matched with a buyer.[3]

3 Value Functions and Prices

3.1 Centralized market value functions

The value of being a buyer in the second subperiod/centralized market is,

$$W_b(z) = \max_{x,y,z_{+1}} \left[x - y + \beta V_b(z_{+1})\right] \tag{3}$$

$$\text{s.t.} \quad x + \gamma z_{+1} = y + z, \tag{4}$$

where z represents the buyer's real balances[4] and V_b is the value of being a buyer in the first subperiod/decentralized market. According to (3), the buyer chooses his net consumption of general goods and his real balances for the next period, subject to the budget constraint (4). By substituting the buyer's budget constraint, (4), into his objective function, (3), the buyer's value function in the centralized market can be written as,

$$W_b(z) = z + \max_{z_{+1}} \left[-\gamma z_{+1} + \beta V_b(z_{+1})\right]. \tag{5}$$

Note that, from (5), $W_b(z) = z + W_b(0)$, and the choice of z_{+1} is independent of z, the real balances with which the buyer enters the centralized market.

It is important to distinguish between buyers who will be participating in the subsequent decentralized market and those who will not. Below, I will identify a buyer who participates with a superscript "a"—a standing for active—and a buyer who does not with a hat "$\hat{}$".

[3] More formally, let $M(b, s)$ represent a strictly increasing, constant return matching function with $M_{11} < 0$ and $M_{22} < 0$ and $M_{11}M_{22} - M_{12} > 0$, where b and s represent the measures of buyers and sellers, respectively, in the market. Since $s = 1$ and the number of "active" buyers is equal to 1, the matching probabilities for a seller, α_s, and a buyer, α_b, are $\alpha_s = M(b, s)/s = M(1, 1) \equiv M(b, s)/b = \alpha_b \equiv \alpha$.

[4] Note that $\gamma z_{+1} = \left(\phi/\phi_{+1}\right) z_{+1}$ represents how much to produce in the current period to have z_{+1} in the next period. I will assume, for simplicity, that only the seller receives a monetary transfer or tax, T, in the centralized market. None of the results are affected if it is, instead, assumed that only the buyer receives the monetary transfer or that both the buyer and seller receive a transfer in the centralized market.

By similar reasoning, the value of being a seller in the second subperiod is

$$W_s(z^s) = z^s + T + \max_{z^s_{+1}} \left[-\gamma z^s_{+1} + \beta V_s(z^s_{+1}) \right], \tag{6}$$

where T represents the real monetary transfer or tax that the seller receive receives or pays and z^s represents his real balances. Note that $W_s(z^s) = z^s + W_s(0)$.

3.2 Decentralized prices

The terms of trade in a bilateral match in the decentralized market are denoted (q, d), where q is quantity of special good produced by the seller and consumed by the buyer and d is the real dollars transferred by the buyer to the seller. Prices in the decentralized market are determined by a simple bargaining protocol: Buyers make take-it-or-leave-it offers. The offer made by a buyer in a match who holds z units of real balances to a seller who holds z^s units of real balances is given by the solution to,

$$\max_{(q,d)} \left\{ \varepsilon u(q) + \lambda W^a_b(z-d) + (1-\lambda) \hat{W}_b(z-d) \right\} \tag{7}$$

$$\text{s.t.} \quad -c(q) + W_s(z^s + d) \geq W_s(z^s) \tag{8}$$

$$d \leq z \tag{9}$$

According to (7)-(9), the buyer chooses (q, d) so as to maximize his expected utility, subject to the constraints that the offer must be acceptable from the seller's point of view, (8), and the buyer cannot offer to transfer more money than what he holds, (9). Using the linearity of W_b and W_s, problem (7)–(9) can be compactly rewritten as,[5]

$$\max_{q,d \leq z} \{\varepsilon u(q) - d\} \quad \text{s.t.} \quad -c(q) + d \geq 0. \tag{10}$$

The solution for q to this program is,

$$q(z, \varepsilon) = \begin{cases} q^*(\varepsilon) & \text{if} \quad \varepsilon \leq \bar{\varepsilon}(z) \\ \bar{q}(z) & \text{if} \quad \varepsilon > \bar{\varepsilon}(z) \end{cases}, \tag{11}$$

where $q^*(\varepsilon)$ is the value of q that satisfies $\varepsilon u'(q) = c'(q)$, $\bar{\varepsilon}(z)$ is the critical value of ε that satisfies $c[q^*(\varepsilon)] = z$, and $\bar{q}(z)$ is the value of q that satisfies $c(q) = z$. Note that $\bar{\varepsilon}(z)$ and $\bar{q}(z)$ are increasing in z. In words, if the realization of the preference shock is low, buyers can buy the quantity they would like to consume without being constrained by their money holdings. In contrast, if the realization of the preference shock is above the threshold $\bar{\varepsilon}$, then buyers are constrained by their money holdings

[5] Note that $\varepsilon u(q) + \lambda W^a_b(z-d) + (1-\lambda) \hat{W}_b(z-d) = \varepsilon u(q) - d + \lambda W^a_b(z) + (1-\lambda) \hat{W}_b(z)$, and, from the maximization point of view, $W^a_b(z)$ and $\hat{W}_b(z)$ are constants. The constraint (8) can be rewritten as $-c(q) + d + W_s(z^s) \geq W_s(z^s)$.

and cannot buy as much as they would like. Note that $q_z(z,\varepsilon) = 0$ for all $\varepsilon \leq \bar{\varepsilon}(z)$ and $q_z(z,\varepsilon) = 1/c'(q)$ for all $\varepsilon > \bar{\varepsilon}(z)$. Similarly, the solution for d is:

$$d(z,\varepsilon) = \begin{cases} d^*(\varepsilon) & \text{if} \quad \varepsilon \leq \bar{\varepsilon}(z) \\ z & \text{if} \quad \varepsilon > \bar{\varepsilon}(z) \end{cases}, \tag{12}$$

where $d^*(\varepsilon)$ is the value of d that satisfies $c[q^*(\varepsilon)] = d$. For convenience, define $z^* = c[q^*(1)]$. Note that the terms of trade, (q,d), are not a function of the seller's real balances, z^s.

3.3 Decentralized market value functions

Let $V_b^a(z)$ denote the value function of a buyer in the decentralized market before he is (potentially) matched. Consider a match in the decentralized market, where the buyer is holding z units of real balances and his realization for the preference shock shock—which is learned when he is matched—is ε. His expected utility in the event of trade, $\mathcal{V}_b^a(z,\varepsilon)$, is

$$\begin{aligned} \mathcal{V}_b^a(z,\varepsilon) &= \varepsilon u[q(z,\varepsilon)] + \lambda W_b^a(z-d) + (1-\lambda)\hat{W}_b(z-d) \\ &= \varepsilon u[q(z,\varepsilon)] + z - d + \lambda W_b^a(0) + (1-\lambda)\hat{W}_b(0). \end{aligned} \tag{13}$$

The buyer consumes $q(z,\varepsilon)$ and delivers $d(z,\varepsilon)$ real dollars to the seller; as a result the buyer carries over $z-d$ real dollars into the centralized market. With probability λ, the buyer is active in the subsequent decentralized market, and with complementary probability he is not. Note that there is a simple linear relationship between $W_b^a(0)$ and $\hat{W}_b(0)$. In particular, a buyer who is excluded from the subsequent decentralized market will learn, in the next period, if he will be able to participate in the decentralized market two periods from now, or

$$\hat{W}_b(0) = \beta\left[\lambda W_b^a(0) + (1-\lambda)\hat{W}_b(0)\right],$$

which implies to

$$\hat{W}_b(0) = \frac{\beta\lambda}{1-\beta(1-\lambda)} W_b^a(0). \tag{14}$$

The buyer's expected utility prior to being matched in the decentralized market is given by

$$\begin{aligned} V_b^a(z) &= \alpha \int \max\left\{\mathcal{V}_b^a(z,\varepsilon), W_b^a(z)\right\} dF(\varepsilon) \\ &\quad +(1-\alpha)W_b^a(z). \end{aligned} \tag{15}$$

Equation (15) has the following interpretation. With probability α the buyer meets a seller; he then receives a preference shock, ε, drawn from $F(\varepsilon)$. If he trades, then he receives $\mathcal{V}_b^a(z,\varepsilon)$; if he does not, then he gets $W_b^a(z)$.

Let $V_s(z^s)$ denote the value function of a seller in the decentralized market. Assume that all buyers bring z real dollars into the decentralized market. The value of being a seller in the decentralized market is given by,

$$V_s(z^s) = \alpha \int \{-c[q(z,\varepsilon)] + W_s[z^s + d(z,\varepsilon)]\} dF(\varepsilon)$$
$$+(1-\alpha)W_s(z^s). \tag{16}$$

The interpretation of (16) is similar to (13) except that sellers suffer disutility of production and receive money from buyers. The seller's value function in the decentralized market, (16), can be rewritten as,

$$V_s(z^s) = \alpha \int \{-c[q(z,\varepsilon)] + d(z,\varepsilon)\} dF(\varepsilon) + W_s(z^s) \tag{17}$$

From constraint (10), with an equality, (17) can be simplified to $V_s(z^s) = W_s(z^s)$; and from the linearity of $W_s(z^s)$ we have $V_s(z^s) = z^s + V_s(0)$. As well, the seller's value function in the centralized market, (6), implies that the seller will not hold money in the decentralized market if $\gamma \geq \beta$. Since it is not possible to have $\gamma < \beta$, the seller will optimally choose $z^s = 0$.

4 Buyers' choices

Define the total surplus of match as $S(z,\varepsilon) = \varepsilon u[q(z,\varepsilon)] - c[q(z,\varepsilon)]$. It can be checked that $\partial S(z,\varepsilon)/\partial\varepsilon \equiv S_\varepsilon(z,\varepsilon) = u[q(z,\varepsilon)] > 0$; that is, the match surplus is increasing in the quality of the match. Furthermore, for all $\varepsilon > \bar{\varepsilon}(z)$,

$$\frac{\partial S(z,\varepsilon)}{\partial z} \equiv S_z(z,\varepsilon) = \frac{\varepsilon u'[q(z,\varepsilon)] - c'[q(z,\varepsilon)]}{c'[q(z,\varepsilon)]} > 0,$$

and for all $\varepsilon \leq \bar{\varepsilon}(z)$, $S_z(z,\varepsilon) = 0$; that is, the match surplus is increasing in real balances and strictly increasing when the match surplus is not maximized. Money allows agents to extract larger gains from trade by increasing the total surplus. Using this notation, the Bellman equation (15) can be rewritten as:

$$V_b^a(z) = \alpha \int \max\{S(z,\varepsilon) - \theta W_b^a(0), 0\} dF(\varepsilon) + W_b^a(z), \tag{18}$$

where

$$\theta = 1 - \frac{\lambda}{1-\beta(1-\lambda)} \tag{19}$$

The following proposition describes some important properties of the active buyer's value function in the centralized market. I define the nominal interest rate, i, as $1+i = (1+r)\gamma$ or $i = (\gamma-\beta)/\beta$.

Lemma 1 *The value function for an active buyer in the centralized market is uniquely determined by*

$$rW_b^a(0) = \max_{\hat{z}\in[0,c(q^*)]} \left\{-i\hat{z} + \alpha \int_{\varepsilon=0}^{1} \max\left\{S(\hat{z},\varepsilon) - \theta W_b^a(0), 0\right\} dF(\varepsilon)\right\} \qquad (20)$$

and $W^b(0) > 0$.

Proof. Substitute $V_b^a(z)$ from (18) into (5) and rearrange to get (20). To show that $W_b^a(0)$ is uniquely determined, notice the following:

(i) $\hat{z}$ can be restricted to be chosen from the interval $[0, c(q^*(1))]$ since for all $\hat{z} \geq c(q^*(1))$, $\int [S(\hat{z},\varepsilon) - \theta W_b^a(0)] dF(\varepsilon) = \int [u(q^*(\varepsilon)) - c(q^*(\varepsilon)) - \theta W_b^a(0)] dF(\varepsilon) =$ a constant, i.e., the buyer has no (strict) incentive to choose $\hat{z} > c(q^*(1))$.

(ii) If I define the right-hand side of (20) as $\mathcal{W}(W_b^a(0))$, then from the theorem of the maximum, $\mathcal{W}(W_b^a(0))$ is continuous in $W_b^a(0)$. As well, $\mathcal{W}(W_b^a(0))$ is weakly decreasing in $W_b^a(0)$; $\mathcal{W}(W_b^a(0)) \to 0$ as $W_b^a(0) \to \infty$, and $\mathcal{W}(0) = \max_{\hat{z}} \left\{-i\hat{z} + \alpha \int S(\hat{z},\varepsilon) dF(\varepsilon)\right\} > 0$ since $S_z(0,\varepsilon) = \infty$ for all $\varepsilon \in (0,1]$.

(iii) Part (ii), in conjunction with the fact that the left-hand side of (20) is a strictly increasing function of $W_b^a(0)$, implies that $W_b^a(0)$ is unique and strictly greater than zero. ■

The first term on the right-hand side of (20) represents the opportunity cost of holding real balances and the second term represents the expected payoff of participating in the decentralized market, which is the match surplus minus the opportunity cost of making a trade. The right-hand side of (20) nicely describes the trade-off that a buyer faces when he chooses his real balances, which is the cost of holding the real balances versus the benefit associated of having the real balances.

5 The Inflation Tax Effect

In this section, it is assumed that the buyer participates in the decentralized market in every period. This can be accomplished by assuming that $b = 1$, i.e., there is no queue to get into the decentralized market.

The buyer's decision problem regarding the amount of real balances to accumulate in the centralized market, z, is given by the $\hat{z}$ that solves the right hand side of (20) when $\lambda = 1$,[6] i.e.,

$$\max_{z\in[0,c(q^*)]} \left\{-iz + \alpha \int_{\varepsilon=0}^{1} S(z,\varepsilon) dF(\varepsilon)\right\}. \qquad (21)$$

Note that when $\lambda = 1$, matched buyers always trade in the decentralized market for any value of $\varepsilon > 0$; this reflects the fact that there is no opportunity cost of accepting a trade. The actual realization of ε determines how much agents produce

[6] If $\lambda = 1$, then $\theta = 0$, see equation (19).

and consume in a match in the decentralized market. The solution to (21) is given by

$$\frac{i}{\alpha} = \int_{\varepsilon=0}^{1} S_z\left(z,\varepsilon\right) dF\left(\varepsilon\right),$$

which can be rewritten as

$$\frac{\gamma-\beta}{\beta\alpha} = \int_{\bar{\varepsilon}(z)}^{1} \frac{\left\{\varepsilon u'\left[q(z,\varepsilon)\right] - c'\left[q(z,\varepsilon)\right]\right\}}{c'\left[q(z,\varepsilon)\right]} dF(\varepsilon). \tag{22}$$

A definition for an equilibrium in this environment is now provided.

Definition 1 *A monetary equilibrium is a $z > 0$ that satisfies (22).*

Lemma 2 *For all $\gamma \geq \beta$, a monetary equilibrium exists and is unique. Furthermore, z is decreasing with γ and $\lim_{\gamma\to\beta} \bar{\varepsilon}\left(z\right) = 1$.*

Proof. The right-hand side of (22) is strictly decreasing in z for all $z \leq z^* = c[q^*(1)]$. As $z \to 0$, the right-hand side of (22) goes to ∞, and when $z \to z^*$ the right-hand side of (22) goes to 0. Therefore, there is a unique $z \in [0, z^*]$ that satisfies (22). ■

As inflation increases, buyer's reduce their real balances due to the standard inflation tax effect. As a consequence, some buyers will now be constrained in what they can purchase in some matches, i.e., in those matches where $\varepsilon \in (\bar{\varepsilon}(z), 1]$.

Consider a social planner who maximizes the sum of surpluses in all matches by choosing how much to trade. Note that a strictly positive surplus can be generated for any $\varepsilon > 0$. Therefore, the problem that the social planner solves is

$$\max_{q(\varepsilon)} \alpha \int_{\varepsilon=0}^{1} \left\{\varepsilon u\left[q(\varepsilon)\right] - c[q(\varepsilon)]\right\} dF(\varepsilon)$$

The social planner will choose $q(\varepsilon) = q^*(\varepsilon)$ for all $\varepsilon \in [0, 1]$.

Proposition 1 *The monetary equilibrium is efficient iff $\gamma = \beta$.*

Proof. Trade will always be efficient—$q = q^*(\varepsilon)$ for all $\varepsilon \in [0, 1]$—when $\bar{\varepsilon}\left(z\right) = 1$ in (22). From (22), $\bar{\varepsilon}(z) = 1$ iff $\gamma = \beta$. ■

Here, the Friedman rule is optimal and achieves the first-best allocation. The Friedman rule drives the cost of holding real balances to zero and, as a result, buyers will always carry sufficient balances to purchase the efficient level of output for any $\varepsilon \in [0, 1]$. Inflation, i.e., $\gamma > \beta$, generates a misallocation of resources in this environment because buyers reduce their real balances which prevents them from exploiting all the gains from trade in some matches, i.e., in those matches where ε is "high."

6 The Hot Potato Effect

The previous section essentially assumes that there is no opportunity cost associated with accepting a trade for the buyer since $b = 1$. In this section, it is assumed that $b > 1$, which implies that if a buyer trades in the decentralized market, then he will, with positive probably, be excluded from trading in the decentralized market in subsequent periods.

Buyers will choose an optimal "consumption rule" that determines when they will trade in the decentralized market. More specifically, buyers will choose a *reservation preference shock level*, ε_R, such that when they are matched in the decentralized market, they will trade if $\varepsilon \geq \varepsilon_R$; otherwise, they will not. Using this notation, equation (20) can be rewritten as

$$rW_b^a(0) = \max_{z \in [0, c(q^*)], \varepsilon_R} \left\{ -iz + \alpha \int_{\varepsilon_R}^1 \left[S(z, \varepsilon) - \theta W_b^a(0) \right] dF(\varepsilon) \right\}. \tag{23}$$

The decision problem of the buyer regarding his choice of real balances and reservation preference shock level is given by the z and ε_R, respectively, that solves the right-hand side of (23). The solution is given by

$$-(\gamma - \beta) + \alpha\beta \int_{\varepsilon_R}^1 S_z(z, \varepsilon) F(\varepsilon) = 0 \tag{24}$$

and

$$-S(z, \varepsilon_R) + \theta W_b^a(0) = 0. \tag{25}$$

Note that equation (24) can be rewritten as

$$\frac{\gamma - \beta}{\beta\alpha} = \int_{\max\{\bar{\varepsilon}(z), \varepsilon_R\}}^1 \frac{\varepsilon u'[q(z, \varepsilon)] - c'[q(z, \varepsilon)]}{c'[q(z, \varepsilon)]} dF(\varepsilon), \tag{26}$$

because $S_z = 0$ for $\varepsilon < \bar{\varepsilon}(z)$. Using (23) and (25), one can conclude that in any equilibrium, the reservation price shock level, ε_R, and real money balances, z, must satisfy

$$(1 - \beta) S(z, \varepsilon_R) = \theta A, \tag{27}$$

where

$$A \equiv \left\{ -(\gamma - \beta) z + \alpha\beta \int_{\varepsilon_R}^1 \left[S(z, \varepsilon) - S(z, \varepsilon_R) \right] dF(\varepsilon) \right\}$$

It must also be the case that, in any equilibrium, the buyer's choice of ε_R must be consistent with λ, that is

$$\lambda(\varepsilon_R) = \alpha \frac{1 - F(\varepsilon_R)}{b - 1 + \alpha(1 - F(\varepsilon_R))}. \tag{28}$$

Note that $\lambda' = (1 - b) F'(\varepsilon_R) / [b - 1 + \alpha(1 - F(\varepsilon_R))]^2 < 0$, i.e., as one would expect, as buyers become more choosy, the probably of exiting the queue decreases.

An equilibrium in this environment is given by,

Definition 2 *An equilibrium is a triplet* $(z, \varepsilon_R, \lambda)$ *that satisfies (26), (27) and (28)*

I will focus on a monetary policy that is close to the Friedman rule; a monetary policy that is close to the Friedman rule is characterized by $\gamma \approx \beta$, where $\gamma > \beta$.

Proposition 2 *Assume* $\gamma \approx \beta$. *Then, a monetary equilibrium exists. As* γ *increases, both* z *and* ε_R *decrease in the Pareto-dominant equilibrium.*

Proof. Equation (23) implies that $\varepsilon_R < 1$ for all $z > 0$, i.e., if $\varepsilon_R = 1$, then the right-hand side of (23) is non-positive and the left-hand side is strictly positive, a contradiction. Since $\varepsilon_R < 1$ and $\lim \bar{\varepsilon}(z) \to 1$ as $\gamma \to \beta$, $\max\{\bar{\varepsilon}(z), \varepsilon_R\} = \bar{\varepsilon}(z)$ when $\gamma > \beta$ but $\gamma - \beta$ is arbitrarily small. Hence, when $\gamma - \beta$ is arbitrarily small, there exists a unique z that satisfies equation (26), where $\bar{\varepsilon}(z) > \varepsilon_R$; as a result, the right-hand side is strictly decreasing in z for $\bar{\varepsilon}(z) < 1$. Given such a z, (27) determines ε_R. Even though the left-hand side of (27), $(1-\beta) S(z, \varepsilon_R)$ is strictly increasing in ε_R, there may be multiple solutions since the right-hand side, θA, may not be monotonic is ε_R: θ is strictly increasing in ε_R and A is strictly decreasing. Figure 1 depicts the graphs $(1-\beta) S(z, \varepsilon_R)$ and θA when there exists multiple equilibria. Note that the value θA is strictly positive at $\varepsilon_R = 0$ and equal to zero at $\varepsilon_R = 1$. Therefore, at the Pareto-dominant equilibrium—given by the intersection of $(1-\beta) S(z, \varepsilon_R)$ and θA at the lowest value of ε_R—the slope difference in the slopes of $(1-\beta) S(z, \varepsilon_R)$ and θA, i.e., $(1-\beta) S_\varepsilon(z, \varepsilon_R) - \frac{\partial(\theta A)}{\partial \varepsilon_R}$, must be positive. (Of course, if (27) has a unique solution, then$(1-\beta) S_\varepsilon(z, \varepsilon_R) - \frac{\partial(\theta A)}{\partial \varepsilon_R}$ evaluated ε_R is positive.) Total differentiating equations (24) and (27), we get

$$-d\gamma + \alpha\beta \int_{\varepsilon_R}^{1} S_{zz}(z, \varepsilon)\, dF(\varepsilon)\, dz - \alpha S_z(z, \varepsilon_R)\, dz = 0 \tag{29}$$

and

$$\begin{aligned}(1-\beta) S_z(z, \varepsilon_R)\, dz + (1-\beta) S_\varepsilon(z, \varepsilon_R)\, d\varepsilon_R = -\theta z d\gamma \\ +\theta \left\{ -(\gamma - \beta) + \alpha\beta \int_{\varepsilon_R}^{1} S_z(z, \varepsilon) F(\varepsilon) \right. \\ \left. +\alpha\beta \int_{\varepsilon_R}^{1} S_z(z, \varepsilon_R) F(\varepsilon) \right\} dz + \frac{\partial(\theta A)}{\partial \varepsilon_R} d\varepsilon_R.\end{aligned} \tag{30}$$

Plugging equation (24) into (30), and recognizing that when $\gamma \approx \beta$, $S_z(z, \varepsilon) = 0$, equations (29) and (30) can be rewritten as

$$\frac{dz}{d\gamma} = \frac{1}{\alpha\beta \int_{\varepsilon_R}^{1} S_{zz}(z, \varepsilon)\, dF(\varepsilon)\, dz} < 0$$

and

$$\frac{d\varepsilon_R}{d\gamma} = -\frac{z}{(1-\beta) S_\varepsilon(z, \varepsilon_R) - \frac{\partial(\theta A)}{\partial \varepsilon_R}}.$$

Since $(1-\beta) S_{\varepsilon}(z, \varepsilon_R) - \frac{\partial(\theta A)}{\partial \varepsilon_R} > 0$ at the Pareto-dominant equilibrium, $\frac{d\varepsilon_R}{d\gamma} < 0$. ■

Note that inflation now has two effects. There is the usual inflation tax effect according to which buyers lower their real balances to reduce their exposure to the inflation tax. The other effect associated with an increase in inflation—which is new to the literature—that has some buyers purchasing output when they would not otherwise at lower inflation rates. This is reminiscent of the "hot potato effect," where people spend their balances on goods more rapidly in order to avoid the inflation tax.

The social planner wants to maximize the sum of the surpluses in all matches. As in the previous section, the planner will set $q(\varepsilon) = q^*(\varepsilon)$ and $\varepsilon_R = 0$. The rationale for these settings is that the planner would like buyers to trade as soon as they find a match with a positive surplus since there is always a unit measure of buyers in the decentralized market. Hence, from a social perspective, there is no opportunity cost associated with accepting *any* trade that generates a non-negative surplus.

Proposition 3 *A monetary equilibrium is always inefficient. A deviation from the Friedman rule will increase social welfare in the Pareto-dominant equilibrium.*

Proof. Equation (11) implies that $q(\varepsilon) = q^*(\varepsilon)$ for all $\varepsilon \in [0,1]$ iff $\bar{\varepsilon}(z) = 1$ and equation (26) implies that $\bar{\varepsilon}(z) = 1$ iff $\gamma = \beta$. When $\gamma = \beta$, $i = 0$ and $q(\varepsilon) = q^*(\varepsilon)$ for all $\varepsilon \in [0,1]$; hence, equation (23) implies that $\varepsilon_R > 0$. Therefore, the equilibrium is inefficient. Let $\mathcal{W}(\gamma)$ measure social welfare as a function of γ, i.e.,

$$\mathcal{W}(\gamma) = \alpha \int_{\varepsilon_R(\gamma)}^{1} S(z(\gamma), \varepsilon) dF(\varepsilon).$$

Then

$$\mathcal{W}'(\gamma) = -\alpha S(z, \varepsilon_R) f(\varepsilon_R) \frac{d\varepsilon_R}{d\gamma} + \alpha \int_{\varepsilon_R(\gamma)}^{1} S_z(z, \varepsilon) dF(\varepsilon) \frac{dz}{d\gamma}.$$

Therefore, at the Pareto-dominate equilibrium

$$\mathcal{W}'(\beta) = -\alpha S(z^*, \varepsilon_R) f(\varepsilon_R) \left. \frac{d\varepsilon_R}{d\gamma} \right|_{\gamma=\beta} > 0,$$

since $\frac{d\varepsilon_R}{d\gamma} < 0$ and $S_z(z^*, \varepsilon) = 0$ when $\gamma = \beta$. ■

At the Friedman rule, buyers are too choosy as they fail to fully internalize the effect of their trading strategies on buyers in the queue. A buyer who chooses to keep on searching for a better match prevents a buyer in the queue—who has exactly the same trading opportunities—from entering the decentralized market. Inflation has a welfare improving role as it gives buyers an incentive to exit the decentralized market more rapidly.

7 Concluding Comments

Although the notion that buyers will not transact in small surplus trades is novel to monetary economics, it is a rather natural extension that comes to us via labor search literature. The externality that creates an opportunity cost associated with the buyer accepting a trade generates a meaningful conversation between the hot potato effect *and* the Friedman rule. As well, it offers an interesting result: A little bit of inflation—that is, a departure from the Friedman rule—may be a good thing when buyers are too choosy from a social perspective. An increase in inflation will cause buyers to lower their "reservation surplus," which increases the number of trades and welfare.

The analysis conducted above assumes a particular pricing policy in decentralized markets: buyer-take-all pricing. One would conjecture that the hot potato effect will still be operative for other pricing schemes, but the result that a departure from the Friedman rule is optimal may no longer be valid. The hot potato effect depends on the existence of an opportunity cost of accepting a trade and that inflation acts as a search cost. Since both of these concepts are independent of the nature of the pricing scheme in decentralized markets, the hot potato effect will be operative for alternative pricing schemes. The optimality of a departure from the Friedman rule depends on buyers choosing $z = z^*$ at the Friedman rule. Although for many pricing schemes, e.g., competitive price posting, buyer-take-all, and buyers and sellers payoffs are monotone in the size of the total surplus, $z = z^*$ at the Friedman rule, some pricing schemes, e.g., generalized Nash Bargaining, have buyers choosing $z < z^*$. In these cases, it is not obvious that a departure from the Friedman rule will increase social welfare.

References

[1] Ennis, Huberto M., forthcoming. "Avoiding the Inflation Tax," *International Economic Review*.

[2] Kiyotaki, Nobuhiro and Randall Wright, 1991. "A Contribution to the Pure Theory of Money," *Journal of Economic Theory* 53, 215-235.

[3] Lagos, Ricardo and Guillaume Rocheteau, 2005. "Inflation, Output and Welfare," *International Economic Review* 46, 495-522.

[4] Tommasi, Mariano, 1994. "The Consequences of Price Instability on Search Markets: Toward Understanding the Effects of Inflation," *American Economic Review* 84, 1385-1396.

[5] Tommasi, Mariano, 1999. "On High Inflation and the Allocation of Resources," *Journal of Monetary Economics* 44, 401-421.

[6] Rocheteau, Guillaume and Randall Wright, 2005. "Money in Search Equilibrium, in Competitive Equilibrium and in Competitive Search Equilibrium," *Econometrica* 73, 175-202.

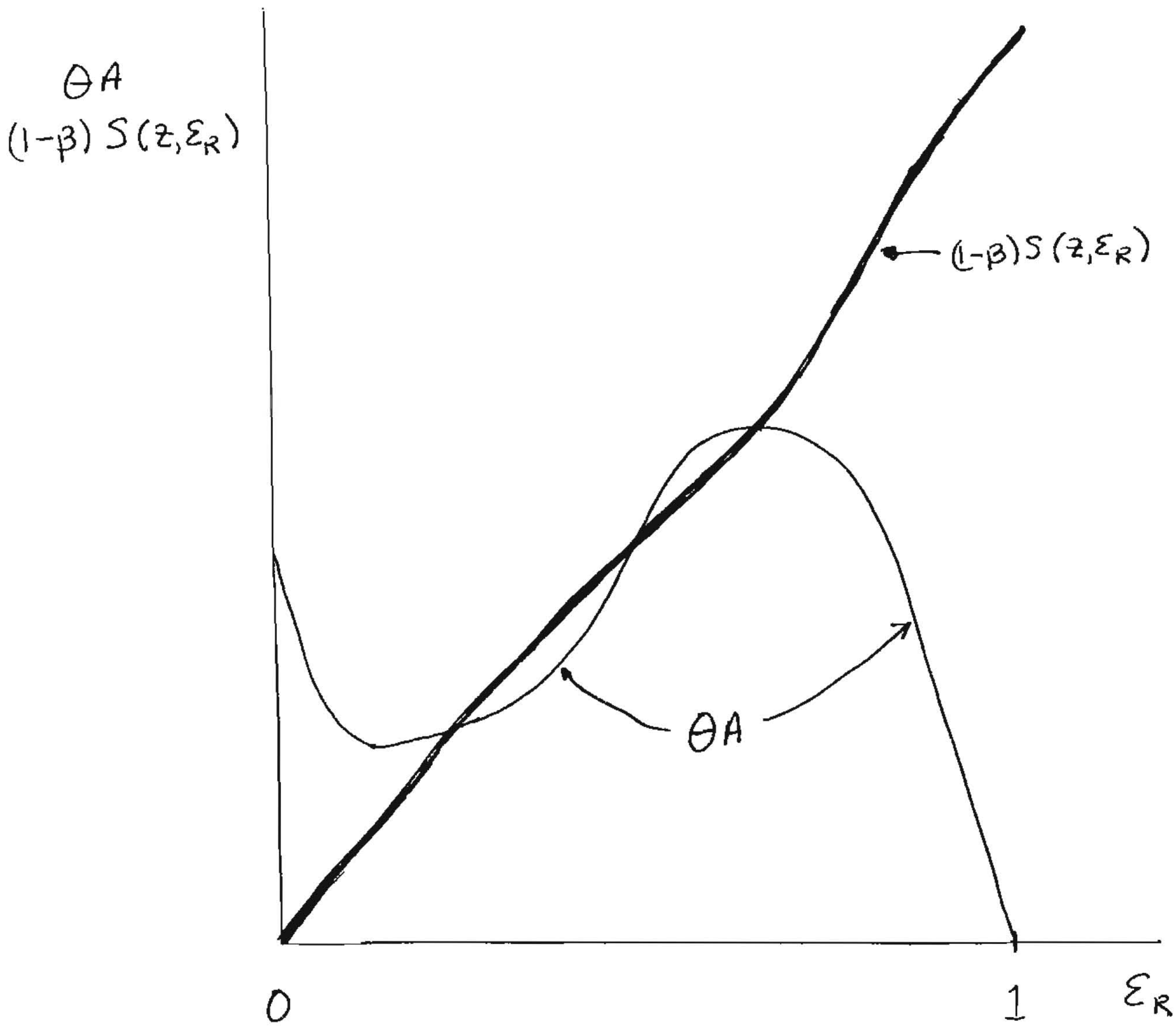

Figure 1