Financial Fragility and the Exchange Rate Regime

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Abstract: We study financial fragility, exchange rate crises, and monetary policy in an open economy version of a Diamond-Dybvig model. The banking system, the exchange rate regime, and central bank credit policy are seen as parts of a mechanism intended to maximize social welfare; if the mechanism fails, banking crises and speculative attacks become possible. We compare currency boards, fixed rates, and flexible rates with and without a lender of last resort. A currency board cannot implement a socially optimal allocation; in addition, bank runs are possible under a currency board. A fixed exchange rate system may implement the social optimum but is more prone to bank runs and exchange rate crises than a currency board. A flexible rate system implements the social optimum and eliminates runs, provided the exchange rate and central bank lending policies are appropriately designed.

JEL classification: F3, E5, G2

Key words: exchange rates, financial crises, central bank policy

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1. Introduction

The 1994 Mexican crisis and the 1997 Thai crisis are the latest examples of an age-old predicament: weak financial systems easily undermine fixed exchange rates. Fragile banks prevent the monetary authority from using tight money and high interest rates to defend the peg; bank runs may imply a shift out of domestic money and toward foreign assets; bankrupt banks may require money-financed bailouts. In all cases, the net effect is reserve erosion and a likely collapse of the fixed exchange rate.

Such links are borne out by more than anecdotal evidence from Mexico and

Thailand. In their analysis of a large number of banking and currency crises, Kaminsky and Reinhart [3] find that the occurrence of the former helps predict the emergence of the latter. In a similar vein, Sachs, Tornell and Velasco [9] find that a weak banking system (proxied by a previous lending boom) is the best predictor of whether a country was hit by the "Tequila effect" in early 1995.

These policy experiences have given new urgency to the question of what exchange rate arrangements help limit the dangers associated with financial fragility. If fixed exchange rate systems are vulnerable, does adopting a currency board help? Or should countries move in the other direction and adopt flexible rates? Indeed, financial considerations can have such weight that Calvo [1] has argued that we need a new theory of exchange rate regime choice based on financial structure rather than, as is conventional, the degree of price rigidity or the source of stochastic shocks.

Furthermore, for a given exchange rate regime, what are the proper complementary monetary and fiscal policies? Having the Central Bank serve as the lender of last resort is a common arrangement. But this is hard to do under fixed exchange rates and capital mobility, and impossible under a currency board. For instance, Argentina's strict convertibility law prevents the Central Bank from issuing domestic credit to troubled banks; this may have opened the way to a bank panic in 1995. Given this loss of monetary independence, it is sometimes suggested that governments should instead use fiscal policy (that is, their powers of taxation) to build a "war chest" to be spent at times of bank trouble.

Questions also arise about the proper management of the financial system. Is increasing reserve requirements on banks (as Argentina has done) part of the solution? Are foreign currency deposits in the domestic banking system de-stabilizing? Observers such as Sachs [8] certainly think so.

In this paper we develop a formal framework in which such claims can be analyzed and evaluated. Our approach is novel in that it starts from the microfoundations of a country's financial system. More specifically, we study commercial banking, Central Bank policies, and exchange rate regimes as *mechanisms* whose purpose is to implement socially desirable allocations in an economy with incomplete financial markets.

Given that banking fragility is a core issue, we focus on a monetary,[1] open economy version of the benchmark model by Diamond and Dybvig [2]. In that model banks are essentially maturity transformers that take liquid deposits and invest part of the proceeds in illiquid assets. In doing so they pool risk and

[1] More precisely, money enters the utility function. In addition, for most of the paper bank deposits are assumed to be denominated in domestic currency.

enhance welfare, but also create the possibility of self-fulfilling bank runs.

We embed this banking story in a general equilibrium macroeconomic model which can operate under a variety of exchange rate and monetary arrangements or *regimes.* We consider currency boards, fixed exchange rates with Central Bank credit to commercial banks, and flexible exchange rates. When appropriate we analyze the functioning of these regimes with and without a lender of last resort. We can also study the effects of a variety of precautionary measures, including high reserve requirements, "war chests" and foreign currency deposits. In all cases we investigate whether self-fulfilling runs on the bank and/or the currency can occur in equilibrium.

Our main results are:

1. A currency board, which by design mimics the operation of a non-monetary economy, is vulnerable to self-fulfilling bank runs but (predictably) not to currency crises.

2. A fixed exchange rate regime with limited Central Bank credit to commercial banks yields higher welfare than a currency board for the reasons stressed by Friedman [5] nearly half a century ago: since (up to a point) additional fiat money raises utility and costs nothing to produce, it is optimal to issue it in positive quantities. However, this regime is more prone to bank runs than a currency

board. The intuition is that, with fixed exchange rates, the liabilities of the banking system as a whole are, implicitly, obligations in international currency. Hence bank runs are possible to the extent that the banking system's implicit liabilities are greater than its *internationally* liquid assets. We show that the gap between the two is greater when Central Bank credit to the banking system is positive, which implies that bank runs are more likely.

3. Introducing unlimited domestic credit to the banking system at times of trouble (that is, a lender of last resort) into a fixed exchange rate system precludes self-fulfilling bank runs only at the cost of creating a balance of payments crisis (the central bank runs out of reserves) if and when the bank is run on. If a balance of payments crisis occurs, the corresponding real allocations (and therefore welfare) are the same as they would be in the event of a bank run.

4. Both a policy of high bank reserves and one of large international reserves (the "war chest" approach) can eliminate the bad (run) equilibrium, but only at the cost of inducing a worse allocation and hence lowering welfare (relative to the good equilibrium without these policies). Whether either policy is desirable depends on agents' risk aversion, and on the probability the economy would land in the bad equilibrium when one is feasible. Moreover, we show that a policy of high bank reserves unambiguously dominates one of large international reserves.

The intuition is that increasing the international liquidity of the banking system has a social opportunity cost. Under a policy of high reserve requirements, banks internalize this cost; under the war chest approach, they do not.

5. The combination of flexible exchange rates and a lender of last resort dominates all other policy regimes, for it can implement the preferred real allocation while ruling out self-fulfilling bank runs. The intuition for this last result is simple. An equilibrium bank run occurs if and only if each bank depositor expects others will run and exhaust the available resources. Under a fixed rates regime, those who run to the bank withdraw domestic currency, which in turn they use to buy dollars at the Central Bank. If one depositor expects this sequence of actions will cause the Central Bank to run out of dollars, then it is a best response for her to run as well, and the pessimistic expectations become self-fulfilling. On the other hand, under a flexible rates regime plus a lender of last resort there is always enough domestic currency at the commercial bank to satisfy those who run. But since the Central Bank is no longer compelled to sell all the available reserves (including the illiquid investments), those who run face a depreciation, while those who do not run know that there will still be dollars available when they desire to withdraw them at a later date. Hence, running to the bank is no longer a best response, pessimistic expectations are not self-fulfilling, and the depreciation does

not happen in equilibrium.

6. Introducing dollar-denominated bank deposits makes no difference to the feasible equilibria in the cases of a currency board and of a fixed exchange rate with limited domestic credit. But the Central Bank can no longer serve as a lender of last resort. That means that the benefits of a flexible exchange rate regime vanish if deposits are dollarized, and welfare is consequently reduced.

The paper is organized as follows. Section 2 describes the economic environment. Section 3 studies a currency board, while 4 analyzes the case of a fixed exchange rate with limited domestic credit. The effects of high reserve requirements and war chests are considered in 5. Section 6 contains the analysis of flexible exchange rates. Section 7 considers dollarized bank deposits, while 8 concludes.

2. The Economic Environment

We will study a small open economy populated by a large number of ex ante identical agents. There are three periods indexed by $t = 0, 1, 2$. Each agent is born at $t = 0$ with a quantity $e > 0$ of an endowment good. The endowment good can be invested at home in a *long term* constant returns technology whose yield per unit invested is $r < 1$ units of consumption in period 1, and $R > 1$

units of consumption in period 2. Alternatively, the endowment can be invested in the world market, where one unit invested at $t = 0$ yields one unit of a foreign currency ("dollars") in either period 1 or period 2. The price of consumption in the world market is fixed and normalized at one dollar.

At $t = 1$ each domestic agent discovers her "type". With probability λ she is "impatient" and derives utility only from period 1 consumption. With probability $(1 - \lambda)$ she turns out to be "patient". Patient agents derive utility from the real value of holdings of a domestic currency ("pesos") in period 1 and from consumption in period 2. Pesos are costlessly created and/or destroyed by a Central Bank.

Some investment in notation will help formalizing the above assumptions. Let x denote the typical agent's consumption in period 1 if she turns out to be impatient. Let M and y denote, respectively, the quantity of pesos acquired in period 1 and her period 2 consumption if she turns out to be patient. Finally, let E_t the peso price of consumption in period $t = 1, 2$, which must equal the exchange rate. Then the expected utility of the representative agent can be represented by:

$$\lambda g(x) + (1 - \lambda) g(\chi(M/E_2) + y) \tag{2.1}$$

Note that real money balances are measured by the quantity of dollars obtained at the end of period 1 deflated by the price of consumption in period 2. This is the natural assumption, since patient agents will hold such balances until the end of period 2.

The function $g(.)$ is smooth, strictly increasing, strictly concave, and satisfies Inada conditions. Assume also that $\chi(.)$ is smooth, strictly concave, and satisfies $\chi(0) = 0, \chi'(0) = \infty$, and $\chi'(\overline{m}) = 0$ for some $\overline{m} > 0$. This defines $\overline{m}$ as the satiation level of money.

Two further assumptions complete the description of the economic environment. As in conventional in models of this type, and particularly in [2], the realization of each agent's type is private information to that agent.

We will also assume that domestic residents (including the Central Bank) can invest but not borrow in the world market. This is a natural assumption if we are to study currency crises (the same assumption, for instance, is made in [4]), for such a crisis is by definition a situation of extreme scarcity of foreign exchange, and such extreme illiquidity could not occur if solvent domestic residents could borrow abroad all they desired at the world rate of interest.

This setup is similar to the classic Diamond-Dybvig formulation in [2], since there is uncertainty about the timing of consumption, and also a pattern of asset returns such that agents would prefer to invest in the world market if they knew they were impatient, and in the illiquid technology if they knew they were patient. A new feature of the model is that agents are assumed to demand domestic currency. Since only the domestic Central Bank can print pesos, the analysis of this model must include an assumption on the *exchange rate - monetary regime,* that is, a specification of the way pesos are issued and exchanged for dollars. The analysis of alternative regimes is the subject of our ensuing discussion.

3. A Currency Board

This section analyzes regimes in which the Central Bank follows a very simple rule: it stands ready to exchange dollars for pesos at a fixed exchange rate and, in addition, it is committed not to create or destroy pesos in any other way. In other words, the Central Bank functions as a *currency board.* Under a currency board the amount of base money in circulation is exactly equal to the foreign reserves of the Central Bank. Hence there cannot be a "balance of payments crisis."

Given a currency board regime, the private sector can be organized in various ways. We will examine two of them, one resembling autarky and another resembling a banking system. Under plausible conditions, a banking system may be

Pareto superior to autarky. However, it may also be subject to *bank runs*.

3.1. Autarky under a Currency Board

Under autarky each private agent acts in isolation from the others, interacting only with the Central Bank and the rest of the world. We shall assume without loss of generality that the currency board guarantees that the exchange rate will be one peso for each dollar. Then, the typical agent must choose how to divide his period 0 endowment between the long term technology and the world asset, how much of the long term asset to liquidate in period 1, and how much to consume and to devote to peso holdings in each period contingent on his type realization. Formally, each agent maximizes 2.1, given $E_2 = 1$, subject to:

$$k + b \leq e \tag{3.1}$$

$$x \leq b + rk \tag{3.2}$$

$$M \leq b + rl \tag{3.3}$$

$$y \leq M + R(k - l) \tag{3.4}$$

$$l \leq k \tag{3.5}$$

$$k, b, x, y, M, l \geq 0 \tag{3.6}$$

where b denotes investment in the world market, k investment in the long term asset, and l liquidation of the long term asset in period 1 if the agent turns out to be patient. Equation 3.1 is the period 0 constraint on investment. Equation 3.2 says that, if the agent turns out to be impatient, she consumes in period 1 the liquidation value of her investments. On the other hand, 3.3 says that if she turns out to be patient she must decide in period 1 the quantity of pesos to acquire and, correspondingly, how much of the long term asset to liquidate. The value of the remaining long term investment plus her peso holdings in the last period determines her consumption then; this is all reflected in 3.4. Equation 3.5 says that liquidation cannot exceed the investment in the long term (illiquid) technology, and 3.6 contains the obvious nonnegativity constraints.

It should be clear that under autarky the agent faces idiosyncratic uncertainty. This distorts the allocation of investment and results in costly liquidation of the long term asset. This situation can be improved upon if agents can pool their resources in some kind of insurance scheme, as we will see next.

3.2. A Banking System

Now suppose that agents act collectively; their coalition will be called a "bank" (in fact, a commercial bank) for reasons that will be clear shortly. The objective of the bank is to pool the resources of the economy in order to maximize the welfare of its representive member. Ideally this would require assigning consumption and peso holdings to each agent contingent on the realization of her type. In choosing such a contingent allocation, the bank is restricted not only by resource feasibility constraints but also by fact that type realizations are private information. This implies that the bank must find some way of eliciting such information.

Examining the bank's options is greatly simplified applying the Revelation Principle,[2] which implies that attention can be restricted to feasible type contingent allocations that give no agent an incentive to misrepresent her type. To express the last condition correctly, it is necessary to make a more precise assumption what the bank can and cannot observe. The simplest assumption, which will be maintained hereon, is that the commercial bank can observe each agent's transactions with the domestic banking system (the bank itself and the Central Bank),

[2] The Revelation Principle applied to the bank's problem is that the Bayesian Nash equilibria of any game that the depositors may play can be replicated by the truthful equilibria of a game in which each depositor is asked to report her types. See [6] for an excellent introduction to the Revelation Principle.

while consumption and world transactions cannot be monitored.

The above discussion implies that the bank will choose an allocation that maximizes 2.1, with $E_2 = 1$, subject to 3.1 and

$$\lambda x + (1-\lambda)M \leq b + rl \tag{3.7}$$

$$(1-\lambda)y \leq (1-\lambda)M + R(k-l) \tag{3.8}$$

$$\chi(M) + y \geq x \tag{3.9}$$

$$l \leq k \tag{3.10}$$

$$k, b, x, y, M, l \geq 0 \tag{3.11}$$

The above problem will be denoted by P1, and its solution denoted by tildes. Equation 3.7 is the feasibility constraint in period 1. The solution of P1 assigns $\tilde{x}$ units of consumption to each impatient and $\tilde{M}$ pesos to each patient; this is financed by liquidating the short term asset, $\tilde{b}$, and possibly some portion of the long term asset. Note that 3.7 incorporates the assumption that the bank can acquire pesos from the Central Bank at an exchange rate of one. To understand 3.8, it is easiest to assume that the bank requires patient agents to return in period 2 the pesos they were given in period 1. The bank then resells the pesos to the Central Bank, and in addition it liquidates the remainder of the long term investment. The proceeds from these operations are used to finance the period 2 consumption of the patient.

Finally, equations 3.10 and 3.11 are self explanatory. Equation 3.9 is the *incentive compatibility* or truth telling constraint for patient agents. If a patient agent reports her type honestly she will receive $\tilde{M}$ pesos in the first period and consume $\tilde{y}$ in the second, whose value is $g\left[\chi(\tilde{M})+\tilde{y}\right]$. If she lies she will only be given $\tilde{x}$ units of consumption in period 1; given our assumptions, the best she can do then is to exchange them for $\tilde{x}$ dollars, which can be used in period 2 to buy $\tilde{x}$ units of consumption. The value of lying is then $g\left(\tilde{x}\right)$, which 3.9 requires not to be larger than the value of telling the truth.

P1 is a standard maximization problem and its analysis yields useful properties of the solution. It can be shown that $\tilde{l}=0$ –that is, there is no liquidation in period 1 of the long term investment. This should be obvious, since the bank faces no aggregate uncertainty, and liquidating the long term asset in period 1 is costly. Given this fact, it can be shown that the value of P1 is superior to the value of the autarkic solution. This also should be intuitive, since the bank pools resources to prevent the inefficient liquidation of the long term asset; in contrast, the long term asset must be liquidated with positive probability in autarky.

The other features of the solution follow from marginal optimality conditions. The optimal quantity of pesos to give patient agents is determined by

$$\chi'(\tilde{M}) = R - 1 > 0 \tag{3.12}$$

Equation 3.12 says that the marginal utility of giving pesos to the patient must equal the marginal cost of pesos to the bank, which is $R-1$, and implies that $\tilde{M}$ is less than $\overline{m}$. Given this result, the choices of x and y must satisfy the following equation, which can be thought of as a *transformation curve*:

$$R\lambda\tilde{x} + (1-\lambda)\tilde{y} = eR - (R-1)(1-\lambda)\tilde{M} \tag{3.13}$$

The final optimality condition is that the *social indifference curve* be tangent to the above transformation curve:

$$\frac{\lambda g'(\tilde{x})}{(1-\lambda)g'\left[\chi\left(\tilde{M}\right)+\tilde{y}\right]} = \frac{\lambda R}{(1-\lambda)} \tag{3.14}$$

or $g'(\tilde{x}) = Rg'\left[\chi(\tilde{M}) + \tilde{y}\right]$. (Note that, since $R > 1$ and $g(.)$ is concave, 3.14 guarantees that the incentive constraint 3.9 does not bind.)

Equations 3.12, 3.13, and 3.14 completely determine the optimal allocation of consumption and peso holdings by the two types. Next we discuss how this allocation can be implemented in a decentralized fashion.

3.3. Demand deposits and bank runs

The previous subsection identified the solution of P1 as the best allocation that the bank can in principle achieve given the environment, including the existence of a currency board. The next question is how that allocation will be implemented in practice and, more generally, how the banking system may try to accomplish any given objective. To answer such questions, we shall assume that the commercial bank and its depositors establish a particular contract. Also, we shall assume that the Central Bank and the commercial bank agree on procedures about how dollars and pesos will be exchanged and, possibly, a lending-borrowing relationship. The contract between the bank and its depositors and the operating rules of the banking system, including Central Bank policy, is what we call a *regime.* We shall see that each regime induces a game played by the depositors. The analysis of a regime then reduces to the analysis of the equilibria of the induced game.

One natural way in which the bank can implement the solution of P1 is via *demand deposits.* Demand deposits are contracts that stipulate that each agent must turn over her endowment to the bank in period 0. The bank invests $\tilde{b}$ in the world liquid asset and $\tilde{k}$ in the long term technology. In return, the agent is promised some payments from the bank in periods 1 and 2, depending on her reported type.

We shall impose some additional assumptions on the problem. First, we will assume that all payments from the bank to its depositors are denominated and are given *in pesos.* This implies that the agent is entitled to withdraw either $\tilde{x}$ pesos in period 1, or $\tilde{M}$ pesos in period 1 and $\tilde{y} - \tilde{M}$ pesos in period 2, depending on the report of her type. Also, in order to buy consumption in the world market, the agent must go to the Central Bank to exchange pesos into dollars.

The second assumption is that both the bank and the Central Bank must respect *sequential service constraints.* These constraints require, loosely speaking, that both the commercial bank and the Central Bank attend the requests of depositors on a first come-first served basis. The existence of sequential service constraints can be justified by more primitive features of the environment, as suggested by [10].

We will assume that in period 1 depositors arrive to the bank in random order. Upon arrival, each agent reports her type realization and withdraws either $\tilde{x}$ or $\tilde{M}$ pesos, assuming the bank is still open. If the agent withdraws $\tilde{x}$ pesos she must exchange them for dollars in period 1 at the Central Bank; to do this, she "walks" to the Central Bank to join a "line" waiting there. In contrast, if the agent withdraws $\tilde{M}$ pesos she must hold them until period 2.

The commercial bank services requested withdrawals sequentially, as long as it has assets to liquidate, in whose case it liquidates them as needed to obtain dollars, which it exchanges for pesos at the Central Bank. If the bank exhausts its assets, it closes and disappears.

After all depositors have visited the bank, the Central Bank starts selling dollars, at an exchange rate of unity, to the depositors that have withdrawn $\tilde{x}$ pesos. We will assume that the Central Bank closes if it runs out of dollars (this cannot happen under a currency board, but will happen in other regimes).

Finally, if the bank did not close in period 1, in period 2 the bank liquidates all of its remaining investments and sells the resulting dollar proceeds to the Central Bank. It pays $\tilde{y} - \tilde{M}$ pesos plus any profits to agents that reported to be patient in period 1. These agents exchange the pesos thus accumulated for dollars at the Central Bank, which they use to buy consumption.

The regime just described is, in short, a demand deposit system with a currency board. Given this regime, the consumers are engaged in an (anonymous) game. The outcomes of this regime are then given by the *equilibria* of the game. An equilibrium is a description of the strategies of each depositors and aggregate outcomes such that the aggregate outcomes are implied by the depositors' strategies and each depositor strategy is optimal for her given the aggregate outcomes.[3]

The following can now be proven:

Proposition 3.1. *There is an* honest *equilibrium. In this equilibrium, in period 1 impatient depositors retire* $\tilde{x}$ *pesos and patient ones retire* $\tilde{M}$*. The bank does not fail, and pays* $\tilde{y} - \tilde{M}$ *to patient depositors in the second period. The Central Bank ends period 1 with* $(1 - \lambda)\tilde{M}$ *dollars, which it uses to redeem the money supply in period 2.*

Proof. (Sketch) Under the hypothesized behavior it is easy to check that the commercial bank does not fail, and that the Central Bank accumulates $(1 - \lambda)\tilde{M}$ dollars in period 1. Only optimal behavior by depositors needs to be checked. Impatient depositors get $\tilde{x}$ units of consumption if they tell their type; if they lie, they get $\tilde{M}$ pesos in period 1, which they must carry to period 2 when pesos will be of no use for them. Hence it is optimal for the impatient not to lie. Patient types get $\tilde{M}$ pesos in period 1 and $\tilde{y} - \tilde{M}$ more pesos if they behave honestly. Since they can exchange pesos at an exchange rate of one, the value of honesty

[3] This description is intentionally vague. This is because we have assumed a large number of depositors, who have measure zero. Given this, the equilibrium definition must ensure that depositors assume that their impact on aggregate outcomes is negligible. In such case the appropriate equilibrium concept is that of [7].

is $g\left[\chi(\tilde{M})+\tilde{y}\right]$. If they lie, they obtain $\tilde{x}$ pesos in period 1, which they must convert to dollars. Hence they can consume $\tilde{x}$ in period 2, and the value of lying is $g(\tilde{x})$. Since $g(.)$ is increasing, 3.9 ensures that the patient do not lie either. ■

The above Proposition clarifies the role of a banking system in a currency board regime (and, in fact, in most other regimes). A banking system may implement an allocation that improves upon what agents can achieve in isolation.

However, the banking system may attain such improvement only by holding less *internationally* liquid assets than its implicit liabilities. Consequently, the banking system may be subject to a *run*. In particular, suppose that

$$\tilde{x} > \tilde{b} + r\tilde{k} \tag{3.15}$$

In such case, a run emerges in equilibrium:

Proposition 3.2. *If 3.15 holds, there is an equilibrium in which all agents claim to be impatient and the bank fails in period 1. If 3.15 does not hold, there cannot be equilibrium bank runs .*

Proof. First we prove sufficiency. Suppose that every depositor claims to be impatient whatever her type. Then, 3.15 implies that the bank fails in period 1 after liquidating all investments. The Central Bank does not fail, but closes after selling $\tilde{b}+r\tilde{k}$ dollars.

Impatient depositors are obviously behaving optimally. To check that it is optimal for each patient agent to claim to be impatient if everybody else does the same, it is enough to consider the options open to those lucky patient agents that arrive to the bank early enough to have a choice of withdrawing $\tilde{x}$ or $\tilde{M}$ in the first period. In the posited equilibrium, the bank closes in period 1, and hence no further payments will be given in period 2. Also, the Central Bank will hold no dollars in period 2, and hence the real value of pesos then will be zero. It follows that a withdrawal of $\tilde{M}$ is worthless for both types, confirming the optimality of claiming impatience.

To prove necessity, suppose that a number $\vec{\lambda}$ of depositors claim to be impatient in period 1. Clearly it is enough to restrict attention to the case in which $\vec{\lambda} > \lambda$. We shall argue by contradiction and show that if 3.15 does not hold the bank is able to pay at least $\tilde{y}$ to honest patient types in period 2, which implies that it cannot be optimal for the patient to lie.

Let $\vec{l}$ denote the amount of liquidation of the long term asset; it is defined by $\vec{\lambda}\tilde{x}+\left(1-\vec{\lambda}\right)\tilde{M}=\tilde{b}+r\vec{l}$. Using this and $\lambda\tilde{x}+(1-\lambda)\tilde{M}=\tilde{b}$, one concludes that $r\vec{l}=(\vec{\lambda}-\lambda)(\tilde{x}-\tilde{M})$. Now, it is easy to check that the bank will be able to pay $\tilde{y}$ to the $\left(1-\vec{\lambda}\right)$ reportedly patient depositors if $\tilde{M}+R(\tilde{k}-\vec{l})/\left(1-\vec{\lambda}\right)\geq\tilde{y}$.

This inequality will hold, from the definition of $\tilde{y}$ and the last equation for $\vec{l}$, if $r\tilde{k} \geq (1-\lambda)(\tilde{x}-\tilde{M})$. But the latter is equivalent to the failure of 3.15 . The proof is complete. ■

The proposition states that 3.15 is not only sufficient but also necessary for the existence of a bank run. Now, 3.15 is a condition on the solution of P1. One may wish to describe conditions on the fundamental parameters of the economy are associated with the possibility of bank runs. Such conditions can in fact be characterized as follows.

A little manipulation shows that 3.15 is equivalent to:

$$\tilde{x} > \frac{er + (1-r)(1-\lambda)\tilde{M}}{1-(1-r)\lambda} \equiv \tilde{x}^{run} \tag{3.16}$$

Using the above fact, it can be shown that a necessary and sufficient condition for the existence of a bank run is:

Proposition 3.3. *Let* $\tilde{y}^{run} = (1-R)\tilde{M} + R(e - \lambda\tilde{x}^{run})/(1-\lambda)$. *Then, a bank run is possible if and only if*

$$g'(\tilde{x}^{run}) > Rg'(\chi(\tilde{M}) + \tilde{y}^{run}) \tag{3.17}$$

Proof. The conditions above ensure that 3.16 holds. ■

We conclude that:

Corollary 3.4. *Equilibrium bank runs may occur in a currency board regime.*

The proof is easy and left to the reader.

3.4. Summary

Under a currency board, there can be no attacks on Central Bank reserves. A banking system operating with demand deposits can emerge and improve upon autarky. However, this arrangement may be prone to runs. In fact, since in a currency board all monetary holdings must be backed by "real" reserves (in this case, holdings of the liquid asset), this case is quite analogous to the original Diamond-Dybvig setup in [2]. Both honest and run equilibria were possible in that context, and the same is true here.

4. Fixed Exchange Rates and Central Bank Credit

Under a currency board, each peso in circulation is backed by a dollar in the Central Bank's vault. This implies that, in period 1 pesos are created at a positive cost, which is equal to the opportunity cost of the Central Bank's reserve holdings.

However, printing pesos is free for the Central Bank. This suggests that the currency board is inefficient and that a better allocation may be attainable.

This section starts by showing that the above conjecture is true, and that a regime in which the Central Bank gives positive credit to the commercial bank may attain a better allocation than under a currency board. This regime can be decentralized, as with the currency board, with a demand deposit system.

As with a currency board, bank runs are possible. In addition, now *balance of payments crises* become possible. Whether bank runs or balance of payments crises are observed depends on the specific arrangements between the Central Bank and commercial banks. However, both can be caused by the international illiquidity of the banking system as a whole.

Finally, this section shows that the international liquidity of the banking system is *less* under a fixed exchange rate system with Central Bank credit than under a currency board. As a result, conventional fixed exchange rate systems are more prone to runs than currency board systems.

4.1. The Optimal Supply of Central Bank Credit

To analyze a fixed exchange rate regime with Central Bank credit it is helpful to start by studying a *social optimum* problem. Consider what the bank and the Central Bank can ideally achieve if they acted together, assuming that the exchange rate will be fixed at one. The Revelation Principle applies to this social problem, and the options open are given by all feasible incentive compatible allocations. Hence the social optimum must maximize 2.1 (with $E_2 = 1$) subject to 3.1 and

$$\lambda x \leq b \tag{4.1}$$

$$(1 - \lambda) y \leq Rk \tag{4.2}$$

$$x, y, M, k, b \geq 0 \tag{4.3}$$

and the incentive compatibility condition 3.9. We shall call this problem P2 and its solutions will be identified with a bar.

In setting up P2, we have assumed that pesos can be provided at no cost to satisfy the peso demands of the patient types. We have also assumed that the exchange rate in period 2 is one. We shall see that both assumptions turn out to be justified, in the sense that the resulting allocation can be implemented.

The solution of P2 can be intuitively understood as follows. First, since providing pesos is effectively *gratis*, the optimal allocation ensures that the real quantity of pesos is the satiation level:

$$\chi'(\bar{M}) = \chi'(\overline{m}) = 0 \tag{4.4}$$

Second, the transformation curve is now given by

$$R\lambda\bar{x} + (1-\lambda)\bar{y} = eR \tag{4.5}$$

Note the difference with the transformation curve under a currency board, given by 3.13: in a fixed exchange rate system with optimal provision of pesos, the economy "saves" $(R-1)(1-\lambda)\tilde{M}$, which is the opportunity cost of providing pesos for the patient types to hold between periods 1 and 2.

The final optimality condition is that

$$g'(\bar{x}) = Rg'\left[\chi(\bar{M}) + \bar{y}\right] \tag{4.6}$$

which says that the social marginal rate of substitution must equal the slope of the transformation curve. The three conditions 4.4, 4.5, and 4.6 characterize the social optimum.

Now we shall prove that the optimal allocation can be decentralized in a fixed exchange rate system with Central Bank credit. Suppose now that the commercial bank acts independently of the Central Bank and maximizes the welfare of the representative depositor. Assume also that pesos can be exchanged for dollars at a unity exchange rate. Finally, suppose that the Central Bank offers to lend pesos to the commercial bank in period 1, to be repaid without interest in period 2. The Revelation Principle applies once more and implies that the bank's problem is to maximize 2.1, with $E_2 = 1$, subject to 3.1, and

$$\lambda x + (1-\lambda)M \leq b + h \tag{4.7}$$

$$(1-\lambda)y \leq (1-\lambda)M + Rk - h \tag{4.8}$$

$$x, y, M, k, b, h \geq 0 \tag{4.9}$$

and the incentive compatibility constraint 3.9, where h denotes the bank's borrowing from the Central Bank.

As stated, there is a difficulty with this argument, in that the commercial bank will choose to invest all of the economy's endowment in the long term asset and borrow from the Central Bank all it needs to service withdrawals in period 1. This is easily corrected, however, if we assume that the commercial bank must respect a kind of *reserve requirement*:

$$\lambda x \leq b \tag{4.10}$$

With the reserve requirement, the bank's problem is to maximize 2.1 subject to 3.1, 4.7, 4.8, 4.9, 3.9, and 4.10. The solution of this problem can be easily seen to coincide with the social optimum described above.

Some features of the social optimum are worth noting. It can be checked that the solution results in a Pareto improvement over the currency board allocation derived from P1. This implies, in particular, that $\bar{x} > \tilde{x}$: impatient agents are entitled to more consumption in period 1 than in a currency board.[4] Lastly, the optimal supply of credit from the Central Bank to the commercial banks is $\bar{h} = (1 - \lambda)\bar{M} = (1 - \lambda)\overline{m}$. That is, the amount the Central Bank lends to the bank should be just enough to cover the demand for pesos of patient agents in period 1.

[4] Whether $\overline{y} > \widetilde{y}$ as well depends on the shape of the $\chi(.)$ function. Overall incomehas risen (the transformation curve has moved away from the origin), which would tend to increase $\overline{y}$ relative to $\widetilde{y}$. But $\chi\left(\overline{M}\right) > \chi\left(\widetilde{M}\right)$, which, ceteris paribus, would tend to reduce $\overline{y}$ relative to $\widetilde{y}$.

Summarizing, in a fixed exchange rate system with a reserve requirement and optimal Central Bank credit the commercial bank will try to implement a socially optimal allocation. Next we shall study how this implementation may occur via a demand deposit system.

4.2. Demand Deposits and Bank Runs

As with the currency board, a demand deposit in a fixed exchange rate regime is a contract that requires depositors to surrender their period 0 endowments to the bank. The bank invests $\bar{b}$ in the short term asset and $\bar{k}$ in the long term technology. In return, depositors get a promise of withdrawing, at their own discretion, either $\bar{x}$ pesos in period 1 or $\bar{M}$ pesos in period 1 and $\bar{y}$ pesos in period 2. Note that we maintain the assumption that deposits are denominated in pesos.

We shall also maintain our assumptions on sequential service. The main difference with the currency board case is that now the existence of Central Bank credit makes it necessary to describe the credit arrangements between the Central Bank and the commercial bank. We shall examine some alternatives in turn, and argue that the main difference they make is whether there may be bank runs or balance of payments crises.

4.3. A Regime with Limited Domestic Credit and no Balance of Payments Crises

Suppose that the Central Bank offers to lend $\bar{h}$ dollars to the commercial bank in period 1 at no interest, but it restricts the commercial bank to use this line of credit only for withdrawals of the reportedly patient types. The reason for this rule is that the Central Bank wants to defend its reserves. An agent who reports that she is impatient will try to sell the pesos she withdraws within the period to the Central Bank. Hence, requiring the commercial bank to use the credit line only for patient withdrawals ensures that the Central Bank does not run out of dollars and that a balance of payments crisis is therefore not possible.

Given this kind of arrangement, in period 1 the commercial bank services withdrawals of size $\bar{M}$ by borrowing from the Central Bank, and withdrawals of size $\bar{x}$ by liquidating $\bar{b}$ or $\bar{k}$. The commercial bank is assumed to close if it liquidates all of its assets. In period 2, if the commercial bank did not close, it liquidates the rest of its assets. It uses the proceeds to repay the loan from the Central Bank; finally, it pays $\bar{y} - \bar{M}$ plus any profits to each reportedly patient depositor.

As in the currency board case, this *modus operandi* induces an anonymous game played by the depositors. Now it can be shown that:

Proposition 4.1. *There is an* honest *equilibrium whose outcome is the socially optimal allocation. In this equilibrium, neither the bank nor the Central Bank close.*

Proof. If all agents report their types honestly, in period 1 the bank services withdrawals of $\lambda \bar{x}$ by liquidating the world asset and $(1-\lambda)\bar{M}$ by borrowing from the Central Bank. This is feasible for the bank, given 4.10 and $\bar{h} = (1-\lambda)\bar{M}$. Now, 4.8 ensures that the bank can repay the Central Bank's loan and service deposits in period 2. Hence the bank does not close.

In period 1, the Central Bank receives $\bar{b}$ dollars from the bank, which are (just) enough to attend the demands of the impatient types. In period 2, the Central Bank receives $R\bar{k}$ dollars from the bank, which is enough to satisfy the demand for dollars of the patient types.

The rest of the proof is obvious and left to the reader. ■

Hence this system may implement the optimal allocation. However,

Proposition 4.2. *Assume that*

$$\bar{x} > \bar{b} + r\bar{k} \tag{4.11}$$

Then there is an equilibrium in which all depositors claim to be impatient and the bank fails in period 1. Conversely, if 4.11 does not hold there cannot be an equilibrium with a bank run.

Proof. First we prove the sufficiency of 4.11. Suppose all consumers attempt to withdraw $\bar{x}$. Then, given the assumption that the bank cannot use the Central Bank credit line to service impatient withdrawals, 4.11 implies that the bank fails in period 1. Under the same conditions, there is no Central Bank lending in period 1.

In period 1, the Central Bank receives $\bar{b} + r\bar{k}$ dollars from the bank. This is just enough to sell dollars to the depositors that hold pesos. Hence the Central Bank does not close in period 1.

Impatient depositors are obviously behaving optimally. To check that it is optimal for each patient agent to claim to be impatient if everybody else does the same, one need only check the optimal actions of a patient depositor that arrives to the bank early enough to have an option to withdraw $\bar{x}$ or $\bar{M}$. If she withdraws $\bar{x}$, he can obtain $\bar{x}$ dollars from the Central Bank, which he can use in period 2 to buy $\bar{x}$ units of consumption. If she withdraws $\bar{M}$ pesos he must hold them until period 2. But in period 2 pesos will have no value because the Central Bank will have no dollars with which to redeem pesos. Hence the patient agents will lie about their type.

The proof of the necessity of 4.11 is analogous to that of the necessity of 3.15 in a currency board. The details are left to the reader. ■

The condition 4.11 is obviously analogous to 3.15. Now, it turns out that:

Proposition 4.3. *If 3.15 is satisfied, so is 4.11. The converse is not true.*

Proof. Using 3.1, 4.11 can be rewritten as $\bar{x} > er/(1-(1-r)\lambda)$. Using the definition of x^{run} given in 3.16, this is equivalent to $\bar{x} + (1-r)(1-\lambda)\tilde{M}/(1-\lambda(1-r)) > x^{run}$. Since $\bar{x} > \tilde{x}$ the proposition follows immediately. ■

Hence, if runs are possible in a currency board they are also possible in a fixed exchanger rate regime, but the converser is not true. There is a sense, therefore, in which a fixed exchange rate regime is more conducive to a bank run than a currency board. On the other hand, the fixed exchange rate regime can implement an allocation (if the "good" equilibrium obtains) that improves upon the best currency boards can attain.

The question of which regime is to be preferred is subtle. If parameter values are such that a run equilibrium can occur under a currency board (that is, if 3.15 is satisfied), then shifting to a fixed exchange rate with domestic credit may improve the features of the good (honest) equilibrium without changing the number of equilibria that are feasible. In this case a currency board is clearly inferior.

If, on the other hand, only one equilibrium (the honest one) exists under a currency board, moving to a fixed exchange rate with limited domestic credit may (again, for some parameter values) open up the possibility of a new, bad equilibrium. In this case, which regime is preferred depends on an assessment of the likelihood that the economy would land in the bad equilibrium if one is feasible. Unfortunately, our analysis (as well as existing economic theory) is not informative on how an equilibrium will be selected when there are many of them.

4.4. Fixed Exchange Rates With a Lender of Last Resort

In the preceding subsection the Central Bank did not act as a *lender of last resort* in the sense that, if there was a bank run, the Central Bank did not extend credit to the commercial bank. We shall now examine the consequences of dropping that assumption. However, we shall still assume that the Central Bank tries to maintain a fixed exchange rate regime in the sense that it will defend the peso until it is impossible for the economy to obtain more dollars.

Such a regime is best formalized as follows. In addition to the line of credit of $\bar{h}$, the Central Bank offers to lend an unlimited quantity of pesos to the commercial bank in case more than λ customers claim to be impatient. If such emergency credit is used, though, we assume that the Central Bank obtains control over the long term asset in period 1, and liquidates the asset as needed to sell dollars to agents claiming impatience.

In period 1, the commercial bank pays reportedly patient types by borrowing from the "normal" line of credit, and it pays withdrawals of the impatient by liquidating the world asset first, and then drawing on the emergency credit. Because emergency credit is unlimited, the bank does not close. The Central Bank sells dollars at an unity exchange rate, using first the dollars obtained from the bank, and then liquidating the long term asset. If the long term asset is completely liquidated in period 1, we say that there is a "balance of payments crisis."

Under these assumptions one can show that if all depositors act honestly the social optimum is implemented:

Proposition 4.4. *The fixed exchange rate system with a lender of last resort has an* honest *equilibrium.*

The proof is similar to previous ones and left to the reader.

Now, the same conditions under which there was a *bank run* in the previous regime imply that there is a *balance of payments crisis* with a lender of last resort:

Proposition 4.5. *If 4.11 holds, there is an equilibrium in which all depositors* *claim to be impatient and the Central Bank is unable to service all dollar demands in period 1.*

Proof. If all depositors claim to be impatient, they all arrive to the Central Bank with $\bar{x}$ pesos in period 1. The Central Bank obtains $\bar{b}$ dollars from the bank, and $r\bar{k}$ from liquidating the long term asset. Given 4.11, the Central Bank runs out of dollars and closes.

Clearly, the bank does not close in period 1. Since all deposits are paid in the first period, the bank has no further liabilities *vis a vis* depositors in period 2.[5]

It remains to check that each depositor is acting optimally. This is easy and left to the reader. ■

One conclusion from the analysis is that, with fixed exchange rates, whether the effects of international illiquidity lead to bank runs or balance of payments crises depend on the credit arrangements between the Central Bank and the commercial bank. However, the root source of financial fragility is the same. The socially optimal allocation implies that investment in the world liquid asset is less than the implicit short term dollar liabilities of the banking system. This has been

[5] The bank has no resources to repay its debt to the Central Bank in period 2 –but this is not important, since by then the Central Bank has become bankrupt. Alternatively, one can assume that the debt is cancelled when the Central Bank fails to return the long term asset to the commercial bank.

identified before as a potential problem under fixed rates. However, our analysis contrasts with previous ones in showing that international illiquidity has benefits as well as costs: when the system is internationally illiquid as in the example of this section, the good equilibrium is associated with a real allocation which is preferred to the allocation that would prevail under a currency board.

The analysis of this subsection assumes that the Central Bank fights a devaluation *like a dog*. This is implied, in particular, in the assumption that the Central Bank stops selling dollars in the first period only after the long term asset has been liquidated. One may ask what would happen with a less committed Central Bank. For example, the Central Bank procedure may call for stopping the sale of dollars before the long term asset has been exhausted. Such a rule, in fact, would amount to a rule for when to devalue the peso and, hence, best delayed until flexible exchange rates are considered. Before that, we shall study some policy measures that may be used to prevent runs under fixed rates.

5. Narrow Banking versus Large International Reserves

Given the possibility of bank runs and balance of payments crises with fixed rates, some observers have suggested that commercial banks should be required to hold enough liquidity to cover all of their demandable debt. In this section we shall show that this position, which has been termed *narrow banking,* can in fact eliminate equilibrium runs, but at the cost of implementing an inefficient allocation.

We shall also study a regime in which the government imposes a tax-transfer scheme to finance an "emergency" dollar fund to be used to fight runs. As discussed in the introduction, this policy has been advocated in the wake of the Argentine bank run of 1995. It turns out such a policy of *large international reserves* also eliminates runs, but is also inefficient. In fact, we show that narrow banking dominates a policy of large international reserves.

5.1. Narrow Banking

To analyze the implications of narrow banking, suppose that the Central Bank fixes the exchange rate at one. Also, commercial banks are allowed to borrow pesos in period 1 to be repaid in period 2 at no interest. Finally, in order to ensure the "stability" of the financial system, commercial banks are constrained by

$$x \leq b \tag{5.1}$$

which ensures that the bank's total amount of demandable debt is completely backed by the world liquid asset. This restriction is the crucial difference between the fixed exchange rate system with Central Bank credit and a narrow banking system.

Given narrow banking, the commercial bank will choose an allocation to maximize 2.1 subject to 3.1, 4.7, 4.9, 3.9, the reserve requirement 5.1, and

$$(1-\lambda)\, y \leq (1-\lambda)M + Rk - h + [b - \lambda x - (1-\lambda)M] \tag{5.2}$$

which is the same as 4.8 except for the term in square brackets, which allows for the possibility that the bank may choose not to spend in period 1 all of the dollars obtained from liquidating the short term asset.

Solving the bank's problem is a standard exercise. Here we will only discuss the relevant aspects of the solution, which will be identified with asterisks.

First, under narrow banking the bank provides the satiation level of pesos to patient agents, that is, $M^* = \overline{m}$. This is as expected, because in period 1 the marginal cost of pesos to the bank is zero. However, it can also be shown that $h^* = 0$, that is, that Central Bank credit is not necessary. These facts do not contradict each other because it turns out that 4.7 (the first period resource constraint) is not binding: at the optimum, the bank's holdings of the world liquid asset are more than sufficient to finance *all* period 1 withdrawals. In other words, in period 1 the bank has "excess liquidity," which is of course a consequence of the imposition of narrow banking.

On the other hand, the reserve requirement 5.1 is strictly binding at the solution. Now one can use the bank's feasibility constraints to show that the optimum values x^* and y^* must satisfy

$$[R - (1-\lambda)]\, x^* + (1-\lambda) y^* = eR \tag{5.3}$$

which should be thought of as the "transformation curve" in a narrow banking regime. Comparing 5.3 against 4.5 makes clear why narrow banking is costly: the "price" of x under narrow banking is $R - (1-\lambda)$, which is larger than λR, the price of x when narrow banking is not imposed. Intuitively, this occurs because, under narrow banking, an increase of x of, say, dx, requires increasing b by more than λdx, the difference being an increase in required reserves.

The last optimality condition is that the social marginal rate of substitution must be equal to the marginal rate of transformation, or

$$\frac{\lambda g'(x^*)}{(1-\lambda)g'[\chi(\overline{m})+y^*]}=\frac{R-(1-\lambda)}{1-\lambda} \tag{5.4}$$

Comparisons with the optimal allocation under fixed exchange rates are now straightforward. It can be shown that $x^* < \bar{x}$, while y^* may be greater or less than $\bar{y}$, depending on the relative strength of familiar income and substitution effects associated with the increase in the "price" of x.

The narrow banking solution can be implemented as a demand deposit system, as described before. The reserve requirement 5.1 would then imply that neither bank runs nor balance of payments crises can occur. However, the resulting allocation is not socially optimal.[6] In other words, completely avoiding bank runs is costly: there is no free lunch here.

5.2. Large Reserve Requirements

Instead of forcing banks to hold a large quantity of the world liquid asset, suppose that the Central Bank builds a "war chest" of dollars to be used in case of a bank run. To do this, it imposes a lump sum tax of T dollars on banks in period 0. If

[6] This follows from the fact that the bank is now solving the same problem as in the fixed rates case studied above, but with the tighter constraint $x \leq b$ instead of $x\lambda \leq b$.

there is no run, it returns the T dollars to the banks in period 2 as a lump sum transfer. We can think of this as a situation in which the government uses "fiscal" as opposed to "monetary" policy to defend the integrity of the banking system.

Consider first the commercial bank's problem. Given the tax-transfer policy of the Central Bank, the commercial bank must choose an allocation to maximize 2.1 subject to

$$k + b \leq e - T, \tag{5.5}$$

the period 1 constraint 4.7,

$$(1 - \lambda)\, y \leq Rk + T - h + (1 - \lambda)M, \tag{5.6}$$

the reserve requirement 4.10, the nonnegativity constraints, and incentive compatibility.

The relevant aspects of the solution, which will be identified by hats, are the following. First, the bank provides the satiation level of pesos to patient agents in period 1 –that is, $\widehat{M} = \overline{m}$. This should be obvious since the bank's cost of pesos between periods 1 and 2 is zero.

Second, the optimal choices $\widehat{x}$ and $\widehat{y}$ must satisfy

$$R\lambda\widehat{x} + (1-\lambda)\widehat{y} = eR - (R-1)T \tag{5.7}$$

which is the bank's transformation curve in this case. The last term on the RHS captures the cost for the bank of the tax-transfer scheme, but the prices of x and y that the bank perceives are the same as with a fixed exchange rate with no taxes.

The last optimality condition now is that the slope of the bank's objective function be equated to the slope of its perceived transformation curve:

$$\frac{\lambda g'(\widehat{x})}{(1-\lambda)\, g'[\chi(\overline{m}) + \widehat{y}]} = \frac{R\lambda}{1-\lambda} \tag{5.8}$$

The resulting allocation clearly depends on the size of the "war chest" T. To enable comparison with the narrow banking case, consider the case in which

$$b + T = \widehat{x} \tag{5.9}$$

so that the *economy's* liquid assets are sufficient to back the total amount of demandable debt.

Clearly, if 5.9 holds, the resulting allocation can be decentralized in a demand deposit system that cannot exhibit equilibrium runs. However, the outcome is not the same as with narrow banking. To see this, use 5.9 in 5.7 to obtain

$$[R - (1 - \lambda)]\widehat{x} + (1 - \lambda)\widehat{y} = eR \tag{5.10}$$

which is a *social* transformation curve in this case. Comparing 5.3 and 5.10 implies that the economy faces the same set of options between x and y as with narrow banking. However, 5.8 must also hold. This implies that, in the system with high international reserves, the social marginal rate of substitution between x and y is *not* equated to the social marginal rate of transformation at $(\widehat{x}, \widehat{y})$.

The intuition is clear: because taxes and transfers are lump sum, the bank does not internalize the implicit cost of building the war chest T. This cost exists because the Central Bank keeps all of T in a liquid form, instead of investing T optimally. The divergence between the social and private returns to investment creates a distortion that does not exist under narrow banking.

In short, using “fiscal” policy to ensure that no bank runs occur is inferior to using “monetary” instruments such as reserve requirements.

6. Flexible Exchange Rates

Up to this point we have assumed that the Central Bank has a policy of defending the exchange rate until it is no longer feasible. In this section we shall drop such assumption and analyze a flexible rate system. Our main result is that flexible rates result in the implementation of the social optimum. Moreover, the possibility of a devaluation implies that there are no equilibrium runs.

6.1. Flexible Rates With a Lender of Last Resort

We shall focus on the question of how the social optimum –that is, the solution to P2– can be implemented by a demand deposit system. To examine this issue, we shall retain all of the assumptions of the case of fixed rates with a lender of last resort, except that the Central Bank is no longer committed to selling dollars to depositors at a fixed exchange rate of one for as long as this is possible. Instead, we will assume that the Central Bank allows exchange rates to change according to financial events.

Consider the following mechanism. The commercial bank and depositors agree on the same contract as under fixed rates. When period 1 begins, depositors arrive to the commercial bank in sequence, and each withdraws either $\bar{x}$ or $\bar{M}$ pesos. The commercial bank services withdrawals of $\bar{M}$ pesos by borrowing from the

Central Bank; withdrawals of $\bar{x}$ pesos are financed by first liquidating the world liquid asset and selling the dollar proceeds to the Central Bank at an exchange rate of one, and then by borrowing from the Central Bank. After all depositors have visited the commercial bank, the Central Bank sells dollars to depositors at an exchange rate equal to $E_1 = Max\{(\lambda^r/\lambda), 1\}$ where λ^r denotes the fraction of agents reporting impatience. Hence, if $\lambda^r > \lambda$ there will be a devaluation.

This devaluation rule can be interpreted in one of two equivalent ways. The first is that the Central Bank, after observing the actual pattern of withdrawals at the commercial bank, announces a devaluation. This rule is consistent with the existence of a sequential service constraints at the Central Bank, since it will be seen that the devaluation is such that the Central Bank will not run out of dollars. A second interpretation, perhaps more appealing, is that the Central Bank does not "fix" the exchange rate, but there is no sequential service at the Central Bank. Instead, the exchange rate is determined by an auction. In the auction, the Central Bank offers the dollars obtained from the commercial bank, and each of the $(1 - \lambda^r)$ depositors that reported impatience offers the $\bar{x}$ pesos withdrawn from the commercial bank.

The devaluation rule implies that each agent that reportedly impatient agents receives $\bar{x}/E_1$ dollars in the first period, which can be used for purchasing consumption in the world market.

In period 2 the bank liquidates the long term asset and sells the resulting $R\bar{k}$ dollars to the Central Bank at an exchange rate of Q pesos per dollar. Here $Q = 1 + [(\lambda^r - \lambda)\bar{x}]/R\bar{k}$ and, therefore, it may depend on what happened in period 1; the reason for this devaluation rule will be discussed shortly. The commercial bank then repays its debt to the Central Bank and distributes its remaining pesos to the $(1 - \lambda^r)$ agents that reported patience in period 1. Finally, these agents use their pesos to purchase dollars from the Central Bank at an exchange rate E_2 determined in an auction.

Given this procedure, it can be proven that the optimal allocation is an equilibrium:

Proposition 6.1. *There is an* honest *equilibrium whose outcome is the solution to P2. In this equilibrium the exchange rate is one at all times.*

Proof. Suppose all agents report their types honestly. Then $\lambda^r = \lambda$, and hence $E_1 = Q = 1$. It can be shown that in period 2 the commercial bank distributes exactly $R\bar{k}$ pesos to the patient depositors. Since in period 2 the Central Bank sells the $R\bar{k}$ dollars obtained from the bank, the exchange rate at which those dollars are sold must also be one.

The rest of the proof follows previous arguments and is left to the reader. ■

The Proposition makes clear that this regime may implement the social optimum. In addition, if this equilibrium prevails, the outcome is the same as in *honest* equilibria with fixed rates.

In spite of the equivalence of their honest outcomes, flexible rates and fixed rates are rather different: under the proposed flexible rate regime, the possibility of a peso devaluation eliminates the incentives for any runs:

Proposition 6.2. *Runs cannot occur in equilibrium.*

Proof. Clearly, if a run is to occur, λ^r must be greater than λ. We will show that this implies that lying is not optimal for patient depositors.

Given the rules in this regime, neither the bank nor the Central Bank fails in any period. Now, Q has been defined so that E_2 is exactly one. To see this, note that in period 2 the debt of the commercial bank to the Central Bank is $(\lambda^r - \lambda)\bar{x} + (1 - \lambda^r)\bar{M}$. Hence the commercial bank distributes $QR\bar{k} - (\lambda^r - \lambda)\bar{x} - (1 - \lambda^r)\bar{M}$ to $(1 - \lambda^r)$ depositors, each of which arrives to period 2 with $\bar{M}$ pesos. As a consequence, depositors go to the Central Bank with a total of $QR\bar{k} - (\lambda^r - \lambda)\bar{x}$ pesos to buy $R\bar{k}$ dollars. The definition of Q ensures that these two quantities are equal, and hence $E_2 = 1$.

If a patient depositor lies, she gets $\bar{x}/E_1 = \lambda\bar{x}/\lambda^r < \bar{x}$ units of consumption. If she tells the truth, she gets $\bar{M}$ pesos in period 1, whose real value is $\overline{m}$ since $E_2 = 1$. In addition, in period 2 her consumption will be at least $\bar{y}$, as implies by the following argument. Since she will receive $[QR\bar{k}-(\lambda^r-\lambda)\bar{x}-(1-\lambda^r)\bar{M}]/(1-\lambda^r)$ from the bank, her consumption will equal to $[QR\bar{k} - (\lambda^r - \lambda)\bar{x}]/(1 - \lambda^r)$ pesos. This is equal to $R\bar{k}/(1 - \lambda^r)$, given $E_2 = 1$ and the definition of Q. Since $\lambda^r > \lambda$, $R\bar{k}/(1-\lambda^r) > R\bar{k}/(1-\lambda) = \bar{y}$. Since $\lambda\bar{x}/\lambda^r < \bar{x} < \chi(\bar{M})+\bar{y} < \chi(\bar{M})+R\bar{k}/(1-\lambda^r)$, a patient agent has no incentive to lie about her type. The proof is complete. ■

The intuition for the success of the flexible exchange rate regime in preventing runs is that the *devaluation-lending* policy of the Central Bank guarantees that patient agents will collect their promised consumption and enjoy the satiation level of money if they behave honestly. This guarantee is feasible, in particular, because in a run the nominal claims of depositors reporting impatience in the first period are deflated by a devaluation, which makes it unnecessary to liquidate the long term asset. Hence, the possibility of devaluation acts as a coordination device that eliminates runs and renders actual devaluation not an equilibrium phenomenon.

6.2. Discussion

From the analysis it should be apparent that, to be successful, a flexible rate system must be coupled with an appropriate Central Bank policy. This policy includes acting as a lender of last resort and selecting exchange rates appropriately.

As in the fixed rate case, the Central Bank acts as a lender of last resort since it lends pesos to the commercial bank in a run. If that is not the case, runs may still occur. For example, if the Central Bank only allows the bank to borrow $(1-\lambda)\bar{M}$ in the first period to satisfy the withdrawals of honest patient agents, there is an equilibrium in which the commercial bank fails. (We leave the details to the reader.) This result is of some interest, because it means that flexible rates and Central Bank lending are not sufficient to prevent bank runs: the lending policy of the Central needs to be carefully designed.

There is some latitude in designing the devaluation policy to ensure the prevention of runs. There are constraints, though. One is that the devaluation policy should deliver to patient agents (along an out of equilibrium trajectory) a level of real peso holdings close to the satiation level.[7] This is necessary to ensure that patient agents do not profit from participating in a run. The devaluation

[7] This is not an issue in the devaluation policy that we consider, for the price level (and the exchange rate) is always one in period 2.

policy described in the previous subsection is one of the many possibilities that accomplish that goal.[8]

7. Dollarizing the Banking System

Bank accounts denominated in foreign currency (e.g. dollars) are a common —and controversial– feature of many countries' financial systems. Some observers view them as an irreplaceable mechanism to make deposits attractive in countries with long histories of currency instability. Others, such as Sachs [8], stress that Central Banks cannot print dollars, and hence cannot serve as a lender of last resort to back such deposits; in this view, then, the dollarization of deposits contributes to the potential instability of the banking system. The framework of this paper is perfectly suited to analyze such conflicting claims.

It is straightforward to see that the analysis of a currency board regime in Section 3 is unchanged if deposits are denominated in dollars instead of pesos. The only mechanical difference is that the commercial bank, instead of selling the proceeds of both kinds of investment at the Central Bank and using the resulting

[8] Here is an alternative devaluation policy, assuming that $\bar{y} \geq \bar{x}$. In period 2, the Central Bank sells pesos to the bank at an exchange rate of one, and later places only $R\bar{k} - (\lambda^r - \lambda)\bar{x}$ dollars for sale to reportedly patient depositors. Since the latter will arrive to the auction with $R\bar{k} - (\lambda^r - \lambda)\bar{x}$ pesos, p_2 will be one. It can also be shown that, if $\bar{y} \geq \bar{x}$, each patient depositor will consume at least $\bar{y}$. Hence patient types will not lie.

pesos to pay off depositors, meets withdrawals directly with dollars. Hence, the solution to problem P1 is the same as before, and so are the possible real allocations: the honest equilibrium remains feasible, and so does the run equilibrium if parameters are such that the system is sufficiently illiquid internationally. We therefore have:

Proposition 7.1. *In a currency board system in which deposits are denominated in foreign currency, there is an honest equilibrium, and there is a bank run equilibrium if and only if condition 3.15 is satisfied.*

Proof. Same as in Propositions 3.1 and 3.2. ■

Consider now what happens if the Central Bank extends credit to the commercial bank. If such credit is limited as in sections 4.1 to 4.3, the analysis also remains essentially unchanged. If deposits are in dollars and pesos are only printed to satisfy money demand by patient depositors, the mechanics of dollar bank transactions are the same as in the currency board case just analyzed. In addition, the solution to P2 is unchanged, and so are the possible equilibrium allocations: the honest equilibrium is always feasible, and equilibrium bank runs can occur if the system is sufficiently illiquid internationally. We therefore have:

Proposition 7.2. *In a fixed exchange rate system with limited domestic credit,* *if deposits are denominated in foreign currency there is an honest equilibrium, and there is a bank run equilibrium if and only if condition 4.11 is satisfied.*

Proof. Same as in Propositions 4.1 and 4.2. ■

We conclude that:

Corollary 7.3. *In a currency board and in a fixed exchange rate system with limited domestic credit, the conditions under which honest and run equilibria occur are the same regardless of the currency in which deposits are denominated.*

The difference that dollar deposits make becomes clear when we consider the Central Bank's potential role as lender of last resort. In the event of a bank run, the commercial bank will liquidate b and then k, but at no point will any of the resulting dollars go through the Central Bank. Hence, the monetary authority has no dollars in its vault with which to serve as lender of last resort. In addition, it cannot print pesos and sell them to the public in exchange for dollars, for in the event of a run all depositors declare themselves to be impatient, and impatient agents have zero demand for pesos. We conclude that there is nothing the Central Bank to stop the run, and such runs can occur in equilibrium if the relevant conditions are satisfied.

Notice, furthermore, that bank runs occur regardless of the exchange rate regime that is in place. This is because it is not flexible exchage rates *per se* that were able to rule out runs in Section 6 above, but the coupling of exchange rate flexibility with a lender of last resort. Since such a lender cannot exist if deposits are dollarized, the beneficial effects of floating vanish. This result gives credence to the concerns that dollarization of the banking system's liabilities can be destabilizing.

8. Summary and Conclusions

This paper provides a detailed and formal account of the possible interactions between bank fragility and the exchange rate and monetary regimes. In a world in which banks play a well defined microeconomic role, we show that different such regimes induce different real consumption allocations, and can also limit financial fragility to widely varying degrees. In particular, we find that a flexible exchange rate system, coupled with a policy by the central bank to serve as lender of last resort to commercial banks, can both implement the first best allocation and eliminate self-fulfilling bank runs.

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