SELECTED TECHNIQUES OF SEASONAL ADJUSTMENT

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COMPUTATIONAL STEPS OF SELECTED METHODS OF SEASONAL ADJUSTMENT

The tables appearing on the following four pages were compiled from published descriptions as follows:

Census Method II, Seasonal Adjustment on Electronic Computers, Organization for Economic Cooperation and Development, pp. 389-398 and Electronic Computers and Business Indicators, Occasional Paper 57, National Bureau of Economic Research, Inc. pp. 248-257.

X-9 and X-10 Versions of Census II, mimeographed material from Office of Chief Economic Statistician, Bureau of the Census.

BLS Method, "BLS Seasonal Factor Method," Abe Rothman, 1960 Proceedings of the Business and Economic Statistics Section, American Statistical Association, pp. 2-12.

Regression Method, "The Practice of Seasonal Adjustment with Regression Equations," Deutsche Bundesbank, Frankfurt, October, 1960.

BASIC CALCULATION STEPS USED IN SELECTED METHODS OF SEASONAL ADJUSTMENT

CENSUS METHOD II

Computation of Preliminary Seasonally Adjusted Series

*1. Adjustment for trading days is optional. If used, daily averages become original data.

2. Compute ratio of original to average of preceding and following months.

*3. Develop an uncentered 12-month moving average (MA) of original.

*4. Center 12-month MA.

*5. Calculate ratio of original to centered 12-month MA.

*6. Identify extreme values of step 5 and replace with more representative ones as follows:

a. Compute 5-term MA for each month. To get MA for first two years, average the first two ratios available. MA for last two years are obtained similarly.

b. For each month, compute 2-sigma control limits about 5-term MA. All ratios falling outside limits are extreme.

c. Replace extremes as follows: (1) Ratio falling first of series, average of first three ratios; (2) Ratio falling in middle, average extreme ratio and preceding and following ones; (3) Ratio falling at end, average extreme and two preceding ratios.

X-9 VERSION OF CENSUS METHOD II

Computation of Preliminary Seasonally Adjusted Series

*1. Steps 1-5 are the same as Census Method II.

*6. Omit step 6 in Census II and substitute the following:

a. Compute a 5-term MA for each month of data in step 5. To get MA for first two years, repeat MA of third year. MA for last two years are obtained similarly.

b. For each month, compute 2-sigma control limits about 5-term MA. All ratios falling outside limits are extreme.

c. Replace extremes as follows: (1) Ratio falling first of series average of second, third, and fourth ratios; (2) Falling second, average of first, third, and fourth ratios; (3) Falling middle, average two preceding and two following; (4) Falling next to last or last, similar to beginning.

X-10 VERSION OF CENSUS METHOD II

Computation of Preliminary Seasonally Adjusted Series

*1. Steps 1-5 are the same as Census Method II.

*6. Omit step 6 in Census II and substitute the following:

a. Compute a 5-term MA for each month of data in step 5. To get MA for first two years, repeat MA of third year. MA for last two years are obtained similarly.

b. For each month, compute 2-sigma control limits about 5-term MA. All ratios falling outside limits are extreme.

c. Replace extremes as follows: (1) Ratio falling first of series, average of second, third and fourth ratios; (2) Falling second, average of first, third, and fourth ratios; (3) Falling middle, average two preceding and two following; (4) Falling next to last or last, similar to beginning.

BUREAU OF LABOR STATISTICS METHOD

First Iteration

*1. Develop a centered 12-month moving average (MA) of original. Six values at each end are computed by a series of steps. Preliminary estimate of trend-cycle (TC).

*2. Compute ratio of original to centered 12-month MA. First approximation of seasonal-irregular (SI).

*3. For each calendar month, compute a 5-term weighted moving average (WMA) of SI ratios in step 2. Unforced seasonals, first approximation.

*4. Force total to 1200.

*5. Compute seasonally adjusted series.

*6. Divide seasonally adjusted series by 12-month MA of original. First approximation of irregular (I) with some TC.

Second Iteration

*7. Smooth I ratios in step 6 by 7-month WMA after extending I for three months at each end. Measure of residual TC. Multiply resulting values by TC of step 1.

*8. Compute ratio of original to TC of previous step. Second approximation of SI.

*9. For each calendar month, compute 5-term WMA of SI ratios. Unforced seasonals, second approximation.

*10. Force total to 1200.

REGRESSION METHOD OF DEUTSCHE BUNDESBANK

Basic Method

*1. Develop an uncentered 12-month MA of original (a). This is used to represent trend (t).

2. Basic analysis based on following additive relationship:
an=tn + Pn + En
Original values=trend values + seasonal component + residual component.

3. Other symbols:
a'=seasonal values (regression values)
a"=residual values (a-a')
a*=seasonally adjusted values.

*4. Standard regression equation used for each month: a'=tB + A, where B slope; A=Y intercept:

$$A = \frac{\Sigma t \, \Sigma at - \Sigma t^2 \, \Sigma a}{(\Sigma t)^2 - n \Sigma t^2}$$

$$B = \frac{\Sigma a - nA}{\Sigma t}$$

*5. With A and B values, compute seasonal values (a') for each original values: a'=tB + A.

*6. Graphical checking of computations. Using (t) on the X axis and (a) on the Y axis, plot the values and the regression line. Visual proof of the correct computation of A and B.

*7. Compute residual values (a") by comparing regression values (a') with original values (a): a"=a-a'. If a" greater than 0, there is superseasonal present; if a" is less than 0, there is subseasonal; if 0, there is is only purely seasonal.

CENSUS METHOD II

*6. (Continued)

d. Six missing ratios (due to step 4) at beginning are supplied by extending first available ratios for corresponding months back to initial month of series. Six missing at end supplied similarly.

e. Force total to 1200.

f. Compute 3-term of 3-term MA of ratios in 6e for each month. Supply missing values at each end. The results are preliminary seasonal factors.

(Step 7 follows on the next page)

X-9 VERSION OF CENSUS METHOD II

*6. (Continued)

d. For each month, compute a 3-term MA. Missing values supplied for first year-average first three ratios; similar for end.

e. Compute a centered 12-month MA. Missing values--repeat first available ratio six times. Similar for end. Divide into step 6d.

f. For each month, compute a 3-term MA. Missing values-use value in step 6e corresponding to the month missing.

g. Six factors missing at end (due to step 4) are obtained by using the factor for the same month of the first or last available year. These are preliminary seasonal factors.

X-10 VERSION OF CENSUS METHOD II

*6. (Continued)

d. For each month, compute a 7-term MA of ratios in step 6c. Missing ratios supplied in first 3 years by averaging first three years available. Similar for last years. MA values computed by using these estimates.

e. For each month, compute the average, without regard to sign, of year-to-year percent changes in MA of step 6d.

f. For each month, divide step 6d into step 6c. Estimate of I.

g. For each month, compute average, without regard to sign, of year-to-year percent changes in I.

h. For each month, compute ratio of step 6g to step 6e. Designated as Moving Seasonality Ratios.

i. For each month, depending upon size of ratio in step 6h, MA of ratios yielded by step 6c is computed using term indicated in the table at the end of the instructions. Missing ratios supplied.

j. Compute a centered 12-month MA of 6i. Missing values--repeat first available ratio six times. Similar for end. Divide into step 6i.

BUREAU OF LABOR STATISTICS METHOD

*11. Compute seasonally adjusted series.

*12. Divide seasonally adjusted series by TC of step 7. Second approximation of I.

Third Iteration

*13. Smooth I ratios in step 12 by 7-month WMA after extending I for 3 months at each end. Multiply smoothed I by TC in step 7. Result is final TC unless extreme values are present.

*14. Compute ratio of original to TC of previous step. Third approximation of SI.

*15. For each calendar month, compute 5-term WMA of SI in step 14. Unforced seasonal, third approximation.

*16. Force total to 1200. These are final factors unless extreme values are present.

*17. Compute seasonally adjusted series.

*18. Divide seasonally adjusted series by TC of step 13. Final I unless extreme values are present.

Fourth Iteration

*19. Test for extreme values and, if found, replace with substitute values. Tests involve developing, smoothing and analyzing irregular component to determine whether values fall outside ± 2.8 sigma limits. Replacements are calculated by multiplying TC by S for a given month.

REGRESSION METHOD OF DEUTSCHE BUNDESBANK

8. Compute seasonally adjusted values (a) by adding residual values (a") to corresponding trend values: $a^* = t + a''$.

9. To decompose time series: (seasonal) $p = a' - t$; (residual) $E = a'' = a - a'$.

Refinement of Trend Translation.

*10. Test whether trend is "true," i.e., whether seasonal fluctuations around trend are distorted.

*11. If refinement is necessary, improved trend values (t_1) are obtained by smoothing trend values (t):

$$t_1 = \frac{a - A_o}{B_o}$$

Where A_o and B_o are original regression equation coefficients.

*12. New improved trend values (t_1) are then used as basis of a refined correlation between the trend and original values. Calculation techniques for various values are the same as in the Basic Method.

Regression Method Concluded

CENSUS METHOD II	X-9 VERSION OF CENSUS METHOD II	X-10 VERSION OF CENSUS METHOD II	BUREAU OF LABOR STATISTICS METHOD
		*6. (Continued)	Fifth Iteration
		k. For each month, compute a 3-term MA. Missing values--use value in step 6j corresponding to the month missing.	Repeat basic steps 1-6 of First Iteration using replacements for extreme values. This iteration develops first approximation to time series components.
		l. Six factors missing at end (due to step 4) are obtained by using the factor for the same month of the first or last available year. These are preliminary seasonal factors.	Sixth Iteration Steps 7-12 of Second Iteration are repeated still using replacement values as original values.
			Seventh Iteration
*7. Compute preliminary seasonally adjusted series.	*7. Same as Census II.	*7. Same as Census II	Steps 13-18 of Third Iteration are repeated. After derivation of final measures, replacement values are replaced with original values and final seasonally adjusted series is derived.
Final Seasonally Adjusted Series	Final Seasonally Adjusted Series	Final Seasonally Adjusted Series	
*8. Develop a 15-month WMA of preliminary seasonally adjusted series supplying missing values.	*8. Same as Census II.	*8. Same as Census II.	Bureau of Labor Statistics Method Concluded
*9. Compute ratio of original to 15-month WMA	*9. Same as Census II	*9. Same as Census II.	
*10. Compute ratio of preliminary seasonally adjusted series to its 15-month WMA.	*10. Same as Census II.	*10. Omit from Census II.	
*11. Compute month-to-month percent changes of step 10 and average without regard to sign. Measures average amplitude of I.	*11. Same as Census II.	*11. Omit from Census II.	
*12. Identify extreme values in step 9 and replace in the same manner as explained in steps 6a-6c above. Force total total to 1200.	*12. Omit step 12 in Census II and apply steps 6a-6c above to the results of step 10.	*12. Omit steps 12 and 13 in Census II and substitute as explained above in steps 6a-6k using the results in step 9 above. These are final seasonal factors.	

CENSUS METHOD II	X-9 VERSION OF CENSUS METHOD II	X-10 VERSION OF CENSUS METHOD II
*13. Final seasonal factors are derived as follows: If irregular in step 11 averages under 2, use a 3-term MA of a 3-term MA; If I is 2 or more, use a 3-term MA of a 5-term MA. Missing values at each end are supplied.	*13. Final seasonal factors are derived as follows: If I in step 11 averages under 2, use 3-term MA; if I is 2 or more, compute a 5-term MA. Missing values at each end are supplied. Then perform steps 6e and 6f. These are final seasonal factors.	*13. See step 12.
*14. Project seasonals in step 13 for year ahead on basis of the seasonal factors for the last two years.	*14. The remaining steps are identical to the steps in Census II.	*14. The remaining steps are identical to the steps in Census II.
15. Compute seasonally adjusted series.		
16. Compute ratio of final seasonally adjusted series to average of preceding and following month as test of residual seasonal.		
17. Develop an uncentered 12-month MA of seasonally adjusted series.		
18. Compute ratio of uncentered 12-month MA of final series to similar average of original series to provide test for bias.		
19. Calculate ratio of each month to the preceding January in final series as test for residual seasonal of more than a month's duration.		
Measures of Irregular (I), Cyclical (C), Seasonal (S)		
20. Compute 15-month WMA of final series-yields (C).		
21. Compute month-to-month percentage changes in original (O), seasonal factors (S), final seasonally adjusted series (CI), cyclical (C), and ratio of original to 12-month WMA.		
22. Compute ratio of final series to 15-month WMA of final series. Yields estimate of I. Calculate month-to-month percentage changes in I.		
23. Derive mean of percentage changes in original $\overline{(O)}$, irregular $\overline{(I)}$, cyclical $\overline{(C)}$, seasonal $\overline{(S)}$, and seasonally adjusted $\overline{(CI)}$.		
24. Using averages in step 23, calculate: $\overline{I/C}$, $\overline{I/S}$, $\overline{S/C}$, $\overline{I/O}$, $\overline{C/O}$, $\overline{S/O}$.		
25. Compute ratio of $\overline{I/C}$ with percentage changes taken 2, 3, 4, and 5 months apart. The interval corresponding to the last $\overline{I/C}$ ratio that is less than 1.00 is the "number of months for cyclical dominance." Calculate MA of final series, using this number as its period.		
26. Derive average duration of run for CI, I, C, and CI, smoothed in step 25.		
27. Compute, without regard to sign, ratio of 12-month MA of month-to-month percent changes in I to 12-month MA of month-to-month percent changes in C.		

TERM OF MOVING AVERAGE FOR DIFFERENT SEASONALITY RATIOS IN X-10. SEE STEP 6i.

Moving Seasonality Ratio step 6h	Average of 6c Values	Number of beginning or ending 6c values averaged to extend MA
0-1.49	None (Leave 6c values unchanged)	---
1.50-2.49	3-term moving average	2
2.50-4.49	5-term moving average	2
4.50-6.49	9-term moving average	3
6.50-8.49	15-term moving average	3
8.50 and over	Arithmetic average of all 6c values	---

An asterisk (*) identifies a step involved in derivation of seasonal factors.

RESEARCH DEPARTMENT
FEDERAL RESERVE BANK OF ATLANTA
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BUREAU OF THE CENSUS SEASONAL
ADJUSTMENT TECHNIQUE*
(METHOD II)

I. Computation of Preliminary Seasonally Adjusted Series

1. Original observations. Where an adjustment for the number of working or trading days is made, these figures are shown after adjustment and all subsequent computations are based on these adjusted figures (Table I of sample "print-out").

2. Ratios of the original observations for each month to the average of the original observations for the preceding and following months are computed. Arithmetic means of these ratios for each month are given at the bottom of the table (Table 2).

3. A twelve-month moving average of the original series is computed. This curve provides a measure of the trend-cycle component of the series. It also provides annual averages of the original series (Table 3).

4. The twelve-month moving average is centered--that is, a two-month moving average of the twelve-month moving average is computed. This operation places the moving average values at mid-months. The first value of the centered moving average is placed at the seventh month of the original series. Thus six moving average values will be missing at the beginning and at the end of the series (Table 4).

5. Ratios of the original observations to the centered twelve-month moving average are computed. This computation results in a series which shows primarily the seasonal and irregular components of the original series (Table 5).

6. This step will provide a method for identifying extreme items among the ratios computed by step 5, substituting more representative ratios for these extreme ratios and fitting smooth curves to all ratios for each month.

   a. Fit a five-term moving average to the ratios for each month. This results in the loss of moving average values for the first two and the last two years for which ratios are available. To obtain moving averages for the first two years, use the average of the first two ratios as the estimated value of the ratio for each of the two years preceding the first year available. This is equivalent to weighting the first three years' ratios by 2/5, 2/5, and 1/5, respectively,

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*"A Description of the United States Bureau of the Census Method of Adjustment of Series of Monthly Data for Seasonal Variations," Seasonal Adjustment on Electronic Computers, pp. 391-398. This description is the same as contained in Electronic Computers and Business Indicators, Occasional Paper No. 57, National Bureau of Economic Research, 1957, pp. 248-257.

to obtain the first year's moving average value, and to weighting the first four years' ratios by 3/10, 2/10, 2/10, and 2/10, respectively, to obtain the second year's moving average value. Moving average values for the last two years are obtained in a similar manner.

b. For each month, compute two-sigma control limits about the five-term moving average line. All ratios falling outside these limits are designated as extreme.

c. Replace extreme ratios as follows: for an extreme ratio falling at the first point in the series, substitute the average of the first three ratios of the series; for an extreme ratio falling in the middle of the series, substitute the average of the extreme ratio and the preceding and following ratios; for an extreme ratio falling at the end of the series, substitute the average of the extreme ratio and the two preceding ratios.

d. The six missing ratios at the beginning of the series are supplied by extending the first available ratios for the corresponding months back to the initial month of the series. The six missing ratios at the end are supplied similarly.

e. For each year, center the twelve ratios (i.e., adjust the twelve ratios so that their sum will be 1,200) by division of the twelve items by their arithmetic mean. If the initial year is incomplete, use as the ratio for any missing month the value of the average ratio for the same month in the next two years in centering the initial year's ratios. Treat the terminal year's ratios in a similar manner.

f. For each month, compute a three-term moving average of a three-term moving average of the centered ratios yielded by step 6e, above. This will result in the loss of two moving average values at the beginning and two at the end. To obtain the values missing at the beginning, use the average of the first two centered ratios as the estimated value of the centered ratio for each of the two years preceding the first year available. This is equivalent to weighting the first three years' centered ratios by 9/18, 7/18, and 2/18, respectively, to obtain the first year's moving average value, and to weighting the first four years' centered ratios by 5/18, 7/18, 4/18, and 2/18, respectively, to obtain the second year's moving average value. The missing values at the end are obtained in a similar way. The values of these twelve curves constitute the preliminary seasonal adjustment factors (Table 6).

7. These seasonal factors are divided into the corresponding figures of the original series, month by month; i.e., the seasonal factor for January, 1947, is divided into the original observation for January, 1947; the factor for January, 1948, is divided into the original observation for January, 1948. Similarly, the factor for February, 1947, is divided into the original observation for February, 1947; the factor for February, 1948, into the original observation for February, 1948; and so on. This yields the preliminary seasonally adjusted series (Table 7).

II. Computation of Final Sesonally Adjusted Series

8. Compute a weighted fifteen-month moving average (Spencer's fifteen-term formula) of the preliminary seasonally adjusted series. The weights are as follows: -3/320, -6/320, -5/320, 3/320, 21/320, 46/320, 67/320, 74/320, 67/320, 46/320, 21/320, 3/320, -5/320, -6/320, -3/320. This is equivalent to a weighted five-month moving average (weights are -3/4, 3/4, 1, 3/4, -3/4) of a five-month moving average, of a four month moving average, of a four-month moving average of the data.

    To obtain values for the beginning points of this curve, use the average of the first four values of the preliminary seasonally adjusted series as the estimated value of this series for each of the seven months preceding the first month available. The values for the end are supplied similarly.

    The preliminary seasonally adjusted series contains the cyclical, trend, and irregular components of the series with only a trace of the seasonal component. The weighted fifteen-month moving average can be used in place of a twelve-month moving average because there is no significant seasonal factor to suppress. The weighted fifteen-month moving average is much more flexible than a twleve-month moving average and will, therefore, provide a better measure of the trend-cycle component; it is also much smoother than a simple five-month moving average (Table 8).

9. Ratios of the original observations to the weighted fifteen-month moving average are computed (Table 9).

10. Compute the ratios of the preliminary seasonally adjusted series (step 7) to its weighted fifteen-month moving average (step 8). Month-to-month changes in these ratios are computed and averaged without regard to sign. This yields a preliminary measure of the average amplitude of the irregular component.

11. This step will provide a method for identifying extreme items among the ratios computed by step 9, substituting more representative ratios for these extreme ratios, and fitting smooth curves to all ratios for each month.

a. Fit a five-term moving average to the ratios for each month. This results in the loss of moving average values for the first two and the last two years. To obtain moving averages for the first two years, use the average of the first two ratios as the estimated value of the ratio for each of the two years preceding the first year available. This is equivalent to weighting the first three years' ratios by 2/5, 2/5, and 1/5, respectively, to obtain the first year's moving average value, and to weighting the first four years' ratios by 3/10, 3/10, 2/10, and 2/10, respectively, to obtain the second year's moving average value. The moving average values for the last two years are obtained in a similar manner.

b. For each month, compute two-sigma control limits about the five-term moving average line. All ratios falling outside these limits are designated as "extreme."

c. Replace extreme ratios as follows: for an extreme ratio falling at the first point in the series, substitute the average of the first three ratios of the series; for an extreme ratio falling at the end of the series, substitute the average of the extreme ratio and the two preceding ratios (Table 10).

d. For each year center the twelve ratios (i.e., adjust the twelve ratios so that their sum will be 1,200) by division of the twelve items by their arithmetic mean. If the initial year is incomplete, use as the ratio for any missing month the value of the average ratio for the same month in the next two years in centering the initial year's ratios. Treat the terminal year's ratios in a similar manner (Table 11).

e. If the average irregular amplitude, computed in step 10 above, is under 2, use step 11f; if it is 2 or more, use step 11g.

f. For each month compute a three-term moving average of a three-term moving average of the centered ratios yielded by step 11d, above. This will result in the loss of two moving average values at the beginning and two at the end. To obtain the values missing at the beginning, use the average of the first two centered ratios as the estimated value of the centered ratio for each of the two years preceding the first year available. This is equivalent to weighting the first three year's centered ratios by 9/18, 7/18, and 2/18, respectively, to obtain the first year's moving average value, and to weighting the first four year's centered ratios by 5/18, 7/18, 4/18, and 2/18, respectively, to obtain the second year's moving average value. The missing values at

the end are obtained in a similar way. These smoothed ratios constitute the final seasonal adjustment factors. This series is identified later by the symbol S (Table 12).

g. For each month compute a three-term moving average of a five-term moving average of the centered ratios yielded by step 11d, above. This will result in the loss of three moving average values at the beginning and three at the end. To obtain the values missing at the beginning, use the average of the first two centered ratios as the estimated value of the centered ratio for each of the three years preceding the first year available. This is equivalent to weighting the first four year's centered ratios by 6/15, 6/15, 2/15, and 1/15, respectively, to obtain the first year's moving average value; to weighting the first five years' centered ratios by 9/30, 9/30, 6/30, 4/30, and 2/30, respectively, to obtain the second year's moving average value; and to weighting the first six years' centered ratios by 5/30, 7/30, 6/30, 6/30, 4/30, and 2/30, respectively, to obtain the third year's moving average value. The missing values at the end are obtained in a similar way. These smoothed ratios constitute the final seasonal adjustment factors. This series is later identified by symbol S (Table 12).

h. Estimates of the seasonal factors one year ahead are given at the bottom of Table 12. These estimates are made by adding to the seasonal factor for the end year, one-half the trend between the factor for that year and the preceding year. If X=seasonal adjusment factor for year N, then X_{N+1} is estimated by the equation

$$X_{N+1} = \frac{3X_N - X_{N-1}}{2}$$

12. These seasonal factors are divided into the corresponding figures of the original series, month by month; i.e., the seasonal factor for January, 1947, is divided into the original observation for January, 1947; the factor for January, 1948, is divided into the original observation for January, 1948. Similarly, the factor for February, 1947, is divided into the original observation for February, 1947; the factor for February, 1948, into the original observation for February, 1948; and so on. This yields the final seasonally adjusted series. This series is later identified by the symbol CI (Table 13).

13. The ratios of the final seasonally adjusted series to the averages of the final seasonally adjusted series for the preceding and following months are computed. This is a rough test for residual seasonality, similar to that made on the original observations described in step 2 above. Arithmetic means of these ratios for each month are given at the bottom of the table (Table 14).

14. Compute an uncentered twelve-month moving average of the final seasonally adjusted series. This step is required to carry out the test described in step 15. It also provides annual averages of the seasonally adjusted series (Table 15).

15. Compute ratios of the uncentered twelve-month moving average of the standard seasonally adjusted series to the uncentered twelve-month moving average of the original series. This is a test of the effect of the seasonal adjustment on the level of the series, showing whether the adjustment has resulted in significant differences between the level of adjusted and the unadjusted series for any twelve-month period (Table 16).

16. Using the final seasonally adjusted series, compute the ratio of the value of each month, from February through the following January, to that of the preceding January. Such a table of ratios will disclose repetitive patterns in successive years of more than one month's duration (Table 17).

III. Measures of the Irregular, Cyclical and Seasonal Components

17. Compute a weighted fifteen-month moving average (Spencer's fifteen-term formula) of the final seasonally adjusted series. The weights are as follows: -3/320, -6/320, -5/320, 3/320, 21/320, 46/320, 67/320, 74/320, 67/320, 46/320, 21/320, 3/320, -5/320, -6/320, -3/320. This is equivalent to a weighted five-month moving average (weights are -3/4, 3/4, 1, 3/4, -3/4), of a five-month moving average, of a four-month moving average, of a four-month moving average of the data.

    To obtain values for the beginning points of this curve, use the average of the first four values of the final seasonally adjusted series as the estimated value of this series for each of the seven months preceding the first month available. The values for the end are supplied similarly.

    The final seasonally adjusted series contains the cyclical, trend, and irregular components of the series. The weighted fifteen-month moving average can be used in place of a twelve-month moving average because there is no seasonal factor to suppress. The weighted fifteen-month moving average is much more flexible than a twelve-month moving average and will therefore provide a better measure of the trend-cycle component; it is also much smoother than a simple five-month moving average, and it fits the data about as closely as does the five-month moving average. This series is identified by the symbol C (Table 18).

18. Compute the month-to-month percentage changes in the original series (Table 19).

19. Compute the month-to-month percentage changes in the final seasonal adjustment factors (Table 20).

20. Compute the month-to-month percentage changes in the final seasonally adjusted series (Table 21).

21. Compute the month-to-month percentage changes in the ratios (step 9) of the original observations to the weighted fifteen-month moving average (Table 22).

22. Compute the ratios of the final seasonally adjusted series (step 12) to its weighted fifteen-month moving average (step 17). This provides a measure of the irregular component of the series. This series is identified by the symbol I (Table 23).

23. Compute the month-to-month percentage changes in the irregular component (Table 24).

24. Compute the month-to-month percentage changes in the weighted fifteen-month moving average of the final seasonally adjusted series (Table 25).

25. Compute the average, without regard to sign, of the percentage changes in steps 18, 19, 20, 23, and 24. This operation yields measures of the average monthly amplitude of the original series, the seasonal component, the seasonally adjusted series, the irregular component, and the cyclical component, respectively. The symbols used to represent these averages are original, $\overline{O}$; irregular, $\overline{I}$; cyclical, $\overline{C}$; seasonal, $\overline{S}$; and seasonally adjusted, $\overline{CI}$ (Table 27).

26. Compute the following ratios of the average monthly amplitudes of step 25:

    a. Irregular component to cyclical component ($\overline{I}/\overline{C}$);

    b. Irregular component to seasonal component ($\overline{I}/\overline{S}$);

    c. Seasonal component to cyclical component ($\overline{S}/\overline{C}$);

    d. Irregular component to original series ($\overline{I}/\overline{O}$);

    e. Cyclical component to original series ($\overline{C}/\overline{O}$);

    f. Seasonal component to original series ($\overline{S}/\overline{O}$);

    See Table 27.

27. Compute the ratio of the average monthly amplitude of the irregular to the cyclical components when percentage changes are taken between entries two, three, four, and five months apart (Table 27).

    The interval corresponding to the last $\overline{I}/\overline{C}$ ratio that is less than 1.00 is designated as "Number of Months for Cyclical Dominance," and a moving average of the seasonally adjusted data is computed, using this interval as its period (Table 26).

28. The average duration of run, that is, the average number of months the series moves before changing direction, is computed for the following:

    a. Seasonally adjusted series;

    b. Irregular component;

    c. Cyclical component;

    d. Seasonally adjusted series smoothed by moving average with period as given by number of months for cyclical dominance;

    See Table 27.

29. Compute the ratios of a) the twelve-month moving average of the month-to-month percentage changes in the irregular component (step 23) to b) the twelve-month moving average of the month-to-month percentage changes in the cyclical component (step 24). In the computation of these moving averages, the signs of the percentage changes are disregarded (Table 28).

IV. Notes**

30. Where the average monthly amplitude of the irregular component is 4.0 or larger (on the basis of the preliminary seasonally adjusted series) and for special purposes, two additional tables are computed and inserted between Tables 10 and 11. In the first one, the stable adjustment factors are computed by averaging the modified ratios of step 11c for each month and then centering the average so that their sum will be 1,200. In the second table, these stable factors are divided into the corresponding values of the original data, yielding a seasonally adjusted series based on a constant seasonal pattern. These two additional tables do not affect the computations in any other tables.

**Electronic Computers and Business Indicators, Occasional Paper No. 57, National Bureau of Economic Research, 1957, p. 252.

Listing of Tables Prepared by Census II Method

Table Number	Title of Table
1	Original series
2	Ratios of original to preceding and following Averages of ratios
3	Uncentered 12-month moving average of original
4	Centered 12-month moving average of original
5	Ratios of original to 12-month moving average
9	Ratios of original to weighted 15-month moving average
10	Modified ratios, original/WTD 15-month moving average Stable-seasonal adjustment factors Stable-seasonal adjusted series
11	Centered ratios, original/WTD 15-month moving average
12	Final seasonal adjusted factors, 3*5-month moving averages Estimated seasonal factors one year ahead
13	Final seasonally adjusted series
14	Ratios, final adjusted to preceding and following Averages
15	Uncentered 12-month moving average, final adjustment
16	Ratios, 12-month moving average, final adjustment to original
17	Ratios, each month to preceding January, final adjustment
18	Weighted 15-month moving average of final adjustment
19	Percent change from preceding month, original
20	Percent change from preceding month, seasonal
21	Percent change from preceding month, final adjustment
22	Percent change from preceding month, S-I ratios
23	Irregular component
24	Percent change from preceding month, irregular
25	Percent change from preceding month, cyclical
26	2-month moving average, final adjusted series
27	I, C, & S components, their relations, & average duration of run
28	Ratios, 12-month moving averages of irregular and cycle amplitudes

Listing of Tables Prepared under the 1401 Version of Census II Method (Philadelphia Program)

Table Number	Title of Table
1	Original series
2	Ratios of original to preceding and following Averages
3	Uncentered 12-month moving average of original
4	Centered 12-month moving average of original
5	Ratio of original to 12-month moving average
6	Preliminary seasonal factors
7	Preliminary adjusted series
8	Weighted 15-month moving average of preliminary series
9	Ratios of original to weighted 15-month moving average
10	Percent change from preceding month, original
11	Percent change from preceding month, S-I ratios
12	Modified ratios of original/WTD, 15-month moving average
13	Centered ratios of original/WTD, 15-month moving average
14	Final seasonal adjusted factors, 3*3-month moving averages Estimated seasonal factors one year ahead
15	Final seasonally adjusted series
16	Percent change from preceding month, seasonal
17	Percent change from preceding month, final adjustment
18	Ratios, final adjustment to preceding and following Averages
19	Uncentered 12-month moving average, final adjustment
20	Ratios, 12-month moving averages, final adjustment to original
21	Ratios, each month to preceding January, final adjustment
22	Weighted 15-month moving average of final series
23	Irregular component
24	Percent change from preceding month, irregular
25	Percent change from preceding month, cyclical
26	Ratios, 12-month moving averages of irregular and cycle amplitudes
27	2-month moving average, final adjusted series
28	I, C, & S components, their relations, and average duration of run

BUREAU OF THE CENSUS SEASONAL
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The X-9 Version of Census Method II*

This procedure replaces steps 6 and 11 of Census Method II as described in the foregoing description, "A Description of the United States Bureau of the Census Method of Adjustment of series of Monthly Data for Seasonal Variations," Seasonal Adjustment on Electronic Computers.

6. This step will provide a method for identifying extreme items among the ratios computed by step 5, substituting more representative ratios for these extreme ratios and fitting smooth curves to all ratios for each month.

   a. Fit a five-term moving average to the ratios for each month. This results in the loss of moving average values for the first two and the last two years for which ratios are available. To obtain moving average values for the first two years, repeat the moving average value of the third year. This is equivalent to weighting the first five years' ratios by 1/5, 1/5, 1/5, 1/5, and 1/5 to obtain the first and second years' moving average values. Moving average values for the last two years are obtained in a similar manner.

   b. For each month, compute two-sigma control limits about the five-term moving average line. All ratios falling outside these limits are designated as extreme.

   c. Replace extreme ratios for each month as follows: for an extreme ratio falling at the first point in the series, substitute the average of the second, third, and fourth ratios; for an extreme ratio falling at the second point of the series, substitute the average of the first, third, and fourth ratios; for an extreme ratio falling in the middle of the series, substitute the average of the two preceding and two following ratios; for an extreme ratio falling at the next to last or last point, follow a procedure similar to that for the beginning of the series (Table 5A, "Modified Ratios, Original/12-month Moving Average").

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*"Specifications for the X-9 Version of Census II Method Seasonal Adjustment Program," Bureau of the Census, Office of Chief Economic Statistician, March 6, 1962.

d. For each month, compute a three-term moving average of the modified ratios yielded by step 6c. This results in the loss of moving average values for the first and last years for which ratios are available. To obtain the moving average value for the first year, use the average of the first three ratios as the estimated value for the ratio preceding the first year available. This is equivalent to weighting the first three years' ratios by 4/9, 4/9, and 1/9, respectively, to obtain the first year's moving average value. The missing value at the end is obtained in a similar way (Table 6B, "Preliminary Uncentered Seasonal Factors").

e. For the entire series, compute a centered twelve-month moving average (a two-term of a twelve-term moving average) of the preliminary uncentered seasonal factors yielded by step 6d (Table 6C, "Preliminary Centering Factors"). For the six missing values at the beginning of the centered twelve-month moving average, repeat the first available value six times. The six missing values at the end are obtained in a similar way. The values computed in step 6d are divided by these values (Table 6D, "Preliminary Centered Seasonal Factors").

f. For each month, compute a three-term moving average of the preliminary centered seasonal factors yielded by step 6e. This results in the loss of moving average values for the first and last years. To obtain the moving average value for the first year, use the first 6e value as an estimated value for the year preceding the first year for which a value is available. This is equivalent to weighting the first two years' values by 2/3 and 1/3, respectively, to obtain the first year's moving average value. The missing value at the end is obtained in a similar way.

To obtain the six factors missing at the beginning of the series (due to the use of the twelve-term moving average in step 4), repeat the factor from the same month of the first available year. Fill in the six missing factors at the end of the series in a similar way (Table 6E, "Preliminary Seasonal Factors").

Continue with step 7 of "A Description of the United States Bureau of the Census Method of Adjustment of series of Monthly Data for Seasonal Variations," Seasonal Adjustment on Electronic Computers.

11. This step will provide a method for identifying extreme items among the ratios computed by step 9, substituting more representative ratios for these extreme ratios, and fitting smooth curves to all ratios for each month.

a. Fit a five-term moving average to the ratios for each month. This results in the loss of moving average values for the first two and the last two years for which ratios are available. To obtain moving average values for the first two years, repeat the moving average value of the third year. This is equivalent to weighting the first five years' ratios by 1/5, 1/5, 1/5, 1/5, and 1/5 to obtain the first and second years' moving average values. Moving average values for the last two years are obtained in a similar manner.

b. For each month, compute two-sigma control limits about the five-term moving average line. All ratios falling outside these limits are designated as extreme.

c. Replace extreme ratios for each month as follows: for an extreme ratio falling at the first point in the series, substitute the average of the second, third, and fourth ratios; for an extreme ratio falling at the second point of the series, substitute the average of the first, third and fourth ratios; for an extreme ratio falling in the middle of the series, substitute the average of the two preceding and two following ratios; for an extreme ratio falling in the next to last or last point, follow a procedure similar to that for the beginning of the series (Table 10, "Modified Ratios, Original/Weighted 15-Month Moving Average").

d. If the average irregular amplitude, computed in step 10 above, is under 2, use step 11e; if it is 2 or more, use step 11f.

e. For each month, compute a three-term moving average of the modified ratios yielded by step 11c. This results in the loss of moving average values for the first and last years for which ratios are available. To obtain the moving average value for the first year, use the average of the first three ratios as the estimated value for the ratio preceding the first year available. This is equivalent to weighting the first three years' ratios by 4/9, 4/9, and 1/9, respectively, to obtain the first year's moving average value. The missing value at the end is obtained in a similar way (Table 10D, "Final Uncentered Seasonal Factors").

f. For each month, compute a five-term moving average of the modified ratios yielded by step 11c. This results in the loss of moving average values for the first two and last two years for which ratios are available. To obtain moving average values for the first two years, use the average of the first four ratios as the estimated value for the ratios for each of the two years preceding the first year available. This is equivalent to weighting

the first four years' ratios by 6/20, 6/20, 6/20, and 2/20, respectively, to obtain the first year's moving average value and to weighting the first four years' ratios by 5/20, 5/20, 5/20, and 5/20 to obtain the second year's moving average value. The missing values at the end are obtained in a similar way (Table 10D, "Final Uncentered Seasonal Factors").

g. For the entire series, compute a centered twelve-month moving average (a two-term of a twelve-term moving average) of the final uncentered seasonal factors Yielded by step 11e or 11f (Table 10E, "Final Centering Factors"). For the six missing values at the beginning of the centered twelve-month moving average, repeat the first available value six times. The six missing values at the end are obtained in a similar way. The values computed in step 11e or 11f are divided by these values (Table 11, "Final Centered Seasonal Factors").

h. For each month, compute a three-term moving average of the final centered seasonal factors yielded by step 11g. This results in the loss of moving average values for the first and last years. To obtain the moving average value for the first year, use the first 11g value as an estimated value for the year preceding the first year for which a value is available. This is equivalent to weighting the first two years' values by 2/3 and 1/3, respectively, to obtain the first year's moving average value. The missing value at the end is obtained in a similar way (Table 12, "Final Seasonal Factors").

i. Estimates of the seasonal factors one year ahead are given at the bottom of Table 12. These estimates are made by adding to the seasonal factor for the end year, one-half the trend between the factor for that year and the preceding year. If X=seasonal adjustment factor for year n, then $X_n + 1$ is estimated by the equation $X_n + 1 = \frac{3X_n - X_n - 1}{2}$.

Continue with step 12 of "A Description of the United States Bureau of the Census Method of Adjustment of series of Monthly Data for Seasonal Variations," Seasonal Adjustment on Electronic Computers.

NOTE: In these specifications, no description is given for Tables 6A, 10A, 10B, and 10C. In the Census Bureau's printout, Tables 10A and 10B are the "Stable-Seasonal Factors" and "Stable-Seasonal Adjusted Series" described in step 30 of Occasional Paper No. 57. They are printed out regardless of the size of the irregular component, not only when the average monthly amplitude of the irregular component is 4.0 or larger as originally specified. Tables 6A and 10C are the Moving Seasonality Ratios described in the specificiations for X-10. In X-9, these ratios do not play a role in the

selection of the seasonal factor curves; however, they are useful as a descriptive measure of the type of seasonality present in each month.

BUREAU OF THE CENSUS SEASONAL
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The X-10 Version of Census Method II*

This procedure** replaces steps 6, 10, 11, of Census Method II as described in the foregoing description, "A Description of the United States Bureau of the Census Method of Adjustment of Series of Monthly Data for Seasonal Variations," Seasonal Adjustment on Electronic Computers.

6. This step will provide a method for identifying extreme items among the ratios computed by step 5, substituting more representative ratios for these extreme ratios and fitting smooth curves to all ratios for each month.

   a. Fit a five-term moving average to the ratios for each month. This results in the loss of moving average values for the first two and last two years for which ratios are available. To obtain moving average values for the first two years, repeat the moving average value of the third year. This is equivalent to weighting the first five years' ratios by 1/5, 1/5, 1/5, 1/5, and 1/5 to obtain the first and second year's moving average values. Moving average values for the last two years are obtained in a similar manner.

   b. For each month, compute two-sigma control limits about the five-term moving average line. All ratios falling outside these limits are designated as extreme.

   c. Replace extreme ratios for each month as follows: for an extreme ratio falling at the first point in the series, substitute the average of the second, third and fourth ratios; for an extreme ratio falling at the second point of the series, substitute the average of the first, third, and fourth ratios; for an extreme ratio falling in the middle of the series, substitute the average of the two preceding and two following ratios; for an extreme ratio falling at the next to last or last point, follow a procedure similar to that for the beginning of the series (Table 5A, "Modified Ratios, Original/12-Month Moving Average").

   d. For each month, compute a seven-term moving average of the modified ratios yielded by step 6c. This results in the loss of moving average values for the first three and the last three years for which ratios are available. To obtain

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*"Specifications for the X-10 Version of the Census Method II Seasonal Adjustment Program," Bureau of the Census, Office of Chief Economic Statistician, March 6, 1962.

moving average values for the first three years, use the average of the first three ratios as the estimated value for the ratios for each of the three years preceding the first year available. Then the moving average values for the first three years are computed by including these estimated ratios in the moving average (see part (1) of Note at end of specifications). The missing values at the end are obtained in a similar way.

e. For each month, compute the average, without regard to sign, of the year-to-year percentage changes in the moving average values of step 6d. This average is an estimate of the change in the seasonal component for a particular month and is referred to as $\overline{S}_y$.

f. For each month, divide the moving average values in step 6d into the modified ratios from step 6c. The resulting series is an estimate of the irregular component.

g. For each month, compute the average, without regard to sign, of the year-to-year percentage changes in the irregular component yielded by step 6f. This average is an estimate of the change in the irregular component and is referred to as $\overline{I}_y$.

h. For each month, compute the ratio of the 6g value to the 6e value, $\overline{I}y/\overline{S}y$. These ratios are designated Moving Seasonality Ratios (Table 6A, "Moving Seasonality Ratios").

i. For each month, depending upon the size of the moving seasonality ratio computed in step 6h, an average of the modified ratios yielded by step 6c is computed, as specified in the table below. When a moving average is selected and computed, there is a loss of moving average values at the beginning and end. The number of values lost depends upon the length of the moving average selected. To obtain the moving average values at the beginning, a specified number of beginning ratios are averaged to obtain estimated ratios for the years preceding the first available ratio. Then the moving average values for the first years are computed by including these estimated ratios in the average. The number of ratios to be averaged, in order to obtain the estimated ratios, is shown in the last column of the table (See part (1) of Note at end of specifications). The moving average values missing at the end are obtained in a similar way.

Moving seasonality ratio step 6h	Average of 6c values	No. of beginning or ending 6c values average to extend the moving average
0 to 1.49	None (Leave 6c values unchanged.)	---
1.50 to 2.49	3-term moving average	2
2.50 to 4.49	5-term moving average	2
4.50 to 6.49	9-term moving average	3
6.50 to 8.49	15-term moving average	3
8.50 and over	Arithmetic average of all 6c values	---

The values obtained in this step are printed out (Table 6B, "Preliminary Uncentered Seasonal Factors").

j. For the entire series, compute a centered twelve-month moving average (a two-term of a twelve-term moving average) of the preliminary uncentered seasonal factors yielded by step 6i (Table 6C, "Preliminary Centering Factors"). For the six missing values at the beginning of the centered twelve-month moving average, repeat the first available value six times. The six missing values at the end are obtained in a similar way. The values computed in step 6i are divided by these values (Table 6D, "Preliminary Centered Seasonal Factors").

k. For each month, compute a three-term moving average of the preliminary centered seasonal factors yielded by step 6j. This results in the loss of moving average values for the first and last years. To obtain the moving average value for the first year, use the first 6j value as an estimated value for the year preceding the first year for which a value is available. This is equivalent to weighting the first two years' values by 2/3 and 1/3, respectively, to obtain the first year's moving average value. The missing value at the end is obtained in a similar way. To obtain the six factors missing at the beginning of the series (due to the use of the twelve-term moving average in step 4), repeat the factor from the same month of the first available year. Fill in the six missing factors at the end of the series in a similar way (Table 6E, "Preliminary Seasonal Factors").

Continue with step 7 of "A Description of the United States Bureau of the Census Method of Adjustment of Series of Monthly Data for Seasonal Variations," Seasonal Adjustment on Electronic Computers.

10. Delete step 10.

11. This step will provide a method for identifying extreme items among the ratios computed by step 9, substituting more representative ratios for these extreme ratios, and fitting smooth curves to all ratios for each month.

    a. Fit a five-term moving average to the ratios for each month. This results in the loss of moving average values for the first two and the last two years for which ratios are available. To obtain moving average values for the first two years, repeat the moving average value of the third year. This is equivalent to weighting the first five years' ratios by 1/5, 1/5, 1/5, 1/5, and 1/5 to obtain the first and second years' moving average values. Moving average values for the last two years are obtained in a similar manner.

    b. For each month, compute two-sigma control limits about the five-term moving average line. All ratios falling outside these limits are designated as extreme.

    c. Replace extreme ratios for each month as follows: For an extreme ratio falling at the first point in the series, substitute the average of the second, third, and fourth ratios; for an extreme ratio falling at the second point of the series, substitute the average of the first, third and fourth ratios; for an extreme ratio falling in the middle of the series, substitute the average of the two preceding and two following ratios; for an extreme ratio falling at the next to last or last point, follow a procedure similar to that for the beginning of the series (Table 10, "Modified Ratios, Original/Weighted 15-Month Moving Average").

    d. For each month, compute a seven-term moving average of the modified ratios yielded by step 11c. This results in the loss of moving average values for the first three and the last three years for which ratios are available. To obtain moving average values for the first three years, use the average of the first three ratios as the estimated values for the ratios for each of the three years preceding the first year available. Then the moving average values for the first three years are computed by including these estimated ratios in the moving average (See part 1 of Note at end of specifications). The missing values at the end are obtained in a similar way.

    e. For each month, compute the average, without regard to sign, of the year-to-year percentage changes in the moving average values of step 11d. This average is an estimate of the

change in the seasonal component for a particular month and is referred to as $\bar{S}y$.

f. For each month, divide the moving average values in step 11d into the modified ratios from step 11c. The resulting series is an estimate of the irregular component.

g. For each month, compute the average, without regard to sign, of the year-to-year percentage changes in the irregular component series yielded by step 11f. This average is an estimate of the change in the irregular component and is referred to as $\bar{I}y$.

h. For each month, compute the ratio of the 11g value to the 11e values, $\bar{I}y/\bar{S}y$. These ratios are designated Moving Seasonality Ratios (Table 10C, "Moving Seasonality Ratios").

i. For each month, depending upon the size of the moving seasonality ratio computed in step 11h, an average of the modified ratios yielded by step 11c is computed as specified in the table below. When a moving average is selected and computed, there is a loss of moving average values at the beginning and end. The number of values lost depends upon the length of the moving average selected. To obtain the moving average values at the beginning, a specified number of beginning ratios are averaged to obtain estimated ratios for the years preceding the first available ratio. Then the moving average values for the first years are computed by including these estimated ratios in the average. The number of ratios to be averaged, in order to obtain the estimated ratios, is shown in the last column of the table (See part 1 of Note at end of specifications). The moving average values missing at the end are obtained in similar way.

Moving seasonality ratio step 11h	Average of 11c values	No. of beginning or ending 11c values average to extend the moving average
0 to 1.49	None (Leave 11c values unchanged.)	---
1.50 to 2.49	3-term moving average	2
2.50 to 4.49	5-term moving average	2
4.50 to 6.49	9-term moving average	3
6.50 to 8.49	15-term moving average	3
8.50 and over	Arithmetic average of all 11c values	---

The values obtained in this step are printed out (Table 10D, "Final Uncentered Seasonal Factors").

j. For the entire series, compute a centered twelve-month moving average (a two-term of a twelve-term moving average) of the final uncentered seasonal factors yielded by step 11i (Table 10E, "Final Centering Factors"). For the six missing values at the beginning of the centered twelve-month moving average, repeat the first available value six times. The six missing values at the end are obtained in a similar way. The values computed in step 11i are divided by these values (Table 11, "Final Centered Seasonal Factors").

k. For each month, compute a three-term moving average of the final centered seasonal factors yielded by step 11j. This results in the loss of moving average values for the first and last years. To obtain the moving average value for the first year, use the first 11j value as an estimated value for the year preceding the first year for which a value is available. This is equivalent to weighting the first two years' values by 2/3 and 1/3, respectively, to obtain the first year's moving average value. The missing value at the end is obtained in a similar way (Table 12, "Final Seasonal Factors").

l. Estimates of the seasonal factors one year ahead are given at the bottom of Table 12. These estimates are made by adding to the seasonal factor for the end year, one-half the trend between the factor for that year and the preceding year. If X_n = seasonal adjustment factor for year n, then X_{n+1} is estimated by the equation $X_{n+1} = \frac{3X_n - X_{n-1}}{2}$.

Continue with step 12 of "A Description of the United States Bureau of the Census Method of Adjustment of series of Monthly Data for Seasonal Variations," Seasonal Adjustment on Electronic Computers.

NOTE: 1. No implicit weights are given for steps 6d, 6i, 11d, or 11i, as are given for steps 6a, 6k, etc., because when the series is shorter than the moving average, the weights vary with the length of the series. The original Method II was programmed to accept series with a minimum of 72 months (six years) of data. For the 15-term moving average, different sets of weights are required for 14, 13,.....6-year series; for the 9-term, sets for 8, 7, and 6-year series are required; and for the 7-term, sets for 6-year series are needed. The purpose in using a 15-term moving average with a series as short as six years is that it is a convenient way to fit a straight line within the framework of the method.

2. In these specifications, no description is given for Tables 10A and 10B. In the Census Bureau's printout, Tables 10A and 10B are the

"Stable-Seasonal Factors" and "Stable-Seasonal Adjusted Series" described in step 30 of Occasional Paper No. 57. They are printed out regardless of the size of the irregular component, not only when the average monthly amplitude of the irregular component is 4.0 or larger, as originally specified.

**The technique for selecting the seasonal factor curves on the basis of the moving seasonality ratios, which is incorporated in X-10, was developed by Stephen N. Marris, Head of Statistics Division of the Organization for Economic Cooperation and Development, Paris, France, and is described in Seasonal Adjustment on Electronic Computers, pages 257-309, OECD (Paris 1961). The Bureau of the Census and the Organization for Economic Cooperation and Development have cooperated in further theoretical and empirical development of this technique during the past two years. The X-10 program differs slightly from that described in the OECD paper.

THE SEASONAL ADJUSTMENT METHOD OF THE BUREAU OF LABOR STATISTICS*

Detailed Listing of Steps

The following steps describe the method used by the Bureau of Labor Statistics in developing seasonal factors. The "Table No." reference preceding a description refers to the table in the print-out provided by the electronic computer** program. The BLS method may involve four or seven iterations, depending on extreme values detected in the original data. The table numbers have been assigned so that the first digit indicates the iteration; the third digit identifies the type of information contained in the table as follows:

Table X01 always refers to trend-cycle

X02 to seasonal-irregular ratios

X03 to unforced seasonals

X04 to forced seasonals

X05 to irregular movements

X07 to extreme values

X08 to deseasonalized original values

X09 to original data

The computer program used with the BLS method permits selection of either a complete or partial record (print-out) of the values developed. The partial record includes the final trend-cycle, seasonal, and irregular components, the detected extreme original values and their substituted values, the deseasonalized series, and the centered 12-month moving average. Tables included in the partial record (short print-out) are identified by an asterisk immediately preceding the table number. The complete record (long print-out) includes all the tables shown.

*Table 101: 12-month moving average. This is a centered moving average of the original values (table 709), developed as a first approximation to the trend-cycle component. A centered moving average would being six months later than the original series. However, the difference has been reduced to three

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*"The BLS Factor Method," Abe Rothman, Proceedings of the 1960 American Statistical Association, pp. 8-11.

**I.B.M. 650 basic installation.

months by the following series of steps. (Corresponding operations are applied at the end of the series. All operations in the entire procedure are symmetical with respect to the time scale.)

a. Seasonal-irregulars are computed as described for table 102. These seasonal-irregulars begin with the following January, the first month for which the 12-month moving average is available.

b. Unforced seasonals are computed as described for table 103. These begin with January.

c. Forced seasonals are computed as described for table 104. These begin with January.

d. A seasonally adjusted series is computed by dividing the original values (table 709) by the forced seasonal factors (step c). For the first six months of the original series, the seasonal factor for the same month of the following year is used. The adjusted series begins with July.

e. The average of the first three seasonally adjusted values (those for July, August, and September) is multiplied by the seasonal factors (step c) for April, May, and June of the following year to provide synthetic original values for the three months preceding the beginning of the original series. The centered 12-month moving average of this extended original series is printed as table 101.

Table 102: Seasonal-irregular, first approximation. The original values (table 709) are divided by their 12-month moving average (table 101).

Table 103: Unforced seasonal, first approximation. For each calendar month, the seasonal-irregular ratios (table 102) are arranged by year and a weighted average is secured. The weights .30, .30, .20, .10, .10 are applied to the first five seasonal-irregulars. (the underline weight is applied to the term (year) whose seasonal is being computed.) For the second term, the weights .24, .26, .20, .16, .14 are applied to the same first five values. For the third and all subsequent terms up to the last two, the weights .17, .20, .26, .20, .17 are applied to a centered group of five years. The

next-to-last term applies weights of .14, .16, .20, .26, .24 to the last five values. The last term applies weights of .10, .10, .20, .30, .30, to these same five end values. The weights for the central term are a compromise between a pattern with uniform weights (.20) and one with weights associated with a 3 x 3 moving average (.11, .22, .33, .22, .11). The actual pattern is very close to the average of these two patterns but is a little flatter in shape.

Table 104: Forced seasonal, first approximation. Each unforced seasonal (table 103) is multiplied by an adjustment factor which is the ratio of 1200 to the sum of the unforced seasonals in the whole calendar year. This makes the average of the seasonal factors equal to 100.

Table 105: Irregular, first approximation. A seasonally adjusted series is computed by dividing the original values (table 709) by the forced seasonal factors (table 104). This, in turn, is divided by the 12-month moving average (table 101) to produce an estimate of the irregular component which also includes some residual trend-cycle. For the partial year at each end of the series, the seasonal factors of the adjacent year are used.

Table 201: Moving average, modified once. The irregulars (table 105) are extended three months at each end by tapering the first and last values to 100 percent. The extended series of irregulars, arranged in normal time sequence, is then smoothed by a weighted 7-month moving average to remove the irregular part and leave only the residual trend-cycle. The weighting pattern used, .090, .127, .183, .200, .183, .127, .090, is the average of a pattern with equal weights (.143) and a pattern associated with a 3-term of a 3-term of a 3-term (3x3x3) moving average (1, 3, 6, 7, 6, 3, 1 equal to .037, .111, .222, .259, .222, .111, .037).

Table 202. Seasonal irregular, second approximation. The original values (table 709) are divided by the improved estimate of trend cycle (table 201).

Table 203: Unforced seasonal, second approximation. This is a weighted 5-term moving average of the seasonal-irregulars (table 202) for each calendar month, using the same weights as for table 103.

Table 204: Forced seasonal, second approximation. Each unforced seasonal (table 203) is multiplied by an adjustment factor which is the ratio of 1200 to the sum of the unforced seasonals in the whole calendar year.

Table 205: Irregular, second approximation. A seasonally adjusted series is computed by dividing the original values (table 709) by the forced seasonal factors (table 204). This is in turn divided by the trend-cycle (table 201) to estimate the irregular component. For the partial year at each end of the series, the seasonal factors of the adjacent year are used.

*Table 301: Moving average, modified twice (final trend if no extremes). The irregulars (table 205) are smoothed in the same way described for table 201. The smoothed series of irregulars is multiplied by the previous estimate of trend-cycle (table 201) to produce table 301 as an improved estimate. This table gives the final trend-cycle component if there are no extreme values (revealed in next iteration).

Table 302: Seasonal-irregular, third approximation. The original values (table 709) are divided by the latest estimate of trend-cycle (table 301).

Table 303: Unforced seasonal, third approximation. This is a weighted 5-term moving average of the seasonal-irregulars (table 302) for each calendar month, using the same weights as for table 103.

*Table 304: Forced seasonal, third approximation (final if no extremes). Each unforced seasonal (table 303) is multiplied by an adjustment factor which is the ratio of 1200 to the sum of the unforced seasonals in the whole calendar year. This table gives the final seasonal component if there are no extreme values.

*Table 305: Irregular, third approximation (final if no extremes). A seasonally adjusted series is computed by dividing the original values (table 709) by the forced seasonal factors (table 304). This, in turn, is divided by the trend-cycle (table 301) to yield the irregular component. This table gives the final irregular component if there are no extreme values. For the partial year at each end of the series, the seasonal factors of the adjacent year are used.

*Table 308: Seasonally adjusted series (final if no extremes). The original values (table 709) are divided by the forced seasonal factors (table 304). For the partial year at each end of the series, the seasonal factors are taken from the corresponding months of the adjacent year.

*Table 407: Extreme values - tests and replacement values. This table contains the results of the series of steps designed to determine whether the series contains any extreme values. If any are found, the procedure provides replacement values. If no extreme

values are found, tables 407 through 708 are omitted. The test for extreme values includes the following steps:

a. The irregulars (table 305) are smoothed in the same way described for table 201, except that the central weight is zero instead of .200. The "mid-zero" weight pattern provides a trend-cycle which minimizes the effect of an extreme value on the test criterion.

b. The smoothed series of irregulars (step a) is multiplied by the latest trend-cycle (table 301) to produce the test trend-cycle. These values are uniformly 20 percent too low, because the weights used in step a add to only .800.

c. The original values (table 709) are divided by the test trend-cycle (step b) to yield test seasonal-irregulars, which are uniformly 25 percent too high.

d. The test seasonal-irregulars (step c) are smoothed by a weighted 5-term moving average for each calendar month to produce test seasonals, using the following "mid-zero" weights. For the first year, the weights are 0, .43, .29, .14, .14. For the second year they are .32, 0, .27, .22, .19. For the third and subsequent years up to the last two, they are .23, .27, 0, .27, .23. For the next-to-last year, they are .19, .22, .27, 0, .32. For the last year they are .14, .14, .29, .43, 0. (These weights are proportional to those for table 103 except that the target year always receives zero weight.) The test seasonals, like the test seasonal-irregulars, are uniformly 25 percent too high.

e. The test seasonal-irregulars (step c) are divided by the test seasonals (step d) to produce test irregulars.

f. The mean and standard deviation are computed for the entire (all months of all complete calendar years) distribution of test irregulars (step e). Control limits are set at the mean $\pm 2.8\sigma$ and are designed to provide a probability of about 50 percent that all "good" values will fall inside the limits. The 2.8σ and the 50 percent probability are based on the assumption that all values in the original series are "good" and belong to the series. However, since an original value not really belonging in the series may be encountered, a discriminating test is needed which will detect the non-belonging observation without rejecting

too many acceptable values. The 2.8σ is the point in the distribution which will, in 50 percent of the cases, reject no values; in the other 50 percent, it will reject one or more (usually one) values. Initially, different sigma limits were calculated based on the length of the series. However, since our computer program handled series of from 6-12 years, and the limits varied by .2 sigma, the single limit of 2.8 sigma was considered close enough for 6-12 year series.

g. Particular months whose test irregulars (step e) fall outside the control limits (step f) are designated as extreme and are listed in table 407. The replacement value for each extreme value is obtained by multiplying the test trend-cycle (step b) by the test seasonal (step d). This provides a value whose extreme irregularity has been removed.

*Table 501: 12-month moving average (extremes replaced). The set of original values (table 709) is modified by substituting for each extreme value the replacement value given in table 407. Table 501 is a centered moving average of these modified original values with extensions at the ends of series computed the same way as for table 101.

Table 502: Seasonal-irregular, first approximation (extremes replaced). The modified original values are divided by the 12-month moving average (table 501).

Table 503: Unforced seasonal, first approximation (extremes replaced). This is a weighted 5-term moving average of the seasonal-irregulars (table 502) for each calendar month, using the same weights as for table 103.

Table 504: Forced seasonal, first approximation (extremes replaced). Each unforced seasonal (table 503) is multiplied by an adjustment factor which is the ratio of 1200 to the sum of the unforced seasonals in the whole calendar year.

Table 505: Irregular, first approximation (extremes replaced). A seasonally adjusted series is computed by dividing the modified original values by the forced seasonal factors (table 504). This is in turn divided by the trend-cycle (table 501) to estimate the irregular component. For the partial year at each end of the series, the seasonal factors of the adjacent year are used.

Table 601: Moving average, modified once (extremes replaced). The irregulars (table 505) are smoothed in the same way described for table 201. The smoothed series of irregulars is multiplied by

the previous estimate of trend-cycle (table 501) to produce table 601 as an improved estimate.

Table 602: Seasonal-irregulars, second approximation (extremes replaced). The modified original values are divided by the latest estimate of trend-cycle (table 601).

Table 603: Unforced seasonal, second approximation (extremes replaced). This is a weighted 5-term moving average of the seasonal-irregulars (table 602) for each calendar month, using the same weights as for table 103.

Table 604: Forced seasonal, second approximation (extremes replaced). Each unforced seasonal (table 603) is multiplied by an adjustment factor which is the ratio of 1200 to the sum of the unforced seasonals in the whole calendar year.

Table 605: Irregular, second approximation (extremes replaced). A seasonally adjusted series is computed by dividing the modified original values by the forced seasonal factors (table 604) This is in turn divided by the trend-cycle (table 601) to estimate the irregular component. For the partial year at each end of the series, the seasonal factors of the adjacent year are used.

*Table 701: Final trend-cycle (extremes replaced). The irregulars (table 605) are smoothed in the same way described for table 201. The smoothed series of irregulars is multiplied by the previous estimate of trend-cycle (table 601) to produce this final estimate.

Table 702: Final seasonal-irregular (extremes replaced). The modified original values are divided by the final trend-cycle (table 701).

Table 703: Final unforced seasonal (extremes replaced). This is a weighted 5-term moving average of the final seasonal-irregulars (table 702) for each calendar month, using the same weights as for table 103.

*Table 704: Final seasonal (extremes replaced). Each unforced seasonal (table 703) is multiplied by an adjustment factor which is the ratio of 1200 to the sum of the unforced seasonals in the whole calendar year.

*Table 705: Final irregular (extremes replaced). A seasonally adjusted series is computed by dividing the actual original values (table 709) by the final seasonal factors (table 704). This is in turn divided by the final trend-cycle (table 701) to yield the final irregular component. For the partial year at each end of the series, the seasonal factors of the adjacent year are used.

*Table 708: Seasonally adjusted series. The original values (table 709) are divided by the final seasonal factors (table 704). For the partial year at each end of the series, the seasonal factors are taken from the corresponding months of the adjacent year.

*Table 709: Original series. This is the monthly series of original values.

Listing of Tables Prepared

Table Number	Title of Table
101	12-month moving average
102	Seasonal irregular, first approximation
103	Unforced seasonal, first approximation
104	Forced seasonal, first approximation
105	Irregular, first approximation
201	Moving average modified once
202	Seasonal irregular, second approximation
203	Unforced seasonal, second approximation
204	Forced seasonal, second approximation
205	Irregular, second approximation
301	Moving average modified twice *final trend if no extremes*
302	Seasonal irregular, third approximation
303	Unforced seasonal, third approximation
304	Forced seasonal, third approximation *final if no extremes*
305	Irregular, third approximation *final if no extremes*
308	Seasonally adjusted series *final if no extremes*
407	Extreme values - tests and replacement values
501	12-month moving average *extremes replaced*
502	Seasonal irregular, first approximation *extremes replaced*
503	Unforced seasonal, first approximation *extremes replaced*
504	Forced seasonal, first approximationn *extremes replaced*
505	Irregular, first approximation *extremes replaced*
601	Moving average modified once *extremes replaced*
602	Seasonal irregular, second approximation *extremes replaced*
603	Unforced seasonal, second approximation *extremes replaced*
604	Forced seasonal, second approximation *extremes replaced*
605	Irregular, second approximation *extremes replaced*
701	Final trend cycle *extremes replaced*
702	Final seasonal irregular *extremes replaced*
703	Final unforced seasonal *extremes replaced*
704	Final seasonal,*extremes replaced*
705	Final irregular *extremes replaced*
708	Seasonally adjusted series
709	Original series