

**BOARD OF GOVERNORS OF THE FEDERAL RESERVE SYSTEM**  
**DIVISION OF RESEARCH AND STATISTICS**

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**To:** Federal Open Market Committee  
**From:** Flint Brayton and Dave Reifschneider  
**Subject:** Revised Bluebook Estimates of the Equilibrium Real Rate — Overview

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**Strictly Confidential (FR)**  
**Class II – FOMC**

**Introduction**

For some time, estimates of the equilibrium real interest rate ( $R^*$ ) have regularly appeared in the Bluebook and in other FOMC-related material. Unfortunately, a recent survey of Bluebook readers and informal conversations suggest that the usefulness of these measures to the Committee has been impaired by two factors: (1) the lack of clarity and uniformity in the concept of  $R^*$ ; and the incomplete characterization of the uncertainty associated with estimates. In response, the staff has greatly revised the computation and the presentation of  $R^*$  estimates for the December Bluebook.

In the new presentation, all estimates are derived as answers to one of two specific forecasting questions. The first question is short-run in nature and relates to the stimulus required from monetary policy to close the output gap in twelve quarters. The second question is longer-run in nature, and concerns the projected level of the real funds rate consistent with output at potential seven years in the future, long enough for all transitory factors currently buffeting the economy to have faded away.

As part of this overhaul, the staff has also reviewed and modified the forecasting models used to estimate  $R^*$ . The revised presentation features estimates derived from three models — a simple equation for the output gap, a small economic model, and the large-scale FRB/US model. It also incorporates estimates of  $R^*$  derived

from financial data as well as measures that are consistent with the staff Greenbook projection. Although we regard the value of  $R^*$  implicit in the Greenbook projection as the best available measure, the estimates derived from the three econometric models and the bond market are useful benchmarks against which to compare the staff estimate. We also use the three models to generate confidence intervals for  $R^*$  that account for different sources of uncertainty, including model specification, equation coefficients, and errors in the real-time measurement of potential output.

The rest of this memo is organized as follows. In the next section, we discuss two different concepts of  $R^*$  and consider how these concepts can be used in monetary policymaking. We then turn to the limitations of  $R^*$ , and explain why the equilibrium real rate by itself is not an adequate guide for setting the real funds rate. After briefly discussing the models and procedures used to estimate  $R^*$ , we finish with an overview of the new  $R^*$  exhibit that will appear in the December Bluebook. An accompanying memo, "Revised Bluebook Estimates of the Equilibrium Real Rate — Technical Documentation," provides background information about our estimation procedures and results.

### **Defining $R^*$**

Economists use the terms "equilibrium" and "natural" real rate of interest in various ways, depending on the issue under discussion. The Board staff, in discussions related to monetary policy decisionmaking, has tended to define  $R^*$  as "the real funds rate at which the output gap would gradually return to zero, barring further disturbances to aggregate demand and supply." Published papers in this field typically use similar definitions.<sup>1</sup>

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<sup>1</sup> The quote is taken from Thomas Laubach and John Williams, "Estimates of a Time-Varying Equilibrium Real Federal Funds Rate," memorandum to Members of the Board (December 14, 2000). Examples of recent papers by Federal Reserve System economists that define  $R^*$  in a similar manner include: Antulio Bomfim (1997), "The Equilibrium Fed funds Rate and the Indicator Properties of Term Structure Spreads," *Economic Inquiry*, 35 (4), 830-46; Thomas Laubach and John C. Williams (2003), "Measuring the Natural Rate of Interest," *Review of Economic and Statistics*, 85 (4), 1063-1070; and Todd Clark and Sharon Kozicki (2004), "Estimating Equilibrium Real Interest Rates in Real Time," Federal Reserve Bank of Kansas City Research Working Paper 04-08. There is also a considerable body of published work, carried out by economists within the System and elsewhere, that deals with the related issue of how estimates of  $R^*$  should be used in monetary policymaking; see, for example, Athanasios Orphanides and John C. Williams,

This definition addresses a key question facing policymakers: What level of the real funds rate is consistent with full resource utilization? But left vague is a critical dimension of the problem — the time required for the output gap to close. Although the standard practice is to say that full resource utilization is achieved in the “medium term,” whether this state should be considered as something likely to occur two, five, or ten years into the future is not clear. To be more precise about the time dimension of  $R^*$ , we believe it is necessary to settle on one or more specific questions that define  $R^*$  as the answer.

Of course, many questions can be asked about the relationship between the stance of monetary policy and economic slack. But one of the most basic questions is whether the current funds rate is consistent with making significant progress in the near term toward restoring a normal degree of resource utilization in labor and product markets. We therefore define the following *short-run concept of  $R^*$ : the value of the real funds rate that, if sustained, would be projected to close the output gap twelve quarters in the future*. This concept, which we denote as  $R^*_{SR}$ , has the advantage of being the answer to a well-defined forecasting problem that can be addressed with a variety of economic models.

Some may object that this definition is insufficiently short-run in nature and that  $R^*_{SR}$  would be better defined as the real rate projected to close the output gap within eight or even four quarters. However, the lags in the monetary transmission mechanism are sufficiently long that closing the output gap within two years would occasionally entail extremely large fluctuations in the funds rate, and closing the output gap within one year might at times be impossible without wildly unrealistic movements in the funds rate. Given these constraints, defining  $R^*_{SR}$  using a three-year window seems consistent with the goal of determining the setting of the funds rate that would make substantial near-term progress in achieving full employment.

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“Robust Monetary Policy Rules With Unknown Natural Rates,” *Brookings Papers on Economic Activity*, 2, 63-145. Finally, there is a somewhat different concept of  $R^*$  that is in common use in academic circles — specifically, the real rate of interest consistent with instantaneous market clearing in the absence of wage and price frictions. Estimates of this concept — which has been discussed in papers by Michael Woodford among others — can be derived in a straightforward manner only in the context of macroeconomic models with strict micro-theoretic foundations.

Not all monetary policy questions concerning  $R^*$  are so oriented toward conditions over the near term. For example, policymakers may be interested in comparing the current stance of monetary policy to that prescribed by the Taylor rule. Such a comparison requires an estimate of the intercept in the Taylor rule, which conceptually equals the value of the real funds rate consistent with output at potential and inflation at its target once all transitory influences on aggregate demand and supply have passed. To address these sorts of questions, we therefore propose the following *medium-run concept of  $R^*$* : *the value of the real funds rate projected to prevail in seven years under the assumption that monetary policy will act to eliminate economic slack in three years and to hold output at potential thereafter*. This concept, which we denote by  $R^*_{MR}$ , is similar to  $R^*_{SR}$  in that it answers a well-defined forecasting problem.

As might be expected, both measures of the equilibrium real rate are strongly influenced by fundamental factors such as fiscal policy and trend productivity growth, with the difference being that  $R^*_{SR}$  is more closely tied to the current state of these factors whereas  $R^*_{MR}$  depends only on their projected values in the longer run. Another difference between the two concepts is that the value of  $R^*_{SR}$  — but not the value of  $R^*_{MR}$  — is strongly influenced by the current amount of slack in the economy: Because the output effects of even transitory shocks take time to dissipate and because monetary policy works only with a substantial lag, a large starting output gap requires a low real funds if the gap is to be closed in twelve quarters.<sup>2</sup>

In principle, one could compute an equilibrium real rate based on an even longer view than that used to define  $R^*_{MR}$ . Such a measure might equal the expected level of the real funds rate once the economy reaches a steady state in which all markets clear, all flow variables (output, spending, and the factors of production) expand along their long-run growth paths, and all stocks are stable as ratios to an appropriate scale variable, such as GDP. However, such a concept would not be of much practical help in

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<sup>2</sup> A link exists as well between the Taylor rule and  $R^*_{SR}$ , although the connection is less clear than the link to  $R^*_{MR}$ . The Taylor rule can be written as  $R = R^*_{MR} + \alpha \text{ xgap} + \beta \text{ pgap}$ , where  $R$  is the real funds rate,  $\text{xgap}$  is the output gap, and  $\text{pgap}$  is the difference between the actual and target rates of inflation. Given the dependence of the short-run equilibrium funds rate on the initial level of the output gap, the sum of the first two terms in the Taylor rule,  $R^*_{MR} + \alpha \text{ xgap}$ , is likely to be highly correlated with  $R^*_{SR}$  as long as the value of the coefficient  $\alpha$  in the rule is set to yield closure of the output gap within twelve quarters on average.

monetary policymaking: Achievement of full stock equilibrium can take decades, especially if underlying investment and borrowing flows are not projected to stabilize in the near future, as is now the case for the deficits in both the federal budget and the current account. For this reason, we focus only on estimates of  $R^*_{SR}$  and  $R^*_{MR}$ .<sup>3</sup>

The two proposed concepts of  $R^*$  are inherently real-time in nature: Today's estimates obviously do not and cannot incorporate information unavailable today. Therefore, if we wish to study how  $R^*$  has changed over time, or to compare its historical path to the real funds rate path actually chosen by policymakers, past estimates of  $R^*$  should be computed on a comparable basis — that is, with only the information that was available at a given point in the past. As a practical matter, such real-time estimates are usually impossible to obtain in full because of the lack of comprehensive real-time datasets. The closest feasible approximation to the real-time forecasting problem is to compute the value of, say,  $R^*_{SR}$  in 1990:Q1 using data through 1989:Q4 as currently published. We take this approach in producing the revised model-based estimates of the equilibrium real rate.

### **Limitations of $R^*$ for monetary policymaking**

Estimates of  $R^*$  are not intended to be complete prescriptions for setting the funds rate; rather, they are among the set of indicators that might be considered by the Committee in deciding how best to meet the dual objectives of price stability and full employment. An obvious limitation of  $R^*$  is its exclusive real-side orientation: Most importantly, although setting the real funds rate equal to the short-run or even the medium-run estimate of the equilibrium real rate would be expected to ensure full resource utilization eventually, such  $R^*$  policies would not necessarily achieve the Committee's inflation objectives. Instead, such policies would allow the inflation rate to

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<sup>3</sup> The estimates of  $R^*$  previously published in the Bluebook did not conform to the short-run concept of  $R^*$  because they were designed to exclude the effects of transitory factors likely to influence aggregate spending over the next few years. However, the old estimates did not conform to  $R^*_{MR}$ , either, even though they were intended to be medium-term in nature. In the case of the statistical filter estimate, the poor correspondence resulted from a model misspecification that caused the variability of the medium-run value of  $R^*$  to be overstated, effectively making it a short-run measure. In the case of the FRB/US estimate, the problem was that it was not computed using medium-term projections of fiscal policy, trend growth, and capital stocks, but instead was estimated using the current values of these slow-moving variables.

- 6 -

drift over time because they would make no provision for offsetting the inflation effects of shocks to aggregate demand and supply. At a minimum, price stability requires a policy that raises the real funds rate above  $R^*$  when inflation is undesirably high and does the opposite when inflation is too low.

A second limitation is that the estimates of  $R^*_{SR}$  and  $R^*_{MR}$  only crudely approximate a trajectory for the real funds rate that policymakers might find desirable, even in the absence of inflation concerns. For example, on our assessment real activity is presently being restrained by transitory factors that appear to be dissipating only gradually. In response to that situation, the Committee might prefer to have the real funds rate climb for some time and then settle down at some longer-term sustainable rate. But rather than fully outlining this path,  $R^*_{SR}$  at best measures only the average value of the desired real rate over the first twelve quarters, and  $R^*_{MR}$  describes only its endpoint.

Another factor that limits the usefulness of  $R^*$  is the uncertainty associated with estimates of its value. Some of this imprecision reflects our ignorance about the true nature of the economy — that is, our ignorance about both the specification and the coefficients of our models of real activity. Such uncertainty is especially germane for monetary policy, because it concerns not only the level of aggregate spending that is consistent with a given level of interest rates (additive uncertainty) but also the shift in aggregate spending generated by a given change in the funds rate (slope uncertainty).  $R^*$  uncertainty also arises because of errors in the measurement of potential output, especially on a contemporaneous basis: Although we can infer with some accuracy what the level of potential output was, say, five years ago by taking account of the subsequent behavior of inflation and other factors, we have more difficulty gauging how much economic slack exists currently and thus judging what size of a change in the funds rate is needed to close the output gap. Finally, because we do not know what future disturbances will hit the economy, we can never be sure in advance what setting of the real funds rate will, after the fact, prove to have been consistent with full resource utilization.

Overall, in no sense can setting the real funds rate equal to  $R^*$  be interpreted as “optimal.” As already noted, such policies ignore inflation and are generally too

simple to describe the full trajectory for the real funds rate that would be needed to stabilize output. They also might at times deliver a speed of closure of the output gap that Committee members would find unacceptably slow. Finally,  $R^*$  policies take no account of any other considerations, such as uncertainty, that may be germane to setting policy. In short, such policies do not substitute for a comprehensive strategy that balances the Committee's objectives and that takes into account all relevant factors.<sup>4</sup>

### Three forecasting models

After reviewing the forecasting models used to compute the estimates of  $R^*$  that have appeared in previous Bluebooks, we have modified the specifications of the pair that had been used and added a third model to the group. We believe that, although this expanded group by no means includes all the models that could be used to estimate  $R^*$ , it yields estimates of uncertainty likely to provide a realistic sense of the range of uncertainty that might be estimated from a larger set of models. We leave open the possibility of expanding or altering our set of forecasting models at some future date.

**Single-equation model.** This model consists of an equation in which the output gap depends only on a constant term and lagged values of the output gap and the real funds rate. This equation is similar in spirit to the statistical filter that has heretofore been employed to derive one set of estimates of  $R^*$  for the Bluebook, in that both rely on a simple characterization of the relationship between output and the real funds rate. But the two approaches differ in the details of this characterization. One difference is that the new equation includes additional lags of the output gap and the real funds rate. Not only are these lags statistically significant, but they also affect the fundamental nature of the model's long-run value of the equilibrium real rate. In the statistical filter, the

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<sup>4</sup> An  $R^*_{SR}$  policy would also create a type of time-inconsistency problem. By definition, setting the real funds rate at  $R^*_{SR}$  and holding it there would be predicted to close the output gap in twelve quarters, barring any further economic disturbance. In the next quarter, the same setting for the real funds rate would be expected to close the output gap in *eleven* quarters, assuming no new shocks to the economy. But if at that point one recalculated  $R^*_{SR}$  according to the definition laid out in the text, one would ordinarily obtain a different estimate because the new calculation would show the value of the real funds rate needed to close the output gap in the standard *twelve* quarters. Thus, under an  $R^*_{SR}$  policy, one would choose to change the stance of monetary policy even though economic conditions had unfolded exactly as expected because the effective planning period had advanced one period. A side effect of such an ever-advancing endpoint is that the elimination of economic slack might take appreciably longer than three years.

relationship between the output gap and the real funds rate is subject to both transitory and permanent shocks. However, the estimated variance of permanent shocks falls to zero with the added lags in the new output gap equation, indicating that all shocks are transitory and that the long-run value of  $R^*$  is a constant (estimated to be about 2-1/4 percent).

**Small model.** This five-equation model permits  $R^*$  to be affected by changes in trend GDP growth, fiscal policy, and stock prices. In the model's key equation, the output gap responds gradually to movements in the real bond rate, trend GDP growth, the ratio of the high-employment federal budget surplus to potential output, and the equity premium. The structure of the equation implies that in the long run  $R^*$  moves one-for-one with changes in the trend rate of GDP growth; a sustained increase of one percentage point in the ratio of the federal surplus to GDP reduces long-run  $R^*$  about 30 basis points, and an increase of one percentage point in the equity premium boosts  $R^*_{LR}$  about 3/4 percentage point. Each of these economic determinants of  $R^*$  is modeled with a simple time-series equation. A final equation specifies that the real bond rate depends on the real federal funds rate, the expected value of the real funds rate seven years into the future (that is,  $R^*_{MR}$ ), and the lagged output gap.

**FRB/US.** FRB/US is the staff's large-scale model of the U.S. economy. The estimates of  $R^*$  from FRB/US reported in previous Bluebooks were based on a special flow-equilibrium version of the model that is ill-suited for calculating the new short-run concept of  $R^*$ . The standard version of FRB/US — the one routinely used in simulations reported in the Greenbook and Bluebook — is more easily applied to this task. Estimates of the equilibrium real rate using FRB/US depend on a broad array of economic factors, some of which take the form of projected values of the model's exogenous variables. These projections are based on several simple forecasting rules that are appropriate for the three-year period relevant for  $R^*_{SR}$  but are less sensible at longer horizons. Thus, we do not compute  $R^*_{MR}$  with FRB/US.



### **Greenbook-consistent estimates of $R^*$**

If the Greenbook projection were derived from an explicit economic model, that model could also be used to generate estimates of  $R^*$ . Although such a formal “staff” model does not exist, we are able to derive estimates of  $R^*$  that incorporate most of the information in the Greenbook by using the FRB/US model in place of the unknown staff model. Specifically, we first produce an FRB/US baseline that exactly matches the Greenbook forecast and then run a simulation in which the model’s multiplier properties are used to determine the value of the real funds rate that satisfies the short-run definition of  $R^*$ .<sup>5</sup> To construct a consistent set of historical values of  $R^*_{SR}$  associated with past Greenbooks, we employ not the current version of FRB/US but rather the version in use at the time. Thus, our historical Greenbook-consistent estimates of  $R^*$  are the same as those that would have been calculated at the time the original Greenbook forecast was published and so are “real-time” in nature.

### **The revised Bluebook exhibit**

Associated with the new procedures for estimating the equilibrium real rate is a revised presentation of this information in the Bluebook, shown here as chart 1. The chart has two parts. The upper panel is a figure that plots current and historical estimates of the short-run concept of  $R^*$ ; the bottom panel is a table summarizing the latest estimates of both the short- and medium-run concepts of  $R^*$  for the current quarter only. Except for changes arising from revisions before the close of the forecast, the chart is the same as the one that will appear in the December Bluebook.

As noted earlier, some notion of the degree of uncertainty associated with the various measures of  $R^*$  is a prerequisite for these estimates to be useful to policymakers. Information of this type is represented in the upper panel by three shaded “confidence” bands, which have been derived using the econometric models. The simplest source of uncertainty is the availability of more than one model with which to estimate  $R^*$ . The

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<sup>5</sup> Because the Greenbook forecast extends only two and a half years (at most) into the future, we are unable to compute Greenbook-consistent estimates of  $R^*_{MR}$ . Also, because the Greenbook forecast period is always less than twelve quarters, we must modify our  $R^*_{SR}$  estimation procedures somewhat; see the accompanying technical memo for details.

- 10 -

magnitude of this type of uncertainty, shown by the inner red band, is approximated by the range spanned by the set of point estimates of short-run  $R^*$  from the three models. In the current quarter, this range runs from 1.9 to 2.6 percent and is 0.7 percentage point wide. The width of the inner band has averaged 1-1/4 percentage points since 1990 but has varied considerably over time.

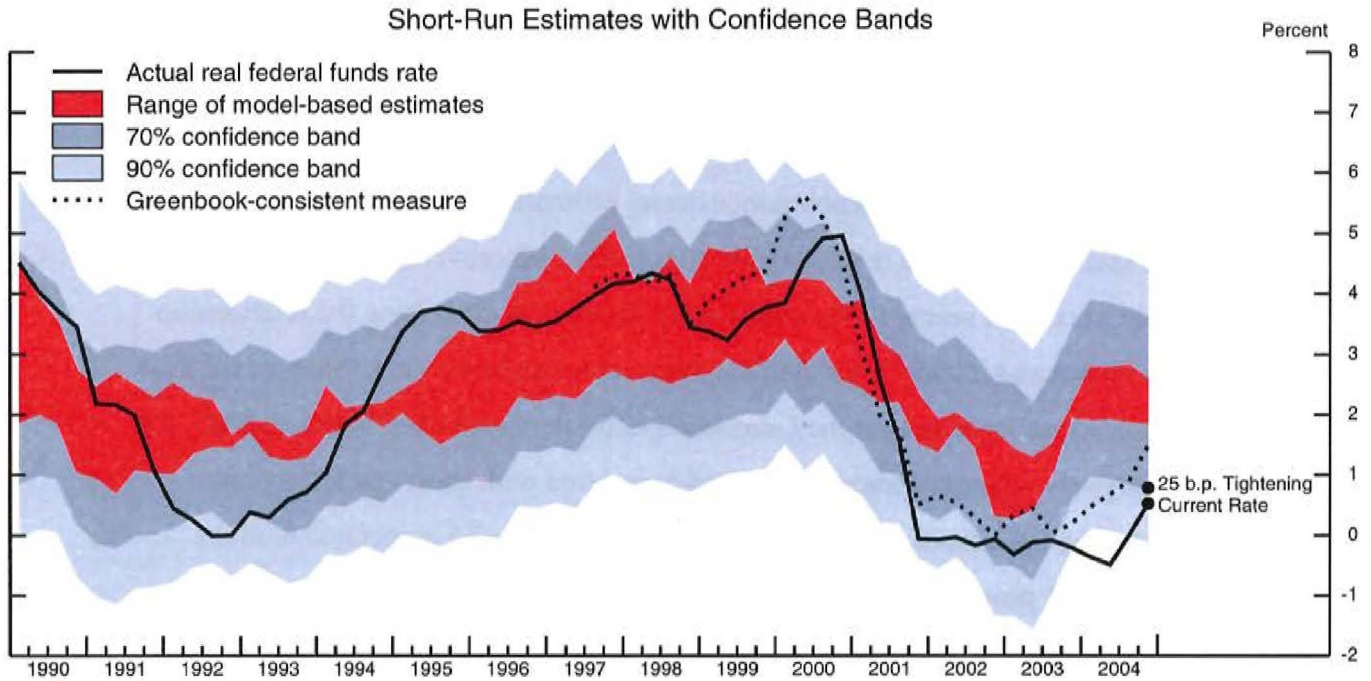
A more complete characterization of uncertainty about  $R^*$  takes account of the degree to which each model's estimate of short-run  $R^*$  is itself imprecise. Two primary sources for this imprecision are uncertainties associated with the estimation of each model's coefficients and those associated with the measurement of potential output. The combined effect of these two sources of uncertainty, together with the uncertainty of not knowing which of the three models is correct, provides a broad-based measure of  $R^*$  uncertainty. The magnitude of this type of uncertainty is summarized by the two blue bands in the figure: As constructed, the short-run value of  $R^*$  has a probability of 70 percent of being within the dark-blue band, and a 90 percent probability of being within the light-blue bank. For the current quarter, the 70 percent confidence band extends from 0.7 percent to 3.6 percent, while the 90 percent band ranges from -0.1 percent to 4.4 percent. Over history, the width of these two bands has on average been about 3 percentage points and 4-3/4 percentage points, respectively.

As noted earlier, the fact that the economy is subject to unpredictable shocks is another source of uncertainty: Even a "good" estimate of  $R^*$  is likely with hindsight to be wrong because shocks to aggregate demand and supply that could not have been anticipated will occur after the estimate is made. However, in revising the Bluebook exhibit we have elected to focus only on uncertainty associated with *predictions* of  $R^*$ , not uncertainty about what  $R^*$  will actually turn out to have been in hindsight. Accordingly, all the uncertainty estimates presented in the new exhibit exclude the effects of future shocks to the economy. But an argument can be made that the latter source of uncertainty is also pertinent to monetary policy, and so should be incorporated into the confidence bands. If we did so, the bands would widen considerably; for example, the 70 percent confidence interval would almost double.

Another perspective on the current and expected stance of policy is provided by the Greenbook-consistent estimate of short-run  $R^*$  (dotted black line). The current estimate is 1.5 percent, a value that is somewhat below the midpoint of the confidence intervals. Thus, the Greenbook projection is that, for a given level of the real funds rate, output will tend to be weaker relative to potential than it is in the models' forecasts. However, the fact that the Greenbook-consistent estimate is within the 70 percent range suggests that the difference is not terribly significant. Moreover, over the past several years the Greenbook seems to have been more accurate at describing the relationship between output and interest rates than the three econometric models. Support for this conclusion seems strongest for the Greenbook-consistent estimates from late 2001, for which the twelve-quarter forecasting period associated with short-run  $R^*$  is now observed. The Greenbook-consistent estimate of short-run  $R^*$  associated with the December 2001 projection was 1/2 percent, a value at the lower end of the 70 percent model-based range at the time. With hindsight, the value of short-run  $R^*$  for this date was likely less than the average value of the actual real funds rate from the fourth quarter of 2001 to the third quarter of 2004 (-1/4 percent), given that the staff currently estimates that output remains below potential.

Current estimates of the medium-run concept of  $R^*$  are reported in the table for two of the forecasting models, together with an estimate derived from the indexed securities market. The model-based estimates average about 2-1/2 percent and in both cases are higher than the corresponding short-run estimate of  $R^*$ . In large part, this pattern reflects the current economic slack: With output below potential, short-run  $R^*$  is held down by the need for extra stimulus from interest rates to boost output to potential in the twelve-quarter planning interval. By definition, this factor is absent in the medium-run estimates. The 70 percent confidence bands for the medium-run estimates of  $R^*$  — which are defined to exclude the effects of future economic shocks — range from 1-1/2 to 3-1/4 percent. This range encompasses the market-based estimate of  $R^*_{MR}$ , which currently stands at only 1-3/4 percent.

Chart 1  
Equilibrium Real Federal Funds Rate



Notes: The real federal funds rate is constructed as the difference between the quarterly average of the actual nominal funds rate and the log difference of the core PCE price index over the previous four quarters. For the current quarter, the nominal funds rate used is the target federal funds rate as of the close of the Bluebook.

Short-Run and Medium-Run Measures for 2004:Q4

	Current Estimate	Previous Bluebook
<b>Short-Run Measures</b>		
Greenbook-consistent measure	1.5	1.1
FRB/US model	2.1	1.8
Small structural model	2.6	2.6
Single-equation model	1.9	1.9
Confidence intervals for three model-based estimates		
70% confidence interval	0.7 - 3.6	
90% confidence interval	-0.1 - 4.4	
<b>Medium-Run Measures</b>		
TIPS-consistent measure	1.8	1.8
Small structural model	2.7	2.7
Single-equation model	2.2	2.2
Confidence intervals for two model-based estimates		
70% confidence interval	1.6 - 3.2	
90% confidence interval	0.9 - 3.7	

Notes: The figures in the "Previous Bluebook" column indicate the estimates for the current quarter as of the previous Bluebook that would have been reported using the new procedures. Confidence intervals and bands reflect uncertainties about model specification, coefficients, and the level of potential output.