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# Metro Level Evidence on the Convexity of the U.S. Phillips Curve

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## 1. Introduction and Summary of Findings

Is the U.S. Phillips Curve convex? Does upward pressure on inflation rise increasingly as unemployment falls below the natural rate of unemployment? If so, should monetary policy makers act pre-emptively and raise interest rates sooner rather than later? The evidence for the convexity of the Phillips Curve is rather mixed. Most studies find a convex wage inflation Phillips Curve, but few studies find a convex price inflation Phillips Curve

My research using metro level inflation and unemployment data suggests that the price Phillips Curve is "alive", since labor market slack is always economically and statistically significant. Although the fit of convex Phillips Curves is sometimest better than the fit of linear curves, the degree of convexity in the Phillips curve is modest, and is not economically significant.

## 2. Review of the Existing Evidence

The evidence for the convexity of the U.S. Phillips Curve is rather mixed. Most studies find a convex wage inflation Phillips Curve, but few studies find a convex price inflation Phillips Curve (**Tables 1 and 2**). From a policy viewpoint, convexity of the price Phillips Curve is more important since the pass through from wage inflation to price inflation is not that strong. For example, Peneva and Rudd (2015) "find little evidence that changes in labor costs have had a material effect on price inflation in recent years, even for compensation measures where some degree of pass through to prices still appears to be present".

Many researchers argue that aggregate data may not be sufficiently informative about the convexity of the wage and price Phillips Curves, especially when the Fed successfully

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targeted inflation during the Great Moderation period. They suggest that U.S. regional or metro data may be more informative, and many recent papers adopt this approach.

Two recent papers, which have been cited by Chair Yellen, convey a flavor of the recent findings. Kumar and Orrenius (2016) use state level data to detect convexity in the wage Phillips Curve. They find "strong evidence that the wage-price Phillips curve is nonlinear and convex; declines in the unemployment rate below the average unemployment rate exert significantly higher wage pressure than changes in the unemployment rate above the historical average.". The estimated wage Phillips curve is twice as steep when the unemployment rate is low than when it is high. This means that the upward pressure on wages from a fall in the unemployment rate is twice as large when the rate is low than when it is high.

Nalewaik (2016) uses long time series of aggregate data from the 1960s and a model with different inflation regimes to jointly model U.S. wage and price inflation. In contrast to Kumar and Orrenius (2016), he finds a relatively linear wage Phillips Curve, and a convex price inflation Phillips Curve. He reports finding "a sharp steepening of the (price) Phillips curve after labor market slack becomes sufficiently negative, so the effect of slack on inflation becomes much larger after labor markets tighten beyond a certain point."

## 3. Metro Level Data

Since it is difficult to identify convexity in the Phillips Curve using aggregate data covering the Great Moderation period, I exploit the greater time series and cross section variation in inflation and unemployment rates at the metro level. I use semi-annual core CPI inflation ($\pi^c$) and unemployment ($u$) data from the mid-1980's for a panel of 27 large U.S. metros. I also use quarterly data for about half of these metros. Inflation is measured as the deviation of the metro level, year-on-year core CPI inflation rate from the long-term (10-year) expected inflation rate in the Survey of Professional Forecasters ($\pi_m - \bar{\pi}^e$). Labor market slack is measured as the difference between the metro unemployment rate and the CBO's estimate of the natural rate of unemployment or NAIRU for the U.S. ($u_m - u^{NRU}$).

**4. Models and Results**

Inflation depends on expected long term inflation ($\overline{\pi}^e$), past inflation and lagged measures of labor market slack and lagged changes in slack. I estimate a variety of Phillips Curves with linear, linear spline and convex slack effects since theory does not specify the functional form of the Phillips Curve when it is convex. In the spline specifications, the two terms are $u_m - u^{NRU}$ and $\min(0, u_m - u^{NRU})$. In the non-linear specifications, convexity may be captured by using $\ln(u_m / u^{NRU})$ or $ugap_m / u$ as the slack terms.

Inter alia, I estimate a broader class of models, use higher frequency data and take account of more factors than other researchers do. For example, heterogeneous dynamic panel data models with multiple unobserved common factors are estimated. Some semi-annual estimation results are presented and discussed in the Appendix.

First, I find that the price Phillips Curve is still "alive", in the sense that labor market slack is always economically and statistically significant. In addition, there is no compelling evidence of a significant decline in the effect of slack on inflation in the metro-level dataset. Second, the fit of convex Phillips Curves is sometimes better than the fit of linear Phillips Curves. Third, despite this, the degree of convexity in the Phillips curve is modest, and is not economically significant.

**5. Does Convexity Matter**

Two related ways – one informal, the other more formal - of assessing the importance of the convexity of the Phillips Curve are considered. First, I check whether the estimated linear and convex Phillips Curves are very far apart when slack is negative (the unemployment rate is below the NAIRU)? The answer is no – the estimated linear and convex Phillips Curves are close when slack is negative in the historically relevant range, i.e. - 0% to -2% in the metro panel, and 0% to -1% at the aggregate level. Three different estimated Phillips Curves are plotted in **Figure 1** – a linear curve (the red line) and two convex curves (the blue and green lines). The unemployment gap is measured on the horizontal axis and the deviation of inflation from long term expected inflation on the vertical axis. When unemployment is low, the unemployment gap is negative and inflation is high. The rise in inflation is greater the more convex the Phillips Curve. Generally, when unemployment is relatively low, we observe (negative) slack values between 0 and -1%. Within this range, the differences in the inflation rates associated with the linear and convex Phillips Curves is very small, so the effect of convexity is not economically significant.

Second, I check whether the results of simulating an exogenous fall in slack in a simple, three equation IS-PC-MR model differ significantly when the Phillips Curve is linear vs. when it is convex? The dynamics of the IS curve are based on estimates from before the Great Recession. The two Phillips curve are based on the quarterly linear and convex (slack = $ugap_{m,t}/\ u_{m,t}$) DCCE estimates. An inertial Taylor Rule is used, with inertia coefficient of 0.85 and equal weights on the deviation of inflation from target and the unemployment gap. The NAIRU and inflation target are assumed to fixed, and inflation expectations are either constant or slowly adjusting. The results of simulating the effects of a temporary decline in the unemployment rate suggest that the degree of convexity in the Phillips Curve is modest (**Figure 2**). IIn the simulations, a short-term shock that reduces the unemployment rate by one percentage point boosts core CPI inflation by 30 basis points (bps) when the Phillips Curve is linear, and less than 40 bps when it is convex. If inflation expectations adjust modestly, the effects might be 15 bps higher. Similar results hold in more elaborate models.

**Conclusion**

The degree of convexity in the price Phillips Curve appears to be relatively small, and not economically significant. Labor market slack is always economically and statistically significant. Although the fit of convex Phillips Curves is sometimes better than the fit of linear curves, the degree of convexity is modest.

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**Table 1: Recent Studies of the Wage Phillips Curve in the U.S**

| Study | Data | Main Model | Finding |
|---|---|---|---|
| Daly & Hobijn (2014) | CPS y/y wage growth data, 1986 to 2012 | No model | **Suggestive** - Nominal downward rigidities increase in recessions; convex PC |
| Fisher & Koenig (2014) | Quarterly ECI wage & salary growth, 1984 Q1 to 2014 Q2 | Linear model with lagged level and inverse of unemployment rate | **Strong** - convex PC |
| Donayre & Panovska (2016) | Quarterly aggregate data; earnings of production & non-supervisory workers; 1965-1984 | Three regime threshold regression model depending on unemployment rate | **Strong** - convex PC with significantly different regime dynamics |
| Kumar & Orrenius (2016) | Annual state level CPS ORG average hourly wage, 1982 to 2013 | Fixed effects panel model with linear unemployment spline | **Strong** - convex PC |
| Nalewaik (2016) | Annual data, core PCE inflation and growth in non-farm business sector hourly compensation, 1961 to 2015 | Two equation, two regime Markov Switching model with squared low unemployment rate term; one regime is non-stationary | **Weak** - limited convexity in wage PC; |

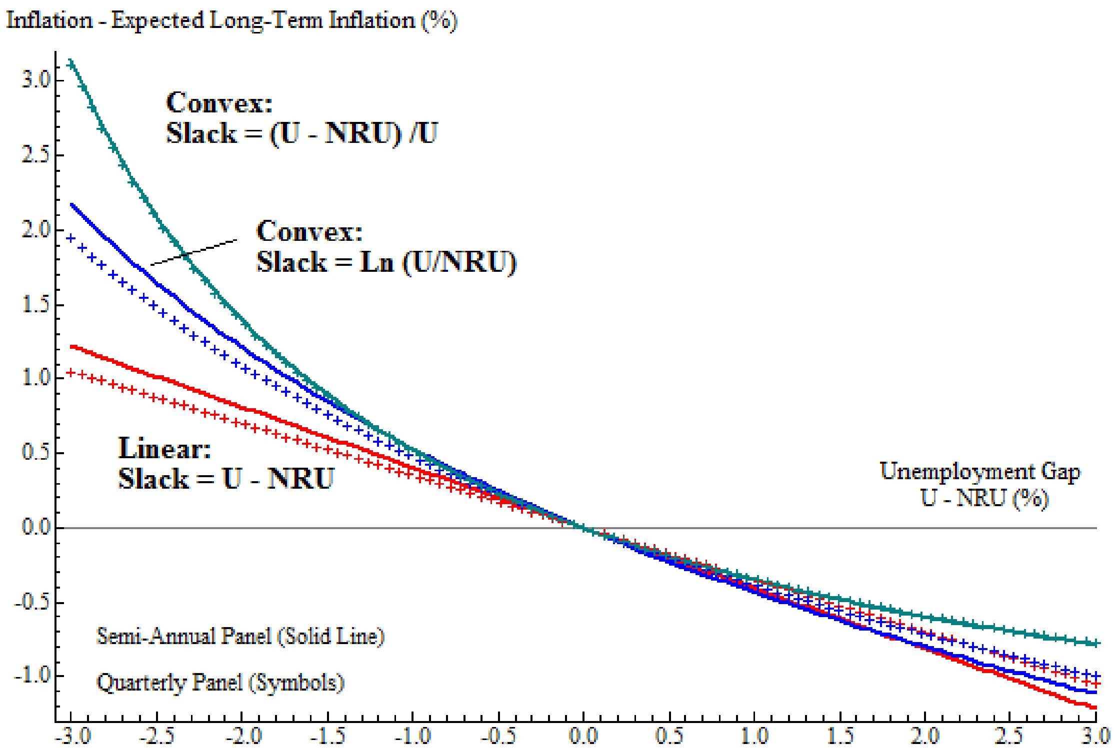
Notes: ECI = employment cost index, PC = Phillips Curve and y/y = year-over year. Additional evidence of downward nominal ECI wage rigidity is provided by Fallick, Lettau and Wascher (2016).

**Table 2: Recent Studies of the Price Phillips Curve in the U.S.**

| Study | Data and Sample | Main Model | Finding |
|---|---|---|---|
| Laxton, Rose & Tambakis (1999) | Quarterly CPI inflation, 1968 Q1 to 1997 Q1 | Two equation model; PC with time varying coefficient on convex unemployment gap term and random walk NRU | **Weak** - convex PC, but fit only marginally better than for linear model |
| Ball & Mazumder (2011) | Quarterly data, y/y headline and core (median) CPI, 1960q1 | Linear model where slope of PC varies with level and/or variance of inflation | **Mixed** - prefer model with varying slope to convex PC model; fit of linear and convex PC models similar |
| Nalewaik (2016) | Annual data, core PCE inflation and growth in non-farm business sector hourly compensation, 1961 to 2015 | Two equation, two regime Markov Switching model with squared low unemployment rate term; one regime is non-stationary | **Strong** - convex PC |
| Albuquerque & Baumann (2017) | Quarterly data, y/y PCE inflation, 1992 Q1 – 2015 Q1 | Time varying parameter model using unemployment gap and labor market tracking index etc. | **Weak** - prefer time varying parameter to convex PC model; fit of linear and convex PC models similar. |
| Detmeister & Babb (2017) | Annual metro data, core CPI inflation, 1984 to 2016 | Fixed effects panel model | **Weak** - some convexity but not economically significant |
| Murphy (2017) | Sem-annual and quarterly metro data, core CPI inflation, 1984 to 2016 | Fixed effects and dynamic correlated common effects panel models. | **Weak** - some convexity but not economically significant |

Notes: See Table 1.

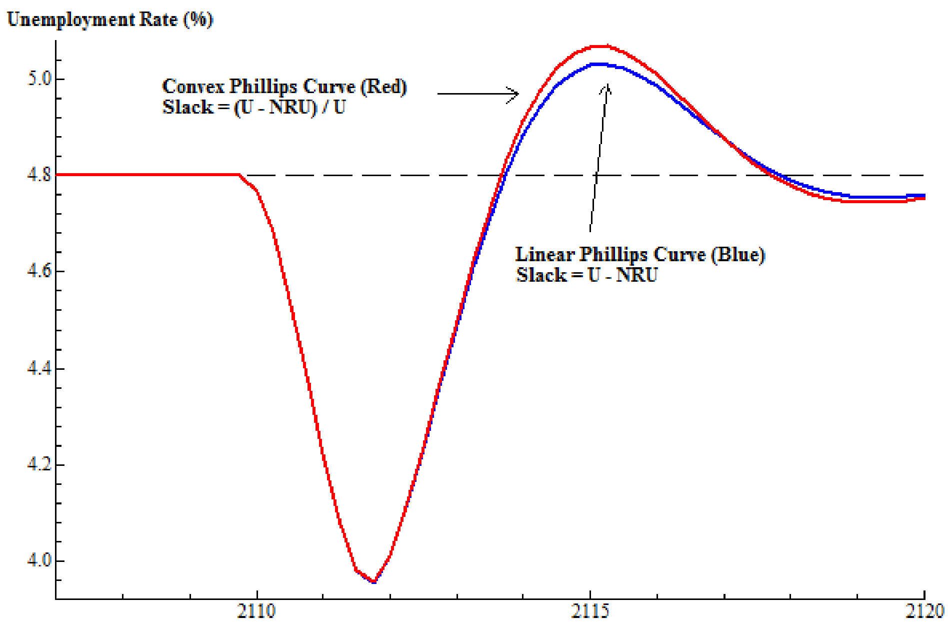
**Figure 1: Is the Convexity of the Phillips Curve Important?**



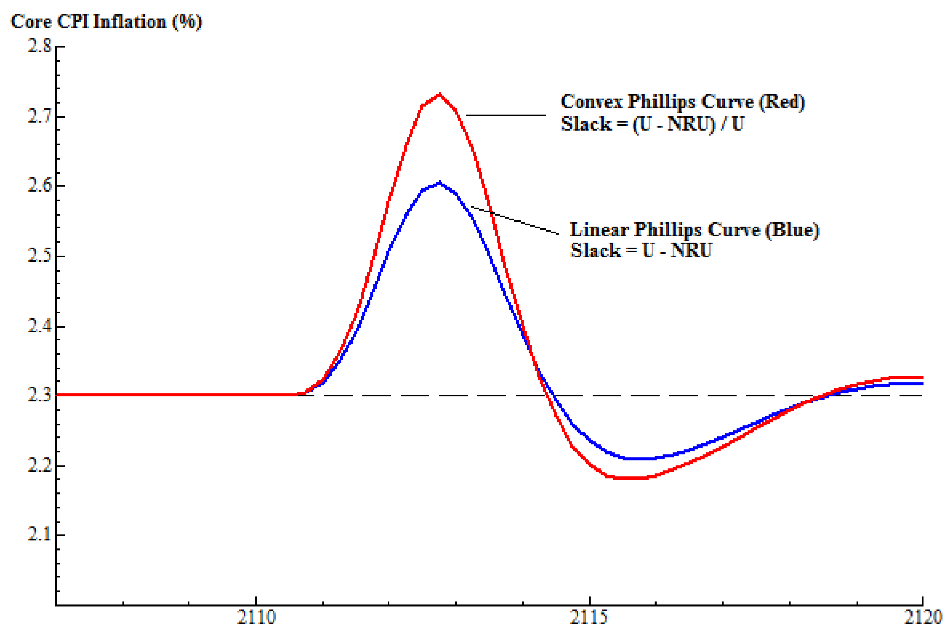

Source: Murphy (2017)

**Figure 2: Simulated Effects of a Temporary Fall in the Unemployment Rate**

**(a) Time Path of the Unemployment Rate**




**(b) Time Path of Inflation – Linear (Blue Line) and Convex (Red Line) Phillips Curves**




Note: Long-term inflation expectations are anchored at 2.3%. Source: Murphy (2017).

## Appendix: Some Econometric Results

### Data and Models

The effect of labor market slack on inflation are identified using the using time series and cross-section variation in unemployment and core CPI inflation rates at the metro level. The models are formulated in term of the deviations of inflation from survey based, long run expected inflation and the deviation of the unemployment rate from the NAIRU.

- $\tilde{\pi}^c_{m,t} = \pi^c_{m,t} - \bar{\pi}^e_t$ = Devistion of core year-on-year CPI inflation in metro *m* from long-term expected inflation in the Survey of Professional Forecasters (SPF).
- $ugap_{m,t} = u_{m,t} - u^{NRU}_t$ = Unemployment gap, the deviation from the CBO's natural rate of unemployment or NAIRU.
- $ugap^{neg}_{m,t} = min(0, ugap_{m,t})$ = Negative unemployment gap (i.e. tight labor market).

Models with linear, linear spline and convex labor market slack effects estimate. The base linear spline model is:

$$\tilde{\pi}^c_{m,t} = \beta_0 + \beta_1\tilde{\pi}^c_{m,t-1} + \beta_2\tilde{\pi}^c_{m,t-2} + \beta_3 ugap_{m,t-2} + \beta_4 ugap^{neg}_{m,t-2} + \beta_5 \Delta u_{m,t-2} + v_{m,t}$$

where:

$\tilde{\pi}^c_{m,t} = \pi^c_{m,t} - \bar{\pi}^e_t$ = Devistion of core year-on-year CPI inflation in metro *m* from long-term expected inflation in the Survey of Professional Forecasters (SPF).
$ugap_{m,t} = u_{m,t} - u^{NRU}_t$ = Unemployment gap, the deviation from the CBO's natural rate of unemployment or NAIRU.
$ugap^{neg}_{m,t} = min(0, ugap_{m,t})$ = Negative unemployment gap (i.e. tight labor market).

The model is an expectations augmented Phillips Curve, as opposed to a New Keynesian Phillips Curve, with priors: $\beta_3 < 0, \beta_4 < 0 \left(since\ ugap^{neg}_{m,t} < 0\right)\ and\ \ \beta_5 < 0.$ The data are I(0), and the choice of lags is based on limited pre-searching. Other convex specifications for the effect of slack use $ugap_{m,t}$ and $ugap^{neg}_{m,t}$ squared as in Nalewaik (2016), $ugap_{m,t}/\ u_{m,t}$ as in Debelle and Vickery (1998), or log slack, $ln(u_{m,t}/u^{NRU}_t)$.

The Phillips Curves are estimated using pooled OLS, one and two-way fixed effects and dynamic common correlated effects (DCCE) estimators. The DCCE estimator (Chudik and Pesaran, 2015) is the most general one and has many advantages:

$$\tilde{\pi}^{core}_{m,t} = \beta_{m,0} + \beta_{m,1}\tilde{\pi}^{core}_{m,t-1} + \beta_{m,2}\tilde{\pi}^{core}_{m,t} + \beta_{m,3}(ugap_{m,t-2}/u_{m,t-2}) + \sum_j \gamma_{m,j} f_{j,t} + v_{m,t}$$

It provides consistent estimates of the mean effects in dynamic, heterogeneous panel data models with weakly exogenous variables and cross section dependence. The cross section dependence is modelled in a flexible way (as unobserved factors $f_{j,t}$), which are "partialed out" by adding current and lagged cross section averages of the dependent regressors and other related covariates to the individual equations.

Some representative regression results are set out in **Table A**. Consider the linear spline results initially. The pooled OLS and FE results are very similar - inflation is highly persistent; lagged labor market slack and changes in slack are economically and statistically significant. The linear spline term in lagged slack is significant suggesting that the Phillips Curve is convex.

However, the pooled OLS and FE results do not account of any common omitted factors, such as imported core goods inflation, driving metro-level inflation. The DCCE results, which do, are rather different. Inflation is not as persistent and lagged labor market slack, but not the lagged change in slack, is significant. The spline term is insignificant, which suggests that the Phillips Curve is linear. Other convex specifications need to be examined before reaching this conclusion. The fit of the two convex models is about the same as that of the linear / linear spline models.

Similar results are obtained using quarterly data for approx. 13 metros and in sub-samples. Lagged labor market slack is always economically and statistically significant. The linear spline term is also insignificant in the DCCE results. Convex Phillips Curve models fit marginally better.

The effects of slack are fairly stable in the sub-samples. Changes in lagged slack are also statistically significant in the quarterly models, but are hard to identify in the sub-samples. Results hold up to various robustness checks – breaks in CPS-based unemployment series, threshold effects, alternative measures of expected inflation etc.

**Table A: Linear Spline and Convex Phillips Curve Specifications**

Dependent Variable: $\tilde{\pi}^c_{m,t} = \pi^c_{m,t} - \bar{\pi}_t$. Sample: 24 to 27 Metros, 1985 or 1986 H1 to 2016 H2 (Semi-Annual).

| Regressors | Linear Spline in $ugap_m$ | | | Slack = $ugap_m/u_m$ | | | Slack = $ln(u_m/u^{NRU})$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | OLS | FE | DCCE | OLS | FE | DCCE | OLS | FE | DCCE |
| $\tilde{\pi}^c_{m,t-1}$ | 0.867*** | 0.836*** | 0.670*** | 0.866*** | 0.835*** | 0.679*** | 0.872*** | 0.843*** | 0.672*** |
| | (0.027) | (0.025) | (0.032) | (0.027) | (0.025) | (0.033) | (0.028) | (0.025) | (0.032) |
| $\tilde{\pi}^c_{m,t-2}$ | -0.244*** | -0.256*** | -0.331*** | -0.244*** | -0.256*** | -0.334*** | -0.241*** | -0.252*** | -0.336*** |
| | (0.023) | (0.022) | (0.030) | (0.024) | (0.022) | (0.024) | (0.024) | (0.023) | (0.027) |
| $ugap_{m,t-2}$ | -0.017 | -0.032** | -0.319*** | -0.471*** | -0.633*** | -1.587*** | - | - | - |
| | (0.012) | (0.012) | (0.053) | (0.069) | (0.064) | (0.275) | - | - | - |
| $ugap^{NEG}_{m,t-2}$ | -0.195*** | -0.235*** | - | -0.170*** | -0.160*** | - | - | - | - |
| | (0.036) | (0.039) | - | (0.030) | (0.021) | - | - | - | - |
| $\Delta u_{m,t-2}$ | -0.172*** | -0.163*** | - | -0.170*** | -0.160*** | - | | | |
| | (0.029) | (0.021) | - | (0.030) | (0.021) | - | | | |
| $ugap_{m,t-2}/u_{m,t-2}$ | - | - | - | -0.471*** | -0.633*** | -1.587*** | | | |
| | - | - | - | (0.069) | (0.064) | (0.275) | - | - | - |
| $\ln(u_{m,t-2}/u^{NRU}_{t-2})$ | - | - | - | - | - | - | -0.423*** | -0.564*** | -1.763*** |
| | - | - | - | - | - | - | (0.072) | (0.065) | (0.275) |
| $\Delta \ln u_{m,t-2}$ | - | - | - | - | - | - | -1.093*** | -1.027*** | - |
| | - | - | - | - | - | - | (0.206) | (0.159) | - |
| Metro Fixed effects | - | Yes | - | - | Yes | - | - | Yes | - |
| Adjusted $R^2$ | 0.621 | 0.609 | 0.620 | 0.620 | 0.607 | 0.607 | 0.618 | 0.604 | 0.615 |
| SE | 0.671 | 0.661 | 0.607 | 0.672 | 0.662 | 0.612 | 0.674 | 0.665 | 0.608 |
| No of Observations | 1679 | 1679 | 1599 | 1679 | 1679 | 1599 | 1679 | 1679 | 1599 |

Notes: Standard errors are shown in parentheses. The superscripts *, ** and *** denote significance at the 10%, 5% and 1% levels respectively. FE denotes fixed effects estimators. The dynamic correlated common effects (DCCE) estimates use three lags of the cross section averages. Source: Murphy (2017).