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DIVISION OF MONETARY AFFAIRS
FOMC SECRETARIAT

Date: December 4, 2017
To: Federal Open Market Committee
From: James A. Clouse
Subject: Supporting Documents for DSGE Models Update

The attached documents support the update on the projections of the DSGE models.

The Current Outlook in EDO:
December 2017 FOMC Meeting
Class II FOMC – Restricted (FR)

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November 30, 2017

1 The EDO Forecast from 2018 to 2020

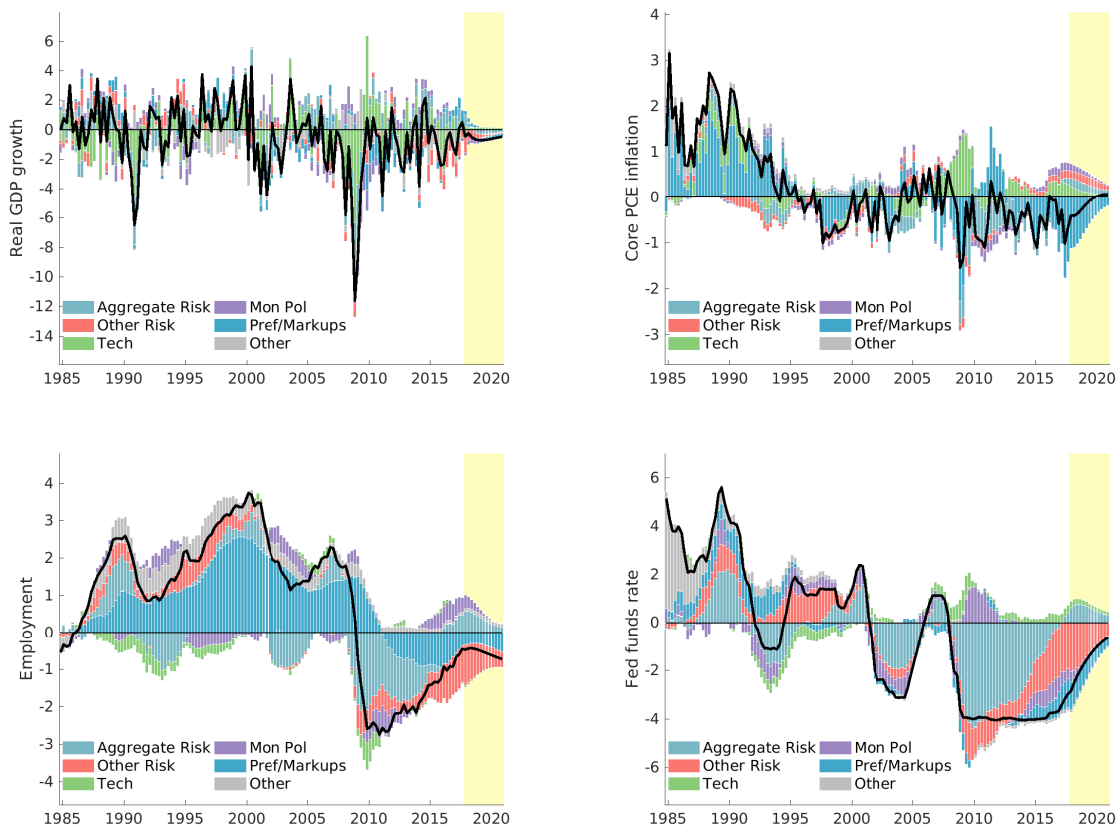
The EDO models forecast is conditional on data through the third quarter of 2017 and on a preliminary Tealbook forecast for the fourth quarter of 2017.

Real GDP growth is 2.6 percent, on average, over the projection horizon, somewhat below its long-run value of 3 percent. Inflation reaches the Committee's 2 percent objective in the fourth quarter of 2019 and then slightly overshoots the target thereafter. Below-trend real GDP growth is driven by the slow fading of risk premium shocks and accommodative monetary policy. For inflation, the EDO model interprets the weakness in inflation over the past few years as driven by negative wage markup shocks and expects them to dissipate only gradually over the projection horizon.

The output gap is estimated to be currently negative 1 percent. The output gap closes very slowly and remains at negative 0.3 percent by the end of 2020. The real natural rate of interest is projected to increase from 1 percent in the fourth quarter of 2017 to 1.5 percent at the end of 2020, 0.6 percentage point below its steady-state value of 2.1 percent. According to the EDO model, capital-specific risk premium shocks inferred from a combination of weaker-than-expected investment and output data with stronger-than-expected consumption data over the past several years have been holding down the output gap and the real natural rate. As these shocks slowly dissipate, the output gap closes and the real natural rate rises. Consistent with the gradual return of inflation and the output gap to their long-run values, the federal funds rate is projected to increase gradually over the forecast horizon, reaching 3.5 percent by the end of 2020. At the end of the projection horizon, the federal funds rate is still below its long-run value of 4.1 percent, reflecting the inertia in the policy rule and the persistently negative output gap even at the end of the projection horizon.

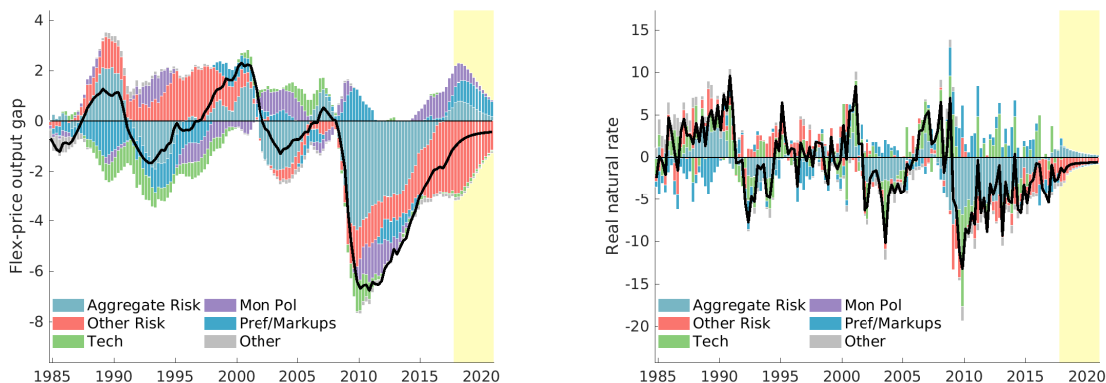
*Cristina Fuentes-Albero is affiliated with the Division of Research and Statistics of the Federal Reserve Board. Sections 2 and 3 contain background material on the EDO model, as in previous rounds. These sections were co-written with Hess Chung and Jean-Philippe Laforte.

Figure 1: Recent History and Forecasts



The EDO models projection of real GDP growth in this round is slower for the next three years than it was in September 2017. The downward revision in the real GDP growth projection is mostly driven by risk premium shocks. Core PCE inflation is, on average, 4 basis points lower over the forecast horizon in this round than in September, also resulting from more negative wage markup shocks. The output gap has revised down, on average, 13 basis points since September. The projection of the real natural rate of interest has been revised down 18 basis points, on average, since September. And, consistent with the lower inflation path, the path of the federal funds rate is lower this round than in September.

Figure 2: Recent History and Forecasts: Latent Variables



2 An Overview of Key Model Features

Figure 3 provides a graphical overview of the model. While similar to most related models, EDO has a more detailed description of production and expenditure than most other models.¹

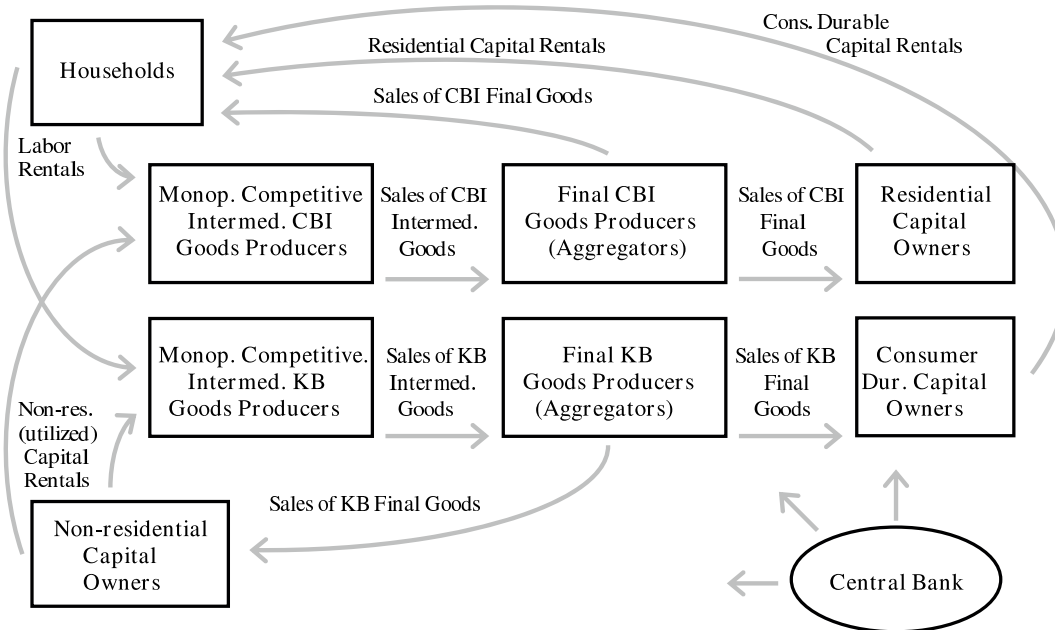
Specifically, the model possesses two final good sectors in order to capture key long-run growth facts and to differentiate between the cyclical properties of different categories of durable expenditure (for example, housing, consumer durables, and nonresidential investment). For example, technological progress has been faster in the production of business capital and consumer durables (such as computers and electronics).

The disaggregation of production (aggregate supply) leads naturally to some disaggregation of expenditures (aggregate demand). We move beyond the typical model with just two categories of (private domestic) demand (consumption and investment) and distinguish between four categories of private demand: consumer nondurable goods and nonhousing services, consumer durable goods, residential investment, and nonresidential investment. The boxes surrounding the producers in the figure illustrate how we structure the sources of each demand category. Consumer nondurable goods and services are sold directly to households; consumer durable goods, residential capital goods, and nonresidential capital goods are intermediated through capital-goods intermediaries (owned by the households), who then rent these capital stocks to households. Consumer nondurable goods and services and residential capital goods are purchased (by households and residential capital goods owners, respectively) from the first of economy’s two final goods-producing sectors, while consumer durable goods and nonresidential capital goods are purchased (by consumer durable and residential capital goods owners, respectively) from the second sector. In addition to consuming the nondurable goods and services that they purchase, households supply labor to the intermediate goods-producing firms in both sectors of the economy.

The remainder of this section provides an overview of the main properties of the model. In

¹Chung, Kiley, and Laforte (2010) provide much more detail regarding the model specification, estimated parameters, and model properties.

Figure 3: Model Overview



particular, the model has five key features:

- A New-Keynesian structure for price and wage dynamics. Unemployment measures the difference between the amount workers are willing to be employed and firms’ employment demand. As a result, unemployment is an indicator of wage and, hence, price pressures as in Galí (2011).
- Production of goods and services occurs in two sectors, with differential rates of technological progress across sectors. In particular, productivity growth in the investment and consumer durable goods sector exceeds that in the production of other goods and services, helping the model match facts regarding long-run growth and relative price movements.
- A disaggregated specification of household preferences and firm production processes that leads to separate modeling of nondurables and services consumption, durables consumption, residential investment, and business investment.
- Risk premiums associated with different investment decisions play a central role in the model. These include, first, an aggregate risk premium, or natural rate of interest, shock driving a wedge between the short-term policy rate and the interest rate faced by private decisionmakers

(as in Smets and Wouters (2007)) and, second, fluctuations in the discount factor/risk premiums faced by the intermediaries financing household (residential and consumer durable) and business investment.

2.1 Two-sector production structure

It is well known (for example, Edge, Kiley, and Laforte (2008)) that real outlays for business investment and consumer durables have substantially outpaced those on other goods and services, while the prices of these goods (relative to others) has fallen. For example, real outlays on consumer durables have far outpaced those on other consumption while prices for consumer durables have been flat and those for other consumption have risen substantially; as a result, the ratio of nominal outlays in the two categories has been much more stable, although consumer durable outlays plummeted in the Great Recession. Many models fail to account for this fact.

EDO accounts for this development by assuming that business investment and consumer durables are produced in one sector and other goods and services in another sector. Specifically, production by firm j in each sector s (where s equals kb for the sector producing business investment and consumer durables and cbi for the sector producing other goods and services) is governed by a Cobb-Douglas production function with sector-specific technologies:

$$X_t^s(j) = (Z_t^m Z_t^s L_t^s(j))^{1-\alpha} (K_t^{u,nr,s}(j))^\alpha, \text{ for } s = cbi, kb. \quad (1)$$

In 1, Z^m represents (labor-augmenting) aggregate technology, while Z^s represents (labor-augmenting) sector-specific technology; we assume that sector-specific technological change affects the business investment and consumer durables sector only. L^s is labor input and $K^{u,nr,s}$ is capital input (that is, utilized *nonresidential business* capital (and hence the nr and u terms in the superscript)). Growth in this sector-specific technology accounts for the long-run trends, while high-frequency fluctuations allow for the possibility that investment-specific technological change is a source of business cycle fluctuations, as in Fisher (2006).

2.2 The structure of demand

EDO differentiates between several categories of expenditure. Specifically, business investment spending determines nonresidential capital used in production, and households value consumer non-durables goods and services, consumer durable goods, and residential capital (for example, housing). Differentiation across these categories is important, as fluctuations in these categories of expenditure can differ notably, with the cycles in housing and business investment, for example, occurring at different points over the last three decades.

Valuations of these goods and services, in terms of household utility, is given by the following utility function:

$$\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \zeta^{cnn} \ln(E_t^{cnn}(i) - hE_{t-1}^{cnn}(i)) + \zeta^{cd} \ln(K_t^{cd}(i)) + \zeta^r \ln(K_t^r(i)) - \Lambda_t^{Lpref} \Theta_t^H \sum_{s=cbi, kb} \int_0^1 \zeta^{l,s} L_t^s(i)^{\frac{1+\sigma_N}{1+\sigma_h}} di \right\}, \quad (2)$$

where E^{cnn} represents expenditures on consumption of nondurable goods and services, K^{cd} and K^r represent the stocks of consumer durables and residential capital (housing), Λ_t^{Lpref} represents a labor supply shock, Θ_t is an endogenous preference shifter whose role is to reconcile the existence of a long-run balance growth path with a small short-term wealth effect², L^{cbi} and L^{kb} represent the labor supplied to each productive sector (with hours worked causing disutility), and the remaining terms represent parameters (such as the discount factor, relative value in utility of each service flow, and the elasticity of labor supply). Gali, Smets, and Wouters (2011) state that the introduction of the endogenous preference shifter is key in order to match the joint behavior of the labor force, consumption, and wages over the business cycle.

By modeling preferences over these disaggregated categories of expenditure, EDO attempts to account for the disparate forces driving consumption of nondurables and durables, residential investment, and business investment —thereby speaking to issues such as the surge in business investment in the second half of the 1990s or the housing cycle in the early 2000s recession and the most recent downturn. Many other models do not distinguish between developments across these categories of spending.

2.3 Risk premiums, financial shocks, and economic fluctuations

The structure of the EDO model implies that households value durable stocks according to their expected returns, including any expected service flows, and according to their risk characteristics, with a premium on assets that have high expected returns in adverse states of the world. However, the behavior of models such as EDO is conventionally characterized under the assumption that this second component is negligible. In the absence of risk adjustment, the model would then imply that households adjust their portfolios until expected returns on all assets are equal.

Empirically, however, this risk adjustment may not be negligible and, moreover, there may be a variety of factors, not explicitly modeled in EDO, that limit the ability of households to arbitrage away expected return differentials across different assets. To account for this possibility, EDO features several exogenous shocks to the rates of return required by the household to hold the assets in question. Following such a shock —an increase in the premium on a given asset, for example —households will wish to alter their portfolio composition to favor the affected asset, leading to changes in the prices of all assets and, ultimately, to changes in the expected path of production underlying these claims.

²The endogenous preference shifter is defined as $\Theta_t^H = Z_t \Lambda_t^{cnn}$, where $Z_t = \frac{Z_{t-1}^{1-\nu}}{\Lambda_t^{cnn}}$ and Λ_t^{cnn} is the shadow price of nondurable consumption. The importance of the short-term wealth effect is determined by the parameter $\nu \in (0, 1]$.

The “sector specific” risk shocks affect the composition of spending more than the path of GDP itself. This occurs because a shock to these premiums leads to sizable substitution across residential, consumer durable, and business investment; for example, an increase in the risk premiums on residential investment leads households to shift away from residential investment and toward other types of productive investment. Consequently, it is intuitive that a large fraction of the non-cyclical, or idiosyncratic, component of investment flows to physical stocks will be accounted for by movements in the associated premiums.

Shocks to the required rate of return on the nominal risk-free asset play an especially large role in EDO. Following an increase in the premium, in the absence of nominal rigidities, the households’ desire for higher real holdings of the risk-free asset would be satisfied entirely by a fall in prices, that is, the premium is a shock to the natural rate of interest. Given nominal rigidities, however, the desire for higher risk-free savings must be offset, in part, through a fall in real income, a decline which is distributed across all spending components. Because this response is capable of generating co-movement across spending categories, the model naturally exploits such shocks to explain the business cycle. Reflecting this role, we denote this shock as the “aggregate risk-premium.”

Movements in financial markets and economic activity in recent years have made clear the role that frictions in financial markets play in economic fluctuations. This role was apparent much earlier, motivating a large body of research (for example, Bernanke, Gertler, and Gilchrist (1999)). While the range of frameworks used to incorporate such frictions has varied across researchers studying different questions, a common theme is that imperfections in financial markets—for example, related to imperfect information on the outlook for investment projects or earnings of borrowers—drives a wedge between the cost of riskless funds and the cost of funds facing households and firms. Much of the literature on financial frictions has worked to develop frameworks in which risk premiums fluctuate for endogenous reasons (for example, because of movements in the net worth of borrowers). Because the risk-premium shocks induces a wedge between the short-term nominal risk-free rate and the rate of return on the affected risky rates, these shocks may thus also be interpreted as a reflection of financial frictions not explicitly modeled in EDO. The sector-specific risk premiums in EDO enter the model in much the same way as does the exogenous component of risk premiums in models with some endogenous mechanism (such as the financial accelerator framework used Boivin, Kiley, and Mishkin (2010)), and the exogenous component is quantitatively the most significant one in that research.³

2.4 Labor market dynamics in the EDO model

This version of the EDO model assumes that labor input consists of both employment and hours per worker. Workers differ in the disutility they associate with employment. Moreover, the labor market is characterized by monopolistic competition. As a result, unemployment arises in equilibrium – some workers are willing to be employed at the prevailing wage rate, but cannot find employment because firms are unwilling to hire additional workers at the prevailing wage.

³Specifically, the risk premiums enter EDO to a first-order (log)linear approximation in the same way as in the cited research if the parameter on net worth in the equation determining the borrowers cost of funds is set to zero; in practice, this parameter is often fairly small in financial accelerator models.

As emphasized by Gali (2011), this framework for unemployment is simple and implies that the unemployment rate reflects wage pressures: When the unemployment rate is unusually high, the prevailing wage rate exceeds the marginal rate of substitution between leisure and consumption, implying that workers would prefer to work more.

The new preference specification and the incorporation of labor force participation in the information set impose discipline in the overall labor market dynamics of the EDO model. The estimated short-run wealth effect on labor supply is relatively attenuated with respect to previous versions of the EDO model. Therefore, the dynamics of both labor force participation and employment are more aligned with the empirical evidence.

In addition, in our environment, nominal wage adjustment is sticky, and this slow adjustment of wages implies that the economy can experience sizable swings in unemployment with only slow wage adjustment. Our specific implementation of the wage adjustment process yields a relatively standard New Keynesian wage Phillips curve. The presence of both price and wage rigidities implies that stabilization of inflation is not, in general, the best possible policy objective (although a primary role for price stability in policy objectives remains).

While the specific model on the labor market is suitable for discussion of the links between employment and wage/price inflation, it leaves out many features of labor market dynamics. Most notably, it does not consider separations, hires, and vacancies, and is hence not amenable to analysis of issues related to the Beveridge curve.

The decline in employment during the Great Recession primarily reflected, according to the EDO model, the weak demand that arose from elevated risk premiums that depressed spending, as illustrated by the light blue and red bars in figure 1. The role played by these demand factors in explaining the cyclical movements in employment is only determinant during the 1980s and during the Great Recession. As apparent in figure 1, the most relevant drivers of employment in the remaining of the sample are labor supply (preference) and markup shocks as shown by the blue bars. Specifically, favorable supply developments in the labor market are estimated to have placed upward pressure on employment until 2010; these developments have reversed, and some of the currently low level for employment growth is, according to EDO, attributable to adverse labor market supply developments. As discussed previously, these developments are simply exogenous within EDO and are not informed by data on a range of labor market developments (such as gross worker flows and vacancies).

2.5 New Keynesian price and wage Phillips curves

As in most of the related literature, nominal prices and wages are both “sticky” in EDO. This friction implies that nominal disturbances—that is, changes in monetary policy—have effects on real economic activity. In addition, the presence of both price and wage rigidities implies that stabilization of inflation is not, in general, the best possible policy objective (although a primary role for price stability in policy objectives remains).

Given the widespread use of the New Keynesian Phillips curve, it is perhaps easiest to consider the form of the price and wage Phillips curves in EDO at the estimated parameters. The price

Phillips curve (governing price adjustment in both productive sectors) has the form

$$\pi_t^{p,s} = 0.22\pi_{t-1}^{p,s} + 0.76E_t\pi_{t+1}^{p,s} + .017mc_t^s + \theta_t^s \quad (3)$$

where mc is marginal cost and θ is a markup shock. As the parameters indicate, inflation is primarily forward looking in EDO.

The wage (w) Phillips curve for each sector has the form

$$\Delta w_t^s = 0.01\Delta w_{t-1}^s + 0.95E_t\Delta w_{t+1}^s + .012\left(mrs_t^{c,l} - w_t^s\right) + \theta_t^w + adj. costs. \quad (4)$$

where mrs represents the marginal rate of substitution between consumption and leisure. Wages are primarily forward looking and relatively insensitive to the gap between households' valuation of time spent working and the wage.

The top right panel of figure 1 presents the decomposition of inflation fluctuations into the exogenous disturbances that enter the EDO model. As can be seen, aggregate demand fluctuations, including aggregate risk premiums and monetary policy surprises, contribute little to the fluctuations in inflation according to the model. This is not surprising: In modern DSGE models, transitory demand disturbances do not lead to an unmooring of inflation (so long as monetary policy responds systematically to inflation and remains committed to price stability). In the short run, inflation fluctuations primarily reflect transitory price and wage shocks, or markup shocks in the language of EDO. Technological developments can also exert persistent pressure on costs, most notably during and following the strong productivity performance of the second half of the 1990s, which is estimated to have lowered marginal costs and inflation through the early 2000s. More recently, disappointing labor productivity readings over the course of 2011 have led the model to infer sizable negative technology shocks in both sectors, contributing noticeably to inflationary pressure over that period (as illustrated by the blue bars in figure 1).

2.6 Monetary authority and a long-term interest rate

We now turn to the last agent in our model, the monetary authority. It sets monetary policy in accordance with an Taylor-type interest rate feedback rule. Policymakers smoothly adjust the actual interest rate R_t to its target level \bar{R}_t

$$R_t = (R_{t-1})^{\rho^r} (\bar{R}_t)^{1-\rho^r} \exp[\epsilon_t^r], \quad (5)$$

where the parameter ρ^r reflects the degree of interest rate smoothing, while ϵ_t^r represents a monetary policy shock. The central bank's target nominal interest rate, \bar{R}_t depends the deviation of output from the level consistent with current technologies and “normal” (steady-state) utilization of capital and labor (\tilde{X}^{pf} , the “production function” output gap). Consumer price inflation also enters the target. The target equation is

$$\bar{R}_t = \left(\tilde{X}_t^{pf} \right)^{r^y} \left(\frac{\Pi_t^c}{\bar{\Pi}_*^c} \right)^{r^\pi} R_*. \quad (6)$$

In equation (6), R_* denotes the economy's steady-state nominal interest rate, and r^y and r^π denote the weights in the feedback rule. Consumer price inflation, Π_t^c , is the weighted average of inflation in the nominal prices of the goods produced in each sector, $\Pi_t^{p,cbi}$ and $\Pi_t^{p,kb}$:

$$\Pi_t^c = (\Pi_t^{p,cbi})^{1-w_{cd}} (\Pi_t^{p,kb})^{w_{cd}}. \quad (7)$$

The parameter w^{cd} is the share of the durable goods in nominal consumption expenditures.

The model also includes a long-term interest rate (RL_t), which is governed by the expectations hypothesis subject to an exogenous term premiums shock:

$$RL_t = \mathcal{E}_t [\Pi_{\tau=0}^N R_\tau] \cdot \Upsilon_t. \quad (8)$$

where Υ is the exogenous term premium, governed by

$$Ln(\Upsilon_t) = (1 - \rho^\Upsilon) Ln(\Upsilon_*) + \rho^\Upsilon Ln(\Upsilon_{t-1}) + \epsilon_t^\Upsilon. \quad (9)$$

In this version of EDO, the long-term interest rate plays no allocative role; nonetheless, the term structure contains information on economic developments useful for forecasting (for example, Edge, Kiley, and Laforte (2010)), and hence RL is included in the model and its estimation.

2.7 Summary of model specification

Our brief presentation of the model highlights several points. First, although our model considers production and expenditure decisions in a bit more detail, it shares many similar features with other DSGE models in the literature, such as imperfect competition, nominal price and wage rigidities, and real frictions like adjustment costs and habit-persistence. The rich specification of structural shocks (to aggregate and investment-specific productivity, aggregate and sector-specific risk premiums, and markups) and adjustment costs allows our model to be brought to the data with some chance of finding empirical validation.

Within EDO, fluctuations in all economic variables are driven by 13 structural shocks. It is most convenient to summarize these shocks into five broad categories:

- Permanent technology shocks: This category consists of shocks to aggregate and investment-specific (or fast-growing sector) technology.
- A labor supply shock: This shock affects the willingness to supply labor. As was apparent in our earlier description of labor market dynamics and in the presentation of the structural drivers below, this shock captures the dynamics of the labor force participation rate in the sample and those of employment. While EDO labels such movements labor supply shocks, an alternative interpretation would describe these as movements in the labor force and employment that reflect structural features not otherwise captured by the model.

- Financial, or intertemporal, shocks: This category consists of shocks to risk premiums. In EDO, variation in risk premiums —both the premium households receive relative to the federal funds rate on nominal bond holdings and the additional variation in discount rates applied to the investment decisions of capital intermediaries —are purely exogenous. Nonetheless, the specification captures aspects of related models with more explicit financial sectors (for example, Bernanke, Gertler, and Gilchrist (1999)), as we discuss in our presentation of the model’s properties below.
- Markup shocks: This category includes the price and wage markup shocks.
- Other demand shocks: This category includes the shock to autonomous demand and a monetary policy shock.

3 Estimation: Data and Properties

3.1 Data

The empirical implementation of the model takes a log-linear approximation to the first-order conditions and constraints that describe the economy’s equilibrium, casts this resulting system in its state-space representation for the set of (in our case, 13) observable variables, uses the Kalman filter to evaluate the likelihood of the observed variables, and forms the posterior distribution of the parameters of interest by combining the likelihood function with a joint density characterizing some prior beliefs. Since we do not have a closed-form solution of the posterior, we rely on Markov-Chain Monte Carlo (MCMC) methods.

The model is estimated using 13 data series over the sample period from 1984:Q4 to 2015:Q3. The series are the following:

1. The growth rate of real gross domestic product (ΔGDP);
2. The growth rate of real consumption expenditure on nondurables and services (ΔC);
3. The growth rate of real consumption expenditure on durables (ΔCD);
4. The growth rate of real residential investment expenditure (ΔRes);
5. The growth rate of real business investment expenditure (ΔI);
6. Consumer price inflation, as measured by the growth rate of the Personal Consumption Expenditure (PCE) price index ($\Delta P_{C,total}$);
7. Consumer price inflation, as measured by the growth rate of the PCE price index excluding food and energy prices ($\Delta P_{C,core}$);
8. Inflation for consumer durable goods, as measured by the growth rate of the PCE price index for durable goods (ΔP_{cd});
9. Hours, which equals hours of all persons in the nonfarm business sector from the Bureau of Labor Statistics (H);
10. Civilian employment-population ratio, defined as civilian employment from the Current Population Survey (household survey) divided by the noninstitutional population, age 16 and over (N);

11. Labor force participation rate;
12. The growth rate of real wages, as given by compensation per hour in the non-farm business sector from the Bureau of Labor Statistics divided by the GDP price index (ΔRW); and
13. The federal funds rate (R).

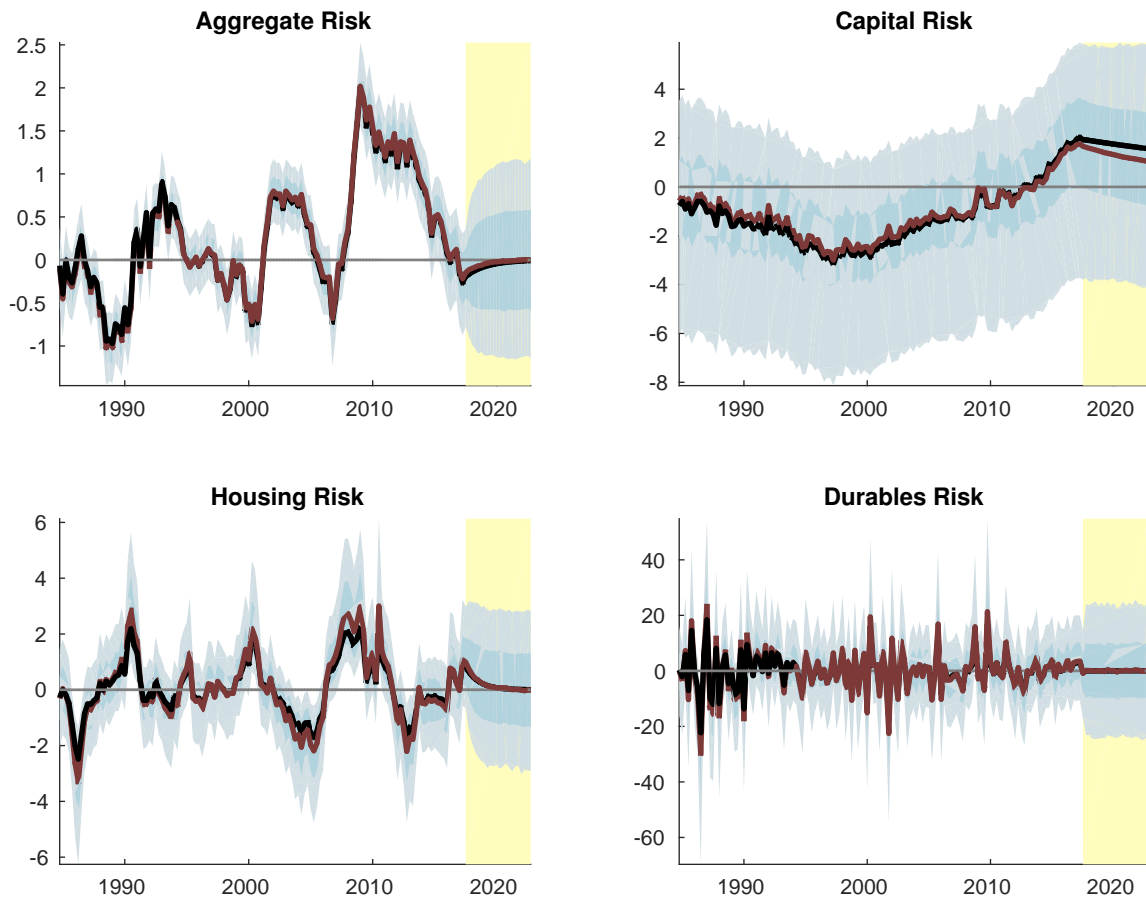
Our implementation adds measurement error processes to the likelihood implied by the model for all of the observed series used in estimation except the short-term nominal interest rate series.

3.2 Estimates of latent variable paths

Figures 4, 5, and 6 report estimates of the model's persistent exogenous fundamentals (for example, risk premiums and autonomous demand). These series have recognizable patterns for those familiar with U.S. economic fluctuations. For example, the risk premiums jump at the end of 2008, reflecting the financial crisis and the model's identification of risk premiums, both economy-wide and for housing, as key drivers.

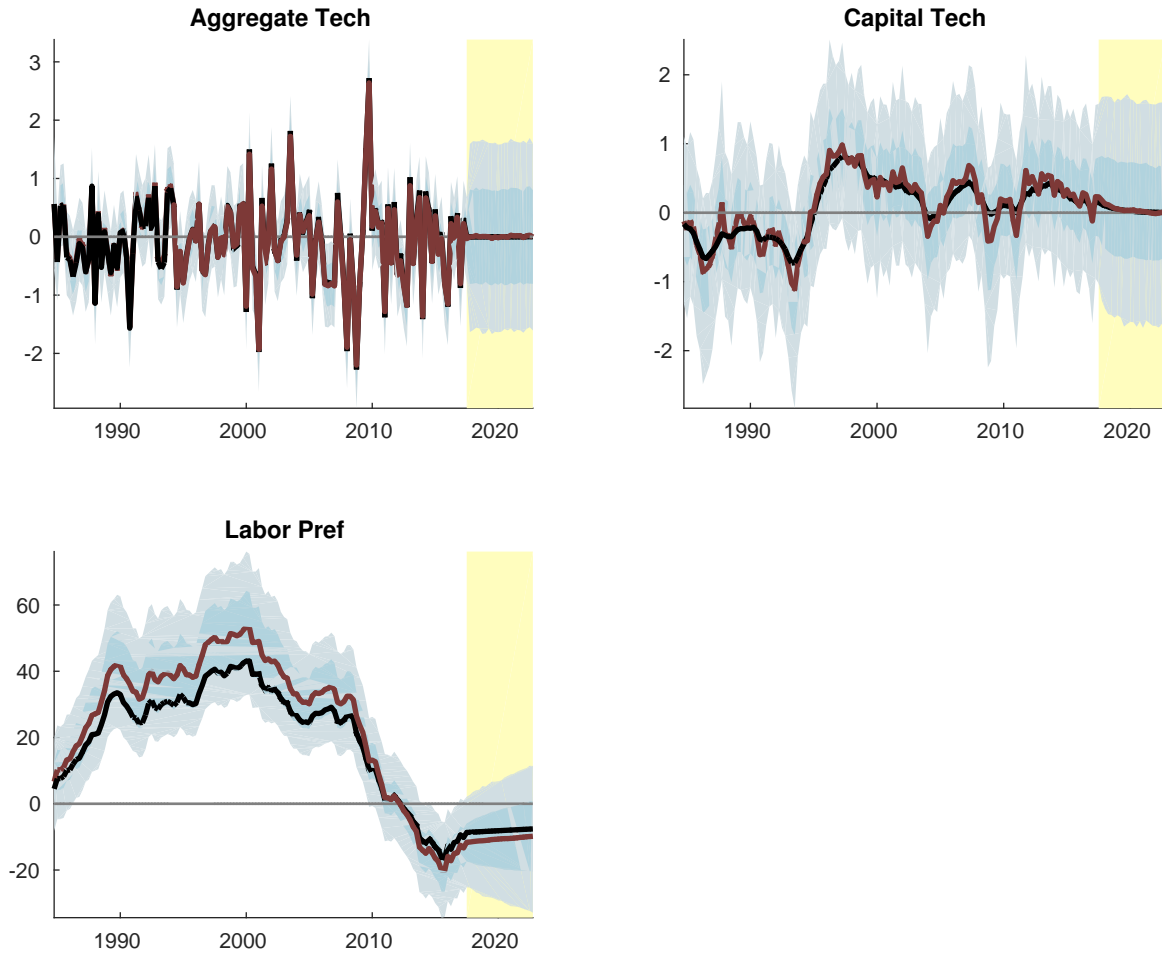
Of course, these stories from a glance at the exogenous drivers, yield applications for alternative versions of the EDO model and future model enhancements. For example, the exogenous risk premiums can easily be made to have an endogenous component, following the approach of Bernanke, Gertler, and Gilchrist (1999) (and, indeed, we have considered models of that type). At this point, we view incorporation of such mechanisms in our baseline approach as premature, pending ongoing research on financial frictions, banking, and intermediation in dynamic general equilibrium models. Nonetheless, the EDO model captured the key financial disturbances during the last several years in its current specification, and examining the endogenous factors that explain these developments will be a topic of further study.

Figure 4: Model Estimates of Risk Premiums



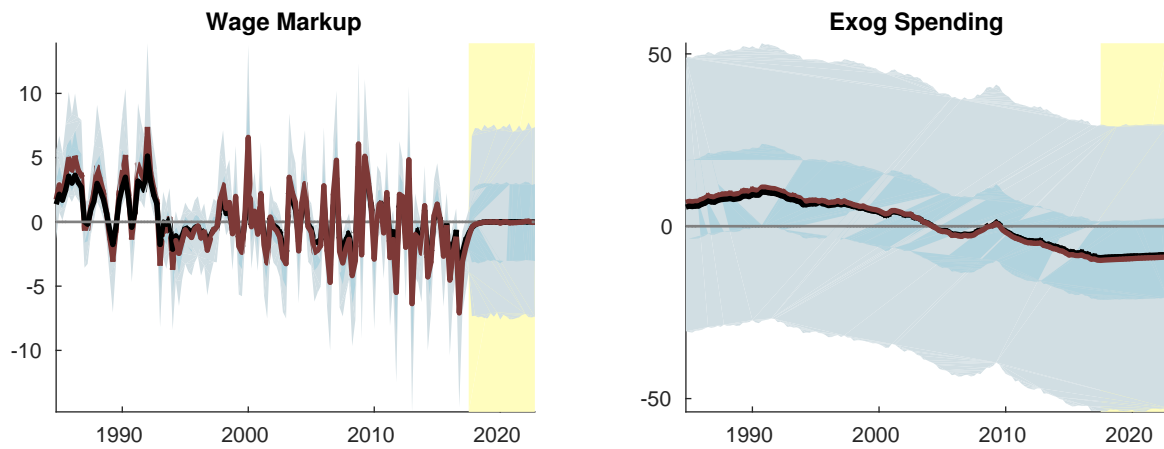
Black line: modal parameters. Red line: posterior median. Dark blue intervals: 68 percent credible set. Light blue intervals: 95 percent credible set.

Figure 5: Model Estimates of Key Supply-side Variables



Black line: modal parameters. Red line: posterior median. Dark blue intervals: 68 percent credible set. Light blue intervals: 95 percent credible set.

Figure 6: Model Estimates of Selected Other Exogenous Drivers



Black line: modal parameters. Red line: posterior median. Dark blue intervals: 68 percent credible set. Light blue intervals: 95 percent credible set.

References

- [Bernanke, Gertler, and Gilchrist (1999)] Bernanke, B., M. Gertler, and S. Gilchrist. 1999. The financial accelerator in a quantitative business cycle framework, in: John B. Taylor and Michael Woodford, Editor(s), *Handbook of Macroeconomics*, Elsevier, 1999, volume 1, part 3, pages 1341-1393.
- [Boivin, Kiley, and Mishkin (2010)] Boivin, J., M. Kiley, and F.S. Mishkin. 2010. How Has the Monetary Transmission Mechanism Evolved Over Time? In B. Friedman and M. Woodford, eds., *The Handbook of Monetary Economics*, Elsevier.
- [Chung, Kiley, and Laforte (2010)] Chung, H., M. Kiley, and J.P. Laforte. 2010. Documentation of the Estimated, Dynamic, Optimization-based (EDO) model of the U.S. economy: 2010 version. Finance and Economics Discussion Series, 2010-29. Board of Governors of the Federal Reserve System (U.S.).
- [Edge, Kiley, and Laforte (2008)] Edge, R., M. Kiley, and J.P. Laforte. 2008. Natural rate measures in an estimated DSGE model of the U.S. economy. *Journal of Economic Dynamics and Control*, vol. 32(8), pages 2512-2535.
- [Edge, Kiley, and Laforte (2010)] Edge, R., M. Kiley, and J.P. Laforte. 2010. A comparison of forecast performance between Federal Reserve staff forecasts, simple reduced-form models, and a DSGE model. *Journal of Applied Econometrics* vol. 25(4), pages 720-754.
- [Fisher (2006)] Fisher, Jonas D. M. 2006. The Dynamic Effects of Neutral and Investment-Specific Technology Shocks. *Journal of Political Economy*, University of Chicago Press, vol. 114(3), pages 413-451.
- [Gali (2011)] Gali, J. 2011. The Return Of The Wage Phillips Curve. *Journal of the European Economic Association*, vol. 9(3), pages 436-461.
- [Gali, Smets, and Wouters (2011)] Gali, J., F. Smets, and R. Wouters. 2011. Unemployment in an Estimated New Keynesian Model. *NBER Macroeconomics Annual* vol. 26(1), pages 329-360.
- [Smets and Wouters (2007)] Smets, F., and R. Wouters. 2007. Shocks and Frictions in the US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, American Economic Association, vol. 97(3), pages 586-606.

New York Fed DSGE Model: Research Directors Draft

December 1, 2017

Forecast Summary

The New York Fed model forecasts are obtained using data released through 2017Q3, augmented for 2017Q4 with the New York Fed staff forecasts (as of November 22) for real GDP growth and core PCE inflation, and with values of the federal funds rate, the 10-year Treasury yield and the spread between Baa corporate bonds and 10-year Treasury yields based on 2017Q4 averages up to November 22.

Based on this information, we project real GDP growth of 2.6 percent in 2017 on a Q4/Q4 basis, significantly stronger than the forecasts of 2.3 and 2 percent reported in September and June respectively. This projection fully reflects the current New York Fed staff judgmental forecast, which is somewhat more optimistic than the model's unconditional assessment of a 2.3 percent growth rate for this year. In 2018, GDP growth is anticipated to decline to 2 percent, the same as in September. Further into the future, however, the model forecasts a very gradual strengthening of activity, with GDP growth expected to reach 2.2 percent in 2019 and 2020, a slight improvement with respect to September. Consistent with this somewhat more solid growth prospects, inflation is also forecast to be higher in the medium term than expected in September, at 1.5 percent in both 2017 and 2018. However, its progress towards the FOMC's longer-run goal of 2 percent will remain glacial according to the model, with core PCE inflation only reaching 1.6 percent at the end of 2020.

Notwithstanding this modest improvement in the outlook, the output gap is currently estimated to be somewhat larger in 2017Q4 than projected in September: -1.1 percent compared to -0.9 percent. As in that round, the gap is expected to close very gradually over the course of the next several years, shrinking to -0.6 percent at the end of 2020. The natural rate of interest is also estimated to be somewhat lower at the end of 2017 than in September, but it is expected to continue recovering gradually over the next three years, as previously anticipated, reaching 1.3 percent at the end of 2020. The federal funds rate is projected to increase alongside its natural counterpart, reaching 2.9 percent by the end of 2020. This path translates into approximately four rate hikes in 2018, two more in 2019 and only one more in 2020.

The projections for all the variables are surrounded by notable uncertainty. For instance, the 68 percent posterior probability interval for GDP growth includes negative readings for all three years between 2018 and 2020. In comparison, the posterior probability intervals for inflation are tighter, with their upper bound never exceeding 3 percent throughout the forecast horizon.

The model attributes the above average real GDP growth rate in 2017 to continued improvement in financial conditions, as captured by positive contributions of both the financial and marginal efficiency of investment shocks. These positive forces were partly offset by low TFP growth in the first half of the year, but this drag from productivity appears to have abated in the last two quarters, contributing to the recent pickup in economic growth. As for inflation, the model attributes its recent weakness to a confluence of several factors, which continue to hold it below target over the forecast horizon. These factors include the lingering effects of the financial headwinds that have hampered the recovery, whose impact on inflation is estimated to be very persistent, as well as negative shocks to wage and price markups, which in the model capture some of the more transitory influences on inflation dynamics.

The Model and Its Transmission Mechanism

General Features of the Model

The New York Fed DSGE model is a medium-scale, one-sector dynamic stochastic general equilibrium model which is based on the New Keynesian model with financial frictions used in Del Negro et al. (2015). The core of the model is based on the work of Smets and Wouters (2007) and Christiano et al. (2005): It builds on the neo-classical growth model by adding nominal wage and price rigidities, variable capital utilization, costs of adjusting investment, and habit formation in consumption. The model also includes credit frictions as in the *financial accelerator* model developed by Bernanke et al. (1999), where the actual implementation of the credit frictions follows closely Christiano et al. (2014); and it allows for a time-varying inflation target following Del Negro and Schorfheide (2012). In contrast to these papers, the model features both a deterministic and a stochastic trend in productivity. Finally, it accounts for forward guidance in monetary policy by including anticipated policy shocks as in Laseen and Svensson (2011). More details on the model are in the New York

Fed DSGE Model Documentation, available upon request.

In this section, we briefly describe the microfoundations of the model, including the optimization problem of the economic agents and the nature of the exogenous processes. The innovations to these processes, which we refer to as “shocks,” are the drivers of macroeconomic fluctuations. The model identifies these shocks by matching the model dynamics with numerous quarterly data series: real GDP and GDI growth, real consumption growth, real investment growth, real wage growth, hours worked, inflation as measured by the personal consumption expenditures deflator and the GDP deflator, the federal funds rate (FFR), the 10-year nominal Treasury bond yield, 10-year survey-based inflation expectations, the Baa/10-year Treasury bond yield spread, and data on total factor productivity. In addition, from 2008Q4 to 2015Q2, we use market expectations of future federal funds rates. Model parameters are estimated from 1960Q1 to the present using Bayesian methods. structure of the model, data sources, and results of the estimation procedure can be found in Del Negro et al. (2015).

The economic units in the model are households, intermediate-goods producing firms, banks, entrepreneurs, capital-goods producers and the government. (Figure 1 describes the interactions among the various agents, the frictions and the shocks that affect the dynamics of this economy.)

Households derive utility from leisure, supply labor services to firms, and set wages in a monopolistically competitive fashion. random disturbance, which we call supply” shocks (this shock is “leisure” capture exogenous movements in labor supply due and labor market imperfections. The labor market is subject to frictions because of nominal wage rigidities. In addition, we allow for exogenous disturbances to wage mark-ups, labeled “wage mark-up” shocks, which capture exogenous changes in the degree of competitiveness in the labor market, or other exogenous movements in the labor supply.

Households, who discount future utility streams, also have to choose how much to consume and save. Their savings take the form of deposits to banks and purchases of government bills. Household preferences feature habit persistence, a characteristic that affects their consumption smoothing decisions. In addition, “discount factor” shocks drive an exogenous wedge between the change in the marginal utility of consumption and the riskless real return. These shocks possibly capture phenomena like deleveraging, or increased risk aversion.

Monopolistically competitive firms produce intermediate goods, which a competitive firm aggregates into the single final good that is used for both consumption and investment. The

production function of intermediate producers is subject to “total factor productivity” (TFP) shocks, which affect both the temporary and the permanent component of the level of total factor productivity. Intermediate goods markets are subject to price rigidities. Together with wage rigidities, this friction is quite important in allowing demand shocks to be a source of business cycle fluctuations, as countercyclical mark-ups induce firms to produce less when demand is low. Inflation evolves in the model according to a standard, forward-looking New Keynesian Phillips curve with indexing, which determines inflation as a function of marginal costs, expected future inflation, past inflation, and “price mark-up” shocks. The latter capture exogenous changes in the degree of competitiveness in the intermediate goods market. In practice, these shocks capture unmodeled inflation pressures, such as those arising from fluctuations in commodity prices.

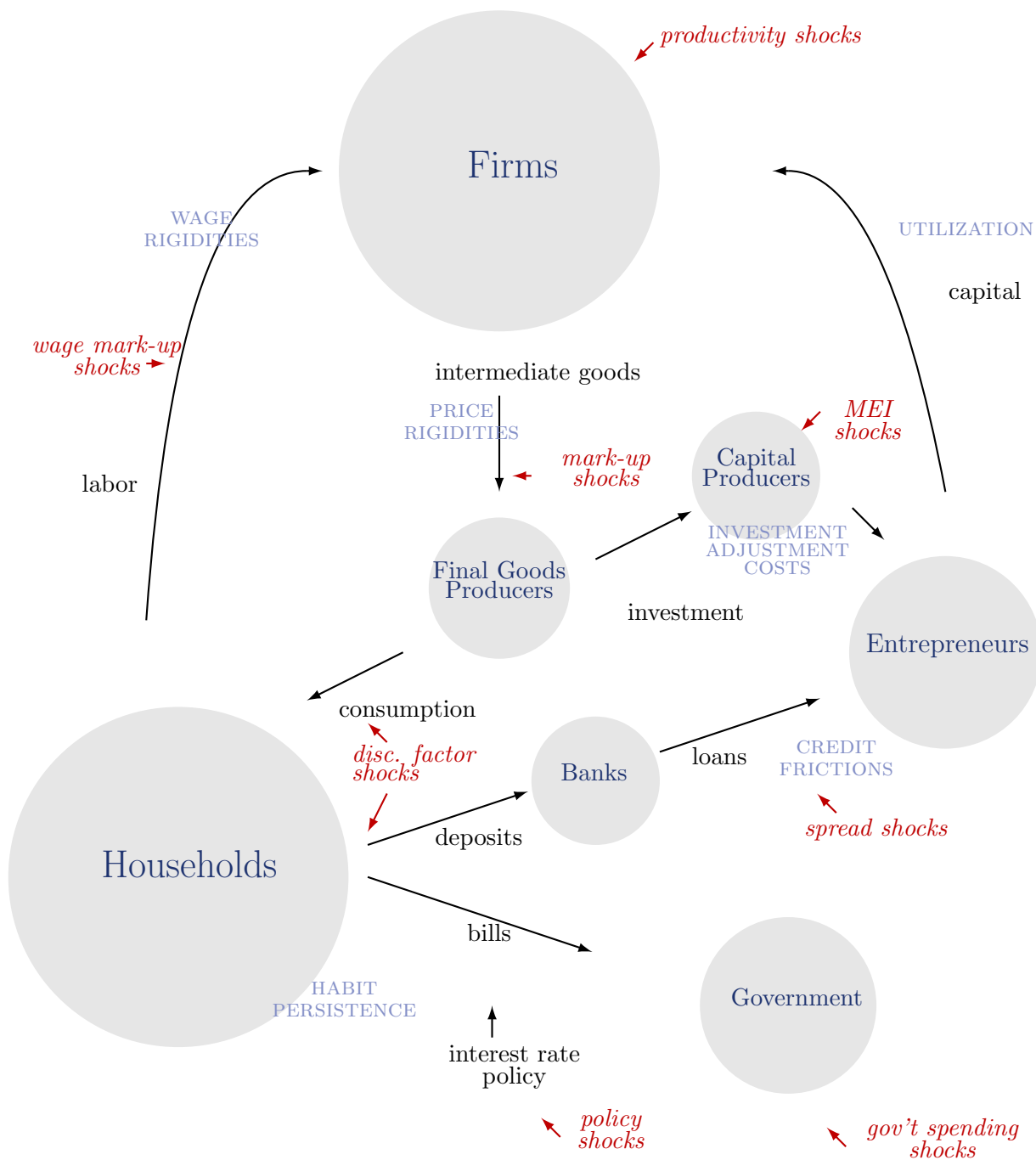
Financial intermediation involves two actors, *banks* and *entrepreneurs*, whose interaction captures imperfections in financial markets. These actors should not be interpreted in a literal sense, but rather as a device for modeling credit frictions. Banks take deposits from households and lend to entrepreneurs. Entrepreneurs use their own wealth and the loans from banks to acquire capital. They then choose the utilization level of capital and rent the capital to intermediate good producers. Entrepreneurs are subject to idiosyncratic disturbances in their ability to manage the capital. Consequently, entrepreneurs’ revenue may not be enough to repay their loans, in which case they default. Banks protect against default risk by pooling loans to all entrepreneurs and charging a spread over the deposit rate. Such spreads vary endogenously as a function of the entrepreneurs’ leverage, but also exogenously depending on the entrepreneurs’ riskiness. Specifically, mean-preserving changes in the volatility of entrepreneurs’ idiosyncratic shocks lead to variations in the spread (to compensate banks for changes in expected losses from individual defaults). We refer to these exogenous movements as “spread” shocks. Spread shocks capture financial intermediation disturbances that affect entrepreneurs’ borrowing costs. Faced with higher borrowing costs, entrepreneurs reduce their demand for capital, and investment drops. With lower aggregate demand, there is a contraction in hours worked and real wages. Wage rigidities imply that hours worked fall even more (because nominal wages do not fall enough). Price rigidities mitigate price contraction, further depressing aggregate demand.

Capital producers transform general output into capital goods, which they sell to the entrepreneurs. Their production function is subject to investment adjustment costs: producing capital goods is more costly in periods of rapid investment growth. It is also subject to exoge-

nous changes in the “marginal efficiency of investment” (MEI). These MEI shocks capture exogenous movements in the productivity of new investments in generating new capital. A positive MEI shock implies that fewer resources are needed to build new capital, leading to higher real activity and inflation, with an effect that persists over time. Such MEI shocks reflect both changes in the relative price of investment versus that of consumption goods (although the literature has shown the effect of these relative price changes to be small), and most importantly financial market imperfections that are not reflected in movements of the spread.

Finally, the *government* sector comprises a monetary authority that sets short-term interest rates according to a Taylor-type rule and a fiscal authority that sets public spending and collects lump-sum taxes to balance the budget. Exogenous changes in government spending are called “government” shocks; more generally, these shocks capture exogenous movements in aggregate demand. All exogenous processes are assumed to follow independent AR(1) processes with different degrees of persistence, except for mark-up shocks which have also a moving-average component, disturbances to government spending which are allowed to be correlated with total factor productivity disturbances, and exogenous disturbances to the monetary policy rule, or “policy” shocks, which are assumed to be i.i.d.

Figure 1: Model Structure



The Model's Transmission Mechanism

In this section, we illustrate some of the key economic mechanisms at work in the model's equilibrium. We do so with the aid of the impulse response functions to the main shocks hitting the economy, which we report in Figures 6 to 11.

We start with the shocks most closely associated with the Great Recession and the severe financial crisis that characterized it: the discount factor shock and the spread shock. The discount factor shock reflects a sudden desire by households to cut down on their consumption and save more. This shift may capture the fact that households want to reduce their debt level, or increased pessimism about future economic conditions. Figure 6 shows the impulse responses of the variables used in the estimation to a one-standard-deviation innovation in the discount factor shock. Such a shock results in a decline in consumption (fourth panel in left column), and hence in aggregate demand, which leads to a fall in output growth (top left panel), hours worked (top right panel), and real wage growth. The implied reduction in marginal costs puts downward pressure on inflation (second and third rows). In addition, the discount factor shock implies an increase in the credit spread (fifth panel in left row), which weighs negatively on investment. Monetary policy typically attempts to mitigate the decline in activity and inflation by lowering the FFR, but it cannot fully offset the macroeconomic effects of the shock.

The other key shock, the spread shock, stems from an increase in the perceived riskiness of borrowers, which induces banks to charge higher interest rates for loans, thereby widening credit spreads. As a result of this increase in the expected cost of capital, entrepreneurs' borrowing falls, hindering their ability to channel resources to the productive sector via capital accumulation. Figure 7 shows the impulse responses to a one-standard-deviation innovation in the spread shock. This leads to a reduction in investment and consequently to a reduction in output growth (top left panel) and hours worked (top right panel). The fall in the level of hours is fairly sharp in the first year and persists for many quarters afterwards. Of course, the effects of this same shock on GDP growth, which roughly mirrors the change in the level of hours, are more short-lived. Output growth returns to its steady state level less than three years after the shock hits, but it barely moves above it after that, implying no catch up of the level of GDP towards its previous trend (bottom left panel). The persistent drop in the level of economic activity due to the spread shock also leads to a prolonged decline in real marginal costs, and, via the New Keynesian Phillips curve, in inflation. Finally, policymakers endogenously respond to the change in the inflation and real

activity outlook by cutting the federal funds rate (right panel on the third row).

Similar considerations hold for the MEI shock, which represents a direct hit to the ‘technological’ ability of entrepreneurs to transform investment goods into productive capital, rather than an increase in their funding cost. The impulse responses to MEI shocks, shown in Figure 8, also feature a decrease in investment, output and hours worked, as well as in real wages, although these are less persistent than in the case of spread shocks.

Another shock that plays an important role in the model is the stationary TFP shock (the model features shocks to both the level and the growth rate of productivity – we discuss here the former). As shown in Figure 9, a positive TFP shock has a large effect on output growth, but it drives hours down on impact. This negative response of hours is due to the presence of nominal rigidities, which prevent aggregate demand from expanding enough to absorb the increased ability of the economy to supply output. With higher productivity, marginal costs and thus the labor share fall, leading to lower inflation. These dynamics make the TFP shock particularly suitable to account for the first phase of the recovery, in which GDP growth was above trend, but hours and inflation remained weak.

The last shock that plays a relevant role in the current economic environment is the price mark-up shock, whose impulse response is depicted in Figure 10. This shock is an exogenous source of inflationary pressures, stemming from changes in the market power of intermediate goods producers. As such, it leads to higher inflation and lower real activity, as producers reduce supply to increase their desired markup. Compared to those of the other prominent supply shock in the model, the TFP shock, the effects of markup-shocks are less persistent. GDP growth falls on impact after mark-ups increase, but returns above average after about one year, and the effect on the level of output is absorbed in a little over four years. Inflation is sharply higher, but only for a few quarters, leading to a temporary spike in the nominal interest rate, as monetary policy tries to limit the pass-through of the shock to inflation. Unlike in the case of TFP shocks, however, hours fall immediately, mirroring the behavior of output.

Forecasts

	Unconditional Forecast							
	2017		2018		2019		2020	
	Dec.	Sep.	Dec.	Sep.	Dec.	Sep.	Dec.	Sep.
Real GDP	2.3	2.3	1.9	2.0	2.2	2.0	2.3	2.1
Growth (Q4/Q4)	(1.5,3.1)	(0.9,3.6)	(-0.8,4.3)	(-0.8,4.5)	(-0.6,4.8)	(-0.8,4.7)	(-0.6,5.0)	(-0.8,4.8)
Core PCE	1.3	1.2	1.3	1.3	1.4	1.5	1.5	1.6
Inflation (Q4/Q4)	(1.1,1.5)	(0.9,1.6)	(0.5,2.1)	(0.4,2.2)	(0.4,2.4)	(0.4,2.5)	(0.3,2.7)	(0.4,2.8)
Federal Funds	1.2	1.4	2.1	2.1	2.6	2.6	2.9	2.9
Rate (Q4)	(1.2,1.2)	(0.6,2.2)	(0.7,3.6)	(0.7,3.8)	(0.9,4.4)	(0.9,4.5)	(1.0,4.9)	(1.0,4.9)
Real Natural	0.3	0.5	0.8	0.9	1.1	1.1	1.3	1.2
Rate (Q4)	(-0.9,1.6)	(-1.0,2.0)	(-0.9,2.5)	(-0.8,2.6)	(-0.8,2.9)	(-0.7,3.0)	(-0.7,3.2)	(-0.7,3.2)
Output	-1.2	-0.9	-1.2	-0.7	-0.9	-0.6	-0.7	-0.5
Gap (Q4)	(-2.5,0.0)	(-2.3,0.6)	(-3.5,0.8)	(-3.3,1.5)	(-4.0,1.7)	(-4.0,2.2)	(-4.4,2.5)	(-4.4,2.7)

	Conditional Forecast							
	2017		2018		2019		2020	
	Dec.	Sep.	Dec.	Sep.	Dec.	Sep.	Dec.	Sep.
Real GDP	2.6	2.3	2.0	2.0	2.2	2.0	2.2	2.1
Growth (Q4/Q4)	(2.6,2.6)	(1.3,3.2)	(-0.6,4.4)	(-0.8,4.4)	(-0.6,4.8)	(-0.8,4.7)	(-0.6,4.9)	(-0.8,4.8)
Core PCE	1.5	1.4	1.5	1.3	1.5	1.5	1.6	1.7
Inflation (Q4/Q4)	(1.5,1.5)	(1.2,1.6)	(0.7,2.2)	(0.5,2.2)	(0.4,2.5)	(0.4,2.6)	(0.4,2.8)	(0.4,2.9)
Federal Funds	1.2	1.4	2.2	2.1	2.6	2.6	2.9	2.9
Rate (Q4)	(1.2,1.2)	(0.6,2.2)	(0.7,3.7)	(0.7,3.8)	(0.9,4.5)	(0.9,4.5)	(1.1,4.9)	(1.0,4.9)
Real Natural	0.3	0.5	0.8	0.9	1.1	1.1	1.3	1.2
Rate (Q4)	(-1.0,1.5)	(-0.9,2.0)	(-0.9,2.5)	(-0.9,2.6)	(-0.8,2.9)	(-0.7,3.0)	(-0.6,3.2)	(-0.7,3.2)
Output	-1.1	-0.9	-1.0	-0.8	-0.8	-0.6	-0.6	-0.6
Gap (Q4)	(-2.3,0.2)	(-2.2,0.5)	(-3.1,1.0)	(-3.2,1.4)	(-3.8,1.9)	(-3.9,2.1)	(-4.2,2.5)	(-4.3,2.6)

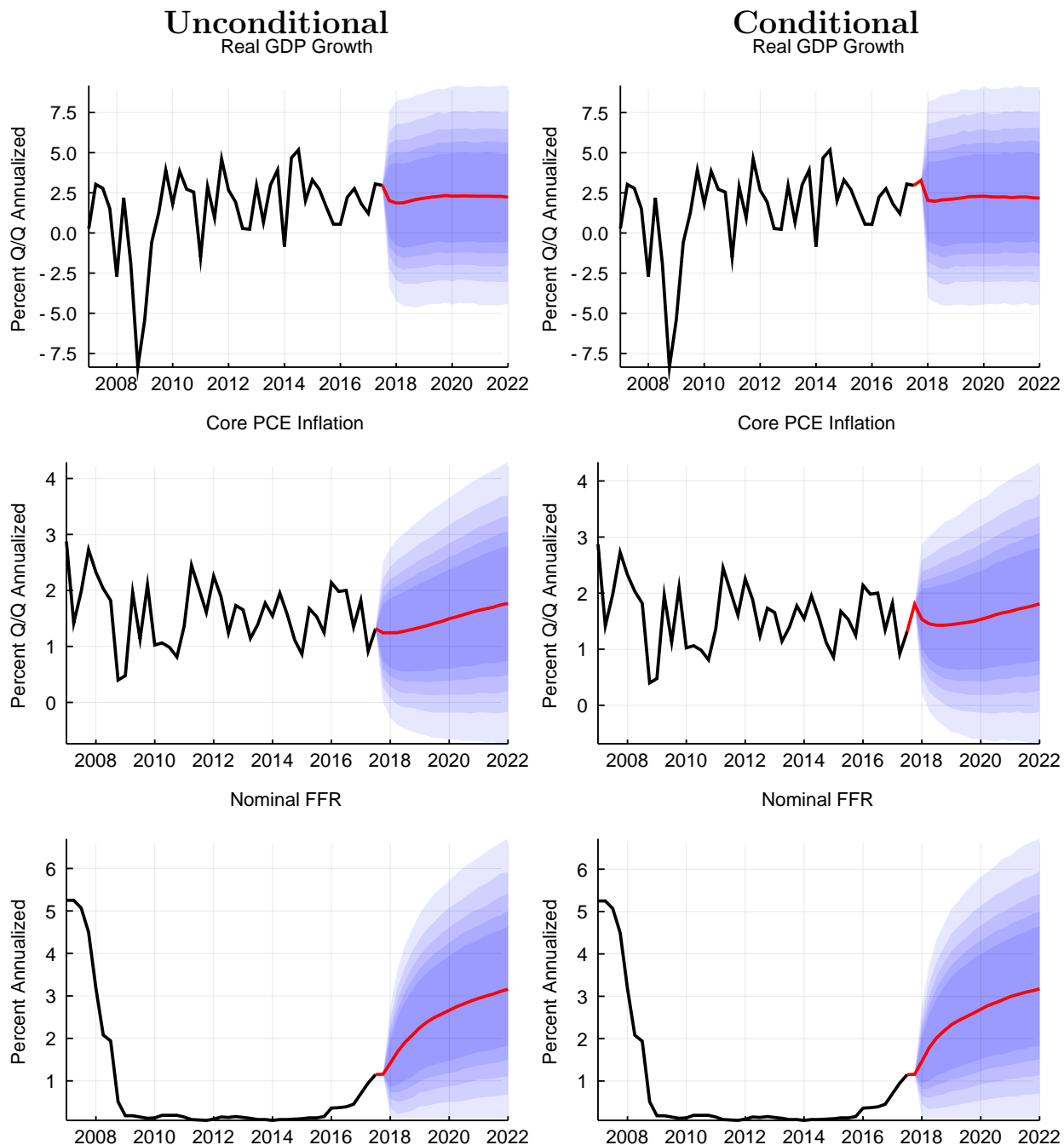
*The unconditional forecasts use data up to 2017Q3, the quarter for which we have the most recent GDP release, as well as the federal funds rate, 10-year Treasury yield, and spreads data for 2017Q4. In the conditional forecasts, we further include the 2017Q4 New York Fed projections for GDP growth and core PCE inflation as additional data points. Numbers in parentheses indicate 68 percent probability intervals.

The table above presents annual forecasts for real GDP growth, core PCE inflation, the real natural rate, and the output gap for 2017-2020, with 68 percent probability intervals. We include two sets of forecasts. The *unconditional* forecasts use data up to 2017Q3, the quarter for which we have the most recent GDP release. These forecasts also use federal funds rate, 10-year Treasury yield, and spreads data for 2017Q4 by taking the average realizations for the quarter up to the forecast date. In the *conditional* forecasts, we further include the 2017Q4 New York Fed staff projections as of November 22 for GDP growth (2.6 percent) and core PCE inflation (1.5 percent) as additional data points. Treating the 2017Q4 staff forecasts as data allows us to incorporate information about the current quarter into the DSGE forecasts for the subsequent quarters. In addition to providing the current forecasts,

the table reports the forecasts included in the DSGE memo forwarded to the FOMC in advance of its December 2017 meeting.

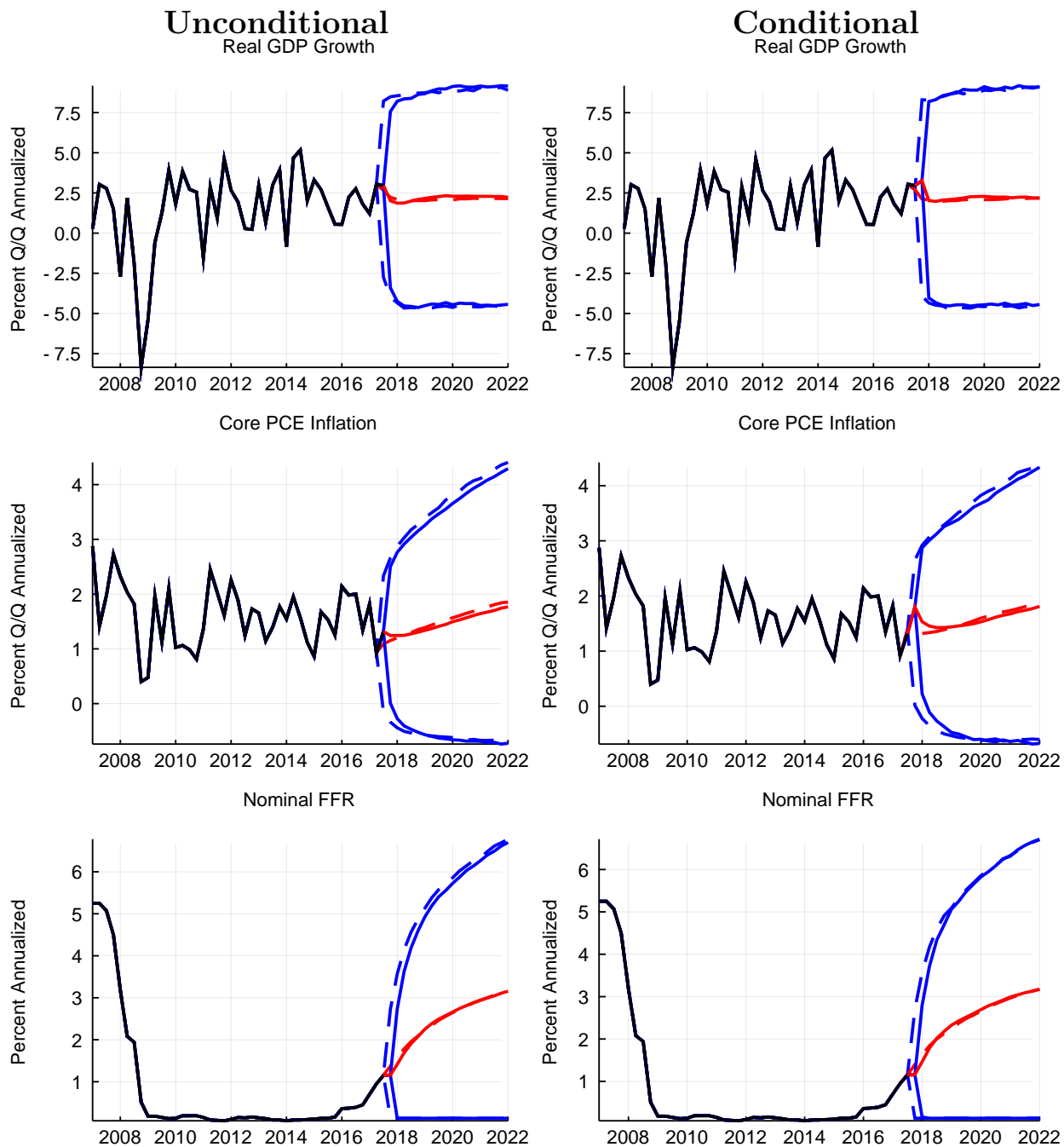
Figure 2 presents quarterly forecasts, both unconditional (left panels) and conditional (right panels). In the graphs, the black line represents data, the red line indicates the mean forecast, and the shaded areas mark the 50, 60, 70, 80 and 90 percent probability intervals for the forecasts, reflecting both parameter and shock uncertainty. Figure 3 compares the current forecasts with the September forecasts.

Figure 2: Forecasts



Black lines indicate data, red lines indicate mean forecasts, and shaded areas mark the uncertainty associated with our forecast as 50, 60, 70, 80, and 90 percent probability intervals.

Figure 3: Change in Forecasts



Solid (dashed) red and blue lines represent the mean and the 90 percent probability intervals of the current (previous) forecast.

Interpreting the Forecasts

We use the shock decomposition shown in Figure 4 to interpret the forecasts. This figure quantifies the relevance of the most important shocks for output growth, core PCE inflation, and the federal funds rate (FFR) from 2007 onwards. In each of the three panels, the solid line (black for realized data, red for mean forecast) shows the variable in deviation from its steady state. The bars represent the contribution of each shock to the deviation of the variable from steady state, computed as the counterfactual values (in deviations from the mean) obtained when all other shocks are zero. Some of the shocks have been aggregated in this decomposition. For example, the bars labeled “financial” (in purple) capture the effect of shocks to the spread as well as to the discount factor.

Seen through the lens of this decomposition, the evolution of the economy over the past few years and its forecast through 2020 can be described as follows. Between 2010 and 2014, persistent headwinds from the financial crisis, which are captured in the model by the financial (purple) and MEI (azure) shocks, held back the pace of the recovery. These sources of drag on the economy were also accompanied by a sequence of negative TFP shocks (orange bars), as was also apparent from the extraordinarily weak readings on both TFP and labor productivity over this period. During the course of 2014, the financial headwinds appeared to be abating, providing positive contributions to GDP growth that helped to lift it over its potential, hence also helping to close the output gap and increase the natural rate of interest (Figure 5). However, this improvement in financial conditions suffered a setback since the summer of 2015, pushing growth once again below steady state. More recently, monetary policy shocks are estimated to be depressing growth and to continue to do so throughout the forecast horizon, while financial and MEI shocks will provide a somewhat offsetting force.

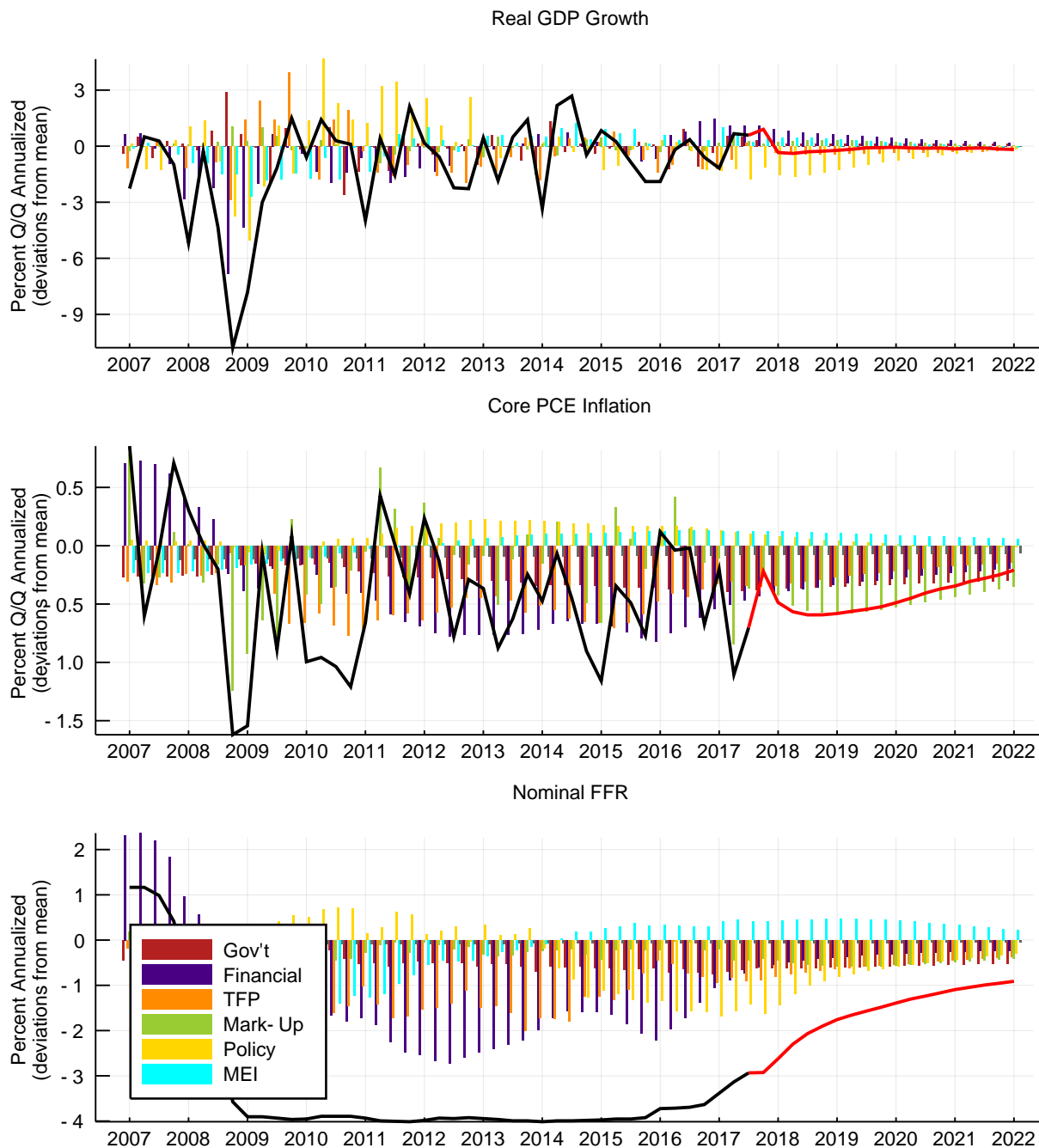
The oscillations in the contribution of financial shocks to economic developments are also evident in the historical decomposition of inflation, with the purple bars becoming negative after the financial crisis and then contributing even more negatively beginning in 2011. Starting in 2016, these effects began to diminish very gradually, but are still projected to keep inflation below steady state throughout the forecast horizon. In addition, the model sees mark-up shocks (green bars), which capture the effect of exogenous changes in marginal costs such as those connected with fluctuations in commodity prices, as a further negative drag on inflation. This drag is especially pronounced in 2017 and it is projected to persist throughout the forecast horizon. Beginning in 2011, inflation was also pulled down by

negative government spending and TFP shocks.

In equilibrium, the negative impact of financial shocks on the economy is partly cushioned by the endogenous response of monetary policy, in the form of a reduction in the policy rate. In the case of financial shocks, for instance, this endogenous response is captured by the purple bars in the interest rate panel, which indicate that the federal funds rate was lowered throughout the recovery in response to the financial headwinds. In fact, this endogenous adjustment of the policy instrument was decreasing during 2014, when the effects of the headwinds were abating, but was dialed back up again in 2015 as financial conditions deteriorated again. In addition, the negative impact of exogenous shocks can be offset through expansionary monetary policy. In particular, forward guidance about the future path of the federal funds rate (captured by anticipated policy shocks whose effects are included in the yellow bars) played an important role in counteracting the financial headwinds between 2009 and 2013, lifting both output and inflation. However, the positive effect of this policy accommodation on the level of output has been negligible over the most recent quarters, and it is forecasted to be a drag on output growth over the forecast horizon.

Figure 5 shows the output gap—computed as the percent difference between output and its “natural” level, namely the one that would prevail in the absence of nominal rigidities and mark-up shocks—and the natural rate of interest through history. The natural rate of interest is projected to increase slowly over time, reflecting the continuing restraining effect of financial headwinds and lower productivity growth. This path for the real natural rate is roughly in line with that for the real policy rate, implying that monetary policy is not especially accommodative over the forecast horizon. However, policy is expected to be slightly more accommodative in the near term, as indicated by the modest gap between the ex-ante real interest rate and the real natural rate in 2017. The model’s estimate of the output gap suggests that slack persists and will be absorbed only gradually over time. This measure of underutilization of resources also reflects low marginal costs of production for firms, a key driver of the inflation projections. The model’s estimate of firms marginal costs suggests that these have not recovered much over the last few years, owing to the weakness in real wage growth. The output gap thus closes only gradually, which explains the slow return of inflation to target.

Figure 4: Shock Decomposition



The shock decomposition is presented for the conditional forecast. The solid lines (black for realized data, red for mean forecast) show each variable in deviation from its steady state. The bars represent the shock contributions; specifically, the bars for each shock represent the counterfactual values for the observables (in deviations from the mean) obtained by setting all other shocks to zero.

Figure 5: Output Gap and the Natural Interest Rate

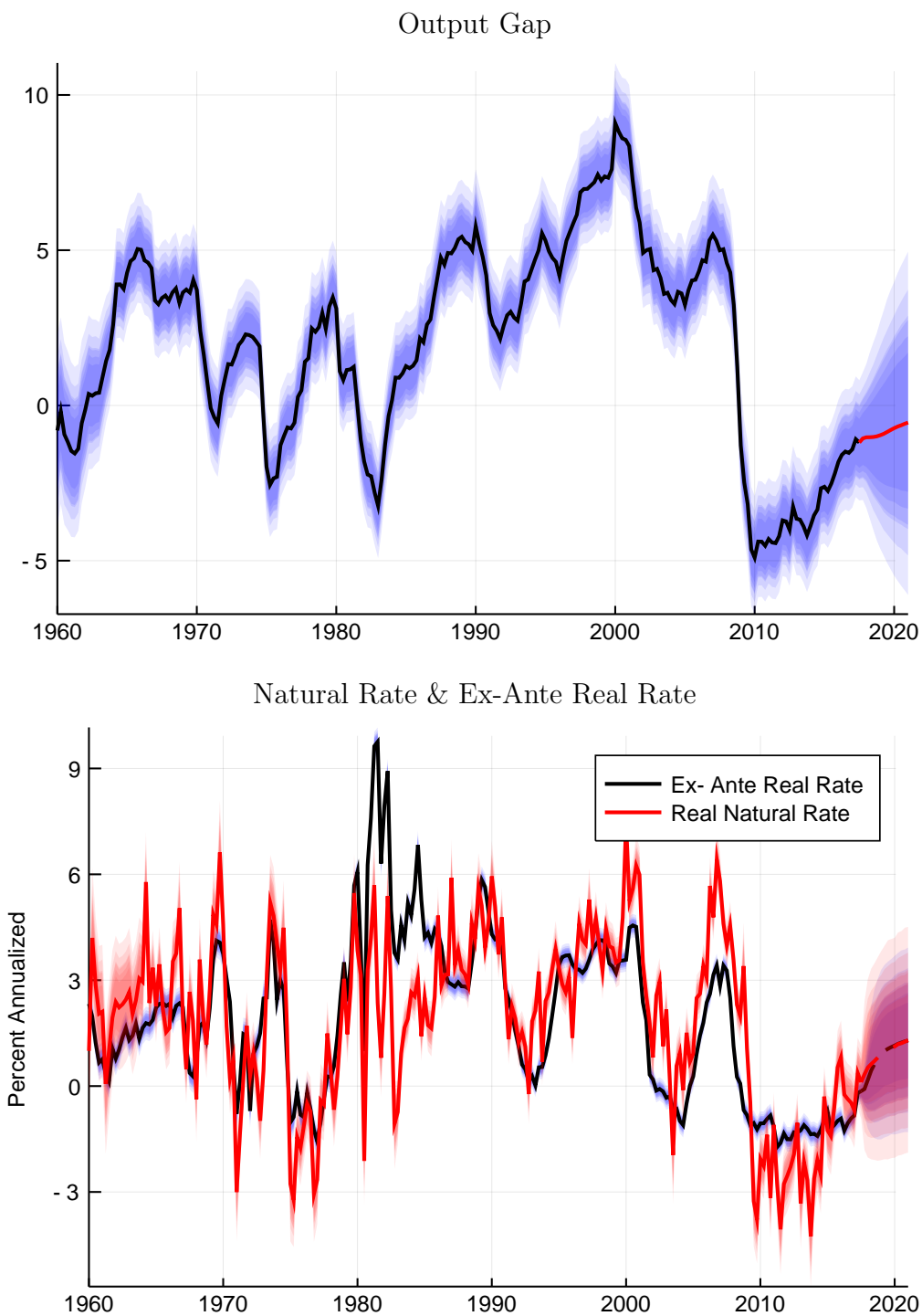


Figure 6: Responses to a Discount Factor Shock

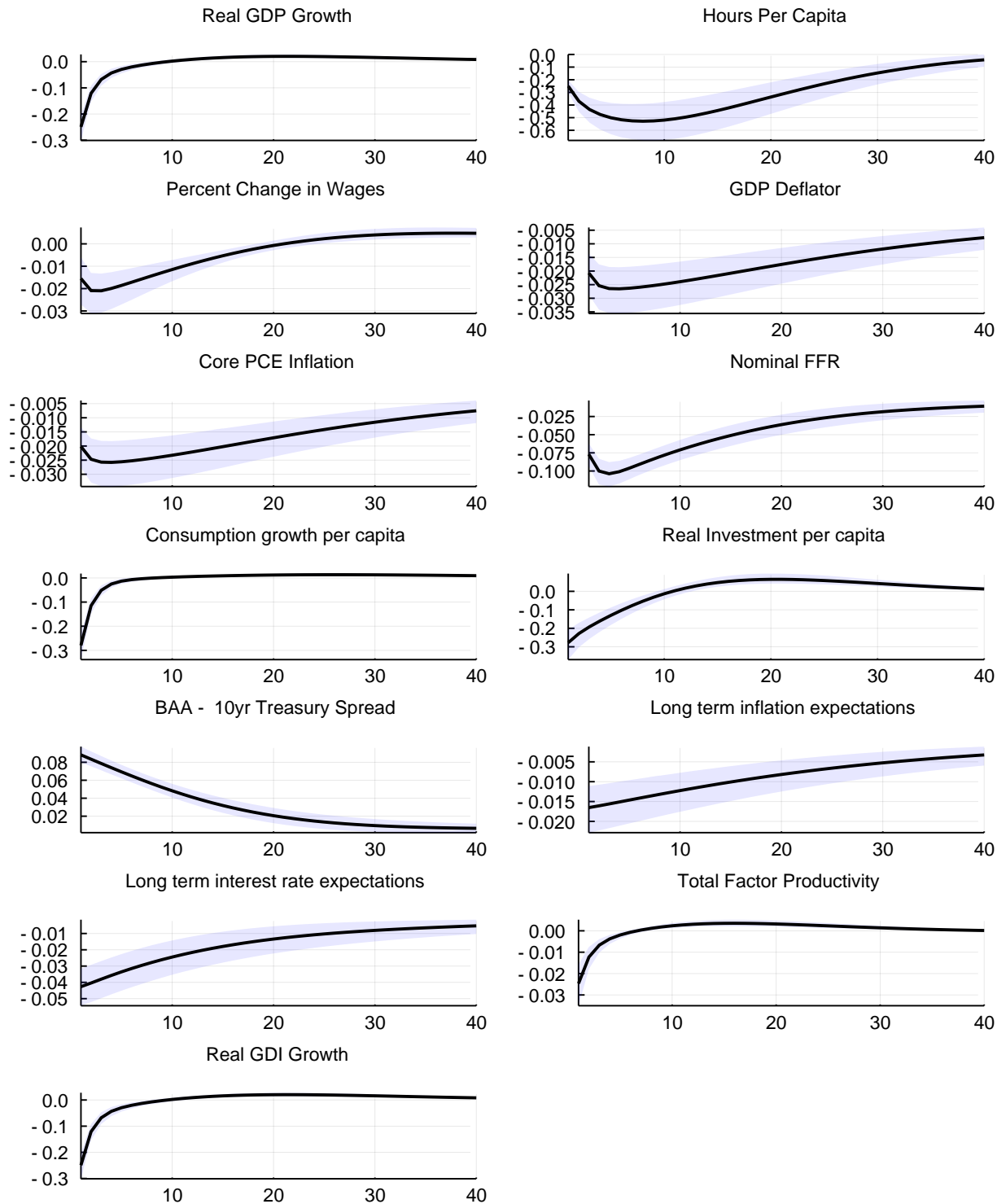


Figure 7: Responses to a Spread Shock

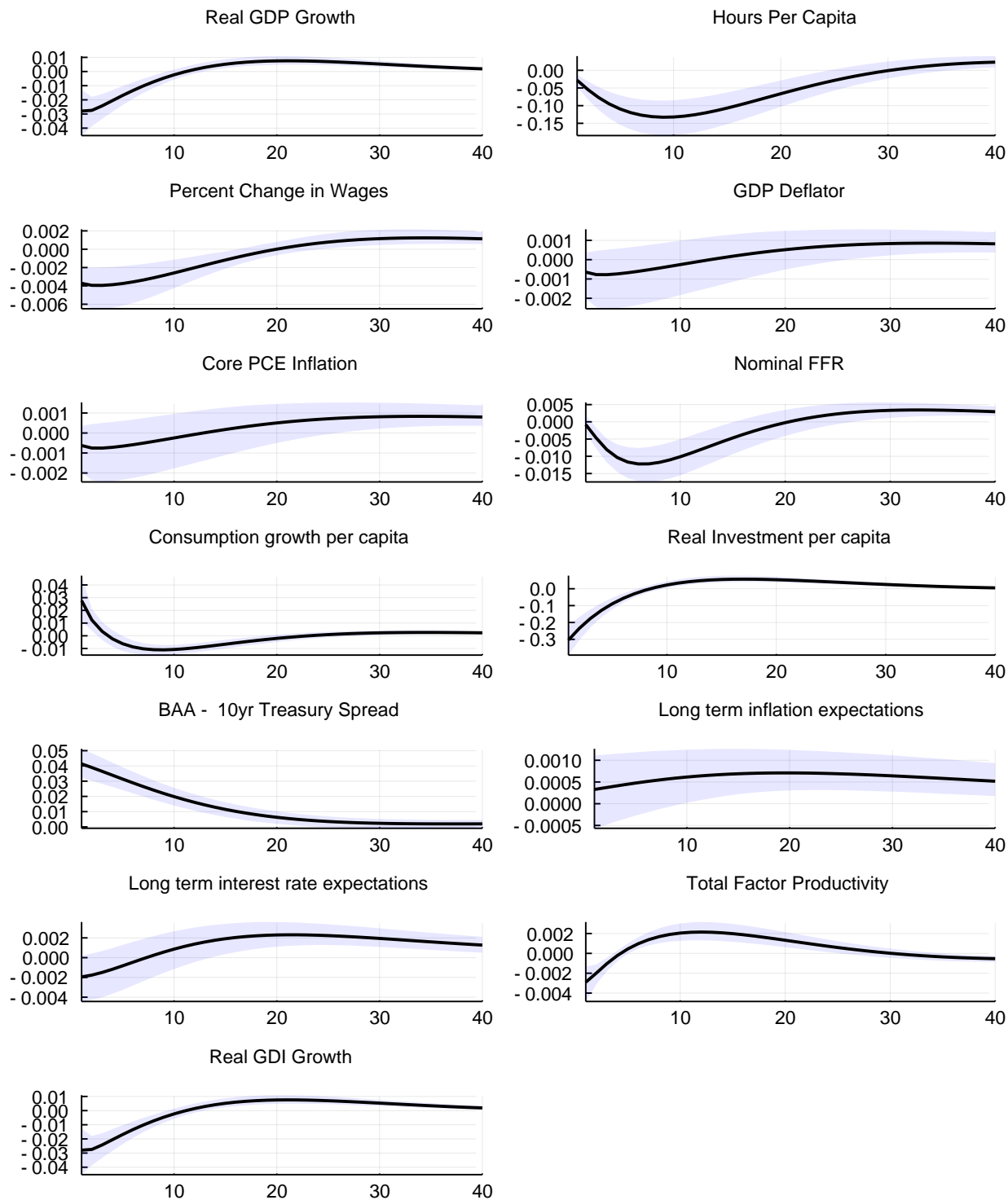


Figure 8: Responses to an MEI Shock

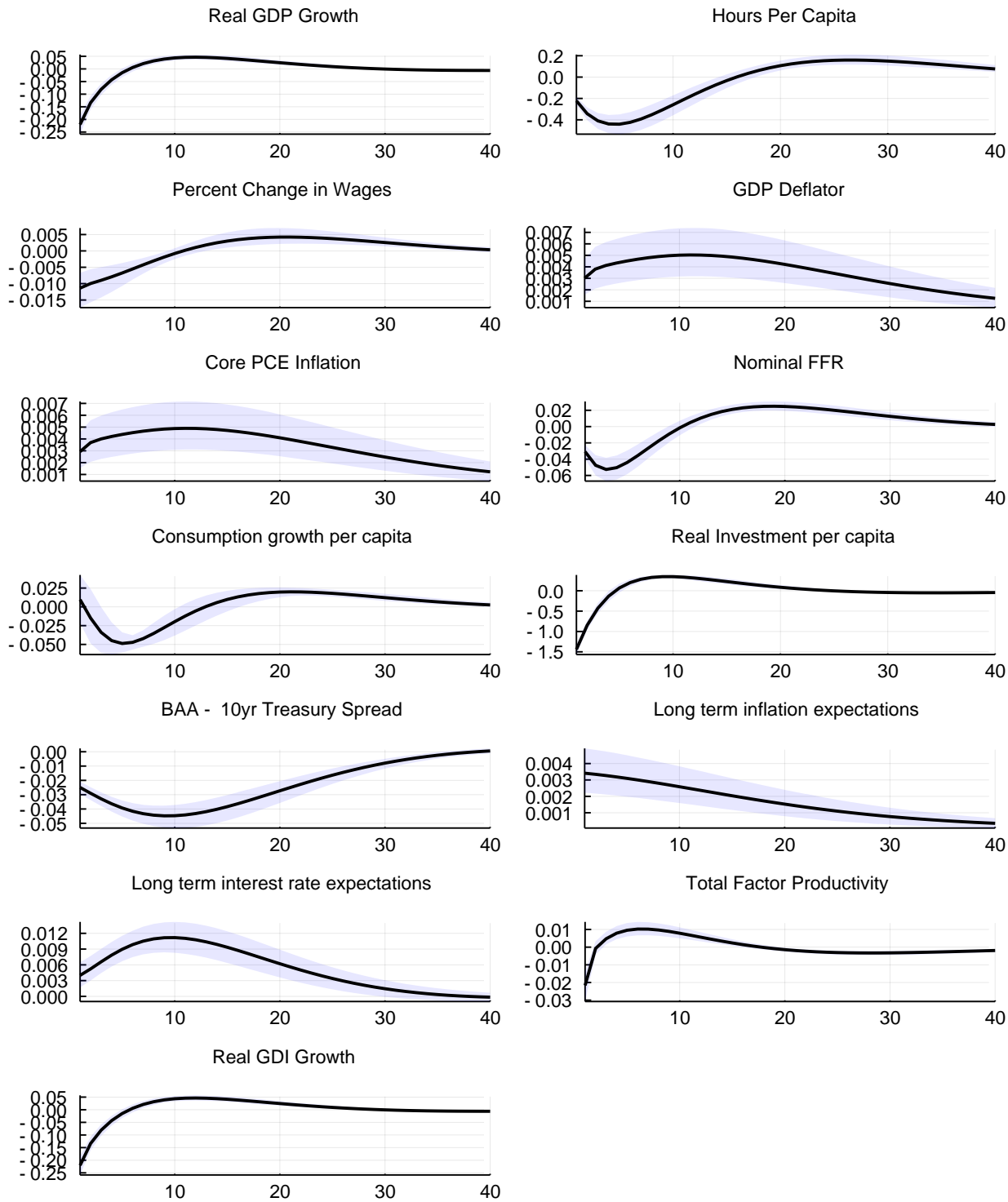


Figure 9: Responses to a TFP Shock

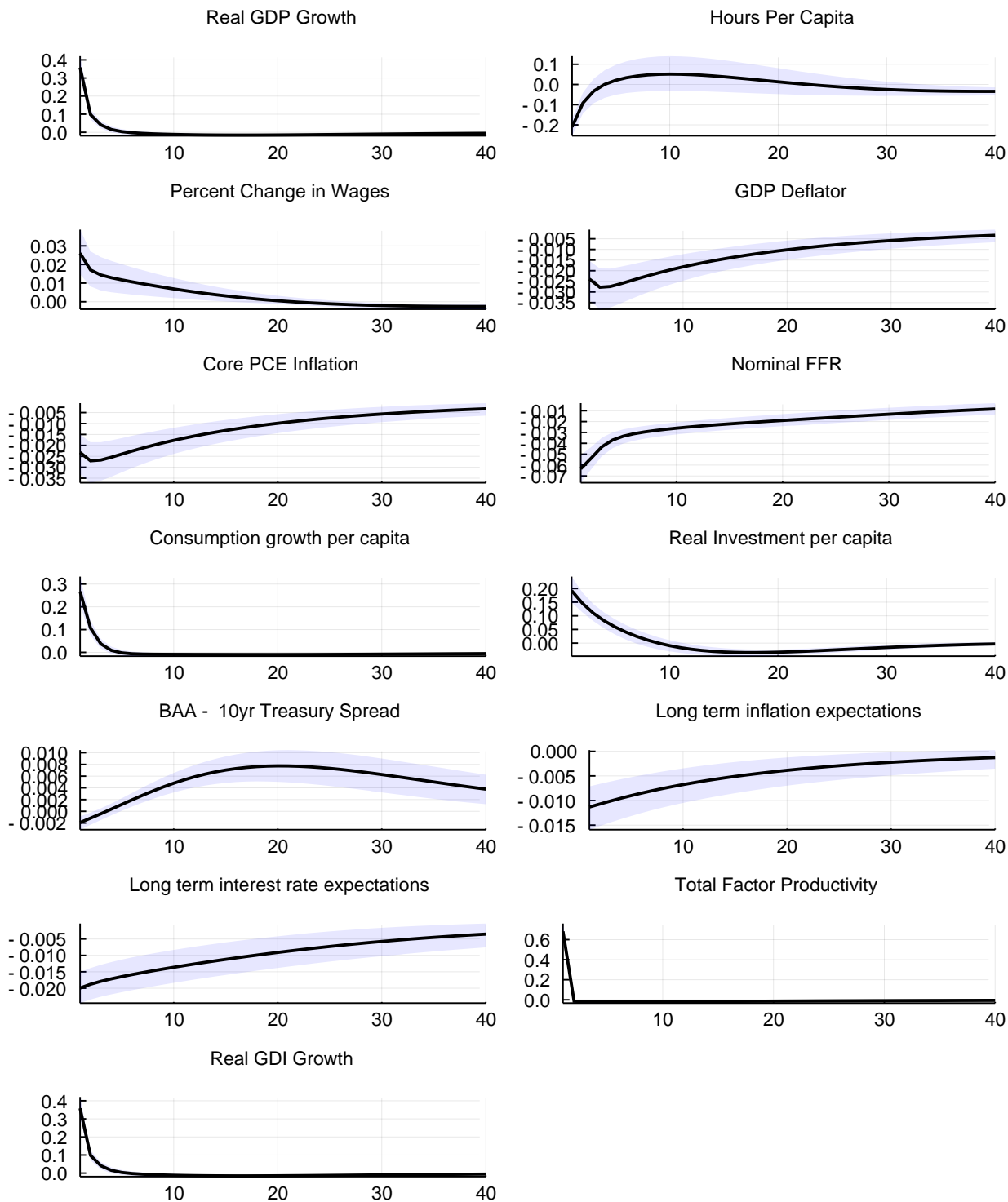


Figure 10: Responses to a Price Mark-up Shock

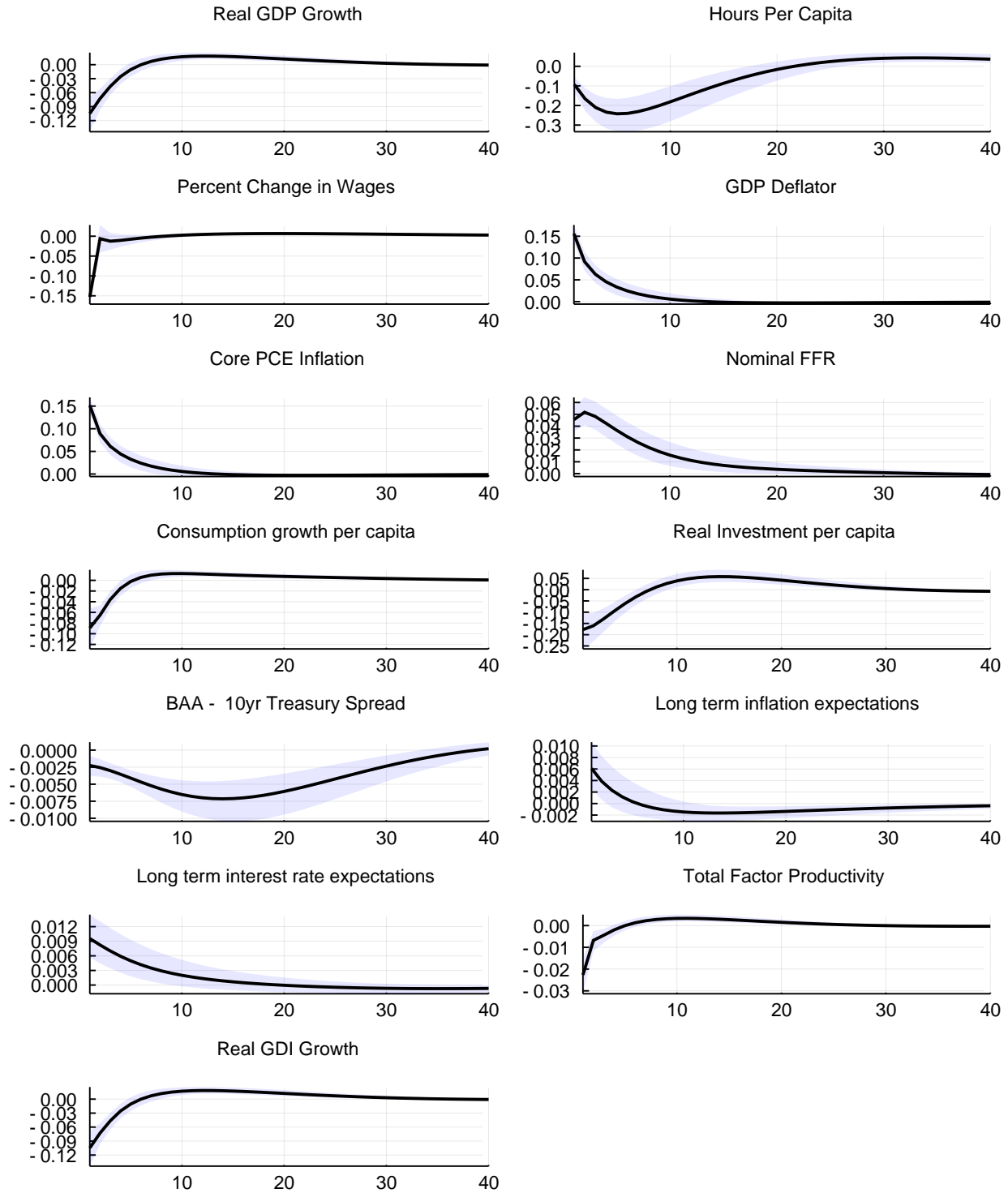
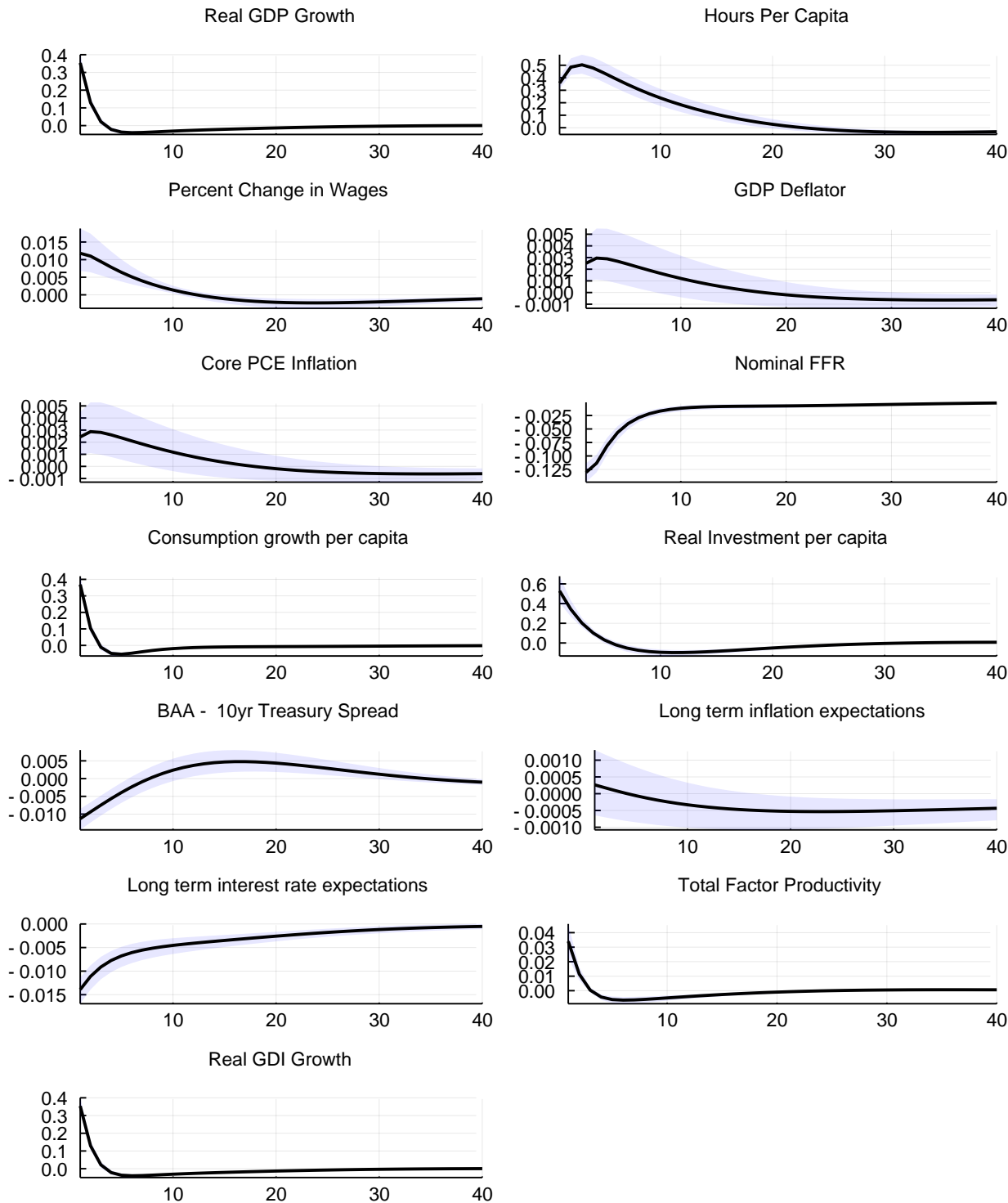


Figure 11: Responses to a Monetary Policy Shock



References

- BERNANKE, B. S., M. GERTLER, AND S. GILCHRIST (1999): “The Financial Accelerator in a Quantitative Business Cycle Framework,” in *Handbook of Macroeconomics*, ed. by J. B. Taylor and M. Woodford, Amsterdam: North-Holland, vol. 1C, chap. 21, 1341–93.
- CHRISTIANO, L. J., M. EICHENBAUM, AND C. L. EVANS (2005): “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,” *Journal of Political Economy*, 113, 1–45.
- CHRISTIANO, L. J., R. MOTTO, AND M. ROSTAGNO (2014): “Risk Shocks,” *American Economic Review*, 104, 27–65.
- DEL NEGRO, M., M. P. GIANNONI, AND F. SCHORFHEIDE (2015): “Inflation in the Great Recession and New Keynesian Models,” *American Economic Journal: Macroeconomics*, 7, 168–196.
- DEL NEGRO, M. AND F. SCHORFHEIDE (2012): “DSGE Model-Based Forecasting,” *Federal Reserve Bank of New York Working Paper*.
- LASEEN, S. AND L. E. SVENSSON (2011): “Anticipated Alternative Policy-Rate Paths in Policy Simulations,” *International Journal of Central Banking*, 7, 1–35.
- SMETS, F. AND R. WOUTERS (2007): “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” *American Economic Review*, 97, 586 – 606.

Detailed Philadelphia (PRISM) Forecast Overview

December 2017

Keith Sill

Forecast Summary

The FRB Philadelphia DSGE model denoted PRISM, projects that real GDP growth will run at a fairly strong pace over the forecast horizon with real output growth peaking at a bit under 3.4 percent in mid-2019. Core PCE inflation edges up to reach 2 percent at the end of 2020. The funds rate rises to 2.2 percent in 2018Q4 and rises steadily to reach 3.6 percent at the end of 2020. The current gap between the level of output and its trend level remains significant in the estimated model and, absent any shocks, the model continues to predict a fairly rapid recovery to the trend level. The relatively slow pace of growth and low inflation that have characterized U.S. economic performance over the past few years require the presence of shocks to offset the strength of the model's internal propagation channels.

The Current Forecast and Shock Identification

The PRISM model is an estimated New Keynesian DSGE model with sticky wages, sticky prices, investment adjustment costs, and habit persistence. The model is similar to the Smets & Wouters 2007 model and is described more fully in Schorfheide, Sill, and Kryshko 2010. Unlike in that paper though, we estimate PRISM directly on core PCE inflation rather than projecting core inflation as a non-modeled variable. Details on the model and its estimation are available in a Technical Appendix that was distributed for the June 2011 FOMC meeting or is available on request.

The current forecasts for real GDP growth, core PCE inflation, and the federal funds rate are shown in Figures 1a-1c along with 68 percent probability coverage intervals. The forecast uses data through 2017Q3 supplemented by a 2017Q4 nowcast. The model takes the 2017Q4 nowcast for output growth of 2.6 percent as given and the projection begins with 2018Q1. PRISM anticipates that output growth rises to a 3 percent pace in 2018Q1, with growth then edging up to 3.3 percent in mid-2019. Overall, the growth forecast for this round is similar to the September projection. While output growth is fairly robust going forward, core PCE inflation stays contained and runs at a pace slightly below the 2 percent target until mid-2020 when it reaches 2 percent. Based on the 68 percent coverage interval, the model sees a minimal chance of deflation or recession (measured as negative quarters of real GDP growth) over the next 3 years. The federal funds rate is determined by an estimated policy rule and the funds rate rises from 1.2 percent in 2017Q4, 2.2 percent in 2018Q4, 3.1 percent in 2019Q4, and 3.6 percent in 2020Q4. This path for the funds rate is similar to that in the September projection.

The key factors driving the projection are shown in the forecast shock decompositions (Figures 2a-2e) and the smoothed estimates of the model's primary shocks (shown in Figure 3, where they are normalized by standard deviation). Over the last few quarters real GDP growth has been running close to the model's trend rate. Positive shocks to government spending, investment, and labor supply have offset negative contributions from the model's other shocks (TFP, markups, financial, and monetary policy). As these shocks unwind, output growth maintains edges up to a slightly above steady state pace over the next few years. Over the course of the recession and recovery PRISM estimated a series of large positive shocks to leisure (negative shocks to labor supply) that have a persistent effect on hours worked and so pushed hours well below steady state. As these shocks unwind hours worked continue to rebound over the forecast horizon and so support higher output growth. Similarly, the unwinding of investment shocks contribute to output growth over the forecast horizon.

After strong performance in early 2016, consumption growth (Figure 2d) pulled back to a below trend pace through mid 2017. This was largely driven by negative contributions from TFP shocks and investment shocks. Consumption is projected to gradually rise toward trend over the next three years. A gradual unwinding of investment shocks and higher interest rates keeps consumption growth below steady state until 2020. Financial shocks that boost consumption in turn weaken investment growth (Figure 2d-e). However, strong investment shocks pushed investment growth to an above-trend pace in the second half of 2016 and into 2017. The model now forecasts above-trend growth in investment (gross private domestic + durable goods consumption) in 2018 as the gradual unwinding of MEI shocks (see Figures 2e and 3) are partially offset by the effects of financial shocks: the unwinding of the discount factor shocks leads to a downward pull on investment growth over the next three years.

The forecast for core PCE inflation continues to be a story of upward pressure from the unwinding of negative labor supply shocks and MEI shocks being offset by downward pressure from the waning of discount factor shocks. Negative discount factor shocks have a strong and persistent negative effect on marginal cost and inflation in the estimated model. But labor supply shocks that push down aggregate hours also serve to put upward pressure on the real wage and hence marginal cost. The effect is persistent -- as the labor supply shocks unwind over the forecast horizon they exert a waning upward push to inflation. On balance the effect of these opposing forces keep inflation close to, or slightly below, target over the next 3 years.

The federal funds rate is projected to rise fairly quickly over the forecast horizon. The model attributes the current level of the funds rate primarily to a combination of monetary policy, discount factor and MEI shock dynamics. Looking ahead, the positive contribution from labor supply shocks is more than offset by discount factor shock dynamics over the medium term, but as these shocks wane the funds rate gradually rises to 3.6 percent by the end of 2020.

References

Schorfheide, Frank, Keith Sill, and Maxym Kryshko. 2010. "DSGE model-based forecasting of non-modelled variables." *International Journal of Forecasting*, 26(2): 348-373.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in U.S. Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97(3): 586-606.

Figure 1a

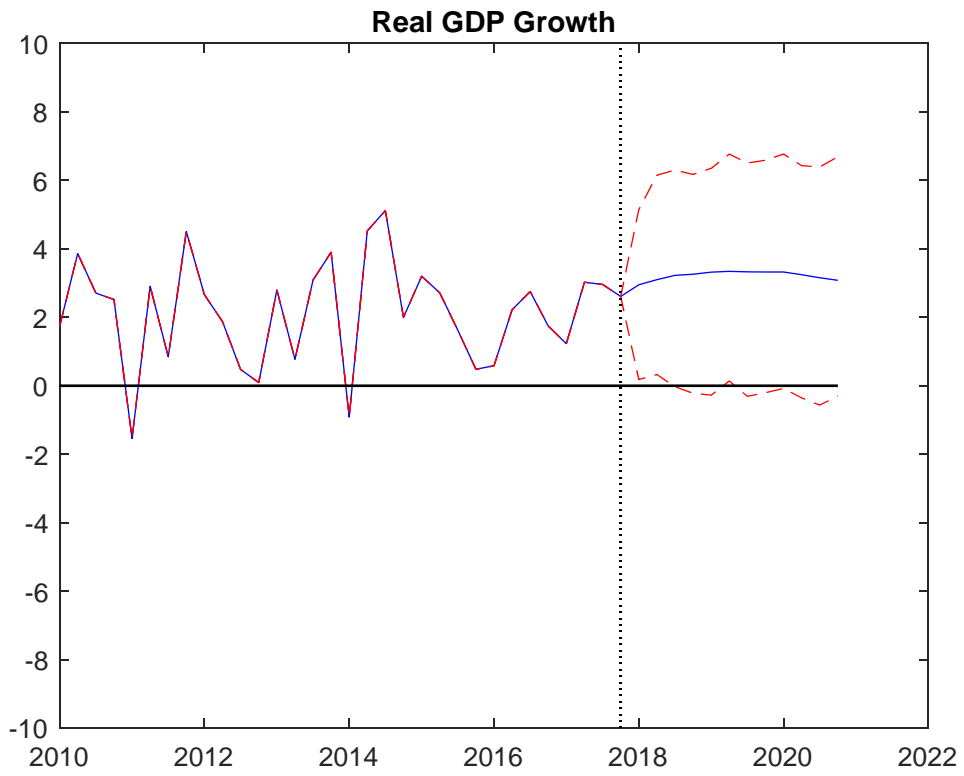


Figure 1b

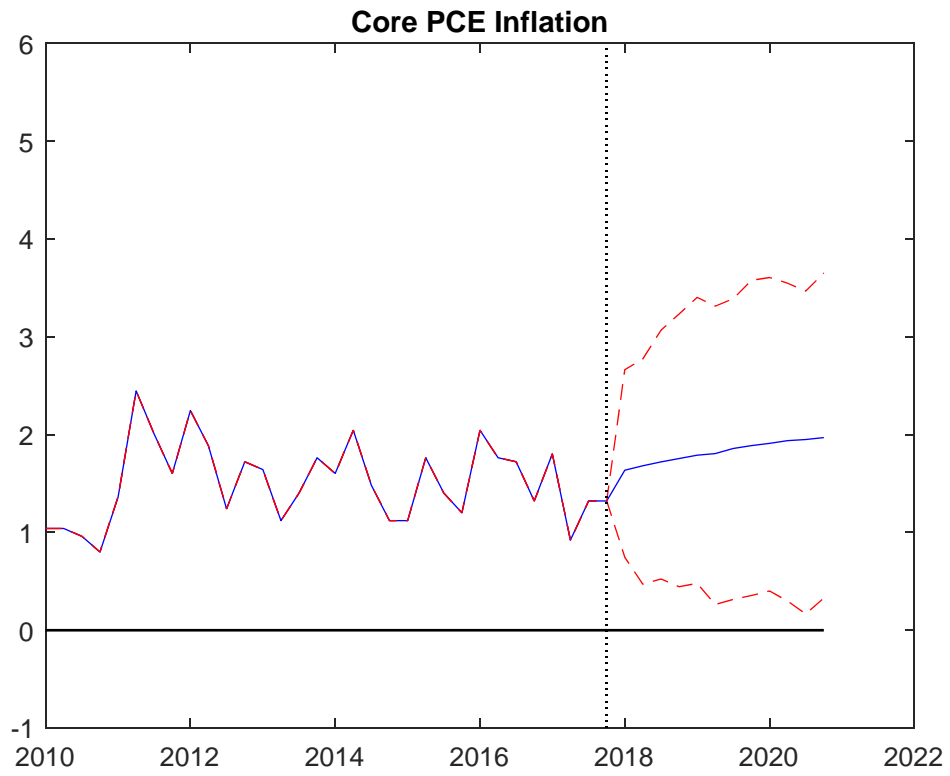


Figure 1c

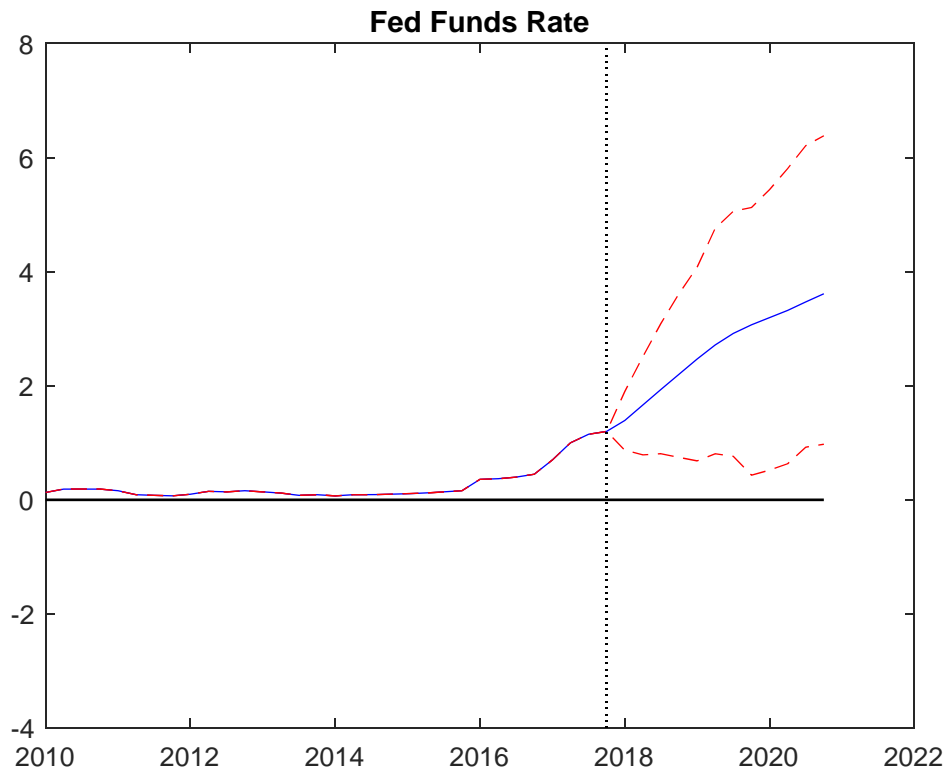
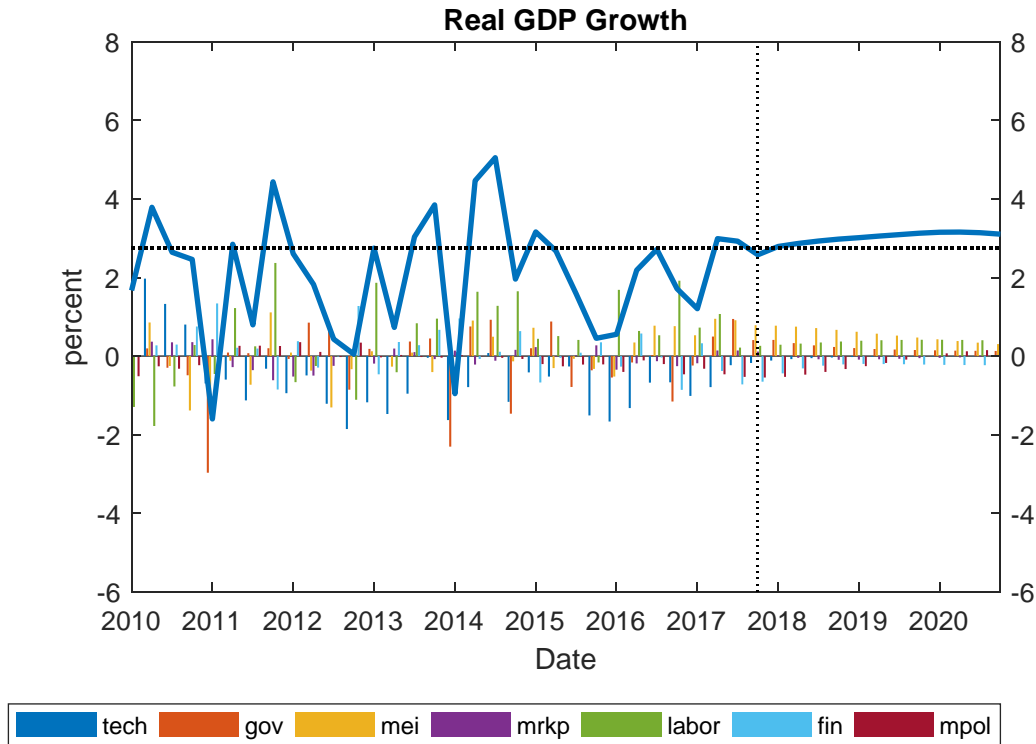


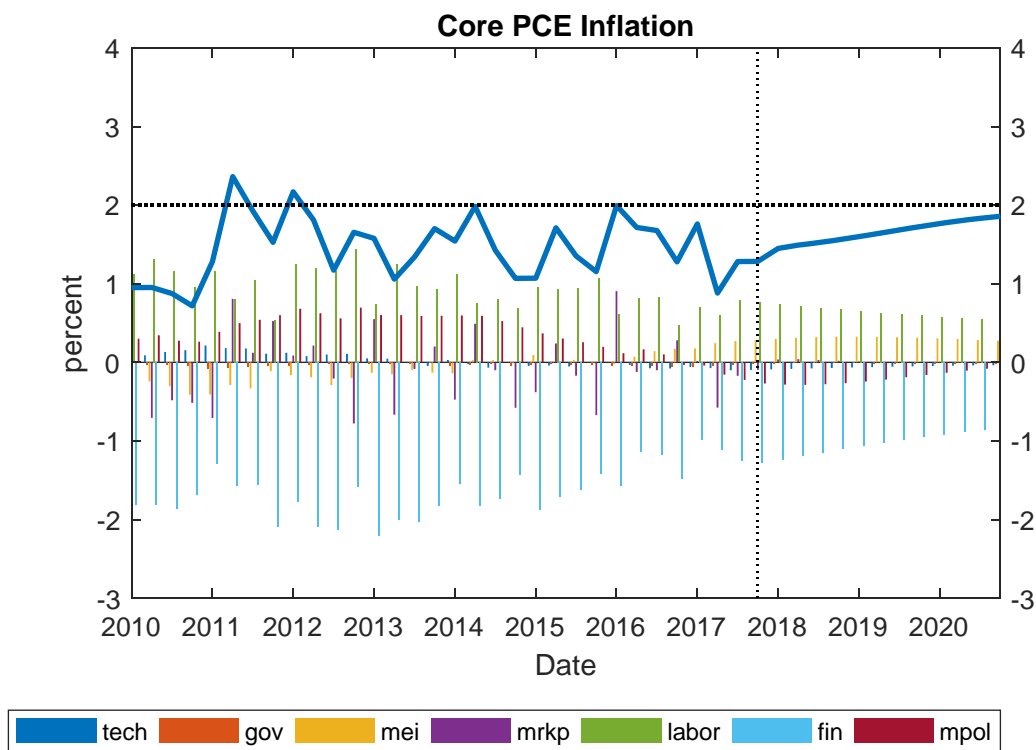
Figure 2a
Shock Decompositions



shocks:

- TFP: Total factor productivity growth shock
- Gov: Government spending shock
- MEI: Marginal efficiency of investment shock
- MrkUp: Price markup shock
- Labor: Labor supply shock
- Fin: Discount factor shock
- Mpol: Monetary policy shock

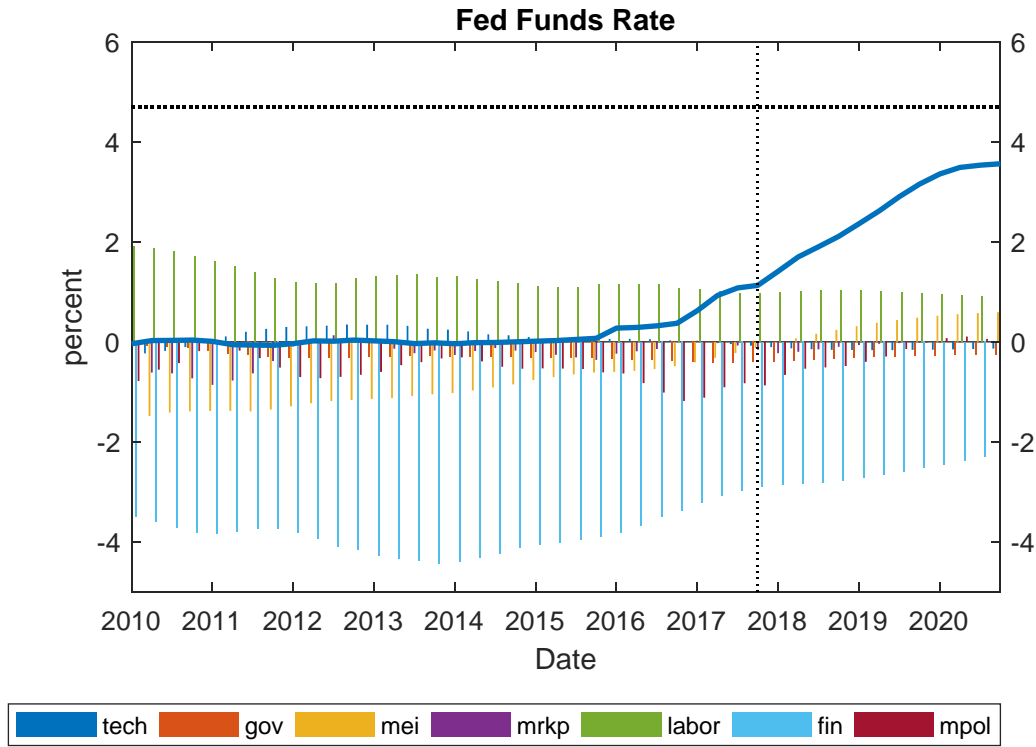
Figure 2b
Shock Decompositions



shocks:

- TFP: Total factor productivity growth shock
- Gov: Government spending shock
- MEI: Marginal efficiency of investment shock
- MrkUp: Price markup shock
- Labor: Labor supply shock
- Fin: Discount factor shock
- Mpol: Monetary policy shock

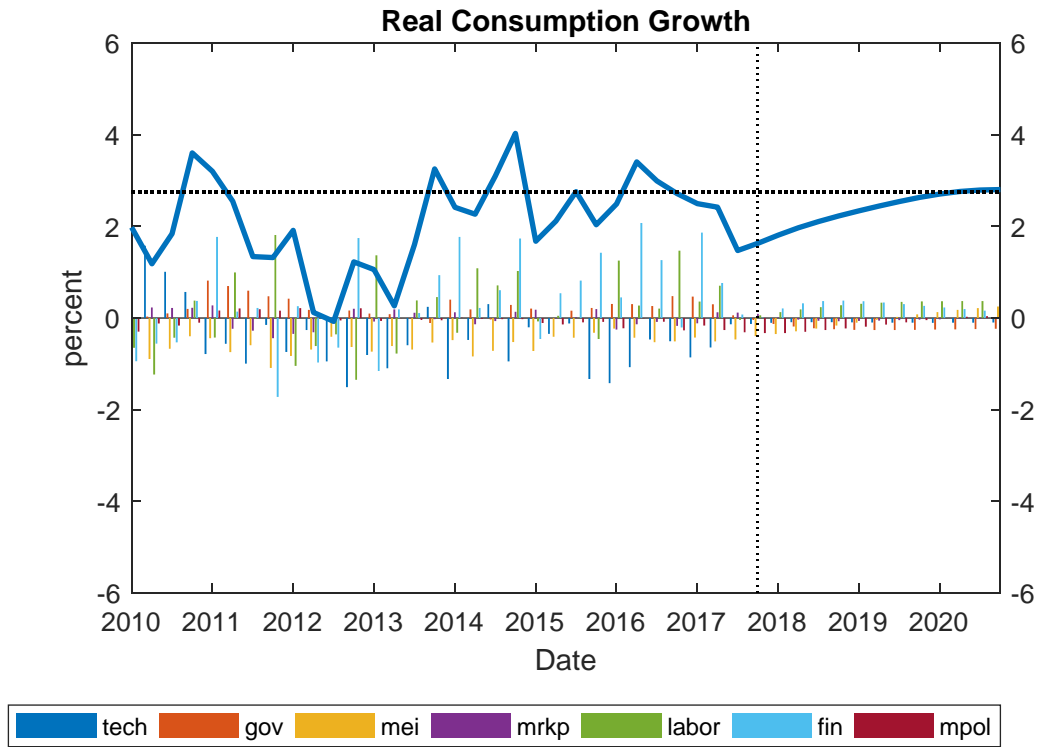
Figure 2c
Shock Decompositions



shocks:

- TFP: Total factor productivity growth shock
- Gov: Government spending shock
- MEI: Marginal efficiency of investment shock
- MrkUp: Price markup shock
- Labor: Labor supply shock
- Fin: Discount factor shock
- Mpol: Monetary policy shock

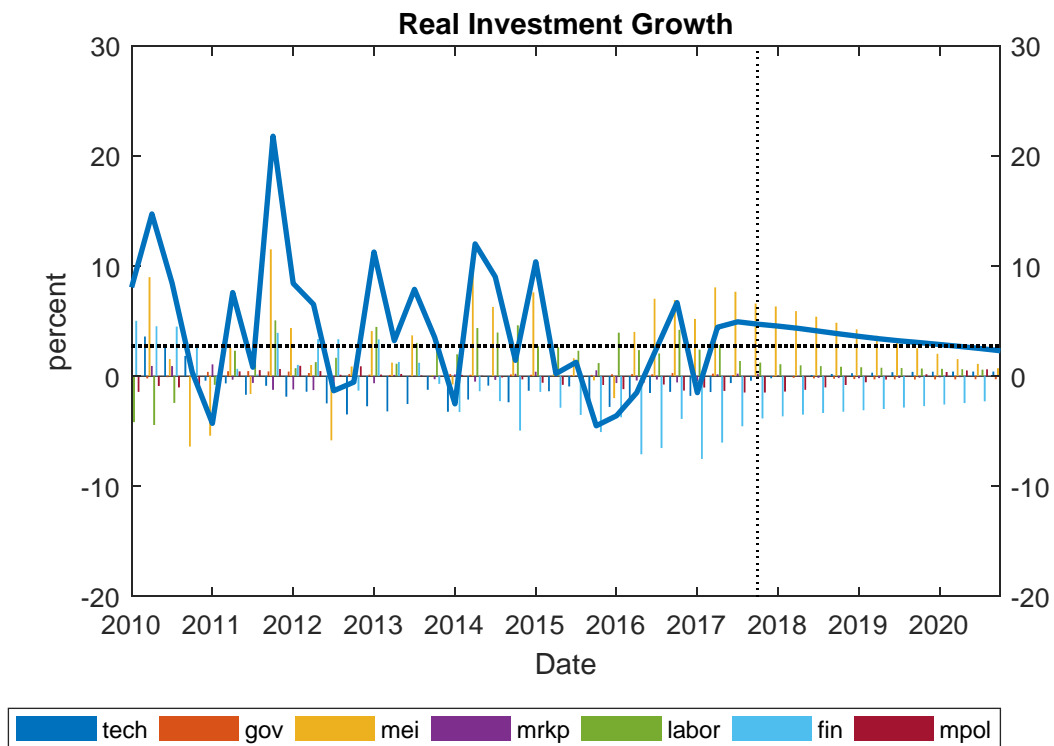
Figure 2d
Shock Decompositions



shocks:

- TFP: Total factor productivity growth shock
- Gov: Government spending shock
- MEI: Marginal efficiency of investment shock
- MrkUp: Price markup shock
- Labor: Labor supply shock
- Fin: Discount factor shock
- Mpol: Monetary policy shock

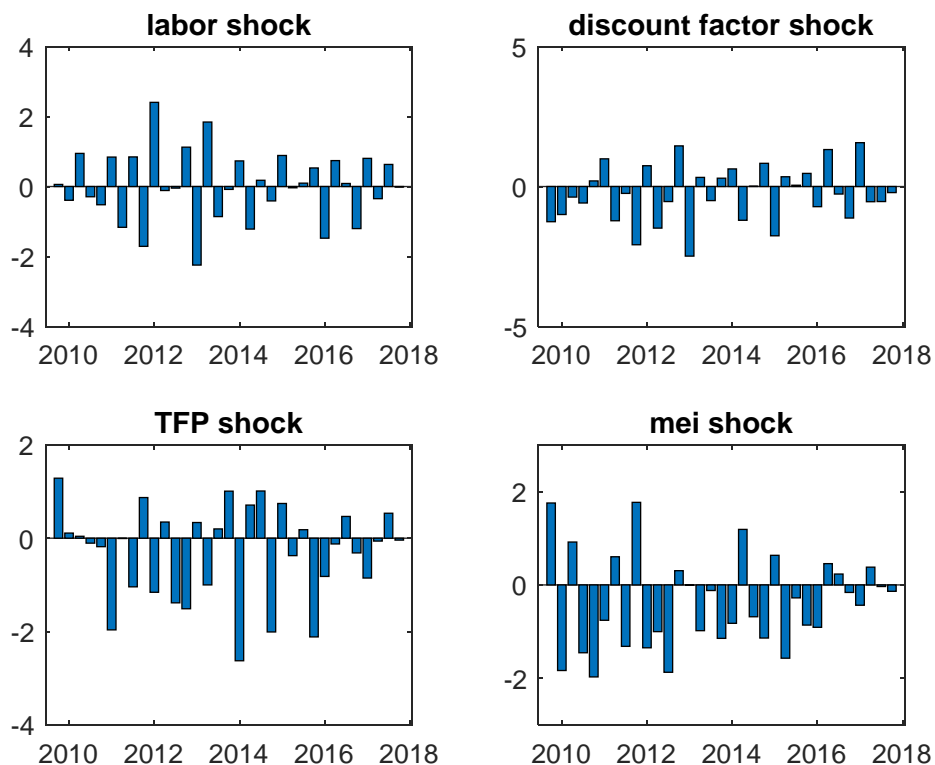
Figure 2e
Shock Decompositions



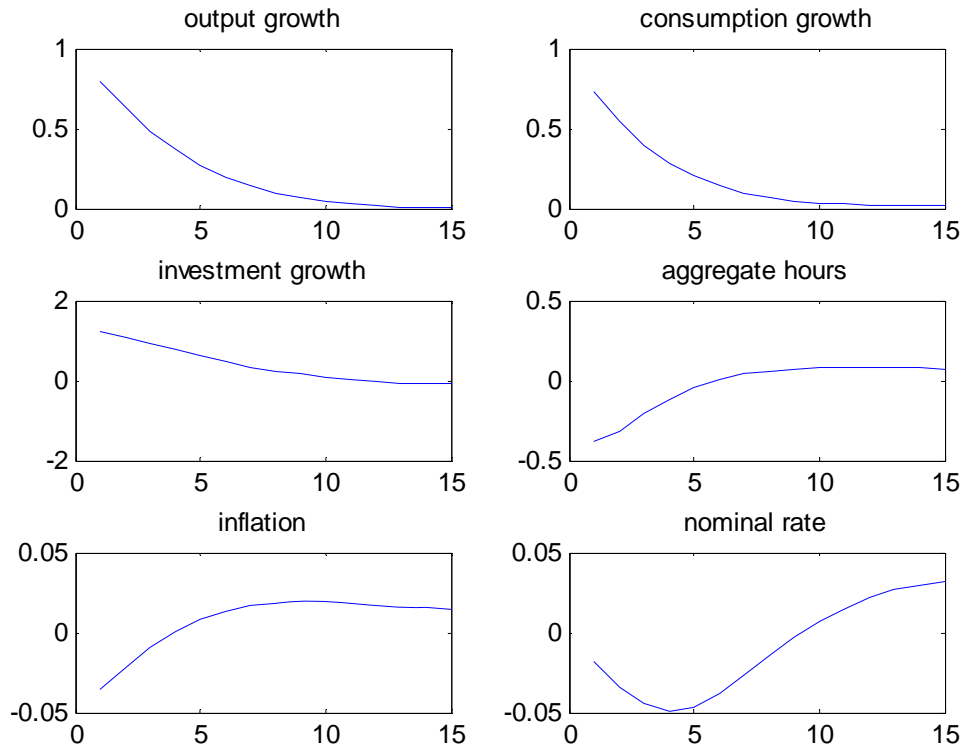
shocks:

- TFP: Total factor productivity growth shock
- Gov: Government spending shock
- MEI: Marginal efficiency of investment shock
- MrkUp: Price markup shock
- Labor: Labor supply shock
- Fin: Discount factor shock
- Mpol: Monetary policy shock

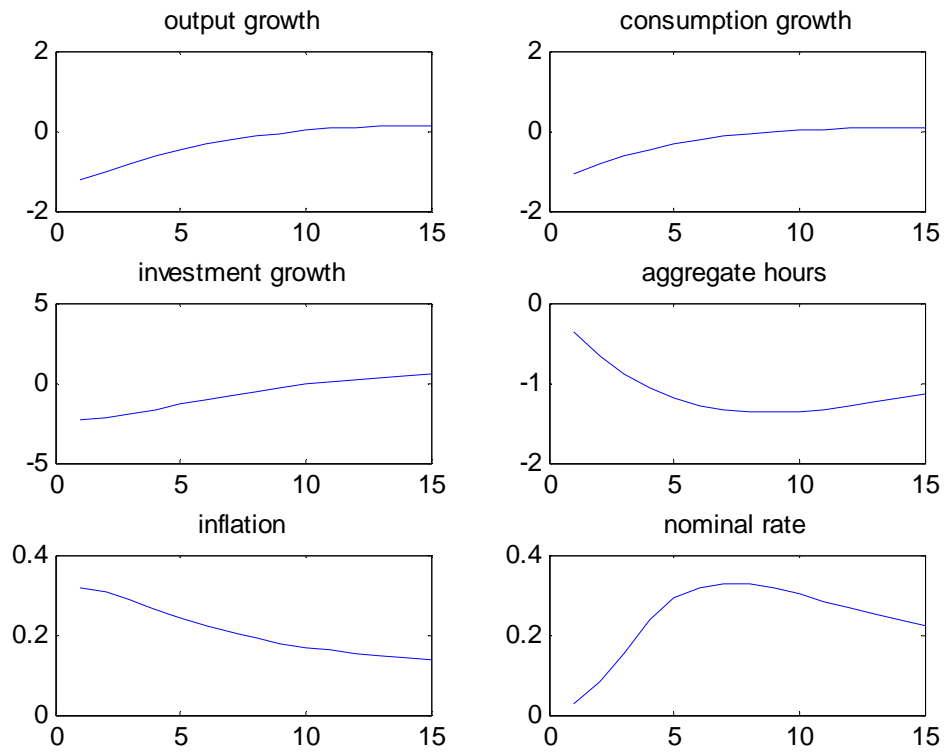
Figure 3
Smoothed Shock Estimates for Conditional Forecast Model
(normalized by standard deviation)



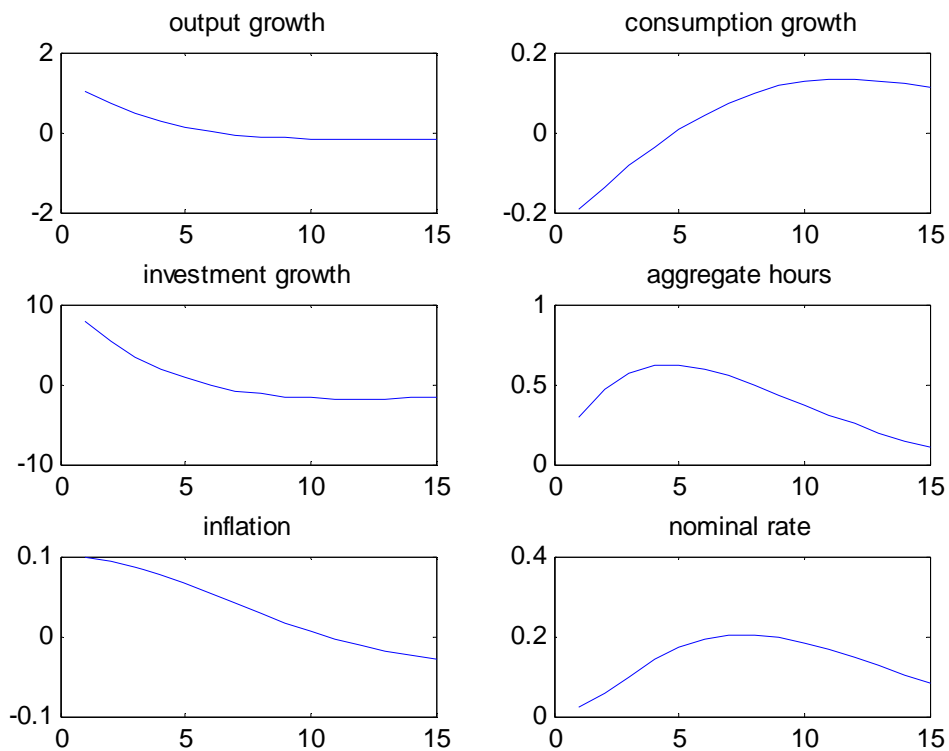
Impulse Responses to TFP shock



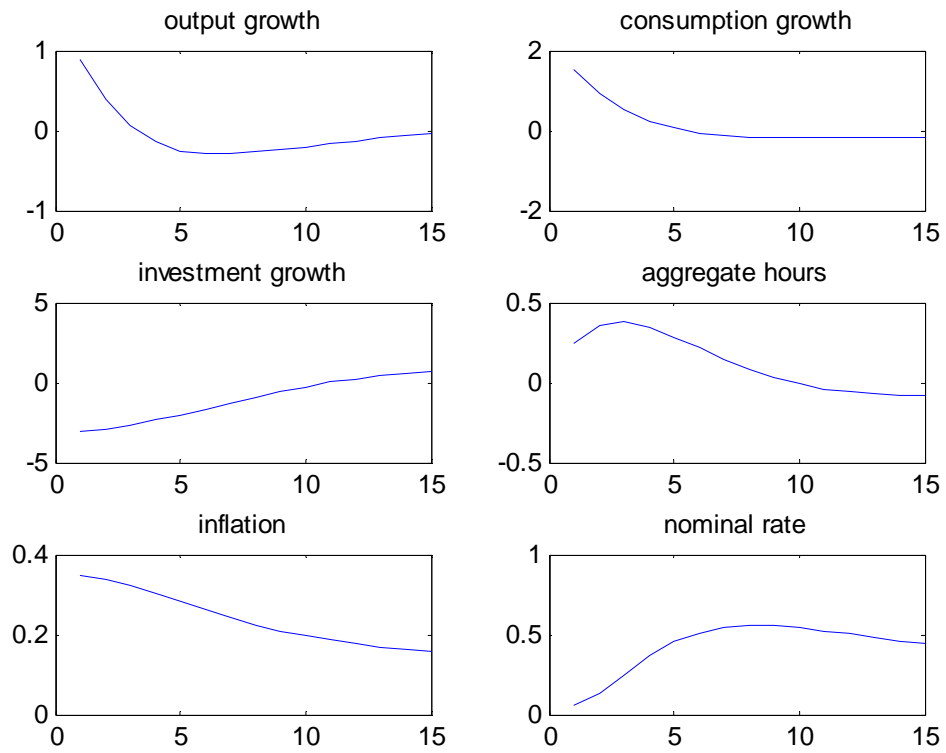
Impulse Response to Leisure Shock



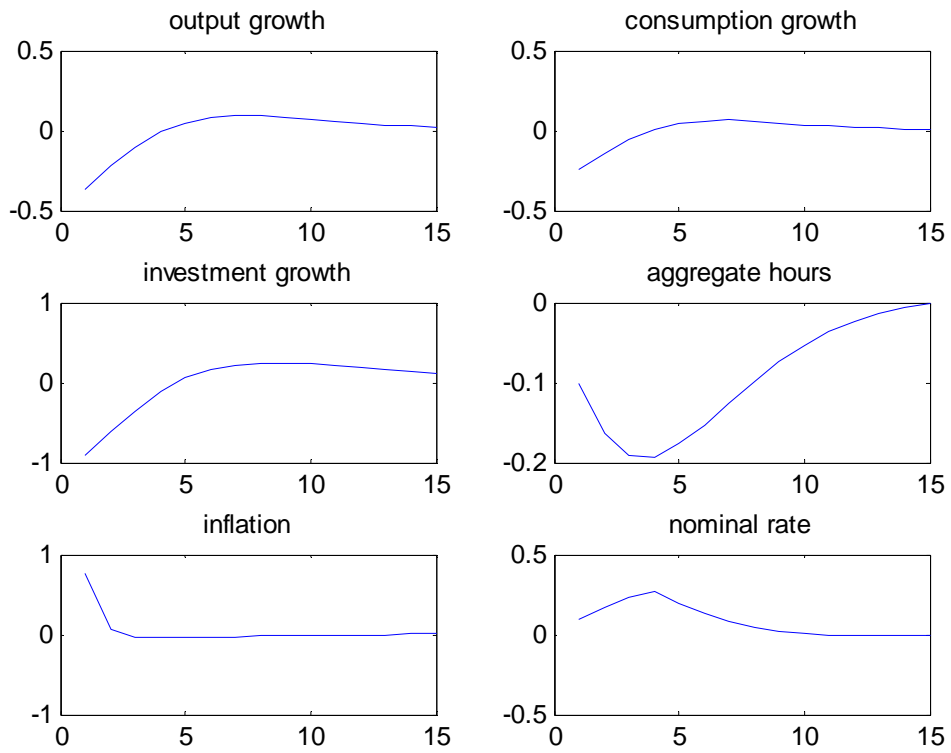
Impulse Responses to MEI Shock



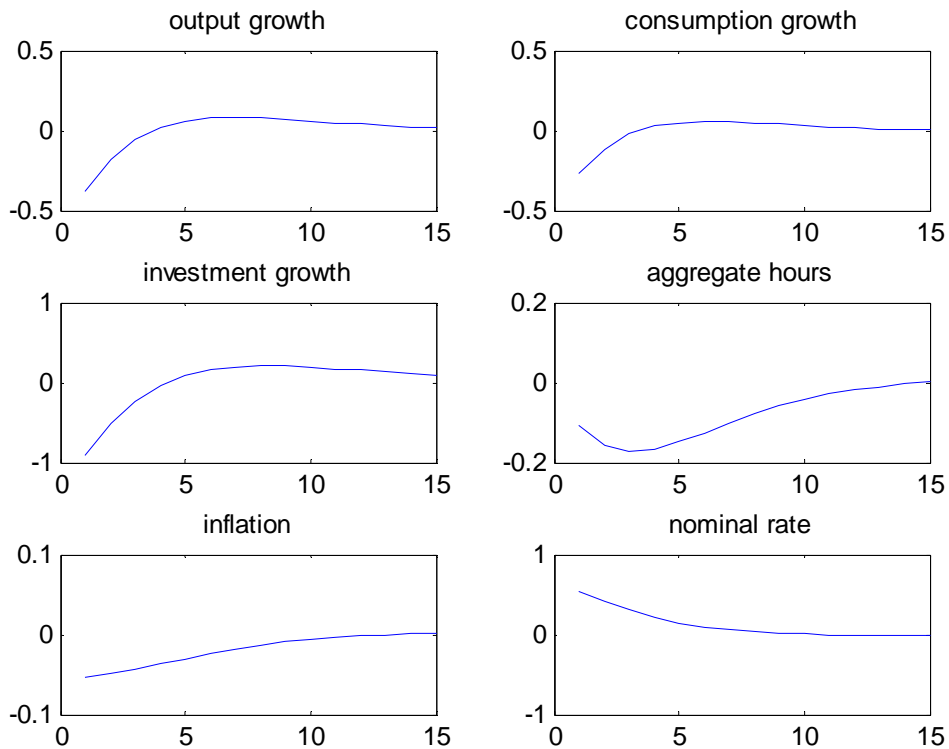
Impulse Responses to Financial Shock



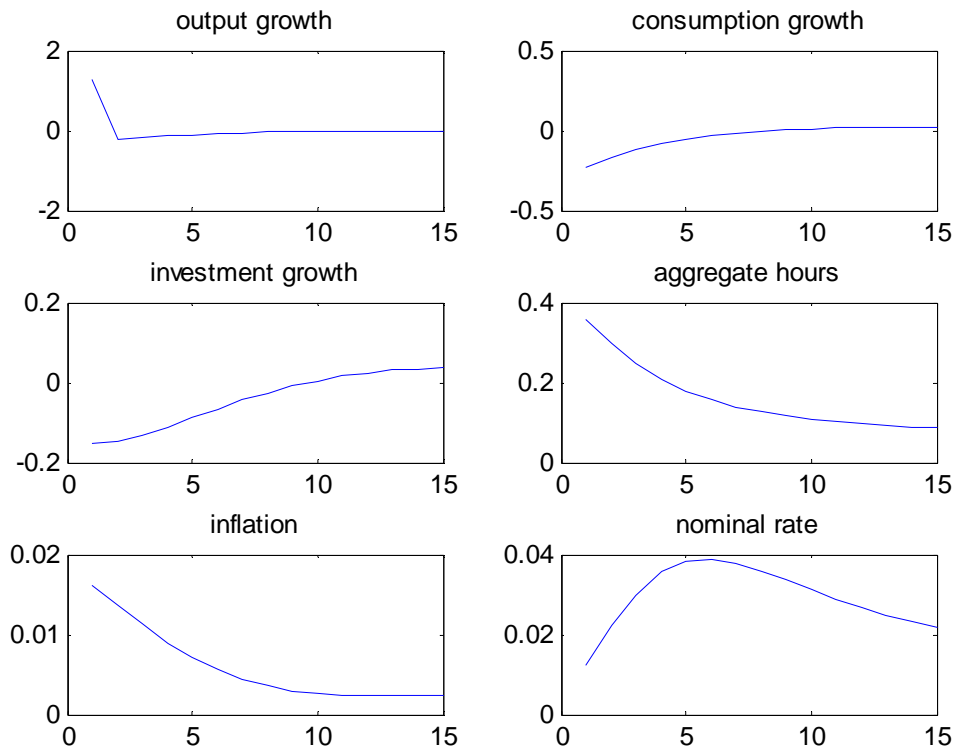
Impulse Responses to Price Markup Shock



Impulse Responses to Unanticipated Monetary Policy Shock



Impulse Responses to Govt Spending Shock



Research Directors' Guide to the Chicago Fed DSGE Model*

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November 30, 2017

This guide describes the construction and estimation of the Chicago Fed's DSGE model, which we use both for internal forecasting and for creating our contributions to the System DSGE memo distributed quarterly to the FOMC. The model has been in use and under ongoing development since 2010. Originally, it was largely based on [Justiniano, Primiceri, and Tambalotti \(2010\)](#). We published results based on simulations from the estimated model in [Campbell, Evans, Fisher, and Justiniano \(2012\)](#) and in [Campbell, Fisher, Justiniano, and Melosi \(2016\)](#).

The model contains many features familiar from other DSGE analyses of monetary policy and business cycles. External habit in preferences, i -dot costs of adjusting investment, price and wage stickiness based on [Calvo's \(1983\)](#) adjustment probabilities, and partial indexation of unadjusted prices and wages using recently observed price and wage inflation. The features which distinguish our analysis from many otherwise similar undertakings are

- **Forward Guidance Shocks:** An interest-rate rule which depends on recent (and expected future) inflation and output and is subject to stochastic disturbances governs our model economy's monetary policy rate. Standard analysis prior to the great recession restricted the stochastic disturbances to be unforecastable. Our model deviates from this historical standard by including forward guidance shocks, as in [Laséen and Svensson \(2011\)](#). A j -quarter ahead forward guidance shock revealed to the public at time t influences the interest-rate rule's stochastic intercept only at time $t + j$. Each period, the model's monetary authority reveals a vector of these shocks with one element for each quarter from the present until the end of the forward guidance horizon. The

*The views expressed herein are the authors'. They do not necessarily represent those of the Federal Reserve Bank of Chicago, the Federal Reserve System, or its Board of Governors.

vector’s elements may be correlated with each other, so the monetary authority could routinely reveal persistent shifts in the interest-rate rule’s stochastic intercept. However, the forward guidance shocks are serially uncorrelated *over time*, as is required for them to match the definition of “news.”

- **Investment-Specific Technological Change:** As in the Real Business Cycle models from which modern DSGE models descend (King, Plosser, and Rebelo, 1988a), stochastic trend productivity growth both short-run and long-run fluctuations. Our model features two such stochastic trends, one to Hicks-neutral productivity (King, Plosser, and Rebelo, 1988b) and one to the technology for converting consumption goods into investment goods (as in Fisher (2006)). This investment-specific technological change allows our model to reproduce the dynamics of the relative price of investment goods to consumption goods, which is a necessary input into the formula we use to create Fisher-ideal chain-weighted index of real GDP.
- **A Mixed Calibration-Bayesian Estimation Empirical Strategy:** Bayesian estimation of structural business cycle models attempts to match all features of the data’s probability distribution using the model’s parameters. Since no structural model embodies Platonic “truth,” this exercise inevitably requires trading off between the model’s ability to replicate first moments with its fidelity to the business cycles in second moments. Since the criteria for this tradeoff are not always clear, we adopt an alternative “first-moments-first” strategy. This selects the values of model parameters which govern the model’s steady-state growth path, such as the growth rates of Hicks-neutral and investment-specific technology, to match estimates of selected first moments. These parameter choices are then fixed for Bayesian estimation, which chooses values for model parameters which only influence second moments, such as technology innovation variances. (Since we employ a log linear solution of our model and all shocks to its primitives have Gaussian distributions, our analysis has no non-trivial implications for third and higher moments of the data.)

The guide proceeds as follows. The next section presents the model economy’s primitives, while Section 2 presents the agents’ first-order conditions. Section 3 gives the formulas used to remove nominal and technological trends from model variables and thereby induce model stationarity, and Sections 4 and 5 discuss

the stationary economy's steady state and the log linearization of its equilibrium necessary conditions around it. Section 6 discusses measurement issues which arise when comparing model-generated data with data measured by the BEA and BLS. Section 7 describes our mixed Calibration-Bayesian Estimation empirical strategy and presents the resulting parameter values we use for model simulations and forecasting.

1 The Model's Primitives

Eight kinds of agents populate the model economy:

- Households,
- Investment producers,
- Competitive final goods producers,
- Monopolistically-competitive differentiated goods producers,
- Labor Packers,
- Monopolistically-competitive guilds,
- a Fiscal Authority and
- a Monetary Authority.

These agents interact with each other in markets for

- final goods used for consumption
- investment goods used to augment the stock of productive capital
- differentiated intermediate goods
- capital services
- raw labor
- differentiated labor
- composite labor

- government bonds
- privately-issued bonds, and
- state-contingent claims.

The households have preferences over streams of an aggregate consumption good, leisure, and the real value of the fiscal authority’s bonds in their portfolios. Our specification for preferences displays balanced growth. They also feature *external* habit in consumption; which creates a channel for the endogenous propagation of shocks. Our bonds-in-the-utility-function preferences follow those of Fisher (2015), and they allow us to incorporate a persistent spread between the monetary policy rate and the return on productive capital. The aggregate consumption good has a single alternative use, as the only input into the linear production function operated by investment producers. These firms sell their output to the households. In turn, households produce capital services from their capital stocks, which they then sell to differentiated goods producers. Producers of final goods operate a constant-returns-to-scale technology with a constant elasticity of substitution between its inputs, which are differentiated goods produced by the monopolistically-competitive firms. These firms operate technologies with affine cost curves (a constant fixed cost and linear marginal cost), which employs capital services and composite labor as inputs. The labor packers produce composite labor using a constant-returns-to-scale technology with a constant elasticity of substitution between its inputs, the differentiated labor sold by guilds. Each of these produces differentiated labor from the raw labor provided by the households with a linear technology, and they sell their outputs to the labor packers. There is a nominal unit of account, called the “dollar.” The fiscal authority issues one-period nominally risk-free bonds, provides public goods through government spending, and assesses lump-sum taxes on households. The monetary authority sets the interest rate on the fiscal authority’s one-period bond according to an interest-rate rule.

All non-financial trade is denominated in dollars, and all private agents take prices as given with two exceptions: the monopolistically-competitive differentiated-goods producers and guilds. These choose output prices to maximize the current value of expected future profits taking as given their demand curves and all relevant input prices. Financial markets are complete, but all securities excepting equities in differentiated-goods producers are in zero net supply. These producers’ profits

and losses are rebated to the households (who own the firms' equities) lump-sum period-by-period, as are the profits and losses of the guilds. Given both a process for government spending and taxes and a rule for the monetary authority's interest rate choice, a competitive equilibrium consists of allocations and prices that are consistent with households' utility maximization, firms' profit maximization, guilds' profit maximization, and market clearing.

The economy is subject to stochastic disturbances in technology, preferences, and government policy. Without nominal rigidities, the economy's real allocations in competitive equilibrium can be separated from inflation and other dollar-denominated variables. Specifically, monetary policy only influences inflation. To connect real and nominal variables in the model and thereby consider the impact of monetary policy on the business cycle, we introduce Calvo-style wage and price setting. That is, nature endows both differentiated goods producers and guilds with *stochastic* opportunities to incorporate all available information into their nominal price choices. Those producers and guilds without such an opportunity must set their prices according to simple indexing formulas. These two pricing frictions create two forward-looking Phillips curves, one for prices and another for wages, which form the core of the new Keynesian approach to monetary policy analysis.

The model economy is stochastic and features complete markets in state-contingent claims. To place these features on a sound footing, we base all shocks on a general Markovian stochastic process s_t . Denote the history of this vector from an initial period 0 through τ with $s^\tau \equiv (s_0, s_1, \dots, s_\tau)$. The support of s^τ is Σ^τ , and the probability density of s^τ given s_t for some $t < \tau$ is $\mathfrak{D}(s^\tau | s_t)$. (The Hebrew letter \mathfrak{D} , pronounced "samekh," corresponds to the Greek letter σ .) All model shocks are implicit functions of s_t , and all endogenous variables are implicit functions of s^t . We refer to all such implicit functions as "state-contingent sequences." We use braces to denote such a sequence. For example, $\{X_t\}$ represents the state-contingent sequence for a generic variable X_t .

1.1 Households

Our model's households are the ultimate owners of all assets in positive net supply (the capital stock, differentiated goods producers, and guilds). They provide labor and divide their current after-tax income (from wages and assets) between current

consumption, investment in productive capital, and purchases of financial assets, both those issued by the government and those issued by other households. The individual household divides its current resources between consumption and the available vehicles for intertemporal substitution (capital and financial assets) to maximize a discounted sum of current and expected future felicity.

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \varepsilon_{t+\tau}^b \left(U_{t+\tau} + \varepsilon_{t+\tau}^s L \left(\frac{B_{t+\tau}}{P_{t+\tau} R_{t+\tau}} \right) \right) \right]$$

with

$$U_t = \frac{1}{1 - \gamma_c} \left((C_t - \varrho \bar{C}_{t-1}) (1 - H_t^{1+\gamma_h}) \right)^{(1-\gamma_c)} \quad (1)$$

The function $L(\cdot)$ is strictly increasing, concave, and differentiable everywhere on $[0, \infty)$. In particular, $L'(0)$ exists and is finite. Without loss of generality, we set $L'(0)$ to one. The argument of $L(\cdot)$ equals the real value of government bonds in the household's portfolio: their period $t + 1$ redemption value B_t divided by their nominal yield R_t expressed in units of the consumption good with the nominal price index P_t . The time-varying coefficient multiplying this felicity from bond holdings, ε_t^s , is the liquidity preference shock introduced by [Fisher \(2015\)](#). A separate shock influences the household's discounting of future utility to the present, ε_t^b . Specifically, the household discounts a certain utility in $t + \tau$ back to t with $\beta^\tau \mathbb{E}_t [\varepsilon_{t+\tau}^b / \varepsilon_t^b]$. In logarithms, these two preference shocks follow independent autoregressive processes.

$$\ln \varepsilon_t^b = (1 - \rho_b) \ln \varepsilon_*^b + \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b, \eta_t^b \sim \mathbb{N}(0, \sigma_b) \quad (2)$$

$$\ln \varepsilon_t^s = (1 - \rho_s) \ln \varepsilon_*^s + \rho_s \ln \varepsilon_{t-1}^s + \eta_t^s, \eta_t^s \sim \mathbb{N}(0, \sigma_s). \quad (3)$$

A household's wealth at the beginning of period t consists of its nominal government bond holdings, B_t , its net holdings of privately-issued financial assets, and its capital stock K_{t-1} . The household chooses a rate of capital utilization u_t , and the capital services resulting from this choice equal $u_t K_{t-1}$. The cost of increasing utilization is higher depreciation. An increasing, convex and differentiable function $\delta(U)$ gives the capital depreciation rate. We specify this as

$$\delta(u) = \delta_0 + \delta_1 (u - u_*) + \frac{\delta_2}{2} (u - u_*)^2.$$

A household can augment its capital stock with investment, I_t . Investment requires paying adjustment costs of the “i-dot” form introduced by [Christiano, Eichenbaum, and Evans \(2005\)](#). Also, an *investment demand shock* alters the efficiency of investment in augmenting the capital stock. Altogether, if the household’s investment in the previous period was I_{t-1} , and it purchases I_t units of the investment good today, then the stock of capital available in the *next* period is

$$K_t = (1 - \delta(u_t)) K_{t-1} + \varepsilon_t^i \left(1 - S \left(\frac{A_{t-1}^K I_t}{A_t^K I_{t-1}} \right) \right) I_t. \quad (4)$$

In (4), A_t^K equals the productivity level of capital goods production, described in more detail below, and ε_t^i is the investment demand shock. In logarithms, this follows a first-order autoregression with a normally-distributed innovation.

$$\ln \varepsilon_t^i = (1 - \rho_i) \ln \varepsilon_*^i + \rho_i \ln \varepsilon_{t-1}^i + \eta_t^i, \eta_t^i \sim \mathbb{N}(0, \sigma_i) \quad (5)$$

1.2 Production

The producers of investment goods use a linear technology to transform the final good into investment goods. The technological rate of exchange from the final good to the investment good in period t is A_t^I . We denote $\Delta \ln A_t^I$ with ω_t , which we call the *investment-specific technology shock* and which follows first-order autogression with normally distributed innovations.

$$\omega_t = (1 - \rho_\omega) \omega_* + \rho_\omega \omega_{t-1} + \eta_t^\omega, \eta_t^\omega \sim \mathbb{N}(0, \sigma_\omega^2) \quad (6)$$

Investment goods producers are perfectly competitive.

Final good producers also operate a constant-returns-to-scale technology; which takes as inputs the products of the differentiated goods producers. To specify this, let Y_{it} denote the quantity of good i purchased by the representative final good producer in period t , for $i \in [0, 1]$. The representative final good producer’s output then equals

$$Y_t \equiv \left(\int_0^1 Y_{it}^{\frac{1}{1+\lambda_t^p}} di \right)^{1+\lambda_t^p}.$$

With this technology, the elasticity of substitution between any two differentiated products equals $1 + 1/\lambda_t^p$ in period t . Although this is constant across products within a time period, it varies stochastically over time according to an ARMA(1, 1) in logarithms.

$$\ln \lambda_t^p = (1 - \rho_p) \ln \lambda_*^p + \rho_p \ln \lambda_{t-1}^p - \theta_p \eta_{t-1}^p + \eta_t^p, \eta_t^p \sim \mathbb{N}(0, \sigma_p) \quad (7)$$

Given nominal prices for the intermediate goods P_{it} , it is a standard exercise to show that the final goods producers' marginal cost equals

$$P_t = \left(\int_0^1 P_{it}^{-\frac{1}{\lambda_t^p}} di \right)^{-\lambda_t^p} \quad (8)$$

Just like investment goods firms, the final goods' producers are perfectly competitive. Therefore, profit maximization and positive final goods output together require the competitive output price to equal P_t . Therefore, we can define inflation of the nominal final good price as $\pi_t \equiv \ln(P_t/P_{t-1})$.

The intermediate goods producers each use the technology

$$Y_{it} = (K_{it}^e)^\alpha (A_t^Y H_{it}^d)^{1-\alpha} - A_t \Phi \quad (9)$$

Here, K_{it}^e and H_{it}^d are the capital services and labor services used by firm i , and A_t^Y is the level of neutral technology. Its growth rate, $\nu_t \equiv \ln(A_t^Y/A_{t-1}^Y)$, follows a first-order autoregression.

$$\nu_t = (1 - \rho_\nu) \nu_* + \rho_\nu \nu_{t-1} + \eta_t^\nu, \eta_t^\nu \sim \mathbb{N}(0, \sigma_\nu), \quad (10)$$

The final term in (9) represents the fixed costs of production. These grow with

$$A_t \equiv A_t^Y (A_t^I)^{\frac{\alpha}{1-\alpha}}. \quad (11)$$

We demonstrate below that A_t is the stochastic trend in equilibrium output and consumption, measured in units of the final good. We denote its growth rate with

$$z_t = \nu_t + \frac{\alpha}{1-\alpha} \omega_t \quad (12)$$

Similarly, define

$$A_t^K \equiv A_t A_t^I \tag{13}$$

In the specification of the capital accumulation technology, we labelled A_t^K the “productivity level of capital goods production.” We demonstrate below that this is indeed the case with the definition in (13).

Each intermediate goods producer chooses prices subject to a [Calvo \(1983\)](#) pricing scheme. With probability $\zeta_p \in [0, 1]$, producer i has the opportunity to set P_{it} without constraints. With the complementary probability, P_{it} is set with the indexing rule

$$P_{it} = P_{it-1} \pi_{t-1}^{\iota_p} \pi_\star^{1-\iota_p}. \tag{14}$$

In (14), π_\star is the gross rate of price growth along the steady-state growth path, and $\iota_p \in [0, 1]$.¹

1.3 Labor Markets

Households’ hours worked pass through two intermediaries, guilds and labor packers, in their transformation into labor services used by the intermediate goods producers. The guilds take the households’ homogeneous hours as their only input and produce differentiated labor services. These are then sold to the labor packers, who assemble the guilds’ services into composite labor services.

The labor packers operate a constant-returns-to-scale technology with a constant elasticity of substitution between the guilds’ differentiated labor services. For its specification, let H_{it} denote the hours of differentiated labor purchased from guild i at time t by the representative labor packer. Then that packer’s production of composite labor services, H_t^s are given by

$$H_t^s = \left(\int_0^1 (H_{it})^{\frac{1}{1+\lambda_t^w}} di \right)^{1+\lambda_t^w}.$$

As with the final good producer’s technology, an ARMA(1, 1) in logarithms governs

¹To model firms’ price-setting opportunities as functions of s_t , define a random variable u_t^p which is independent over time and uniformly distributed on $[0, 1]$. Then, firm i gets a price-setting opportunity if either $u_t^p \geq \zeta_p$ and $i \in [u_t^p - \zeta_p, u_t^p]$ or if $u_t^p < \zeta_p$ and $i \in [0, u_t^p] \cup [1 + u_t^p - \zeta_p, 1]$.

the constant elasticity of substitution between any two guilds' labor services.

$$\ln \lambda_t^w = (1 - \rho_w) \ln \lambda_*^w + \rho_w \ln \lambda_{t-1}^w - \theta_w \eta_{t-1}^w + \eta_t^w, \eta_t^w \sim \mathbb{N}(0, \sigma_w^2) \quad (15)$$

Just as with the final goods producers, we can easily show that the labor packers' marginal cost equals

$$W_t = \left(\int_0^1 (W_{it})^{-\frac{1}{\lambda_t^w}} di \right)^{-\lambda_t^w}. \quad (16)$$

Here, W_{it} is the nominal price charged by guild i per hour of differentiated labor. Since labor packers are perfectly competitive, their profit maximization and positive output together require that the price of composite labor services equals their marginal cost.

Each guild produces its differentiated labor service using a linear technology with the household's hours worked as its only input. A [Calvo \(1983\)](#) pricing scheme similar to that of the differentiated goods producers constrains their nominal prices. Guild i has an unconstrained opportunity to choose its nominal price with probability $\zeta_w \in [0, 1]$. With the complementary probability, W_{it} is set with an indexing rule based on π_{t-1} and last period's trend growth rate, z_{t-1} .

$$W_{it} = W_{it-1} (\pi_{t-1} e^{z_{t-1}})^{\iota_w} (\pi_* e^{z_*})^{1-\iota_w}. \quad (17)$$

In (17), $z_* \equiv \nu_* + \frac{\alpha}{1-\alpha} \omega_*$ is the unconditional mean of z_t and $\iota_w \in [0, 1]$.

1.4 Fiscal and Monetary Policy

The model economy hosts two policy authorities, each of which follows exogenously-specified rules that receive stochastic disturbances. The fiscal authority issues bonds, B_t , collects lump-sum taxes T_t , and buys "wasteful" public goods G_t . Its period-by-period budget constraint is

$$G_t + B_{t-1} = T_t + \frac{B_t}{R_t}. \quad (18)$$

The left-hand side gives the government's uses of funds, public goods spending and the retirement of existing debt. The right-hand side gives the sources of funds, taxes

and the proceeds of new debt issuance at the interest rate R_t . We assume that the fiscal authority keeps its budget balanced period-by-period, so $B_t = 0$. Furthermore, the fiscal authority sets public goods expenditure equal to a stochastic share of output, expressed in consumption units.

$$G_t = (1 - 1/g_t)Y_t, \quad (19)$$

with

$$\ln g_t = (1 - \rho_g) \ln s_*^g + \rho_g \ln g_{t-1} + \eta_t^g, \eta_t^g \sim \mathbb{N}(0, \sigma_g^2). \quad (20)$$

The monetary authority sets the nominal interest rate on government bonds, R_t . For this, it employs a Taylor rule with interest-rate smoothing and forward guidance shocks.

$$\ln R_t = \rho_R \ln R_{t-1} + (1 - \rho_R) \ln R_t^n + \sum_{j=0}^M \xi_{t-j}^j. \quad (21)$$

The monetary policy disturbances in (21) are $\xi_t^0, \xi_{t-1}^1, \dots, \xi_{t-M}^M$. The public learns the value of ξ_{t-j}^j in period $t - j$. The conventional unforecastable shock to current monetary policy is ξ_t^0 , while for $j \geq 1$, these disturbances are *forward guidance shocks*. We gather all monetary shocks revealed at time t into the vector ε_t^1 . This is normally distributed and *i.i.d.* across time. However, *its elements may be correlated with each other*. That is,

$$\varepsilon_t^1 \equiv (\xi_t^0, \xi_t^1, \dots, \xi_t^M) \sim \mathbb{N}(0, \Sigma^1). \quad (22)$$

The off-diagonal elements of Σ^1 are not necessarily zero, so forward-guidance shocks need not randomly impact expected future monetary policy at two adjacent dates independently. Current economic circumstances influence R_t through the notional interest rate, R_t^n .

$$\ln R_t^n = \ln r_* + \ln \pi_t^* + \frac{\phi_1}{4} \mathbb{E}_t \sum_{j=-2}^1 (\ln \pi_{t+j} - \ln \pi_t^*) + \frac{\phi_2}{4} \mathbb{E}_t \sum_{j=-2}^1 (\ln Y_{t+j} - \ln y^* - \ln A_{t+j}). \quad (23)$$

The constant r_* equals the real interest rate along a steady-state growth path, and π_t^* is the central bank's intermediate target for inflation. We call this the *inflation-drift shock*. It follows a first-order autoregression with a normally-distributed innovation. Its unconditional mean equals π_* , the inflation rate on a steady-state growth path.

$$\ln \pi_t^* = (1 - \rho_\pi)\pi_* + \rho_\pi \ln \pi_{t-1}^* + \eta_t^\pi, \eta_t^\pi \sim \mathbb{N}(0, \sigma_\pi^2) \quad (24)$$

Allowing π_t^* to change over time enables our model to capture the persistent decline in inflation from the early 1990s through the early 2000s engineered by the Greenspan FOMC.

1.5 Other Financial Markets and Equilibrium Definition

All households participate in the market for nominal risk-free government debt. Additionally, they can buy and sell two classes of privately issued assets without restriction. The first is one-period nominal risk-free *private* debt. We denote the value of household's net holdings of such debt at the beginning of period t with B_{t-1}^P and the interest rate on such debt issued in period t maturing in $t + 1$ with R_{t+1}^P . The second asset class consists of a complete set of *real* state-contingent claims. As of the end of period t , the household's ownership of securities that pay off one unit of the aggregate consumption good in period τ if history s^τ occurs is $Q_t(s^\tau)$, and the nominal price of such a security in the same period is $J_t(s^\tau)$.

We define an equilibrium for our economy in the usual way: Households maximize their utility given all prices, taxes, and dividends from both producers and guilds; final goods producers and labor packers maximize profits taking their input and output prices as given; differentiated goods producers and guilds maximize the market value of their dividend streams taking as given all input and financial-market prices; differentiated goods producers and guilds produce to satisfy demand at their posted prices; and otherwise all product, labor, and financial markets clear.

2 First Order Conditions

In this section we present the first-order conditions associated with the optimization problems that the agents in our model solve.

2.1 Households

Given initial financial asset holdings, a stock of productive capital, investment in the previous period (which influences investment adjustment costs), and the external habit stock; households' choices of consumption, capital investment, capital utilization, hours worked, and financial investments maximize utility subject to the constraints of the capital accumulation and utilization technology and a sequence of one-period budget constraints. To specify these budget constraints, denote the nominal wage-per-hour paid by labor guilds to households with W_t^h , the nominal rental rate for capital services with R_t^k , the nominal price of investment goods with P_t^I , and the dividends paid by labor guilds added to those paid by differentiated good producers with D_t . With this notation, writing the period t budget constraint with uses of funds on the left and sources of funds on the right yields

$$C_t + \frac{P_t^I I_t}{P_t} + \frac{B_t}{R_t P_t} + \frac{B_t^P}{R_t^P P_t} + \frac{T_t}{P_t} \leq \frac{B_{t-1}}{P_t} + \frac{B_{t-1}^P}{P_t} + \frac{W_t^h H_t}{P_t} + \frac{R_t^k u_t K_{t-1}}{P_t} + \frac{D_t}{P_t} \quad (25)$$

Denote the Lagrange multiplier on (25) with $\beta^t \Lambda_t^1$, and that on the capital accumulation constraint in (4) with $\beta^t \Lambda_t^2$. With these definitions, the first-order conditions for a household's utility maximization problem are

$$\begin{aligned}
 \Lambda_t^1 &= \varepsilon_t^b \left((C_t - \varrho \bar{C}_{t-1}) (1 - \varepsilon_t^h H_t^{1+\gamma_h}) \right)^{-\gamma_c} (1 - \varepsilon_t^h H_t^{1+\gamma_h}) \\
 \Lambda_t^1 \frac{W_t^h}{P_t} &= (1 + \gamma_h) \varepsilon_t^b \left((C_t - \varrho \bar{C}_{t-1}) (1 - \varepsilon_t^h H_t^{1+\gamma_h}) \right)^{-\gamma_c} (C_t - \varrho \bar{C}_{t-1}) \varepsilon_t^h H_t^{\gamma_h} \\
 \frac{\Lambda_t^1}{R_t P_t} - \varepsilon_{t+q}^b L' \left(\frac{B_t}{R_t P_t} \right) \frac{\varepsilon_t^s}{R_t P_t} &= \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}^1}{P_{t+1}} \right. \\
 \frac{\Lambda_t^1}{R_t^P P_t} &= \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}^1}{P_{t+1}} \right. \\
 \Lambda_t^2 &= \beta \mathbb{E} \left[\Lambda_{t+1}^1 \frac{R_{t+1}^k u_{t+1}}{P_{t+1}} + \Lambda_{t+1}^2 (1 - \delta(u_{t+1})) \right] \\
 \frac{\Lambda_t^1 R_t^k}{P_t} &= \Lambda_t^2 \delta'(u_t) \\
 \Lambda_t^1 &= \varepsilon_t^i \Lambda_t^2 \left((1 - S_t(\cdot)) - S'_t(\cdot) \frac{i_t}{i_{t-1}} \right) \\
 &\quad + \beta \mathbb{E}_t \left[\varepsilon_{t+1}^i e^{(1-\gamma_C)z_{t+1}} \lambda_{t+1}^2 S'_{t+1}(\cdot) \frac{i_{t+1}^2}{i_t^2} \right]
 \end{aligned}$$

In equilibrium, $\bar{C}_t = C_t$ always.

2.2 Goods Sector

2.2.1 Final Goods Producers

The nominal marginal cost of final goods producers equals the right-hand side of (8). A producer of final goods maximizes profit by shutting down if P_t is less than this marginal cost and can make an arbitrarily large profit if P_t exceeds it. When (8) holds, an individual final goods producer's output is indeterminate.

Final goods producers' demand for intermediate goods takes the familiar constant-elasticity form. If we use Y_t to denote total final goods output, then the amount of differentiated good i demanded by final goods producers is

$$Y_{it} = Y_t \left(\frac{P_{it}}{P_t} \right)^{-\frac{1+\lambda_t^p}{\lambda_t^p}}.$$

Given the choice of a reset price, we wish to calculate the overall price level. All intermediate goods producers with a price-setting opportunity choose \tilde{P}_t . The

remaining producers use the price-indexing rule in (14). The aggregate price level is given by

$$P_t = \left[(1 - \zeta_p) \tilde{P}_t^{\frac{1}{\lambda_{p,t-1}}} + \zeta_p \left((\pi_{t-1})^{\lambda_p} (\pi_*)^{1-\lambda_p} P_{t-1} \right)^{\frac{1}{\lambda_{p,t-1}}} \right]^{\lambda_{p,t-1}}$$

where \tilde{P}_t is the optimal reset price

2.2.2 Intermediate Goods Producers

Intermediate goods producers' cost minimization reads as follows:

$$\begin{aligned} & \max_{H_{t,i}, K_{t,i}^e} W_t H_{t,i}^d + R_t^k K_{t,i}^e \\ & \text{s.t. } Y_{t,i} = \varepsilon_t^a (K_{t,i}^e)^\alpha (A_t^y H_{t,i}^d)^{1-\alpha} - A_t \Phi \end{aligned}$$

We get the following optimal capital-labor ratio.

$$\frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} = \frac{(K_{t,i}^e)^\alpha}{H_{t,i}^d}$$

Notice how for each firm, the idiosyncratic capital to labor ratio is not a function of any firm-specific component. Hence, each firm has the same capital to labor ratio. In equilibrium,

$$K_t^e = u_t K_{t-1}$$

To find the marginal cost, we differentiate the variable part of production with respect to output, and substitute in the capital-labor ratio.

$$MC_{t,i} = (\varepsilon_t^a)^{-1} (A_t^y)^{-(1-\alpha)} W_t^{1-\alpha} R_t^{k\alpha} \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)}$$

Again, notice that each firm has the same marginal cost.

Given cost minimization, a differentiated goods producer with an opportunity to adjust its nominal price does so to maximize the present-discounted value of profits

earned until the next opportunity to adjust prices arrives. Formally,

$$\begin{aligned} \max_{\tilde{P}_{t,i}} E_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\beta^s \Lambda_{t+s}^1 P_t}{\Lambda_t^1 P_{t+s}} [\tilde{P}_{t,i} X_{t,s}^y - MC_{t+s}] Y_{t+s,i} \\ \text{s.t. } Y_t(i) = \left(X_{t,s}^y \frac{\tilde{P}_{t,i}}{P_t} \right)^{\frac{\lambda_{p,t}}{1-\lambda_{p,t}}} Y_t \\ \text{where } X_{t,s}^y = \left\{ \begin{array}{ll} 1 & : s = 0 \\ \prod_{l=1}^s \pi_{t+l-1}^{\lambda_p} \pi_*^{1-\lambda_p} & : s = 1, \dots, \infty \end{array} \right\} \end{aligned}$$

This problem leads to the following price-setting equation for firms that are allowed to reoptimize their price:

$$0 = E_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\beta^s \Lambda_{t+s}^1 P_t}{\Lambda_t^1 P_{t+s}} Y_{it+s} \left[\lambda_{p,t+s} MC_{t+s} - X_{t,s} \tilde{P}_{it} \right]$$

It can be shown that the producers that are allowed to reoptimize choose the same price. So henceforth, $\tilde{P}_{it} = \tilde{P}_t$.

2.2.3 Investment Goods Producers

Characterizing the profit-maximizing choices of investment goods and final goods producers is straightforward. If $P_t^I > P_t/A_t^I$, then each investment goods producer can make infinite profit by choosing an arbitrarily large output. On the other hand, if $P_t^I < P_t/A_t^I$, then investment goods producers maximize profits with zero production. Finally, their profit-maximizing production is indeterminate when

$$P_t^I = P_t/A_t^I. \tag{26}$$

The relative price of investment to consumption is equal to $(A_t^I)^{-1}$. Making this substitution into the household f.o.c and noting that $P_t Y_t^I$ is an intermediate input that will not be reflected in the aggregate resource constraint, it suffices to substitute the relative price $(A_t^I)^{-1}$ in the constraint for the household.

2.3 Labor Sector

2.3.1 Labor Packers

The labor packers choose the the labor inputs supplied by guilds, pack them into a composite labor service to be sold to the intermediate goods producers. Formally, labor packers' problem reads as follows:

$$\begin{aligned} \max_{H_t^s, H_{it}} & W_t H_t^s - \int_0^1 W_{it} H_{it} di \\ \text{s.t.} & \left[\int_0^1 H_{it}^{\frac{1}{1+\lambda_{w,t}}} di \right]^{1+\lambda_{w,t}} = H_t^s \end{aligned}$$

We obtain the following labor demand equation for guild i :

$$H_{it} = \left(\frac{W_{it}}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} H_t \quad (27)$$

As for the goods sector, we can show that aggregate wage is given by the following equation:

$$W_t = \left[(1 - \zeta_w) \tilde{W}_t^{-\frac{1}{\lambda_{w,t}}} + \zeta_w \left((e^{z_{t-1}} \pi_{t-1})^{\lambda_w} (\pi_* e^{z_*})^{1-\lambda_w} W_{t-1} \right)^{-\frac{1}{\lambda_{w,t}}} \right]^{-\lambda_{w,t}}$$

where \tilde{W} is the optimal reset wage for guilds.

2.3.2 Guilds

Each guild with an opportunity to set its nominal price does so to maximize the current value of the stream of dividends returned to the household. Formally, their problem reads

$$\begin{aligned} \max_{\tilde{W}_{it}} & E_t \sum_{s=0}^{\infty} \zeta_w^s \left(\frac{\beta^s \Lambda_{t+s}^1 P_t}{\Lambda_t^1 P_{t+s}} \right) [X_{t+s}^l \tilde{W}_{it} - W_{t+s}^h] H_{it+s} \\ \text{s.t.} & H_{it+s} = \left(\frac{X_{t,s}^l \tilde{W}_{it}}{W_{t+s}} \right)^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} H_{t+s} \\ \text{where } X_{t,s}^l &= \begin{cases} 1 & : s = 0 \\ \prod_{j=1}^s \left(\pi_{t+j-1} \frac{A_{t+j-1}}{A_{t+j-2}} \right)^{1-\lambda_w} (\pi e^\gamma)^{\lambda_w} & : s = 1, \dots, \infty \end{cases} \end{aligned}$$

\tilde{W}_t is the optimal reset wage. This optimal wage is chosen by the guilds who are allowed, with probability ζ_w , to change their prices in a given period. Also, we index the nominal wage inflation rate with ι_w .

This maximization problem gives a wage-setting equation that reads as follows:

$$0 = E_t \sum_{s=0}^{\infty} \zeta_w^s \frac{\beta^s \Lambda_{t+s}^1 P_t}{\Lambda_t^1 P_{t+s}} H_{it+s} \frac{1}{\lambda_{w,t+s}} \left((1 + \lambda_{w,t+s}) W_{t+s}^h - X_{t,s}^l \tilde{W}_{it} \right)$$

It can be shown that the guilds that are allowed to reoptimize choose the same wage. So henceforth, $\tilde{W}_{it} = \tilde{W}_t$.

3 Detrending

To remove nominal and real trends, we deflate nominal variables by their matching price deflators, and we detrend any resulting real variables influenced permanently by technological change. All scaled versions of variables are the lower-case counterparts.

$$\begin{aligned} c_t &= \frac{C_t}{A_t} & i_t &= \frac{I_t}{A_t A_t^I} \\ k_t &= \frac{K_t}{A_t A_t^I} & k_t^e &= \frac{K_t^e}{A_t A_t^I} \\ w_t &= \frac{W_t}{A_t P_t} & \tilde{w}_t &= \frac{\tilde{W}_t}{A_t P_t} \\ \tilde{p}_t &= \frac{\tilde{P}_t}{P_t} & \pi_t &= \frac{P_t}{P_{t-1}} \\ y_t &= \frac{Y_t}{A_t} & mc_t &= \frac{MC_t}{P_t} \\ r_t^k &= \frac{R_t^k A_t^I}{P_t} & w_t^h &= \frac{W_t^h}{A_t P_t} \\ \lambda_t^1 &= \Lambda_t^1 A_t^{\gamma_C} & \lambda_t^2 &= \Lambda_t^2 A_t^{\gamma_C} A_t^I \\ \varepsilon_t^s &= A_t^{\gamma_C} \varepsilon_t^s & & \end{aligned}$$

3.1 Detrended Equations

The detrended equations describing our model are listed in the following sections.

Households' FOC

$$\begin{aligned} \lambda_t^1 &= \varepsilon_t^b \left[\left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \left(1 - \varepsilon_t^h h_t^{1+\gamma_h} \right) \right]^{-\gamma_c} \left(1 - \varepsilon_t^h h_t^{1+\gamma_h} \right) \\ \lambda_t^1 w_t^h &= (1 + \gamma_h) \varepsilon_t^b \left[\left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \left(1 - \varepsilon_t^h h_t^{(1+\sigma_h)} \right) \right]^{-\gamma_c} \left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \varepsilon_t^h h_t^{\gamma_h} \\ \frac{\lambda_t^1}{R_t^P} &= \beta E_t \left[\frac{\lambda_{t+1}^1 e^{-\gamma_C z_{t+1}}}{\pi_{t+1}} \right] \\ \frac{\lambda_t^1}{R_t} - L'(0) \frac{\varepsilon_t^b \varepsilon_t^s}{R_t} &= \beta E_t \frac{\lambda_{t+1}^1}{\pi_{t+1}} e^{-z_{t+1} \gamma_C} \\ \lambda_t^1 &= \varepsilon_t^i \lambda_t^2 \left((1 - S_t(\cdot)) - S_t'(\cdot) \frac{i_t}{i_{t-1}} \right) + \beta E_t \left[\varepsilon_{t+1}^i e^{(1-\gamma_C) z_{t+1}} \lambda_{t+1}^2 S_{t+1}'(\cdot) \frac{i_{t+1}^2}{i_t^2} \right] \\ \lambda_t^2 &= \beta E_t \left[e^{-\gamma_C z_{t+1} - \omega_{t+1}} \left(\lambda_{t+1}^1 r_{t+1}^k u_{t+1} + \lambda_{t+1}^2 (1 - \delta(u_{t+1})) \right) \right] \\ \lambda_t^1 r_t^k &= \lambda_t^2 \delta'(u_t) \\ k_t &= (1 - \delta(u_t)) k_{t-1} e^{-z_t - \omega_t} + \varepsilon_t^i (1 - S(\cdot)) i_t \\ k_t^e &= u_t k_{t-1} e^{-z_t - \omega_t} \end{aligned}$$

Final Goods Price Index

$$1 = \left[(1 - \zeta_p) \bar{p}_t^{\frac{1}{1-\lambda_{p,t}}} + \zeta_p (\pi_{t-1}^{\zeta_p} \pi^*(1-\zeta_p) \pi_t^{-1})^{\frac{1}{1-\lambda_{p,t}}} \right]^{1-\lambda_{p,t}}$$

Intermediate Goods Firms: Capital-Labor Ratio

$$\frac{k_t^e}{h_t^d} = \frac{\alpha}{1 - \alpha} \frac{w_t}{r_t^k}$$

Intermediate Goods Firms: Real Marginal Costs

$$mc_t = \frac{w_t^{1-\alpha} (r_t^k)^\alpha}{\varepsilon_t^a \alpha^\alpha (1 - \alpha)^{1-\alpha}}$$

Intermediate Goods Firms: Price-Setting Equation

$$0 = E_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \lambda_{t+s}^1 \frac{\tilde{y}_{t,t+s}}{\lambda_{p,t+s} - 1} \left(\frac{A_{t+s}}{A_t} \right)^{1-\gamma_C} \left[\lambda_{p,t+s} m c_{t+s} - \tilde{X}_{t,s}^p \tilde{p}_t \right]$$

where

$$\tilde{X}_{t,s}^p = \begin{cases} 1 & : s = 0 \\ \frac{\prod_{j=1}^s \pi_{t+j-1}^{1-\iota_p} \pi_*^{\iota_p}}{\prod_{j=1}^s \pi_{t+j}} & : s = 1, \dots, \infty \end{cases}$$

$\tilde{y}_{t,t+s}$ denotes the time $t + j$ output sold by the producers that have optimized at time t the last time they have reoptimized. Since it can be shown that optimizing producers all choose the same price, then we do not have to carry the i -subscript.

Labor Packers: Aggregate Wage Index

$$w_t = \left[(1 - \zeta_w) \tilde{w}_t^{-\frac{1}{\lambda_{w,t}}} + \zeta_w \left(e^{\iota_w z_{t-1} - z_t} e^{(1-\iota_w) z_*} \pi_{t-1}^{\iota_w} \pi_t^{-1} \pi_*^{1-\iota_w} w_{t-1} \right)^{-\frac{1}{\lambda_{w,t}}} \right]^{-\lambda_{w,t}}$$

Guilds: Wage-Setting Equation

$$0 = E_t \sum_{s=0}^{\infty} \zeta_w^s \beta^s \lambda_{t+s}^1 \left(\frac{A_{t+s}}{A_t} \right)^{1-\gamma_C} \frac{\tilde{h}_{t,t+s}}{\lambda_{w,t+s}} \left((1 + \lambda_{w,t+s}) w_{t+s}^h - \tilde{X}_{t,s}^l \tilde{w}_t \right)$$

where

$$\tilde{X}_{t,s}^l = \begin{cases} 1 & : s = 0 \\ \frac{\prod_{j=1}^s (\pi_{t+j-1} e^{z_{t+j-1}})^{1-\iota_w} (\pi \gamma)^{\iota_w}}{\prod_{j=1}^s \pi_{t+j} e^{z_{t+j}}} & : s = 1, \dots, \infty \end{cases}$$

$\tilde{h}_{t,t+s}$ denotes the time $t + j$ labor supplied by the guild that have optimized at time t the last time they have reoptimized. Since it can be shown that optimizing guilds all choose the same wage, then we do not have to carry the i -subscript.

Monetary Authority

$$R_t = R_{t-1}^{\rho_R} \left[r_* \pi_t^* \left(\prod_{j=-2}^1 \frac{\pi_{t+j}}{\pi_t^*} \right)^{\frac{\psi_1}{4}} \left(\prod_{j=-2}^1 \frac{y_{t+j}}{y^*} \right)^{\frac{\psi_2}{4}} \right]^{1-\rho_R} \prod_{j=0}^M \xi_{t-j,j}$$

The Aggregate Resource Constraint

$$\frac{y_t}{g_t} = c_t + i_t$$

Production Function

$$y_t = \varepsilon_t^a (k_t^e)^\alpha (h_t^d)^{1-\alpha} - \Phi$$

Labor Market Clearing Condition

$$h_t = h_t^d$$

4 Steady State

We normalize most shocks and the utilization rate:

$$\begin{aligned} u_* &= 1 & \varepsilon^i &= 1 \\ \varepsilon^a &= 1 & \varepsilon^b &= 1 \end{aligned}$$

Next, we set the following restriction on adjustment costs:

$$\begin{aligned} S(\cdot_*) &\equiv 0 \\ S'(\cdot_*) &\equiv 0 \end{aligned}$$

4.1 Prices and Interest Rates

Given β , z_* , γ_C , and π_* , we can solve for the steady-state nominal interest rate on private bonds R_*^P by using the FOC on private bonds:

$$R_*^P = \frac{\pi_*}{(\beta e^{-\gamma_C z_*})} \quad (28)$$

From the definition of $\delta(u)$, we have

$$\begin{aligned} \delta(1) &= \delta_0 \\ \delta'(1) &= \delta_1. \end{aligned}$$

Next, given ω_* , δ_0 , and the above, we can solve for the real return on capital r_*^k using the FOC on capital:

$$r_*^k = \frac{e^{\gamma_C z_* + \omega_*}}{\beta} - (1 - \delta_0) \quad (29)$$

4.2 Ratios

Moving to the production side, we can use the aggregate price equation to solve for \tilde{p}_* :

$$\tilde{p}_* = 1$$

Using this result and given $\lambda_{p,*}$, we can use the price Phillips curve to solve for mc_* :

$$mc_* = \frac{1}{1 + \lambda_{p,*}} \quad (30)$$

Given values for α and ε_*^a , we can use the marginal cost equation to solve for w_* :

$$w_* = (mc_* \alpha^\alpha (1 - \alpha)^{1-\alpha} (r_*^k)^{-\alpha})^{\frac{1}{1-\alpha}} \quad (31)$$

The definition of effective capital gives us a value for k_*^e in terms of k_* :

$$k_*^e = k_* e^{-z_* - \omega_*}$$

Calculating y_* using the labor share of output $1 - \alpha$:

$$y_* = \frac{w_* h_*}{1 - \alpha}$$

Using capital shares based off our value of α , we can calculate the output to capital ratio as follows:

$$\begin{aligned} \frac{y_*}{k_*^e} &= \frac{r_*^k}{\alpha} \\ \frac{y_*}{k_*} &= e^{-z_* - \omega_*} \frac{r_*^k}{\alpha} \end{aligned}$$

Using the capital accumulation equation, we can get a value for $\frac{i_*}{k_*}$

$$\frac{i_*}{k_*} = 1 - (1 - \delta_0) e^{-z_* - \omega_*}$$

Using the resource constraint, we can get $\frac{c_*}{k_*}$:

$$\frac{c_*}{k_*} = \frac{y_*}{k_* s_*^g} - \frac{i_*}{k_*}$$

These ratios will give us the remaining steady-state levels and ratios:

$$\begin{aligned} k_* &= y_* \left(\frac{y_*}{k_*} \right)^{-1} & i_* &= \frac{i_*}{k_*} k_* \\ c_* &= \frac{c_*}{k_*} k_* & g_* &= g_y y_* \end{aligned}$$

4.3 Liquidity Premium

Using the aggregate wage equation, we can get the following for \tilde{w}_* :

$$\tilde{w}_* = w_*$$

Combining this result with the wage Phillips curve, we get the following:

$$w_*^h = \frac{w_*}{1 + \lambda_{w,*}}$$

We can use the FOC for consumption and the labor supply to pin down ε^h and λ_*^1

$$\begin{aligned} \varepsilon^b \left[c_* \left(1 - \frac{\rho}{e^z} \right) \right]^{-\gamma_c} \left(1 - \varepsilon^h h_*^{(1+\gamma_h)} \right) - \lambda_*^1 &= 0 \\ -(1 + \gamma_h) \varepsilon^b c_*^{(1-\gamma_c)} \left(1 - \frac{\rho}{e^z} \right)^{(1-\gamma_c)} \left(1 - \varepsilon^h h_*^{(1+\gamma_h)} \right)^{-\gamma_c} \varepsilon^h h_*^{\gamma_h} + \lambda_*^1 w_*^h &= 0 \end{aligned}$$

Finally, the government bond rate is calculated from

$$\begin{aligned} \lambda_*^1 - \varepsilon_*^b \varepsilon_*^s &= \beta R_* \frac{\lambda_*^1}{\pi_*} e^{-\gamma_c z} \\ \underbrace{\frac{\pi_*}{\beta e^{-\gamma_c z}}}_{R_*^P} - \varepsilon_*^b \varepsilon_*^s \frac{\pi_*}{\beta e^{-\gamma_c z} \lambda_*^1} &= R_* \end{aligned}$$

Noting that $R_*^P = \frac{\pi_*}{\beta e^{-\gamma_c z}}$ we can write

$$\frac{R_*^P - R_*}{R_*^P} = \frac{\varepsilon_*^b \varepsilon_*^s}{\lambda_*^1}.$$

This is the liquidity premium in steady state.

5 Log Linearization

Hatted variables refer to log deviations from steady-state ($\hat{x} = \ln \left(\frac{x_t}{x_*} \right)$):

$$\ln \varepsilon_t^j = \rho_j \ln \varepsilon_{t-1}^j + \eta_t^j$$

In the cases of z_t , ω_t , and ν_t , we have that $\hat{x} = x_t - x_*$ as these variables are already in logs.

Households' First Order Conditions

$$\hat{\varepsilon}_t^b - \hat{\lambda}_t^1 - \gamma_c \frac{1}{1 - \frac{\rho}{e^z}} \hat{c}_t + \gamma_c \frac{\frac{\rho}{e^z}}{1 - \frac{\rho}{e^z}} (\hat{c}_{t-1} - \hat{z}_t) \quad (32)$$

$$\begin{aligned} \hat{\lambda}_t^1 + \hat{w}_t^h - \hat{\varepsilon}_t^b - \hat{\varepsilon}_t^h - \frac{1 - \gamma_c}{1 - \frac{\rho}{e^z}} \hat{c}_t + (1 - \gamma_c) \frac{\frac{\rho}{e^z}}{1 - \frac{\rho}{e^z}} (\hat{c}_{t-1} - \hat{z}_t) \\ - \left(\gamma_h + \gamma_c (1 + \gamma_h) \frac{\varepsilon^h h_*^{1+\gamma_h}}{(1 - \varepsilon^h h_*^{1+\gamma_h})^2} \right) \hat{h}_t = 0 \end{aligned} \quad (33)$$

$$\hat{\lambda}_t^1 = \frac{R_*^P - R_*}{R_*^P} (\hat{\varepsilon}_t^s + \hat{\varepsilon}_t^b) + \frac{R_*}{R_*^P} (\hat{R}_t + E_t[(\hat{\lambda}_{t+1}^1 - \hat{\pi}_{t+1} - \gamma_C \hat{z}_{t+1})]) \quad (34)$$

$$\hat{\lambda}_t^1 = E_t [\hat{\lambda}_{t+1}^1 - \gamma_C \hat{z}_{t+1} + \hat{R}_t - \hat{\pi}_{t+1}] \quad (35)$$

$$\hat{\lambda}_t^1 = (\ln \varepsilon_t^i + \hat{\lambda}_t^2) - S''' (\hat{i}_t - \hat{i}_{t-1}) + \beta e^{(1-\gamma_C)\gamma} S'' E_t (\hat{i}_{t+1} - \hat{i}_t) \quad (36)$$

$$\begin{aligned} \lambda_*^2 \hat{\lambda}_t^2 = \beta e^{-\gamma_C z_* - \omega_*} [\lambda_*^1 u_* r_*^k E_t (-\gamma_C \hat{z}_{t+1} - \hat{w}_{t+1} + \hat{\lambda}_{t+1}^1 + \hat{r}_{t+1}^k + \hat{u}_{t+1})] + \\ + \beta e^{-\gamma_C z_* - \omega_*} [(1 - \delta_0) \lambda_*^2 E_t (-\gamma_C \hat{z}_{t+1} - \hat{w}_{t+1} + \hat{\lambda}_{t+1}^2) - \lambda_*^2 \delta_1 u_* E_t \hat{u}_{t+1}] \end{aligned} \quad (37)$$

$$\hat{\lambda}_t^1 = \hat{\lambda}_t^2 + \frac{\delta_2}{\delta_1} u_* \hat{u}_t - \hat{r}_t^k \quad (38)$$

$$\hat{k}_t = \left(1 - \frac{\varepsilon_*^i i_*}{k_*} \right) (\hat{k}_{t-1} - \hat{z}_t - \hat{w}_t) + \frac{\varepsilon_*^i i_*}{k_*} (\hat{\varepsilon}_t^i + \hat{i}_t) - \delta_1 u_* e^{-z_* - \omega_*} \hat{u}_t \quad (39)$$

$$\hat{k}_t^e = \hat{u}_t + \hat{k}_{t-1} - \hat{z}_t - \hat{w}_t \quad (40)$$

Capital-Labor Ratio

$$\hat{k}_t^e = \hat{w}_t - \hat{r}_t^k + \hat{h}_t^d \quad (41)$$

Real Marginal Costs

$$\widehat{mc}_t = (1 - \alpha) \hat{w}_t + \alpha \hat{r}_t^k - \hat{\varepsilon}_t^a \quad (42)$$

The New Keynesian Phillips Curve for Inflation

$$\begin{aligned} \hat{\pi}_t = \frac{(1 - \beta \zeta_p e^{(1-\gamma_C)z_*})(1 - \zeta_p)}{(1 + \beta l_p e^{(1-\gamma_C)z_*}) \zeta_p} \left[\frac{\lambda_{p,*}}{1 + \lambda_{p,*}} \hat{\lambda}_{p,t} + \widehat{mc}_t + \right. \\ \left. + \frac{l_p}{1 + \beta l_p e^{(1-\gamma_C)z_*}} \hat{\pi}_{t-1} + \frac{\beta e^{(1-\gamma_C)z_*}}{1 + \beta l_p e^{(1-\gamma_C)z_*}} E_t \hat{\pi}_{t+1} \right] \end{aligned} \quad (43)$$

Wage Mark-Up

$$\hat{\mu}_t^w = \hat{w}_t - \hat{w}_t^h \quad (44)$$

The New Keynesian Phillips Curve for Wages

$$\begin{aligned} \hat{w}_t = & \frac{1}{1 + \beta e^{(1-\gamma_C)z_*}} \hat{w}_{t-1} + \frac{\beta e^{(1-\gamma_C)z_*}}{1 + \beta e^{(1-\gamma_C)z_*}} \hat{w}_{t+1} + \frac{\beta e^{(1-\gamma_C)z_*}}{1 + \beta e^{(1-\gamma_C)z_*}} (E_t \hat{\pi}_{t+1} + E_t \hat{z}_{t+1}) + \\ & \frac{\iota_w}{1 + \beta e^{(1-\gamma_C)z_*}} (\hat{\pi}_{t-1} + \hat{z}_{t-1}) - \frac{1 + \iota_w \beta e^{(1-\gamma_C)z_*}}{1 + \beta e^{(1-\gamma_C)z_*}} (\hat{\pi}_t + \hat{z}_t) + \\ & \frac{1 - \beta \zeta_w e^{(1-\gamma_C)z_*}}{1 + \beta e^{(1-\gamma_C)z_*}} \frac{1 - \zeta_w}{\zeta_w} \left[\frac{\lambda_{w,*}}{1 + \lambda_{w,*}} \hat{\lambda}_{w,t} - \hat{\mu}_t^w \right] \end{aligned} \quad (45)$$

The Aggregate Resource Constraint

$$\frac{y_*}{g_*} (\hat{y}_t - \hat{g}_t) = \frac{c_*}{c_* + i_*} \hat{c}_t + \frac{i_*}{c_* + i_*} \hat{i}_t \quad (46)$$

The Production Function

$$\hat{y}_t = \frac{1}{mc_*} (\ln \varepsilon_t^a + \alpha \hat{k}_t^e + (1 - \alpha) \hat{h}_t^d) \quad (47)$$

Labor Market Clearing Condition

$$\hat{h}_t = \hat{h}_t^d \quad (48)$$

Monetary Authority's Reaction Function

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left[(1 - \psi_1) \hat{\pi}_t^* + \frac{\psi_1}{4} \left(\sum_{j=-2}^1 \hat{\pi}_{t+j} \right) + \frac{\psi_2}{4} \left(\sum_{j=-2}^1 \hat{y}_{t+j} \right) \right] + \sum_{j=0}^M \hat{\xi}_{t-j,j} \quad (49)$$

6 Measurement

6.1 National Income Accounts

The model economy's basic structure, with the representative household consuming a single good and accumulating capital using a different good, differs in some important ways from the accounting conventions of the Bureau of Economic Analysis (BEA) underlying the National Income and Product Accounts (NIPA). In particular

1. The BEA treats household purchases of long-lived goods inconsistently. If classifies purchases of residential structures as investment and treats the service flow from their stock as part of Personal Consumption Expenditures (PCE) on services. The BEA classifies households purchases of all other durable goods as consumption expenditures. No service flow from the stock of household durables enters measures of current consumption. In the model, all long-lived investments add to the productive capital stock.
2. The BEA treats all government purchases as government consumption. However, government at all levels makes purchases of investment goods on behalf of the populace. In the model, these should be treated as additions to the single stock of productive capital.
3. The BEA sums PCE and private expenditures on productive capital (Business Fixed Investment and Residential Investment), with government spending, inventory investment, and net exports to create Gross Domestic Product. The model features only the first three of these.

To bridge these differences, we create four *model consistent* NIPA measures from the BEA NIPA data.

1. Model-consistent GDP. Since the model's capital stock includes both the stock of household durable goods and the stock of government-purchased capital, a model-consistent GDP series should include the value of both stocks' service flows. To construct these, we followed a five-step procedure.
 - (a) We begin by estimating a constant (by assumption) service-flow rate by dividing the nominal value of housing services from NIPA Table 2.4.5 by the beginning-of-year value of the residential housing stock from the

BEA's Fixed Asset Table 1.1. We use annual data and average from 1947 through 2014. The resulting estimate is 0.096. That is, the annual value of housing services equals approximately 10 percent of the housing stock's value each year.

- (b) In the second step, we estimate estimate constant (by assumption) depreciation rates for residential structures, durable goods, and government capital. We constructed these by first dividing observations of value lost to depreciation over a calendar year by the end-of-year stocks. Both variables were taken from the BEA's Fixed Asset Tables. (Table 1.1 for the stocks and Table 1.3 for the deprecation values.) We then averaged these ratios from 1947 through 2014. The resulting estimates are 0.021, 0.194, and 0.044 for the three durable stocks.
- (c) In the third step, we calculated the average rates of real price depreciation for the three stocks. For this, we began with the nominal values and implicit deflators for PCE Nondurable Goods and PCE Services from NIPA Table 1.2. We used these series and the Fisher-ideal formula to produce a chain-weighted implicit deflator for PCE Nondurable Goods and Services. Then, we calculated the price for each of the three durable good's stocks in consumption units as the ratio of the implicit deflator taken from Fixed Asset Table 1.2 to this deflator. Finally, we calculated average growth rates for these series from 1947 through 2014. The resulting estimates equal 0.0029, -0.0223, and 0.0146 for residential housing, household durable goods, and government-purchased capital.
- (d) The fourth combines the previous steps' calculations to estimate constant (by assumption) service-flow rates for household durable goods and government-purchased capital. To implement this, we assumed that all *three* stocks yield the same financial return along a steady-state growth path. These returns sum the per-unit service flow with the appropriately depreciated value of the initial investment. This delivers two equations in two unknowns, the two unknown service-flow rates. The resulting estimates are 0.29 and 0.12 for household durable goods and government-purchased capital.
- (e) The fifth and final step uses the annual service-flow rates to calculate real

and nominal service flows from the real and nominal stocks of durable goods and government-purchased capital reported in Fixed Asset Table 1.1. This delivers an annual series. Since the stocks are measured as of the end of the calendar year, we interpret these as the service flow values in the *next* year's first quarter. We create quarterly data by linearly interpolating between these values.

With these real and nominal service flow series in hand, we create nominal model-consistent GDP by summing the BEA's definition of nominal GDP with the nominal values of the two service flows. We create the analogous series for model-consistent real GDP by applying the Fisher ideal formula to the nominal values and price indices for these three components.

2. Model-consistent Investment. The nominal version of this series sums nominal Business Fixed Investment, Residential Investment, PCE Durable Goods, and government investment expenditures. The first three of these come from NIPA Table 1.1.5, while government investment expenditures sums Federal Defense, Federal Nondefense, and State and Local expenditures from NIPA Table 1.5.5. We construct the analogous series for real Model-consistent Investment by combining these series with their real chain-weighted counterparts found in NIPA Tables 1.1.3 and 1.5.3 using the Fisher ideal formula. By construction, this produces an implicit deflator for Model-consistent investment as well.
3. Model-consistent Consumption. The nominal version of this series sums nominal PCE Nondurable Goods, PCE Services, and the series for nominal services from the durable goods stock. The first two of these come from NIPA Table 1.1.5. We construct the analogous series for real Model-consistent consumption by combining these series with their real chain-weighted counterparts using the Fisher ideal formula. The two real PCE series come from NIPA Table 1.1.3. Again, this produces an implicit deflator for Model-consistent consumption as a by-product.
4. Model-consistent Government Purchases. Conceptually, the model's measure of Government Purchases includes all expenditures not otherwise classified as Investment or Consumption: Inventory Investment, Net Exports, and actual Government Purchases. We construct the nominal version of this series simply

by subtracting nominal Model-consistent Investment and Consumption from nominal Model-consistent GDP. We calculate the analogous real series using “chain subtraction.” This applies the Fisher ideal formula to Model-consistent GDP and the *negatives* of Model-consistent Consumption and Investment.

Our empirical analysis requires us to compare model-consistent series measured from the NIPA data with their counterparts from the model’s solution. To do this, we begin by solving the log-linearized system above, and then we feed the model specific paths for all exogenous shocks starting from a particular initial condition. for a given such simulation, the growth rates of Model-consistent Consumption and Investment equal

$$\begin{aligned}\Delta \ln C_t &= z_* + \Delta \hat{c}_t + z_t \text{ and} \\ \Delta \ln I_t &= z_* + \omega_* + \Delta \hat{i}_t + z_t + \omega_t\end{aligned}$$

The measurement of GDP growth in the model is substantially more complicated, because the variables Y_t and y_t denote model output *in consumption units*. In contrast, we mimic the BEA by using a chain-weighted Fisher ideal index to measure model-consistent GDP. Therefore, we construct an analogous chain-weighted GDP index from model data. Since such an ideal index is invariant to the units with which nominal prices are measured, we can normalize the price of consumption to equal one and employ the prices of investment goods and government purchases relative to current consumption. Our model identifies the first of these relative prices as with investment-specific technology. However, the model characterizes only government purchases *in consumption units*, because private agents do not care about their division into “real” purchases and their relative price. For this reason, we use a simple autoregression to characterize the evolution of the price of government services in consumption units. Denote this price in quarter t with P_t^g . We construct this for the US economy by dividing the Fisher-ideal price index for model-consistent government purchases by that for model-consistent consumption. Then, our model for its evolution is

$$\ln(P_t^g/P_{t-1}^g) = \mu_g + \theta_{gg1} \ln(P_{t-1}^g/P_{t-2}^g) + \theta_{gg2} \ln(P_{t-2}^g/P_{t-3}^g) + \varepsilon_t^{gg}. \quad (50)$$

Here, $\varepsilon_t^{gg} \sim \mathcal{N}(0, \sigma_{gg}^2)$. Given an arbitrary normalization of P_t^g to one for some time period, simulations from (??) can be used to construct simulated values of P_t^g for

all other time periods. With these and a simulation from the model of all other variables in hand, we can calculate the simulation's values for Fisher ideal GDP growth using

$$\frac{Q_t}{Q_{t-1}} \equiv \sqrt{\dot{Q}_t^P \dot{Q}_t^L}, \quad (51)$$

where the Paasche and Laspeyres indices of quantity growth are

$$\dot{Q}_t^P \equiv \frac{C_t + P_t^I I_t + P_t^G (G_t/P_t^G)}{C_{t-1} + P_t^I I_{t-1} + P_t^G (G_{t-1}/P_{t-1}^G)} \text{ and} \quad (52)$$

$$\dot{Q}_t^L \equiv \frac{C_t + P_{t-1}^I I_t + P_{t-1}^G (G_t/P_t^G)}{C_{t-1} + P_{t-1}^I I_{t-1} + P_{t-1}^G (G_{t-1}/P_{t-1}^G)}. \quad (53)$$

In both (52) and (53), P_t^I is the relative price of investment to consumption. In equilibrium, this always equals A_t^I .

The above gives a complete recipe for *simulating* the growth of model-consistent real GDP growth. However, we also embody its insights into our estimation with a log-linear approximation. For this, we start by removing stochastic trends from all variables in (52) and (53), and we proceed by taking a log-linear approximation of the resulting expression. Details are available from the authors upon request.

6.2 Hours Worked Measurement

Empirical work using DSGE models like our own typically measure labor input with hours worked per capita, constructed directly from BLS measures of hours worked and the civilian non-institutional population over age 16. However, this measure corresponds poorly with business cycle models because it contains underlying low frequency variation. This fact led us to construct a new measure of hours for the model using labor market trends produced for the FRB/US model and for the Chicago Fed's in-house labor market analysis.

We begin with a multiplicative decomposition of hours worked per capita into hours per worker, the employment rate of those in the labor force, and the labor-force participation rate. The BLS provides CPS-based measures of the last two rates for the US as a whole. However, its measure of hours per worker comes from the Establishment Survey and covers only the private business sector. If we use hours per worker in the business sector to approximate hours per worker in the economy

as a whole, then we can measure hours per capita as

$$\frac{H_t}{P_t} = \frac{H_t^E}{E_t^E} \frac{E_t^C}{L_t^C} \frac{L_t^C}{P_t^C}.$$

Here, H_t and P_t equal total hours worked and the total population, H_t^E/E_t^E equals hours per worker measured with the *Establishment* survey, E_t^C/L_t^C equals one minus the *CPS* based unemployment rate, and L_t^C/P_t^C equals the *CPS* based labor-force participation rate. Our measure of *model-relevant* hours worked deflates each component on the right-hand side by an exogenously measured trend. The trend for the unemployment rate comes from the Chicago Fed's Microeconomics team, while those for hours per worker and labor-force participation come from the FRB/US model files.

6.3 Inflation

Our empirical analysis compares model predictions of price inflation, wage inflation, inflation in the price of investment goods relative to consumption goods, and inflation expectations with their observed values from the U.S. economy. We describe our implementations of these comparisons sequentially below.

6.3.1 Price Inflation

Our model directly characterizes the inflation rate for Model-consistent Consumption. In principle, this is close to the FOMC's preferred inflation rate, that for the implicit deflator of PCE. However, in practice the match between the two inflation rates is poor. In the data, short-run movements in food and energy prices substantially influences the short-run evolution of PCE inflation. Our model lacks such a volatile sector, so if we ask it to match observed short-run inflation dynamics, it will attribute those to transitory shocks to intermediate goods' producers' desired markups driven by λ_t^p .

To avoid this outcome, we adopt a different strategy for matching model and data inflation rates, which follows that of [Justiniano, Primiceri, and Tambalotti \(2013\)](#). This relates three observable inflation rates – core CPI inflation, core PCE inflation, and market-based PCE inflation – to Model-consistent consumption inflation using

auxiliary observation equations. For core PCE inflation, this equation is

$$\pi_t^{p1} = \pi_* + \pi_*^{p1} + \beta_1^{p1} \hat{\pi}_t + \beta_2^{p1} \pi_t^D + \varepsilon_t^{p1} \quad (54)$$

In (54) as elsewhere, π_* equals the long-run inflation rate. The constant π_*^{p1} is an adjustment to this long-run inflation rate which accounts for possible long-run differences between realized inflation and the FOMC's goal of π_* . The right-hand side's inflation rates, $\hat{\pi}_t$ and π_t^D equal Model-consistent consumption inflation and PCE Durables inflation. We refer to the coefficients multiplying them, β_1^{p1} and β_2^{p1} , as the *inflation loadings*. We include PCE Durables inflation on the right-hand side of (54) because the principle adjustment required to transform Model-consistent inflation into core PCE inflation is the replacement of the price index for durable goods services with that for durable goods purchases. The disturbance term ε_t^{p1} follows a first-order autoregression with autocorrelation φ_{p1} and normally distributed innovations with mean zero and standard deviation σ_{p1} .

The other two observed inflation measures, market-based PCE inflation and core CPI inflation, have identically specified observation equations. We use $p2$ and $p3$ in superscripts to denote these equations parameters and error terms, and we use the same expressions as subscripts to denote the parameters governing the evolution of their error terms. We assume that the error terms ε_t^{p1} , ε_t^{p2} , and ε_t^{p3} are independent of each other at all leads and lags.

To produce forecasts of inflation with these these three observation equations, we must forecast their right-hand side variables. The model itself gives forecasts of $\hat{\pi}_t$. The forecasts of durable goods inflation come from a second-order autoregression.

$$\pi_t^D = \theta_0^D + \theta_1^D \pi_{t-1}^D + \varepsilon_t^D \quad (55)$$

Its innovation is normally distributed and serially uncorrelated with standard deviation σ_D .

6.3.2 Wage Inflation

Although observed wage inflation does not feature the same short-run variability as does price inflation, it does include the influences of persistent demographic labor-market trends which we removed ex ante from our measure of hours worked.

Therefore, we follow the same general strategy of relating observed measures of wage inflation to the model's predicted wage inflation with a error-augmented observation equation. For this, we employ two measures of compensation per hour, Earnings per Hour and Total Compensation per Hour. In parallel with our notation for inflation measures, we use $w1$ and $w2$ to denote these two wage measures of wage inflation. The observation equation for Earnings per Hour is

$$\pi_t^{w1} = z_* + \pi_*^{w1} + \beta^{w1} \hat{\pi}_t^w + \varepsilon_t^{w1} \quad (56)$$

Just as with the price inflation measurement errors, ε_t^{w1} follows a first-order autoregression with autocorrelation φ_{w1} and innovation standard deviation σ_{w1} . The observation equation for Total Compensation per Hour is analogous to (56).

6.3.3 Relative Price Inflation

To empirically ground investment-specific technological change in the model, we use an error-augmented observation equation to relate the relative price of investment to consumption, both model-consistent measures constructed from NIPA and Fixed Asset tables as described above, with the model's growth rate of the rate of technological transformation between these two goods, ω_t .

$$\pi_t^{C/I} = \omega_t + \varepsilon_t^{C/I}$$

Here, we use the superscript C/I to indicate that the variables characterize the price of Consumption relative to Investment. The measurement error $\varepsilon_t^{C/I}$ follows a first-order autoregression with autocorrelation $\varphi_{C/I}$ and normally-distributed innovations with standard deviation $\sigma_{C/I}$.

6.3.4 Inflation Expectations

We also discipline our model's inferences about the state of the economy by comparing expectations of one-yea and 10-year inflation from the Survey of Professional Forecasters with the analogous expectations from our model. Just as with all of the other inflation measures, we allow these two sets of expectations to differ from each other by including serially correlated measurement errors. The

observation equations are

$$\begin{aligned}\pi_t^{e4} &= \pi_* + \pi_*^{e4} + \frac{1}{4} \sum_{i=1}^4 \mathbb{E}_t [\hat{\pi}_{t+i}] + \varepsilon_t^{e4} \\ \pi_t^{e40} &= \pi_* + \pi_*^{e40} + \frac{1}{40} \sum_{i=1}^{40} \mathbb{E}_t [\hat{\pi}_{t+i}] + \varepsilon_t^{e40}\end{aligned}$$

The two measurement errors follow mutually-independent first-order autoregressions with autocorrelations φ_{e4} and φ_{e40} and innovation standard deviations σ_{e4} and σ_{e40} .

6.4 Interest Rates and Monetary Policy Shocks

Since our model features forward guidance shocks, it has non-trivial implications for the current policy rate as well as for expected future policy rates. We use two distinct but complementary approaches to disciplining the parameters governing their realizations, the elements of Σ_1 , using data. The first method compares the model's monetary policy shocks to high-frequency interest-rate innovations informed by event studies, such as that of [Gürkaynak, Sack, and Swanson \(2005\)](#). Those authors applied a factor structure to innovations in implied expected interest rates from futures prices around FOMC policy announcement dates. Specifically, they show that the vector of M implied interest rate changes following an FOMC policy announcement, Δr , can be written as

$$\Delta r = \Lambda f + \eta$$

Where f is a 2×1 vector of factors, Λ is a $M \times 2$ matrix of factor loadings, and η is an $M \times 1$ vector of mutually independent shocks. Denoting the 2×2 diagonal variance covariance matrix of f with Σ_f and the $M \times M$ diagonal variance-covariance matrix of η with Ψ , we can express the observed variance-covariance matrix of Δr as $\Lambda \Sigma_f \Lambda' + \Psi$.

Our model has implications for this same variance covariance matrix. For this, use the model's solution to express the changes in current and future expected interest rates following monetary policy shocks as $\Delta r = \Gamma_1 \varepsilon^1$. Here, ε^1 is the vector which collects the current monetary policy shock with $M - 1$ forward guidance shocks, and Γ_1 is an $M \times M$ matrix. In general, Γ_1 does *not* simply equal the identity matrix, because current and future inflation and output gaps respond to the

monetary policy shocks and thereby influence future monetary policy “indirectly” through the interest rate rule. Given this solution for Δr , we can calculate its variance-covariance matrix as $\Gamma_1 \Sigma_1 \Gamma_1'$. Equating these two expressions and solving for Σ_1 yields

$$\Sigma_1 = \Gamma_1^{-1} (\Lambda \Sigma_f \Lambda' + \Psi) \Gamma_1'^{-1}.$$

The second approach to disciplining Σ_1 is more traditional: directly compare quarterly observations of the current policy rate and expected future interest rates – from market prices, surveys of market participants, or both – with their implied values from the model given a particular realization of the vector of monetary policy shocks. We use both methods in the estimation procedure described below.

7 Calibration and Bayesian Estimation

As we noted in the introduction, we follow a two-stage approach to the estimation of our model’s parameters. In a calibration stage, we set the values of selected parameters so that the model has empirically-sensible implications for long-run averages from the U.S. economy. In this stage, we also enforce several normalizations and a judgemental restriction on one of the measurement error variances. In the second stage, we estimate the model’s remaining parameters using standard Bayesian methods.

We employ standard prior distributions, but those governing monetary policy shocks deserve further elaboration. Our estimation requires the variance-covariance matrix of monetary policy shocks to be consistent with the factor-structure of interest rate innovations used by [Gürkaynak, Sack, and Swanson \(2005\)](#), as described above. Therefore, we parameterize Σ_1 in terms of Λ , Σ_f , Ψ , and the model parameters which influence Γ_1 . We then center our priors for Λ , Σ_f , and Ψ at their estimates from event-studies. However, we do not require our estimates to equal their prior values. Our Bayesian estimation procedure employs quarterly data on expected future interest rates, the posterior likelihood function includes Λ , Σ_f , and Ψ as free parameters. It is well known that Λ and Σ_f are not separately identified, so we impose two scale normalizations and one rotation normalization on Λ . The rotation normalization requires that the first factor, which we label “Factor A”, is

the only factor influence the current policy rate. That is, the second factor, “Factor *B*” influences only future policy rates. [Gürkaynak, Sack, and Swanson \(2005\)](#) call Factors *A* and *B* the “target” and “path” factors.

Our estimation’s sample period begins in the first quarter of 1993 and ends in the fourth quarter of 2016. Of course, the FOMC substantially changed its operating procedures in the aftermath of its persistent stay at the Zero Lower Bound, so it would be unwise to imagine the data from this entire period being generated from our model with time-invariant parameters. For this reason, we estimate the model twice. For the *first sample*, which runs from 1993Q1 through 2008Q3, we estimate all model parameters while allowing for four quarters of forward guidance. For the *second sample*, we estimate the parameters governing monetary policy shocks allowing for ten quarters of forward guidance, adjust the average rate of Hicks-neutral productivity growth to bring potential GDP growth rate from its first-sample value of 3 percent down to 2 percent, and hold all other model parameters fixed at their first-sample posterior-mode values.

We report the results of our two-stage two-sample estimation in a series of tables. Table 1 reports our most notable calibration targets. The long-run policy rate equals 1.1 percent on a quarterly basis. We target a two percent growth rate of per capita GDP. Given an average population growth rate of one percent per year, this implies that our potential GDP growth rate equals three percent. The other empirical moments we target are a nominal investment to output ratio of 26 percent and nominal government purchases to output ratio of 15 percent. Finally, we target a capital to output ratio of approximately 10 on a quarterly basis.

Table 2 lists the parameters which we calibrate along with their given values. The table includes many more parameters than there are targets in Table 1. This is because Table 1 omitted calibration targets which map one-to-one with particular parameter values. For example, we calibrate the steady-state capital depreciation rate (δ_0) using standard methods applied to data from the Fixed Asset tables. It is also because Table 2 lists several parameters which are normalized prior to estimation. Most notable among these are the three factor loadings listed at the table’s bottom.

Tables 3 and 4 report prior distributions and posterior modes for the model’s remaining parameters, for the first and second samples respectively.

Table 1: Calibration Targets

Description	Expression	Value
Fixed Interest Rate (quarterly, gross)	R	1.011
Per-Capita Steady-State Output Growth Rate (quarterly)	Y_{t+1}/Y_t	1.005
Investment to Output Ratio	I_t/Y_t	0.260
Capital to Output Ratio	K_t/Y_t	10.763
Fraction of final good output spent on public goods	G_t/Y_t	0.153

Table 2: First Sample Calibrated Parameters

Parameter	Symbol	Value
Discount Factor	β	0.986
Steady-State Measured TFP Growth (quarterly)	z_*	0.489
Investment-Specific Technology Growth Rate	ω_*	0.371
Elasticity of Output w.r.t Capital Services	α	0.401
Steady-State Wage Markup	λ_*^w	1.500
Steady-State Price Markup	λ_*^p	1.500
Steady-State Scale of the Economy	H_*	1.000
Steady-State Inflation Rate (quarterly)	π_*	0.500
Steady-State Depreciation Rate	δ_0	0.016
Steady-State Marginal Depreciation Cost	δ_1	0.039
Nominal Output over Nominal Private Purchases	g_*	0.847
Std. Dev Long-Run Inflation Expectations Measurement Error	σ_{e40}	0.010
Long-Run Inflation Expectations (Constant CPI Adjustment)	π_*^{e40}	0.122
Average Earnings Constant	π_*^{w1}	-0.237
Average Total Compensation Constant	π_*^{w2}	-0.202
Loading Compensation	β_1^{w2}	1.000
Loading Core PCE	β_1^{p1}	1.000
Constant for Relative Price Inflation	π_*^G	0.252
Loading 0 Factor A	$\lambda_{0,1}$	0.981
Loading 0 Factor B	$\lambda_{0,2}$	0.000
Loading 4 Factor B	$\lambda_{4,2}$	0.951

Table 3: First Sample Estimated Parameters

Parameter	Symbol	Density	Prior		Posterior
			Mean	Std.Dev	Mode
Depreciation Curve	$\frac{\delta_2}{\delta_1}$	G	1.0000	0.150	0.499
Active Price Indexation Rate	ι_p	B	0.5000	0.150	0.280
Active Wage Indexation Rate	ι_w	B	0.5000	0.150	0.082
External Habit Weight	λ	B	0.7500	0.025	0.790
Labor Supply Elasticity	γ_H	N	0.6000	0.050	0.591
Price Stickiness Probability	ζ_p	B	0.8000	0.050	0.833
Wage Stickiness Probability	ζ_w	B	0.7500	0.050	0.904
Adjustment Cost of Investment	φ	G	3.0000	0.750	4.326
Elasticity of Intertemporal Substitution	γ_c	N	1.5000	0.375	1.915
Interest Rate Response to Inflation	ψ_1	G	1.7000	0.150	1.833
Interest Rate Response to Output	ψ_2	G	0.2500	0.100	0.488
Interest Rate Smoothing Coefficient	ρ_R	B	0.8000	0.100	0.791
Autoregressive Coefficients of Shocks					
Discount Factor	ρ_b	B	0.5000	0.250	0.850
Inflation Drift	ρ_π	B	0.9900	0.010	0.998
Exogenous Spending	ρ_g	B	0.6000	0.100	0.920
Investment	ρ_i	B	0.5000	0.100	0.759
Liquidity Preference	ρ_s	B	0.6000	0.200	0.841
Price Markup	ρ_{λ_p}	B	0.6000	0.200	0.687
Wage Markup	ρ_{λ_w}	B	0.5000	0.150	0.668
Neutral Technology	ρ_ν	B	0.3000	0.150	0.496
Investment Specific Technology	ρ_ω	B	0.3500	0.100	0.407
Moving Average Coefficients of Shocks					
Price Markup	θ_{λ_p}	B	0.4000	0.200	0.608
Wage Markup	θ_{λ_w}	B	0.4000	0.200	0.306
Standard Deviations of Innovations					
Discount Factor	σ_b	U	0.5000	2.000	1.187
Inflation Drift	σ_π	I	0.0150	0.0075	0.094
Exogenous Spending	σ_g	U	1.0000	2.000	2.500

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

First Sample Estimated Parameters (Continued)

Parameter	Symbol	Density	Prior		Posterior
			Mean	Std.Dev	Mode
Investment	σ_i	I	0.2000	0.200	0.618
Liquidity Preference	σ_s	U	0.5000	2.000	0.390
Price Markup	σ_{λ_p}	I	0.1000	1.000	0.069
Wage Markup	σ_{λ_w}	I	0.1000	1.000	0.031
Neutral Technology	σ_ν	U	0.5000	0.250	0.504
Investment Specific Technology	σ_ω	I	0.2000	0.100	0.183
Relative Price of Cons to Inv	$\sigma_{\frac{c}{i}}$	I	0.0500	2.000	0.215
Monetary Policy					
Unanticipated	σ_{μ_0}	N	0.0050	0.0025	0.012
1Q Ahead	σ_{μ_1}	N	0.0050	0.0025	0.012
2Q Ahead	σ_{μ_2}	N	0.0050	0.0025	0.007
3Q Ahead	σ_{μ_3}	N	0.0050	0.0025	0.009
4Q Ahead	σ_{μ_4}	N	0.0050	0.0025	0.010
Total Earnings					
Loading 1	β_1^{w1}	N	0.8000	0.100	0.824
Standard Deviation	σ_{w1}	I	0.0500	0.100	0.147
AR(1) Coefficient	φ_{w1}	B	0.4000	0.100	0.624
Total Compensation					
Standard Deviation	σ_{w2}	I	0.0500	0.100	0.169
AR(1) Coefficient	φ_{w2}	B	0.4000	0.100	0.343
Core PCE					
Constant	π_*^{p1}	N	-0.1000	0.100	-0.087
Loading 2	β_2^{p1}	N	0.0000	1.000	0.014
Standard Deviation	σ_{p1}	I	0.0500	0.100	0.048
AR(1) Coefficient	φ_{p1}	B	0.2000	0.100	0.091
Market-Based Core PCE					
Constant	π_*^{p2}	N	-0.1000	0.100	-0.123
Loading 1	β_1^{p2}	N	1.0000	0.100	1.102
Loading 2	β_2^{p2}	N	0.0000	1.000	0.028
Standard Deviation	σ_{p2}	I	0.0500	0.100	0.039

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

First Sample Estimated Parameters (Continued)

Parameter	Symbol	Density	Prior		Posterior
			Mean	Std.Dev	Mode
AR(1) Coefficient	φ_{p2}	B	0.2000	0.100	0.128
Core CPI					
Constant	π_*^{p3}	N	0.0500	0.100	0.047
Loading 1	β_1^{p3}	N	1.0000	0.100	0.804
Loading 2	β_2^{p3}	N	0.0000	1.000	0.119
Standard Deviation	σ_{p3}	I	0.1000	0.100	0.076
AR(1) Coefficient	φ_{p3}	B	0.4000	0.200	0.597
PCE Durable Goods Inflation					
Constant	θ_*^D	N	-0.3500	0.100	-0.356
1st Lag Coefficient	θ^{D1}	N	0.4500	0.200	0.430
2nd Lag Coefficient	θ^{D2}	N	0.4000	0.200	0.362
Standard Deviation	σ_D	I	0.2000	2.000	0.287
Relative Price Inflation					
1st Lag Coefficient	θ^{G1}	N	0.0000	0.500	0.279
2nd Lag Coefficient	θ^{G2}	N	-0.1000	0.500	0.006
Standard Deviation	σ_G	I	0.5000	2.000	0.811
Factor A					
Loading 1	$\lambda_{1,1}$	N	0.6839	0.200	1.256
Loading 2	$\lambda_{2,1}$	N	0.5224	0.200	0.857
Loading 3	$\lambda_{3,1}$	N	0.4314	0.200	0.361
Loading 4	$\lambda_{4,1}$	N	0.3243	0.200	0.032
Standard Deviation	σ_{F_1}	N	0.1000	0.0750	0.041
Factor B					
Loading 1	$\lambda_{1,2}$	N	0.3310	0.200	0.698
Loading 2	$\lambda_{2,2}$	N	0.6525	0.200	1.162
Loading 3	$\lambda_{3,2}$	N	0.8059	0.200	1.199
Standard Deviation	σ_{F_2}	N	0.1000	0.0750	0.072

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

Table 4: Second Sample Estimated Parameters

Parameter	Symbol	Prior		Posterior
		Mean	Std.Dev	Mode
Total Earnings				
Constant	β_*^{w1}	-0.2370	0.200	-0.096
Loading 1	β_1^{w1}	0.8242	0.200	0.252
Standard Deviation	σ_{w1}	0.1468	0.100	0.183
AR(1) Coefficient	φ_{w1}	0.6239	0.200	0.529
Total Compensation				
Constant	β_*^{w2}	-0.2023	0.200	-0.142
Standard Deviation	σ_{w2}	0.1687	0.100	0.233
AR(1) Coefficient	φ_{w2}	0.3430	0.200	0.351
Core PCE				
Loading 2	β_2^{p1}	0.0281	0.100	0.232
Standard Deviation	σ_{p1}	0.0481	0.100	0.143
AR(1) Coefficient	φ_{p1}	0.0913	0.150	0.256
Market PCE				
Constant	π_*^{p2}	-0.1230	0.100	-0.114
Loading 1	β_1^{p2}	1.1022	0.150	0.358
Loading 2	β_2^{p2}	0.0139	0.100	0.219
Standard Deviation	σ_{p2}	0.0755	0.100	0.121
AR(1) Coefficient	φ_{p2}	0.5972	0.150	0.527
CPI				
Constant	π_*^{p3}	0.0475	0.100	-0.022
Loading 1	β_1^{p3}	0.8039	0.150	0.305
Loading 2	β_2^{p3}	0.1192	0.100	0.206
Standard Deviation	σ_{p3}	0.0388	0.100	0.085
AR(1) Coefficient	φ_{p3}	0.1278	0.150	0.220
Durable Goods Inflation				
Constant	π_*^D	-0.4500	0.200	-0.463
Standard Deviation	σ_D	0.5000	0.150	0.291
Relative Price Inflation				

Second Sample Estimated Parameters (Continued)

Parameter	Symbol	Prior		Posterior
		Mean	Std.Dev	Mode
Constant	π_*^G	0.8900	0.400	-0.146
Standard Deviation	σ_G	0.8143	0.080	1.023
Factor A				
Loading 0	$\lambda_{0,1}$	0.0180	0.250	0.158
Loading 1	$\lambda_{1,1}$	0.0574	0.250	0.356
Loading 2	$\lambda_{2,1}$	0.1941	0.250	0.496
Loading 3	$\lambda_{3,1}$	0.3996	0.250	0.623
Loading 4	$\lambda_{4,1}$	0.6520	0.250	0.820
Loading 6	$\lambda_{6,1}$	1.2266	0.250	0.990
Loading 7	$\lambda_{7,1}$	1.5237	0.250	1.122
Loading 8	$\lambda_{8,1}$	1.8139	0.250	1.102
Loading 9	$\lambda_{9,1}$	2.0914	0.250	1.087
Loading 10	$\lambda_{10,1}$	2.3523	0.250	2.515
Standard Deviation	σ_{F_1}	0.0442	0.100	0.072
Factor B				
Loading 0	$\lambda_{0,2}$	-0.0181	0.300	0.029
Loading 1	$\lambda_{1,2}$	0.2211	0.300	0.039
Loading 2	$\lambda_{2,2}$	0.3679	0.300	0.070
Loading 3	$\lambda_{3,2}$	0.4424	0.300	0.095
Loading 4	$\lambda_{4,2}$	0.4612	0.300	0.123
Loading 5	$\lambda_{5,2}$	0.4370	0.300	0.138
Loading 6	$\lambda_{6,2}$	0.3817	0.300	0.167
Loading 7	$\lambda_{7,2}$	0.3032	0.300	0.184
Loading 9	$\lambda_{9,2}$	0.1074	0.300	0.229
Standard Deviation	σ_{F_2}	0.0334	0.100	0.429
Standard Deviations of Monetary Policy Innovations				
Unanticipated	σ_{μ_0}	0.0061	0.005	0.011
1Q Ahead	σ_{μ_1}	0.0021	0.005	0.010
2Q Ahead	σ_{μ_2}	0.0004	0.005	0.010
3Q Ahead	σ_{μ_3}	0.0019	0.005	0.009

Second Sample Estimated Parameters (Continued)

Parameter	Symbol	Prior		Posterior
		Mean	Std.Dev	Mode
4Q Ahead	σ_{μ_4}	0.0001	0.005	0.010
5Q Ahead	σ_{μ_5}	0.0025	0.005	0.010
6Q Ahead	σ_{μ_6}	0.0019	0.005	0.010
7Q Ahead	σ_{μ_7}	0.0011	0.005	0.010
8Q Ahead	σ_{μ_8}	0.0001	0.005	0.009
9Q Ahead	σ_{μ_9}	0.0014	0.005	0.010
10Q Ahead	$\sigma_{\mu_{10}}$	0.0028	0.005	0.010

References

- Calvo, G. A. (1983). Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics* 12, 383–398. [1](#), [9](#), [10](#)
- Campbell, J. R., C. L. Evans, J. D. Fisher, and A. Justiniano (2012). Macroeconomic effects of Federal Reserve forward guidance. *Brookings Papers on Economic Activity Spring*, 1–54. [1](#)
- Campbell, J. R., J. D. M. Fisher, A. Justiniano, and L. Melosi (2016). Forward guidance and macroeconomic outcomes since the financial crisis. In M. Eichenbaum and J. Parker (Eds.), *NBER Macroeconomics Annual 2016*, Volume 31. University of Chicago Press. [1](#)
- Christiano, L., M. Eichenbaum, and C. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113(1), 1–45. [7](#)
- Fisher, J. (2015, March-April). On the structural interpretation of the smets-wouters “risk premium” shock. *Journal of Money, Credit and Banking* 47(2-3), 511–516. [4](#), [6](#)
- Fisher, J. D. M. (2006). The dynamic effects of neutral and investment-specific technology shocks. *Journal of Political Economy* 114(3), 413–451. [2](#)
- Gürkaynak, R., B. Sack, and E. Swanson (2005). Do actions speak louder than words? *International Journal of Central Banking* 1(1), 55–93. [35](#), [36](#), [37](#)
- Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010). Investment shocks and business cycles. *Journal of Monetary Economics* 57(2), 132 – 145. [1](#)
- Justiniano, A., G. E. Primiceri, and A. Tambalotti (2013, April). Is there a trade-off between inflation and output stabilization? *American Economic Journal: Macroeconomics* 5(2), 1–31. [32](#)
- King, R. G., C. I. Plosser, and S. T. Rebelo (1988a). Production, growth and business cycles I. the basic neoclassical model. *Journal of Monetary Economics* 21, 195–232. [2](#)

King, R. G., C. I. Plosser, and S. T. Rebelo (1988b). Production, growth and business cycles II. new directions. *Journal of Monetary Economics* 21, 309–341.

[2](#)

Laséen, S. and L. E. Svensson (2011). Anticipated alternative policy-rate paths in policy simulations. *International Journal of Central Banking*. [1](#)