

Forecasting the Effects of Reduced Defense Spending

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I. INTRODUCTION

The end of the Cold War provides the United States with an opportunity to cut its defense spending significantly. Indeed, the Bush Administration's 1992-1997 Future Years Defense Program (presented in 1991 and therefore referred to as the "1991 plan") calls for a 20 percent reduction in real defense spending by 1997. Although expenditures related to Operation Desert Storm have delayed the implementation of the 1991 plan, policymakers continue to call for defense cutbacks. In fact, since Bush's plan was drafted prior to the collapse of the Soviet Union, it seems likely that the Clinton Administration will propose cuts in defense spending that are even deeper than those specified by the 1991 plan. This paper draws on both theoretical and empirical economic models to forecast the effects that these cuts will have on the U.S. economy.

A. Economic Theory

Economic theory suggests that in the short run, cuts in defense spending are likely to have disruptive effects on the U.S. economy. Productive resources—both labor and capital—must shift out of defense-related industries and into nondefense industries. The adjustment costs that this shift entails are likely to restrain economic growth as the defense cuts are implemented.

Economic theory is less clear, however, about the likely long-run consequences of reduced defense spending. The neoclassical macroeconomic model (a simple version of which is presented by Barro, 1984) assumes that all goods and services are produced by the private sector. Rather than hiring labor, accumulating capital, and producing defense services itself, the government simply purchases these services from the private sector. Thus, according to the neoclassical model, the direct effect of a permanent

\$1 cut in defense spending acts to decrease the total demand for goods and services in each period by \$1. Of course, so long as the government has access to the same production technologies that are available to the private sector, this prediction of the neoclassical model does not change if instead the government produces the defense services itself.¹

A permanent \$1 cut in defense spending also reduces the government's need for tax revenue; it implies that taxes can be cut by \$1 in each period. Households, therefore, are wealthier following the cut in defense spending; their permanent income increases by \$1. According to the permanent income hypothesis, this \$1 increase in permanent income induces households to increase their consumption by \$1 in every period, provided that their labor supply does not change.

However, the wealth effect of reduced defense spending may also induce households to increase the amount of leisure that they choose to enjoy. If households respond to the increase in wealth by taking more leisure, then the increase in consumption from the wealth effect only amounts to $\$(1 - \alpha)$ per period, where α is a number between zero and one. That is, the increase in wealth is split between an increase in consumption and an increase in leisure. In general, therefore, the wealth effect of a cut in defense spending acts to increase private consumption, and hence total demand, by $\$(1 - \alpha)$ per period.

The increase in leisure from the wealth effect, meanwhile, translates into a decrease in labor supply. This decrease in labor supply, in turn, translates into a decrease in the total supply of goods and services. In fact, the increase in leisure acts to decrease the total supply of goods by $\$\alpha$ per period (Barro, 1984, Ch. 13). Thus, the number α measures

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¹ See Wynne (1992), however, for a more general version of the neoclassical model in which the government may have access to different technologies from those used by the private sector. Wynne's model also distinguishes between the goods and services that the government produces itself and those that it purchases from the private sector.

the magnitude of the wealth effect's impact on leisure and the supply of goods relative to its impact on consumption and the demand for goods. The higher α is, the larger the decrease in supply and the smaller the increase in demand.

Combining the direct effect of the permanent \$1 cut in defense spending, which decreases total demand by \$1 per period, with the wealth effect, which increases total demand by $\$(1 - \alpha)$ per period, shows that the permanent cut in defense spending decreases the total demand for goods by $\$\alpha$ per period. Likewise, the direct effect implies no change in supply, while the wealth effect implies a decrease in supply of $\$\alpha$ per period; when combined, the two effects imply a decrease in supply of $\$\alpha$ per period. Altogether, both total demand and total supply decrease by $\$\alpha$ in each period, so that the permanent cut in defense spending reduces real output by $\$\alpha$ in each period.

Before moving on, it is important to emphasize that although the neoclassical model predicts that total output (GNP) will fall in response to a permanent cut in defense spending, this result does not imply that households would be better off without the spending cut. While the permanent \$1 cut in defense spending reduces total GNP by $\$\alpha$ per period, it also makes available to households \$1 per period that would otherwise be allocated to defense. Private GNP, defined as total GNP less all government spending, therefore increases by $\$(1 - \alpha)$ per period. Since private GNP accounts for the goods and services that are available to the private sector, it is a better measure of welfare than total GNP; the rise in private GNP indicates that households are better off after the defense cuts, even though total GNP is lower. In fact, the increase in private GNP underestimates the welfare gain from reduced defense spending since it does not take into account the increase in leisure resulting from the wealth effect of the spending cut.

So far, the analysis has assumed that the cut in defense spending will be used to reduce taxes. Provided that the Ricardian equivalence theorem applies, however, the results do not change if instead the cuts are used to reduce the government debt. Suppose that, in fact, the permanent \$1 cut in defense spending is initially used to reduce the government debt. According to the Ricardian equivalence theorem, households recognize that by reducing its debt, the government is reducing its need for future tax revenues by an equal amount. Thus, using the cut in defense spending to reduce the

government debt today simply means that tax cuts of more than \$1 per period will come in the future. Under Ricardian equivalence, household wealth does not depend on the precise timing of the tax cuts. The magnitude of the wealth effect, and hence the changes in aggregate supply and demand, are the same whether the cut in defense spending is used to reduce the federal debt or to reduce taxes.

Central to the Ricardian equivalence theorem is the assumption that households experience the same change in wealth from a reduction in government debt as they do from a cut in taxes. If this assumption is incorrect, then a cut in defense spending can have very different long-run effects from those predicted by the neoclassical model under Ricardian equivalence.

Most frequently, the relevance of the Ricardian equivalence theorem is questioned based upon the observation that households have lifetimes of finite length (Bernheim, 1987). Suppose, for instance, that while individuals recognize that a reduction in government debt today implies that taxes will be lower in the future, they also expect that the future tax cuts will occur after they have died. In this extreme case, individuals who are alive today experience no change in wealth if the cuts in defense spending are used to reduce the government debt. Only the direct effect of the defense cut is present; the wealth effect is missing. Since households do not experience an increase in permanent income, neither their consumption nor their labor supply changes. The decrease in total demand resulting from the direct effect of the spending cut leads to a condition of excess supply.

In response to excess supply, output falls in the short run, and the real interest rate falls as well. In the long run, however, the lower real interest rate leads to increases in both investment and output. Thus, a departure from Ricardian equivalence can explain why cuts in defense spending might increase, rather than decrease, total GNP in the long run, provided that the cuts are used to reduce the government debt.

Whether or not the Ricardian equivalence theorem applies to the U.S. economy is a controversial issue. There are many theoretical models in which household wealth is affected by a decrease in government debt in exactly the same way that wealth is affected by a decrease in taxes, so that Ricardian equivalence applies (see Barro, 1989, for a survey of these models). On the other hand, there are many other

models in which the wealth effect from a cut in government debt differs from the wealth effect from a cut in taxes, so that Ricardian equivalence does not hold (see Bernheim, 1987, for a survey of these models). Overall, economic theory provides no clear answer as to the relevance of the Ricardian equivalence theorem. Consequently, economic theory does not provide a clear answer as to the long-run effects of cuts in defense spending either. Instead, empirical models must be used to forecast the effects of reduced defense spending.

B. Previous Empirical Estimates

A detailed study by the Congressional Budget Office (CBO, 1992) forecasts the effects of the 1991 plan for the U.S. economy. The CBO's conclusions are based on results from two large-scale macroeconomic forecasting models: the Data Resources, Inc. (DRI) Quarterly Macroeconomic Model and the McKibbin-Sachs Global (MSG) Model. Both of these econometric models incorporate short-run adjustment costs of changes in defense spending and long-run non-Ricardian effects of changes in the government debt into their forecasts for real economic activity. The models predict, therefore, that the cuts proposed by the 1991 plan will reduce growth in the U.S. economy in the short run. The models also predict that if the cuts in defense spending are used to reduce the federal debt, then the real interest rate will fall and investment and output will increase in the long run as the non-Ricardian effects kick in. Thus, while the CBO predicts that the 1991 plan will reduce total GNP by approximately 0.6 percent throughout the mid-1990s, their forecasts also show positive effects on total GNP by the end of the decade, leading to a long-run increase in total GNP of almost 1 percent.

The Congressional Budget Office's econometric models draw heavily on economic theory to obtain their conclusions. As noted above, however, there is considerable debate in the theoretical literature concerning the possible channels through which defense spending influences aggregate activity in the long run. Models that assume that the Ricardian equivalence theorem holds indicate that cuts in defense spending will reduce output in the long run. On the other hand, models in which Ricardian equivalence does not apply predict that defense cuts may increase output in the long run, provided that the proceeds from the cuts are used to reduce the government debt. The CBO's models both assume that Ricardian equivalence does not hold in the U.S. economy. Hence, their forecasts show significant long-run gains in total GNP from the 1991 plan. But

these forecasts will be on target only to the extent that their underlying—and controversial—assumption about Ricardian equivalence is correct.

C. An Alternative Forecasting Strategy

This paper takes an approach to forecasting the effects of reduced defense spending that differs significantly from the approach taken by the CBO. Rather than using a large-scale econometric model, it uses a much smaller vector autoregressive (VAR) model like those developed by Sims (Jan. 1980, May 1980). As emphasized by Sims (Jan. 1980), VAR models require none of the strong theoretical assumptions that the DRI and MSG models rely on so heavily. The approach taken here, therefore, recognizes that economic theory provides no clear answer as to the likely long-run effects of reduced defense spending. Moreover, as documented by Lupoletti and Webb (1986), VAR models typically perform as well as the larger models when used as forecasting tools, especially over long horizons. Thus, there are both theoretical and practical reasons to prefer the VAR approach to the CBO's.

Forecasts from the VAR model, like those from the CBO's models, show that the 1991 plan will lead to weakness in aggregate output in the short run. Unlike the CBO's models, however, the VAR does not predict that there will be a long-run increase in total GNP resulting from the cuts in defense spending, even if the cuts are used to reduce the federal debt. This result, which is consistent with the neo-classical model under Ricardian equivalence, suggests that the larger models rely on the incorrect assumption that there are strong non-Ricardian effects of changes in the government debt in the U.S. economy.

Although the VAR forecasts for total GNP are considerably more pessimistic than the CBO's forecasts, they do not imply that the defense cuts called for by the 1991 plan are undesirable. Private GNP, in contrast to total GNP, is forecast by the VAR to increase in the long run as a result of the 1991 plan. This result, which is again consistent with the neo-classical model under Ricardian equivalence, indicates that the 1991 plan will make more resources available to the private sector in the long run. As noted by Garfinkel (1990) and Wynne (1991), this gain in resources can be used to increase private consumption and investment, making American households better off in the long run.

The VAR is introduced in the next section. Section III presents the forecasts generated by the VAR

and compares them to the forecasts given by the CBO. Section IV summarizes and concludes.

II. DESCRIPTION OF THE MODEL

The basic model is an extension of the four-variable VAR developed by Sims (May 1980) and is designed specifically to capture the effects of defense spending on aggregate economic activity. There are six variables in the model: the growth rate of real defense spending (RDEF), the growth rate of real U.S. government debt (RDEBT), the nominal six-month commercial paper rate (R), the growth rate of the broad monetary aggregate (M2), the growth rate of the implicit price deflator for total GNP (P), and the growth rate of real total GNP (Y). All of the variables except for the interest rate are expressed as growth rates so that all may be represented as stationary stochastic processes. Using growth rates for these variables avoids the problems, discussed by Stock and Watson (1989), associated with including nonstationary variables in the VAR.

Using vector notation, the model can be written as

$$X_t = \sum_{s=1}^k B_s X_{t-s} + u_t \quad (1)$$

where the 6x1 vector X_t is given by

$$X_t = [RDEF_t, RDEBT_t, R_t, M2_t, P_t, Y_t]' \quad (2)$$

and where the B_s are each 6x6 matrices of regression coefficients. In order to obtain information about the long-run effects of changes in defense spending, the system (1) is estimated using a long data set that extends from 1931 through 1991. All data are annual (quarterly data are unavailable for dates prior to World War II); their sources are given in the appendix. The lag length $k=4$ is chosen on the basis of the specification test recommended by Doan (1989).

Once the system (1) is estimated, impulse response functions can be used to trace out the effects of changes in defense spending and the government debt on total GNP and, in particular, to forecast the effects of the 1991 plan. For the purpose of generating impulse response functions, the ordering of variables shown in equation (2) reflects the assumption that policy decisions that change defense spending are made before the contemporaneous values of the other variables are observed. Monetary policy actions, which are best captured as changes in R_t (McCallum, 1986), are made after decisions

that affect defense spending and the government debt, but before money, prices, or output are observed. Money, prices, and output are then determined in succession, given that fiscal policy has determined RDEF and RDEBT and monetary policy has determined R_t .²

III. FORECASTS FROM THE VAR

The VAR results are foreshadowed in Figure 1, which plots the series Y, RDEF, and RDEBT over the 60-year sample period. Panel B reveals that there were significant cuts in defense spending following World War II, the Korean War, and the Vietnam War. In each case, the defense cuts were accompanied by slow growth in total GNP (panel A). In light of this past relationship, it seems likely that the VAR will associate the defense cuts called for by the 1991 plan with slower total GNP growth, at least in the short run.

By comparing the behavior of RDEBT (panel C) to that of Y (panel A), however, it is difficult to see the negative long-run relationship between output and government debt predicted by models in which the Ricardian equivalence theorem does not apply. Growth in the real value of U.S. government debt was negative for much of the 1950s and 1960s and positive for much of the 1970s and all of the 1980s.

² More formally, the variables in equation (2) are organized as a Wold causal chain to produce the impulse response functions. See Sims (1986) for a detailed discussion of the Wold causal chain approach as well as other strategies for identifying impulse response functions in VAR models.

Figure 1A

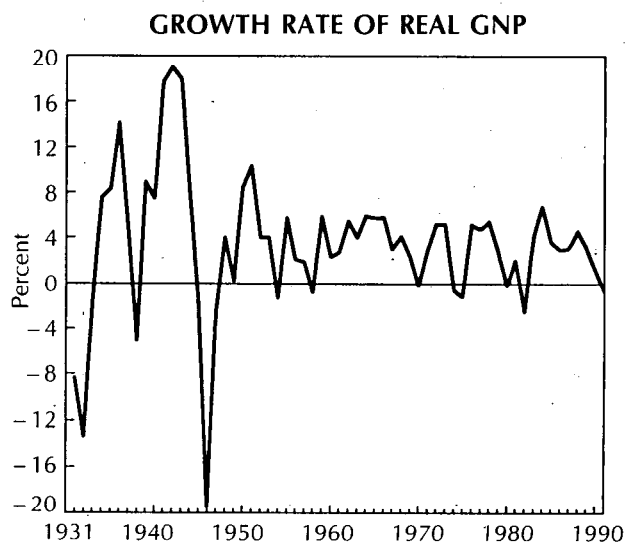


Figure 1B

GROWTH RATE OF REAL DEFENSE SPENDING

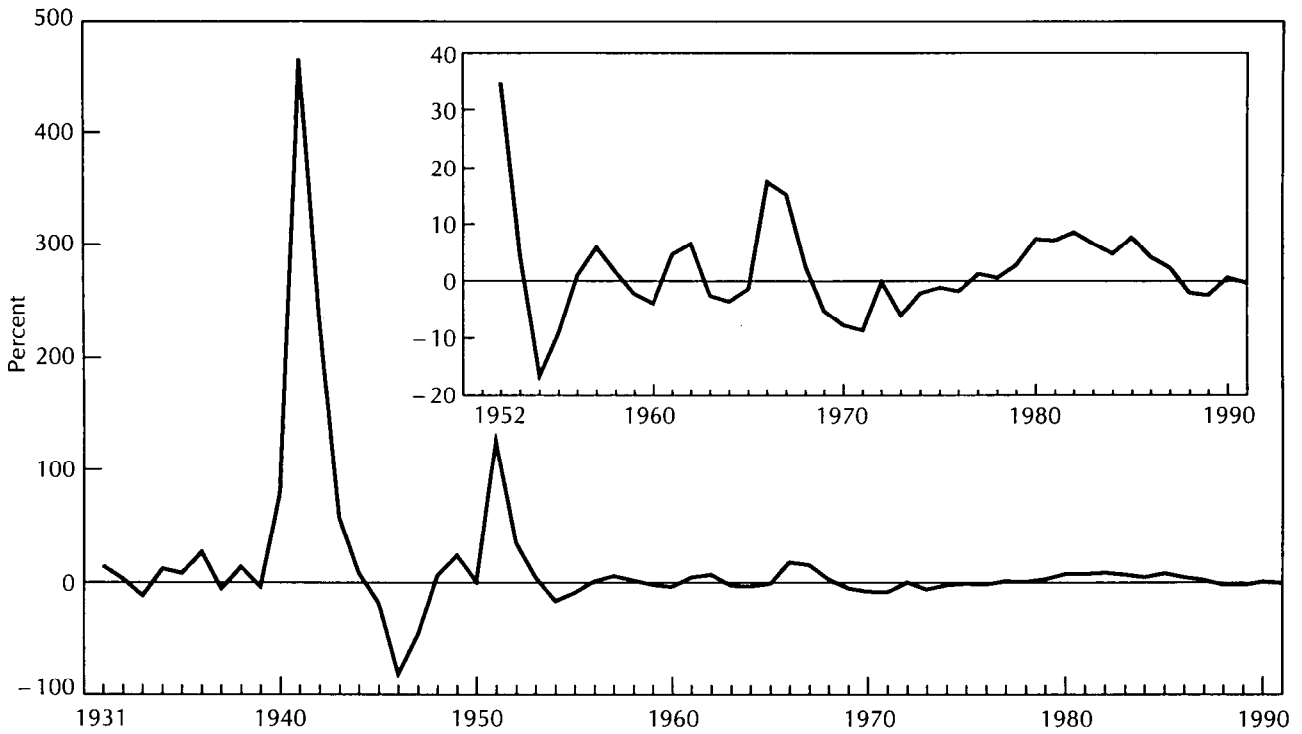
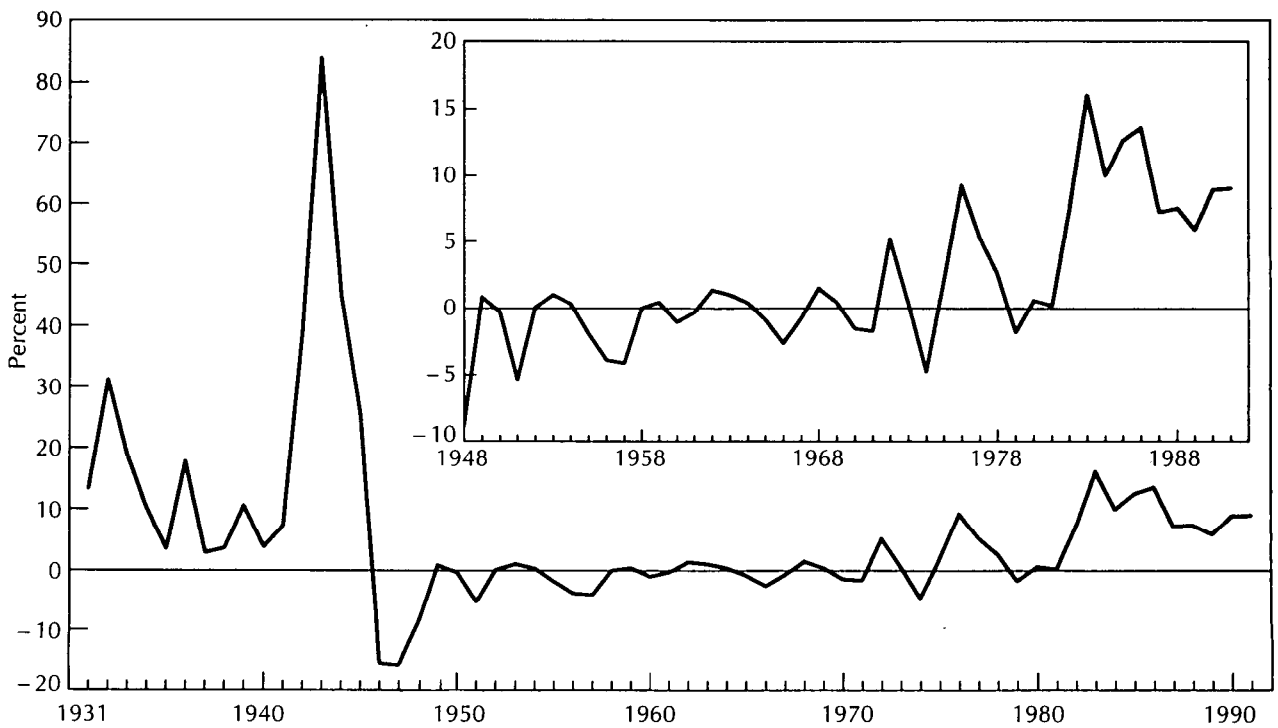


Figure 1C

GROWTH RATE OF REAL GOVERNMENT DEBT



Yet it does not appear that the growth rate of output was substantially different before and after 1970, as these models predict. Thus, it seems likely that the VAR will not find strong non-Ricardian effects of changes in government debt in the U.S. economy.

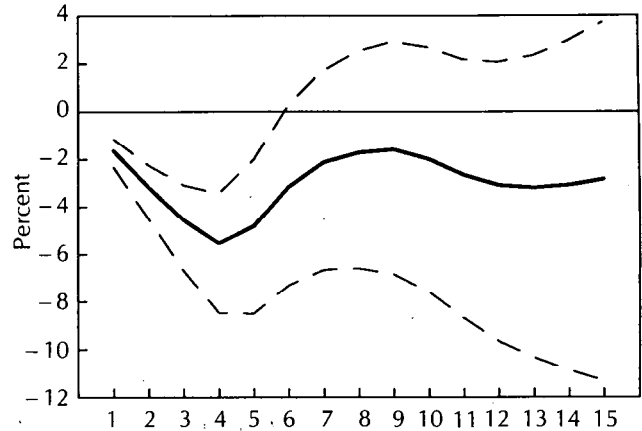
Figure 2, panel A, shows the cumulative impulse response function of Y to a one-time one-standard deviation (20 percent) decrease in RDEF, computed using the VAR described by equations (1) and (2).³ The graph shows the cumulative change in the level of total GNP through the end of each year that results from a decrease in RDEF in the first year. It indicates that the decrease in RDEF yields a contemporaneous decrease in total GNP of approximately 1.5 percent. The effect of the shock to RDEF peaks at 5.5 percent after four years before settling down to a long-run decrease of about 2.5 percent. The confidence interval reveals that the initial decrease in total GNP due to the decrease in defense spending is statistically significant. In fact, the hypothesis that changes in RDEF do not influence Y (more formally, the hypothesis that changes in RDEF do not Granger-cause changes in Y) can be rejected at the 99 percent confidence level.⁴ Thus, the model indicates that in response to a decrease in defense spending, total GNP will fall in both the short run and the long run.

Figure 2, panel B, plots the cumulative impulse response function of Y to a one-time one-standard deviation (3 percent) decrease in RDEBT. Consistent with the non-Ricardian assumptions that are embedded in the DRI and MSG models, this impulse response function indicates that a decrease in RDEBT will increase total GNP in the long run. In addition, the hypothesis that changes in RDEBT do not influence changes in Y can be rejected at the 98 percent confidence level. However, at no time are the effects of RDEBT on Y very large; except in period 2, the confidence interval always includes zero. Overall, therefore, Figure 2 is consistent with the neoclassical model under Ricardian equivalence, which predicts that a reduction in the size of the federal debt will not have a large effect on output and that a reduction in defense spending will permanently reduce total GNP.

³ Note that each of the impulse response functions in Figure 2 traces out the effects of a decrease in one of the variables on GNP. Impulse response functions more typically examine the effects of an increase in one of the model's variables. Here, however, it is the effects of decreases in defense spending and government debt that are of interest, so the direction of the shock is reversed.

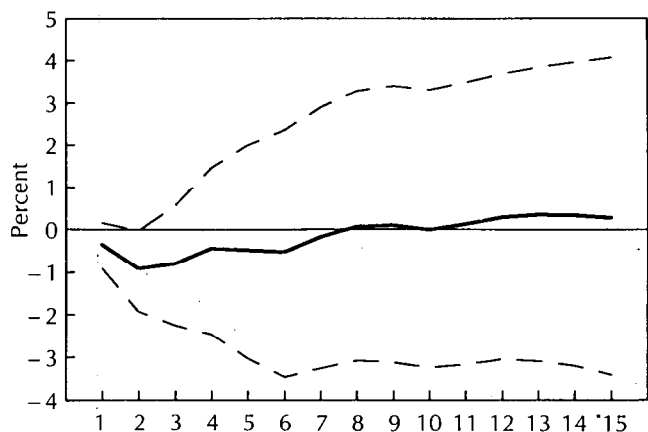
⁴ Here and below, the likelihood ratio test for block exogeneity described by Doan (1989) is used to test for the absence of Granger causality.

Figure 2A
CUMULATIVE RESPONSE OF GROWTH RATE OF REAL GNP TO GROWTH RATE OF REAL DEFENSE SPENDING
 (with 95 percent confidence interval)



Forecasts from the VAR are generated by constraining future values of RDEF and RDEBT as called for by the 1991 plan. The constrained values of RDEF and RDEBT translate into constrained values of the shocks to these two variables. Hence, the VAR forecasts are essentially linear combinations of the impulse response functions shown in Figure 2. The impulse response functions suggest that the short-run forecasts from the VAR will be similar to those given by the CBO's models, but the long-run forecasts will be quite different. All of the models

Figure 2B
CUMULATIVE RESPONSE OF GROWTH RATE OF REAL GNP TO GROWTH RATE OF REAL GOVERNMENT DEBT
 (with 95 percent confidence interval)



predict that cuts in defense spending will reduce total GNP in the short run. But the VAR forecasts none of the long-run gains in total GNP that the large-scale models do.

The table (right) compares the forecasts of the effects of the 1991 plan on total GNP generated by the VAR model to those generated by the DRI and MSG models. All three sets of forecasts compare the predicted behavior of total GNP under a base case, in which real defense spending is essentially held constant as a fraction of total GNP (except for the small decreases called for by the Budget Enforcement Act), to the behavior of total GNP when defense spending is cut as called for by the 1991 plan and the proceeds are used to reduce the federal debt. The figures in the table represent the predicted differences, in percentages, between the level of total GNP under the 1991 plan and the level of total GNP in the base case. Details about these two alternative paths for defense spending are provided in the CBO's report (1992, Table 3, p. 10), as are the forecasts from the DRI and MSG models (Figure 3, p. 14 and Table 4, p. 15).

The table shows that the VAR model is consistently more pessimistic than the CBO's models about both the short-run and long-run effects of the 1991 plan. While the DRI and MSG models predict that the short-run costs of reduced defense spending will be 0.5 to 0.7 percent of total GNP, the VAR estimates these costs at 1 to 1.8 percent of total GNP. While the DRI and MSG models expect long-run benefits from the debt reduction to begin offsetting the short-run costs in the mid-1990s, the VAR predicts that the costs of the 1991 plan will peak at 2.4 percent of total GNP in the late 1990s. Finally, while both the DRI and MSG models predict gains in total GNP by the year 2000, the VAR model predicts that there will be a permanent loss of 1.9 percent of total GNP from the 1991 plan.

In order to check the robustness of the VAR forecasts, several kinds of alternative model specifications can be considered. Although the causal ordering used in equation (2) is to be preferred based on economic theory, it would be troublesome if other orderings yielded vastly different results. Similar forecasts are obtained, however, when RDEF and RDEBT are placed last, rather than first, in the ordering. The model does not include some variables that may nonetheless be useful in forecasting GNP growth. Following the suggestion of Dotsey and Reid (1992), an oil price series can be added to the model, but again the results do not change. Nor do the

Forecasts of the Effects of the 1991 Plan on GNP

Year	Model			
	DRI (total GNP)	MSG (total GNP)	VAR (total GNP)	VAR (private GNP)
1992	-0.2	-0.3	-0.4	-0.2
1993	-0.7	-0.6	-1.0	-0.3
1994	-0.6	-0.5	-1.4	-0.5
1995	-0.6	-0.5	-1.8	-0.4
1996	-0.6	-0.3	-2.2	-0.4
1997	-0.6	-0.2	-2.4	-0.3
2000	0.0	0.5	-2.0	-0.1
2010	N/A	0.8	-1.9	0.3
2015	N/A	0.9	-1.9	0.3

Notes: The effects are expressed as percentage differences between GNP under the 1991 plan for reductions in defense spending and GNP under the base case of no change in real defense spending. DRI is the Data Resources Model, MSG is the McKibbin-Sachs Model, and VAR is the vector autoregressive model. Details about the two alternative paths for defense spending, as well as the forecasts from the DRI and MSG models, are taken from CBO (1992). N/A indicates that the forecast is not available.

results change if nondefense government spending or Barro and Sahasakul's (1986) marginal tax rate series is added as a seventh variable. Since Figure 1 reveals that the behavior of the model's variables was most dramatic during and shortly after World War II, it is useful to know the extent to which the results depend on the data from these years. When the six-variable VAR is reestimated with quarterly data from 1947 through 1991, the 1991 plan is predicted to reduce total GNP in the long run by 2.7 percent, a figure that is even larger than that generated by the original model. Finally, the forecasts are insensitive to changes in the lag length from $k=4$ to $k=3, 5$, or 6 . The VAR forecasts, therefore, are quite robust to changes in model specification; in all cases, cuts in defense spending are predicted to reduce total GNP substantially in the long run, even when cuts are used to reduce the federal debt.

To emphasize the point that the VAR forecasts, although considerably more pessimistic than the CBO's forecasts, do not imply that the defense cuts called for by the 1991 plan are undesirable, the table also presents forecasts from a VAR model that is identical to model (1), except that the growth rate of total GNP is replaced by the growth rate of private GNP. Private GNP, like total GNP, is predicted to fall in the short run as the 1991 plan is implemented. In the long run, however, private GNP is expected to increase by 0.3 percent. The 1991 plan reduces total GNP, but it also makes available to the private sector resources that would otherwise be allocated

to defense. The VAR forecasts show that on net, private GNP increases, making American households better off from the 1991 plan in the long run.

IV. SUMMARY AND CONCLUSIONS

The Bush Administration's 1992-1997 Future Years Defense Program (the "1991 plan") calls for the first significant cuts in defense spending in the United States since the end of the Vietnam War. Economic theory indicates that these defense cuts are likely to restrain economic growth in the short run as productive resources shift out of defense-related activities and into nondefense industries. Economic theory is less clear, however, about the long-run consequences of reduced defense spending. Models that assume that the Ricardian equivalence theorem holds find that a permanent decrease in defense spending decreases aggregate output in the long run. On the other hand, models that assume that Ricardian equivalence does not apply predict that a permanent decrease in defense spending increases output in the long run, provided that the proceeds from the spending cut are used to reduce the federal debt.

The large-scale econometric models employed by the Congressional Budget Office (1992) rely on the theoretical assumption that Ricardian equivalence does not hold in the U.S. economy. Thus, the CBO's models predict that while the 1991 plan will reduce total GNP in the short run as the economy adjusts to a lower level of defense spending, they also predict that the non-Ricardian effects of reducing the government debt will generate an increase in total GNP in the long run.

As an alternative to the CBO's large-scale models, this paper uses a much smaller VAR model to forecast the macroeconomic effects of the 1991 plan. Unlike the larger models, the VAR requires no strong theoretical assumption about whether or not Ricardian equivalence holds in the U.S. economy. The VAR, therefore, recognizes that economic theory provides no clear answer as to the likely long-run effects of reduced defense spending.

In fact, results from the VAR suggest that the Ricardian equivalence theorem does apply to the U.S. economy. Changes in government debt are found to have only small effects on aggregate output. Forecasts from the VAR, which show that the 1991 plan is likely to reduce total GNP in both the short run and long run, are more consistent with the neoclassical model presented by Barro (1984), in which Ricardian equivalence holds, than with those of competing models in which Ricardian equivalence does not apply.

Although the VAR forecasts are considerably more pessimistic than the CBO's forecasts, they do not imply that the defense cuts called for by the 1991 plan are undesirable. In fact, both the neoclassical model and the VAR forecasts suggest that as the cuts in defense spending are implemented, growth in total GNP is likely to be a misleading measure of household welfare. Although the 1991 plan reduces total GNP, it also makes available to the private sector resources that would otherwise be allocated to defense. The VAR forecasts show that on net, private GNP increases. As noted by Garfinkel (1990) and Wynne (1991), this net gain can be used to increase private consumption or private investment, making American households better off in the long run.

APPENDIX

DATA SOURCES

Defense Spending: Data for 1930 through 1938 are national security outlays reported in Table A-I of Kendrick (1961). Data for 1939 to 1991 are government purchases of goods and services, national defense, from Table 3.7a of the *Survey of Current Business*, Department of Commerce, Bureau of Economic Analysis. The nominal data are deflated using the implicit price deflator for GNP reported in Table 7.4 of the same publication.

Government Debt: Debt before 1941 is total gross debt at the end of the fiscal year reported in the *Bulletin of the Treasury*. Debt for 1941-1991 is total outstanding debt, also at the end of the fiscal year, reported in the same publication. Nominal debt was deflated to real terms using the implicit price deflator for GNP.

Interest Rate: The six-month commercial paper rate is taken from Table H15 of the *Statistical Release*, Board of Governors of the Federal Reserve System.

Monetary Aggregate: The money supply series before 1959 is the M4 aggregate reported in Table 1 of Friedman and Schwartz (1970). The money series for 1959 to 1991 is the M2 series reported in Table 1.21 of the *Federal Reserve Bulletin*, Board of Governors of the Federal Reserve System.

Price Deflator: The implicit price deflator for GNP is from Table 7.4 of the *Survey of Current Business*, Department of Commerce, Bureau of Economic Analysis.

Gross National Product: Nominal figures for GNP are taken from Table 1.1 of the *Survey of Current Business*, Department of Commerce, Bureau of Economic Analysis. Nominal GNP was deflated to real terms using the implicit price deflator for GNP.

Nondefense Government Spending: Nondefense spending is government purchases of goods and services from Table 1.1 of *The National Income and Product Accounts* (Department of Commerce, Bureau of Economic Analysis), less the defense spending series described above.

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A Simple Model of Irving Fisher's Price-Level Stabilization Rule

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INTRODUCTION

It is now well understood that a price-level stabilization policy is more ambitious than a zero-inflation policy. Targeting zero inflation means that the central bank brings inflation to a halt but leaves the price level where it is at the end of the inflation. By contrast, targeting stable prices means that the central bank ends inflation and also rolls back prices to some fixed target level. By reversing inflated prices and restoring them to their pre-existing level, a price-stabilization policy eradicates the upward drift of prices that can occur under a zero-inflation policy. It follows that a stable-price policy is more stringent than a zero-inflation policy.

The history of monetary thought abounds with price-level (as opposed to inflation-rate) stabilization rules. Not all of these policy rules were sound; some would have destabilized prices rather than stabilizing them. Notoriously flawed was John Law's 1705 proposal to back the quantity of money dollar for dollar with the nominal value of land. His rule guaranteed that changes in the price of land would induce equiproportional changes in the money stock. Equally flawed was the celebrated *real bills doctrine* advanced by the antibullionist writers during the Bank Restriction period of the Napoleonic Wars. It tied money's issue to the "needs of trade" as represented by the nominal quantity of commercial bills presented to banks as loan collateral. It failed to note that since the nominal volume of bills supplied (or loans demanded) varies directly with general prices, accommodating the former with money creation meant accommodating prices as well. Seen by their proponents as price-stabilizing, both rules in fact would have expanded or contracted the money supply in response to shock-induced price-level changes, thus underwriting or validating those changes (see Mints, 1945, pp. 30, 47-48).

Ruling out such inherently fallacious schemes leaves the remaining valid ones. These fall into two categories. The first consists of non-activist policy rules that fix the money stock or its rate of change at a constant level. Milton Friedman's k -percent rule,

which would establish the money stock at a fixed level when output's growth rate is zero, is perhaps the best known example of this type of rule. The second includes activist feedback rules which dictate predetermined corrective responses of the money supply and/or central-bank interest rates to price deviations from target. The proposals of David Ricardo, Knut Wicksell, and Irving Fisher exemplify this type of rule. Given England's 1810 regime of inconvertible paper currency and floating exchange rates, Ricardo (1810) advocated lock-step money-stock contraction in proportion to price-level increases as proxied by exchange-rate depreciation and the premium (excess of market price over mint price) on gold. Wicksell (1898, p. 198) proposed an interest-rate feedback rule: raise the bank interest rate when prices are rising, lower it when prices are falling, and keep it steady when prices are neither rising nor falling. Fisher suggested not one rule but two. His 1920 *compensated dollar plan* called for the policymakers to adjust the gold weight of the dollar equiproportionally to changes in the preceding month's general price index. In essence, he posited the relationship: dollar price of goods = dollar price of gold x gold price of goods. Official adjustments in the dollar price of gold, he thought, would offset fluctuations in the world gold price of goods (as proxied by the preceding month's general price index), thus stabilizing the dollar price of goods. His second rule (1935) was more conventional. Much like stabilization rules proposed today, it dictated automatic variations in the money stock to correct price-level deviations from target.

This article examines Fisher's second policy rule, particularly its dynamic properties. Fisher himself failed to investigate these properties. He provided no analytical model in support of his rule. Such a model is needed to show (1) that the rule would indeed force prices to converge to target, (2) how fast they would converge, and (3) whether the resulting path is oscillatory or monotonic. Lastly, only the model can demonstrate rigorously whether Fisher's rule is capable of outperforming rival rules such as the constant money-stock rule.

The following paragraphs attempt to provide the missing model underlying Fisher's scheme. In so doing, they contribute three innovations to the stabilization literature. First, they express Fisher's scheme in equations, something not done before. They represent his proposal in the form of price-change equations and policy-reaction functions suggested by A. W. Phillips' (1954, 1957) classic work on closed-loop feedback control mechanisms.

Second, they thus extend Phillips' analysis to encompass monetary models of price-level stabilization. Heretofore, Phillips' work has been applied exclusively to the design of output-stabilizing fiscal rules in Keynesian multiplier-accelerator models. (See the texts of Allen, 1959, 1967; Meade, 1972; Nagatani, 1981; and Turnovsky, 1977, for examples.) In finding a new use for Phillips' work, the article incorporates his notions of proportional, derivative, and integral control into Fisher's policy-response functions. The result is to show how the money stock in Fisher's scheme can be programmed to respond automatically (1) to the discrepancy between actual and target prices, (2) to the speed with which that discrepancy is rising or falling, and (3) to the cumulative value of the discrepancy over time beginning with the inauguration of his scheme.

Last but not least, the article compares within a single model the relative performance of Fisher's activist feedback rule with that of a non-activist constant money-stock rule. Policymakers of course must be convinced that Fisher's rule dominates rival candidate rules before they would consider adopting it. A related issue concerns the doctrinal accuracy of the model. To ensure that the model faithfully captures Fisher's thinking, one must outline his scheme to determine the appropriate variables and equations to use.

FISHER'S SCHEME

Fisher presented his proposal in his 1935 book *100% Money*. He argued that the monetary authorities could stabilize prices at a fixed target level via open market operations.

If money became scarce, as shown by a tendency of the price level to fall, more could be supplied instantly; and if superabundant, some could be withdrawn with equal promptness. . . . The money management would thus consist . . . of buying [government securities] whenever the price level threatened to fall below the stipulated par and selling whenever it threatened to rise above that par. (P. 97)

He reasoned that price movements stem from excess money supplies or demands. Since money is employed for spending, these excess supplies spill over into the commodity market in the form of excess aggregate demand for goods, thus putting upward pressure on prices. Prices continue to rise until the surplus money is absorbed by higher cash balances needed to purchase the same real output at elevated prices. Likewise, excess money demands, manifested in increased hoarding and decreased spending, cause aggregate demand contractions and downward pressure on prices in the goods market. Prices and the associated need for transaction balances continue to fall until money demand equals money supply. In either case, appropriate variations of the money stock could, Fisher thought, correct the resulting price-level deviations from target. In other words, the Federal Reserve expands the money stock when prices fall below target and contracts the money stock when prices rise above target. Clearly, money constitutes the policy instrument and the price level the goal variable in Fisher's scheme.

A policy instrument of course is only as good as the Fed's ability to control it. In Fisher's view this ability was absolute—or at least it could be made so by the 100 percent reserve regime advocated in his book. As he saw it, the Fed exercises direct control over the high-powered monetary base. And since a 100 percent reserve regime renders the base and the money stock one and the same aggregate, it follows that tight command of the one constitutes perfect regulation of the other. In sum, when deposit money is backed dollar for dollar with bank reserves as prescribed in his book, there can be no slippage in money-stock control to disqualify money as the policy instrument. For that matter, little slippage would occur in a fractional reserve system or even a system of no reserve requirements as long as deposit money bore a stable relationship to high-powered money.

Nor did Fisher see policy lags as a problem. He knew that his rule to be effective required two things: prompt direct response of prices to money and equally prompt feedback response of money to prices. He was sanguine about both, although less so about the former. In *Stabilizing the Dollar* (1920), he stated that prices seem to follow money with "a lag of one to three months" (p. 29). As for money's corrective response to price misbehavior, he found virtually no lag at all. Money, he insisted, can be "supplied instantly" or "withdrawn with equal promptness" in reaction to price deviations from target. His scheme admits of no significant delays to retard the Fed from achieving its desired target setting of the

money stock. Such setting occurs immediately. Accordingly, the model below omits all policy lags. On this point he was quite clear.

He was not so clear on other matters, however. For example, he did not specify the exact price indicator to which the Fed should react. Should it adjust the money stock in response to the differential between actual and target prices? To changes in that differential? To both? Suppose it has missed the target in every period since it initiated its policy. Should it forgive these past misses? Or should it allow them to influence current policy by linking the money stock to the cumulative sum of the past price errors? Which price indicator and associated policy response yields the smoothest and quickest path to price stability? Fisher did not say. Moreover, as noted above, he offered no proof that his feedback rule could in fact deliver price stability or that it would outperform other candidate rules.

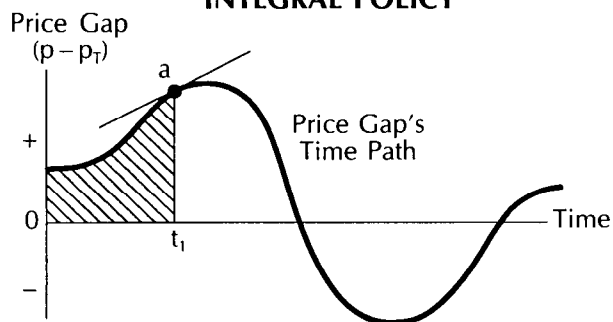
MODELING FISHER'S SCHEME

Addressing these issues requires an explicit analytical framework. The one supplied here employs the four-step technique pioneered by A. W. Phillips (1954, 1957) in his celebrated analysis of stabilization policy.

Step one models how the price level would behave if uncorrected by policy. Consistent with Fisher's exposition, the model treats prices as moving in response to excess money supplies and demands.

Step two incorporates policy-response functions embodying elements of *proportional*, *derivative*, and *integral control*. A proportional feedback control rule adjusts the money stock in response to current price deviations from target. Derivative control adjusts money in response to the deviation's rate of change. Integral control adjusts money in response to the cumulative sum of deviations over time. It seeks to correct the time integral of all past misses from target. For example, suppose prices since the inauguration of corrective policy have fluctuated about target. Or, what is the same thing, the discrepancy between actual and target prices $p - p_T$ has fluctuated about zero, as shown in the figure. The application of proportional policy at time t_1 requires money-stock contraction in proportion to the price gap $t_1 a$. Derivative policy contracts the money stock by a fixed proportion of the rate of price rise indicated by the slope of the tangent to the curve at time t_1 . Finally, integral policy aimed at correcting cumulative deviations from target contracts the money stock in

PROPORTIONAL, DERIVATIVE, AND INTEGRAL POLICY



Proportional policy contracts the money stock in proportion to the price gap $t_1 a$. Derivative policy contracts the money stock in proportion to the rate of price rise as indicated by the slope of the tangent to the curve at point a . Integral policy contracts the money stock in proportion to the cumulative value of the price gap over time as indicated by the shaded area under the curve.

proportion to the shaded area under the curve up to time t_1 . In short, proportional policy focuses on current price gaps, derivative policy on the direction of movement of such gaps, and integral policy on the sum of all gaps, past and present. Once applied, each policy produces a different policy-corrected path for prices.

Step three solves the model for these alternative policy-corrected paths. Step four uses a loss function to measure the cost (in terms of reputational damage suffered by the Fed when prices deviate from target) of adhering to each path. This procedure allows one to rank the alternative policy rules according to how smoothly and quickly they stabilize prices. As shown below, at least one of the feedback rules dominates the fixed money-stock rule.

PRICE-CHANGE EQUATION

The first step is to model the non-policy determinants of price-level behavior. To Fisher, price changes emanated from excess money supplies and demands caused, say, by policy mistakes and/or shifts in the amount of money people want to hold at existing prices and real incomes. In his words, prices fall when money is "scarce" relative to the demand for it and rise when money is "superabundant" relative to demand. Accordingly, one seeks the simplest equation that captures his hypothesis.

That equation is $\dot{p} = \alpha(m - kpq)$, where the time derivative \dot{p} denotes a change in prices, m denotes the money supply, the product kpq denotes money demand consisting of velocity's inverse or the Cambridge k times the price level p times real output q ,

and α is a positive goods-market reaction coefficient expressing the speed of response of prices to excess money supply.¹ Let the model's time unit be a calendar quarter. Define $T=1/\alpha$ as the number of quarters required for prices to adjust to clear the market for money balances. Then Fisher's 1920 finding that prices follow money with a lag of close to three months implies that both α and T are approximately equal to unity in magnitude. As for the other items in the equation, output q and the Cambridge k are taken as given constants at their long-run equilibrium values. In his 1935 book, Fisher mentioned no other money-demand determinants such as interest rates or price-change expectations. For that reason they are omitted here.²

POLICY RULES

The next step is to specify the alternative policy-response functions the Fed might use to bring prices to target in the model. These functions are absolutely essential. Without them, prices p would adjust in response to an excess money supply until they reached an equilibrium level $p=m/kq$ different from the target level p_T . Permanent price gaps would result.

Under a *constant money-stock rule*, the Fed merely sets the money stock at the level $m_T=kqp_T$ that equilibrates money supply and demand at the target price level p_T and then leaves it there. Other than setting m_T consistent with p_T , the Fed does nothing

¹ This equation admits of a straightforward derivation. Define an excess supply of money x as the surplus of money m over the demand for it kpq , or $x=m-kpq$. Assume this excess money supply spills over into the commodity market to underwrite an excess demand for goods g . In short, $x=g$. The excess demand for goods in turn puts upward pressure on prices, which rise in proportion to the excess demand, or $\dot{p}=\alpha g$, where α is the factor of proportionality. Substitution of x for g and $m-kpq$ for x yields the equation of the text.

² It is tempting to write Fisher's equation in expectations-augmented form as $\dot{p}=\alpha[m-(kpq-\tau\dot{p}^e)]$ where the term $-\tau\dot{p}^e$ captures the effect of price-change expectations \dot{p}^e on money demand. This term says that people expecting money to depreciate in value will hold smaller cash balances than they would if they expected no such depreciation. One could further assume that people form their expectations rationally such that expected price changes equal actual ones ($\dot{p}^e=\dot{p}$) in this non-stochastic model and the equation reduces to $\dot{p}=[\alpha/(1-\alpha\tau)](m-kpq)$. But such an equation would be inconsistent with Fisher's own views on the subject. True, in his *The Purchasing Power of Money* (1911), he recognized that inflation perceptions affect money demand. When money "is depreciating," he wrote, "holders will get rid of it as fast as possible" (p. 63). But he did not spell out how such perceptions are formed or how they get incorporated into money demand functions. Nor did he believe in rational expectations. He held that people are subject to money illusion and so revise their expectations slowly in the face of actual price changes such that when those changes occur expectations lag behind. Indeed, he coined the phrase "money illusion" and used it as the title of his 1928 book.

else. Thus the money-stock equation is simply $\dot{m}_e=m_T$, where m_e denotes constant money-stock policy.

Activist feedback rules attempt to improve upon the constant money-stock rule. Thus under a *proportional feedback rule* the Fed adjusts the money stock above or below the desired long-run equilibrium level m_T as prices are below or above their target level. In other words, the money stock is set to counter price gaps or deviations from target. The resulting policy-response equation is $\dot{m}_p=m_T-\beta(p-p_T)$, where m_p denotes proportional policy and β is the proportional correction coefficient.

With a *derivative feedback rule* the Fed adjusts the money stock in response to the speed with which the price gap $p-p_T$ is increasing or decreasing in size. Or, what is the same thing (since the target price level p_T is fixed), it adjusts the money stock above or below money's long-run equilibrium level to counter falls or rises in the price level \dot{p} . The resulting policy-response equation is $\dot{m}_a=m_T-\gamma\dot{p}$, where m_a denotes derivative policy and γ is the derivative correction coefficient.

With an *integral feedback rule* the Fed adjusts the money stock to correct cumulative price gaps or the sum of all past policy misses over time. The Fed learns from these price errors. It uses the information given by their integral to determine the current setting of the money stock. Accordingly, the policy equation is $\dot{m}_i=m_T-\delta\int_0^t(p-p_T)dt$, where m_i denotes integral policy, $\int_0^t(\)dt$ denotes the integral operator, and δ is the integral correction coefficient. This rule can be given an alternative expression. When differentiated to get rid of the integral, it becomes $\dot{m}_i=-\delta(p-p_T)$, stating that the Fed sets the money stock's *rate of change* opposite to the direction that prices are currently deviating from target.

Finally, the Fed may employ a *mixed feedback rule* involving various combinations of the foregoing equations. For example, a mixed proportional-derivative rule would yield the policy-response function $\dot{m}_m=m_T-\beta(p-p_T)-\gamma\dot{p}$ embodying gap and gap-change terms together with their policy correction coefficients.

PRICE TIME PATHS AND LOSS FUNCTIONS

The third step is to substitute each of the foregoing policy rules into the Fisherian price-change

equation $\dot{p} = \alpha(m - kpq)$. Doing so produces expressions whose solutions are the policy-corrected time paths of prices.

Thus substitution of the constant money-stock rule into the price-change equation yields $\dot{p} = \alpha kq(p_T - p)$. Solving this first-order expression for the time path of prices gives $p = p_T + (p_0 - p_T)e^{-at}$, where p_0 denotes the (perturbed) price level at time $t=0$, t denotes time, e is the base of the natural logarithm system, and the parameter $a = \alpha kq$ denotes speed of convergence to target. This expression says that prices converge to target at a rate of $a = \alpha kq$ per unit of time. In short, as time passes and t gets large, the last term on the right-hand side of the equation goes to zero so that only the first term p_T remains. In this way the path to price stability terminates when $p = p_T$.

Associated with the path are certain costs to the Fed. The Fed's objective is to keep prices as close to target as possible over time. Society penalizes it for failing to do so. It suffers losses in reputation, credibility, and prestige that vary directly and disproportionately with the duration and size of its policy errors. These losses can be measured by the quadratic cost function expressing the Fed's reputational loss L as the cumulative squared deviation of prices from their desired target level, or $L = \int_0^{\infty} (p - p_T)^2 dt$. Substituting the price path into this loss function and integrating yields the cost of adhering to the non-activist constant money-stock rule, or $L_c = (p_0 - p_T)^2 / 2a$. As a hypothetical numerical example, let $p_T = 1$, $p_0 = 2$, $\alpha = 1$, $k = 1/2$, $q = 100$, and $a = 50$. Then the quantitative measure of the loss is $L_c = 1/100$.

EFFECTIVENESS OF THE ACTIVIST PROPORTIONAL RULE

To compare the foregoing loss with those of the activist feedback policy rules, one must derive the price paths and loss measures associated with the latter rules. Thus the proportional feedback rule yields the price path $p = p_T + (p_0 - p_T)e^{-bt}$ with associated loss measure $L_p = (p_0 - p_T)^2 / 2b$. Here $b = \alpha(kq + \beta)$ is the speed-of-convergence parameter. Since parameter b is larger than parameter a computed above, prices under the proportional rule converge to target faster than they do under the constant money-stock rule. It follows that prices deviate from target for a shorter time under the proportional rule. Consequently that rule yields the smallest loss of the two and thus dominates the constant money-stock rule. Numerically, $L_p = 1/102$,

assuming the proportional correction coefficient $\beta = 1$ and the constants α , k , q , p_0 , and p_T possess their values as assigned above. This loss compares favorably with the corresponding loss of 1/100 associated with the constant money-stock rule. Given a choice between the two rules, the Fed will select the proportional rule.

A word of warning is in order here. The proportional rule's superiority rests heavily on Fisher's assumption of no policy lags. By slowing convergence to target, policy lags could reverse the ranking of the two rules. Such lags would delay the Fed's adjustment \dot{m} of the money stock m to its desired proportional setting m_p . A new equation $\dot{m} = \lambda(m_p - m)$, where the positive coefficient λ represents the policy lag, would have to be added to the model. The resulting reduced-form expression for \dot{p} would be a second-order differential equation whose solution—the time path of prices—would be more complicated than before. Overshooting and oscillations would be a distinct possibility; slower convergence a certainty. These considerations highlight the importance of Fisher's assumption of zero policy lags. Embodied in the model, his assumption ensures the superiority of the proportional rule such that the Fed will select it.

OPTIMAL VALUE OF THE β COEFFICIENT

Given the proportional rule's capability of outperforming the constant money-stock rule, a natural question to ask next is whether the proportional correction coefficient β has been assigned its optimal value. The preceding numerical example assumed $\beta = 1$. But a glance at the proportional rule's loss measure $L_p = (p_0 - p_T)^2 / 2\alpha(kq + \beta)$ suggests that the Fed should make β as large as possible ($\beta \rightarrow \infty$) and L_p negligibly small. That is, optimality considerations would seem to compel instantaneous monetary contraction in amounts sufficient to force prices to target immediately.

Fisher, however, would have rejected this implication of the mathematical formulation of his scheme. He would have condemned the violent monetary contraction implied by high values of β . To him such contraction spelled devastating losses to output and employment. In his *The Purchasing Power of Money* (1911) and his 1926 paper on price changes and unemployment, he ascribed these losses to the failure of sticky nominal wage and interest rates to respond as fast as product prices to monetary shocks. He attributed nominal wage rigidities to fixed contracts and the inertia of custom; nominal interest rate

rigidities to price misperceptions and sluggishly adjusting inflation expectations. Because of these inhibiting forces, a sharp fall in money and prices would transform sticky nominal wage and interest rates into rising real rates, thus depressing economic activity. In his 1933 debt-deflation theory of great depressions, he cited still another reason to fear violent monetary contraction. He argued that price deflation could, by raising the real burden of nominal debt, precipitate a wave of business bankruptcies with all its adverse repercussions for the real economy. To avoid these consequences, Fisher would have recommended relatively gradual monetary contraction implied by moderate values of β (such as $\beta = 1$). Consistent with his views, β 's value here is restricted to unity.

RELATIVE EFFECTIVENESS OF THE DERIVATIVE RULE

As for the other candidate rules, the Fed will reject as inferior to proportional policy the derivative rule. That rule calls for money-stock adjustments opposite to the direction prices are moving. It exerts stabilizing pressure when prices are moving away from target; less so when they are moving toward target. When prices fall toward target, the derivative rule interferes perversely by expanding the money stock. In so doing, it retards convergence and becomes a relatively unattractive option. In symbols, derivative policy results in the price path $p = p_T + (p_0 - p_T)e^{-ct}$ and loss function $L_d = (p_0 - p_T)^2 / 2c$, with speed of convergence denoted by the parameter $c = \alpha k q / (1 + \alpha \gamma)$. This parameter is smaller than its counterparts a and b , signifying slower convergence and larger reputation loss with a derivative rule than with a constant or proportional rule. Numerically, the derivative rule's loss is $L_d = 1/50$, assuming the derivative correction coefficient γ is assigned a value of one and the other constants possess the same magnitudes as defined above. This loss is twice that of the constant money-stock rule and more than twice that of the proportional rule. The Fed, therefore, would hardly opt for a derivative rule.

RELATIVE EFFECTIVENESS OF THE INTEGRAL RULE

Nor would the Fed opt for an integral rule that seeks to correct the sum of all past and present misses of target. The past misses would continue to influence policy even when the current price level was close to target. Too strong a response to them would cause overshooting. Conversely, too weak a response would cause prices to move too slowly to target. Suppose past misses totaled 5 when the current miss was 0.

Integral policy in this case would push the price level below target. And if the same past misses were exactly counterbalanced by a current miss of -5 , integral policy would fail at that moment to put corrective pressure on prices. Of course continuation of the current miss would eventually activate corrective pressure. But this pressure would be slow in coming. Thus while stabilizing prices in the long run, integral policy might do so sluggishly and via damped oscillatory paths.

Integral policy yields the time path $p = p_T + A_1 e^{r_1 t} + A_2 e^{r_2 t}$. Here A_1, A_2 are constants of integration determined by initial conditions. And $r_1, r_2 = (-\alpha k q / 2) \pm \sqrt{(\alpha k q)^2 - 4\alpha \delta} / 2$ are the characteristic roots of the left-hand or homogeneous part of the second-order expression $\ddot{p} + (\alpha k q)\dot{p} + \alpha \delta p = \alpha \delta p_T$, obtained by differentiating the Fed's policy-reaction function to eliminate the integral and substituting the result into the Fisherian price-change equation. The roots r_1, r_2 possess negative real parts ($-\alpha k q / 2$) thus ensuring convergence. Damped oscillations about target occur if $(\alpha k q)^2 < 4\alpha \delta$ or, in other words, if the integral correction coefficient δ is larger than $\alpha k^2 q^2 / 4$. Monotonic convergence results from smaller values of that coefficient. But convergence, whether cyclical or monotonic, is slower under the integral rule than under the proportional and constant money-stock rules. The above-mentioned characteristic roots reveal as much. Dwarfed by the speed-of-convergence parameters of the other rules, their relatively small size renders the integral rule inferior to its rivals. Confirmation comes from the loss function, which shows a large reputational loss associated with the integral rule. One computes the integral policy's loss function as $L_i = -[A_1^2 / 2r_1] - [2A_1 A_2 / (r_1 + r_2)] - [A_2^2 / 2r_2]$, where $A_1 = [\dot{p}_0 - r_2(p_0 - p_T)] / (r_1 - r_2)$ and $A_2 = [r_1(p_0 - p_T) - \dot{p}_0] / (r_1 - r_2)$. Although hard to evaluate analytically, this function yields to numerical computation. Let $\alpha = \delta = p_T = 1, \dot{p}_0 = -1, p_0 = 2, k = 1/2$, and $q = 100$ as before. Then the function in this hypothetical illustrative example produces a numerical value of 26, several hundred times the losses under the other rules.

RELATIVE EFFECTIVENESS OF A MIXED RULE

Finally, the Fed might try a mixed rule embodying proportional and derivative elements. The mixed rule yields the price path $p = p_T + (p_0 - p_T)e^{-dt}$ and associated loss function $L_m = (p_0 - p_T)^2 / 2d$, where the speed-of-convergence parameter $d = \alpha(\beta + kq) / (1 + \alpha \gamma)$. With the coefficient values as

assigned above, the loss L_m is $2/102$, ranking the mixed rule inferior to the proportional and constant money-stock rules, but superior to the derivative and integral rules. This ranking, however, depends on the values assigned to the coefficients. Giving γ a value of $1/52$ instead of 1 would reverse the order of the mixed and constant money-stock rules. In general, the mixed rule ranks below the constant money-stock rule if $\gamma > \beta/\alpha kq$ and above it if that inequality is reversed.

CONCLUSION

Fisher's feedback policy rule delivers price stability in the simple money-demand-and-supply model presented here. And it does so whether the rule is expressed in terms of proportional, derivative, integral, or mixed control. All the rules yield paths

that converge to target, albeit at different speeds. Proportional policy yields the quickest and smoothest path, followed by mixed, derivative, and integral policies in that order. Of these four activist feedback rules, only the proportional always outperforms the nonactivist constant money-stock rule. For this reason, proportional policy's loss measure is the smallest of the lot. Indeed, one can rank the loss measures to show that the proportional feedback rule dominates the constant money-stock rule, which in turn dominates the derivative and integral rules. While the mixed rule may, at certain values of the γ coefficient, outrank the constant money-stock rule, it can never dominate the proportional rule. Provided policy lags are short or nonexistent, these results create a presumption in favor of a proportional feedback rule.

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Money Market Futures

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INTRODUCTION

Money market futures are futures contracts based on short-term interest rates. Futures contracts for financial instruments are a relatively recent innovation. Although futures markets have existed over 100 years in the United States, futures trading was limited to contracts for agricultural and other commodities before 1972. The introduction of foreign currency futures that year by the newly formed International Monetary Market (IMM) division of the Chicago Mercantile Exchange (CME) marked the advent of trading in financial futures. Three years later the first futures contract based on interest rates, a contract for the future delivery of mortgage certificates issued by the Government National Mortgage Association (GNMA), began trading on the floor of the Chicago Board of Trade (CBT). A host of new financial futures have appeared since then, ranging from contracts on money market instruments to stock index futures. Today, financial futures rank among the most actively traded of all futures contracts.

Four different futures contracts based on money market interest rates are actively traded at present. To date, the IMM has been the site of the most active trading in money market futures. The three-month U.S. Treasury bill contract, introduced by the IMM in 1976, was the first futures contract based on short-term interest rates. Three-month Eurodollar time deposit futures, now one of the most actively traded of all futures contracts, started trading in 1981. More recently, both the CBT and the IMM introduced futures contracts based on one-month interest rates. The CBT listed its 30-day interest rate futures contract in 1989, while the Chicago Mercantile Exchange introduced a one-month LIBOR futures contract in 1990.

This article provides an introduction to money market futures. It begins with a general description of the organization of futures markets. The next section describes currently traded money market futures contracts in some detail. A discussion of the relationship between futures prices and underlying

spot market prices follows. The concluding section examines the economic function of futures markets.

AN INTRODUCTION TO FUTURES MARKETS

Futures Contracts

Futures contracts traditionally have been characterized as exchange-traded, standardized agreements to buy or sell some underlying item on a specified future date. For example, the buyer of a Treasury bill futures contract—who is said to take on a “long” futures position—commits to purchase a 13-week Treasury bill with a face value of \$1 million on some specified future date at a price negotiated at the time of the futures transaction; the seller—who is said to take on a “short” position—agrees to deliver the specified bill in accordance with the terms of the contract. In contrast, a “cash” or “spot” market transaction simultaneously prices and transfers physical ownership of the item being sold.

The advent of cash-settled futures contracts such as Eurodollar futures has rendered this traditional definition overly restrictive, however, because actual delivery never takes place with cash-settled contracts. Instead, the buyer and seller exchange payments based on changes in the price of a specified underlying item or the returns to an underlying security. For example, parties to an IMM Eurodollar contract exchange payments based on changes in market interest rates for three-month Eurodollar deposits—the underlying deposits are neither “bought” nor “sold” on the contract maturity date. A more general definition of a futures contract, therefore, is a standardized, transferable agreement that provides for the exchange of cash flows based on changes in the market price of some commodity or returns to a specified security.

Futures contracts trade on organized exchanges that determine standardized specifications for traded contracts. All futures contracts for a given item specify the same delivery requirements and one of a limited number of designated contract maturity dates, called settlement dates. Each futures exchange has an affiliated clearinghouse that records all transactions and ensures that all buy and sell trades match. The clearing organization also assures the financial

* This paper has benefited from helpful comments by Timothy Cook, Ira Kwall, Jeffrey Lacker, and Robert LaRoche.

integrity of contracts traded on the exchange by guaranteeing contract performance and supervising the process of delivery for contracts held to maturity. A futures clearinghouse guarantees contract performance by interposing itself between a buyer and seller, assuming the role of counterparty to the contract for both parties. As a result, the original parties to the contract need never deal with one another again—their contractual obligations are with the clearinghouse.

Contract standardization and the clearinghouse guarantee facilitate trading in futures contracts. Contract standardization reduces transactions costs, since it obviates the need to negotiate all the terms of a contract with every transaction—the only item negotiated at the time of a futures transaction is the futures price. The clearinghouse guarantee relieves traders of the risk that the other party to the contract will fail to honor contractual commitments. These two characteristics make all contracts for the same item and maturity date perfect substitutes for one another. Consequently, a party to a futures contract can always liquidate a futures commitment, or open position, before maturity through an offsetting transaction. For example, a long position in Treasury bill futures can be liquidated by selling a contract for the same maturity date. The clearinghouse assumes responsibility for collecting funds from traders who close out their positions at a loss and passes those funds along to traders with opposing futures positions who liquidate their positions at a profit. Once any gains or losses are settled, the offsetting sale cancels the commitment created through the earlier purchase of the contract. Most futures contracts are liquidated in this manner before they mature. In recent years less than 1 percent of all futures contracts have been held to maturity, although delivery is more common in some markets.¹

Forward agreements resemble futures contracts in that they specify the terms of a transaction to be undertaken at some future date. For this reason, the terms “forward agreement” and “futures contract” are often used synonymously. There are important differences between the two, however. Whereas futures contracts are standardized agreements, forward agreements tend to be custom-tailored to the needs of users. While a good deal of contract standardization exists in forward markets, items such as delivery dates, deliverable grades, and amounts can all be negotiated separately with each contract.

¹ Based on data from the *Annual Report 1991* of the Commodity Futures Trading Commission.

Moreover, forward contracts are not traded on organized exchanges as are futures contracts and carry no independent clearinghouse guarantee. As a result, a party to a forward contract faces the risk of nonperformance by the other party. For this reason, forward contracting generally takes place among parties that have some knowledge of each other's creditworthiness. Unlike futures contracts, which can be bought or sold at any time before maturity to liquidate an open futures position, forward agreements, as a general rule, are not transferable and so cannot be sold to a third party. Consequently, most forward contracts are held to maturity.

Futures Exchanges

In addition to providing a physical facility where trading takes place, a futures exchange determines the specifications of traded contracts and regulates trading practices. There are 13 futures exchanges in the United States at present. The principal exchanges are in Chicago and New York.

Each futures exchange is a corporate entity owned by its members. The right to conduct transactions on the floor of a futures exchange is limited to exchange members, although trading privileges can be leased to nonmembers. Members have voting rights that give them a voice in the management of the exchange. Memberships, or “seats,” can be bought and sold: futures exchanges routinely make public the most recent selling and current offer price for a seat on the exchange.

Trading takes place in designated areas, known as “pits,” on the floor of the futures exchange through a system of open outcry in which traders announce bids to buy and offers to sell contracts. Traders on the floor of the exchange can be grouped into two broad categories: floor brokers and floor traders. Floor brokers, also known as commission brokers, execute orders for off-exchange customers and other members. Some floor brokers are employees of commission firms, known as Futures Commission Merchants, while others are independent operators who contract to execute trades for brokerage firms. Floor traders are independent operators who engage in speculative trades for their own account. Floor traders can be grouped into different classifications according to their trading strategies. “Scalpers,” for example, are floor traders who perform the function of marketmakers in futures exchanges. They supply

liquidity to futures markets by standing ready to buy or sell in an attempt to profit from small temporary price movements.²

Futures Commission Merchants

A Futures Commission Merchant (FCM) handles orders to buy or sell futures contracts from off-exchange customers. All FCMs must be licensed by the Commodity Futures Trading Commission (CFTC), which is the government agency responsible for regulating futures markets. An FCM can be a person or a firm. Some FCMs are exchange members employing their own floor brokers. FCMs that are not exchange members must make arrangements with a member to execute customer orders on their behalf.

Role of the Exchange Clearinghouse

As noted earlier, each futures exchange has an affiliated exchange clearinghouse whose purpose is to match and record all trades and to guarantee contract performance. In most cases the exchange clearinghouse is an independently incorporated organization, but it can also be a department of the exchange. The Board of Trade Clearing Corporation, the CBT's clearinghouse, is a separate corporation affiliated with the exchange, while the CME Clearing House Division is a department of the exchange.

Clearing member firms act as intermediaries between traders on the floor of the exchange and the clearinghouse. They assist in recording transactions and assume responsibility for contract performance on the part of floor traders and commission merchants who are their customers. Although clearing member firms are all members of the exchange, not all exchange members are clearing members. All transactions taking place on the floor of the exchange must be settled through a clearing member. Brokers or floor traders not directly affiliated with a clearing member must make arrangements with one to act as a designated clearing agent. The clearinghouse requires each clearing member firm to guarantee contract performance for all of its customers. If a clearing member's customer defaults on an outstanding futures commitment, the clearinghouse holds the clearing member responsible for any resulting losses.

² A good description of trading strategies employed by different floor traders can be found in Hieronymus (1971). In addition, detailed descriptions of different trading strategies can also be found in almost any good textbook on futures markets such as Chance (1989), Merrick (1990), or Siegel and Siegel (1990). Silber (1984) presents a comprehensive analysis of scalper trading behavior.

Margin Requirements

Margin deposits on futures contracts are often mistakenly compared to stock margins. Despite the similarity in terminology, however, futures margins differ fundamentally from stock margins. Stock margin refers to a down payment on the purchase of an equity security on credit, and so represents funds surrendered to gain physical possession of a security. In contrast, a margin deposit on a futures contract is a performance bond posted to ensure that traders honor their contractual obligations, and not a down payment on a credit transaction. The value of a futures contract is zero to both the buyer and the seller at the time it is negotiated, so a futures transaction involves no exchange of money at the outset.

The practice of collecting margin deposits dates back to the early days of trading in time contracts, as the precursors of futures contracts were then called. Before the institution of margin requirements, traders adversely affected by price movements frequently defaulted on their contractual obligations, often simply disappearing as the delivery date on their contracts drew near. In response to these events, futures exchanges instituted a system of margin requirements, and also began requiring traders to recognize any gains or losses on their outstanding futures commitments at the end of each trading session through a daily settlement procedure known as "marking to market."

Before being permitted to undertake a futures transaction, a buyer or seller must first post margin with a broker, who, in turn, must post margin with a clearing agent. Margin may be posted either by depositing cash with a broker or, in the case of large institutional traders, by pledging collateral in the form of marketable securities (typically, Treasury securities) or by presenting a letter of credit issued by a bank. Brokers sometimes pay interest on funds deposited in a margin account.

As noted above, clearing member firms ultimately are liable to the clearinghouse for any losses incurred by their customers. To assure the financial integrity of the settlement process, clearing member firms must themselves meet margin requirements in addition to meeting minimum capital requirements set by the exchange clearinghouse.

Daily Settlement

The practice of marking futures contracts to market requires all buyers and sellers to realize any gains or losses in the value of their futures positions at the

end of each trading session, just as if every position were liquidated at the closing price. The exchange clearinghouse collects payments, called variation margin, from all traders incurring a loss and transfers the proceeds to those traders whose futures positions have increased in value during the latest trading session. If a trader has deposited cash in a margin account, his broker simply subtracts his losses from the account and transfers the variation margin to the clearinghouse, which, in turn, transfers the funds to the account of a trader with a short position in the contract. Most brokers require their customers to maintain minimum balances in their margin accounts in excess of exchange requirements. If a trader's margin account falls below a specified minimum, called the maintenance margin, he faces a margin call requiring the deposit of additional margin money. In cases where collateral has been posted in the form of securities rather than in cash, the trader must pay the variation margin in cash. Should a trader fail to meet a margin call, his broker has the right to liquidate his position. The trader remains liable for any resulting losses.

Marking a futures contract to market has the effect of renegotiating the futures price at the end of each trading session. Once the contract is marked to market, the trader begins the next trading session with a commitment to purchase the underlying item at the previous day's closing price. The exchange clearinghouse then calculates any gains or losses for the next trading session on the basis of this latter price.

The following example involving the purchase of a Treasury bill futures contract illustrates the mechanics of the daily settlement procedure. Treasury bill futures prices are quoted as a price index determined by subtracting the futures discount yield (stated in percentage points) from 100. A 1 basis point change in the price of the Treasury bill contract is valued at \$25.³ Thus, if a trader buys a futures contract at a price of 96.25 and the closing price at the end of the trading session falls to 96.20, he must pay \$125 (5 basis points x \$25 per basis point) in variation margin. Conversely, the seller in this transaction would earn \$125, which would be deposited to his margin account. The buyer would then begin the next trading session with a commitment to buy the underlying Treasury bill at 96.20, and any gains or losses sustained over the course of the next trading session would be based on that price.

³ Price quotation and contract specifications for Treasury bill futures are discussed in more detail in the next section.

Final Settlement

Because buying a futures contract about to mature is equivalent to buying the underlying item in the spot market, futures prices converge to the underlying spot market price on the last day of trading. This phenomenon is known as "convergence." At the end of a contract's last trading session, it is marked to market one final time. In the case of a cash-settled contract, this final daily settlement retires all outstanding contractual commitments and any remaining margin money is returned to the traders. If the contract specifies delivery of the underlying item, the clearinghouse subsequently makes arrangements for delivery among all traders with outstanding futures positions. The delivery, or invoice price, is based on the closing price of the last day of trading. Any profit or loss resulting from the difference between the initial futures price and the final settlement price is realized through the transfer of variation margin. The gross return on the futures position is reflected in accumulated total margin payments, which must equal the difference between the final settlement price and the futures price determined at the time the futures commitment was entered into.

Regulation of Futures Markets

The Commodity Futures Trading Commission is an independent federal regulatory agency established in 1974 to enforce federal laws governing the operation of futures exchanges and futures commission brokers. By law, the CFTC is charged with the responsibility to ensure that futures trading serves a valuable economic purpose and to protect the interests of users of futures contracts. The CFTC must approve all futures contracts before they can be listed for trading by the futures exchanges. It also enforces laws and regulations prohibiting unfair and abusive trading and sales practices.

The futures industry attempts to regulate itself through a private self-regulatory organization called the National Futures Association, which was formed in 1982 to establish and help enforce standards of professional conduct. This organization operates in cooperation with the CFTC to protect the interests of futures traders as well as those of the industry. As noted earlier, the futures exchanges themselves can be viewed as private regulatory bodies organized to set and enforce rules to facilitate the trading of futures contracts.

CONTRACT SPECIFICATIONS FOR MONEY MARKET FUTURES

Treasury Bill Futures

The Chicago Mercantile Exchange lists 13-week Treasury bill futures contracts for delivery during the months of March, June, September, and December. Contracts for eight future delivery dates are listed at any one time, making the furthest delivery date for a new contract 24 months. A new contract begins trading after each delivery date.

Delivery Requirements The Treasury bill contract requires the seller to deliver a U.S. Treasury bill with a \$1 million face value and 13 weeks to maturity. Delivery dates for T-bill futures always fall on the three successive business days beginning with the first day of the contract month on which (1) the Treasury issues new 13-week bills and (2) previously issued 52-week bills have 13 weeks left to maturity.⁴ This schedule makes it possible to satisfy delivery requirements for a T-bill futures contract with either a newly issued 13-week bill or an original-issue 26- or 52-week bill with 13 weeks left to maturity. Deliverable bills can have 90, 91, or 92 days to maturity, depending on holidays and other special circumstances. The last day of trading in a Treasury bill futures contract falls on the day before the final settlement date.

Price Quotation Treasury bills are discount instruments that pay no explicit interest. Instead, the interest earned on a Treasury bill is derived from the fact that the bill is purchased at a discount relative to its face or redemption value. Treasury bill yields are quoted on a discount basis—that is, as a percentage of the face value of the bill rather than as a percentage of actual funds invested. Let S denote the current spot market price of a bill with a face value of \$1 million. Then, the discount yield is calculated as

$$\text{Yield} = [(1,000,000 - S)/1,000,000](360/\text{Days}),$$

where *Days* refers to the maturity of the bill. As with other money market rates, calculation of the discount yield on Treasury bills assumes a 360-day year.

⁴ The Treasury auctions 13- and 26-week bills each Monday (except for holidays and special situations) and issues them on the following Thursday; 52-week bills are auctioned every four weeks. Auctions for one-year bills are held on a Thursday and the bills are issued on the following Thursday.

Treasury bill futures prices are quoted as an index determined by subtracting the discount yield of the deliverable bill (expressed as a percentage) from 100:

$$\text{Index} = 100 - \text{Futures Discount Yield}.$$

Thus, a quoted index value of 95.25 implies a futures discount yield for the deliverable bill of $100 - 95.25 = 4.75$ percent. This convention was adopted so that quoted prices would vary directly with changes in the future delivery price of the bill.

Final Settlement Price The final settlement price, also known as the delivery price or invoice cost of a bill, can be expressed as a function of the quoted futures index price using the formulas given above. For a bill with a face value of \$1 million, the resulting expression is

$$S = \$1,000,000 - \$1,000,000(100 - \text{Index})(0.01)(\text{Days}/360),$$

where $(100 - \text{Index})(0.01)$ is just the annualized futures discount yield expressed as a decimal. The CME determines the days to maturity used in this formula by counting from the first scheduled contract delivery date, regardless of when actual delivery takes place. This means that calculation of the invoice cost is based on an assumed 91-day maturity, except in special cases where holidays interrupt the regular Treasury bill auction and delivery schedules.

To illustrate, suppose that the final index price of a traded contract is 95.25 and the deliverable bill has 91 days to maturity as of the first scheduled delivery date. Then, the final delivery price would be

$$\$987,993.06 = \$1,000,000 - \$1,000,000(0.0475)(91/360).$$

Minimum Price Fluctuation The minimum price fluctuation permitted on the trading floor is 1 basis point, or 0.01 percent. Thus, the price of a Treasury bill futures contract may be quoted as 95.25 or 95.26, but not 95.255. The exchange values a 1 basis point change in the futures price at \$25. Note that this valuation assumes a 90-day maturity for the deliverable bill.

Three-Month Eurodollar Time Deposit Futures

Three-month Eurodollar futures are traded actively on three exchanges at present. The IMM was first

to list a three-month Eurodollar time deposit futures contract in December of 1981. Futures exchanges in London and Singapore soon followed suit by listing similar contracts. The London International Financial Futures Exchange (LIFFE) introduced its three-month Eurodollar contract in September of 1982, while the Singapore International Monetary Exchange (SIMEX) introduced a contract identical to the IMM contract in 1984. A special arrangement between the IMM and SIMEX allows for mutual offset of Eurodollar positions initiated on either exchange. Thus, a trader who buys a Eurodollar futures contract at the IMM can undertake an offsetting sale on SIMEX after the close of trading at the IMM.⁵ The Tokyo International Financial Futures Exchange began listing a three-month Eurodollar contract in 1989, but that contract is not traded actively at present. The IMM contract remains the most actively traded of the different Eurodollar contracts by a wide margin.

The IMM Eurodollar contract is the first futures contract traded in the United States to rely exclusively on a cash settlement procedure. Contract settlement is based on a "notional" principal amount of \$1 million, which is used to determine the change in the total interest payable on a hypothetical underlying time deposit. The notional principal amount itself is never actually paid or received.

Expiration months for listed contracts are March, June, September, and December. A maximum of 20 contracts are listed at any one time, making the furthest available delivery date 60 months in the future.

Contract Settlement When a futures contract contains provisions for physical delivery, market forces cause the futures price to converge to the spot market price as the delivery date draws near. Actual delivery of the underlying item never takes place with a cash-settled futures contract, however. Instead, the futures exchange forces the process of convergence to take place by setting the final settlement price equal to the spot market price prevailing at the end of the last day of trading. Final settlement is achieved by marking the contract to market one last time based on the final settlement price determined by the exchange.

Price Quotation Eurodollar time deposits pay a fixed rate of interest upon maturity. The rate of

⁵ See Burghardt et al. (1991) for a more detailed discussion of the LIFFE and SIMEX contracts.

interest paid on the face amount of such a deposit is termed an add-on yield because the depositor receives the face amount of the deposit plus an explicit interest payment when the deposit matures. Like other money market rates, the add-on yield for Eurodollar deposits is expressed as an annualized rate based on a 360-day year. Eurodollar futures prices are quoted as an index determined by subtracting the futures add-on yield from 100.

Final Settlement Price Contract settlement is based on the 90-day London Interbank Offered Rate (LIBOR), which is the interest rate at which major international banks with offices in London offer to place Eurodollar deposits with one another. To determine the final settlement price for its Eurodollar futures contract, the CME clearinghouse randomly polls a sample of banks active in the London Eurodollar market at two different times during the last day of trading: once at a randomly selected time during the last 90 minutes of trading, and once at the close of trading. The four highest and lowest price quotes from each polling are dropped and the remaining quotes are averaged to arrive at the LIBOR used for final settlement.

To illustrate the settlement procedure, suppose that the closing price of a Eurodollar futures contract is 96.10 on the day before the last trading day. As with Treasury bill futures, each 1 basis point change in the price of a Eurodollar futures contract is valued at \$25. Thus, if the official final settlement price is 96.16, then all traders who carry open long positions from the previous day have \$150 (\$25 per basis point x 6 basis points) credited to their margin accounts while traders with open short positions from the previous day have \$150 subtracted from their accounts. Since the contract is cash settled, traders with open positions when the contract matures never bear the responsibility of placing or accepting actual deposits.

Minimum Price Fluctuation The minimum price fluctuation permitted on the floor of the exchange is 1 basis point, which, as noted above, is valued at \$25.

One-Month LIBOR Futures

One-month LIBOR futures began trading on the IMM in 1990. The one-month LIBOR contract resembles the three-month Eurodollar contract described above, except that final settlement is based on the 30-day LIBOR.

Table 1

Three-Month Interest Rate Futures: Contract Specifications

<u>Contract</u>	<u>Three-Month Treasury Bill</u>	<u>Three-Month Eurodollar Time Deposit</u>
Exchange	International Monetary Market Division of the Chicago Mercantile Exchange	International Monetary Market Division of the Chicago Mercantile Exchange
Contract Size	\$1,000,000	\$1,000,000
Delivery Requirements	U.S. Treasury bills with 13 weeks to maturity	Cash settlement with clearing corporation
Delivery Months	March, June, September, December	March, June, September, December
Price Quotation	Index: 100 minus discount yield	Index: 100 minus add-on yield
Minimum Price Fluctuation	\$25 per basis point	\$25 per basis point
Last Day of Trading	One day before first delivery date	Second London business day before the third Wednesday of the delivery month
Delivery Days	Three successive business days beginning with the first day of the contract month on which a 13-week bill is issued and an original-issue one-year bill has 13 weeks left to maturity	Last day of trading

Table 2

One-Month Interest Rate Futures: Contract Specifications

<u>Contract</u>	<u>One-Month LIBOR</u>	<u>Thirty-Day Interest Rate</u>
Exchange	International Monetary Market Division of the Chicago Mercantile Exchange	Chicago Board of Trade
Contract Size	\$3,000,000	\$5,000,000
Delivery Requirements	Cash settlement	Cash settlement
Delivery Months	First five consecutive months starting with current month	First seven calendar months and the next two months in the March, June, September, December trading cycle following the spot month
Price Quotation	Index: 100 minus the LIBOR for one-month Eurodollar time deposits	Index: 100 minus the monthly average federal funds rate
Minimum Price Fluctuation	\$25 per basis point	\$41.67 per basis point
Last Day of Trading	The second London bank business day immediately preceding the third Wednesday of the contract month	The last business day of the delivery month
Delivery Days	Last day of trading	Last day of trading

Contract Settlement Like the three-month Euro-dollar contract, the one-month LIBOR contract is cash settled. Settlement is based on a notional principal amount of \$3 million.

Price Quotation and Minimum Price Fluctuation

Prices on one-month LIBOR futures are quoted as an index virtually identical to that used for three-month Eurodollar futures. The index is calculated by subtracting the 30-day futures LIBOR from 100. The minimum price increment is 1 basis point, which is valued at \$25.

Final Settlement Price As with the three-month Eurodollar contract, the final settlement price for one-month LIBOR contract is based on the results of a survey of primary market participants in the London Eurodollar market.

Thirty-Day Interest Rate Futures

The Chicago Board of Trade's 30-day interest rate futures contract is a cash-settled contract based on a 30-day average of the daily federal funds rate. The CBT lists contracts for six consecutive delivery months at any one time.

Contract Settlement The 30-day interest rate futures contract differs from other interest rate futures in that the settlement price is based on an average of *past* interest rates. Final settlement is based on an arithmetic average of the daily federal funds rate for the 30-day period immediately preceding the contract maturity date, as reported by the Federal Reserve Bank of New York. The notional principal amount of the contract is \$5 million.

Price Quotation As with all other money market futures, prices for 30-day interest rate futures are quoted as an index equal to 100 minus the futures rate. For deferred month contracts—that is, contracts maturing after the current month's settlement date—the futures rate corresponds approximately to a forward interest rate on one-month term federal funds.

In theory, the futures rate for the nearby contract should reflect a weighted average of (1) the average funds rate for the expired fraction of the current month, plus (2) the term federal funds rate for the unexpired fraction of the month. To illustrate, suppose the date is April 21. Twenty days of the month have passed, so the index value for the April contract would reflect

$$100 - \text{Index} = (20/30)(\text{average of the daily federal funds rate for the previous 20 days}) \\ + (10/30)(\text{term federal funds rate for 10 days beginning April 21}).$$

At the same time, the price of the May contract would correspond approximately to the forward rate on a 30-day term federal funds deposit beginning May 1. The correspondence to the 30-day rate is only approximate, however, because the settlement price for the contract is based on a simple arithmetic average, which does not incorporate daily compounding.

Minimum Price Fluctuation The minimum price fluctuation is 1 basis point, valued at \$41.67.

Trading Activity in Money Market Futures

Charts 1 and 2 display a history of trading activity in the four money market futures contracts discussed above. Chart 1 displays total annual trading volume, which is a count of the total number of contracts (not the dollar value) traded for all delivery months. Each transaction between a buyer and a seller counts as a single trade. Chart 2 plots total month-end open interest for all contract delivery months. Month-end open interest is a count of the number of unsettled contracts as of the end of the last trading day of each month. Each contract included in the open interest count reflects an outstanding futures commitment on the part of both a buyer and a seller.

Trading activity in the Treasury bill futures contract grew steadily from the time the contract was first listed in 1976 through 1982, falling thereafter below 20,000 contracts per day on average. The trading history depicted in Charts 1 and 2 suggests that the introduction of the Eurodollars futures contract attracted some trading activity away from Treasury bill futures.

In recent years, the IMM Eurodollar futures contract has become the most actively traded futures contract based on money market rates and is now one of the most actively traded of all futures contracts. Three factors have contributed to the popularity of Eurodollar futures. First, most major international banks rely heavily on the Eurodollar market for short-term funds and act as marketmakers in Eurodollar deposits. Eurodollar futures provide a means of hedging interest rate risk arising from these activities. Second, the phenomenal growth of the market for interest rate swaps during the last decade

Chart 1

AVERAGE MONTH-END OPEN INTEREST

U.S. Money Market Futures

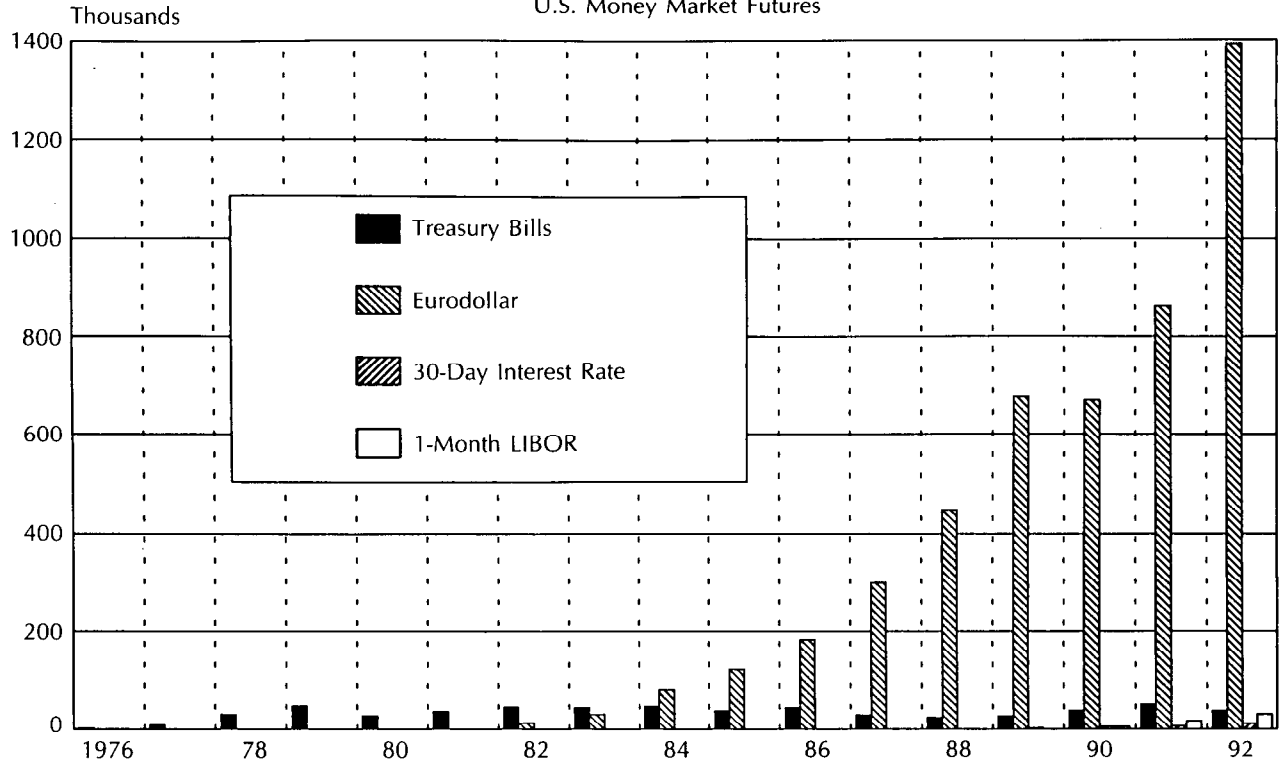
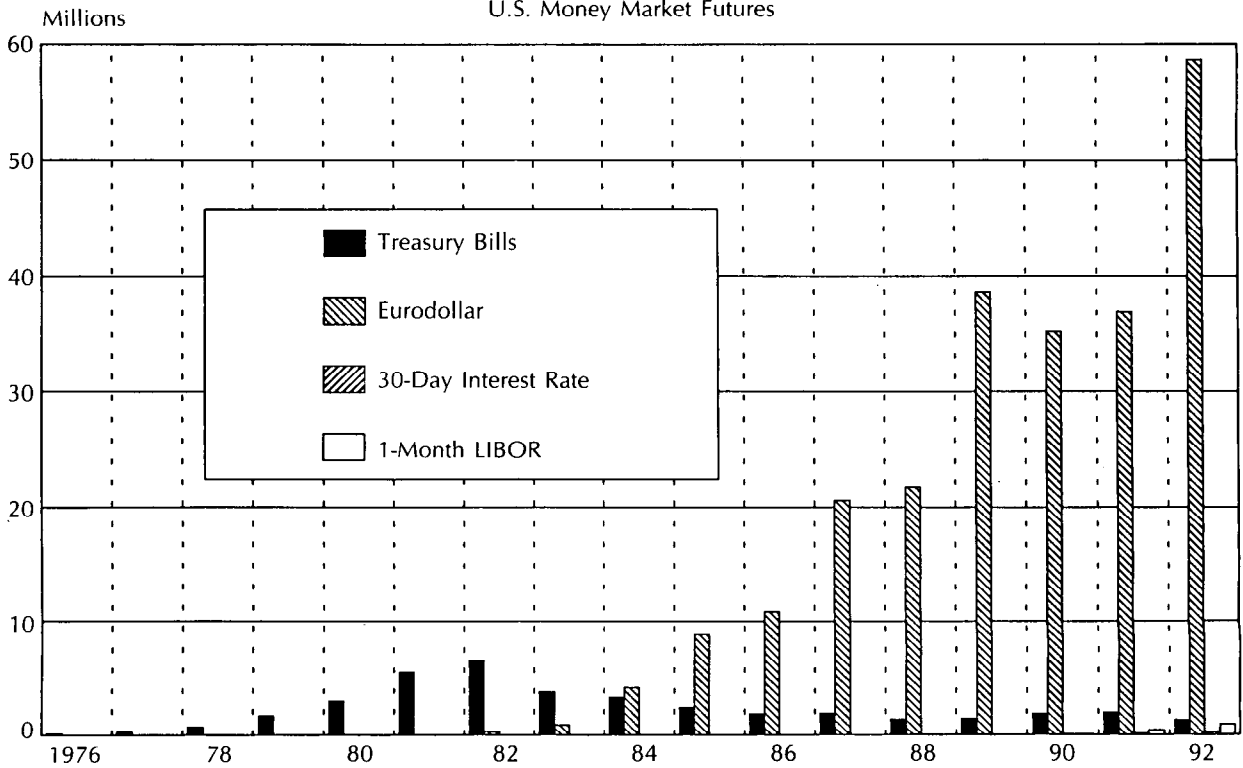


Chart 2

TOTAL ANNUAL TRADING VOLUME

U.S. Money Market Futures



has contributed to the growth of trading in Eurodollar futures.⁶ Most interest rate swap contracts specify payments contingent on three- or six-month LIBOR. Swap market dealers sometimes use Eurodollar futures to hedge their positions in interest rate swaps. Evidence of the widespread use of Eurodollar futures to hedge swap exposures can be found in the fact that the IMM currently lists Eurodollar futures with delivery dates stretching as far as 60 months into the future. In contrast, virtually all other futures contracts list delivery dates only 24 months into the future (Burghardt et al., 1991). Third, it has become common practice for commercial banks to index interest rates on loans to their corporate customers to LIBOR. Such borrowers sometimes use Eurodollar futures to hedge their borrowing costs. (In recent years, however, it has become more common for such borrowers to arrange interest rate swaps.)

The one-month LIBOR contract enables traders to use futures contracts to synthesize maturities corresponding to a wider range of standard maturities in the Eurodollar market. Other than overnight and one-week deposits, standard maturities in the Eurodollar market range from one to six months in one-month increments, nine months, one year, eighteen months, and two to five years in one-year increments. The one-month LIBOR contract allows a trader to synthetically duplicate the interest rate exposure associated with a four-month Eurodollar deposit, as an example, using a combination of a three-month Eurodollar contract and a one-month LIBOR contract. Although trading in the contract has been active since it was introduced in 1990, Charts 1 and 2 show that trading activity in the one-month LIBOR contract has yet to approach that of the more popular three-month Eurodollar futures contract.

The CBT first listed its 30-day interest rate futures contract in 1988. Although there are differences in the way the one-month LIBOR and 30-day interest contracts are priced, both are based on indexes of one-month interbank lending rates. At present, trading volume in the CBT contract is roughly one-third the volume of trading in the one-month LIBOR contract. Past experience has shown that whenever two different exchanges list futures contracts for similar underlying instruments, only one contract survives. Thus, the current outlook for these latter two contracts remains uncertain as of this writing.

⁶ An interest rate swap is a formal agreement between two parties to exchange cash flows based on the difference between two different interest rates.

PRICE RELATIONSHIPS BETWEEN FUTURES AND SPOT MARKETS

Price relationships between futures and spot markets can be explained using arbitrage pricing theory, which is based on the premise that two different assets, or combinations of assets, that yield the same return should sell for the same price. Buying a futures contract on the final day of trading is equivalent to buying the underlying item in the cash market, since delivery is no longer deferred once a futures contract matures. Thus, arbitrage pricing theory predicts that the futures price of an item should just equal its spot market price on the futures contract maturity date: this is just the phenomenon of convergence noted earlier. Buying a futures contract before the contract maturity date fixes the cost of future availability of the underlying item. But the cost of future availability of an item can also be fixed in advance by buying and holding that item. Holding actual physical stocks of a commodity or security entails opportunity costs in the form of interest foregone on the funds used to purchase the item and, in some instances, explicit storage costs. The cost associated with financing the purchase of an asset, along with related storage costs, is known as the cost of carry. Since physical storage can substitute for buying a futures contract, arbitrage pricing theory predicts that the cost of carry should determine the relationship between futures and spot market prices.

Basis and the Cost of Carry

The cost of carry for agricultural and other commodities includes financing costs, warehousing fees, transportation costs, and any transactions costs incurred in obtaining the commodity. Storage costs are negligible for financial assets such as Treasury bills and Eurodollar deposits. Moreover, financial assets often yield an explicit payout, such as interest or dividend payments, that offsets at least a fraction of any financing costs. The convention in financial markets, therefore, is to apply the term net carrying cost to the difference between the interest cost associated with financing the purchase of a financial asset and any explicit interest or dividend payments earned on that asset.

Let $S(0)$ denote the purchase price of an asset at time 0 and $r(0, T)$ denote the market rate of interest at which market participants can borrow or lend over a period starting at date 0 and ending at some future date T .⁷ Assuming, for the sake of convenience,

⁷ This discussion assumes perfect capital markets in which market participants can borrow and lend at the same rate.

that transactions and storage costs are negligible, the cost of purchasing an item and storing it until date T is just the financing cost $r(0, T)S(0)$. Let $y(0, T)$ denote any explicit yield earned on the asset over the same holding period. Then, the net carrying cost for the asset is

$$c(0, T) = [r(0, T) - y(0, T)]S(0).$$

Since physical storage of an item can substitute for buying a futures contract for that item, arbitrage pricing theory would predict that the futures price should just equal the price of the underlying item plus net carrying costs. This result is known as the cost of carry pricing relation. Let $F(0, T)$ denote the futures price of an item at date 0 for delivery at some future date T . Then, the cost of carry pricing relation can be stated formally as:

$$F(0, T) = S(0) + c(0, T).$$

The difference between the spot price of an item and its futures price is known as basis.⁸ Notice that the cost of carry pricing relationship equates basis with the negative of the cost of carry. This relationship is easily demonstrated by rearranging terms in the cost of carry relation to yield

$$S(0) - F(0, T) = -c(0, T).$$

Positive carrying costs imply a negative basis—that is, a futures price above the spot market price. In such instances the buyer of a futures contract pays a premium for deferred delivery, known as contango.

Cash-and-Carry Arbitrage

To see why futures prices should conform to the cost of carry model, consider the arbitrage opportunities that would exist if they did not. Suppose the futures price exceeds the cost of the underlying item plus carrying costs; that is,

$$F(0, T) > S(0) + c(0, T).$$

In this case, an arbitrageur could earn a positive profit of $F(0, T) - S(0) - c(0, T)$ dollars by selling the overpriced futures contract while buying the underlying item, storing it until the futures delivery date, and using it to satisfy delivery requirements.

This type of transaction is known as cash-and-carry arbitrage because it involves buying the underlying item in the cash market and carrying it until the

⁸ Some authors define basis as the difference between the futures price and the spot price. The definition adopted above is the more common.

futures delivery date. Ultimately, the market forces created by arbitrageurs selling the overpriced futures contract and buying the underlying item should force the spread between futures and spot prices down to a level just equal to the cost of carry, where arbitrage is no longer profitable. In practice, arbitrageurs rarely find it necessary to hold their positions to contract maturity; instead, they undertake offsetting transactions when market forces bring the spot-futures price relationship back into alignment.

Example 1: Pricing a Commodity Futures Contract Suppose the current spot price of a commodity is \$100 and the market rate of interest is 10 percent. Assuming that transactions and storage costs are negligible, the cost of carry for this commodity for a period of one year is

$$\begin{aligned} c(0, T) &= (0.10)(\$100) \\ &= \$10. \end{aligned}$$

Thus, the fair futures price for delivery in one year is \$110.

Now consider the opportunity for arbitrage if the futures contract in this example is overpriced. If the futures price for delivery in one year's time is \$115, an arbitrageur could earn a certain profit by selling futures contracts at \$115, borrowing \$100 at 10 percent to buy the underlying commodity, and delivering the commodity in fulfillment of contract requirements at the futures delivery date. The total cost of purchasing and storing the underlying commodity for one year is \$110, while the short position in a futures contract fixes the sale price of the commodity at \$115. Thus, at the end of one year the arbitrageur could close out his position by selling the underlying commodity in fulfillment of contract requirements, thereby earning a \$5 profit net of carrying costs.

Example 2: Pricing an Interest Rate Futures Contract Suppose a long-lived asset that pays a 15 percent annual yield can be purchased for \$100, and assume that the cost of borrowing to finance the purchase of this asset for one year is 10 percent. In this case, the \$10 annual financing cost is more than offset by the annual \$15 yield earned on the asset. The net cost of carry for a one-year holding period is

$$(0.10 - 0.15)\$100 = -\$5.$$

Thus, the fair futures price for delivery in one year is \$95.

The net cost of carry is negative in this last example, resulting in a futures price below the spot market price. This type of price relationship is known as backwardation. It is common for interest rate futures prices to exhibit a pattern of backwardation, although this pattern can be reversed when short-term interest rates are higher than long-term rates.

Reverse Cash-and-Carry Arbitrage

If the futures price of an item fails to reflect full carrying costs, arbitrageurs have an incentive to engage in an operation known as reverse cash-and-carry arbitrage. Reverse cash-and-carry arbitrage involves selling the underlying commodity short while buying the corresponding futures contract. A short sale involves borrowing a commodity or asset for a fixed time period and selling that item in the cash market with the intent of repurchasing it when the commodity is due to be returned to the lender.

In the case of a short sale of an interest-bearing security, a lender typically requires the borrower to return the security plus any interest or dividend payments accruing to the security over the period of the loan. Thus, the net profit resulting from a reverse cash-and-carry operation is determined by the proceeds from the short sale, $S(0)$, plus the interest earned on those proceeds over the holding period, $r(0,T)S(0)$, less the cost of repurchasing the security at date T , $F(0,T)$, and less the interest or dividend that would have been earned by holding the security, which is $y(0,T)S(0)$. The total net profit in this case is just

$$[1 + r(0,T) - y(0,T)]S(0) - F(0,T).$$

Banks active in the Eurodollar market can effect short sales of deposits simply by accepting such deposits from other market participants and investing the proceeds until the deposits mature. Dealers in the Treasury bill market can effect short sales through arrangements known as repurchase agreements. These operations are described in more detail below.

The Phenomenon of "Underpriced" Futures Contracts

Futures prices sometimes fail to reflect full carrying costs, a phenomenon that is most pronounced in commodity markets. At least two different explanations have been offered for this phenomenon: the first deals with impediments to short sales; the second with the implicit convenience yield that accrues to physical ownership of certain assets.

Reverse cash-and-carry arbitrage operations require that market participants be able to effect short sales of the item underlying the futures contract so as to take advantage of an underpriced futures contract. Various impediments to short sales exist in some markets, however. In the stock market, for example, government regulations, as well as stock exchange trading rules, limit the ability of market participants to effect short sales.

The importance of such impediments is mitigated by the fact that it is not always necessary to engage in a short sale to effect a reverse cash-and-carry arbitrage operation. Many firms are ideally situated to take advantage of the opportunities presented by underpriced futures contracts simply by selling any inventories they hold while buying futures contracts to fix the cost of buying back the underlying item. Yet market participants often do not sell their asset holdings to take advantage of "underpriced" futures contracts because ready access to actual physical stores of an item can yield certain implicit benefits. For example, a miller might value having a ready supply of grain on hand to ensure the uninterrupted operation of his milling operations. A futures contract can substitute for physical holdings of the underlying commodity in the sense that it fixes the cost of future availability, but the miller cannot use futures contracts to keep his mill operating in the event that he runs out of grain. Supplies of agricultural commodities can be scarce in periods just preceding harvests, making market participants such as commodity processors willing to pay an implicit convenience yield in return for assured access to physical stores of a commodity at such times. A measure of the implicit convenience yield, call it $y_c(0,T)$, can be obtained by calculating the difference between the cost of storage and the futures price:

$$y_c(0,T) = S(0) + c(0,T) - F(0,T),$$

where the term $c(0,T)$ in the above expression represents the explicit carrying cost.⁹

Pricing Treasury Bill Futures: The Implied Repo Rate

A repurchase agreement, more commonly termed a "repo" or "RP," is a transaction involving the sale of a security with a commitment on the part of the seller to repurchase that security at a higher price

⁹ Siegel and Siegel (1990, Chap. 2) contains a good introductory discussion of these topics. See Williams (1986) for a comprehensive analysis of the price behavior of agricultural futures.

on some future date—usually the next day, although such agreements sometimes cover periods as long as six months. A repurchase agreement can be viewed as a short-term loan collateralized by the underlying security, with the difference between the repurchase price and the initial sale price implicitly determining an interest rate, known as the “repo rate.” Repurchase agreements constitute a primary funding source for dealers in the market for U.S. Treasury securities.

Cash-and-carry arbitrage using Treasury bill futures involves the purchase of a bill that will have 13 weeks to maturity on the contract delivery date. A cash-and-carry arbitrage operation can be viewed as an implicit reverse repurchase agreement, which is just a repurchase agreement from the viewpoint of the lender. A reverse repo entails the purchase of a security with a commitment to sell the security back at some future date. A party entering into a reverse repo effectively lends money while taking the underlying security as collateral. Like a party to a reverse repo, a trader who buys a Treasury bill while selling a futures contract obtains temporary possession of the bill while committing himself to sell it back to the market at some future date. Just as the difference between the purchase price of a bill and the agreed-upon sale price determines the interest rate earned by a party to a reverse repo, the difference between the futures and spot price determines the return to a cash-and-carry arbitrage operation. In effect, the trader “lends” money to the market, earning the difference between the future delivery price and the price paid for the security as implicit interest. The rate of return earned on such an operation is known as the “implied repo rate.”

By market convention, the implied repo rate is expressed as the annualized rate of return that could be earned by buying a Treasury bill at a price $S(0)$ at date 0 and simultaneously selling a futures contract for delivery at date T for a price $F(0, T)$. The formula is

$$irr = \{[F(0, T) - S(0)]/S(0)\}(360/T),$$

where irr denotes the implied repo rate. Note that this formula follows the convention in money markets of expressing annual interest rates in terms of a 360-day year.

The following example illustrates the calculation of the implied repo rate. Suppose that it is exactly 60 days to the next delivery date on three-month Treasury bill futures. A bill with 151 days left to maturity will have 91 days left to maturity on the next futures delivery date and can be used to satisfy

delivery requirements for the nearby futures contract. If the current discount yield on bills with 151 days to maturity is 3.8 percent, the cash price of the bill is

$$\begin{aligned} S(0) &= \$1,000,000 \\ &- \$1,000,000(0.038)(151/360) \\ &= \$984,061.11. \end{aligned}$$

Now suppose that the price of the nearby Treasury bill futures contract is 96.25. An index price of 96.25 implies a futures discount yield for the nearby Treasury bill contract of $100 - 96.25 = 3.75$ percent. Since the deliverable bill will have 91 days to maturity, the future delivery price implied by this yield is

$$\begin{aligned} F(0, 60) &= \$1,000,000 \\ &- \$1,000,000(0.0375)(91/360) \\ &= \$990,520.83. \end{aligned}$$

The implied repo rate in this case is

$$\begin{aligned} irr &= [(\$990,520.83 \\ &- \$984,061.11)/\$984,061.11](360/60) \\ &= 0.0394, \end{aligned}$$

or 3.94 percent.

The cost of carry pricing relation can be used to show that the no-arbitrage price should equate the implied repo rate with the actual repo rate. To see this, note that the cost of carry pricing relation implies that the no-arbitrage price must satisfy

$$F(0, T) - S(0) = c(0, T),$$

Although Treasury bills are interest-bearing securities, the interest earned on a bill is implicit in the difference between the purchase and redemption price. This means that $y(0, T) = 0$, so that total net carrying costs for a Treasury bill must just equal

$$c(0, T) = r(0, T)S(0),$$

where $r(0, T)$ represents the cost of financing the purchase of the bill, expressed as an unannualized interest rate. Substituting these last two expressions into the definition of the implied repo rate gives

$$irr = r(0, T)(360/T).$$

Because repurchase agreements constitute a primary funding source for dealers in the Treasury bill market, $r(0, T)$ should reflect the cash repo rate.¹⁰ Thus, the

¹⁰ Gendreau (1985) found empirical support for the assertion that the repo rate provides the correct measure of carrying costs for Treasury bill futures.

cost of carry pricing relation implies that the implied repo rate should just equal the cash repo rate.

Comparing implied repo rates with actual rates amounts to comparing theoretical futures prices, as determined by the cost of carry model, with actual futures prices. An implied repo rate above the actual three-month repo rate would indicate that futures contracts are relatively overpriced. In this case arbitrage profits could be earned by borrowing money in the cash repo market and implicitly lending the money back out through a cash-and-carry arbitrage to earn the higher implied repo rate.

Conversely, an implied repo rate below the actual rate would indicate that futures contracts are underpriced. In this second case, arbitrageurs would have an incentive to "borrow" money by means of a reverse cash-and-carry futures hedging operation while lending into the cash market through a reverse repo. Such an operation would entail buying an underpriced futures contract and simultaneously entering into a reverse repurchase agreement to lend money into the cash repo market.

The concept of an implied repo rate can also be applied to other types of financial futures. Merrick (1990) and Siegel and Siegel (1990) discuss other applications.

Pricing Eurodollar Futures

Now consider the problem of determining the theoretically correct price of a three-month Eurodollar futures contract maturing in exactly 90 days. Note that a six-month deposit can be viewed as a succession of two three-month deposits. Thus, a bank can synthesize an implicit six-month deposit by placing a three-month deposit and buying a futures contract to fix the rate of return earned when the proceeds of the first deposit are reinvested into another deposit. Arbitrage opportunities will exist unless the return to this synthetic six-month deposit equals the return to the actual six-month deposit.

Let $r(0,3)$ and $r(0,6)$ denote the current (unannualized) three- and six-month LIBOR, respectively. Eurodollar deposits pay a fixed rate of interest over the term of the deposit. For maturities under one year, interest is paid at maturity. Thus, an investor placing \$1 in a 180-day deposit in an account paying an interest rate of $r(0,6)$ receives $\$[1 + r(0,6)]$ at maturity. Similarly, a 90-day deposit will return $[1 + r(0,3)]$ per dollar at maturity. Now let $r_f(3,6)$ denote the interest rate on a three-month deposit to

be placed in three months fixed by buying a Eurodollar futures contract. The condition that a six-month deposit should earn as much as a succession of two three-month deposits requires that

$$1 + r(0,6) = [1 + r(0,3)][1 + r_f(3,6)].$$

The no-arbitrage futures interest rate can thus be calculated from the other two spot rates by rearranging terms to yield

$$r_f(3,6) = [1 + r(0,6)]/[1 + r(0,3)] - 1.$$

As an example, suppose the prevailing three-month LIBOR is quoted at 4.0 percent and the six-month LIBOR at 4.25 percent (in terms of annualized interest rates). Suppose further that the six-month rate applies to a period of exactly 180 days and the three-month rate applies to a period of 90 days. Finally, assume that the nearby Eurodollar contract conveniently happens to mature in exactly 90 days. Then, the no-arbitrage interest rate on a three-month Eurodollar deposit to be made three months in the future is

$$\begin{aligned} r_f(3,6) &= [1 + (0.0425)(180/360)] / \\ &\quad [1 + (0.04)(90/360)] - 1 \\ &= 0.0111. \end{aligned}$$

To express this result as an annualized interest rate just multiply the number obtained above by (360/90). The result is

$$r_f(3,6)(360/90) = 0.0444,$$

which means that the no-arbitrage futures interest rate in this example is 4.44 percent and the theoretically correct index price is 95.56. The same methodology can be used to price one-month LIBOR futures.¹¹

If the futures rate is below the no-arbitrage rate, the interest rate on a synthetic six-month deposit will be less than on an actual six-month deposit. A bank can effect a cash-and-carry arbitrage operation by "buying" a six-month deposit now (that is, by placing a deposit with another bank) while accepting a three-month deposit and selling a Eurodollar futures contract. In this case, arbitrage amounts to lending at the higher spot market rate (by placing a six-month deposit with another bank) while borrowing at the

¹¹ Readers interested in a more detailed exposition of forward interest rate calculations and the pricing of Eurodollar futures should see Burghardt et al. (1991).

lower synthetic six-month rate (obtained by accepting a three-month deposit and selling a futures contract).

Conversely, a futures interest rate above the theoretically correct rate is a signal for banks to enter into a reverse cash-and-carry arbitrage. In this case, a bank would wish to accept a six-month deposit to borrow at the lower spot market rate while placing a three-month deposit and buying the nearby futures contract to lend at the higher synthetic six-month rate.

Daily Settlement and the Cost of Carry

As a concluding comment, it should be noted that the pricing formulas developed in this section do not take account of the effect of variation margin flows. When interest rates fluctuate randomly, the fact that a futures contract is marked to market on a daily basis means that some of the payoff to a futures position will need to be reinvested at different interest rates. Thus, the cost of carry formulas derived above hold exactly only if interest rates are constant or if there are no variation margin payments, as typically is the case with forward agreements (Cox, Ingersoll, and Ross, 1981). In all other cases, the formulas derived above yield theoretical futures prices that only approximate true theoretical futures prices. As an empirical matter, however, the approximation appears to be a close one, so that the cost of carry model is commonly used to price futures contracts as well as forward contracts.¹²

THE ECONOMIC FUNCTION OF FUTURES MARKETS

Hedging, Speculation, and Futures Markets

It is common to categorize futures market trading activity either as hedging or speculation. In the most general terms, a futures hedging operation is a futures market transaction undertaken in conjunction with an actual or planned spot market transaction. Futures market speculation refers to the act of buying or selling futures contracts solely in an attempt to profit from price changes, and not in conjunction with an ordinary commercial pursuit. According to these definitions, then, a dentist who buys wheat futures in anticipation of a rise in wheat prices would be classified as a speculator, while a grain dealer undertaking a similar transaction would be regarded as a hedger.

¹² Chance (1989) reviews the results of studies dealing with the effect of variation margin payments on futures prices.

Speculators have been active participants in futures markets since the earliest days of futures trading. On several occasions, the perception that futures market speculation exerted a destabilizing influence on commodity markets led to attempts to restrict or ban futures trading.¹³ But despite the association of speculative activity with futures trading, it is widely accepted that futures markets evolved primarily in response to the needs of commodity handlers, such as dealers in agricultural commodities and processing firms, who used futures contracts in conjunction with their routine business transactions. The same types of market forces appear to underlie the recent growth of trading in financial futures, the heaviest users of which are financial intermediaries such as commercial banks, securities dealers, and investment funds that routinely use futures contracts to hedge cash transactions in financial markets.

While it is widely accepted that futures markets evolved to facilitate hedging, the motivation behind observed hedging behavior in futures markets has been the topic of considerable debate among economists. Risk transfer traditionally has been viewed as the primary economic function of futures markets. According to this view, the economic purpose of futures markets is to provide a means for transferring the price risk associated with owning an item to someone else. A number of economists have come to question this traditional view in recent years, however, arguing that the desire to transfer price risk cannot fully explain why market participants use futures contracts.

The discussion that follows examines the hedging uses of money market futures and reviews three different views of the economic function of futures markets in an effort to provide some insight into the reasons firms use futures markets. All three theories are based on the premise that futures markets evolved to facilitate hedging on the part of firms active in underlying spot markets, but the different theories each emphasize different characteristics of futures contracts and futures markets to explain why hedgers use futures contracts. This review is of more than academic interest. Futures hedging operations are complex and multifaceted transactions, and each

¹³ One of the most drastic efforts to curb futures trading involved the arrest of nine prominent members of the Chicago Board of Trade following the enactment of the Illinois Elevator Bill of 1867. The act classified the sale of contracts for future delivery as gambling except in cases where the seller actually owned physical stocks of the commodity being sold. Those provisions were soon repealed, however, and the exchange members never came to trial (Hieronymus, 1971, Chap. 4).

theory provides important insights into different aspects of hedging behavior.

Future Markets as Markets for Risk Transfer

In conventional usage, the term "hedging" refers to an attempt to avoid or lessen the risk of loss by matching a risk exposure with a counterbalancing risk, as in hedging a bet. A futures hedge can be viewed as the use of futures contracts to offset the risk of loss resulting from price changes. A short (cash-and-carry) hedging operation, for example, combines a short futures position with a long position in the underlying item to fix the future sale price of that item, thereby protecting the hedger from the risk of loss resulting from a fall in the value of his holdings. Reverse cash-and-carry arbitrage, which combines a short position in an item with a long futures position, represents an example of a long hedge. The long futures position offsets the risk that the price of the underlying item might rise before the hedger can buy the item back to return to the owner. More generally, a long hedge combines a long futures position with a planned future purchase of an item to produce an offsetting risk that protects the hedger from the risk of an increase in the future purchase price of the item.

Most textbook hedging examples rely on this traditional definition of hedging to motivate descriptions of hedging operations. Thus, a dealer in Treasury securities might sell Treasury bill futures to offset the risk that an unanticipated change in market interest rates will adversely affect the value of his securities holdings. Note that the short hedge in this example effectively shortens the maturity of the interest-bearing asset being hedged. In contrast, a long hedge fixes the return on a future investment, thereby lengthening the effective maturity of an existing interest-earning asset.

This traditional definition of hedging accords with the view that the primary function of futures markets is to facilitate the transfer of price risk. The party buying the futures contracts in the above example might be an investor planning to buy Treasury bills at some future date or a speculator hoping to profit from a decline in market interest rates. In the first case the risk exposure is transferred from one hedger to another who faces an opposite risk. In the second, the risk is willingly assumed by the speculator in the hope of earning windfall gains.

Other common hedging operations involving money market futures can also be viewed as being

motivated by the desire to transfer price risk. For example, commercial banks, savings and loans, and insurance companies use interest rate futures to protect their balance sheets and future earnings from potentially adverse effects of changes in market interest rates.¹⁴ In addition, nonfinancial firms sometimes use interest rate futures to fix interest rates on anticipated future investments and borrowing rates on future loans.

The Liquidity Theory of Futures Markets

Working (1962) and Telser (1981, 1986) contend that the hedging behavior of firms cannot be understood by looking at risk avoidance alone as the primary motivation for hedging. Instead, they argue that the hedging behavior of optimizing firms is best understood when hedging is viewed as a temporary, low-cost alternative to planned spot market transactions. According to this line of reasoning, futures markets exist primarily because they provide market participants with a means of economizing on transactions costs, and not solely because futures contracts can be used to transfer price risk. Williams (1986) has termed this view the liquidity theory of futures markets.

Working's and Telser's arguments rest on the observation that market participants need not use futures contracts to insure themselves against price risk. As noted in the earlier discussion on arbitrage pricing, spot purchases (or short sales) of an item can substitute for buying (selling) a futures contract to fix the cost of future availability (future sale price) of an item. Moreover, forward contracts can also be used to transfer price risk. Because they can be custom-tailored to the needs of a hedger, forward contracts would appear to offer a better means of insuring against price risk than futures contracts. Contract standardization, while contributing to the liquidity of futures markets, practically insures that futures contracts will not be perfectly suited to the needs of any one hedger.¹⁵ It would seem, then, that a hedger interested solely in minimizing price risk would have little incentive to use futures contracts,

¹⁴ Brewer (1985) and Kaufman (1984) discuss the problem such firms face in managing interest rate risk.

¹⁵ Since planned transaction dates rarely coincide with standardized futures delivery dates, most hedgers must unwind their futures positions before the contracts mature. As a result, hedging operations must rely upon the predictability of the spot-futures price relationship, or basis. Although theory predicts that behavior of basis should be determined by the cost of carry, changes in the spot-futures price relationship are not always predictable in practice. Thus, a futures hedge involves "basis risk," which is much easier to avoid with forward contracts.

a conclusion which suggests that the view of futures markets as markets for transferring price risk is incomplete.

Although it makes futures contracts less suited to insuring against price risk, contract standardization, along with the clearinghouse guarantee, facilitates trading in futures contracts and reduces transactions costs. By focusing attention on these characteristics of futures contracts, Working (1962) and Telser (1981, 1986) are able to explain why dealers and other intermediaries who perform the function of marketmakers in spot markets tend to be the primary users of futures contracts. Market-making activity requires dealers to constantly undertake transactions that change the composition of their holdings. Securities dealers, for example, must stand ready to buy and sell securities in response to customer orders. As they do, their cash positions change continually, along with their exposure to price risk. Similarly, the assets and liabilities of commercial banks change continually as they accept deposits and offer loans to their customers. Thus, financial intermediaries such as commercial and investment banks hedge using futures contracts because the greater liquidity and lower transactions costs in futures markets mean that a futures hedge can be readjusted frequently with relatively little difficulty and at minimal cost.

To illustrate these concepts, consider the situation faced by an investor who holds a three-month Treasury bill but wishes to lengthen the effective maturity of his holding to six months. The investor could sell the three-month bill and buy a six-month bill, or he could buy a futures contract for a three-month Treasury bill deliverable in three months. A long hedging operation of this type effectively converts the three-month bill into a synthetic six-month bill. The preferred strategy will depend on the relative costs of the two alternatives. Since transactions costs in futures markets tend to be lower than those in underlying spot markets, the futures hedge is often the more cost-effective alternative.

Futures Markets as Implicit Loan Markets

Williams (1986) argues that futures markets are best viewed as implicit loan markets, which exist because they provide an efficient means of intermediating credit risk. Recall that a firm that needs to hold physical inventories of some item for a fixed period has two choices. First, it can make arrangements to borrow the item directly, often by pledging some form of collateral such as cash or securities to secure the loan. Second, it can buy the item in

the spot market and hedge by selling the appropriate futures contract. In either case, the firm will have temporary use of the item and will be required to return (deliver) that item at some set future date. Depending on one's view, therefore, a short hedger is either extending a loan of money collateralized by the item underlying the futures contract or borrowing the underlying commodity using cash as collateral.

A natural question to ask at this juncture is why a firm would choose to engage in a cash-and-carry hedging operation to synthesize an implicit loan of an item rather than borrowing the item outright. The answer lies with the advantages that futures contracts have in the event of default or bankruptcy. Consider the consequences of a default on the part of a firm that loans out securities while borrowing cash. Suppose firm A enters into a repurchase agreement with firm B. If firm A defaults on its obligations, firm B cannot always be assured that the courts will permit it to keep the security collateralizing the loan because the "automatic stay" provisions of the U.S. Bankruptcy Code may prevent creditors from enforcing liens against a firm that enters into bankruptcy proceedings. Thus, when Lombard-Wall, Inc. entered bankruptcy proceedings in 1982, its repurchase agreement counterparties could neither use funds obtained through a repurchase agreement or sell underlying repo securities without first obtaining the court's permission (Lumpkin, 1993). In such cases, creditors may be forced to settle for a fraction of the amounts owed them.

Subsequent amendments to the U.S. Bankruptcy Code have clarified the steps needed to perfect a collateral interest in securities lending arrangements, making it possible for investors to avoid many of the difficulties Lombard-Wall's counterparties encountered with the Bankruptcy Court. Nevertheless, collateralized lending agreements are never riskless. A party to a reverse RP, for example, faces the risk that the market value of the underlying security might fall below the agreed-upon repurchase price. Moreover, parties to mutual lending arrangements sometimes fraudulently pledge collateral to several different creditors. In either case, a lender is exposed to the risk of loss in the event of a default on the part of a borrower. Finally, the Bankruptcy Code amendments do not apply to all types of lending. For example, Eurodollar deposits cannot be collateralized under existing banking laws.

The consequence of a default is quite different when a firm uses futures contracts to synthesize an

implicit loan. Because synthesizing a loan through the use of futures contracts involves no exchange of principal, the risk exposure associated with a futures contract in the event of a default is much smaller than the exposure associated with an outright loan. Thus, a futures hedging operation amounts to a collateralized lending arrangement in which the collateral is never at risk in the event of a default. A position in a futures contract does create credit-risk exposure when changes in market prices change the value of the contract; however, the resulting exposure is a small fraction of the notional principal amount of the contract, and the exchange clearinghouse risks losing only the change in value in the futures contract resulting from price changes in the most recent trading session. Here, daily settlement, or marking to market, of futures contracts provides an efficient means of enforcing contract performance. In the event that a firm fails to meet a margin call, the clearinghouse can order its futures position to be liquidated and claim the firm's margin deposit to offset any losses accruing to the futures position. If the defaulting firm subsequently enters formal bankruptcy proceedings, the futures margin is exempt from the automatic stay imposed by the Bankruptcy Code.¹⁶ Thus, a futures clearinghouse is entitled to seize a trader's margin deposit to offset any trading losses without being required to first appeal to the Bankruptcy Court.

Although forward contracts can also be used to synthesize implicit loans in much the same way as futures contracts, Williams (1986) argues that a crucial difference between futures contracts and forward agreements lies with their legal status in the event of default and bankruptcy proceedings. While forward agreements sometimes specify margin deposits, such deposits have not, until very recently, been exempt from the automatic stay provisions of the Bankruptcy Code.¹⁷

¹⁶ Williams (1986) cites a precedent-setting legal decision that exempted margin deposits from the automatic stay provisions of the Bankruptcy Code.

¹⁷ Recent amendments to the Bankruptcy Code exempt margin deposits on certain types of forward contracts from the automatic stay. See Gooch and Pergam (1990) for a detailed description of these amendments.

These observations led Williams to conclude that futures markets are best viewed as markets for intermediating short-term loans, which resemble money markets. Although Williams' rationale for the existence of futures markets differs in emphasis from that of Working and Telser, the two theories are not inconsistent. While Williams acknowledges that futures markets have certain advantages over other markets stemming from greater liquidity and lower transactions costs, he argues that Working and Telser place too much importance on contract standardization and transactions costs as primary reasons for the existence of futures markets. In the end, however, both theories question the traditional view that the primary function of futures markets is to accommodate the transfer of price risk.

Since Williams published his work, the Bankruptcy Code has been amended to exempt certain repurchase agreements and forward agreements from the automatic stay provisions applicable to most other liabilities of bankrupt firms. As a result, such contracts now have a legal status similar to that of futures contracts in the event of bankruptcy. Williams' theory would thus predict that repurchase agreements and forward contracts should become more widely used, which is what has happened in recent years. Rather than replacing futures contracts, however, the growth of over-the-counter derivatives such as interest rate swaps and Forward Rate Agreements appears to be driving an accompanying increase in trading in futures contracts, especially Eurodollar futures, which derivatives dealers use to hedge their swap and forward contract exposures. Thus, even when forward agreements and collateralized lending arrangements carry the same legal status as futures contracts in the event of a default, each type of contract appears to offer certain advantages to different types of users. Still, Williams' research highlights an important aspect of futures contracts and futures markets not addressed by earlier work in this area.

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