Credit Rationing by Loan Size in Commercial Loan Markets

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I. INTRODUCTION

Ample evidence exists suggesting that banks ration credit with respect to loan size.[1] For example, Evans and Jovanovic (1989) find evidence of loan size rationing in data from the National Longitudinal Survey of Young Men.[2] Further, the Federal Reserve Board's quarterly Survey of Terms of Bank Lending consistently indicates that the average interest rate charged on commercial loans (i.e., the rate per dollar lent) is inversely related to loan size. This evidence suggests two questions. First, why might loan size rationing occur? Second, why might loan size rationing have the particular interest rate and loan size pattern reported in the Survey of Terms of Bank Lending? Economists generally believe that higher average interest rates are charged on smaller loans because small borrowers are greater credit risks or because loan administration costs are being spread over a smaller base. This paper presents a counterexample to that belief. It shows that, even if credit risk and loan administration costs are the same for all borrowers, a lender with market power and imperfect information about borrowers' characteristics still will offer quantity-dependent loan interest rates of exactly the type reported in the Survey of Terms of Bank Lending.[3]

The quantity-dependent loan interest rates that we derive are a form of second-degree price discrimination. Price discrimination is said to occur in a market when a seller offers different units of a good to buyers at different prices. This type of pricing is commonly used by private firms, governments and public utilities. For example, many firms have "bulk rate" pricing schemes, whereby they offer lower marginal rates for large quantity purchases. The income tax rates in the U.S. federal income tax schedule depend on the level of reported income; higher marginal tax rates are levied on higher-income taxpayers. In addition, the price per unit of electricity often depends on how much is used.

Both market power—a firm's ability to affect its product's price—and imperfect information regarding borrowers' characteristics are essential for producing the loan size-interest rate patterns observed in commercial loan markets.[4] To see why, suppose that a lender has market power and perfect information about borrowers' loan demand. In this case, we would observe first-degree (or "perfect") price discrimination: the lender would charge each borrower the most he/she is willing to pay and would lend to all that are willing to pay at least the marginal cost of the loan. Suppose instead that a lender has imperfect information and operates in a competitive market. Milde and Riley (1988, p. 120) have shown that such a lender may not ration credit, even if borrowers can send the lender a signal about their characteristics.

In this paper, we provide an explicit analysis of the information aspects of price discrimination in

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[1] Jaffee and Stiglitz (1990) present alternative definitions of "credit rationing." Broadly defined, credit rationing occurs when there exists an excess demand for loans because quoted interest rates differ from those that would equate the demand and supply of loans.

[2] This evidence is contrary to most recent theoretical models of credit rationing. That literature derives loan *quantity* rationing, whereby some borrowers obtain loans while other observationally identical borrowers do not, in the spirit of Stiglitz and Weiss (1981). While some quantity rationing does occur, the evidence suggests that size rationing is more common.

[3] Of course, the theory we will present is not inconsistent with differential credit risk and loan administration costs, although these factors are not necessary to obtain the observed interest rate-loan size pattern.

[4] See Jaffee and Modigliani (1960) for an early distinction between types of price discrimination and credit rationing.

commercial loan markets. We interpret a lender's price discrimination with respect to loan size as a form of credit rationing that limits borrowing by all but the largest borrowers. Further, because we show that such credit rationing arises from rational, profit-maximizing lender behavior, our analysis has normative implications. We find that small borrowers are more credit constrained than large borrowers and thus bear a larger share of the distortion induced by the market imperfections.[5] In the next section, we describe a simple prototype economy with a single lender and many different types of borrowers about whom the lender has limited information. We then present the lender's profit-maximization problem. Section III follows, describing the loan size-total repayment schedule that solves the lender's problem and explaining why the solution involves credit rationing with respect to loan size. Section IV concludes.

II. A Simple Model Economy

Consider an endowment economy with a single lender that may be thought of either as a local monopolist or as a price leader in the industry.[6] Suppose also that there are n types of borrowers, where n is a positive and finite number. There are N_i borrowers of each type i ($i = 1,\ldots,n$) who live for only two periods. The borrowers may be thought of as privately owned firms that differ only with respect to their fixed endowments of physical good.[7] All firms have the same first-period endowment: $w_1^i = 0$ for all i;[8] however, higher-index firms have larger second-period endowment: $w_2^{i+1} > w_2^i$. In addition, each firm's second-period endowment is positive and known with certainty at the beginning of the first period.

We assume that the welfare of each type i firm (i.e., borrower) is represented by a utility function, $u(x_1^i, x_2^i)$, where x_t^i is the amount of period t good consumed by the owner of the firm, for $t = 1,2$. The utility function $u(\bullet)$ indicates the satisfaction that the owner gets from various combinations of consumption in the two time periods. We assume that the owner's utility function is twice differentiable, strictly increasing and strictly concave. These mathematical properties imply that the owner prefers more consumption to less and prefers relatively equal levels of consumption in the two time periods. We also assume that x_t^i is a normal good, which means that owner i's demand for good x increases with his/her income. Given these assumptions and the endowment pattern specified, all firms will borrow in the first period and higher-index firms will be larger borrowers.[9]

The economy's single lender wishes to maximize profit, which is the difference between revenues (i.e., funds received from loan repayments) and costs (funds lent). Assume that the lender's capital at time 1, measured in units of physical good, is sufficient to support its lending policy, and suppose that the following information restriction exists: the lender and all borrowers know the utility function, the endowment pattern, and the number of borrowers of each type, but cannot identify the type of any individual borrower. Thus, a borrower's type is private information. This information restriction prevents perfect price discrimination by the lender but allows for the possibility of imperfect discrimination via policies that result in borrowers correctly sorting themselves into groups by choosing the loan package designed for their type.[10] Finally, we assume that borrowers are unable to share loans.

The lender's problem is to choose a total repayment (i.e., principal plus interest) schedule for period 2, denoted by $P(q)$, such that any firm that borrows amount q in period 1 must repay amount P in period 2. Let $R_i(q)$ denote the reservation outlay for loans of size q by a type i borrower; that is, $R_i(q)$ indicates

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[5] Price discrimination in loan markets is facilitated by banks' use of "base rate pricing" practices: banks quote a prime rate (the base) and price other loans off that rate. With a base rate pricing scheme, banks price loans competitively for large borrowers with direct access to credit markets, while they act as price-setters on loans to smaller borrowers. Goldberg (1982, 1984) finds substantial evidence for such pricing practices.

[6] The changing of the prime rate has been interpreted by banking industry insiders as an example of price leadership and called "the biggest game of follow-the-leader in American business" [Leander (1990)].

[7] This interpretation is consistent with Prescott and Boyd (1987), which models the firm as a coalition of two-period lived agents with identical preferences and endowments; the coalition in our model consists of only one agent.

[8] We assume $w_1^i = 0$ for simplicity to guarantee that firms borrow in the first period.

[9] Because endowment patterns are deterministic, there is no default risk in this model if the lender induces each type of borrower to self-select the "correct" loan size-interest rate package. We will specify self-selection constraints to ensure that all agents prefer the "correct" package. Consequently, we obtain price discrimination in the form of quantity discounts despite the absence of differences in default risk across borrowers.

[10] With complete information about borrowers' endowments, the lender would use perfect price discrimination, offering each borrower a loan at the highest interest rate the borrower would willingly pay.

the maximum amount a type i borrower is willing to pay at time 2 for a time 1 loan of size q. Let $R_i'(q)$ denote the derivative of $R_i(q)$, which is the inverse demand for loans of size q. The inverse demand curve gives, for each loan size q, the total repayment amount that the lender must request for the borrower to choose that particular loan size. Further assume that the lowest-index group borrows nothing ($q_0 = 0$) and that the reservation value from borrowing zero is zero for all groups [$R_i(0) = 0$]. The lender's two-period profit-maximization problem can now be stated as follows:

$$\max_{(q_1,P(q_1)),\ldots,(q_n,P(q_n))} \sum_{i=1}^{n} N_i[P(q_i)-q_i] \tag{1}$$

subject to

$$R_i(q_i)-P(q_i) \geq R_i(q_j)-P(q_j)$$
$$\text{for all i and all } j \neq i. \tag{2}$$

Equation (1) is the lender's profit function, which is the aggregate amount repaid at time 2 by all borrowers (i.e., the lender's total revenue) minus the aggregate amount lent at time 1 (i.e., the lender's total cost). Equation (2) summarizes constraints for all types of borrowers that would induce a borrower of type i to willingly select a loan of size q. These constraints indicate that borrower i's gain from choosing a loan of size q_i [the left-hand side of (2)] must be at least as great as the gain received from choosing a loan of some other size q_j [the right-hand side of (2)]. If (2) is satisfied, then only a type i borrower would prefer a loan of size q_i with total repayment $P(q_i)$. By choosing loan size q_i, a type i borrower reveals his/her type to the lender. Thus, the lender's two-period problem is to choose an amount to lend at time 1, q_i, and a total repayment schedule for time 2, $P(q_i)$, for every type of borrower.

III. Properties of the Optimal Solution

We can solve the lender's profit-maximization problem as follows. (A formal derivation of the solution appears in the appendix.) When the lender is maximizing profit, equation (2) is satisfied with equality because the lender need only ensure that borrower i is no worse off by selecting loan size q_i instead of any other loan size q_j, $j \neq i$. Using this fact and the assumptions that $q_0 = 0$ and $R_i(0) = 0$, and making successive substitutions into (2), one can show that

$$P(q_i) = \sum_{j=1}^{i} [R_j(q_j)-R_j(q_{j-1})]. \tag{3}$$

Equation (3) gives the lender's profit-maximizing repayment schedule, P(q), for the loan sizes $q_1,\ldots,q_n$.

The profit-maximizing loan sizes now can be determined as follows. Define

$$M_i = \sum_{j=i}^{n} N_j, \; i = 1,\ldots,n,$$

where M_i measures the total number of borrowers of types i through n; thus $M_{n+1} = 0$ because n is the highest endowment group. Substituting (3) into (1), differentiating with respect to q_i and using the definition of M_i yields

$$R_i'(q_i) = \left[\frac{M_{i+1}}{N_i+M_{i+1}}\right] R_{i+1}'(q_i) + \left[\frac{N_i}{N_i+M_{i+1}}\right] \text{ for } i = 1,\ldots,n, \tag{4}$$

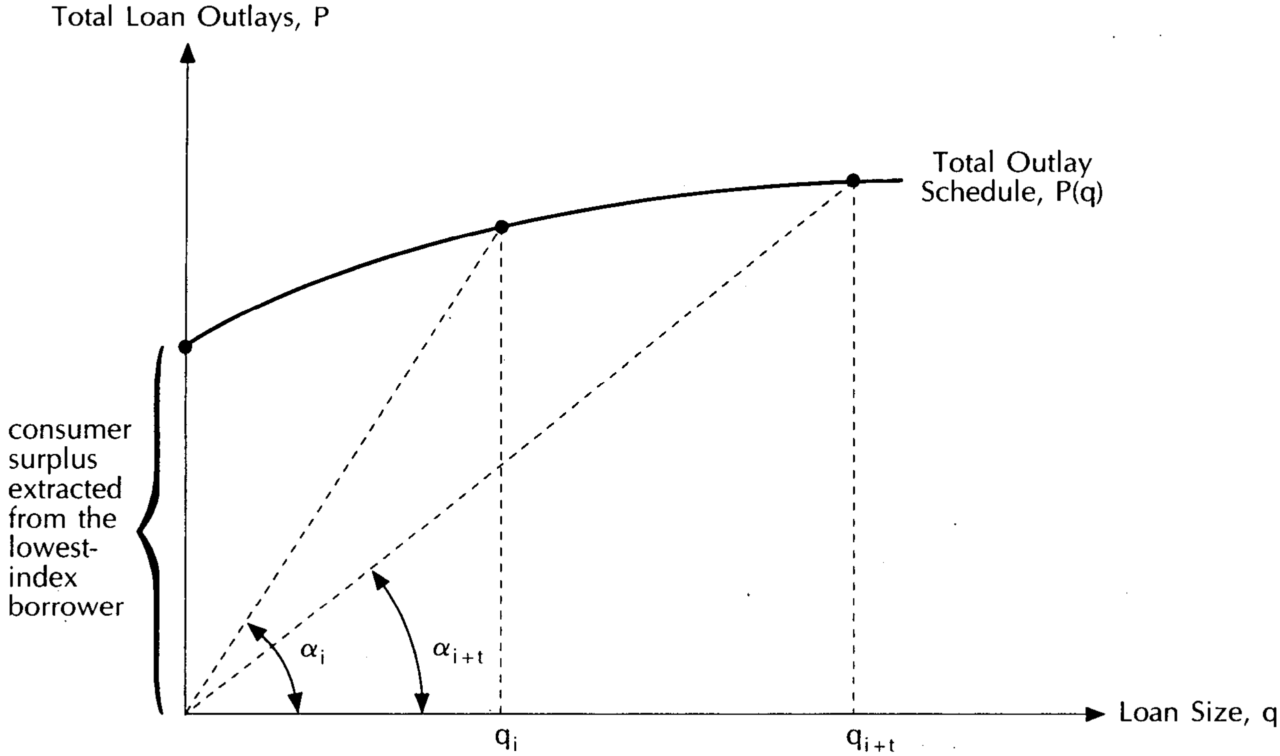
which can be solved for the lender's choice of loan sizes. Thus, equations (3) and (4) together give the solution to the lender's profit-maximization problem. This solution, which takes the form of a quantity-dependent interest rate schedule, is illustrated in Figure 1.

Equation (4), the formula for the optimal loan sizes, has the following properties. It indicates that the loan size, q_i, offered to borrowers of type $i = 1,\ldots,n-1$ is strictly less than the size available in a perfectly competitive market for all groups except the largest. To see why, observe that equation (4) indicates that the profit-maximizing loan size for each group should be chosen so that the implicit marginal value of a loan of size q_i to type i borrowers, $R_i'(q_i)$, equals a weighted average of the implicit marginal value of the loan to the next highest group, $R_{i+1}'(q_i)$, and the marginal cost of lending, which is one. The weights are $M_{i+1}/(N_i + M_{i+1})$ and $N_i/(N_i + M_{i+1})$, respectively. In the perfectly competitive market, the lender instead would equate the loan's marginal value to its marginal cost.

Observe that a profit-maximizing lender will provide the perfectly competitive loan size to the largest borrowers, those in group i = n, because $M_{n+1} = 0$, which implies that $R_n'(q_n) = 1$ for group n. However, for all other borrower types the weight on the first term on the right-hand side of equation (4) is positive. This indicates that the marginal value of a loan to the next highest borrower (i.e., the next highest endowment firm) must be considered if the lender is to maximize profit. Thus, the implicit marginal price of a loan to group

Figure 1

OPTIMAL QUANTITY–DEPENDENT INTEREST RATE SCHEDULE



Note: Unlike a typical demand function, the total outlay schedule in Figure 1 slopes upward. This occurs because the loan outlay schedule, $P(q_i) = p_i q_i$, is the total amount that a borrower pays for a loan of size q_i. In contrast, an ordinary demand function represents the size of a loan requested as a function of price only (p_i). The total outlay schedule in Figure 1 is "quantity-dependent" in the sense that any quantity increase implies a decrease in the average interest rate charged by the lender, $\alpha_i = P(q_i)/q_i$. Thus, in Figure 1 the average interest rate charged on a loan of size q_{i+t} is lower than the average interest rate charged on a (smaller) loan of size q_i. Of course total outlays are higher for the larger loan (q_{i+t}) than the smaller loan (q_i). The average price will be the perfectly competitive price (i.e., a constant, or uniform, per unit price) only when the outlay schedule is a straight line through the origin.

$i = 1,\ldots,n-1$ borrowers exceeds the marginal cost of the loan.

Equation (4) and $M_{n+1} = 0$ indicate that borrowers of type n (those with the largest endowment) clearly obtain the same loan size that they would receive in a perfectly competitive market. However, the degree of credit rationing experienced by borrowers from all other groups, $i = 1,\ldots,n-1$, is regressive (i.e., inversely related to their index). To establish that the pattern of distortion is regressive, we prove in the appendix that our assumptions on preferences and net worth imply that $R'_{i+1}(q_i) > R'_i(q_i)$, which means that higher-index borrowers have a higher implicit value for a loan of size q_i than lower-index borrowers. This result and the restrictions on the distribution of borrower types (i.e., on the N_i) mean that equation (4) implies that low-index (small) borrowers are relatively more constrained than high-index (large) borrowers.[11] This is confirmed by the first term on the right-hand side of equation (4), which is relatively higher for low-index groups.[12]

The final result pertains to the welfare properties of the discriminatory price and quantity scheme given

[11] See Spence (1980, p. 824) for a discussion of constraints on the distribution of consumer types.

[12] For example, suppose $N_i = 10$ for all borrower groups. Further, consider an economy with only two different borrower groups, $i = 1,2$. Let $M_2 = 0.1$ and $M_3 = 0.9$. Then clearly $M_2/(N_1 + M_2) = 0.1/10.1$, which exceeds $M_3/(N_2 + M_3) = 0.9/10.9$, showing that the implicit marginal price of the loan to group 1 is higher than the implicit marginal price to group 2; the marginal cost is one in both cases. This pricing pattern is a general feature of the policy.

by equations (3) and (4). For any single price different from marginal cost, there is a discriminatory outlay schedule that benefits, or at least does not harm, all borrowers and the lender without side payments.[13] In other words, if the borrowers and lender were given a choice between (i) any single interest rate policy that differs from the competitive interest rate and (ii) a quantity-dependent array of interest rates, with one rate appropriate for each group, then they would all prefer or at least be indifferent to the latter policy without coercion. This result indicates that there exists some quantity-dependent interest rate policy that makes all individuals at least as well off as any uniform interest rate policy, except for the single rate that prevails in a competitive market.

Two other features of the solution warrant discussion. Because imperfect information prevents perfect price discrimination, the lender must ensure that the loan size-interest rate package designed for each group satisfies equation (2). The ordering of loan sizes so that $q_i \geq q_{i-1}$ for all i, which is illustrated in Figure 1, is necessary for this constraint to be satisfied. This condition states that the lender must offer loans to high-index (i.e., large-endowment) borrowers that are at least as large as those offered to low-index borrowers. Further, $P(q)/q$ is weakly decreasing in q, which indicates that large borrowers pay lower average interest rates than small borrowers; the declining sequence of α_i in Figure 1 illustrates this. These features of the solution stem from the lender's need to ensure that each group selects the "correct" loan size-interest rate package. The lender must make the selection of a small loan undesirable for high-index borrowers. It does this by allowing the average interest rate to fall with loan size, thus letting larger borrowers keep some of their gains from trade. The lender must also ensure that small borrowers do not select loans designed for large borrowers. Such loan sharing is ruled out by assumption here.

We interpret the preceding results on loan size and interest rate distortions as credit rationing. All but the largest borrowers are prohibited from obtaining loans as large as they would choose if the lender had no market power and all agents had perfect information. Further, the lower a borrower's net worth, the more troublesome (i.e., distorting) the loan size constraints imposed. These theoretical predictions appear to be consistent with the empirical results noted in the introduction. The intuition behind them is as follows. The model consists of numerous borrowers who differ along a single dimension, namely, second-period endowment. The lender has market power and wishes to maximize profit. It knows the distribution of borrower types in the economy, but does not know the identity of any particular borrower. This information restriction prohibits policies such as perfect price discrimination. However, the lender can exploit the correlation of borrowers' market choices with their endowment and does so by offering a discriminatory interest rate schedule that rations loan sizes to all but the largest group. The information implicitly revealed by borrowers' choices allows the lender to partially offset its inability, because of imperfect information about borrower characteristics, to design borrower-specific interest-rate schedules. Thus, the quantity constraints, which we interpret as credit rationing, arise endogenously as an optimal response to the information restriction in an imperfectly competitive market.

IV. CONCLUSION

This paper has presented a theoretical model of a commercial loan market characterized by imperfect information and imperfect competition. The model shows that a profit-maximizing lender operating in such a market will choose to price discriminate (or credit ration) by setting an inverse relationship between the loan sizes offered and the interest rates charged. This loan size-interest rate pattern is consistent with empirical evidence regarding commercial lending. In addition, it is a good example of how, as Friedrich von Hayek argued, the price system can economize on information in a way that brings about desirable results. Hayek (1945, pp. 526-27) noted that "the most significant fact about [the price] system is the economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action." The analysis here shows that a lender with imperfect information about borrower types can set an interest rate schedule that reveals borrowers' characteristics through their borrowing decisions. Interestingly, all loan market participants—the lender and all borrowers—are at least as well off with this discriminatory interest rate schedule as they would be if faced with any uniform interest rate other than the competitive rate.

[13] See Spence (1980, pp. 823-24) for a formal proof.

APPENDIX

We adapt an argument in Villamil (1988) to show that our model is a special case of the widely used Spence nonuniform pricing model. Recall that $R_i(q) = pq$ is the borrowers' reservation outlay function, where p denotes the "reservation interest rate" that a borrower would be willing to pay for a loan of size q. We prove that the assumptions of our model imply reservation outlay functions that satisfy Spence's (1980, pp. 821-22) assumptions. We suppress the q_i and p_i notation because it is unnecessary; indeed, we prove our result for every nonnegative loan amount q. In equilibrium each q is associated with a particular p. Thus, the index i is implicit.

Spence's assumptions are

S.1: Borrower types can be ordered so that for all q, $R_{i+1}(q) > R_i(q)$ and $R'_{i+1}(q) > R'_i(q)$.

S.2: Firms need not borrow, and if they do not, $P(0) = 0$ and $R_i(0) = 0$.

Property S.1 implies that borrowers' reservation outlay schedules can be ordered so that a schedule representing $R_{i+1}(q)$ as a function of q lies above a schedule representing $R_i(q)$ and has a steeper slope. From S.2, it follows that the consumer surplus of a borrower of type i from a loan of size $q \geq 0$, $R_i(q) - P(q)$, is at least as great as the reservation price for purchasing nothing, which is zero. The following proposition shows that our model satisfies these assumptions.

Proposition: The assumptions on preferences and endowments made in Section II imply reservation outlay functions for consumption in excess of endowment in the first period that satisfy S.1 and S.2.

Proof: Let p denote the per unit price of date t+1 good in terms of date t good. Let q denote the amount borrowed, i.e., the amount of first-period consumption in excess of w_1, and let $h_i(p)$ denote the excess demand for first-period consumption by a type i borrower. From the assumptions that $u(\bullet)$ is concave and that consumption is a normal good, $h_i(p)$ is single-valued and decreasing in p where $h_i(p) > 0$. Thus, for all $q \geq 0$, $h_i(p)$ has an inverse that we shall denote by $R'_i(q)$. From the assumptions on preferences and net worth, $h_{i+1}(p) > h_i(p)$, and consequently, $R'_{i+1}(q) > R'_i(q)$ for all $q \geq 0$. Further, letting $R_i(q) = \int_0^q R'_i(z)dz$, we have that $R_{i+1}(q) > R_i(q)$ for all $q \geq 0$. Clearly, S.1 is satisfied. Property S.2 is also satisfied because any borrower can refuse to apply for a loan, in which case his/her repayment obligation and reservation outlay are zero [i.e., $P(0) = R_i(0) = 0$].

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In Search of a Stable, Short-Run M1 Demand Function

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Conventional[1] M1 demand equations went off track at least twice during the 1980s, failing to predict either the large decline in M1 velocity in 1982-83 or the explosive growth in M1 in 1985-86. A number of hypotheses were advanced to explain the prediction errors, but none of these were completely satisfactory.[2] As a result, several analysts have concluded that there has been a fundamental change in the character of M1 demand.

In recent years, some economists have sought to fix conventional M1 demand functions by focusing on specifications that pay adequate attention to the long-run nature and short-run dynamics of money demand. As is well known, conventional money demand functions have been estimated using data either in levels or in differences. Recent advances in time series analysis designed to deal with nonstationary data, however, have raised doubts about either specification. This has led several analysts to integrate these two specifications using cointegration[3] and error-correction techniques. In this approach, one first tests for the presence of a long-run, equilibrium (cointegrating) relationship between real money balances and its explanatory variables including real income and interest rates. If the test for cointegration indicates that such a relationship exists, an equilibrium regression is fit using the levels of the variables. The calculated residuals from the long-run money demand regression are then used in an error-correction model, which specifies the short-run behavior of money demand. This approach thus results in a money demand specification which could include both levels and differences of relevant explanatory variables.[4]

Those who have used cointegration techniques to test for the existence of a long-run, equilibrium M1 demand function, however, have found mixed results. For example, Baum and Furno (1990), Miller (1991), and Hafer and Jansen (1991) do not find a long-run equilibrium relationship between real M1, real income, and a short-term nominal interest rate. Other analysts including Hoffman and Rasche (1991), Dickey, Jansen and Thornton (1991), and Stock and Watson (1991), on the other hand, have presented evidence favorable to the presence of a long-run relationship among these variables.[5]

This study examines whether conventional M1 demand functions reformulated using error-correction techniques can explain the short-run behavior of M1. Much of the recent work on M1 demand has focused on the search for a long-run money demand function. In fact, those economists, who have found

[1] The term *conventional* is meant to indicate those money demand specifications in which the demand for real M1 depends only on real income and short-term interest rates. [For examples, see specifications given in Rasche (1987), Mehra (1989) and Hetzel and Mehra (1989)].

[2] See Rasche (1987), Mehra (1989), and Hetzel and Mehra (1989) for a discussion of various hypotheses and reformulated M1 demand regressions.

[3] Let X_{1t}, X_{2t}, and X_{3t} be three time series. Assume that the levels of these time series are nonstationary but first differences are not. Then these series are said to be cointegrated if there exists a vector of constants (α_1, α_2, α_3) such that $Z_t = \alpha_1 X_{1t} + \alpha_2 X_{2t} + \alpha_3 X_{3t}$ is stationary. The intuition behind this definition is that even if each time series is nonstationary, there might exist linear combinations of such time series that are stationary. In that case, multiple time series are said to be cointegrated and share some common stochastic trends. We can interpret the presence of cointegration to imply that long-run movements in these multiple time series are related to each other.

[4] Miller (1991), Mehra (1992), and Baba, Hendry and Starr (1991), among others, have used this approach to estimate money demand functions.

[5] Sample periods, measures of income and interest rates, tests for cointegration, and estimators of cointegrating vectors used in these studies differ. These factors outwardly appear to explain part of different results found in these studies. However, as shown in Stock and Watson (1991), the main reason for the sensitivity to the sample period and estimator used is the presence of multicollinearity between real income and interest rate in the post-World War II period. The presence of this multicollinearity has made it difficult to get reliable estimates of the long-run money demand parameters. Stock and Watson (1991), however, note that the disappearance since 1982 of the trend in interest rates has reduced the extent of this multicollinearity. This may make it possible to get more reliable estimates of the long-run money demand function over the sample period that includes more of post-1982 observations.

a long-run cointegrating relationship between real M1 and its explanatory variables (like real income and interest rates), either have not constructed error-correction models of money demand or have constructed but failed to evaluate them for parameter stability and for explaining M1's short-run behavior.[6]

This study makes the basic assumption that there exists a long-run equilibrium relationship between real M1, real income, and an opportunity cost variable over the postwar period 1953Q1 to 1991Q2.[7] Under this assumption, error-correction models of M1 demand are constructed, tested for parameter stability, and evaluated for predictive ability. The empirical results indicate that these error-correction models do not depict parameter stability, nor do they adequately explain the short-run behavior of M1 in the 1970s and the 1980s. These results imply that the long-run M1 demand functions postulated here and in several recent M1 demand studies are misspecified. This has the policy implication that M1 remains unreliable as an indicator variable for monetary policy.

The plan of this study is as follows. Section I presents the basic error-correction model, reviews the Engle-Granger test of cointegration, and describes a simple procedure for estimating the error-correction model. Section II presents empirical results. Concluding observations are given in Section III.

I. The Model and the Method

Specification of an M1 Demand Model

The general form of the error-correction money demand model estimated here is given below.

$$\ln(rM1)_t = \beta_0 + \beta_1 \ln(rY)_t + \beta_2 (R - RM1)_t + U_t \qquad (1)$$

$$\Delta\ln(rM1)_t = \delta_0 + \sum_{s=1}^{n1} \delta_{1s} \Delta\ln(rM1)_{t-s} + \sum_{s=0}^{n2} \delta_{2s} \Delta\ln(rY)_{t-s} + \sum_{s=0}^{n3} \delta_{3s} \Delta(R - RM1)_{t-s} + \sum_{s=0}^{n4} \delta_{4s} \Delta^2\ln(p)_{t-s} + \delta_5 U_{t-1} + \epsilon_t, \qquad (2)$$

where rM1 is real M1 balances; rY real income; R a short-term nominal interest rate; RM1 the own rate of return on M1; p the price level; U and ϵ, random disturbance terms; ln the natural logarithm; Δ and Δ^2 the first- and the second-difference operators. Equation (1) is a long-run equilibrium M1 demand equation, which says that the long-run equilibrium demand for real M1 balances depends upon real income and an opportunity cost variable measured as the short-term nominal interest rate minus the own rate of return on M1. The parameter β_1 is the long-run real income elasticity and β_2 the long-run (semi-log) opportunity cost parameter. This equation is consistent with models of the transactions demand for money formulated in Baumol (1952) and Tobin (1956).

The presence of the disturbance term U_t in (1) implies that actual real M1 balances momentarily can differ from the long-run equilibrium value determined by factors specified in (1). Equation (2) describes the short-run behavior of M1 demand and is in a dynamic error-correction form, where δ_{is} (i = 2,3,4) measures the short-run responses of real M1 balances to changes in income, opportunity cost and inflation variables. The parameter δ_5 that appears on the disturbance term U_{t-1} is the error-correction coefficient and measures the extent to which actual real M1 balances adjust to clear disequilibrium in the public's long-term money demand holdings. This can be seen in (3), which is obtained by solving (1) for U_{t-1} and then substituting for U_{t-1} in (2).

$$\Delta\ln(rM1)_t = \delta_0 + \sum_{s=1}^{n1} \delta_{1s} \Delta\ln(rM1)_{t-s} + \sum_{s=0}^{n2} \delta_{2s} \Delta\ln(rY)_{t-s} + \sum_{s=0}^{n3} \delta_{3s} \Delta(R - RM1)_{t-s}$$

[6] Only Hoffman and Rasche (1991) estimate the short-run error-correction model for M1, under the long-run specification that real M1 balances depend upon real income and a short-term interest rate. One important exception is the study by Baba, Hendry and Starr (1991), where the postulated long-run M1 demand function is complicated and differs substantially from that used by others. In particular, they assume that real M1 balances depend upon real income, one-month T-bill rate, the spread between long- and short-term rates, learning-adjusted yields on M1 and M2, and a moving standard deviation of holding period yields on long-term bonds. Given this long-run specification, they estimate an error-correction model for M1 and show that the model is stable over the sample period 1960Q3 to 1988Q3 studied there. The evaluation of this money demand model is outside the scope of the present study.

[7] I do, however, reproduce the mixed evidence found in recent studies on the existence of a long-run M1 demand function.

$$+ \sum_{s=0}^{n4} \delta_{4s} \Delta^2 \ln(p)_{t-s}$$

$$+ \delta_5 [\ln(rM1)_{t-1} - \ln(rM1)^*_{t-1}] + \epsilon_t, \quad (3.1)$$

where

$$\ln(rM1)^*_{t-1} \equiv \beta_0 + \beta_1 \ln(rY)_{t-1} + \beta_2 (R-RM1)_{t-1}. \quad (3.2)$$

One can view $rM1^*$ as the long-term equilibrium real M1 balances, and rM1, of course, is actual real M1 balances. Thus, the term $[\ln(rM1) - \ln(rM1)^*]_{t-1}$ measures disequilibrium in the public's long-term real money balances. If the variables included in (1) are nonstationary but cointegrated, then the error-correction parameter is likely to be non-zero, i.e., $\delta_5 \neq 0$ in (3.1).

Another point to highlight is that equation (3.1) can be viewed as a generalization of the conventional partial-adjustment model, because the approach considered here allows separate reaction speeds to the different determinants of money demand (the coefficients δ_{2s}, δ_{3s}, δ_{4s} and δ_5 are different), yet via the error-correction mechanism ensures that actual real M1 balances converge to equilibrium levels in the long run.

The long-run money demand equation (1) is "conventional" in the sense that real M1 demand is assumed to depend only on real income and an opportunity cost variable. In particular, inflation is assumed to have no long-run effect on money demand. In this respect, the specification used here is similar to ones estimated recently in Dickey, Jansen and Thornton (1991), Hoffman and Rasche (1991), and Stock and Watson (1991). However, following Friedman (1959) the potential long-run influence of inflation on M1 demand is also examined (see footnote 11).

Even if inflation has no long-run effect on money demand, it could still influence real M1 balances in the short run because of the presence of adjustment lags.[8] Hence, the inflation variable appears in the short-run money demand equation (2) and is in first differences rather than in levels. This specification reflects the assumption that inflation is nonstationary. However, the consequences of introducing inflation in levels or dropping it altogether from (2) are also examined (see footnote 18).

Estimation of the Error-Correction Model

If the disturbance term U_t is stationary, then the money demand model described above can be estimated in two alternative ways. The first is a two-step procedure given in Engle and Granger (1987). In the first step, the long-run money demand equation (1) is estimated by ordinary least squares and the residuals are calculated. In the second step, the short-run money demand equation (2) is estimated with U_{t-1} replaced by residuals in step one.

An alternative procedure is to estimate (1) and (2) jointly. This can be seen in (4), which is obtained by substituting (3.2) into (3.1).

$$\Delta\ln(rM1)_t = (\delta_0 - \delta_5\beta_0) + \sum_{s=1}^{n1} \delta_{1s} \Delta\ln(rM1)_{t-s}$$

$$+ \sum_{s=0}^{n2} \delta_{2s} \Delta\ln(rY)_{t-s}$$

$$+ \sum_{s=0}^{n3} \delta_{3s} \Delta(R-RM1)_{t-s}$$

$$+ \sum_{s=1}^{n4} \delta_{4s} \Delta^2\ln(p)_{t-s}$$

$$+ \delta_5 \ln(rM1)_{t-1}$$

$$- \delta_5\beta_1 \ln(rY)_{t-1}$$

$$- \delta_5\beta_2 (R-RM1)_{t-1} + \epsilon_t, \quad (4)$$

where all variables are defined as before. As can be seen, the long- and short-run parameters of the money demand model now appear in (4). All of the key parameters of (1) and (2)—such as those pertaining to income and opportunity cost variables—can be recovered from those of (4). The M1 demand equation here is estimated using the second procedure.[9]

Test for Cointegration: Engle-Granger Procedure

An assumption that is necessary to yield reliable estimates of the money demand parameters is that

[8] The empirical work reported in Goldfeld and Sichel (1987) and Hetzel and Mehra (1989) is consistent with the presence of an inflation effect on money demand in the short run.

[9] The money demand model was also estimated using the first procedure, which generated qualitatively similar results on parameter stability and predictive ability.

the nonstationary variables included in (1) or in (4) are cointegrated as discussed in Engle and Granger (1987). Hence, one must first test for a cointegrating relationship between real M1 balances, real GNP and an opportunity cost variable, i.e., test whether U_t is stationary in (1).

Several tests for cointegration have been proposed in the literature [see, for example, Engle and Granger (1987) and Stock and Watson (1991)]. The test for cointegration used here is the one proposed in Engle and Granger (1987) and consists of two steps. The first tests whether each variable in (1) is nonstationary, which is done performing unit root tests on the variables. (The presence of a single unit root in a series implies that the series is nonstationary in levels but stationary in first differences.) The second step tests for the presence of a unit root in the residuals of the levels regressions estimated using the nonstationary variables. To explain further, assume that $\ln(rM1)_t$, $\ln(rY)_t$ and $(R-RM1)_t$ are nonstationary in levels. In order to test whether these variables are cointegrated, one needs to estimate the following regressions:

$$\ln(rM1)_t = \beta_0 + \beta_1 \ln(rY)_t + \beta_2 (R-RM1)_t + U_{1t}, \qquad (5.1)$$

$$\ln(rY)_t = \beta_3 + \beta_4 \ln(rM1)_t + \beta_5 (R-RM1)_t + U_{2t}, \qquad (5.2)$$

$$(R-RM1)_t = \beta_6 + \beta_7 \ln(rM1)_t + \beta_8 \ln(rY)_t + U_{3t}. \qquad (5.3)$$

If the residuals in any one of these regressions are stationary, then these variables are cointegrated.

Data, Definition of Variables, and Alternative Specifications

The money demand regression (4) is estimated using quarterly data over the period 1953Q1 to 1991Q2. Here rM1 is nominal M1 deflated by the implicit GNP deflator; rY real GNP; p the implicit GNP deflator; R the three-month Treasury bill rate; and RM1 the own rate of return on M1. The variable RM1 is defined as a weighted average of the explicit interest rates paid on the components of M1.[10]

[10] The construction of the own rate on M1 is described in Hetzel (1989).

The opportunity cost variable in (1) is not in logarithms, whereas other variables are. This (semi-log) specification implies that the long-run opportunity cost elasticity varies positively with the level of the opportunity cost variable. I consider an alternative double-log specification in which the opportunity cost variable is also in logarithms. This specification implies that the long-term opportunity cost elasticity is constant. Furthermore, following Hoffman and Rasche (1991), the test for cointegration is also implemented including trend in the long-run part of the model (see the appendix in this paper).

II. Empirical Results

Unit Root Test Results

The unit root tests are performed by estimating augmented Dickey-Fuller regressions of the form

$$X_t = a + \rho X_{t-1} + \sum_{s=1}^{k} b_s \Delta X_{t-s} + n_t, \qquad (6)$$

where X_t is the pertinent variable; n_t a random disturbance term; and k the number of lagged changes in X_t necessary to make n_t serially uncorrelated. If ρ equals one, then X_t has a unit root and is nonstationary. Two statistics are calculated to test the null hypothesis $\rho = 1$. The first is the t-statistic, $t_{\hat\rho}$, and the second is the normalized bias statistic, $T(\hat\rho - 1)$, where T is the number of observations. If these statistics have small values, then the null hypothesis is accepted.

Table 1 reports the unit root test results for the logarithm of real M1, the logarithm of real GNP, the level and the logarithm of the opportunity cost variable $(R-RM1)_t$, and the logarithm of the price level. These results indicate that real M1, real GNP and the opportunity cost variable are nonstationary in levels, but stationary in first differences. (The tests indicate the presence of a single unit root in these variables.) The test results for first differences of the logarithm of the price level, however, are mixed. The t-statistic, $t_{\hat\rho}$, indicates that the inflation variable is nonstationary, whereas the other statistic, $T(\hat\rho - 1)$, indicates that it is stationary.

Cointegration Test Results

Given the unit root test results, the logarithm of real M1, the logarithm of real GNP, and the logarithm (or the level) of opportunity cost are included in the cointegration tests. The inflation rate is not included because unit root test results are ambiguous

Table 1

Unit Root Test Results; 1953Q1–1991Q2

X_t	$\hat{\rho}$	$t_{\hat{\rho}}$	$T(\hat{\rho}-1)$	k	$x^2(1)$	$x^2(2)$	Q(36)
	Augmented Dickey-Fuller Statistics						
$\ln(rM1)_t$	.99	−1.3	−1.6	5	.1	5.2	23.2
$\ln(rY)_t$	.99	−.6	−.2	3	.6	1.1	25.6
$(R-RM1)_t$	.95	−2.2	−7.8	6			
$\ln(R-RM1)_t$	.96	−1.9	−5.8	6	1.1	1.1	19.2
$\ln(p)_t$	1.0	−.1	0.0	5	.6	1.0	28.8
$\Delta\ln(rM1)_t$	.65	−3.6*	−53.1*	6	.9	1.6	18.3
$\Delta\ln(rY)_t$	.32	−6.5*	−104.6*	2	.6	1.1	25.5
$\Delta(R-RM1)_t$	.03	−5.7*	−158.8*	6	.5	.9	27.3
$\Delta\ln(R-RM1)_t$	.00	−6.3*	−153.6*	5	1.4	1.4	19.4
$\Delta\ln(p)_t$	.89	−1.8	−16.3*	4	.5	1.0	28.8

Notes: rM1 is real M1 balances; rY real GNP; R − RM1 the difference between the three-month Treasury bill rate (R) and the own rate on M1(RM1); and p the implicit GNP deflator. RM1 is a weighted average of the explicit rates paid on the components of M1. ln is the natural logarithm and Δ the first-difference operator.

Augmented Dickey-Fuller statistics are from the regression

$$X_t = a + \rho X_{t-1} + \sum_{s=1}^{k} b_s \Delta X_{t-s},$$

where X_t is the pertinent variable; k the number of lagged first differences of X_t included to remove serial correlation in the residuals. $t_{\hat{\rho}}$ is the t-statistic and $T(\hat{\rho}-1)$ the normalized bias statistic. Both are used in the test of the null hypothesis that $\rho = 1$. T is the number of observations used in the regression. k is chosen by the final prediction error criterion given in Akaike (1969). $x^2(1)$ and $x^2(2)$ are Godfrey statistics, which test for the presence of first- and second-order serial correlation in the residuals. Q(36) is the Ljung-Box statistic, which tests for the presence of higher-order serial correlation and is based on 36 autocorrelations.

"*" indicates significant at the 5 percent level. The 5 percent critical values for $t_{\hat{\rho}}$ and $T(\hat{\rho}-1)$ statistics are −2.89 and −13.7, respectively. [See Tables 8.5.1 and 8.5.2 of Fuller (1976).]

about its nonstationarity.[11] Table 2 presents cointegration test results using the Engle-Granger procedure. As can be seen, these test results are mixed. For the semi-log specification, the test results indicate that real M1 balances are cointegrated with real income and interest rates, and this conclusion is not sensitive to the particular normalization chosen, i.e., the choice of the dependent variable in the cointegrating regression (compare results in rows 1 through 3 of Table 2). For the double-log specification, the test results indicate cointegration only if the cointegrating regression is normalized on the interest rate variable (compare results in rows 4 through 6 of Table 2).[12] Despite these mixed results, I proceed under the assumption that real M1 is cointegrated with real income and interest rates over the period studied here.

The Engle-Granger procedure also generates point-estimates of the long-run income and opportunity cost coefficients. For the semi-log specification, the point-estimates of the long-run income elasticity range from .31 to .44 and those for the opportunity cost parameter range from −.03 to −.04. For the

[11] Is the inflation variable, when treated as nonstationary and included in the cointegration regression, statistically significant? In order to answer this question, I estimated, following Stock and Watson (1991), the dynamic version of (1) by ordinary least squares. That is, the cointegrating regression (1) was estimated including, in addition, current, past, and future values of first differences of real income, opportunity cost and inflation variables and the current value of the inflation variable. The estimated coefficient on the current value of the (level) inflation variable is small and not statistically significant. This result indicates that the inflation variable does not enter the cointegrating regression (1). (In contrast, real income and opportunity cost variables were statistically significant.)

[12] This explains why Baum and Furno (1990) and Miller (1991) conclude that real M1 is not cointegrated with real income and interest rates. These authors implement the test for cointegration by estimating the cointegration regression normalized on the M1 variable.

Table 2

Cointegration Test Results: Engle-Granger Procedure

Row #	Dependent Variable	Cointegrating Vector: ln(rY)	Cointegrating Vector: (R − RM1)	Cointegrating Vector: ln(R − RM1)	Augmented Dickey-Fuller Statistics: $t_{\hat{d}}$	k	$x^2(1)$	$x^2(2)$
1	ln(rM1)	.31	−.03		−3.58*	5	.6	3.9
2	ln(rY)	.45	−.04		−3.90*	5	.9	4.5
3	(R − RM1)	.44	−.05		−4.83*	5	.3	2.4
4	ln(rM1)	.36		−.15	−2.57	6	.3	1.2
5	ln(rY)	.52		−.22	−2.89	6	.6	1.1
6	ln(R − RM1)	.53		−.29	−4.98*	5	1.6	1.6

Notes: The left part of the table reports estimates of the long-run income and interest rate coefficients from the cointegrating regressions estimated using alternative dependent variables [see equation (6) in the text]. The right part of the table presents statistics from the augmented Dickey-Fuller (ADF) regression that is used to test for the presence of a unit root in the residuals of the relevant cointegrating regression. The ADF regression is of the form

$$\Delta\hat{U}_t = d\,\hat{U}_t + \sum_{s=1}^{k} b_s\,\Delta\hat{U}_{t-s},$$

where $\hat{U}_t$ is the residual from the relevant cointegrating regression. $t_{\hat{d}}$ is the t-statistic that tests the null hypothesis that $\hat{d}=0$. k is the number of lagged differences of $\hat{U}_t$ in the regression and is chosen by the final prediction error criterion. $x^2(1)$ and $x^2(2)$ are Godfrey statistics, which test for the presence of first- and second-order serial correlation in the residuals of the relevant ADF regression.

"*" indicates significant at the 5 percent level. The 5 percent critical value for $t_{\hat{d}}$ is 3.62 [see Table 3 in Engle and Yoo (1987)].

double-log specification, the ranges for income and opportunity cost elasticities are .36 to .53 and −.15 to −.29, respectively.[13]

Figure 1 shows actual and fitted values from the long-run, semi-log money demand function ($\hat{\beta}_1$ = .44, $\hat{\beta}_2$ = −.05, $\hat{\beta}_0$ = −1.5), whereas Figure 2 shows the same for the double-log version ($\hat{\beta}_1$ = .53, $\hat{\beta}_2$ = −.29, $\hat{\beta}_0$ = −2.11). As can be seen, actual and predicted real money balances do not permanently drift away from each other in the long run. However, over several fairly long intervals actual real money balances persistently differ from the levels predicted by these cointegrating regressions. In order to examine whether such misses can be explained by short-run dynamics, error-correction models are estimated.

Error-Correction M1 Demand Regressions

The results of estimating (4) are reported in Table 3. The opportunity cost variable, (R − RM1), is in levels in Equation A and in logarithms in Equation B. Equations A and B include levels, first differences, and second differences of the pertinent variables and are estimated by ordinary least squares. The estimated regressions look reasonable: all estimated coefficients possess theoretically correct signs and are generally statistically significant. The point-estimates of the long-run GNP elasticity range from .48 to .54. The point-estimate of the long-run opportunity cost elasticity is −.23 in Equation B and −.21 in Equation A; the latter elasticity is calculated as the product of the estimated semi-log opportunity cost parameter (−.04) and the sample mean value of the opportunity cost variable (5.19). These point-estimates of the long-run income and opportunity cost elasticities are close to the estimates generated by the (two-step) Engle-Granger procedure (see Table 2). The hypothesis that the long-run income elasticity is .5 could not be rejected.[14]

[13] The point-estimates of the long-run income and interest rate coefficients are sensitive to the normalization chosen. To explain further, consider the cointegration regression (1). One can re-write (1) as

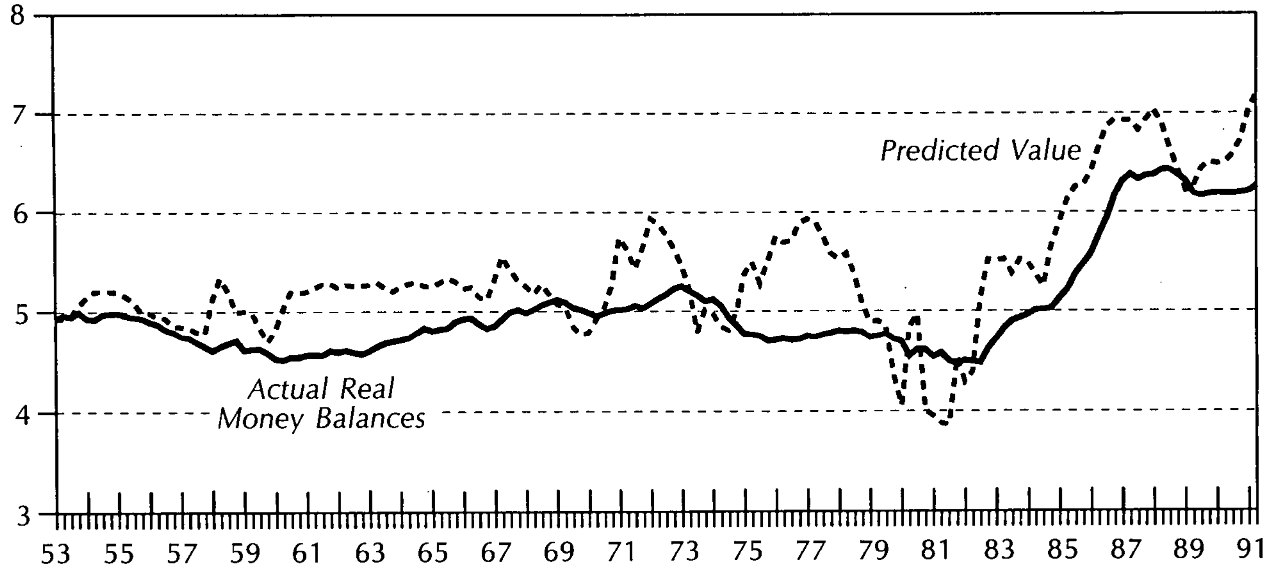
$$\ln(rY)_t = -\beta_0/\beta_1 + (1/\beta_1)\ln(rM1)_t - (\beta_2/\beta_1)(R - RM1)_t,$$

which is the cointegrating regression normalized on the income variable. From this regression, one can recover estimates of the long-run income elasticity β_1 [which is the inverse of the estimated coefficient on $\ln(rM1)_t$] and the long-term interest rate coefficient β_2 [which is the coefficient on $(R-RM1)_t$ divided by the coefficient on $\ln(rM1)_t$]. Another set of point-estimates can be recovered from the cointegration regression normalized on the interest rate variable.

[14] The test of this hypothesis is that the estimated coefficient on $\ln(rY)_{t-1}$ and one-half of the estimated coefficient on $\ln(rM1)_{t-1}$ add up to zero, i.e., $½\,\delta_5 - \delta_5\,\beta_1 = ½\,\delta_5 - ½\,\delta_5 = 0$ in (3). The F-statistic (1,143) that tests the above hypothesis is .09 for Equation A and .08 for Equation B. These F-values are small and indicate that the long-run income elasticity is not different from .5.

Figure 1

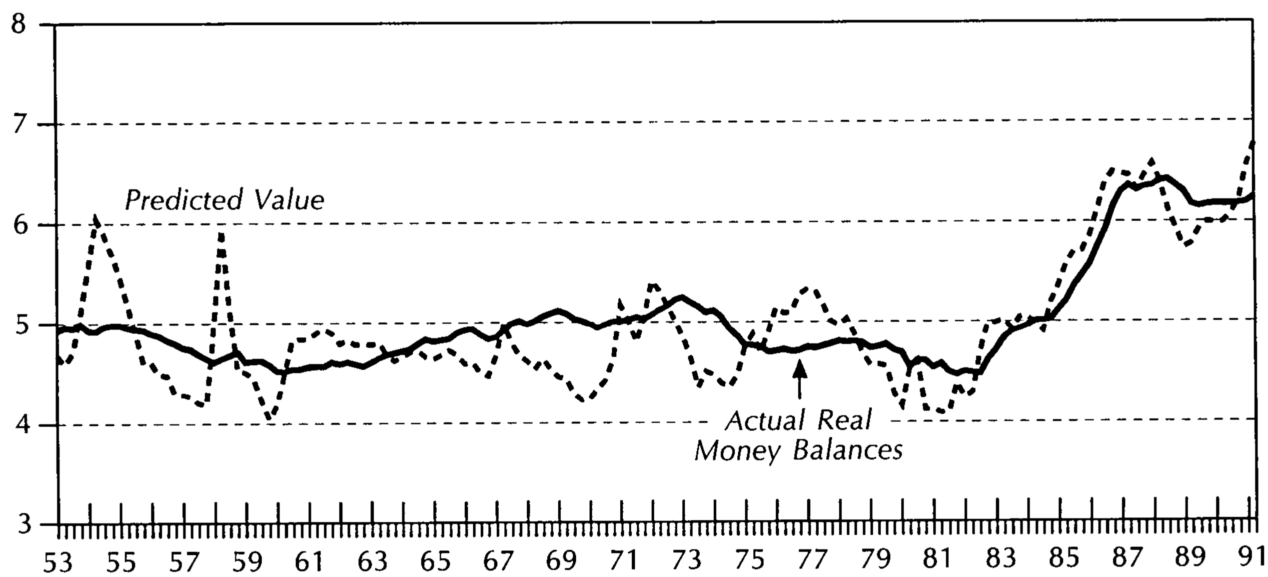
ACTUAL AND PREDICTED VALUES BY THE COINTEGRATING REGRESSION



Cointegrating Regression: ln(rM1) = −1.5 + .44 ln(rY) − .05 (R−RM1)

Figure 2

ACTUAL AND PREDICTED VALUES BY THE COINTEGRATING REGRESSION



Cointegrating Regression: ln(rM1) = −2.11 + .53 ln(rY) − .29 ln(R−RM1)

Table 3

Error-Correction M1 Demand Regressions; 1953Q1–1991Q2

A. Semi-Log Specification

$$\Delta\ln(rM1)_t = \underset{(2.2)}{-.04} \underset{(2.2)}{-.023} \ln(rM1)_{t-1} + \underset{(2.5)}{.011} \ln(rY)_{t-1} - \underset{(2.1)}{.0009} (R-RM1)_{t-1} + \underset{(1.8)}{.11} \Delta\ln(rY)_t + \underset{(5.7)}{.39} \Delta\ln(rM1)_{t-1}$$

$$+ \underset{(3.7)}{.25} \Delta\ln(rM1)_{t-2} - \underset{(0.0)}{.000} \Delta(R-RM1)_t - \underset{(7.9)}{.005} \Delta(R-RM1)_{t-1} - \underset{(6.5)}{.71} \Delta^2\ln(p)_t - \underset{(2.1)}{.26} \Delta^2\ln(p)_{t-1}$$

CRSQ = .68 SER = .00598 DW = 1.96 Q(5) = 3.4 Q(10) = 13.5 N_{ry} = .48 $N_{(R-RM1)}$ = −.04

B. Double-Log Specification

$$\Delta\ln(rM1)_t = \underset{(2.3)}{-.06} - \underset{(2.3)}{.026} \ln(rM1)_{t-1} + \underset{(2.5)}{.014} \ln(rY)_{t-1} - \underset{(2.2)}{.006} \ln(R-RM1)_{t-1} + \underset{(1.7)}{.11} \Delta\ln(rY)_t + \underset{(5.2)}{.39} \Delta\ln(rM1)_{t-1}$$

$$+ \underset{(3.1)}{.24} \Delta\ln(rM1)_{t-2} \underset{(.6)}{-.001} \Delta\ln(R-RM1)_t - \underset{(5.1)}{.023} \Delta\ln(R-RM1)_{t-1} - \underset{(6.0)}{.72} \Delta^2\ln(p)_t - \underset{(2.2)}{.29} \Delta^2\ln(p)_{t-1}$$

CRSQ = .61 SER = .00659 DW = 2.0 Q(5) = 8.5 Q(10) = 16.5 N_{ry} = .54 $N_{\ln(R-RM1)}$ = −.23

Notes: Error-correction regressions are estimated by ordinary least squares. Parentheses contain the absolute value of t-statistics. CRSQ is the corrected R^2; DW the Durbin-Watson statistic; and SER the standard error of regression. Q(5) and Q(10) are Ljung-Box Q-statistics and are based, respectively, on five and ten autocorrelations of the residuals. N_{ry} is the long-term real GNP elasticity and is given by the estimated coefficient on $\ln(ry)_{t-1}$ divided by the estimated coefficient on $\ln(rM1)_{t-1}$. The relevant long-term interest rate coefficient N_{R-RM1} (or $N_{\ln(R-RM1)}$) is given by the coefficient on $(R-RM1)_{t-1}$ [or $\ln(R-RM1)_{t-1}$] divided by the coefficient on $\ln(rM1)_{t-1}$.

Another result to highlight is that the error-correction money demand regressions reported here yield estimates of the long-term opportunity cost (R−RM1) elasticity substantially greater than those given by existing money demand regressions.[15] Hoffman and Rasche (1991), who also use error-correction techniques, report estimates (absolute values) of equilibrium interest elasticities that are of the order .4 to .5 for real M1, versus .21 to .23 reported here.[16]

Evaluating Money Demand Regressions

The money demand regressions reported in Table 3 are now evaluated by examining their structural stability and out-of-sample forecast performance.

The structural stability of these regressions is examined by means of a Chow test, with alternative breakpoints which begin in 1971Q4 and end in 1983Q4 (the start and end dates include periods over which conventional M1 demand functions show instability). The Chow test is implemented using slope dummies on the variables. The restriction that the long-run real GNP elasticity is .5 is imposed. In addition, the stability of the regressions estimated allowing more lags on the explanatory variables than are used in the regressions given in Table 3 is also examined.

Table 4 presents results of the Chow test. F is the F-statistic that tests whether all of the slope dummies plus the one on the constant term are zero. F-statistics for Equations A and B of Table 3 are reported under the columns labeled "Specific." The columns labeled "General" contain results for regressions estimated with more lags on the explanatory variables. As can be seen, the F-values reported there are generally large and thus consistent with the hypothesis that the money demand regressions reported in Table 3 are not stable over the sample period studied.

Equation A of Table 3, which permits varying opportunity cost elasticity, is stable relative to Equation B (compare F-values for Equations A and B under the columns "Specific" in Table 4). This money demand regression depicts parameter stability during the 1970s, but then it breaks down during

[15] For example, a conventional M1 demand equation given in Hetzel and Mehra (1989) was reestimated using data in differences over the period 1953Q1 to 1980Q4. The income elasticity was estimated to be .52 and the opportunity cost elasticity −.04. The estimated income elasticity is close to the value generated using the error-correction model of M1 demand; in contrast, the opportunity cost elasticity is low, i.e., .04 versus .23 given by the error-correction model.

[16] Hoffman and Rasche (1991) do not include the own rate on M1 in defining the opportunity cost variable. This omission could bias upward the coefficient estimated on the interest rate variable and could explain relatively higher estimates of equilibrium interest elasticities reported in their study.

Table 4

Stability Tests; 1953Q1–1991Q2

Breakpoint	Equation A		Equation B	
	General	Specific	General	Specific
	F (26,102)	F (10,134)	F (26,102)	F (10,134)
1971Q4	1.01	1.22	1.91*	4.24*
1972Q4	1.04	1.24	2.09*	4.99*
1973Q4	1.37	1.50	2.75*	5.75*
1974Q4	1.38	1.61	2.46*	5.44*
1975Q4	1.26	.84	2.37*	5.02*
1976Q4	1.53	.76	2.57*	5.17*
1977Q4	1.64*	.88	2.89*	5.84*
1978Q4	1.52	1.09	2.97*	6.05*
1979Q4	1.87*	1.33	2.78*	6.16*
1980Q4	1.89*	1.22	2.51*	3.86*
1981Q4	1.97*	1.74	2.78*	3.19*
1982Q4	1.55	2.05*	1.53*	2.17*
1983Q4	2.00*	2.14*	1.89*	2.24*

Notes: The reported values are the F-statistics that test whether slope dummies when added to Equations A and B are jointly significant. The values reported under the column "Specific" are for Equations A and B reported in Table 3. The values reported under the column "General" are for versions of Equations A and B that are estimated including five lags of first-differenced variables. The breakpoint refers to the point at which the sample is split in order to define the dummies. The dummies take values one for observations greater than the breakpoint and zero otherwise. Parentheses contain degrees of freedom for the F-statistics.

"*" indicates significant at the 5 percent level.

the 1980s. In order to provide a different insight into the timing of predictive failure, I generate out-of-sample predictions of M1 growth conditional on actual values of income and interest rate variables. The predicted values are generated using Equation A of Table 3 and are for forecast horizons one to three years in the future.[17]

The results are reported in Table 5, which contains actual M1 growth as well as prediction errors (with summary statistics) for various forecast horizons. The results presented there suggest two observations. The first is that this regression cannot account for the "missing M1" in 1974-76 and "too much M1" in 1985-86. The explosion in M1 that occurred in 1982-83 is, however, well predicted. The second is that prediction errors do not decline much as the forecast horizon is extended. The root mean squared error (RMSE), which is *2.7* percentage point for one-year horizon, declines slightly to *2.3* percentage point for three-year horizon. This result suggests that short-term misses in M1 are not reversed soon and can persist over periods longer than three years in the future.[18]

The out-of-sample predictions given in Table 5 are further evaluated in Table 6, which presents regressions of the form

$$A_{t+s} = c_0 + c_1 P_{t+s}, \; s = 1,2,3, \tag{7}$$

[17] The forecasts and errors were generated as follows. The money demand model was first estimated over an initial estimation period 1953Q1 to 1970Q4 and then simulated out-of-sample over one to three years in the future. For each of the forecast horizons, the difference between actual and predicted growth was computed, thus generating one observation on the forecast error. The end of the initial estimation period was then advanced four quarters and the money demand function was re-estimated, forecasts generated, and errors calculated as above. This procedure was repeated until it used the available data through the end of 1990.

[18] The short-run M1 demand equations were also estimated excluding inflation or including inflation in levels as opposed to first differences. Such regressions were then examined for their parameter stability and forecast performance. The results were qualitatively similar to those presented in the text. In particular, such M1 demand equations continue to depict parameter instability and fail to explain the weak M1 growth in 1974-76 and the subsequent explosion in 1985-86. The M1 demand equation estimated excluding inflation cannot even explain the explosive growth in 1982-83.

Standard M1 demand equations reported in Hetzel and Mehra (1989) were also estimated and simulated over the updated sample period 1981Q1 to 1991Q2. Such M1 demand regressions continue to underpredict M1 growth in the 1980s.

Table 5

Rolling-Horizon Forecasts of M1 Growth; 1971–1990

	1 Year Ahead			2 Years Ahead			3 Years Ahead		
Year	Actual	Predicted	Error	Actual	Predicted	Error	Actual	Predicted	Error
1971	6.4	9.3	−2.8	—	—	—	—	—	—
1972	8.0	7.3	.7	7.2	8.5	−1.3	—	—	—
1973	5.5	5.4	.1	6.8	5.9	.8	6.7	6.9	−.2
1974	4.7	7.0	−2.3	5.1	6.1	−1.0	6.1	6.3	−.2
1975	4.7	10.5	−5.8	4.7	8.9	−4.3	4.9	7.7	−2.7
1976	5.9	7.7	−1.8	5.3	9.9	−4.5	5.1	8.9	−3.7
1977	7.9	8.7	−.8	6.9	8.4	−1.5	6.2	9.6	−3.4
1978	7.9	7.7	.1	7.9	8.1	−.2	6.7	8.2	−.9
1979	7.0	5.2	1.8	7.4	6.5	.9	7.6	7.1	.5
1980	7.2	4.9	2.2	7.1	5.1	1.9	7.4	6.1	1.3
1981	5.2	3.0	2.2	6.2	3.8	2.4	6.5	4.1	2.3
1982	8.4	7.5	.9	6.8	4.9	1.9	6.9	4.7	2.3
1983	9.9	9.5	.4	9.1	7.7	1.5	7.8	5.8	2.0
1984	5.3	6.0	−.7	7.6	7.7	−.1	7.9	6.9	.9
1985	11.3	7.2	4.1	8.3	6.6	1.7	8.8	7.5	1.3
1986	14.4	8.9	5.4	12.9	6.9	5.9	10.3	6.6	3.7
1987	6.1	11.8	−5.6	10.2	8.1	2.1	10.6	6.9	3.7
1988	4.2	3.9	.3	5.1	9.3	−4.2	8.2	7.3	.9
1989	.6	1.8	−1.2	2.4	3.1	−.7	3.6	6.9	−3.3
1990	4.11	4.7	−.6	2.3	2.9	−.6	2.9	3.5	−.5
Mean Error			−.18			.03			.21
RMSE			2.7			2.5			2.3

Notes: Actual and predicted values are annualized rates of growth of M1 over 4Q to 4Q periods ending in the years shown. The predicted values are generated using money demand Equation A of Table 3 (see footnote 17 in the text for a description of the forecast procedure used). The predicted values are generated under the constraint that the long-run real GNP elasticity is .5.

Table 6

Out-of-Sample Forecast Performance

Error-Correction Equation	1 Year Ahead		2 Years Ahead		3 Years Ahead	
	c_0	c_1	c_0	c_1	c_0	c_1
Semi-Log	2.8	.57	3.9	.43	5.6	.19
(Equation A, Table 3)	(1.7)	(.23)	(1.9)	(.27)	(2.5)	(.32)
Double-Log	2.8	.57	4.1	.39	6.1	.12
(Equation B, Table 3)	(2.1)	(.28)	(2.3)	(.31)	(2.5)	(.34)

Notes: The table reports coefficients (standard errors in parentheses) from regressions of the form $A_{t+s} = c_0 + c_1 P_{t+s}$, where A is actual M1 growth; P predicted M1 growth; and s (= 1,2,3) number of years in the forecast horizon. For Equation A, the values used for A and P are reported in Table 5. For Equation B, the predicted values used are not reported.

where A and P are the actual and predicted values of M1 growth. If these predictions are unbiased, then $c_0 = 0$ and $c_1 = 1$. As can be seen, estimated values of c_1 are less than one and those of c_0 different from zero.[19] These results suggest that the predictions of M1 growth generated by these error-correction models are biased.

III. CONCLUDING OBSERVATIONS

Recent advances in time series analysis designed to deal with nonstationary data have yielded new procedures for estimating long- and short-run econometric relationships. Several analysts have employed these techniques to study M1 demand, and some of them have concluded there exists a long-run equilibrium relationship between real M1, real income, and an opportunity cost variable.

This study also provides evidence consistent with the existence of a stationary linear relationship among these variables. Thus, actual real M1 balances do not drift permanently away from the levels predicted by such cointegrating regressions in the long run. However, in the short run, which can be fairly long, actual real M1 balances differ persistently from the level predicted. The dynamic error-correction models estimated here generally fail the test of parameter stability and do not predict well the short-run changes in M1. In particular, the dynamic models estimated here fail to explain the well-known episodes of "missing M1" in 1974-76 and "too much M1" in 1985-86.[20]

The negative empirical results described above rather suggest that the character of M1 demand has changed in the 1980s. As recently shown in Hetzel and Mehra (1989) and Gauger (1992), the financial innovations of the 1980s caused M1 to become highly substitutable with the savings-type instruments included in M2. Conventional M1 demand equations reformulated here using error-correction techniques yield a high equilibrium interest rate elasticity and thereby capture somewhat better the increase in portfolio substitutions than do the standard (first-differenced) money demand equations. However, the results here suggest that they fail to capture all of the increase in portfolio substitutions. Until that is done, M1 remains unreliable as an indicator variable for monetary policy.

[19] The Ljung-Box Q-statistics (not reported) that test for the presence of higher-order serial correlation in the residuals of (7) were generally small and not statistically significant. This result indicates that the estimated standard errors for coefficients (c_0 and c_1) reported in Table 6 are unbiased.

[20] Additional results presented in the appendix to this paper indicate that these conclusions are robust to some changes in specifications used in the text. In particular, the use of alternative measures of the scale variable and/or the inclusion of trend in money demand regression do not alter qualitatively the results summarized above.

APPENDIX

SENSITIVITY ANALYSIS

Introduction

M1 demand functions reported in the text used real GNP as a scale variable and are estimated without including a linear trend in the long-run part of the model. The results presented there suggested two major conclusions. The first is that the statistical evidence on the existence of a long-run cointegrating relationship among real M1, real income, and a short-term nominal rate is mixed. The second is that short-term M1 demand functions estimated using error-correction techniques depict parameter instability.

This appendix presents additional evidence suggesting that the conclusions stated above are not sensitive to the use of alternative scale measures (such as real personal income or real consumer spending) in money demand equations. Nor do these conclusions change when a linear trend is included in the long-run part of the cointegrating regression. There, however, is one difference. When a linear trend is included in the cointegrating regression, the hypothesis that the long-run real GNP elasticity is unity, not .5, appears consistent with the data. Estimates of the long-run opportunity cost coefficient are, however, unchanged.

Cointegration Test Results: Alternative Scale Measures and Linear Trend

Table A.1 presents cointegration test results with alternative scale variables but with linear trend excluded from cointegrating regressions (as in the text), whereas Table A.2 presents results with linear trend

Table A.1

Cointegration Test Results; Linear Trend Excluded; Different Scale Measures

Row	Dependent Variable	Cointegrating Vector				Augmented Dickey-Fuller Statistics	
		ln(rPY)	ln(rC)	(R−RM1)	$\ln(R-RM1)_t$	$t_{\hat{a}}$	k
1	ln(rM1)	.29		−.03		−3.6*	5
2	ln(rM1)		.29	−.03		−3.6*	5
3	ln(rPY)	.42		−.04		−3.8*	5
4	ln(rC)		.42	−.03		−3.8*	5
5	(R−RM1)	.41		−.05		−4.8*	5
6	(R−RM1)		.41	−.05		−4.8*	5
7	ln(rM1)	.33			−.14	−2.5	6
8	ln(rM1)		.33		−.13	−2.4	6
9	ln(rPY)	.49			−.21	−2.8	6
10	ln(rC)		.48		−.20	−2.6	6
11	ln(R−RM1)	.50			−.29	−4.9*	5
12	ln(R−RM1)		.49		−.27	−3.8*	6

Notes: See notes in Table 2 of the text. rPY is real personal income and rC real consumer spending.

Table A.2

Cointegration Test Results; Linear Trend Included

Row	Dependent Variable	Cointegrating Vector					Augmented Dickey-Fuller Statistics	
		ln(rY)	ln(rPY)	ln(rC)	(R−RM1)	$\ln(R-RM1)_t$	$t_{\hat{a}}$	k
1	ln(rM1)	.61			−.03		−3.4	5
2	ln(rM1)		.85		−.03		−3.2	5
3	ln(rM1)			1.5	−.02		−3.0	5
4	ln(rY)	4.2			−.04		−1.9	3
5	ln(rPY)		4.2		−.04		−1.7	5
6	ln(rC)			3.9	−.02		−1.9	1
7	(R−RM1)	1.02			−.05		−4.6*	5
8	(R−RM1)		1.3		−.05		−4.4*	5
9	(R−RM1)			1.3	−.04		−4.8*	5
10	ln(rM1)	.96				−.17	−3.2	5
11	ln(rM1)		1.2			−.17	−2.6	6
12	ln(rM1)			1.8		−.13	−3.3	5
13	ln(rY)	3.3				−.27	−2.9	5
14	ln(rPY)		3.7			−.26	−2.3	3
15	ln(rC)			3.5		−.14	−3.1	5
16	ln(R−RM1)	1.6				−.29	−5.3*	5
17	ln(R−RM1)		1.9			−.29	−5.3*	5
18	ln(R−RM1)			1.9		−.22	−5.6*	5

Notes: See notes in Table 2 of the text. rY is real GNP; rPY real personal income; and rC real consumer spending.

included in such regressions. The results are presented for alternative scale measures such as real GNP, real personal income, and real consumer spending. As can be seen, these test results indicate cointegration if the test is implemented with cointegrating regressions normalized on the interest rate variable. Otherwise, cointegration test results are sensitive to the particular specification employed. In particular, with cointegrating regressions normalized on real M1, the test results indicate cointegration if linear trend is excluded and if the semi-log specification is employed.

If we focus on specifications which indicate cointegration among real M1, real income (or real consumer spending) and an opportunity cost variable, the resulting point-estimates of the long-run income elasticity are sensitive to the treatment of linear trend. When linear trend is included in cointegrating regressions, it is difficult to reject the hypothesis that the long-term income elasticity is unity. However, when linear trend is excluded, the results instead indicate that the long-term income elasticity is not different from .5. Estimates of the long-term opportunity cost parameter (or elasticity) are not sensitive. In sum, cointegration test results are sensitive to the treatment of linear trend in the nonstationary part of the model and thus provide mixed evidence on the presence of a cointegrating relationship between variables studied here.

Error-Correction M1 Demand Regressions: Tests of Parameter Stability

Despite the mixed evidence on cointegration, error-correction M1 demand regressions were estimated using alternative scale measures and including linear trend in the long-run part of the money demand model. Tables A.3 and A.4 present such regressions for selected measures of income. (In Table A.3, regressions are estimated without including trend and real personal income is used as the income variable. In Table A.4, regressions are estimated including linear trend and real GNP is used as the scale variable. Regressions using other alternative measures considered here are similar and not reported.) As can be seen, estimated regressions look reasonable. The point-estimates of the long-term income elasticity is between 1.04 and 1.09 when linear trend is included in regressions, but is between .44 and .48 if not. The point-estimate of the opportunity cost elasticity, however, is quite robust.

Table A.5 and A.6 present results of implementing the Chow test of stability (as explained in the text). As can be seen, reported regressions do not depict parameter stability over the sample period studied here.

Table A.3

Error-Correction M1 Demand Regressions; Linear Trend Excluded; Real Personal Income as a Scale Variable

C. Semi-Log Specification

$$\Delta\ln(rM1)_t = \underset{(1.1)}{.01} - \underset{(2.2)}{.023}\ln(rM1)_{t-1} + \underset{(2.5)}{.01}\ln(rPY)_{t-1} - \underset{(2.1)}{.001}(R-RM1)_{t-1} + \underset{(2.4)}{.19}\Delta\ln(rPY)_t + \underset{(5.9)}{.40}\Delta\ln(rM1)_{t-1}$$

$$+ \underset{(3.5)}{.24}\Delta\ln(rM1)_{t-2} - \underset{(.8)}{.0005}\Delta(R-RM1)_t - \underset{(7.8)}{.005}\Delta(R-RM1)_{t-1} - \underset{(5.6)}{.64}\Delta^2\ln(p)_t - \underset{(1.8)}{.22}\Delta^2\ln(p)_{t-1}$$

CRSQ = .69 SER = .00589 DW = 2.0 Q(5) = 3.8 Q(10) = 13.3 $N_{rpy} = .44$

$N_{(R-RM1)} = -.22$ [evaluated at the sample mean value of (R − RM1)]

D. Double-Log Specification

$$\Delta\ln(rM1)_t = \underset{(1.1)}{.01} - \underset{(2.5)}{.027}\ln(rM1)_{t-1} + \underset{(2.7)}{.013}\ln(rPY)_{t-1} - \underset{(2.4)}{.006}\ln(R-RM1)_{t-1} + \underset{(3.0)}{.26}\Delta\ln(rPY)_t + \underset{(5.4)}{.40}\Delta\ln(rM1)_{t-1}$$

$$+ \underset{(2.8)}{.21}\Delta\ln(rM1)_{t-2} - \underset{(1.2)}{.005}\Delta\ln(R-RM1)_t - \underset{(5.1)}{.02}\Delta\ln(R-RM1)_{t-1} - \underset{(5.0)}{.62}\Delta^2\ln(p)_t - \underset{(1.7)}{.23}\Delta^2\ln(p)_{t-1}$$

CRSQ = .62 SER = .00648 DW = 2.11 Q(5) = 9.8 Q(10) = 16.9 $N_{rpy} = .48$ $N_{(R-RM1)} = -.22$

Notes: See notes in Table 3 of the text.

Table A.4

Error-Correction M1 Demand Regressions; Linear Trend Included; Real GNP as the Scale Variable

E. Semi-Log Specification

$$\Delta\ln(rM1)_t = \underset{(1.3)}{-.13} - \underset{(2.2)}{.023}\ln(rM1)_{t-1} + \underset{(1.6)}{.024}\ln(rY)_{t-1} - \underset{(2.2)}{.001}\ln(R-RM1)_{t-1} - \underset{(.9)}{.0001}TR_{t-1} + \underset{(1.8)}{.11}\Delta\ln(rY)_t$$

$$+ \underset{(5.8)}{.40}\Delta\ln(rM1)_{t-1} + \underset{(3.8)}{.25}\Delta\ln(rM1)_{t-2} - \underset{(.7)}{.0005}\Delta\ln(R-RM1)_t - \underset{(7.9)}{.006}\Delta\ln(R-RM1)_{t-1}$$

$$- \underset{(6.6)}{.71}\Delta^2\ln(p)_t - \underset{(2.2)}{.26}\Delta^2\ln(p)_{t-1}$$

CRSQ = .68 SER = .00594 DW = 1.98 Q(5) = 3.62 Q(10) = 12.9 N_{ry} = 1.04

$N_{(R-RM1)}$ = −.22 [evaluated at the sample mean value of (R − RM1)]

F. Double-Log Specification

$$\Delta\ln(rM1)_t = \underset{(1.4)}{-.19} - \underset{(2.5)}{.031}\ln(rM1)_{t-1} + \underset{(1.6)}{.034}\ln(rY)_{t-1} - \underset{(2.4)}{.007}\ln(R-RM1)_{t-1} - \underset{(.9)}{.0001}TR_{t-1} + \underset{(1.7)}{.11}\Delta\ln(rY)_t$$

$$+ \underset{(5.2)}{.39}\Delta\ln(rM1)_{t-1} + \underset{(3.1)}{.24}\Delta\ln(rM1)_{t-2} - \underset{(.9)}{.004}\Delta\ln(R-RM1)_t - \underset{(5.0)}{.022}\Delta\ln(R-RM1)_{t-1}$$

$$- \underset{(6.1)}{.74}\Delta^2\ln(p)_t - \underset{(2.3)}{.30}\Delta^2\ln(p)_{t-1}$$

CRSQ = .61 SER = .00659 DW = 2.04 Q(5) = 8.7 Q(10) = 15.6 N_{ry} = 1.09 $N_{(R-RM1)}$ = −.22

Notes: TR is linear trend, and other variables are as defined before. See notes in Table 3 of the text.

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Table A.5

Stability Tests

Breakpoint	Equation C		Equation D	
	General	Specific	General	Specific
	F (26,102)	F (10,134)	F (26,102)	F (10,134)
1971Q4	1.20	1.17	2.63*	4.27*
1972Q4	1.31	1.30	3.03*	5.02*
1973Q4	1.44	1.79	3.21*	5.83*
1974Q4	1.55	1.52	3.12*	5.15*
1975Q4	1.49	1.25	2.99*	5.07*
1976Q4	1.86*	1.39	3.54*	5.33*
1977Q4	2.05*	1.70	3.92*	5.86*
1978Q4	2.15*	1.90*	4.32*	5.80*
1979Q4	1.89*	2.36*	3.64*	6.26*
1980Q4	2.00*	1.95*	2.88*	4.02*
1981Q4	1.92*	1.94*	3.33*	3.84*
1982Q4	1.62	2.16*	2.00*	2.56*
1983Q4	1.38*	2.27*	1.55*	2.60*

Notes: See notes in Table 4 of the text. Equations (specific) C and D are reported in Table A.3.

Table A.6

Stability Tests

Breakpoint	Equation E		Equation F	
	General	Specific	General	Specific
	F (26,102)	F (10,134)	F (26,102)	F (10,134)
1971Q4	1.13	1.75	1.87*	4.58*
1972Q4	1.29	1.98*	2.06*	5.51*
1973Q4	1.86*	3.02*	3.27*	8.15*
1974Q4	1.74*	2.88*	3.14*	8.39*
1975Q4	1.40	1.80	2.71*	7.40*
1976Q4	1.72*	1.46	3.02*	7.06*
1977Q4	1.65*	1.24	3.24*	7.60*
1978Q4	1.38	1.09	3.48*	7.75*
1979Q4	1.83*	1.94*	2.95*	7.51*
1980Q4	1.99*	2.30*	2.54*	4.92*
1981Q4	1.92*	2.63*	2.57*	3.90*
1982Q4	1.76*	2.73*	1.77*	2.01*
1983Q4	1.83*	2.25*	1.71*	2.20*

Notes: The reported values are the F-statistics that test whether slope dummies when added to Equations E and F are jointly significant. The statistics test stability of all coefficients except the one on the trend term. See also notes in Table 4 of the text. Equations (specific) E and F are reported in Table A.4.

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