A Disaggregate Analysis of Discount Window Borrowing

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At its inception, the Federal Reserve discount window was expected to be the principal instrument of central banking operations. Although open market operations have long since displaced the window in this role, discount window borrowing remains an important source of bank reserves. The window provides relief for short-term liquidity pressures that may develop for depository institutions when they are subject to unexpected outflows in their reserve positions. It is not surprising, therefore, that economists continue to study the behavior of discount window borrowing.[1]

The empirical work has focused on the "borrowing function"—the relationship between discount window borrowing and its key determinants—at the aggregate level, with the estimation results typically interpreted as describing the "representative" bank's behavior.[2] A common finding of such studies is that borrowings are positively related to the spread of the federal funds rate over the discount rate. Since depository institutions can obtain reserves only from each other or the Federal Reserve, they come to the window to meet their reserve needs when the alternative cost of reserves—the federal funds rate—rises relative to the discount rate.[3] A second, less intuitive finding is that an individual bank's current level of borrowings is positively related to last period's, typically weekly, level—the phenomenon of positively autocorrelated borrowings. Because adjustment borrowing is usually the cheapest source of reserves for banks, the window is administered so as to make the ability to borrow a privilege for banks and not an automatic right. The usual practice of discount window administration creates an expectation that when a bank borrows, it diminishes its leeway with the Federal Reserve for further borrowings in the near term. In the aggregate, however, a relation contrary to this expectation holds.

The pattern of borrowing as seen in a scatter plot

[1]There are four types of discount window credit: adjustment, seasonal, extended and emergency credit. Adjustment credit, the focus of this article, helps eligible depository institutions, on a short-term basis, meet a temporary need for funds or cushion briefly more persistent fund outflows while effecting orderly balance sheet adjustments. Adjustment credit is provided only when funds are not reasonably available in the money markets or from usual lenders, including institutional funding sources. Seasonal credit is available to institutions of relatively small size that can demonstrate a clear pattern of recurring intra-yearly swings in funding needs that cannot be satisfied from usual sources. Extended credit involves longer term funds to institutions experiencing special difficulties arising from exceptional circumstances or practices involving individual institutions or from liquidity strains affecting a broad range of depository institutions. Such funding is provided only after all other sources of funds, including special industry lenders, has been exhausted, and only where the lending is judged to be in the public interest. In unusual and exigent circumstances, the Board of Governors may authorize a Reserve Bank to provide emergency credit to individuals, partnerships, and corporations that are not depository institutions if credit is not otherwise available and failure to extend credit would adversely affect the economy. Emergency credit has not been used since the 1930s. See Board of Governors of the Federal Reserve System, *The Federal Reserve Discount Window*, 1990, for details.

[2]All depository institutions may borrow from the window; in this article, we use the terms "bank" and "depository institution" interchangeably.

[3]This relationship was proposed by Robert Turner, *Member Bank Borrowing* (Columbus, Ohio: Ohio State University Press, 1938), and formalized by Murray E. Polakoff, "Reluctance Elasticity, Least Cost, and Member Bank Borrowing: A Suggested Integration," *Journal of Finance*, vol. 16 (March 1960), pp. 1-18, and Stephen M. Goldfeld and Edward J. Kane, "The Determinants of Member Bank Borrowing: An Econometric Study," *Journal of Finance*, vol. 21 (September 1966), pp. 499-514.

(Chart 1) relating weekly borrowings to the interest rate spreads observed over those same weeks suggests that borrowing is less sensitive to the spread at high and low values of the spread. In other words, banks, in the aggregate, appear to be less responsive to the opportunity cost of reserves for the more extreme values of the spread. Furthermore, the scatter plot exhibits an unusual funnel configuration suggesting that borrowing becomes more variable at higher levels of spread.

Such anomalies have prompted some researchers to work on improving the aggregate-level specification. This article, however, seeks to shed light on the anomalies by specifying the borrowing function at the individual bank level. Accepting the influence of the federal funds–discount rate spread, we model the effects of discount window administration on individual bank demand for adjustment credit. This specification is estimated for 240 individual commercial banks that borrowed more than a minimum number of times from January 1981 to August 1990. We then simulate individual demand functions and show that the implied aggregate borrowings exhibit the same general pattern and data anomalies as actual aggregate borrowings.

The disaggregate approach is useful because it allows individual banks to have similarly specified individual demand functions but at the same time to exhibit idiosyncratic behavior. This approach confirms the finding of earlier studies that the demand for adjustment borrowing increases with the cost of alternative sources of reserves. It also yields the following conclusions:

- Depository institutions differ in their responses to discount window administration. Each institution lets its borrowing demand reach some minimum level before it exercises its privilege. Generally, this threshold level is proportionately lower for smaller banks than for larger banks. At low levels of the spread, most banks' demand for adjustment borrowing is below their threshold levels. Hence little or no borrowing will be observed in the aggregate for a range of low values; diagrammatically, the borrowings function will be flatter near the origin. Similarly, because banks may not be able to borrow as much as they would ideally like at very high levels of the spread, the borrowings function should be flatter in that region also. Both of these characteristics help explain the nonlinearity of the scatter plot. In other words, our analysis shows that banks' heterogeneous responses to how the discount window is administered are sufficient to explain the observed S-shaped nonlinearity in the aggregate borrowings function.
- The funnel shape of the data comes from another, though related, source. At high levels of the spread, more and more banks are likely to come to the window. Moreover, each bank responds to given rate conditions in its own individual way. Hence high levels of aggregate borrowings are usually correlated with a greater number of disparate institutions visiting the window. This increased heterogeneity of banks at the window causes the aggregate borrowing function to be more variable at high levels of the spread.
- Positively autocorrelated borrowings are observed for individual small banks but not for individual large banks. This result most likely reflects the Federal Reserve's greater tolerance of consecutive-period borrowing by smaller banks. The Federal Reserve recognizes that small banks may have only limited access to national money markets and may need more time to correct unexpected liquidity pressures.
- Lastly, there is evidence that a majority of larger banks conserve on their use of the window if they expect the cost of reserves to rise in the next maintenance period.

Individual bank demand for adjustment borrowing

Individual banks facing a reserve need must decide whether to visit the discount window or to borrow reserves from other depository institutions. All else equal, if the discount rate is below the federal funds rate, banks would prefer to come to the Federal Reserve because adjustment credit would then be the cheapest source of funds. In practice, however, unlimited access is not possible because the window is administered as a privilege and not as an automatic right. Borrowers are expected to seek other reasonably available sources of funds before turning to the window for assistance and to have an appropriate reason for the request. Credit is granted at the discretion of the Reserve Bank and is always secured.

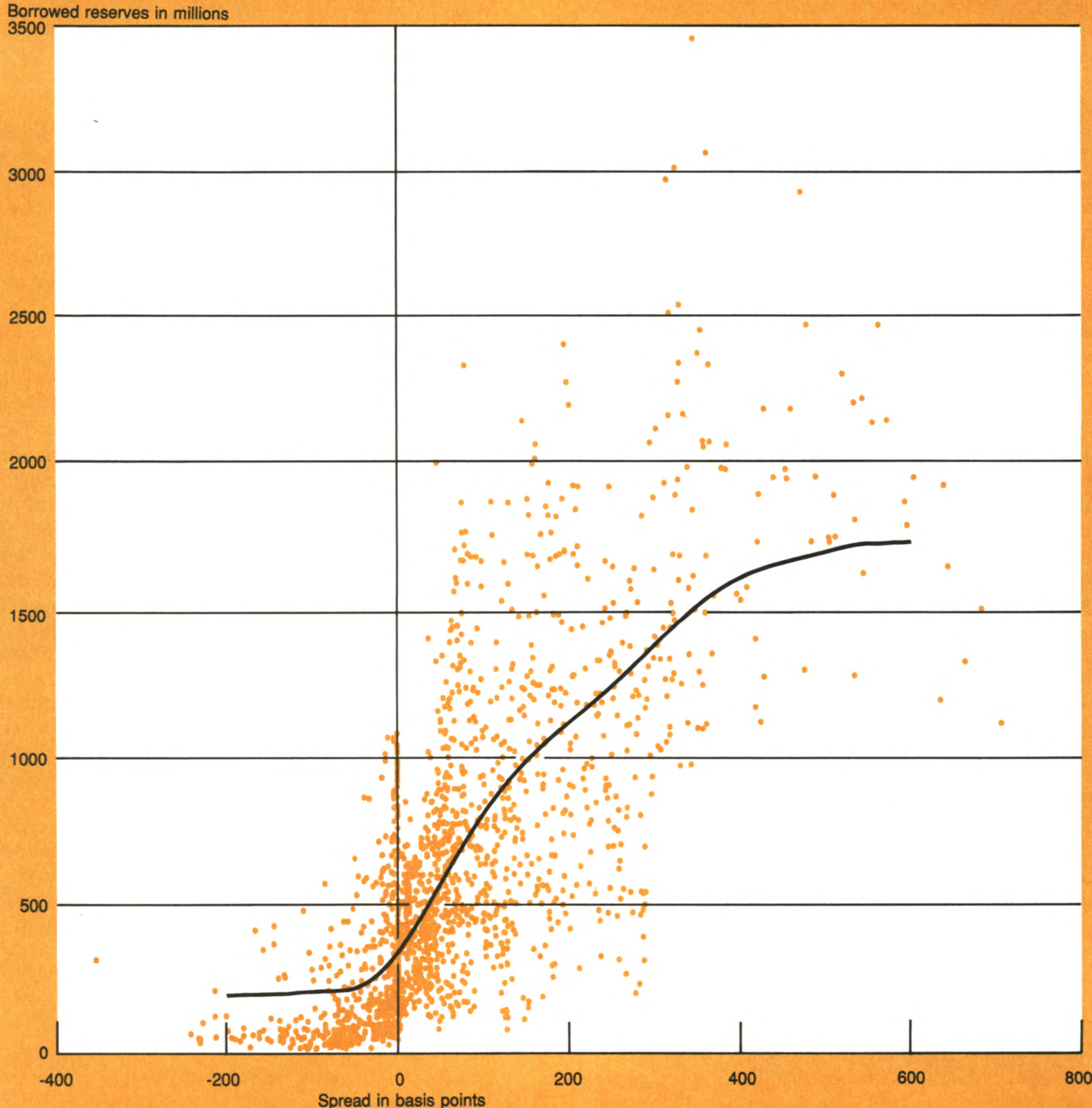
In effect, this practice introduces an implicit cost to discount window use not reflected in the federal funds–discount rate spread. Consequently, the amount that a bank actually borrows, denoted by B^d_{ti}, may not always reflect the bank's unconstrained or notional demand for discount window credit, denoted by B^{*d}_{ti}.

Actual and notional borrowings differ under two sets of circumstances. The nonprice mechanism may lead a bank to wait until its borrowing need exceeds some minimum or threshold level before the bank will actually use its window privilege. Such a reservation level is likely to reflect the implicit cost of borrowing and may depend on, among other considerations, the borrower's recent discount window usage—in particular, the size of the borrowings, the frequency of visits, and the number of consecutive visits. Moreover, even

Chart 1

Relationship of Borrowed Reserves to Spread between Federal Funds Rate and Discount Rate

January 1959 - August 1990



Source: Board of Governors of the Federal Reserve System.

Note: The plotted curve represents predicted borrowed reserves obtained from a nonlinear regression model of the spread. Borrowed reserves consists of adjustment plus seasonal borrowing. The series is adjusted for irregularities in bank borrowing.

though a bank may be willing to visit the discount window, it is not permitted to borrow unlimited amounts of funds. For example, as a general rule, it is inappropriate for a bank to be a net seller of federal funds while borrowing. Alternatively, a bank may not have enough collateral immediately available to secure a large borrowing at the Federal Reserve.[4] Thus, in principle, there exists an upper bound on a bank's borrowing ability that could be less than its notional demand for discount credit. In the current framework, notional demand, B_{ti}^{*d} would equal observed borrowing, B_{ti}^{d}, only when the bank's notional demand exceeds its borrowing threshold and is less than its effective upper bound, should one exist. Individual bank behavior was estimated by the following regression equation:

$$(1)\quad B_{ti}^{*d} = \beta_0 + \beta_1 S_t + \beta_2(\hat{S}_{t+2} - S_t) + \beta_3 \Delta \text{Yield}_t + \gamma B_{t-1,i}^{d} + u_{ti},$$

where B_{ti}^{*d} is notional demand measured as the average daily amount borrowed over the week t.[5] Ordinarily, estimating an individual bank's notional demand for borrowing requires knowing the upper and lower constraints. Unfortunately, these limits are not known. As shown in the appendix, however, the constant term, β_0, can be interpreted to measure the true constant term of the equation (which is greater than or equal to zero because banks cannot lend to the window) as well as the effects of both the threshold and the upper constraint described above. Moreover, since both bounds work to bias downwards only the estimated constant term of the regression, this specification allows for the unbiased estimation of all the exogenous factors influencing individual bank demand for adjustment credit. The algebra in the appendix shows that if the threshold and upper constraint are important behaviorally, then we would expect the estimated constant terms to be significant and negative.

The remaining specification is quite standard. The spread of the weekly effective federal funds rate over the discount rate, S_t, measured in basis points, represents the opportunity cost of borrowing from the window. Of course, each bank's opportunity cost of borrowing is not exactly equal to this interest rate spread; some banks pay more and some pay less. However, their average cost of reserves is probably well represented by the effective funds rate, which is a volume-weighted average. At the very least, the rate spread is an unbiased proxy for the banks' cost of reserves and shows how this cost changes from week to week. On average, we would expect this coefficient estimate to be positive, because the larger the spread, the lower the relative cost of adjustment credit and therefore the higher the notional demand for it.

Recently, it has been argued persuasively that banks' use of the discount window is also a function of their expectations about the future level of interest rates.[6] Optimizing banks will conserve their borrowing privilege and forgo borrowing today in order to visit the window during those future periods when the alternative cost of borrowing is expected to be highest. To incorporate this intertemporal aspect of adjustment demand, we include a measure of the expected change in the federal funds–discount rate spread, $(\hat{S}_{t+2} - S_t)$.[7] If a bank does optimize the use of its privilege over time, then the estimated coefficient β_2 should be negative.

Lagged discount window borrowing, $B_{t-1,i}^{d}$, is also included as an explanatory variable. If banks believe that they deplete their privilege when they borrow, then the estimated coefficient, γ, should be negative. This result may not, however, hold empirically for smaller banks. Discount officers recognize that smaller institutions have more limited access to national money markets than do their larger counterparts. Consequently, smaller banks are given more leeway in their access to the window. For example, money-center and other larger banks are usually expected to need assistance for only a single day (not including holidays or weekends). By contrast, smaller banks may need to borrow over a number of days. For these banks, a single decision to borrow may end up being observed over two consecutive periods and may lead to positive serial correlation.

For technical reasons, general market and economic conditions should be modeled parsimoniously. Hence

[4]The Federal Reserve requires that all extensions of discount window credit be secured to its satisfaction. Assets that are suitable collateral include U.S. government and agency securities, municipal and corporate securities, customer notes based on commercial and agricultural loans, residential real estate notes, and bankers' acceptances. In most instances the collateral is kept at the Federal Reserve Banks.

[5]For instance, if \$70 million was borrowed only for Tuesday, then

$$B_{ti}^{d} = \frac{0+0+0+0+0+70+0}{7} = \$10 \text{ million.}$$

[6]See Marvin Goodfriend, "Discount Window Borrowing, Monetary Policy, and the Post-October 6, 1979 Federal Reserve Operating Procedure," *Journal of Monetary Economics*, vol. 12 (September 1983), pp. 343-56.

[7]The expectation was led forward by two weeks to account for the biweekly reserve maintenance period since 1984. Moreover, the prediction was utilized in differenced form because $\hat{S}_{t+2}$ was highly collinear with S_t. We experimented with other variations that led the prediction by only one week prior to 1984, and the results were essentially similar. The expectation $\hat{S}_t$ was calculated in a standard way from an autoregressive model that included a host of commonly accepted explanatory variables: lags of the federal funds–discount rate spread, the repurchase agreement–federal funds rate spread, and growth in the monetary aggregates.

these factors are proxied very simply by the slope of the Treasury yield curve, as given by the spread of the thirty-year Treasury bond rate over the bond equivalent three-month Treasury bill rate, $\Delta Yield_t$. This variable can be viewed as a simple control variable that allows the behavioral parameters to be measured more efficiently.

Some features of the data should be noted. Because the model describes notional adjustment credit behavior, the sample is restricted to banks that visited the window at least once in six of the possible ten years and that made minimal use of the seasonal and extended credit facility.[8] This restriction yields a consistent sample of 240 commercial banks. The data are weekly observations of daily average adjustment credit borrowing for the period January 1981-August 1990.[9]

Empirical findings

Although the estimation was done at the disaggregate level, the results are presented in summary form to preserve the confidentiality of the data on individual banks. But because smaller institutions—those with assets less than or equal to $1 billion during the first half of 1990—seem to behave differently than larger banks, we present the summary statistics for the two size classifications in separate tables (Tables 1 and 2). In both tables, the first column presents the group average of the estimated coefficient. The coefficient estimates are scaled by dividing the coefficient by each bank's average level of borrowings. Otherwise, larger banks, which tend to borrow considerable amounts because of their size, would exhibit larger unscaled coefficients even though their behavioral response, in a relative statistical significance sense, may not be different from that of a much smaller bank. For the sample, the larger banks' average level of daily borrowing ($28 million) is fourteen times as large as the smaller banks' level ($2 million). Columns 2 and 3 give the percent of the sample of banks that have a positive or negative coefficient value. Columns 4 and 5 provide a representative range of the coefficient values.

Consistent with the model described in the appendix, all the banks, regardless of size, exhibit a negative constant term. If we assume that banks are generally not upper-bound constrained, the estimated constant in Table 1 suggests that smaller banks have borrowing threshold levels that are almost two and one-half times as large as their actual level of average borrowings. For example, suppose that a bank's average level of observed daily borrowings at the window is $2 million. Then its notional demand must on average exceed $5 million before the bank would be willing to come to the window. But even among smaller institutions the range of responses is quite wide (columns 4 and 5 of Table 1). Some banks' threshold levels are virtually nonexistent, while the levels of others are more than double that of their group's average.

After size is taken into account, larger banks with assets greater than $1 billion have somewhat higher threshold levels than do smaller banks (Table 2, row 1). Thus a bank that averages adjustment borrowing of $30 million will need to have notional demand in excess of $85 million before it will consider coming to the window. After this size effect is taken into account, a simple t-test indicates that the larger banks' threshold is significantly higher, but only marginally so ($p-\text{value} = 0.069$).

Our findings suggest that smaller banks for the most part respond positively to a widening of the federal funds–discount rate spread (Table 1, row 2).[10] When the spread widens by 100 basis points, the banks' notional demand increases by 20 percent of their average level of borrowing. Larger banks are somewhat more sensitive to a widening of the spread (Table 2, row 2). A 100 basis point widening increases their notional demand by 25 percent of their average level of borrowing.

Testing the effect of expectations of future interest rate changes on the demand for adjustment borrowing produces the most mixed evidence for both groups of banks. The majority of banks in each sample have negative coefficients, indicating that the banks are making some attempt to optimize intertemporal use of the

[8]Because we were modeling adjustment borrowing behavior, we tried to limit our sample to banks that had made minimal use of other types of borrowing. The six-year cutoff reflects a balance between including large banks in the sample and having a sufficient number of noncensored observations per bank to make the statistical estimation viable. Other cutoff levels were tried, and the results were found to be robust.

[9]The model was estimated by an EM-algorithm procedure that is equivalent to maximum likelihood estimation. (See Takeshi Amemiya, *Advanced Econometrics* [Cambridge: Harvard University Press, 1985], pp. 375-78, for a more detailed description.) In other words, 240 separate equations, one for each bank, were estimated for the sample period. Censored regression models frequently have a large percentage of censored (zero) dependent observations. In the present sample, the average value of censoring was 93 percent, although in some extreme cases it was as high as 98 percent. Alternatively, banks had an outstanding loan balance 7 percent of the time on average. Maximum likelihood estimation projects all censored (zero) observations on a hypothesized normal distribution structure. The technique is most suitable for assessing the primary motives of the notional behavior; thus it is important that the employed explanatory variables provide a reasonable specification. For this reason, we limited our estimation to a parsimonious specification.

[10]The interest rate spread variable was not adjusted in the individual bank equations to reflect the surcharge because such data were not available. This omission should not be a problem because the surcharge was actually triggered in only 9 weeks of the 501-week sample period. Moreover, surcharge corrections to the aggregate borrowing function typically are not significant.

Table 1

Estimates of Individual Borrowing Function for Banks with Assets Less Than or Equal to One Billion Dollars

Summary Statistics

Borrowing Function: $B_t^* = \beta_0 + \beta_1 S_t + \beta_2(\hat{S}_{t+2} - S_t) + \beta_3 \Delta \text{Yield}_t + \gamma B_{t-1}$

Coefficient	Mean	Percent Positive	Percent Negative	First Percentile	Ninety-ninth Percentile
$\frac{\beta_0}{\bar{B}}$	−2.54[-28.47*]	0	100	−6.352	−0.223
$\frac{\beta_1}{\bar{B}}$	0.0020[11.43*]	79	21	−0.0039	0.0081
$\frac{\beta_2}{\bar{B}}$	−0.00036[−1.31]	48	52	−0.0010	0.0091
$\frac{\beta_3}{\bar{B}}$	0.0027[17.06*]	96	4	−0.0015	0.0114
$\hat{\gamma}$	0.261[15.79*]	87	13	−0.405	0.679

Notes: Number of banks with assets less than or equal to $1 billion is 184. The time series sample consists of weekly observations from January 1981 to August 1990, a total of 501 observations. The variable $\bar{B}_j$ (j = 1,...,184) represents average outstanding adjustment borrowing of the j-th bank over the weekly sample. Numbers in brackets are t-statistics for the null hypothesis that $(\frac{\beta}{\bar{B}}) = 0$.

B_t^* = average outstanding adjustment borrowing (millions of dollars)
B_t = max{0,B_t^*} (see equation system A.2 in the appendix)
S_t = federal funds–discount rate spread (basis points)
$\hat{S}_t$ = federal funds–discount rate predicted spread (basis points)
ΔYield$_t$ = spread between thirty-year Treasury bond rate and three-month bill rate (basis points).

*Statistically significant at the 5 percent level.

Table 2

Estimates of Individual Borrowing Function for Banks with Assets Greater Than One Billion Dollars

Summary Statistics

Borrowing Function: $B_t^* = \beta_0 + \beta_1 S_t + \beta_2(\hat{S}_{t+2} - S_t) + \beta_3 \Delta \text{Yield}_t + \gamma B_{t-1}$

Coefficient	Mean	Percent Positive	Percent Negative	First Percentile	Ninety-ninth Percentile
$\frac{\beta_0}{\bar{B}}$	−2.85[-19.50*]	0	100	−5.030	−0.365
$\frac{\beta_1}{\bar{B}}$	0.0025[7.78*]	86	14	−0.0025	0.0084
$\frac{\beta_2}{\bar{B}}$	−0.0085[−2.0*]	39	61	−0.0099	0.0052
$\frac{\beta_3}{\bar{B}}$	0.0031[12.64*]	96	4	−0.00024	0.0085
$\hat{\gamma}$	−0.206[3.89*]	35	65	−1.445	0.470

Notes: Number of banks with assets greater than $1 billion is 56. The time series sample consists of weekly observations from January 1981 to August 1990, a total of 501 observations. The variable $\bar{B}_j$ (j = 1,...,56) represents average outstanding adjustment borrowing of the j-th bank over the weekly sample. Numbers in brackets are t-statistics for the null hypothesis that $(\frac{\beta}{\bar{B}}) = 0$.

B_t^* = average outstanding adjustment borrowing (millions of dollars)
B_t = max{0,B_t^*} (see equation system A.2 in the appendix)
S_t = federal funds–discount rate spread (basis points)
$\hat{S}_t$ = federal funds–discount rate predicted spread (basis points)
ΔYield$_t$ = spread between thirty-year Treasury bond rate and three-month bill rate (basis points).

*Statistically significant at the 5 percent level.

window. This finding applies in particular to the bigger banks. Typically liability-managed institutions with ready access to national money markets, the bigger banks seldom need to borrow for more than occasional overnight needs. Hence, they must consider not only whether to borrow but also in which period to deplete their window privilege. If larger banks expect the spread to fall 10 basis points in the next maintenance period, they will increase their notional demand by less than 9 percent of their average level of borrowings (Table 2, row 3). Alternatively, when rates are expected to rise, banks tend to delay exhausting their window privilege. Thus the rate spread affects the notional demand for adjustment borrowing through two channels: its current level, β_1, and its anticipated change for the next period, β_2. For example, if the current federal funds–discount rate differential is 100 basis points but is expected to narrow in the next period by 10 basis points, then large banks' notional desire for the current period will increase by about 34 (=25 + 9) percent of their average level of borrowings, all else equal.

The most striking difference in discount window behavior between larger and smaller banks appears in the lagged borrowing coefficient. When smaller banks develop a liquidity need, it takes them some time to adjust their positions because of their more limited access to money markets. Recognizing this special circumstance, window officers allow such banks to borrow over several periods. Hence their borrowings are positively serially correlated. On average, 27 percent of current borrowings may arise from this adjustment process. In contrast, the majority of larger banks have a negative coefficient. When these banks utilize their privilege, their notional demand for adjustment credit in the subsequent period falls on average by 21 percent because they recognize that the Federal Reserve discourages sequential visits.

The aggregate borrowing relationship

Our estimates of the individual borrowing functions enable us to examine the implications of individual bank behavior for the aggregate level of adjustment credit. In particular, we simulate individual demand functions and show that the implied aggregate borrowings exhibit the same general pattern and data anomalies as actual aggregate borrowings. A different simulation is done to illustrate each anomaly separately.[11]

Simulation 1: aggregate borrowing nonlinearity

To show that nonlinearity can arise merely from variations in banks' borrowing thresholds and upper bounds, we assume that banks are identical except for these two constraints. More specifically, we assume:

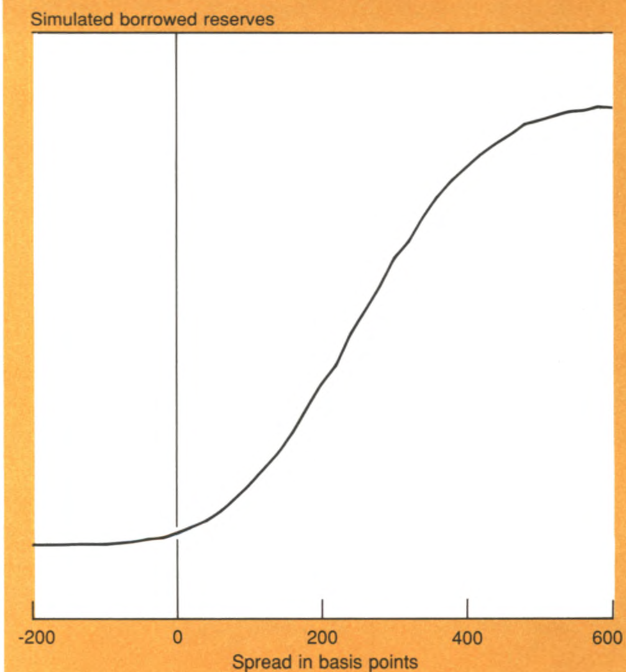
$B^{*d}_{ti} = 2 + 0.01S_t$,

where S_t again denotes the federal funds–discount rate spread. The parameter values are selected to approximate those of the empirical findings. For example, we found that the average of the unscaled coefficient for the interest rate spread, β_1, is about 0.04 for the large bank group and 0.002 for banks with assets less than or equal to $1 billion. The value of 0.01, utilized in our simulations, is a weighted average of the various β_1 estimates. The other parameter values and random shocks are chosen similarly. Individual banks' differing thresholds and upper bounds are given by $I_{ti} = 4 + 1.8\zeta_{ti}$ and $C_{ti} = 5 + 1.2\zeta_{ti}$, respectively, where each ζ_{ti} is randomly drawn from a standard normal distribution.

Chart 2 shows the aggregate borrowing relationship that is the sum of 2,000 such simulated demand functions—a general approximation of the number of weekly reporting banks that visit the window during the sample period. At very low levels of the spread, most banks' notional demands for adjustment credit are less than their individual minimum thresholds, and therefore

Chart 2

Simulation 1: Nonlinearity of Aggregate Borrowed Reserves



[1]For a specific proof of the statistical traits of the aggregate borrowed reserves, see Stavros Peristiani, "The Model Structure of Discount Window Borrowing," *Journal of Money, Credit, and Banking*, vol. 23 (January 1991), Section 2, pp. 13-34.

they do not come to the window. As the spread widens, however, the opportunity cost of borrowing from the window also rises and individual banks' notional demands for adjustment credit increase. Progressively, more and more banks cross their minimum borrowing thresholds and come to the window to meet their reserve needs. At very high levels of the spread, individual banks may start encountering their upper constraints, and therefore the sensitivity of aggregate borrowings to the spread declines. The S-shaped nonlinear profile of Chart 2 suggests that even if all borrowers had identical notional demands, nonlinearity in the aggregate relationship would still arise because of variations in banks' responses to the implicit cost of discount window borrowing.

Simulation 2: aggregate borrowing variability
To demonstrate that the variability of total borrowings increases with the number of borrowers, we allow individual banks to differ in their response sensitivity to the spread variable and to have their own stochastic element. More specifically, it is assumed that

$$B_{ti}^{*d} = 2 + (0.01 + 0.003\xi_i)S_t + u_{ti},$$

where $u_{ti} = \zeta_{ti}$ and ξ_i are again randomly drawn from a standard normal distribution. For comparability, we maintain the threshold and upper-bound values of simulation 1. In other words, for every period, a random shock is added to the bank's notional demand as well as its threshold and upper limit. For greater realism, the spread values are more frequently in the range of 0 to 200 basis points. Chart 3 depicts a scatter plot of aggregate borrowing derived from 2,000 such simulated borrowing functions.

This exercise also shows the aggregate relationship to be nonlinear. In Chart 3, we fit the same aggregate borrowings function to the generated data as was fit to the actual data of Chart 1. Using the R-square coefficient as a criterion of comparability, we find that the nonlinear specification for the generated data yields an R^2 value of .89, while the actual data R^2 value was .81. Hence the generated data are nonlinear in a way that is very similar to the actual data.

Chart 3 also shows a funnel configuration to the data points that is very like the configuration of Chart 1. The statistical literature terms this type of data pattern "heteroskedasticity," and a number of formal statistical tests are proposed for detecting its presence. The computed chi-square test values for the two data sets are both significant and very similar in value (167 for the actual data and 188 for the generated data). At wider spreads, more banks have crossed their minimum borrowing threshold and therefore are willing to come to the window to meet their borrowing need. But as more heterogeneous borrowers come to the window, the cumulative impact of their differences manifests itself as increased variability of aggregate adjustment borrowing.[12] Thus, the greater variability of aggregate borrowings that we see in actual borrowings can arise from behavioral variation across banks and randomness in the banks' notional demands for borrowing.

Conclusion

When plotted against the federal funds–discount rate spread, discount window borrowing exhibits a puzzling pattern. Researchers have tried to explain this relationship in a number of ways, but almost always at the aggregate level. We take a different approach by modeling individual bank behavior to reflect discount window administration. We assume that banks have a continuous notional demand for adjustment credit that is a function of the federal funds–discount rate spread and other variables. But because the window is administered as a privilege and not as an automatic right, banks do not come to the window until their notional demand exceeds some minimum or threshold level. Hence low levels of their notional demand will not be observed, so that actual borrowings and the aggregate borrowings function will tend to be flat for low levels of the spread. Similarly, banks may sometimes be unable to borrow as much as they ideally would like at high levels of the spread. Hence high levels of notional demand will also not be readily observed as actual borrowings. As a consequence, the aggregate borrowings function will tend to flatten out at high levels of the spread as well. This pattern is in fact what a scatter plot of actual adjustment borrowings shows. Thus the nonlinearity of the aggregate borrowings function derives from banks' responses to the fact that the window is administered as a privilege and not as an automatic right.

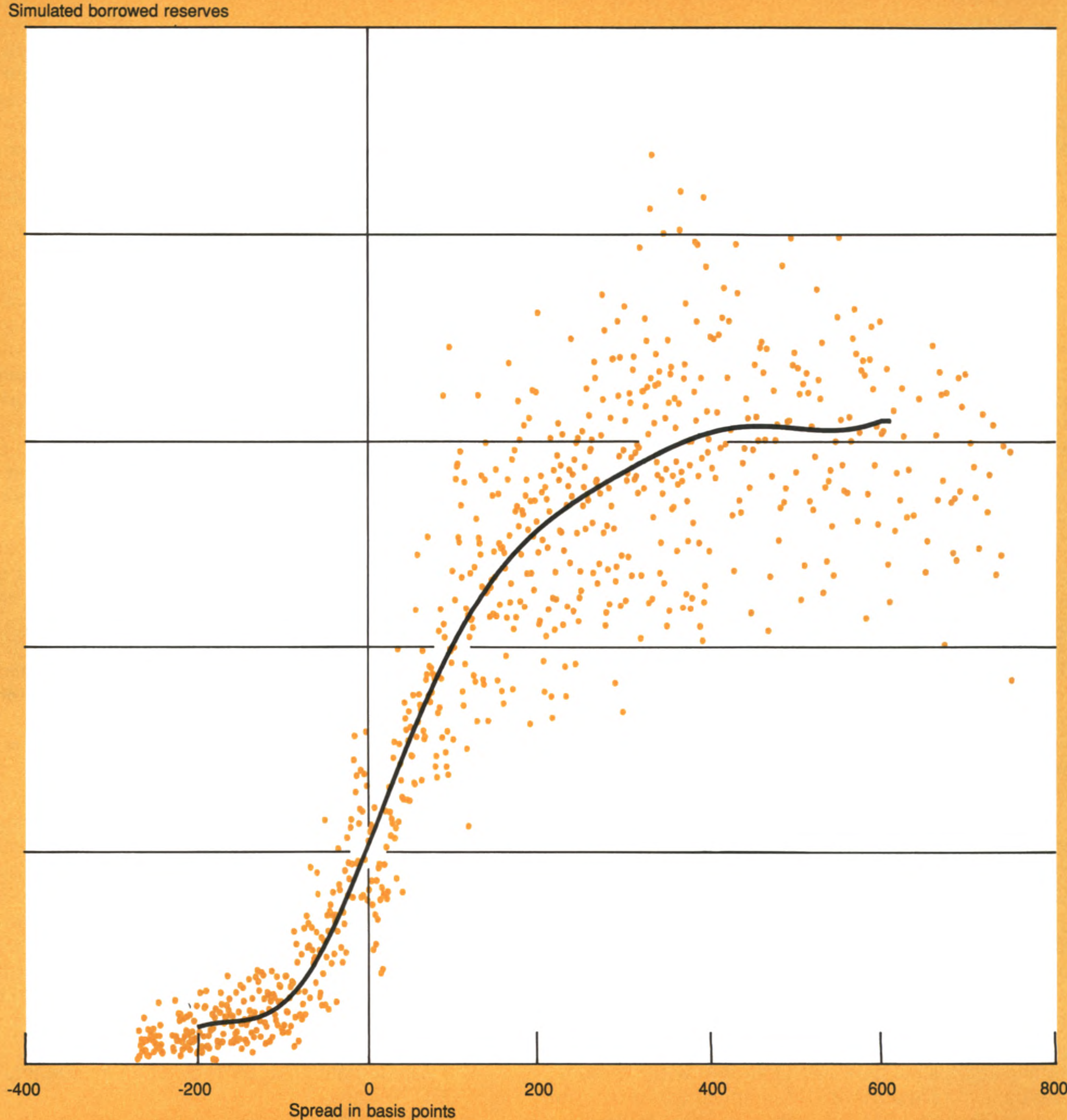
The funnel shape of the data stems from a related source. The behavioral parameters that determine notional borrowing differ from one bank to the next. As the spread between the federal funds rate and the discount rate widens, notional demands also increase, but to varying degrees. More and more institutions are likely to come to the window as the spread widens further. Hence high levels of aggregate borrowing usually arise because a greater number of disparate institutions are coming to the window. The presence of diverse banks, each with its own idiosyncratic behavior, causes the aggregate borrowings function to be more variable at high levels of the spread.

Our disaggregate approach gives an insight into total

[12] In statistical language, the variance of total borrowings is the sum of the individual bank variances plus their covariances. Since the covariances are equal to zero, the variance of total borrowings increases with the number of borrowers.

Chart 3

Simulation 2: Variability in Aggregate Borrowed Reserves



Note: The plotted curve represents predicted simulated aggregate borrowing obtained from a nonlinear regression model of the spread.

discount window borrowing that cannot be easily obtained by studying the relationship at the aggregate level exclusively. Although this approach may not help predict total borrowings any more precisely, it does explain why the relationship is nonlinear and more variable at high levels of the spread. The variability is endogenous to the process and not necessarily due to some episodic instability.

Appendix

Individual bank behavior at the discount window can be formalized by adapting a censored behavior model. In this framework, a reservation or threshold level must be exceeded, ceteris paribus, before the bank will actually use its window privilege.

We assume that individual banks have a notional demand for discount window credit, denoted by B^{*d}_{ti}, which is a function of their reserve needs and the cost of reserves. In practice, however, unlimited access is not possible because the window is administered as a privilege rather than an automatic right. Borrowers are expected to seek other reasonably available sources of funds before turning to the window for assistance, and credit is granted at the discretion of the Reserve Bank. In effect, these restrictions introduce a nonprice consideration to discount window use. Consequently, the bank's notional demand is different from its effective demand, which is observed as actual discount window borrowing.

To model the nonprice aspects of discount window borrowing, we assume that each bank has an unobserved reservation level of borrowing, I_{ti}, that depends on the extent of recent discount window usage (the frequency of visits, the size of the borrowings, the number of consecutive visits) and on other possible economic variables. Similarly, we assume that banks may not be able to borrow as much as they ideally would like. For example, the Federal Reserve requires that all discount window loans be secured. Although banks customarily maintain collateral at Federal Reserve Banks, some banks may not have enough eligible collateral immediately available in a given time period to support a large notional borrowing need. We term those institutions that are not borrowing as much as they ideally would like "collateral constrained" (even though the constraint does not always arise from a scarcity of collateral). The variable C_{ti} denotes this upper bound on a bank's borrowing ability.

Demand for discount window credit can by described by the following equation system:

(A.1.1) $$B^{*d}_{ti} = s_t\beta_i + u_{ti}$$

(A.1.2) $$I_{ti} = z_t\delta_i + \epsilon_{ti}$$

subject to:

(A.1.3)

$$\begin{aligned} B^{d}_{ti} &= C_{ti} && \text{if} \quad B^{*d}_{ti} \geqslant C_{ti} && i = 1,\ldots,n^c_t \\ &= B^{*d}_{ti} && \text{if} \quad I_{ti} < B^{*d}_{ti} < C_{ti} && i = n^c_{t+1},\ldots,n_t \\ &= 0 && \text{if} \quad \text{otherwise} && i = n_{t+1},\ldots,N_t. \end{aligned}$$

Notional demand, B^{*d}_{ti}, consists of a systematic component, s_t, which may include a variety of economic variables, and a random component, u_{ti}. For example, a very simple formulation would let the systematic component s_t consist solely of the spread of the federal funds rate over the discount rate.† The unobserved reservation borrowing level, I_{ti}, is assumed to depend on a number of explanatory variables represented by the vector, z_t, and a random component, ϵ_{ti}. The parameters β_i and δ_i describe individual bank behavior. Note that these parameters are unrestricted. The variable N_t represents the total number of banks that are eligible to borrow at the window in period t, while n_t is the number of banks that actually borrow in period t and n^c_t is the number of borrowing banks that borrow up to their collateral limit.

The first constraint of A.1.3 specifies that if the bank is collateral constrained, then its observed demand for borrowing will be equal to the value of its collateral or perceived upper bound. The third constraint indicates that the observed demand will be equal to zero if the notional demand is below the reservation threshold. If the bank is neither collateral constrained nor below its reservation level, then the observed level of borrowing will be an accurate measure of its notional demand for discount window credit.

The advantage of equation system A.1 is that it realistically allows notional demands to be continuous while observed or effective demands are discontinuous. In this model, notional demand, B^{*d}_{ti}, is observed as actual demand, B^{d}_{ti}, only when the notional demand is greater than the implicit reservation of borrowing, I_{ti}. Thus, when banks do not visit the discount window, it is not because the desired amount of credit is zero but because the amount is below the implicit reservation threshold. This threshold is in turn determined by the banks' recent use of the discount window and by pre-

†The variable s_t is a vector that includes all explanatory information. For instance, in the case of equation 1 of the main text, s_t is equal to $(1, S_t, \hat{S}_{t+2} - S_t, \Delta\text{Yield}_t, B^{d}_{t-1,i})$.

Appendix *(continued)*

vailing and future economic conditions.‡

The equation system given by A.1 could be estimated directly by a two-limit sample selectivity maximum likelihood algorithm. Unfortunately, the two limits, C_{ti} and I_{ti}, are not observed. One practical alternative is to allow for a deterministic reservation index and upper collateral bound such as ($I_{ti} = \bar{I}_i, C_{ti} = \bar{C}_i$). With these simplifications, the model is reduced to the simple censored regression:

(A.2.1) $B^{*d}_{ti} = s_t\beta_i + u_{ti}$

subject to:

(A.2.2)

$$
\begin{aligned}
B^{d}_{ti} &= \bar{C}_i && \text{if} \quad B^{*d}_{ti} \geq \bar{C}_i && i = 1,\ldots,n^c_t \\
&= B^{*d}_{ti} && \text{if} \quad \bar{I}_i < B^{*d}_{ti} < \bar{C}_i && i = n^c_{t+1},\ldots,n_t \\
&= 0 && \text{if} \quad \text{otherwise} && i = n_{t+1},\ldots,N_t.
\end{aligned}
$$

Equation system A.2 appears to impose a fairly crude solution to these apparent unobservables. Even so, we will show that this specification is still capable of capturing the basic effects. Assuming that $C_{ti} = \bar{C}_i$ is somewhat problematic because we are unable to identify observations that reach the collateral limit. The lower threshold, $\bar{I}_i$, is less troubling; although the value of the outcome is not observed, the action is identifiable from the bank's borrowing activities.

The unobservability of both $\bar{I}_i$ and $\bar{C}_i$ poses no significant problems for the estimation because it introduces only a downward bias on the constant coefficient β_{0i}. Let us assume that $s_t = (1,S_t)$. That is, banks arrive at their decision to borrow only by looking at the federal funds–discount rate differential. To estimate the parameters of the model given by equation system A.2, we need to define the likelihood function of the system. The first component of the likelihood reflects the probability that a bank will not borrow in that particular week. More precisely,

$$
\begin{aligned}
P(B^{*}_{ti} - \bar{I}_i \leq 0) &= P(\beta_{0i} + \beta_{1i}S_t - \bar{I}_i \leq -u_{ti}) \\
&= P(u_{ti} \leq -\beta^{I}_{0i} - \beta_{1i}S_t),
\end{aligned}
$$

where $\beta_{0i} = \beta^{I}_{0i} - \bar{I}_i$. The second component of the likelihood represents the possibility that a bank will borrow the maximum allowable. As in the previous case, it can be shown that this is equivalent to

$$P(B^{*d}_{ti} - C_i \geq 0) = 1 - P(u_{ti} \leq -\beta^{C}_{0i} - \beta_{1i}S_t),$$

where $\beta^{C}_{0i} = \beta_{0i} - \bar{C}_i$. The third component of the likelihood accounts for the possibility that the i-th bank will borrow at period t an amount not equal to the collateral limit. Specifically, the likelihood of the system can be defined by:

$$
L = \prod_{t=n_t+1}^{N_t} P(u_{ti} \leq -\beta^{I}_{0i} - \beta_{1i}S_t) \prod_{t=n^c_t+1}^{n_t} f(\beta_{0i},\beta_{1i},\sigma_i;S_t)
\prod_{t=1}^{n^c_t} [1 - P(u_{ti} \leq -\beta^{C}_{0i} - \beta_{1i}S_t)].
$$

An interesting detail of the likelihood is that β_{0i}, the usual constant parameter, is affected by the unknown levels of $\bar{I}_i$ and $\bar{C}_i$. Because a bank will not be at the window most of the time, the maximum likelihood estimate of β_{0i} will reflect more β^{I}_{0i}; thus, the final constant estimate should be expected to achieve a large negative value. Note that even when the bank borrows, the negative bias does not disappear, because the constant parameter for the cases that reach the upper limit is represented by β^{C}_{0i}. Since the collateral limit C_i is unavailable, the last component of the likelihood is not measurable. One solution to this problem is to assume no implicit collateral constraint, a step that would in turn constrain the probability of the third component of the likelihood to 1. Despite all the apparent biases imbedded in the constant coefficient, in all three segments of the likelihood the slope coefficient β_1 is uncontaminated. This finding means that we can obtain unbiased measures of the association between notional borrowing and the federal funds–discount rate spread or any other explanatory variable. Another minor difficulty introduced by equation system A.2.2 is the assumption that banks are always supplied with the amount requested ($B^{*d}_{ti} = B^{d}_{ti}$). In general, there is no guarantee that visiting banks will invariably be granted the full amount $B^{*d}_{ti} = B^{d}_{ti}$. Given the available information, however, it is not feasible to separate these minor residual deductions from the observed amount of individual borrowing.

‡Another interesting interpretation can be derived from the canonical specification of equation system A.1. If $N_{ti} = B^{*d}_{ti} - I_{ti}$, then a bank would borrow if $N_{ti} > 0$. The variable N_{ti} can be viewed as the theoretical benefit or net gain from borrowing. As a consequence, the threshold I_{ti} reflects the opportunity cost of borrowing. For instance, suppose that the i-th bank borrowed at period t. The opportunity cost at period (t+1) would be expected to increase because the administration of the discount window discourages frequent visits.