Bank Diversification: Laws and Fallacies of Large Numbers

by Joseph G. Haubrich

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Conventional wisdom states that large banks are safer than small banks because they can diversify more. This conventional wisdom, however, confuses risk with probability of failure. While the *law* of large numbers does imply that a large bank is less likely to fail than a small bank, equating this tendency with lower risk falls into what Samuelson [1963] termed the *fallacy* of large numbers. A $10 billion bank may be less likely to fail than a $10 million bank, but it may also saddle the investor with a $10 billion loss.

In this article, I hope to clarify what this distinction means for banks. Banks diversify by growing—by adding risks—something distinctly different from the subdivision of risk behind standard portfolio theory. A simple mean-variance example will make the point that a risk-averse bank owner need not value diversification by addition. After that, I take a regulator's perspective and consider how a bank guarantee fund, such as the deposit insurance agency, views bank growth and diversification. After a short review of why diversification by adding risks decreases the probability of bank failure, I look at how such diversification alters the expected value of deposit insurance agency payments, then turn to diversification's impact on the deposit insurance agency's expected utility, using recent results from the theory of *standard* risk aversion.

To concentrate on the cleanest example, this article stays with the case of independent and identically distributed risks. This admittedly ignores the alleged ability of large banks to diversify regionally[1] or the possibly adverse incentives of deposit insurance (Boyd and Runkle [1993], Todd and Thomson [1991]).

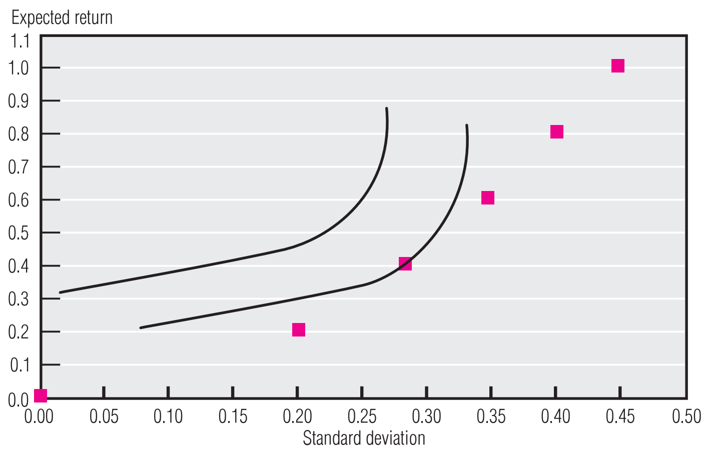
I. A Simple Example

Probably the easiest way to understand the effects of diversification by adding risks is to consider a bank financed exclusively by an owner/investor who cares only about means and variances. With no debt, failure ceases to be an issue; instead, the question is the utility-maximizing portfolio for the bank's owner.

1 Compare Haubrich (1990) with Kryzanowski and Roberts (1993). Even small banks may diversify, however, by selling loans or participating in mortgage pools or other forms of securitization.

FIGURE 1

Bank Opportunity Set



SOURCE: Author.

The owner and sole equity holder has, conveniently for us, sunk his entire wealth W into the bank. He faces the problem of dividing his portfolio between holding S safe government bonds with a certain return of zero and investing in some number K of risky, independent bank loans with returns R_i normally distributed as $N(\mu,\sigma^2)$, that is, with mean μ and variance σ^2. If each loan costs a dollar, the investor's budget constraint is $W = S + K$. These bank loans are indivisible—the bank cannot diversify by spreading one dollar across many loans. Then the return on the portfolio is

$$\tilde{R}_P = \frac{\sum_{i=1}^{K} R_i}{W}.$$

Since $\sum_{i=1}^{K} R_i$ is distributed $N(K\mu, K\sigma^2)$, the portfolio has expected return $E(\tilde{R}_P) = \frac{K}{W}\mu$ and variance $\sigma^2(\tilde{R}_P) = \frac{K}{W^2}\sigma^2$. From this, simple substitution (for this and other standard techniques, see Fama and Miller [1972], chapter 6, section IV) implies that

$$(1) \qquad E(\tilde{R}_P) = \frac{\mu}{\sigma}\sqrt{K(\tilde{R}_P)}.$$

In mean-standard deviation space, equation (1) defines a portfolio opportunity set, or the different risk and return combinations available to the investor. This set is illustrated by figure 1 (for $W = 5$, $\mu = 1$, and $\sigma = 1$). The opportunity set is disjointed, since the decision to add another loan is discrete. Depending on the shape of the indifference curves, the bank owner may choose to put none, all, or some of his wealth into bank loans. Figure 1 shows a typical case with an interior solution, illustrating quite clearly that the bank does not always wish to diversify. Stated another way, the portfolio return is distributed $N(\frac{K}{W}\mu, \frac{K}{W^2}\sigma^2)$, so that as the bank invests in more loans, the standard deviation increases as well as the expected return. Preferences determine which of them matters more.

An all-equity bank offers a nice illustration, but does not provide a very representative case. Even a stylized bank should have deposits.

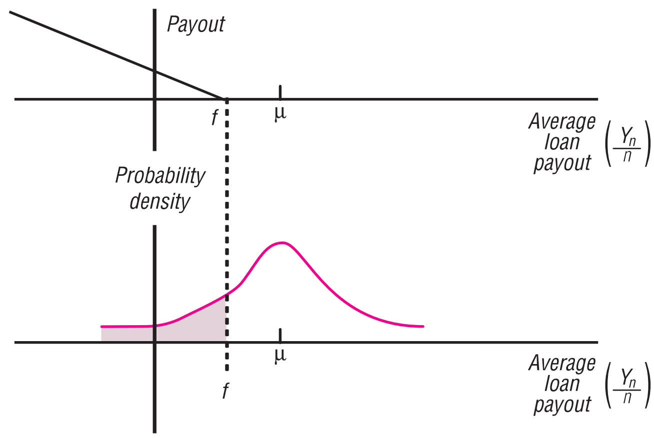
II. Should the Deposit Insurance Agency Want Banks to Diversify?

Allowing banks to take in deposits means allowing banks to fail. The return on assets may not cover the payments promised to depositors. Many countries provide some sort of deposit insurance, which guarantees the deposit. In the case where bank assets cannot cover the payments promised to depositors, the difference becomes a liability for the deposit insurance agency (which may be either public or private). This provides a natural focal point for our discussion, although what happens in reality is much more complicated. Actual banks raise money in many different ways, using several types of preferred stock, subordinated bonds, commercial paper, and both insured and uninsured deposits. What happens in bankruptcy is at best complicated and at worst unknown, because the courts must determine the validity of claims as diverse as offsetting deposits and the source-of-strength doctrine. A detailed consideration of how each class of investors views diversification, then, is beyond the scope of this article. Instead, to make what is admittedly a simple point, I concentrate on the deposit insurance agency, which ultimately bears the liability for bank failures.

The deposit insurance agency steps in if the realization of bank assets Y is too low to repay the face value of the debt F, that is, if $Y < F$. This is a fairly general formulation in that the assets producing Y may be funded by means other than deposits, but it is not completely general because it ignores the possibility that the deposit insurance agency may have priority over some investors. For the rest of the article, however, I will restrict myself to purely deposit-financed banks. This is at the opposite extreme from the discussion in section I, and hence provides a nice comparison.

FIGURE 2

Payout Function and Probability



SOURCE: Author.

What is the face value of the debt, F? With no capital, if the bank funds n projects, each requiring funds f, the face value is the sum of the deposits, $F = n \cdot f$. The payout of bank assets is likewise the sum over the different projects,

$$Y_n = \sum_{i=1}^{n} x_i ,$$

where n indexes the number of projects in which the bank has invested.

The Probability of Bank Failure

How likely is it that this bank will fail? The answer is $Pr(\sum_i x_i < F)$ or

(2) $\quad Pr(Y_n < n \cdot f).$

Assume that the x_i's are independent and identically distributed (i.i.d.), with mean $E(x_i) = \mu$; further assume that $f < \mu$, so that the face value of the debt is smaller than the expected payout of the assets.

We can rewrite expression (2) as

(3) $\quad Pr(\frac{Y_n}{n} < f)$

because the set $\{y : y < n \cdot f\}$ is the same as the set $\{y : \frac{y}{n} < f\}$.

The weak law of large numbers (see Shirayev [1984], theorem 2, p. 323; for a more elementary discussion, see Hogg and Craig [1978, chapter 5]) tells us that provided $E|x_i| < \infty$ and $E x_i = \mu$, then for all $\varepsilon > 0$,

$$Pr\{|\frac{Y_n}{n} - \mu| \geq \varepsilon\} \rightarrow 0 \text{ as } n \rightarrow \infty.$$

In particular, since $f < \mu$, $Pr(\frac{Y_n}{n} < f) < Pr(|\frac{Y_n}{n} - \mu| \geq \mu - f)$. That is, we can represent the values $\frac{Y_n}{n}$ below f as values that are more than $\mu - f$ away from the mean μ. Thus, as Diamond (1984) explicitly states, the weak law of large numbers implies that diversification by adding risks reduces the probability of bank failure.[2]

The Expected Value of the Deposit Insurance Agency's Liabilities

As Samuelson points out, a rational utility maximizer maximizes expected utility, not the probability of success. The probability of each outcome must be weighted by the utility of that outcome. As mentioned before, the failure of a \$10 billion bank may cost the deposit insurance agency more to resolve than that of a \$10 million bank.

In the simplest case of risk neutrality, expected utility corresponds to expected value. The first question, then, equivalent to assuming risk neutrality on the agency's part, concerns the expected value of the deposit insurance agency's payout.[3] Determining the expected payout value becomes a question of finding the expected value of a particular function. The deposit insurance agency must pay

$$(4) \quad \begin{cases} 0 & \text{if } Y_n \geq F, \text{ that is, if } \frac{Y_n}{n} \geq f \\ \text{and} & \\ F - Y_n & \text{if } Y_n < f, \text{ that is, if } \frac{Y_n}{n} < f. \end{cases}$$

Figure 2 plots the function along with a typical density function.

It is worth noting that the expected value of (4) is not a conditional expectation. If the set $A = \{Y_n : Y_n < F\}$, then the expected value of (4) is $P(A)\,E(Y_n | A)$ rather than $E(Y_n | A)$. A simple example will make this clear. Take a four-point distribution, $P(1) = P(2) = P(3) = P(4) = \frac{1}{4}$. Then $E(X) = \frac{1}{4}(1 + 2 + 3 + 4) = \frac{5}{2}$. Now define the function $g(x)$ as $g(x) = \{0, \text{ if } x \geq 2.5, \text{ and } x, \text{ if } x < 2.5\}$. Then $E[g(x)] = \frac{1}{4}(1 + 2) = \frac{3}{4}$, while $E[x | x \leq 2.5] = \frac{3}{2}$.

2 See also Winton (1997).

3 Although the calculation is not particularly difficult, I have not seen it before in the literature.

The question before us is what happens to the expected value of the deposit insurance agency's payments as the bank diversifies. Recall that the deposit insurance agency pays off if $\frac{Y_n}{n} < f$ or, equivalently, $\frac{\sum_{i=1}^{n} x_i}{n} < f$. By the strong law of large numbers, the mean of the partial sums $\sum_{i=1}^{n} x_i$ converges to a mass point on $E(x)$; that is, the sample means approach the true mean. Intuition suggests that the expected value of anything below the mean (and, *a fortiori*, anything below f) will have very little importance, that is, an expected value approaching zero. Put another way, as the bank gets very large the probability gets vanishingly small, and the average loan payoff falls below the amount promised to depositors; thus, the probability of a deposit insurer having to make a payoff gets so low that the expected value of that payoff approaches zero.

To establish this rigorously and to understand what diversification does to the expected value of the deposit insurance agency's payments requires a more formal approach, which is provided in the appendix. The intuition and results are less complicated, however. As a bank makes more loans, the expected value of deposit insurance agency payouts tends toward zero, and so the deposit insurance agency would like to encourage large banks. Diversification by adding loans works.

III. A Risk-Averse Deposit Insurance Agency

When risk aversion enters the picture, however, a deposit insurer can be worse off with larger banks. Strictly speaking, what Samuelson terms the fallacy of large numbers enters only with risk aversion. Applying it to an organization such as a deposit insurance agency, rather than to an individual, requires some justification. One possibility is that a publicly sponsored deposit insurance agency must obtain its funds by taxing people, either indirectly through its assessment on banks or directly by Congressional appropriation. Risk aversion by the deposit insurance agency may then reflect risk aversion on the part of those taxed, or nonlinearities associated with distortionary taxation. Alternatively, the risk aversion may result from the incentives, constraints, and information facing the organization: The managers running it may act risk averse, perhaps because their future income depends on their performance. (Of course, as Kane [1989] points out, this dependency may sometimes promote risk-seeking behavior, as in the FSLIC case.)

Conditions for the Fallacy

Samuelson (1963) shows that if a consumer rejects a bet at *every* wealth level, then he will always reject any independent sequence of those bets. Under the Samuelson condition, if the deposit insurance agency found one bank loan too risky, it would find a portfolio of any number too risky. It would be no happier to insure a large bank with many loans than a small bank with few loans.

Samuelson posits a rather stringent condition. It rules out, for example, constant relative risk-averse (CRRA) utility, because CRRA exhibits decreasing absolute risk aversion (DARA), and so some unacceptable gambles would become acceptable at higher wealth levels. Pratt and Zeckhauser (1987, p. 143) improve considerably on the condition with their notion of *proper* risk aversion. The conditions for proper risk aversion answer the question, "An individual finds each of two independent monetary lotteries undesirable. If he is required to take one, should he not continue to find the other undesirable?" In our problem, if the deposit insurer does not like the risk in a bank with one loan, then it will not like the risk in a bank with two loans. Proper risk aversion shares one defect with Samuelson's condition, however: It is difficult to characterize and difficult to determine whether a particular utility function satisfies the condition.

A slightly stronger condition with a simple characterization is proposed by Kimball (1993), whose *standard* risk aversion implies proper risk aversion. It thus applies a slightly stronger condition than is strictly necessary for the fallacy. If a utility function displays standard risk aversion, then an investor who dislikes a bet will also dislike a collection of such bets.

Kimball (1993) shows that necessary and sufficient conditions for standard risk aversion are (monotone) DARA and (monotone) decreasing absolute prudence. If the utility function in question has a fourth derivative, then these conditions (where, as before, W indicates a person's wealth) become

$$(5) \quad \frac{d}{dW}\left(-\frac{u'}{u''}\right) < 0 \text{ or } u^{(3)} > \frac{(u'')^2}{u'} > 0$$

and

$$(6) \quad \frac{d}{dW}\left(-\frac{u^{(2)}}{u^{(3)}}\right) < 0 \text{ or } u^{(4)} < \frac{(u^{(3)})^2}{u^{(2)}} < 0.$$

A key point here is that the individual finds each independent risk undesirable. (Kimball has a slightly weaker, more technical condition that he calls loss aggravation.) This certainly applies to the problem as we have defined it, because the payoff to the deposit insurance agency is nonpositive—at best, it pays nothing. This is not the only way to structure the problem, however, because the deposit insurance agency collects premiums from banks. A major strand in banking research has been to ascertain whether the insurance premiums are fairly priced, that is, whether they represent a tax or a subsidy on the bank (Pennacchi [1987], Thomson [1987]). The empirical results are mixed, varying by time period and by bank; in any case, they assume risk neutrality and so do not directly answer the question most relevant here. It makes sense, then, to think about both possibilities—the case where the deposit insurance agency finds insuring a single loan undesirable and the case where it finds insuring a single loan desirable.

In the first case, where the deposit insurance agency dislikes insuring an individual loan, expressions (5) and (6) provide sufficient conditions for the agency to dislike insuring any portfolio of such loans. That is, diversification by adding risks does not work; adding risks makes the insurance agency (guarantee corporation) worse off.

In the second case, where the deposit insurance agency likes insuring an individual loan, equations (5) and (6) do not help. Their derivation presupposes that the agency dislikes the risk it bears. For *favorable* bets, Diamond (1984) builds on Kihlstrom, Romer, and Williams (1981) to develop sufficient conditions for when the fallacy of large numbers is not a fallacy.

Diamond poses the problem in terms of risk premiums and notes that adding risks provides true diversification if it reduces the risk premium. That is, diversification works if the incremental premium for adding the second risky loan to the portfolio is lower than for adding the first (identical) risky loan. Kihlstrom, Romer, and Williams show how to handle risk aversion with two sources of uncertainty by defining a new utility function, given initial wealth W_0 and initial risky bet $\tilde{x}_1$, as

$$(7) \qquad v(x_2) = Eu(W_0 + \tilde{x}_1 + x_2).$$

Now $v(x_2)$, as defined in equation (7), can be treated as a utility function, so Diamond's question comes down to whether $u(\cdot)$ is more risk averse than $v(\cdot)$. If it is, then the risk premium for bearing the second risk is lower than for the first, and the fallacy of large numbers is not a fallacy.

Diamond derives two sufficient conditions under which $u(\cdot)$ will be more risk averse than $v(\cdot)$ Using Jensen's inequality, he shows that

$$(8) \qquad u^{(3)} > 0$$

and

$$(9) \qquad u^{(4)} > 0$$

are sufficient conditions when the risk has zero expected value. When the risk is freely chosen, he must append decreasing absolute risk aversion, equation (5), to these conditions. The reason is that a freely chosen gamble increases mean wealth, which requires us to augment the sufficient conditions.

Notice that inequalities (6) and (9) cannot hold simultaneously because (6) demands a negative fourth derivative and (9) demands a positive fourth derivative. The inequalities apply in different situations, however. Inequality (6) concerns unfavorable bets and describes when bearing one such risk makes the agent less willing to bear another. Inequality (9) concerns favorable bets and describes when bearing such a risk makes the agent even more willing to bear another. The conditions really answer two quite different questions. Since each inequality provides a sufficient but not necessary condition, any contradiction between the answers is more apparent than real.

An important caveat is that this is consciously a partial equilibrium analysis, concentrating on the risk of a single bank. If that bank grows by absorbing smaller ones, the total number of loans insured by the system does not change. A bank merger does not change the total loans insured by the agency, but merely redistributes them. In a bank with many loans, the profitable loans may offset the unprofitable, lessening the guarantor's liability. Since the deposit insurance agency does not share in the positive profits, it cannot undertake a similar offset if the loans are in different banks. In an extreme case, if each bank had only one loan, the insurer would make payments on every nonperforming loan. If all loans were in one bank, the insurer would make payments only if the aggregate loan loss were too large.

This is not the only scenario, however. The bank may grow at the expense of nonbank intermediaries or by making loans that would not be made without the guarantee. Either case results in an increase of total loans guaranteed by the deposit insurance agency, increasing its liabilities as it takes on new loans that must get insured.

TABLE 1

Premium Computation

Risk aversion, *a*	Face value	π_1	π_2
0.1	1	0.006	0.018
1.0	1	0.066	0.193
10.0	1	0.392	0.979

SOURCE: Author.

An Exponential Example

A simple example can serve to illustrate some of the subtleties involved. To show what can happen, I use an exponential utility function and an exponential distribution. The exponential distribution keeps the algebra simple because sums of exponentials are gamma distributions.[4] Exponential utility exhibits constant, rather than decreasing, absolute risk aversion. It does not satisfy the sufficiency conditions of Kimball ([5] and [6]) or of Diamond ([8] and [9]).

Whether diversification helps or hurts depends on the risk premium. If the risk premium decreases as the investor adds i.i.d. risks, diversification helps. If the risk premium increases, diversification hurts. The simplicity of the example allows us to calculate the risk premium explicitly.

Recall from equation (4) that, for one loan, the deposit insurance agency pays nothing if the loan's payoff exceeds its face value; otherwise it pays the difference. Denoting this function by $g(x)$ (as in the appendix), the risk premium is defined as the π_1 that satisfies

$$(10)\quad u\{W_0 - E[g(\tilde{x})] - \pi_1\} = Eu[W_0 - g(\tilde{x})].$$

With x following the simplest exponential distribution, e^{-x}, the expected value in (10) becomes

$$Eg(\tilde{x}) = (f - 1) + e^{-f}.$$

Using exponential utility of the form e^{-aW} allows us to solve for π_1:

$$(11)\quad \pi_1 = (f - 1 + e^{-f}) - \frac{1}{a}\log[e^{-f} - \frac{1}{1+a}(e^{-f} - e^{af})].$$

For two loans, $g(x)$ is zero if x exceeds $2f$ and $2f - x$ otherwise. The random variable x, as the sum of two independent exponentials, has a gamma distribution,

$$x \sim \frac{xe^{-x}}{\Gamma(2)} = \frac{x}{2}e^{-x}.$$

The expected value then becomes $Eg(\tilde{x}) = 2(f - 1) + 2(f + 1)e^{-2f}$. Solving for the risk premium implicitly defined by $u\{W_0 - E[g(\tilde{x})] - \pi_2\} = Eu[W_0 - g(\tilde{x})]$ using $\pi_2 = Eg(x) - \frac{1}{a}\log Ee^{ag(x)}$ yields

$$(12)\quad \pi_2 = 2(f - 1) + 2(f + 1)e^{-2f} - \frac{1}{a}\log \{e^{-2f}\frac{1}{1+a^2}e^{2af} [1 - (1 + 2f(1+a))e^{-(1+a)f}] + (1+2f)e^{-2f}\}.$$

To complete the example, set f, the face value of the debt, to 1, and compute the premium for several values of risk aversion, evaluating (11) and (12).

The example in table 1 illustrates that diversification does not work in every case. The required risk premium for two loans is higher than for only one: It even exceeds twice the risk premium for one loan. As risk aversion increases, the risk premiums also increase. Although conditions (5) and (6) are not satisfied, the deposit insurance agency dislikes adding more independent risks to its portfolio.

IV. Conclusion

Discussions of banking have been obscured by a false analogy with portfolio theory. A bank diversifies differently than does a mutual fund, adding risks rather than subdividing them. Using the weak law of large numbers to establish that diversified banks have a lower expected failure rate neglects the deeper question of whether this represents a decrease in economic risk. To clearly pose that question is the main point of this article.

Just because a bank is less likely to fail, it is not necessarily less risky. If the insurer, or owner, is risk neutral, a more complicated argument shows that the bank is less risky in the sense of expected value. With risk aversion, however, the question becomes ambiguous. As a practical matter, sufficient conditions exist, and the combination of exponential utility with exponential distributions provides a tractable framework for further exploration.

4 In general, bank loans are more likely to be negatively skewed, while the exponential is positively skewed, so this example is not meant as a realistic description of actual returns.

Appendix

Let each random variable be defined on the probability space (Ω, F, P) and identify Ω with R, the real numbers, without loss of generality. The random variables are then functions on this space, $X_i(\varpi)$, and define $Z_n(\varpi)$ as

$$Z_n(\varpi) = \sum_{i=1}^{n} \frac{X_i(\varpi)}{n}.$$

Next, define the function $g(\varpi)$ as

$$g(\varpi) = \begin{cases} f - X(\varpi) & \text{if} \quad X(\varpi) \leq f \\ \text{and} & \\ 0 & \text{if} \quad X(\varpi) > f. \end{cases}$$

Note that we can think of the expectation $E[X(\varpi)]$ as a random variable, and so $g(E[X(\varpi)] = g(\mu) = 0$, since f < μ. Further define $g_n(\varpi)$ as $g[Z_n(\varpi)]$.

The value of diversification can then be expressed by saying that as n approaches infinity, the expected value of $g(Z_n)$ approaches zero, or

$$\text{(A1)} \quad \lim_{n \to \infty} \int g_n(\varpi) = g(\mu) = 0.$$

To prove (A1), we use Lebesgue's dominated convergence theorem (Royden [1968], p. 229), which says that if $h(\varpi) \geq 0$ is integrable, if $|g_n(\varpi)| \leq h(\varpi)$, and if $g_n(\varpi) \overset{a.s.}{\to} g(\varpi)$, then

$$\lim_{n \to \infty} \int g_n(\varpi) = g(\varpi).$$

The theorem first requires that we prove $g_n(\varpi) \overset{a.s.}{\to} g(\mu)$. To do so, we use the strong law of large numbers for i.i.d random variables (see Breiman [1992, p. 52, theorem 3.30]), which says that for i.i.d. X_1, X_2, X_3 ..., if $E|X_1| < \infty$ then $\frac{\Sigma X_i}{n} \overset{a.s.}{\to} E(X_1)$, where $\overset{a.s.}{\to}$ denotes almost sure convergence, that is, convergence on all but a set of measure (probability) zero.

Hence, given an ϖ, except for a set of measure zero, we have that for any $\varepsilon > 0$, there exists an N such that if $n > N$, $|Z_n(\varpi) - \mu| < \varepsilon$. Choose $\varepsilon < \mu - f$, which implies that if $|Z_n(\varpi) - \mu| < \varepsilon$, then $Z_n(\varpi) > \mu - \varepsilon > f$. This, with the definition of g, in turn implies that $g_n(\varpi) = 0$. For this ϖ, then, $g_n(\varpi) = g(\mu) = 0$, and, *a fortiori*, $|g_n(\varpi) - g(\mu)| < \varepsilon$. Since $g_n(\varpi) \to g(\mu)$ for each ϖ where $Z_n \to \mu$, the almost sure convergence of the strong law implies the almost sure convergence $g_n(\varpi) \overset{a.s.}{\to} g(\mu)$.

All that remains to be shown is the existence of the integrable bound $h(\varpi)$. For this, use $|X_i(\varpi) + \mu - f|$, which bounds g_n and is integrable because $E|X_1| < \infty$ is a hypothesis of the strong law. Hence, Lebesgue's dominated convergence theorem applies.

As a bank makes more loans, the expected value of deposit insurance agency payouts tends toward zero. Diversification works.

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