Has the Long-Run Velocity of M2 Shifted? Evidence from the P* Model

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Introduction

Since early 1990, M2 has grown more slowly than suggested by its historical relationships with both income and opportunity cost, the latter measured relative to short-term market interest rates. During the first part of this period (1990–91), although historical relationships with its opportunity cost suggested a significant decrease, M2 velocity remained quite close to its long-run average value of about 1.65. During 1992, M2 velocity increased sharply while its opportunity cost apparently decreased further.

This behavior suggests that the long-run velocity of M2, or V-Star (V*), may have risen, perhaps as a result of changes in the money supply process, such as the stricter regulatory environment facing depository institutions. If V* has indeed increased, then the P-Star (P*) model, which assumes no change in M2's long-run velocity, should have persistently underpredicted inflation over the last three years. We find, however, that the model has quite accurately predicted the deceleration of inflation since 1990.

The paper also presents an extensive analysis, based on simulation of the P* model under a variety of alternative hypotheses regarding possible shifts in long-run velocity, that provides little support for the view that V* has changed. Our findings reinforce other recent research concluding that the pickup in M2's velocity may be largely explained by increases in an alternative opportunity cost measure based on long-term market rates.[1] If correct, these results suggest that sluggish M2 growth over the last three years contributed to both the slow pace of economic activity and the significant progress toward price stability. In addition, they suggest the potential for a rebound of M2 growth during 1993 as long-term rates fall and M2 velocity growth decelerates.

I. The P* Model[2]

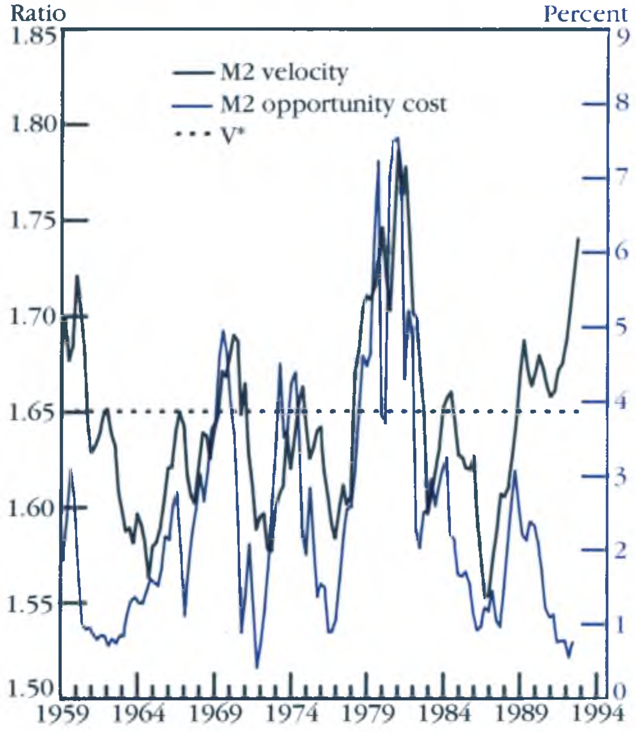
The P* model links the behavior of the price level to the growth of M2 by imposing two hypotheses on the equation of exchange, $MV = PQ$: (i) real output Q_t fluctuates around potential real output Q_t^* over long periods, and (ii)

■ 1 See Feinman and Porter (1992).

■ 2 See Hallman, Porter, and Small (1991).

FIGURE 1

M2 Velocity and Opportunity Cost



SOURCE: Authors' calculations.

velocity V_t has an equilibrium level V*, independent of time, that it tracks in the long run.[3] With these assumptions, P^*_t is defined as the long-run equilibrium price level that could be supported by the current level of the money stock (M_t) if current output (Q_t) settled down to this period's level of potential output (Q^*_t):

$$(1) \qquad P^*_t = \frac{M_t V^*}{Q^*_t}.$$

Our assumptions regarding V_t and Q_t imply that if money remains fixed at M_t, then P_t will fluctuate around P^*_t.

For policymakers, P^*_t provides an index in each period t of the cumulative long-run impact of money on the price level. The difference between the current price level and P^*_t can provide a leading indicator of future acceleration or deceleration of inflation as $P_t \rightarrow P^*_t$. Hallman, Porter, and Small (1991) show that the P* model can be derived as the reduced form of a special case of the expectations-augmented Phillips curve. In this case, changes in the inflation rate follow a simple autoregressive process augmented by the lagged price gap, $p_t - p^*_t$:

$$(2) \qquad \Delta\pi_t = \alpha(p_{t-1} - p^*_{t-1}) + \sum_{1}^{4} \beta_i \Delta\pi_{t-i} + \varepsilon_t,$$

where lower-case letters denote natural logs, π_t is the inflation rate, and $\Delta\pi_t$ is the quarterly change in the inflation rate. The *existence* of P^*_t depends critically on the validity of assumptions (i) and (ii). The assumption that real output fluctuates around a growing level of potential output is not controversial; indeed, measures of potential output are often constructed so as to ensure the validity of this assumption. The velocity assumption is more open to dispute.[4]

The constant velocity assumption of the P* model is motivated, in part, by the tendency of M2's velocity since 1955 to fluctuate around 1.65, trending neither up nor down (see figure 1). Velocity at times has remained above its long-run average for several years, and recent increases do not appear particularly unusual in this respect. The assumption is likewise motivated by the close historical correspondence between M2's velocity and its opportunity cost that prevailed through 1989, also shown in figure 1.[5] During this period, sustained deviations of velocity from its long-run average tended to be accompanied by comparable deviations of opportunity cost from its long-run average.[6] The tendency for M2 opportunity cost to return to its long-run average provided an economic rationale for M2 velocity to do the same. Empirical models

3 Equivalent alternative assumptions are (i) M2 velocity is a stationary stochastic process, or (ii) all shocks to the level of M2 velocity are transitory. In a nonstochastic model, P will converge to P*. For a statement of the modern quantity theory, see Dewald (1988). For antecedents to P*, see Humphrey (1989).

4 See, for example, Kuttner (1990) and Pecchenino and Rasche (1990). As Pecchenino and Rasche note, the inflation dynamics in Kuttner's paper are incorrect because he confuses Q and Q* in the P* model.

5 The opportunity cost shown equals the difference between the three-month Treasury bill rate (on an annualized coupon-equivalent basis) and a share-weighted average of the own rates paid on the components of M2. See Moore, Porter, and Small (1991). Note that their series begins in 1959.

6 M2's velocity and its opportunity cost have moved in opposite directions before. In 1960, velocity rose while opportunity cost fell; in 1983, velocity fell while opportunity cost rose. The duration of the most recent divergence appears unusual, however. Note that the vertical distance between the lines in the figure is not meaningful.

of M2's opportunity cost developed by Federal Reserve Board staff during the 1980s seemed to confirm this long-run behavior.[7] During the past three years, however, M2's velocity and opportunity cost have diverged sharply, with the former increasing as the latter has decreased. This divergence raises the question of whether equilibrium velocity has indeed changed.[8]

II. Using the P* Model to Identify Changes in V*

While the P* model was originally offered as a link between inflation and money growth, its inverse provides a test of one of its primary assumptions: the constancy of long-run M2 velocity.[9] If the long-run velocity of M2 has in fact increased during the last three years, predictions of inflation from the original P* model (which assumes that long-run velocity has not changed) should be inferior to predictions from a model that incorporates the "true" change in V*. This simple insight immediately suggests a testing strategy for evaluating alternative hypotheses regarding putative shifts in V*: Construct the various P_t^* time series corresponding to alternative velocity assumptions; use a battery of goodness-of-fit and forecast accuracy tests to compare the relative forecasting performance of the model under the alternative assumptions; and accept the velocity assumption(s) most consistent with the data or, in other words, the one that yields the best model forecasting performance.[10] Suppose, for example, we learn that V* increased 6 percent in mid-1989, to 1.75 from 1.65, and has remained at that value. Using equation (1), we can construct an alternative time series of P_t^* values that will also have shifted up by 6 percent, consistent with the higher velocity. Use of this new, more accurate measure of the equilibrium price level should improve the accuracy of inflation forecasts from the P* model.

Although the divergence of velocity and opportunity cost shown in figure 1 suggests that V* may have increased, the curves tell us little about the precise form of the change. In our analysis, we consider five alternative hypotheses concerning V* during 1989–92:

- It remained at its 1955–89 average value of 1.65.
- It increased 6 percent in 1989:IIIQ. This quarter was chosen based on the presence of two high-visibility events that marked the end of a decade of regulatory forbearance for undercapitalized depository institutions: passage of the Financial Institutions Reform, Recovery, and Enforcement Act (FIRREA) and the first resolutions of insolvent thrifts by the Resolution Trust Corporation. The depository sector, facing a stricter regulatory environment and the need to improve its capital ratios, might be expected to grow more slowly or even to contract as a result.
- It shifted upward by 2¼ percent each year in 1990 and 1991 and by 2½ percent in 1992. These are approximately the size of the forecast errors from the Federal Reserve Board staff's model of M2 demand based on income and M2's opportunity cost relative to short-term market rates.[11]
- It began increasing at a 1½ percent annual rate in 1990:IQ.
- It began *decreasing* at a ½ percent annual rate in 1990:IQ. This scenario is included for two reasons. First, it directly challenges the widely held conjecture that structural changes affecting depository intermediation during the past three years must have increased M2's long-run velocity. Second, it admits the possibility that the decrease in the inflation rate since 1989 has occurred largely as might have been expected (and perhaps even a bit more rapidly than expected), given the slow growth of M2 and the significant output gap.

■ 7 See, for example, Moore, Porter, and Small (1991). These models typically assumed the existence of a long-run fixed spread between the offering rate on a particular type of deposit and a short-term risk-free market rate (for example, the three-month Treasury bill). A similar assumption was made for money market mutual fund yields. The size of the equilibrium spread presumably depended on both demand and supply factors, including regulatory (capital) requirements facing the intermediary, deposit insurance premiums, and the liquidity of the deposit.

■ 8 It also raises the possibility that M2's opportunity cost was incorrectly measured. Recent research by other Board staff suggests that this may have been the case. A new opportunity cost measure that includes a long-term Treasury rate and a rate on consumer loans appears to track M2 velocity during 1984–92. These models are highly preliminary, however, and do not feature the long-run error-correction behavior of previous Board staff models. See Feinman and Porter (1992).

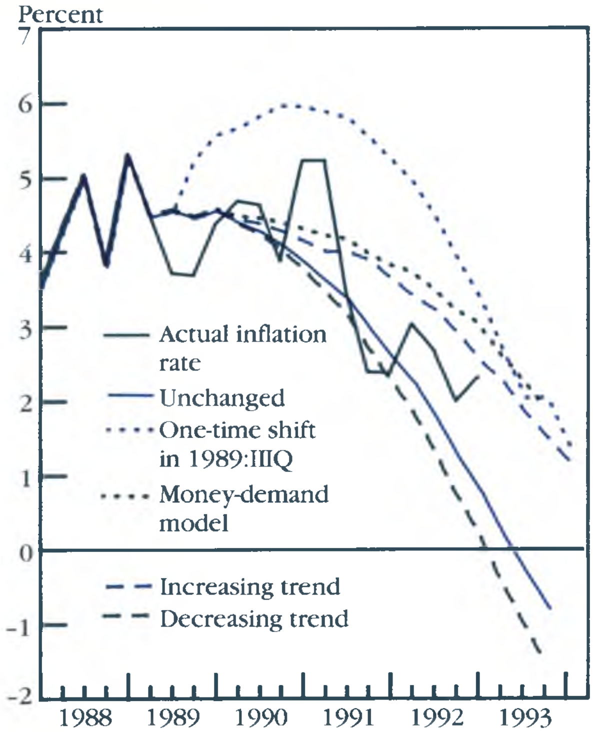
■ 9 The antecedents discussed by Humphrey (1989) also view P*-type models primarily as models of the inflation rate. A constant (or very slowly changing) velocity of money is assumed almost without mention. This is reminiscent of Irving Fisher's quantity theory model. See Laidler (1985), chapter 5.

■ 10 This is somewhat more complicated than stated, since the tests are non-nested. Below, we generate the empirical sampling distribution for each individual statistic.

■ 11 See Feinman and Porter (1992), figure 1.

FIGURE 2

Simulated Inflation Rates from Alternative V* Hypotheses



NOTE: First simulated value under all five hypotheses is 1989:IIIQ.
SOURCE: Authors' calculations.

Each of the V_t^* hypotheses suggests a corresponding P_t^* series, constructed according to equation (1) using the hypothesized V_t^*. Under the null hypothesis that V* has not changed from its 1955–89 level, the inflation-rate path for each P_t^* series is given by equation (2). Actual data are used through 1992:IVQ.[12]

Under the five alternative V* assumptions, dynamic simulation of the P* model, shown in equation (2), yields the five inflation-rate paths shown in figure 2. Each simulation begins in 1989:IIIQ and is nonstochastic; that is, all of the ε_t error terms in equation (2) are set equal to zero over the simulation period. During the past three years, the actual inflation rate generally has been between the rates suggested by the unchanged or declining V* scenarios and those suggested by a trend increase in V*. On balance, the inflation rate appears to have most closely followed the path given by the constant V* hypothesis, at least through 1992:IIIQ. Inflation in 1992:IVQ, however, was higher than forecast by the P* model with V* unchanged.

The nonstochastic simulations shown in figure 2, though suggestive of an unchanged long-run M2 velocity, are not capable of answering our question about a shift in equilibrium velocity. In particular, the simulations assume that no stochastic factors influence the evolution of the inflation rate ($\varepsilon_t = 0$ for all t), including possible random fluctuations in M2 velocity, when M2 velocity in fact has a relatively high variance. From a statistical viewpoint, the data shown in figure 2 represent only one "draw" from the universe of ways velocity and inflation might have evolved under each alternative hypothesis regarding V*. An adequate test must incorporate the inherent randomness and variability of economic variables. Furthermore, comparing the performance of several models (or, in our case, the same model using alternative estimates of P_t^*) solely on observed, actual data leaves unanswered a number of interesting questions, such as:

- Suppose, in fact, that inflation accelerates in 1993. How long might it take before incoming data reveal a change in V*? At what point, if any, will the statistical evidence compel us to reject the hypothesis that the long-run velocity of M2 has not changed?
- Which hypothesis regarding M2 velocity is believed by financial market participants? Are further decreases in long-term market interest rates waiting for clearer signals regarding future M2 velocity?

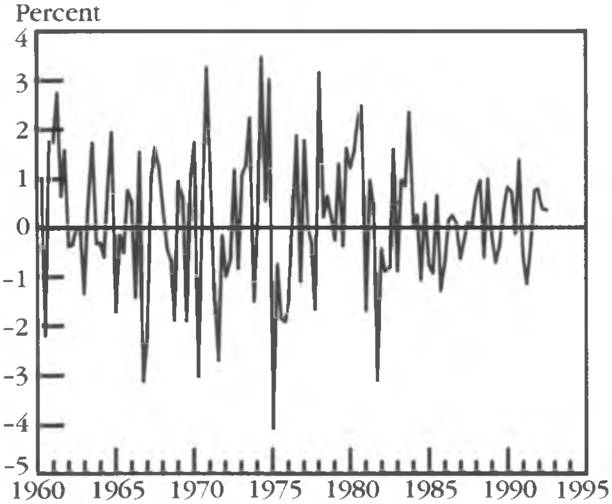
We conducted a simulation study to investigate these issues as well as the overall acceptability of the V* hypotheses.[13] Our simulation design generates, for each of the five V* hypotheses, 1,000 simulated paths for P_t from 1989:IIIQ through 1994:IVQ. Each path is the result of a stochastic simulation of the P* model under the appropriate velocity hypothesis. The stochastic innovations ε_t for the simulations are drawn from a normal distribution scaled to have a mean of zero and a standard deviation of about two-thirds of 1 percent at an annual rate. This corresponds to the smaller post-1986 variance of the residuals from the P* model when estimated over 1960:IIIQ–1992:IIIQ, as shown in figure 3. (A formal statistical test strongly rejects equality of the variance of the residuals before and after 1986.) Although the reason for this smaller variance is not apparent, it may be due to less variance in the expected inflation rate after 1986. Our simulations assume that the future

■ 12 After 1992:IVQ, M2 and Q_t^* are assumed to grow at annual rates of 4.5 percent and 2.5 percent, respectively.

■ 13 The simulation methodology also allows us to address some issues of interest mainly to econometricians, such as assessing how well various statistics perform in detecting the kinds of changes in which we are interested.

FIGURE 3

P* Model Residuals



SOURCE: Authors' calculations.

variance of the random innovations will resemble the smaller post-1986 period.

When the precise specification of alternative hypotheses in a testing situation is uncertain, as it is for hypotheses regarding changes in V*, the choice of an appropriate test statistic is difficult. Some hypotheses suggest tests for omitted dummy variables (such as a discrete shift in the level or a nascent time trend), while others suggest the use of more general tests based on forecast errors. Along each simulated path P_i^*, we computed the values of 12 test statistics, including tests for omitted variables as well as tests for general misspecification based on one-step-ahead forecast errors. Our statistics fall into four categories:

- *Lagrange multiplier (LM) tests* for an omitted variable in equation (1). *Lmshift* tests for a post-1989:IIQ shift dummy variable, *lmtrend* for a time trend beginning in 1990:IQ, and *lmboth* for both the shift and trend.
- *Chow tests* for a change in the forecast error variance, relative to the variance of the disturbance ε_t in the simulations, perhaps due to a change in V*. *Ch4*, *ch8*, and *ch12* are based on the last four, eight, and twelve forecast errors, respectively.
- *Random walk tests* for autocorrelation in the forecast errors due to misspecification of the model, including a structural change. *Rw4*, *rw8*, and *rw12* are based on the last four, eight, and twelve forecast errors, respectively.
- *Binomial tests* for an unusually high number of positive forecast errors, due to the assumed V* being too small. *Bn4*, *bn8*, and *bn12* are based on the last four, eight, and twelve forecast errors, respectively.

The statistics are discussed further in the appendix. For each of the 1,000 replications, we calculated and stored the values of the statistics for each quarter from 1990:IQ through 1994:IVQ.

For any particular quarter within our simulation period, the degree of support for a V* hypothesis may be inferred by comparing the values of the statistics in that quarter to the simulated distributions of possible outcomes. The simulated distributions indicate the range of values of the statistics that could result from random, unobserved influences.[14] If the value of a statistic falls outside the central area of the corresponding simulated distribution, we tend to reject that particular hypothesis.

Our results for 1992:IVQ are shown in table 1 and figure 4. Values of the test statistics calculated from data for 1992:IVQ, the most recent quarter for which we have preliminary gross domestic product (GDP) data, are shown in column 2 of the table. Columns 3-7 display a count of the number of model replications (out of 1,000) wherein a test statistic took on a value less than that shown in the second column. The third column, for example, summarizes our simulations under the hypothesis that V* has not changed from its historical average value of 1.65. Each entry in the column shows the number of replications for which the value of the statistic named in the first column was less than or equal to the 1992:IVQ value, shown in the second column.

Consider, for example, the interpretation of the *lmshift* statistic for 1992:IVQ as summarized by the first row of table 1. The value of this statistic calculated from 1992:IVQ data is 0.026. The third column indicates that the *lmshift* statistic was less than 0.026 in 266 of the 1,000 replications of the unchanged V* scenario. According to this hypothesis, then, 0.026 appears to be neither unusually large nor small. In contrast, the entry in the fourth column tells us that observing an *lmshift* statistic value as small as 0.026 would be highly unusual if V* had in fact increased by a one-time 6 percent shift in 1989:IIIQ. A value that

14 In other words, the distributions shown are the empirical sampling distributions of the statistics.

TABLE 1

Observed Values of Test Statistics in 1992:IVQ and Cumulative Frequency of Occurrence of those Values in Simulation

Test Statistics and 1992:IVQ Values		Number of Replications wherein Value of Statistic Is Less than in 1992:IVQ — V* Hypothesis				
Statistic (1)	Value (2)	H1 No Change (3)	H2 One-Time Shift (4)	H3 Money-Demand-Model Shift (5)	H4 1½ Percent Trend (6)	H5 −½ Percent Trend (7)
LM tests						
lmshift	0.026	266	0	42	135	240
lmtrend	0.233	719	8	78	263	636
lmboth	0.377	573	1	62	229	537
Chow tests						
ch4	4.10	626	278	227	429	576
ch8	12.9	879	351	530	752	866
ch12	15.0	753	84	388	611	727
Random walk tests						
rw4	3.86	960	595	494	752	926
rw8	0.475	536	9	42	184	460
rw12	1.16	727	4	166	377	664
Binomial tests[a]						
bn4	4	947:1,000	612:1,000	541:1,000	719:1,000	979:1,000
bn8	5	648: 869	18: 136	63: 274	170: 473	778: 940
bn12	8	816: 932	16: 95	254: 497	431: 699	895: 974

a. The two values correspond to the value of the statistic being, respectively, either strictly less than, or less than or equal to, the value in column 2.
NOTE: Each entry is the number of replications out of 1,000 trials.
SOURCE: Authors' calculations.

low never occurred in 1,000 replications of the "6 percent shift" scenario.

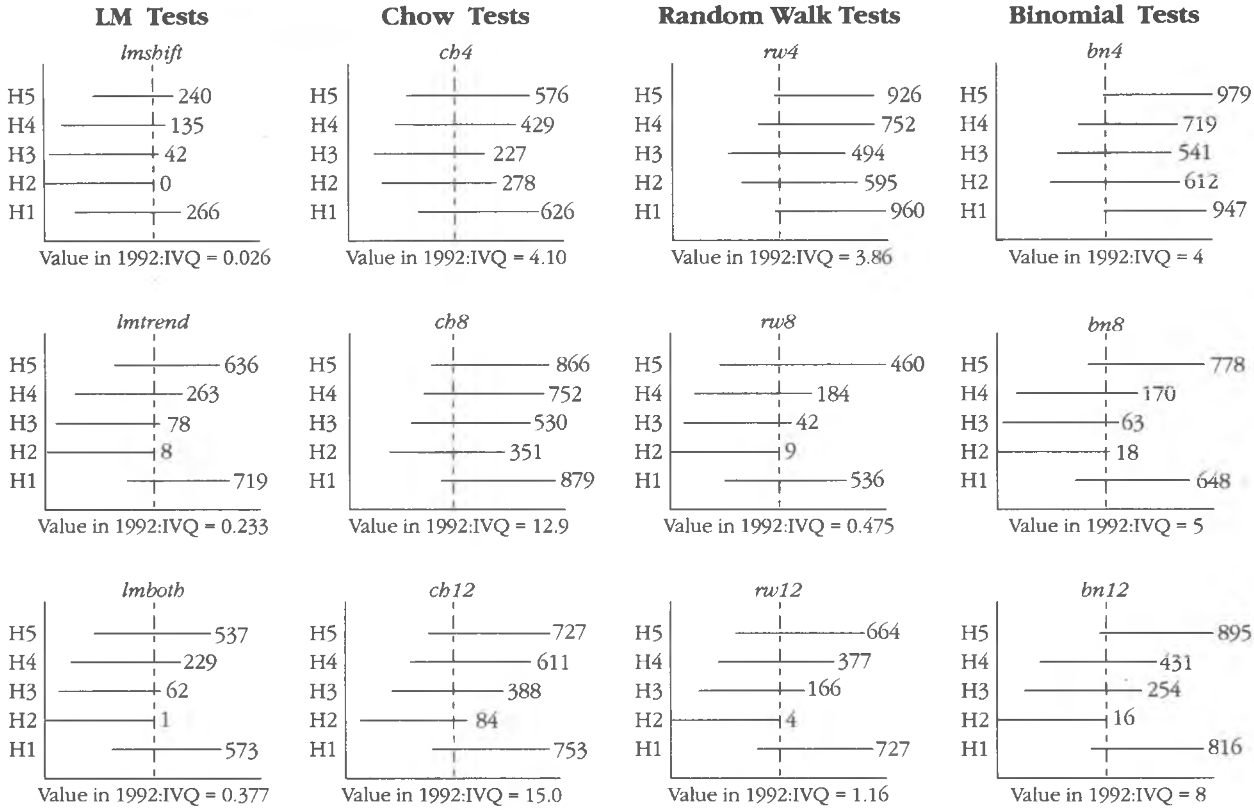
Table 1's test statistics and simulation outcomes are summarized in figure 4, with each panel corresponding to one of the 12 statistics. Each horizontal line segment in each panel represents the 1,000 replications of the P* model under one of the five alternative V* hypotheses, denoted H1–H5. A hypothesis regarding V* is judged more or less acceptable (in other words, consistent with the data) as the horizontal line segments for that hypothesis tend to be centered around the vertical dotted lines denoting the values of the statistics calculated from 1992:IVQ data. Overall, the hypotheses that V* has not changed (H1) or has been decreasing slowly (H5) appear to be highly consistent with the data, with the 1992:IVQ value falling near the midpoint of the distribution of simulated values for a number of the statistics. The hypothesis of a one-time shift in 1989:IIIQ (H2) is soundly rejected. The hypothesis that M2 velocity shifted as suggested by the Federal Reserve Board staff's money-demand model (H3) appears less consistent with the data than the hypothesis of a steady upward trend (H4), which seems fairly plausible. Neither of the trending V* hypotheses (H3 and H4) appear to be as consistent with the data as the unchanged and falling hypotheses (H1 and H5), however.

Market participants' inflation expectations appear to reflect acceptance of a significant increasing trend in M2 velocity, despite the deceleration of inflation over the past three years.[15] The January Blue Chip consensus forecast, for example, calls for the GDP implicit price deflator

15 Chairman Greenspan's latest Humphrey–Hawkins report to the Congress in February of this year appears to endorse this view, as does the FOMC's reduction of its 1993 M2 target growth ranges. To avoid such bias, we use a Blue Chip forecast published before these were announced.

FIGURE 4

Summary of Simulation Experiments for 1992:IVQ



NOTE: Each horizontal line represents 1,000 replications of the P* model under either H1, H2, H3, H4, or H5. Shown after each line is the number of replications wherein the value of the statistic is less than in 1992:IVQ.
SOURCE: Table 1. H1–H5 correspond to columns 3–7 in the table.

to increase at about a 2.7 percent rate during the first half of 1993, versus its 2.1 percent pace in the second half of 1992. The inconsistency between the paths of the price level implied by the Blue Chip forecast and the P* model with an unchanged V* is evident in table 2. Values of our test statistics calculated from projected values of P_t for 1993:IIQ that are based on this forecast are shown in column 2.[16] The entries in column 3 show that many of our statistics will reject the constant V* hypothesis if inflation follows the Blue Chip forecast. The complete set of test results is displayed in figure 5. Ignoring the Chow tests and the *bn4* statistic, the trending V* hypotheses H3 and H4 appear fully consistent with the Blue Chip forecast.[17]

Initially, it may appear somewhat surprising that the statistical support for the constancy of V* is so sharply changed by inclusion of the two additional quarters from the Blue Chip consensus forecast. The reason for this sensitivity is that the consensus inflation forecast is very different from the forecast suggested by the P* model with an unchanged V*. P_t^* is currently more than 8 percent below P_t, so the P* inflation model — equation (2) — forecasts that inflation will continue to decelerate over the next several quarters from its 2.1 percent pace in 1992:IIH. The consensus forecast, by contrast, predicts an acceleration during the first half of 1993. The message of table 2 is that such an acceleration is highly unlikely unless equilibrium velocity has been trending up for some time and has escaped

■ 16 See *Blue Chip Economic Indicators,* Sedona, Arizona, January 10, 1993, p. 5.

■ 17 Neither the Chow tests nor the *bn4* test has much power against the hypothesis being tested, as is evident from examination of table 3.

TABLE 2

Projected Values of Test Statistics in 1993:IIQ and Cumulative Frequency of Occurrence of those Values in Simulation

Test Statistics and 1993:IIQ Values		Number of Replications wherein Value of Statistic Is Less than Projected 1993:IIQ Value				
		V* Hypothesis				
Statistic (1)	Value (2)	H1 No Change (3)	H2 One-Time Shift (4)	H3 Money-Demand-Model Shift (5)	H4 1½ Percent Trend (6)	H5 −½ Percent Trend (7)
LM tests						
lmshift	0.318	791	2	124	368	693
lmtrend	1.40	992	198	318	750	969
lmboth	1.81	982	69	387	774	962
Chow tests						
ch4	10.8	979	885	716	888	971
ch8	17.3	970	731	659	867	964
ch12	22.8	966	514	701	866	954
Random walk tests						
rw4	8.91	1,000	961	817	951	993
rw8	3.54	955	273	184	519	898
rw12	3.63	960	47	199	544	903
Binomial tests[a]						
bn4	4	943:1,000	711:1,000	472:1,000	668:1,000	980:1,000
bn8	6	879: 965	224: 540	149: 439	340: 687	959: 992
bn12	8	824: 952	34: 125	97: 294	279: 550	931: 985

a. The two values correspond to the value of the statistic being, respectively, either strictly less than, or less than or equal to, the value in column 2.
NOTE: Each entry is the number of replications out of 1,000 trials.
SOURCE: Authors' calculations.

detection by our tests for 1992:IVQ.[18] Such an acceleration of inflation would provide significant evidence against the constancy of V*.

III. Evaluating Alternative, Less Specific Hypotheses

At this point, a true believer in higher equilibrium velocity will object that, while our approach mostly rejects the specific shifted and upward-trending V* hypotheses outlined above, this does not conclusively prove that V* has not changed. Simulations with slower growth trends in V* or ones that started later than 1990:IQ, for example, might not be rejected.

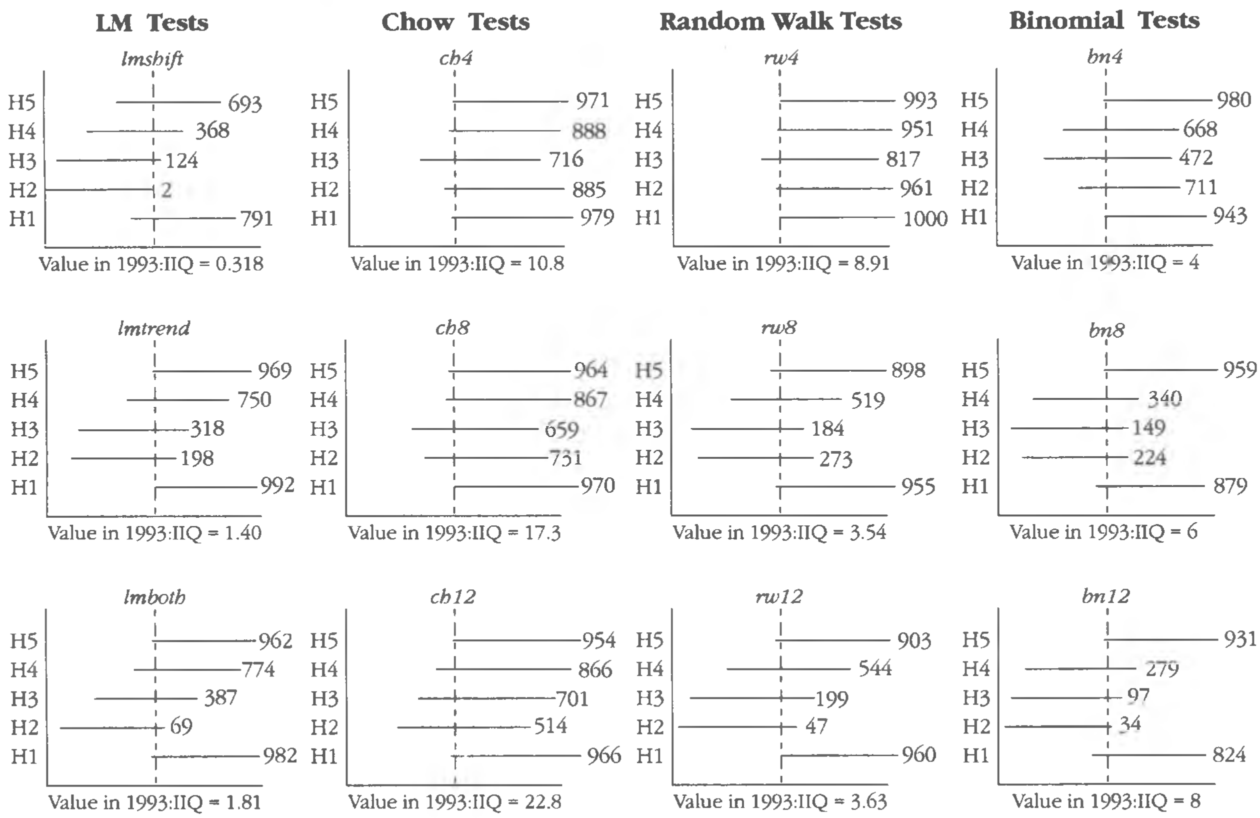
The objection has merit. Our experiments consider only a few specific alternative hypotheses. To evaluate rigorously, using our stochastic simulation method, the evidence for or against a less specific hypothesis—such as "V* shifted sometime in the late 1980s or early 1990s"—would require repeating our experiments using alternative models with shifts beginning in 1989:IVQ, and again with shifts beginning in 1990:IQ, and so on. The number of required simulations increases even further if we allow for a number of trend growth rates, rather than the 1.5 percent annual V* growth used here.

We can, however, address the issue indirectly. Our test statistics should be valuable in detecting

■ 18 Alternatively, it may be that the variance of the innovations has increased. One way to see how inference about the constancy of V* depends on the assumed variance of the innovation process is to note that, if the Blue Chip forecast is correct, the P* model's 1993:IQ forecast of 1.2 percent will miss by about 1.6 percent. Since we have assumed an innovation standard error of 0.6 percent, this is about a two-and-one-half-standard-deviation miss, which is unusual. If the innovation standard deviation were instead (say) 1.6 percent, the forecast error would be only about one standard deviation, which is not so odd.

FIGURE 5

Summary of Simulation Experiments for 1993:IIQ



NOTE: Each horizontal line represents 1,000 replications of the P* model under either H1, H2, H3, H4, or H5. Shown after each line is the number of replications wherein the value of the statistic is less than in 1993:IIQ.
SOURCE: Table 2. H1–H5 correspond to columns 3–7 in the table.

shifts in V* that begin in other time periods or that follow time paths with somewhat different shapes than those considered above. According to table 3, when V* is subjected to a one-time upward shift of 6 percent, within six quarters the best of our test statistics (using the 5 percent critical values shown in the appendix) reject the (false) hypothesis of an unchanged V* in more than half the replications. When V* is subjected to the less dramatic change of increasing at a 1½ percent annual rate, all of our statistics have difficulty detecting this new trend growth until at least three years have passed, as shown in table 4. In part, this slow speed of detection is due to the high underlying variance of V_t.

IV. Conclusion

All models used for policy analysis require periodic revalidation of their underlying assumptions. Of particular concern in the P* model is the assumed constancy of the long-run velocity of M2. Unfortunately, the long-run velocity of M2 is no more amenable to direct observation than other "long-run" variables in economic models. Two of our findings suggest that it has not changed, however. First, the deceleration of inflation over the past three years (at least through 1992:IIIQ) closely resembles the predictions of the P* model based on an unchanged long-run M2 velocity. Second, stochastic simulation of the P* model under five alternative hypotheses regarding putative shifts in V* provides little evidence against the constant V* hypothesis, strong evidence against the hypothesis of a one-time shift following the FIRREA legislation, and somewhat weaker evidence against the hypothesis of an upward trend during the past three years.

These results suggest little reason for policymakers to abandon the P* model when seeking to understand the future adjustment of inflation to money growth. Comparison of the P* model's inflation forecasts to the Blue Chip consensus fore-

TABLE 3

Number of Rejections of Hypothesis "V* Has Not Changed" When V* in Fact Increased 6 Percent in 1989:IIIQ

	LM Tests			Chow Tests			Random Walk Tests			Binomial Tests		
	lmshift	*lmtrend*	*lmboth*	*ch4*	*ch8*	*ch12*	*rw4*	*rw8*	*rw12*	*bn4*	*bn8*	*bn12*
1990												
IQ	299	198	245	252	252	252	207	405	68	0	0	0
IIQ	421	341	347	327	327	327	421	521	472	0	0	0
IIIQ	549	459	491	420	419	419	641	431	657	0	305	305
IVQ	673	591	587	411	452	452	659	765	789	0	264	264
1991												
IQ	735	626	646	359	484	484	609	686	848	0	229	229
IIQ	796	720	733	358	580	580	596	806	855	0	615	615
IIIQ	867	786	796	366	625	609	639	920	826	0	740	545
IVQ	908	812	856	386	590	640	595	886	933	0	727	489
1992												
IQ	917	862	888	380	575	677	598	893	897	0	712	754
IIQ	953	894	907	346	551	713	546	892	952	0	675	695
IIIQ	964	907	932	281	481	709	481	875	978	0	627	765
IVQ	975	939	953	244	488	656	444	868	978	0	583	726
1993												
IQ	981	934	965	206	447	619	354	819	967	0	527	694
IIQ	991	960	969	181	376	561	314	759	967	0	460	875
IIIQ	991	957	979	154	303	515	225	678	949	0	404	836
IVQ	994	966	980	139	281	475	194	616	935	0	353	493
1994												
IQ	995	977	989	125	231	424	180	535	918	185	293	433
IIQ	995	977	991	112	193	379	161	439	834	0	240	370
IIIQ	998	979	993	101	155	305	155	365	765	0	207	311
IVQ	998	988	994	106	161	258	134	320	692	0	171	529

SOURCE: Authors' calculations.

cast suggests that market participants already believe that V* has shifted. In so doing, they apparently are discounting evidence that the steep slope of the yield curve has induced portfolio substitution away from M2 (particularly small time deposits) and toward assets such as bond mutual funds.

Our results also suggest a word of caution. The high variance of V_t means that attempts to distinguish changes in V* from short-run movements in V_t are subject to a high degree of uncertainty. Our tests almost surely would have identified by now a large, discrete shift in V* that occurred other than very recently. However, they might not yet have detected an emerging slow growth trend or a more rapid trend that started later than 1990:IQ. To the extent that inflation responds with a long and variable lag to changes in money growth, this uncertainty reinforces the need for caution and vigilance in the conduct of monetary policy. If M2's long-run equilibrium velocity has in fact shifted or is trending up, continuing slow money growth may yield less progress toward price stability than expected. The stickiness and (later) halting decline of long-term interest rates during the recovery likely reflects, in part, views by financial market participants that V* has increased and that price stability is not yet the rule of the land.

TABLE 4

Number of Rejections of Hypothesis "V* Has Not Changed" When V* in Fact Began Growing at a 1½ Percent Rate in 1990:IQ

	LM Tests			Chow Tests			Random Walk Tests			Binomial Tests		
	lmshift	*lmtrend*	*lmboth*	*ch4*	*ch8*	*ch12*	*rw4*	*rw8*	*rw12*	*bn4*	*bn8*	*bn12*
1990												
IQ	50	50	50	50	50	50	50	50	50	0	0	0
IIQ	52	53	50	53	53	53	52	53	54	0	0	0
IIIQ	54	52	58	52	56	56	49	38	57	0	39	39
IVQ	53	64	60	51	57	57	64	66	79	0	26	26
1991												
IQ	52	65	60	50	58	58	53	46	99	0	15	15
IIQ	61	80	75	61	70	70	78	63	78	0	62	62
IIIQ	69	120	88	67	76	73	115	94	62	0	80	47
IVQ	84	128	104	87	80	84	128	87	120	0	96	39
1992												
IQ	94	182	129	85	95	91	176	141	83	0	122	99
IIQ	135	238	159	102	110	108	188	213	149	0	143	79
IIIQ	181	297	203	118	109	101	223	277	203	0	174	90
IVQ	216	383	263	147	141	118	266	360	249	0	224	120
1993												
IQ	244	431	311	148	156	133	280	405	344	0	271	158
IIQ	317	576	386	182	180	168	343	512	503	0	313	450
IIIQ	378	646	474	210	215	201	334	565	579	0	356	530
IVQ	440	745	557	230	274	244	380	641	675	0	384	290
1994												
IQ	520	823	683	223	302	282	437	700	780	374	417	339
IIQ	569	886	771	256	317	337	478	740	814	0	458	406
IIIQ	645	925	820	266	344	358	454	774	874	0	481	460
IVQ	677	966	867	281	396	400	473	787	927	0	506	757

SOURCE: Authors' calculations.

Appendix—The Test Statistics

The 12 statistics calculated during the simulations for each quarter include tests for omitted variables and for properties of forecast errors.[19] The first three statistics are LM tests for omitted variables in equation (1): *lmshift* tests for a post-1989:IIQ shift dummy, *lmtrend* for a time trend beginning in 1990:IQ, and *lmboth* for both simultaneously. An appropriate test for a 1989:IIIQ shift in equilibrium velocity can be formulated as a test for an omitted variable, where the omitted variable itself is a dummy variable that equals zero until 1989:IIQ and one thereafter. To see this, notice that the variable p^* in equation (1) is defined as $p^* = m2 + v^* - q^*$, where lowercase letters indicate natural logs. A shift or trend in v^* translates directly into an equivalent shift or trend in p^*. If a 6 percent increase in equilibrium velocity causes us to understate p^* by 0.06, this can be handled in equation (1) by adding a constant term equal to -0.06 times α, the coefficient on $p - p^*$. The rationale for the *lmtrend* test is identical.

■ 19 To obtain forecast errors for the tests that need them, we estimate the P* model (using the constant V* version of P_t^*) for each quarter of the simulation period using the simulated P_t series running up through the previous quarter. A single-step forecast error for the quarter is computed and saved, the process is repeated for the next quarter, and so on.

TABLE A-1

95th Percentile of Empirical Sampling Distribution of 12 Test Statistics under Null Hypothesis that V* Is Unchanged from Its Long-Run Value

	LM Tests			Chow Tests			Random Walk Tests			Binomial Tests		
	lmshift	*lmtrend*	*lmboth*	*ch4*	*ch8*	*ch12*	*rw4*	*rw8*	*rw12*	*bn4*	*bn8*	*bn12*
1990												
IQ	0.83	0.89	1.31	8.05	12.72	15.02	3.12	2.25	1.28	3	6	6
IIQ	0.87	0.87	1.38	9.42	13.78	14.86	4.01	2.23	1.34	4	6	7
IIIQ	0.86	0.85	1.29	9.82	13.88	15.73	3.94	2.76	1.93	4	5	7
IVQ	0.86	0.81	1.34	9.99	14.38	17.70	3.72	3.10	2.16	4	6	8
1991												
IQ	0.89	0.93	1.45	10.39	14.64	19.31	4.26	3.65	2.68	4	6	9
IIQ	0.94	0.89	1.42	10.00	15.31	19.67	4.21	4.12	2.90	4	6	8
IIIQ	0.92	0.85	1.41	9.97	15.53	19.95	3.78	3.74	3.08	4	6	8
IVQ	0.94	0.88	1.38	9.64	15.98	19.98	3.82	4.33	3.60	4	6	9
1992												
IQ	0.98	0.84	1.39	9.52	15.85	19.70	3.56	4.13	4.03	4	6	8
IIQ	0.90	0.83	1.44	9.48	15.56	20.63	3.79	3.74	3.89	4	6	9
IIIQ	0.89	0.84	1.42	9.64	16.23	21.27	3.70	3.60	3.80	4	6	9
IVQ	0.87	0.76	1.33	9.25	15.52	21.64	3.40	3.31	3.75	4	6	9
1993												
IQ	0.90	0.82	1.37	9.30	15.21	21.51	3.62	3.55	3.72	4	6	9
IIQ	0.82	0.72	1.38	9.11	15.51	21.49	3.27	3.26	3.26	4	6	8
IIIQ	0.81	0.77	1.36	9.37	15.66	21.51	3.79	3.17	3.30	4	6	8
IVQ	0.80	0.72	1.31	9.29	14.81	21.01	3.72	3.10	3.17	4	6	9
1994												
IQ	0.76	0.67	1.25	9.73	14.87	20.82	3.36	2.96	2.87	3	6	9
IIQ	0.78	0.68	1.23	9.10	14.93	20.49	3.27	2.91	3.15	4	6	9
IIIQ	0.76	0.67	1.27	9.01	15.17	20.65	3.44	2.91	2.98	4	6	9
IVQ	0.78	0.63	1.26	9.31	14.82	20.59	3.65	3.05	2.75	4	6	8

SOURCE: Authors' calculations.

Chow forecast tests have long been used to determine parameter constancy and are, in fact, tests of the constancy of variances. The idea is that if the process generating the data changes at time t but the model used by the forecaster does not, the forecast error variance will increase. The utility of the test is limited by its implicit assumption that the variance of the true disturbances is constant. Our three Chow statistics — *ch4*, *ch8*, and *ch12* — are calculated as the sum of the latest four, eight, or twelve squared forecast errors, respectively, divided by the variance of the simulation innovations.

The *rw* statistics are our own invention, motivated by the idea that a persistent misspecification of the P* model, such as would result from a shift or trend in V*, will lead to positive autocorrelation in the forecast errors. The variance of the sum of K consecutive forecast errors will then be much larger than just K times the innovation variance. The *rw4* statistic is the square of the sum of the four most recent forecast errors, divided by four times the innovation variance; *rw8* and *rw12* are analogous. An *rw* statistic can be written as the sum of a Chow statistic plus a term that measures autocorrelation in the forecast errors. Thus, we expect the *rw* test to be more powerful than the corresponding Chow test when the alternative hypothesis involves positive forecast error autocorrelation.

The binomial statistics (*bn4*, *bn8*, and *bn12*) are simple counts of the number of positive forecast errors made over the corresponding intervals. A correctly specified model should, on average, give about the same number of positive and negative forecast errors. The estimated coefficient in equation (2) is negative, so if V* and P* are understated, we would expect to see an inordinately high number of positive forecast errors.

Table A–1 shows the 95th percentile of the 12 statistics' sampling distributions, based on 1,000 replications, under the null hypothesis that V* has not changed from its 1955–89 value. The number 0.87 in the 1992:IVQ row and *lmshift* column, for example, indicates that the *lmshift* statistic for 1992:IVQ was less than or equal to 0.87 in 950 of the 1,000 replications of the constant V* model.

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