COMPONENT RATIO ESTIMATION OF THE MONEY MULTIPLIER

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This article summarizes a staff analysis that may interest those in the economics and banking professions as well as others. It is more technical than the typical Economic Review article. The analysis and conclusions are those of the author. Studies of this kind do not necessarily reflect the views of the Federal Reserve Bank. The complete study is available as part of a series of Federal Reserve Bank of Atlanta Working Papers. Single copies of this and other studies are available upon request to the Research Department, Federal Reserve Bank of Atlanta, Atlanta, Georgia 30303.

The notion that money growth significantly influences economic activity is the heart of much recent economic doctrine. Accordingly, the Federal Reserve System selects monetary aggregate growth ranges consistent with the goals of price stability, low unemployment, and sustained real economic growth. The Federal Reserve's emphasis on money supply growth necessitates research on procedures for controlling the rate of money growth.

The "multiplier-base" framework is one approach to the analysis of money stock behavior.[1] This framework is so called because the money stock is viewed as a multiple of the "monetary base." Specifically, the money stock (M) is related to the monetary base (B) through the following identity:

$$M = mB$$

The monetary base is the sum of member bank deposits at Federal Reserve Banks, member and nonmember bank vault cash, cash in the hands of the nonbank public, and a reserve adjustment.[2] An important assumption in this framework is that the Federal Reserve is capable of controlling the magnitude of the monetary base. The "money multiplier" (m) is the link connecting the base to the stock of money. This study is primarily concerned with the predictability of the money multiplier. The assumption of a controllable monetary base, combined with a predictable money multiplier, suggests that the Federal Reserve should be able to control the money stock.

Predicting the Money Multiplier. To help explain the determination of the money multiplier, the Definitional-Behavioral technique is employed.[3] Starting with the above "multiplier-base" identity, the multiplier is defined in terms of ten component ratios which specify the influences of the behavior of the U. S. Treasury, commercial banks, and the nonbank public on the money stock (see Appendix). The actual values of these component ratios can be computed at any moment in time. However, the essence of the "multiplier-base" framework is that the public, banks, and U. S. Treasury have a desired value for each ratio under their control. Each desired value depends on the values of other economic, institutional, and policy variables. When actual ratio values differ from desired values, the sectors respond by taking actions to eliminate the discrepancy, bringing the ratios back toward their desired levels.

After the money multiplier formulas

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[1] See, for instance, Karl Brunner and Allan Meltzer, "Some Further Investigations of Demand and Supply Functions of Money," **Journal of Finance,** May 1964, pp. 240-283, and Albert E. Burger, **The Money Supply Process,** Belmont, California, Wadsworth Publishing Company, 1971.
It is important to point out that the Federal Reserve does not use the "multiplier-base" approach to controlling money at this time.

[2] The reserve adjustment accounts for reserve requirement ratio changes and shifts in deposits between classes and sizes of banks over time.
This paper was completed prior to the recent change in the method by which the reserve adjustment magnitude is computed. See Albert E. Burger and Robert H. Rasche, "Revision of the Monetary Base," **Review,** Federal Reserve Bank of St. Louis, July 1977.

[3] Albert Burger, Lionel Kalish, III, and Christopher Babb, "Money Stock Control and Its Implications for Monetary Policy," Reprint No. 72 from the **Review,** Federal Reserve Bank of St. Louis, October 1971, p. 8.

have been developed (see Appendix), the Definitional-Behavioral technique requires specifying the structural relationship between each component ratio and its causal determinants. Each behavioral equation is then estimated with regression analysis using monthly, seasonally unadjusted data from January 1969 to December 1975.[4] (This sample period encompasses most of the period since the September 1968 Amendment to Regulation D of the Federal Reserve Act, which instituted lagged reserve accounting.) A post-sample "forecast" of values of each component ratio for the first nine months of 1976 was constructed using the estimated reduced form regression coefficients, the actual post-sample values of the explanatory variables, and the predicted values of the lagged ratio.

The final step in the Definitional-Behavioral method is to substitute the predicted values for each component ratio from its estimated reduced form equation into the money multiplier formulas (equations (1) and (2) in the Appendix) to calculate the predicted values for the multipliers associated with the narrowly and broadly defined money stocks.[5] Summary results of this final calculation for the sample and post-sample periods are presented in Table 1. Comparing predicted with actual multipliers shows that the model's predictive accuracy in the sample period is comparable for the narrow and broad money multipliers (m_1 and m_2, respectively). The in-sample average monthly prediction error equals 0.37 percent for both multipliers, which is nearly equal to their respective average quarterly prediction errors. The model consistently overpredicts each multiplier before mid-1971 and consistently underestimates them thereafter. This result implies that monthly misses do not cumulate nor do they offset one another. Still, the annualized prediction error for any month would be greater than the annualized error for any quarter.[6]

TABLE 1
SUMMARY STATISTICS FOR THE MULTIPLIER PREDICTION ERRORS[1]

Period	$(m_1^d - \hat{m}_1)/m_1^d$ (percent)	$(m_2^d - \hat{m}_2)/m_2^d$ (percent)
SAMPLE (Monthly)		
ME	0.37	0.37
MAE	0.71	0.74
RMSE	0.78	0.79
SAMPLE (Quarterly)		
ME	0.38	0.37
MAE	0.69	0.70
RMSE	0.69	0.72
POST-SAMPLE (Monthly)		
ME	0.58	1.69
MAE	0.58	1.69
RMSE	0.39	0.41

ME = Mean (Average) Error
MAE = Mean (Average) Absolute Error
RMSE = Root Mean Square Error

[1] m_1^d and m_2^d denote the actual values of the narrow and broad money stocks, respectively, divided by the nonborrowed monetary base. $\hat{m}_1$ and $\hat{m}_2$ denote the predicted values of the multipliers calculated from Appendix equations (1) and (2), respectively, using the predicted values for each component ratio.

In the post-sample period, the average monthly prediction errors for the narrow and broad money multipliers are 0.58 and 1.69 percent, respectively (see Table 1). For the narrow multiplier, the error is only slightly higher than the in-sample prediction error. However, the broad multiplier estimation error is nearly five times as great as the comparable in-sample average error. The model performs much less satisfactorily for the broad than the narrow money multiplier in tracking post-sample movements because the broad multiplier is more sensitive to the public's holdings of time and savings deposits relative to demand deposits (t1)--a ratio that was relatively difficult to predict in the post-sample period.

The Federal Reserve would be interested in estimating the money multiplier because it is the connecting link between the money stock and the presumed controllable nonborrowed monetary base. The

[4] The specification and empirical estimation of the behavioral equation for each component ratio are not discussed in this summary article. A thorough discussion of this important step in the Definitional-Behavioral method can be found in Sections II and III of the Working Paper.

[5] The narrow money stock, M_1, equals currency and demand deposits held by the nonbank public. The broad money stock, M_2, equals M_1 plus time and savings deposits other than large negotiable certificates of deposit (CDs) held by the public.

[6] For example, the narrow multiplier's monthly mean absolute error of 0.71 percent equals an annualized error of 8.5 percent while the quarterly mean absolute error of 0.69 percent equals an annualized error of only 2.8 percent.

FIGURE 1

ACTUAL AND PREDICTED VALUES FOR THE NARROW MONEY STOCK

(January 1969-September 1976)

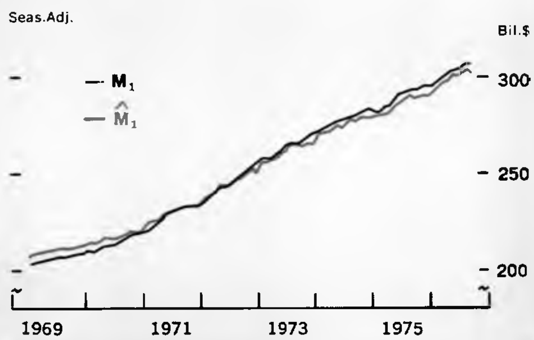
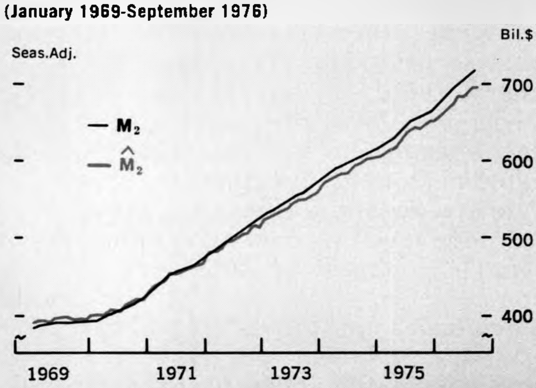


FIGURE 2

ACTUAL AND PREDICTED VALUES FOR THE BROAD MONEY STOCK

(January 1969-September 1976)



ability of the Federal Reserve to control movements in M_1 and M_2 is related to the accuracy of the money multiplier predictions. Multiplying the predicted multipliers by the actual nonborrowed monetary base produces predicted values for the narrow and broad money stocks ($\hat{M}_1$ and $\hat{M}_2$, respectively). These predicted values can be compared to the actual values of each money stock, with the difference between the two measuring the dollar prediction error. These results are graphed in Figures 1 and 2 and summarized in Table 2.[7] The predicted values of the seasonally adjusted narrow and broad money stocks deviate from their actual values by \$1.1 billion and \$2.4 billion, respectively, on both a monthly and quarterly average basis for the 1969-75 period. In the post-sample period, the average monthly M_1 and M_2 prediction errors rise to \$1.8 billion and \$11.7 billion, respectively.[8] The big increase in the average M_2 error reflects the large post-sample (under)prediction error for the broad money multiplier.

[7]Note that the table and figures compare the actual and predicted values for the *seasonally adjusted* money stocks. The predicted seasonally adjusted values were computed in the following manner. The predicted seasonally unadjusted money stock (the predicted seasonally unadjusted multiplier times the actual seasonally unadjusted nonborrowed monetary base) was multiplied by the implicit seasonal factor for that month. This seasonal factor was computed by dividing actual seasonally adjusted money stock by its actual seasonally unadjusted value.

[8]The average *percentage* error between the actual and predicted money stock is, of course, equal to the average percentage error between the actual and predicted money multiplier in both periods.

Multiplier Interest Rate Elasticity. The main issue in this study is the feasibility of money stock control by the Federal Reserve, given its ability to determine the magnitude of the nonborrowed monetary base through open market operations. In conducting an open market purchase, the Federal Reserve induces the commercial banks and the nonbank public to sell U. S. Government securities in exchange for reserves or demand deposits, respectively, by bidding up the price of the securities (or forcing down the yield). In an open market sale, the Federal Reserve prompts just the opposite exchange by forcing down the price of the securities (or forcing up the yield).

In this analysis, the market yield on three-month U. S. Treasury bills (TBR) is used as a proxy for the many different yields on government securities of varying maturities. Estimation of the behavioral equations for the multiplier component ratios revealed that certain ratios were significantly related to the Treasury bill rate.[9] Thus, open market operations, undertaken to control the magnitude of the nonborrowed monetary base, necessarily involve changes in the bill rate. Those changes, in turn, alter the values of the related ratios and, thus, the values of the money multipliers. Are these interest

[9]Specifically, the h, k, t1, t2, e, and b ratios were found to be significantly related to the three-month Treasury bill rate.

TABLE 2
SUMMARY STATISTICS FOR THE MONEY STOCK PREDICTION ERRORS
(billion $)

Period	$M_1 - \hat{M}_1$	$M_2 - \hat{M}_2$
SAMPLE (Monthly)		
ME	1.1	2.4
MAE	1.8	3.9
RMSE	2.0	4.0
SAMPLE (Quarterly)		
ME	1.1	2.4
MAE	1.8	3.7
RMSE	1.8	3.7
POST-SAMPLE (Monthly)		
ME	1.8	11.7
MAE	1.8	11.7
RMSE	1.2	3.0

ME = Mean (Average) Error
MAE = Mean (Average) Absolute Error
RMSE = Root Mean Square Error

TABLE 3
IMPACT AND LONG-RUN INTEREST RATE ELASTICITIES OF THE MONEY MULTIPLIERS[1]

Period	Impact*	Long-Run*
Jan. 1969 - April 1973		
$E(m_1, TBR)$	0.016	0.055
$E(m_2, TBR)$	0.005	-0.018
May 1973 - Dec. 1975		
$E(m_1, TBR)$	0.014	0.034
$E(m_2, TBR)$	0.003	-0.051

[1]The elasticity coefficient of the narrow money multiplier (m_1) with respect to the Treasury Bill Rate (TBR), denoted by $E(m_1, TBR)$, is defined as the *percent* change in m_1 divided by the *percent* change in TBR. The larger the elasticity coefficient the greater the response of the multiplier to a given change in the bill rate.

*Valued at sample means

TABLE 4
IMPACT AND LONG-RUN DISCOUNT RATE ELASTICITIES OF THE MONEY MULTIPLIERS[1]

Period	Impact*	Long-Run*
Jan. 1969 - Dec. 1975		
$E(m_1, DISC)$	-0.016	-0.059
$E(m_2, DISC)$	-0.017	-0.061

[1]The elasticity coefficient of the narrow money multiplier (m_1) with respect to the Federal Reserve Discount Rate (DISC), denoted by $E(m_1, DISC)$, is defined as the *percent* change in m_1 divided by the *percent* change in DISC. The larger the elasticity coefficient the greater the response of the multiplier to a given change in the discount rate.

*Valued at sample means

rate-induced changes in the money multipliers large enough to offset the effects of Federal Reserve policy on the monetary aggregates?

To answer this question, the interest rate elasticity[10] of each multiplier was calculated over the sample period using the money multiplier formulas and the behavioral equation specified for each component ratio. Likewise, the elasticity of each money multiplier with respect to the Federal Reserve Discount Rate (DISC) was calculated. These results are summarized in Tables 3 and 4.[11] The *impact* elasticity measures the immediate or *initial* response of each multiplier to a change in the Treasury bill or discount rate. The *long-run* elasticity measures the full or *complete* response of each multiplier after all subsequent adjustments have taken place. The results confirm the hypothesis that the narrow money multiplier is positively related to the bill rate and inversely related to the discount rate, although the multiplier's response to movements in either rate is small.[12] In both the January 1969-April 1973 and May 1973-December 1975 subperiods,[13] the narrow multiplier's impact interest elasticity is very low and even its long-run response to changes in the bill rate is very inelastic. Likewise, for the full sample period, the narrow multiplier's initial and long-run responses to changes in the discount rate are slight.

In contrast to the narrow multiplier, the broad multiplier's long-run interest rate elasticity is negative, although also very small. A *rise* in the bill rate ultimately

[10]Elasticity measures the degree to which one variable (multiplier) responds to a change in another variable (bill rate).

[11]For a detailed description of the calculation of the money multipliers' interest and discount rate elasticities, see Section V and Appendix II of the Working Paper.

[12]The low interest and discount rate elasticities for the narrow money multiplier found in this study are consistent with the results of previous empirical studies of the money supply process. For a summary of the results of these other studies, see Table 7 of the Working Paper.

[13]The May 1973 suspension of the Regulation Q ceiling rate on 90-day, large negotiable CDs caused a shift in each multiplier's responsiveness to bill rate changes. Therefore, each multiplier's interest rate elasticity was calulated separately for the subperiod prior to ceiling suspension (January 1969-April 1973) and the subperiod after the suspension (May 1973-December 1975).

causes a significant reduction in the public's ratio of time and savings to demand deposits (t1), which leads to a small *decline* in the broad money multiplier. This inverse relationship tends to reinforce the impact of changes in the monetary base on M_2. However, the impact interest elasticity of the broad multiplier in both subperiods is slightly positive. For the total sample period, the broad multiplier's impact and long-run discount rate elasticities are nearly equal to the comparable discount rate elasticities of the narrow multiplier. That is, discount rate movements have similar minor impacts on each multiplier.

These results confirm very low interest and discount rate elasticities for the money multipliers, implying that feedback effects on m_1 and m_2 via changes in the bill rate induced by open market operations will be quite small. Therefore, the use of open market operations by the Federal Reserve to determine the magnitude of the nonborrowed monetary base in an attempt to control M_1 and M_2 will not induce large offsetting changes in the money multipliers. ■

APPENDIX

The narrow money multiplier (m_1) linking the nonborrowed monetary base with the narrow money stock, M_1 (currency and demand deposits held by the nonbank public), is approximated by the following formula:

$$(1)\; m_1 = \frac{1 + k}{\bar{r}^d\,(h+dg) \;+\; \bar{r}^t n(t1+t2) \;+\; (e-b)[h+dg+n\,(t1+t2)\,] \;+\; ck},$$

where h, n, and g are the fractions of total private demand deposits, time and savings deposits, and U.S. Government demand deposits, respectively, held in member banks; k, t1, t2, and d are the ratios of public currency holdings, consumer time and savings deposits, large negotiable CDs, and U.S. Government demand deposits, respectively, to the demand deposit component of the money stock; e and b are the ratios of member bank excess reserves and Federal Reserve Bank borrowings, respectively, to total member bank liabilities subject to reserve requirements; and c is the ratio of currency outside *member* banks to public currency holdings. $\bar{r}^d$ and $\bar{r}^t$ are the average reserve requirement ratios against demand and time and savings deposits, respectively. They are held constant at their August 1954 levels, since the reserve adjustment magnitude included in the nonborrowed base presumably captures changes in reserve requirement ratios.

The general form of equation (1) is

$m_1 = f(\bar{r}^d, \bar{r}^t, h, n, g, d, k, t1, t2, e, b, c)$,

where

$\begin{bmatrix} d \\ g \end{bmatrix}$ = component ratios determined by the behavior of the U. S. Treasury,

$\begin{bmatrix} e \\ b \\ c \end{bmatrix}$ = component ratios determined by the behavior of commercial banks, and

$\begin{bmatrix} h \\ n \\ k \\ t1 \\ t2 \end{bmatrix}$ = component ratios determined by the behavior of the nonbank public.

The broad money multiplier (m_2) linking the nonborrowed monetary base with the broad money stock, M_2 (M_1 plus time and savings deposits less large negotiable CDs held by the nonbank public), is approximated by the following formula:

$$(2)\; m_2 = \frac{1 + k + t1}{\bar{r}^d\,(h+dg) \;+\; \bar{r}^t n(t1+t2) + (e-b)\,[h+dg+n\,(t1+t2)\,] + ck},$$

where all component ratios are defined above.